Knowledge Retention and Mathematical Foundations in Machine Learning Education

Exploring the Role of Prior Mathematical Knowledge in Retaining Core Machine Learning Concepts

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Abstract

As Machine Learning (ML) continues to shape advancements in academia and industry, ensuring effective ML education is essential. This study examines the retention of four core ML concepts - Principal Component Analysis, Gradient Descent, Bayes' Theorem, and Hierarchical Clustering - two years after students completed a university-level ML course. Using a survey-based methodology, it explores how prior mathematical knowledge, perceived difficulty, and confidence influence long-term retention. Results reveal a significant positive correlation between Calculus knowledge and Gradient Descent retention, with weaker correlations for Linear Algebra with PCA and Probability with Bayes' Theorem. Perceived difficulty and confidence also shape retention outcomes. The findings emphasize the need for targeted mathematical refreshers in ML courses to strengthen foundational knowledge and improve retention. This research provides actionable insights for curriculum design, aiming to bridge mathematical gaps, enhance learning outcomes, and sustain student engagement with advanced ML concepts.

1 Introduction

Machine Learning (ML) has become a cornerstone of technological innovation, with applications ranging from data science and artificial intelligence to healthcare, engineering, and social sciences. Defined as "an artificial intelligence technology that enables computer systems to learn from experience and make predictions" [1], ML demonstrates its adaptable potential in addressing complex challenges across diverse industries. Its growing significance and expanding applications [2] underscore the importance of integrating ML education into academic curricula to prepare students for the evolving demands of a technologically driven workforce.

At the core of ML education is mathematics [3], serving as the formal language through which ML concepts are understood and applied [4]. Foundational areas like linear algebra, calculus, and probability underpin ML algorithms and methods [5], enabling computational thinking and problem-solving. As Ralston and Shaw [6] note, "for any science or any engineering discipline, the fundamental principles and theories can only be understood through the medium of mathematics". Despite its importance, mathematics remains widely perceived as difficult [7], contributing to lower retention rates in fields requiring substantial mathematical expertise [8]. This "Mathematics Problem", identified by Rylands and Coady [9], highlights gaps in students' preparedness for quantitative subjects [10], including ML.

1.1 Literature Context and Research Gaps

The challenge, therefore, extends beyond teaching ML concepts to ensuring their long-term retention. Knowledge retention, defined as the ability to preserve learned material over time [11], is vital in STEM (Science, Technology, Engineering, and Mathematics) education, where cumulative learning often determines students' success. However, retention is influenced by various factors, including prior knowledge, confidence, and perceived task difficulty [12]. Prior studies have shown that new information is best retained when linked to existing knowledge [13], enhancing comprehension and recall [14]. This is especially relevant in ML education, where mathematical foundations not only provide frameworks for understanding complex concepts, but also significantly increase students' likelihood of success in computer science [15].

Gaps remain in understanding how prior mathematical knowledge influences the long-term retention of ML concepts. While knowledge retention research emphasizes the natural decline of memory over time [11], most studies focus on short-term periods, typically within a year of course completion [16]. Although actively practicing retrieval enhances long-term recall [17], knowledge decay occurs without reinforcement [18]. This highlights the need for extended assessments to understand retention patterns, particularly in the mathematical foundations underpinning ML, whose long-term impacts remain under-explored. This study addresses this gap by investigating how specific mathematical knowledge influences ML concept retention over a two-year period − an under-examined time frame in current literature.

Additionally, the interplay between perceived difficulty, confidence, and long-term learning outcomes has received limited attention. Wall and Knapp [12] suggest that perceived difficulty significantly influences engagement and performance in technical computing courses, yet its specific impact on knowledge retention in ML education remains unclear. Mangos and Steele-Johnson [19] note that task difficulty perception can be as influential as its actual complexity, particularly with unfamiliar material. This is especially relevant in ML, where abstract mathematical concepts often create barriers to understanding [10], as highlighted by Ralston and Shaw [6], who critique curricula that underemphasize mathematics and its role in overcoming challenges in computational fields.

