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High-accuracy long distance measurements with a mode-filtered frequency comb

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Abstract: Homodyne interferometry with a frequency comb as multi-wavelength source is a powerful method to measure long distances with high accuracy. The measurement principle requires that individual comb modes are spectrally resolved, making hundreds or thousands of accurately known wavelengths available for interferometry. For this reason the method cannot be applied directly to frequency combs with a low repetition rate (e.g. 100 MHz), since the modes are too close to be resolved. In this paper we use cavity mode filtering to increasing the pulse repetition rate of a comb and we apply the filtered comb for mode-resolved absolute distance measurement. Mode-filtering takes place with a single Fabry-Pérot cavity in a Vernier configuration, allowing to set mode spacings ranging from 10s of GHz to more than 100 GHz. Large mode-spacings significantly reduce the requirements on the resolution of the spectrometer. We demonstrate absolute long distance measurement with a mode-filtered frequency comb using a simple array spectrometer for mode-resolved detection. Here a 1 GHz comb is used, that is converted into a 56 GHz comb by mode-filtering. A trade-off between non-ambiguity range and spectral resolution needs to be made when choosing a filter ratio. The pulse-to-pulse distance after filtering is 5.3 mm in this case, so to overcome ambiguity a rough measurement with an accuracy of about 2.5 mm is required. We show that in comparison to a conventional counting interferometer an agreement within 0.5 μm for distances up to 50 m is found. The presented method may enable the field application of low-repetition rate frequency comb lasers, like fiber lasers, for multi-wavelength homodyne interferometry. It relaxes the requirements on the spectral resolution, allowing for simple grating spectrometers as detector.

OCIS codes: (120.2230) Fabry-Perot; (120.3180) Interferometry; (120.3940) Metrology; (140.4050) Mode-locked lasers; (140.4780) Optical resonators; (280.3400) Laser range finder; (320.7090) Ultrafast lasers.

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1. Introduction

In the past decade a wide variety of distance measurement methods based on optical frequency comb technology has been developed [1]. Frequency comb based methods have in common that they provide direct traceability to a stable frequency reference such as a maser or an atomic clock [2]. Since the SI meter is related to the SI second via the speed of light [3], comb-based distance measurement directly profit from highly stable frequency standards. A frequency comb can serve as a frequency reference to a continuous wave (CW) laser, which is then used as a light source for a displacement measurement. Other indirect methods for absolute distance measurement use the optical beat of two CW lasers, whose phase shift of the synthetic wavelength determines the distance [4,5]. In addition, many schemes have been developed using the frequency comb itself as a light source for length metrology. One pioneering method was based on the heterodyne beat between longitudinal modes of a mode-locked laser [6]. An approach based on a time of flight of femtosecond pulses was later proposed [7], followed by several experimental

demonstrations of cross-correlation based measurements [8–12]. Homodyne and heterodyne interferometric schemes utilizing one frequency comb [13, 14] and two frequency combs [15, 16], respectively, have also been demonstrated.

Distance measurement based on homodyne mode-resolved many-wavelength interferometry allows for accurate measurements with a single frequency comb and a wide range of non-ambiguity [14, 17]. This method uses an optical frequency comb as a many-wavelength laser source. The laser illuminates the Michelson interferometer, whose output is then spectrally resolved to individual comb modes. The comb mode spacing is given by a laser cavity length. Mode spacings are typically a few hundred MHz for fiber combs and roughly 80 MHz to 1 GHz for Ti:Sapphire combs. To resolve the individual comb modes a high resolution spectrometer based on a virtually imaged phased array (VIPA) etalon can be used. However, its resolution is still insufficient to resolve mode-spacings of a 100 to a few 100 MHz. The method can thus not be easily exploited by the commonly used fiber combs, which are better suited for potential field applications because of their ease of operation, robustness and smaller sensitivity to environmental disturbances. To enable the application of the mode-resolved method for low repetition rate combs, cavity mode filtering can be applied to increase the repetition rate of the source. This overcomes the limitation of the low repetition rate comb sources and also enables the use of a simple grating-based spectrometer for detection, instead of the rather complex VIPA spectrometer. The first implementation of external Fabry-Pérot cavity to a frequency comb was used for cavity-enhanced spectroscopy [18]. However, cavity filtering frequency comb modes found its application soon after in astro-combs for calibration of high-resolution astronomical spectrographs [19–22]. Experimental work published on this topic nicely describes the cavity filtering model with an effect of cavity-comb offset [23], selection of optimal cavity mirrors radius of curvature [24] or cavity dispersion [25].