1.2 Research Question and Hypotheses

Building on these insights, this study investigates the intersection of mathematical foundations, perceived difficulty, and long-term retention of ML concepts. The central research question guiding this study is:

"To what extent do students who have completed the Machine Learning course (CSE2510) retain core concepts within two years, and how does prior knowledge of specific mathematical topics influence this retention?".

To explore this question, the following hypotheses are proposed:

H_0: Prior knowledge in specific mathematical topics has no significant correlation with the retention of corresponding ML concepts.

H_1: Stronger prior knowledge in specific mathematical topics is positively correlated with better retention of corresponding ML concepts.

H_2: ML topics that rely on advanced mathematical concepts show lower retention scores compared to topics with minimal math prerequisites.

These hypotheses form the analytical framework for examining the role of mathematics in ML education. This research contributes empirical evidence on how prior mathematical knowledge influences long-term retention of ML concepts and explores how perceived difficulty and confidence interact with mathematical foundations to shape learning outcomes. By addressing gaps in existing literature, this study provides actionable insights for curriculum design, emphasizing tailored instructional strategies to effectively transition students' short-term understanding into long-term retention [11].

This paper is structured as follows: Section 2 offers a detailed background, presenting relevant literature and theoretical frameworks that form the basis for this research. The methodology employed is described in Section 3, followed by the experimental setup in Section 4. The results are presented in Section 5, with their implications and significance in Section 6, reflecting on connections to existing research and limitations. Section 7 discusses responsible research practices, and finally, Section 8 concludes with a summary of key contributions and recommendations for future research.

2 Background

This section provides the background for understanding key themes and frameworks in ML education, including the role of mathematical foundations, knowledge retention, and perceived difficulty in shaping student engagement and success. It begins with an overview of the main research themes, emphasizing the multidisciplinary nature of ML and the importance of reinforcing mathematical knowledge for long-term retention and student readiness. It further explores theoretical perspectives on prior knowledge, conceptual understanding, perceived difficulty, and confidence, highlighting their impact on learning outcomes and influence on this study.

2.1 Overview of Main Research Themes

ML education lies at the intersection of computer science, mathematics, and statistics, requiring students to develop a multidisciplinary skill set [6]. Deisenroth *et al.* [4, p. 2] note that ML formalizes intuitive concepts through mathematical frameworks, making mathematical preparation essential for understanding its principles. They further emphasize that ML provides a clear reason for students to engage with mathematics, addressing common concerns about its perceived lack of practical relevance [20]. The challenge thus lies in reinforcing foundational knowledge, as long-term retention of mathematical concepts is crucial for ML mastery. Valderama and Oligo [11] highlight that retention diminishes over time, particularly for concepts requiring higher-order thinking, underscoring the need for refreshers to bridge knowledge gaps and equip students to tackle advanced ML topics effectively.

Furthermore, mathematics is not only foundational to ML, but also integral to broader STEM disciplines [21, 7]. Alnaimi *et al.* [22] describe mathematics as "one of the fundamental pillars of any scientific progress", essential for logical thinking, problem-solving, and reasoning. Similarly, Ralston and Shaw [6] highlight the importance of mathematical readiness for success in computational fields. Moreover, mathematical cognition, encompassing the mental ability to grasp and apply mathematical concepts, is critical for STEM success [3]. As such, addressing gaps in students' mathematical foundations is vital to enhancing their engagement with ML concepts [5].

2.2 Theoretical and Conceptual Frameworks

This research draws on interconnected educational frameworks [23] to explore the relationship between mathematical foundations, perceived difficulty, and long-term retention of ML notions. Conceptual knowledge, defined by Crooks and Alibali [24] as an understanding of principles that structure a domain, provides a foundation for problem-solving and application. In ML, it involves grasping the theoretical underpinnings of algorithms and is crucial for deep understanding, competence, and solving complex problems [25]. Wankhede and Kiwelekar [26] further emphasize that knowledge involves recalling and applying learned material, from particular facts to comprehensive theories, underscoring its role in educational success.