In this paper we demonstrate distance measurement based on a filtered frequency comb with a grating-based spectrometer for detection. Mode-filtering takes place with a single Fabry-Pérot cavity operating at a non-integer filter ratio. In this 'Vernier configuration' comb mode spacing from 10s to 100s of GHz can be achieved with only minor adjustment of the cavity length. Based on the filtered source, homodyne mode-resolved frequency comb interferometry is applied for distance measurement up to 50 m and compared to a counting helium-neon (HeNe) laser interferometer.

2. Measurement principles

2.1. Frequency comb mode selection

An optical frequency comb is the spectrum of a laser whose optical modes are phase locked in such way, that they interfere together to form a train of pulses with the repetition rate frequency f_{rep} and the carrier-envelope offset frequency f_0. The repetition rate directly represents the spacing between neighboring modes in frequency domain. In our experiment we use a Ti:Sapphire frequency comb with 1 GHz repetition rate as the laser source. A single symmetrical Fabry-Pérot cavity made of two mirrors is used to increase the comb mode spacing. Such cavity transmits series of frequencies separated by free spectral range $FSR = c/2L_c$, where c is the speed of light and L_c the optical cavity length. The suppression of unwanted modes is given by the spectral transmission function:

$$T(f, R, L_c) = \frac{(1 - R)^2}{(1 - R)^2 + 4R\sin^2(2\pi f L_c/c)} \tag{1}$$

where R is the mirror reflectivity and f the optical frequency. The desired mode spacing, resolvable by a diffraction grating spectrometer, based on a 50 mm ruled grating was $\approx$ 30 GHz. Such values can be achieved by setting the cavity FSR to an integer multiply m of f_{rep}, $FSR = m \cdot f_{rep}$

as in case of the astro-comb. Here we apply an alternative method, with the cavity FSR set to a non-integer multiply of f_{rep}. This is a scheme that is similar to frequency comb Vernier spectroscopy, see e.g. [26]. As a result we obtain a spectrum consisting of modes spaced by the lowest common multiple of the FSR and f_{rep}

$$iFSR = mf_{rep}, \tag{2}$$

where i and m are integers. An example of the cavity transmission as a function of frequency is shown in Fig. 1(a). The cavity transmission occurs at cavity length

$$L_c = \frac{i}{m} \frac{c}{2f_{rep}}. \tag{3}$$

Since the cavity finesse is finite, non-exact solutions of Eq. (2) can be found. These solutions will result into a set of lateral transmission peaks. The comb spectrum transmitted through a lateral peak will have the same filter ratio m as the central peak, but will have decreased spectral bandwidth, inversely proportional to the finesse and the cavity length detuning. The cavity length can be unintentionally tuned into one of the lateral transmissions because their corresponding cavity lengths differ from L_c only in the nanometer range. This cavity length difference ΔL_c is given by:

$$\Delta L_c \approx \frac{c}{2f} \frac{f_{rep}}{iFSR} = \frac{\lambda}{2m}. \tag{4}$$

Note that the peak to peak distance ΔL_c is given only by wavelength λ and multiplication factor m. Thus measurement of ΔL_c can be used for checking the factor m respectively the cavity length L_c.

Since Eq. (3) holds only if the offset frequency f_0 equals zero, any offset frequency has to be compensated by slightly detuning the cavity by changing the cavity length with:

$$\Delta L_{f_0} \approx \frac{\lambda}{2} \frac{f_0}{FSR}. \tag{5}$$

An example of the cavity transmission as a function of the cavity length and the effect of the offset frequency is shown in Fig. 1(b).

The advantage of the Vernier method is that the filter ratio is not inversely proportional to the cavity length and high filter ratios m can be obtained, via high values of the factor i, with relatively large mirror to mirror distance. This allows to build such a cavity with common optomechanical components and also to adjust the filter ratio ranging from 10 to infinity with small change of cavity length. In practice, the largest filter ratio sufficiently filtering unwanted modes is proportional to the cavity finesse. In our case we used for the measurements filter ratio of $m = 56$ and factor $i = 3$ corresponding to $FSR = 18\frac{2}{3}f_{rep}$. A simulation of the comb mode suppression for such parameters is in Fig. 2. The slight detuning of the cavity to account for nonzero f_0 leads to a small mismatch between the cavity resonance and the comb wavelengths. However, for an optical bandwidth of 14 nm, which is spectral width of the Ti:Sapphire laser in this paper, this only leads to a 0.1 dB decrease of the transmitted signal in the wings of the spectrum.

To keep the cavity in resonance with the comb modes, the cavity length has to be stabilized onto the position of required transmission peak. The stabilization is performed by deriving a control signal from a photodiode that monitors the intensity of the laser light after the light has passed the cavity. The control signal is derived from the photodiode response after modulating the transmission of the cavity by changing the position of one of the cavity mirrors with a harmonic signal. By demodulating the response of the photodiode output a control signal is generated to drive the mirror towards the position of maximum intensity.
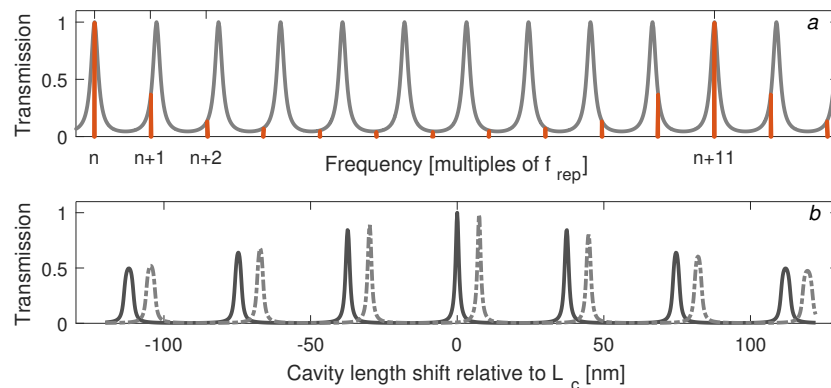
Fig. 1. a) Example of transmission of low finesse $F = 7$ Fabry-Pérot resonator (gray) in frequency domain, frequency components of optical comb (red). The FSR of the resonator is 1.1 times the comb mode spacing and thus every 11th comb mode is resonant with the resonator. b) Total transmission versus cavity length change of Fabry-Pérot resonator for two values of $f_0 = 0$ GHz (solid) and $f_0 = 0.18$ GHz (dotted). Other parameters are: central frequency $f = 365$ THz, $FSR = 1.1$ GHz, $f_{rep} = 1$ GHz and finesse $F = 313$.
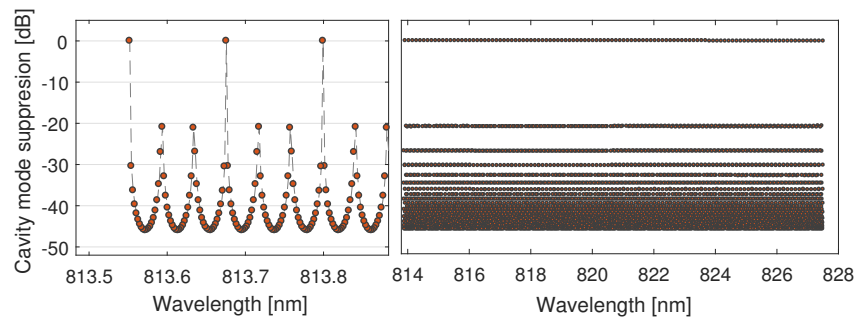

Fig. 2. Simulation of frequency comb mode suppression by filter cavity with $FSR = 18\frac{2}{3}$ GHz. The comb repetition rate is $f_{rep} = 1$ GHz, the offset frequency $f_0 = 180$ MHz and the bandwidth of 14 nm is centered at 820 nm. With a cavity finesse $F = 313$ (mirror reflection 99%), the suppression of unwanted modes remains below 20 dB. The transmission of wanted modes is suppressed at the edges of the spectrum due to the offset frequency only by 0.1 dB. On the left side a small zoomed part of the spectrum is shown, to visualize individual modes. On the right side the mode suppression is shown for the full spectrum.