Prior knowledge has a pivotal role in shaping how students acquire and retain new concepts. Bransford and Johnson [27] and Shapiro [14] show that relevant prior knowledge enhances comprehension and recall by providing a framework for integrating new information. This is particularly relevant in STEM, where mathematical readiness underpins success in mastering complex topics like ML [6]. Hosein and Harle [28] further explore how prior attainment in mathematical courses impact confidence, performance, and self-assessment, linking prior knowledge to academic outcomes. Deisenroth *et al.* [4, p. 2] reinforce the importance of prior preparation, emphasizing the role of mathematical frameworks in formalizing abstract ML concepts.

Furthermore, perceived difficulty and confidence are critical factors in learning outcomes and retention. Wall and Knapp [12] note that perceptions of difficulty strongly influence engagement and performance in technical computing courses. Additionally, confidence, described as belief in one's ability to succeed [22], correlates positively with academic outcomes, especially in mathematics [29]. For instance, Bandura [30] emphasizes that self-efficacy − the belief in one's capacity to complete tasks − drives the effort and persistence needed for mastering challenging material. Riboroso *et al.* [31] further highlight how self-efficacy and anxiety shape mathematical performance, underscoring the psychological dimensions of learning.

Educational research increasingly highlights the interplay of these factors in fostering meaningful learning and retention. Andrews *et al.* [13] argue that connecting new material to existing knowledge enhances long-term understanding, a principle central to cognitive science. Liaw *et al.* [32] emphasize that effective knowledge retention must be intentionally integrated into learning activities, rather than assumed to occur naturally. This is supported by Custers and Ten Cate [33], who describe knowledge retention as following a "negatively accelerated forgetting curve", with longer intervals between acquisition and retrieval leading to diminished retention [34]. These findings underscore the need for educational interventions that not only teach core concepts effectively but also ensure their retention over time.

Integrating these frameworks is essential for understanding how students internalize and apply ML concepts. By examining the role of prior knowledge, conceptual understanding, perceived difficulty, and confidence, this study aims to uncover actionable insights to address gaps in ML education. It also contributes to the broader discourse on STEM education, where the focus is shifting from factual recall to developing scientific reasoning and transferable skills [35].

3 Methodology

This section outlines the methodological approach for investigating the retention of ML concepts and the role of mathematical foundations. It details the study's participants and selection criteria, along with the design and development of the survey instrument, including its structure, question types, and rationale for assessing knowledge retention, perceived difficulty, and confidence. The data collection process and measures to ensure response authenticity are also described.

3.1 Participants and Selection Criteria

The target group for this study comprised students who had successfully completed the Machine Learning course (CSE2510) at Delft University of Technology in the past two academic years, ensuring they had sufficient exposure to core ML concepts. This aligns with prior studies using course completion as a selection criterion [36]. Recruitment was conducted through personal networks and snowball sampling. Most participants were master's students at TU Delft, many of whom had taken advanced ML courses that revisited some of the topics assessed in this study. Although this additional exposure was not explicitly controlled for, participants self-reported their engagement with ML topics since completing the bachelor's course using a five-point Likert scale to account for potential influences.

Ethical approval for the study was obtained from TU Delft's Ethics Committee to ensure adherence to ethical research practices [37]. Participants were informed about the study's purpose, their rights, and the option to withdraw at any time without penalty. To protect privacy and maintain anonymity, no personally identifiable information was collected, and demographic data such as gender and age were excluded as deemed irrelevant to the research. Measures to ensure response integrity included clear instructions emphasizing independent completion without external assistance.

3.2 Survey Development

The survey was designed to assess the retention of core ML concepts and the influence of prior mathematical knowledge, drawing on established research in knowledge retention, perceived difficulty, and confidence in educational contexts. Development involved a literature review, creation of item pools and initial draft of the instrument, pilot testing, and iterative refinements, reflecting best practices in survey design and validation [38]. The finalized survey included six sections: an opening statement, demographics, and sections for each of the four ML concepts – Principal Component Analysis, Gradient Descent, Bayes' Theorem, and Hierarchical Clustering. To mitigate participant fatigue affecting responses to any specific topic, the order of the ML concept sections was randomized, ensuring each topic appeared last in one of the four permutations considered.