2.2. *Distance measurement based on spectral interferometry*

The distance measurement is based on a Michelson interferometer with reference arm and measurement arm. The laser beam of the frequency comb is split into both arms and the recombined output interference signal is spectrally analyzed by a spectrometer based on diffraction grating and linear CCD (charge-coupled device) array. The observed intensities of each individual comb mode are then used for distance calculation. A comb mode (wavelength) experiences a

total phase change Φ according to its optical frequency f and traveled optical distance $2L \cdot n$,

$$\Phi = \frac{4\pi L n f}{c}, \tag{6}$$

where n is the phase refractive index of air. The interference signal carries the information about the phase difference between the arms for each comb mode, therefore the signal taken for a certain distance is modulated as a function of the optical frequency

$$I(f) = I_0 \cos\left(\frac{4\pi L n f}{c}\right). \tag{7}$$

The information about the phase change with frequency $d\Phi/df = 4\pi L n_g/c$ is obtained from the measured spectrum by a cosine fit with $\Phi = C \cdot p + C_2$ and n_g the group refractive index. The group refractive index appears here as a direct result of taking the derivative from total phase to frequency [17]. In this equation C and C_2 are fitting parameters and p is a comb mode label associated to comb mode frequency $f_p = m f_{rep}(Q + p) + f_0$, where Q is a large integer number. The distance L can be then expressed as

$$L = \frac{C}{m f_{rep}} \frac{c}{4\pi n_g}. \tag{8}$$

Since the frequency comb is a periodic pulse train repeating itself with pulse to pulse distance, the interference patterns will also repeat themselves with half the distance

$$\frac{L_{pp}}{2} = \frac{c}{m f_{rep} n_g}. \tag{9}$$

The distance L is defined in a range of non-ambiguity $-\frac{1}{2}L_{pp} < L < \frac{1}{2}L_{pp}$. Since the sign of L cannot by determined from the cosine fit, it is necessary to obtain it otherwise. One method is based on observing interference fringe density with change of measurement arm length. When the length is increasing, increasing fringe density is equivalent to positive L, decreasing fringe density corresponds to negative L. When pulses from both interferometer arms are overlapping, all comb modes interfere constructively - there is no fringe and L equals zero. This holds for distances:

$$L_t = \frac{1}{2} k L_{pp}, \tag{10}$$

where k is an integer and L_t is total single path length difference of both arms. Integer k has to be obtained by approximate distance measurement. This can be done by any method with accuracy better then $L_{pp}/2$. Then a random distance is given by:

$$L_t = \frac{1}{2} k L_{pp} + L, \tag{11}$$

Note that the description above is based on a source with a repetition rate of $m f_{rep}$.

3. Experimental setup

The measurement setup consists of 3 main parts: a frequency comb source combined with a filter cavity, a Michelson interferometer and a grating spectrometer. An overview of the setup is shown in Fig. 3. The frequency comb is generated with a Ti:Sapphire laser emitting an optical spectrum in the 813.5 nm to 827.5 nm wavelength range. The comb mode spacing or the laser repetition rate is 1.012 GHz. From Eq. (1) we estimate the mirror criteria for sufficient spectral filtering of unwanted comb modes. We have chosen the suppression ratio of the closest neighboring

comb mode to be at least 20 dB. To obtain this suppression consistently in the whole region a walk-off between the comb and the cavity modes caused by mirrors dispersion has to be minimized. This is achieved with mirrors having a reflectivity of $R = 99.0\%$ and group delay dispersion (GDD) below 20 fs^2 in the relevant spectral region. A symmetric cavity was built with two concave mirrors both having a radius of curvature of 50 mm. The finesse of the cavity is $F = \pi\sqrt{R}/(1 - R) \approx 300$. The length of the filter cavity is set at 8.0 mm, corresponding to a $FSR = 18\frac{2}{3} f_{rep}$, which due to Vernier selection, results in a filter ratio $m = 56$. This filter ratio was chosen for this experiment, but filter ratios over 300 have also been achieved [27]. The cavity resonance linewidth is approximately 100 MHz (FWHM). The filter cavity is shielded by a foam cover to keep it thermally stable and to suppress coupling of acoustic noise into the cavity. In order to obtain a clean Gaussian beam profile, a single mode optical fiber is used to couple the comb light to the cavity. As the fiber output coupler we use an achromatic collimator. For successful mode-matching of the laser beam and the TEM$_{00}$ mode of the cavity, two achromatic lenses were used. After the cavity the laser beam is collimated with another achromatic lens and 10% of light is directed with a beam-sampler into a photodetector for cavity stabilization. For this purpose, one mirror has been mounted on a piezo-actuator which is driven by electronic feedback loop using a dither locking technique [28]. The rest of light is coupled into a fiber and propagates through an optical fiber circulator into the measurement section in a neighboring laboratory. Typically about 400 μW of comb power is available after the filter cavity, depending on the set filter ratio, providing about 100 μW to the setup for distance measurement after propagation through the fiber circulator.
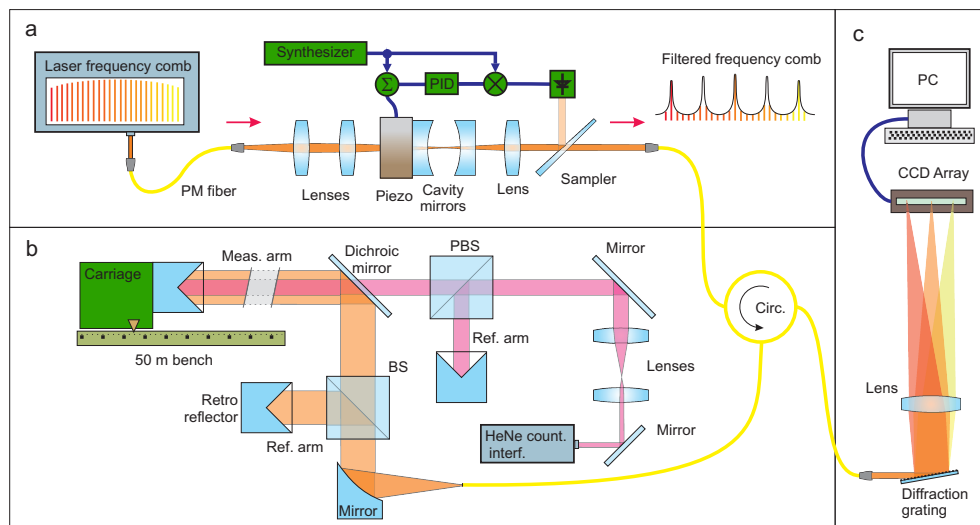

Fig. 3. Experimental setup split into 3 sections. *a*) Filtering part showing the frequency comb laser, cavity mode-matching optics, cavity with piezo actuator, collimation lens and beam sampler. The green parts are the detector and electronics for locking the cavity length. The filtered frequency comb is led through an optical circulator towards the 50 m bench part. *b*) This part shows two Michelson interferometers with a common measurement arm for the filtered frequency comb and the HeNe laser. *c*) Spectral analysis of the interferometer output with a diffraction grating and linear CCD camera.

The measurement section consists of a Michelson interferometer and a commercial Helium-Neon counting interferometer. The Michelson interferometer has been built with 2 inch sized optics, using a non-polarizing beamsplitter and gold coated hollow retroreflectors. The measurement arm has a maximum length of 50 m and consists of a long rail with electric carriage carrying the

retroreflector. The comb beam is collimated into the interferometer with an off-axis parabolic mirror (101.6 mm focal length). A dichroic mirror reflects the comb beam, while it transmits the continuous-wave beam of the HeNe laser. As a result both beams largely propagate through the same volume of air, having a shared measurement arm. The reference arms for the comb and the HeNe laser are different, since the HeNe laser interferometer is based on heterodyne measurements, requiring a polarizing beam splitter.