Assessing Demographics

The demographic section collected information on participant's academic year of ML course completion, grades in key mathematical courses (Linear Algebra, Calculus, and Probability & Statistics), and chosen specialization tracks during their bachelor's studies. The rationale for including grades as a metric was to capture prior mathematical attainment, similar to approaches in previous studies that measured grades in national examinations as a proxy for mathematical ability [28]. Additional questions assessed participants' engagement with ML concepts since completing the course, aligning with research on the role of prior exposure in long-term retention [39]. Higher scores on these items indicated stronger mathematical ability and greater familiarity with ML-related content.

Design of Multiple Choice Questions

The multiple-choice questions (MCQs) assessing core ML concepts were designed to span Bloom's Taxonomy cognitive levels, from recall and understanding to application and analysis [40]. Each section included five MCQs, with the first one assessing recall, the second understanding, the third application, the fourth analysis, and the fifth revisiting understanding. A checklist based on Bloom's Taxonomy guided the categorization, ensuring alignment with its revised framework [41, 42, 43]. The specific mapping between these cognitive levels and the questions is detailed in Table 1. Initially, more questions were created, but a final set of five per topic was chosen to balance survey length, cognitive breadth, and minimize participant fatigue. Most questions were designed by the author, and some were adapted from TU Delft ML course exams[1] or inspired by educational resources such as Medium[2] and Quizizz[3]. This ensured the inclusion of both original items and validated question formats. To mitigate recall of past exam questions, the one- or two-year interval since course completion was deemed sufficient [39]. A "Don't remember" option, informed by Levin-Banchik [44], was included to discourage guessing and ensure validity in assessing retention.

Table 1: Mapping MCQs to Bloom's Taxonomy Cognitive Levels

Cognitive Level	Keywords for Question Design	Question Mapping
Remember	Recognize, Recall	Q1
Understand	Explain, Infer, Compare, Interpret, Classify	Q2, Q5
Apply	Execute	Q3
Analyze	Differentiate, Attribute	Q4

[1] https://studiegids.tudelft.nl/a101_displayCourse.do?course_id=67579&_NotifyTextSearch_

[2] https://kawsar34.medium.com/principal-component-analysis-pca-part-2-ml-interview-question-bank-15-bb2b517f0719

[3] https://quizizz.com/admin/quiz/592ea9414e2f3e1000be3dc9/bayes-theorem

Rationale for Likert-Scale Questions

Likert-scale questions accompanied each MCQ section to assess perceived difficulty, confidence, and the impact of prior mathematical knowledge. A single-item approach was adopted for each dimension, following evidence that such designs can effectively capture key insights without overburdening respondents [45, 46]. Perceived difficulty item creation was inspired by the Perceived Difficulty Assessment Questionnaire (PDAQ), which has demonstrated relevance in similar educational contexts [47]. Similarly, confidence items were influenced by insights from the Instructional Materials Motivation Survey (IMMS) [48, pp. 282–284, 49, 50] and the Mathematics Self-Efficacy and Anxiety Questionnaire (MSEAQ)[4], though these tools were not entirely utilized due to their generalized scope and length. Instead, specific elements were adapted to align with the study's goals, ensuring relevance to ML knowledge while maintaining brevity and clarity. To enrich the data, each ML section included two additional items asking participants whether a refresher on mathematical topics would have improved their understanding, with open-ended responses providing qualitative insights. This combination of quantitative and qualitative approaches offered a comprehensive perspective on participant experiences.

Pilot testing with three representative students assessed the survey's readability, clarity, and completion time. Feedback resulted in minor phrasing adjustments to enhance clarity and flow. Appendix A presents the finalized survey instrument, including all demographic questions, MCQs, and Likert-scale items. Questions sourced from prior ML course exams are marked with a "#", while those adapted from educational resources are marked with an "&".

3.3 Data Collection Procedure

The survey was deployed via Microsoft Forms[5] and distributed electronically. Participants were given ample time to complete the questionnaire and encouraged to provide honest, unbiased responses. A brief study summary and survey link were shared through online platforms to reach eligible participants. Instructions emphasized avoiding preparation or external resources to maintain response authenticity.

4 Experimental Setup

This section outlines the experimental setup designed to examine the relationship between prior mathematical knowledge and retention of core ML concepts. It describes the rationale for selecting specific ML topics based on their mathematical prerequisites, the justification for the sample size, the tools and platforms used for survey deployment and data analysis, as well as the steps taken to ensure methodological rigor.

4.1 Experiment Design

The study examined the relationship between prior mathematical knowledge and retention of four core ML concepts, selected for their distinct mathematical prerequisites. PCA requires linear algebra, Gradient Descent relies on calculus, Bayes' Theorem is rooted in probability & statistics, and Hierarchical Clustering represents a concept with minimal mathematical requirements. The design followed best practices in educational research, categorizing topics by cognitive complexity and linking them to specific learning objectives [52].

Furthermore, in this study, prior mathematical knowledge was the independent variable, serving as the predictor, while retained knowledge of core ML concepts was the dependent variable. Control variables, including perceived difficulty and confidence, were accounted for to isolate the relationship between mathematical knowledge and ML concept retention, minimizing the influence of external factors.

4.2 Sample Size

The sample size was influenced by the study's objectives and practical constraints. In educational research with voluntary participation and limited resources, smaller sample sizes are often observed. For example, Sauro and Dumas [53] had 26 participants in a usability questionnaire comparison, while Schrenzel (2015)[6] studied 23 participants in an experimental group. Similarly, in an educational study on knowledge retention after a medical course, Cheifetz and Phang [55] presented results from 18 surgeons at the one-year measurement point. These cases show that meaningful insights can be obtained even from modest sample sizes. While Creswell [56, p. 146] recommends about 30 participants for correlational studies, time constraints and voluntary participation often result in smaller-than-ideal sample sizes, affecting the generalizability of findings. Strategies such as extended recruitment or offering incentives were considered but limited by available resources. Within these constraints, the study aimed to maximize participation and achieved a final sample size of $N = 28$.

4.3 Survey Platform and Analysis Tools

The survey was hosted on Microsoft Forms[7], chosen for its accessibility and user-friendly interface. Data analysis was conducted using Microsoft Excel[8] and Python programming language, enabling detailed exploration of the relationships between prior mathematical knowledge and ML concept retention. For result analysis, raw MCQ scores $(0 - 5)$ for each topic were averaged and normalized to provide standardized retention scores for each ML concept.

4.4 Validation and Reliability

Inspired by Shih and Chuang [57], the survey validation process included a review of related literature, identification of the four ML categories to be assessed, drafting and revising items, expert consultation for content validity, and iterative refinement based on pilot feedback. Additional procedures, such as those outlined by Erdogan *et al.* [38], were considered but adapted to fit the study's scope and time constraints.

[4] The MSEAQ is described in May's PhD thesis (Appendix B, pp. 70–71). See May [51]. *Mathematics self-efficacy and anxiety questionnaire*. University of Georgia, Athens, GA.

[5] https://www.microsoft.com/en-us/microsoft-365/online-surveys-polls-quizzes

[6] *Knowledge retention over a two year period following completion of an online course on the science of energy balance*. University of Vermont Honors College, Senior Theses. See Schrenzel [54]

[7] https://www.microsoft.com/en-us/microsoft-365/online-surveys-polls-quizzes

[8] https://www.microsoft.com/en-us/microsoft-365/excel

While efforts ensured methodological rigor, the validation process was limited by the lack of advanced procedures, such as large-scale testing, as seen in studies like those by Fiorella *et al.* [58] and Schau *et al.* [59]. These constraints reflect project limitations rather than a disregard for validation practices.