After propagation through the Michelson interferometer, the light from the HeNe laser propagates back through the same path to the HeNe laser head which contains the detector for fringe counting. The comb interferometric output propagates back into the circulator via the same optical path and optical fiber. The fiber circulator directs the light towards the spectrometer, consisting of a 50 mm wide ruled diffraction grating (1800 lines/mm), focusing lens and 3000 pixel CCD line camera sensor. The fiber connector acts as the input slit having a 5 μm mode field diameter. When moving the carriage, the number of interference fringes measured with the line camera changes as described by Eq. (7).

We measured the filter ratio m by measuring the change of piezo voltage, corresponding to a cavity length change, when the cavity was locked on successive transmission peaks, as calculated from Eq. (4). In practice it is more accurate to measure the piezo voltages for locking to n-th peak and the $n+10$th peak, which provides enough accuracy to estimate the filter ratio. For the measurement we used a filter ratio $m = 56$ with cavity $FSR = 18\frac{2}{3}f_{rep}$. This corresponds to a pulse-to-pulse distance $L_{pp} = 5.3$ mm. We verified the filter ratio m by measuring the distance with the HeNe interferometer between two successive positions of the carriage with zero fringes on the camera. Since the distance between the pulses is reduced by mode filtering, the requirements on the a priori knowledge of the distance get tighter, but still a measurement accuracy of a few mm is sufficient to overcome the ambiguity.
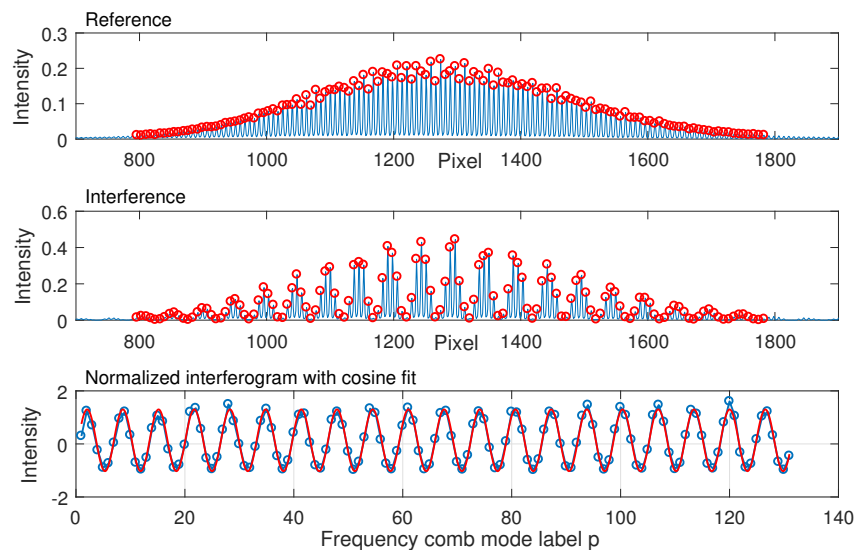
4. Results and discussion

Fig. 4. Top: typical reference measurement-blue, used for finding $I(f)$ positions at CCD and I_0 intensities - red circles. Middle: interference measurement - blue, with values of $I(f) \cdot I_0$ intensities - red circles. Bottom: normalized intensities $I(f)$ - blue circles and cosine fit - red.

The distance measurements were performed quasi-simultaneously with both laser methods for

comparison. Initially, the carriage of the measurement arm was positioned at the beginning of the bench and the counting HeNe interferometer was set to zero and a line camera picture was taken. Then the carriage was moved to a position roughly at 10 m, and again back to a position at zero m. This was done also for 20 m, 30 m, 40 m and 50 m. The measurement positions have been selected arbitrarily, but positions very close to $L = 0$ and $L = L_{pp}/4$ were avoided, as will be discussed below. Due to environmental perturbations, the contrast of the comb interference pattern tends to diminish for too long integration time of the CCD array. Optical path length variations of only $\lambda/4 \approx 200$ nm are sufficient to wash out the fringes. For this reason it is required that the carriage is fixed during the measurement. The method can thus not be used for measuring while the target is being moved. In practice, a 0.4 ms acquisition time of the camera was found as an optimal value.

To be able to normalize the data, not only the interference signal is measured for each position, but also the signals from the reference and measurement arm individually (as obtained with the other arm blocked). For each position, five comb measurements and five readings of the counting interferometer have been recorded. The refractive index of air is determined from data of environmental conditions (temperature, air pressure and humidity) using the Edlén formula [29]. The temperature has been measured at several positions along the measurement bench, simultaneously with the distance measurements.

The sum of the reference data from the measurement and reference arms were used for intensity normalization of interference spectrum as shown in Fig. 4. From its cosine fit the distance L was calculated using Eq. (8) and the method mentioned in Section 2.2 was used for determining its sign. The distance measurements by the HeNe counting interferometer have been used for determining the integer k in Eq. (11) for the comb measurement. However, since the requirements on these measurements are mild, a simple electronic distance meter can also be used to overcome the non-ambiguity.
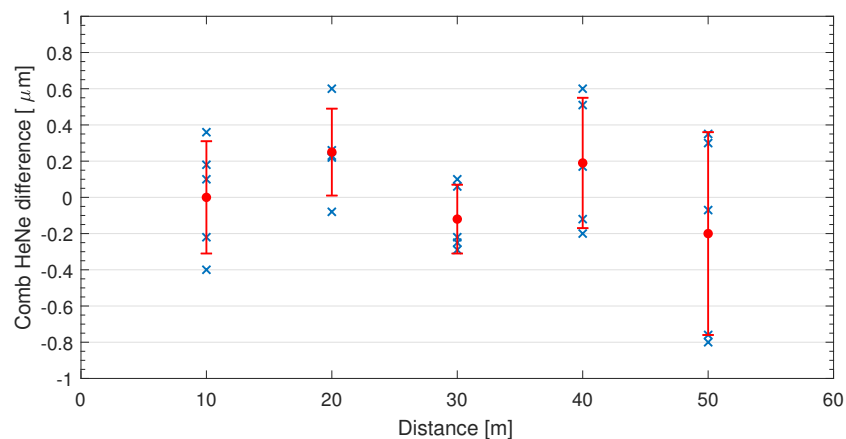

Fig. 5. Measured differences between distance measurement with a filtered frequency comb and a HeNe laser interferometer for distances up to 50 m. The error bars show the standard deviation of the measurements.

The results in Fig. 5 show the differences between the individual measurements done by frequency comb and the average distance as measured with the HeNe counting interferometer. The zero position is based on the average of the five zero measurements. For each individual measurement the agreement between the frequency comb and the HeNe laser is within 0.8 μm. When averaged over five measurements, the largest difference is 250 nm. The standard deviation

doesn't show a clear distance dependence and is on average 0.33 μm. The comparison measurement shows that the mode-filtered frequency comb is a suitable tool for distance measurement, with a relative difference $< 10^{-8}$ with the HeNe laser reading for a distance of 50 m. The observed differences between both methods for a single measurement are caused by environmental effects, like turbulence and vibrations. These effects are not perfectly canceled because of small timing differences between the measurements. When averaging all measurements and all distances, the agreement between both methods even reduces to below 100 nm. The results show that multi-wavelength interferometry with high accuracy can also be achieved with a mode-filtered astro-comb. The possibility to increase the mode spacing enables the application of low-repetition rate combs, like fiber combs for wavelength-resolved distance measurement. Furthermore, spectrometers with a lower resolution can be used, as demonstrated here by using an array spectrometer instead of a VIPA spectrometer.

The measurement uncertainty on the comparison measurement contains several contributions, originating from the HeNe uncertainty, the uncertainty on the comb measurement and the uncertainty due to environmental conditions. The relative uncertainty of the HeNe laser, resulting from wavelength uncertainty and the fringe-counting system is estimated to be 3×10^{-9}. The uncertainty of the comb measurement contains a contribution of the frequency stability (10^{-10} for a measurement time in the ms range), plus an uncertainty of 50 nm, which comes from the uncertainty of the fitting parameter C. This contributions occurs twice, for both the start and the stop position of the carriage. The uncertainty on the refractive index *difference* for the comb and HeNe wavelengths is estimated to be 1.7×10^{-9}. This value is based on the uncertainty in the environmental measurements, being 0.3 °C for air temperature, 7 Pa for pressure and 1.3 % for relative humidity. Finally, there is a contribution to the measurement uncertainty which results from vibrations coupling to the carriage on the measurement bench. Since the comb and HeNe measurements have different integration times and are not perfectly synchronized, timing differences between the measurements of typically 0.1 s occur. As a result path length changes due to vibrations that occur on this time scale do not cancel. This contribution was quantified by performing a time-resolved measurement with the HeNe laser. The HeNe measurement shows a standard deviation of 0.22 μm for a sampling time of 0.1 s, which can be considered as the uncertainty due to vibrations on the comparison measurement. These uncertainties occur at both the start point and the end point of the measurement, leading to a total uncertainty estimate due to vibrations of 0.31 μm. When combining the relative and absolute uncertainty contributions mentioned above as a quadratic sum, a total uncertainty of 0.36 μm for a distance of 50 m is found. For a coverage factor $k = 2$, corresponding to a 95 % coverage interval, the uncertainty equals 0.72 μm.

The uncertainty on the measurement of an absolute distance with either comb or HeNe laser will be dominated by the uncertainty in the refractive index of air. For the uncertainties of the environmental parameters (temperature, pressure, humidity) mentioned above, the relative uncertainty on the refractive index becomes 3.1×10^{-7}. The agreement between both methods is thus much better than the achievable uncertainty on the absolute distance. Since the wavelengths of the Ti:Sapphire laser and the HeNe laser are relatively close and the measurements arms are common path, the uncertainty due to environmental conditions largely drops out in the comparison.

As mentioned above, arbitrary distances were chosen, but path length differences very close to $L = 0$ m or $L = L_{pp}/4$ were avoided. At $L = 0$ m all wavelengths have the same phase (neglecting nonlinear air dispersion), so a typical cosine dependence like in Fig. 4 is not observed. Close to $L = L_{pp}/4$ the Nyquist frequency is approached and each period of the cosine is only determined by 2 datapoints (see discussion in [14]). There are several ways to overcome this issue. A simple solution is to slightly changing f_{rep}, which will change L_{pp} and thus L. Alternatively, a multiplex scheme could be envisaged with two reference retro-reflectors, positioned with a

mutual path length difference of about half of the non-ambiguity range. By using a beamsplitter and two shutters the reference path can be always selected such that the path length differences close to $L = 0$ m or $L = L_{pp}/4$ do not occur.

The measurement results presented in this paper are only based on spectral interferometry, i.e. the distance is determined from the phase change as a function of optical frequency. The actual knowledge of the optical frequencies has not been used. However, the knowledge on the optical frequencies, which are accurately known with an uncertainty limited by the quality of the reference clock, is also available. Using the optical phase fraction of an individual wavelength, which is derived from the curve fit, the accuracy on the distance measurement can be further improved. This could be of interest when the measurement uncertainty is not dominated by environmental effects, e.g. in vacuum or in space.

5. Conclusion

In this paper we have demonstrated that long distances can be measured with high accuracy using a mode-filtered frequency comb and a simple linear array spectrometer. In comparison to a HeNe laser a relative agreement below 10^{-8} has been shown. The method of cavity mode filtering opens up the possibility to use a low repetition rate frequency comb, like a fiber comb, for many-wavelength homodyne interferometry. Since fiber combs are smaller, easier to operate and less sensitive to the environment than Ti:Sapphire-based combs, this could be of interest for potential field applications of this method using fiber combs. When using a filter cavity to generate repetition rates of several 10s of GHz, it is possible to replace the complicated VIPA spectrometer with a linear array spectrometer. This not only simplifies the experimental setup, it also simplifies data processing and analysis. In general, the availability of frequency combs having a mode spacing that is sufficiently large to be resolved by an array spectrometer, will be a step forward towards practical applications of many-wavelength homodyne interferometry for absolute distance measurement. This will be further enhanced by the ongoing development of on-chip semiconductor-based frequency combs, having the potential of significant cost reduction.