5 Results

This section presents the study's results, focusing on how prior mathematical knowledge, perceived difficulty, and confidence relate to long-term retention of the four ML concepts. It examines correlations between grades in foundational mathematics courses and retention, investigates the predictive roles of perceived difficulty and confidence, and provides a comparative analysis across topics. Additionally, performance on math-linked questions is analyzed, and thematic insights from open-ended responses illustrate students' perspectives on retaining ML concepts.

5.1 Correlation Analysis Between Prior Math Knowledge and ML Topic Retention

At the beginning of the results analysis phase, the relationships between students' prior mathematical knowledge and their retention of ML concepts were explored. This was done by analyzing the correlations between grades in foundational mathematics courses (Linear algebra, Calculus, Probability & Statistics) and retention scores for their corresponding math-intensive ML topics (Principal Component Analysis - PCA, Gradient Descent - GD, Bayes' Theorem - BT). Hierarchical Clustering (HC) was excluded for this analysis due to its minimal mathematical prerequisites.

Key findings are illustrated in a targeted heatmap (Figure 1) showing positive Pearson correlation coefficients between math grades and ML topic retention. Linear algebra grades modestly correlate with PCA retention ($r = 0.33$), calculus grades show a moderate correlation with GD retention ($r = 0.52$), and probability grades have a weak correlation with BT retention ($r = 0.11$). Notably, no negative correlations were observed, indicating higher math grades do not adversely affect retention. The complete correlation matrix is provided in Appendix B.1.

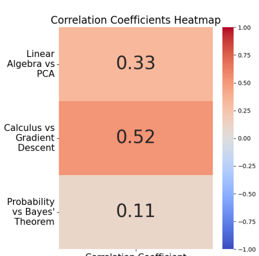

Figure 1: Targeted Correlations Heatmap

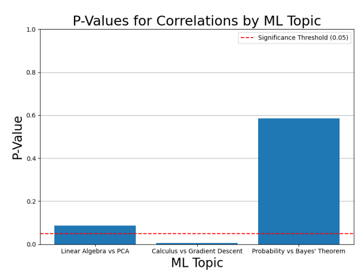

Figure 2: P-Values for Correlations by ML Topic

The statistical significance of these correlations was assessed using Pearson's p-values (Figure 2). Only the calculus-GD correlation was statistically significant ($p = 0.005$), reinforcing the observed relationship, while linear algebra-PCA ($p = 0.087$) and probability-BT ($p = 0.585$) were not significant at the 5% threshold. Scatter plots with regression lines provide a more detailed visualization of these trends (Figure 3). While PCA retention shows a weak positive trend with linear algebra grades, GD retention demonstrates a stronger upward trend with calculus grades, reflecting the higher correlation. BT retention displays negligible variation with probability grades, consistent with its weak correlation.

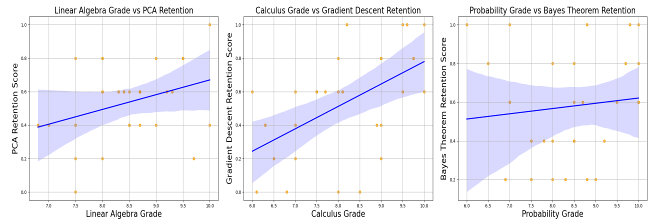

Figure 3: Scatter Plots of Grades vs Retention Scores by ML Topic

Spearman's rank correlation coefficients were calculated to explore potential non-linear but monotonic relationships. The results aligned closely with Pearson's, showing $\rho = 0.29$ (linear algebra-PCA), $\rho = 0.54$ (calculus-GD), and $\rho = 0.18$ (probability-BT).

The findings show that calculus has the strongest and most significant impact on retention of its related ML topic, GD. In contrast, PCA and BT retention are more weakly influenced by their respective math foundations, suggesting other factors like practical applications, perceived difficulty, or confidence may play a role. These results highlight the potential of pedagogical strategies emphasizing mathematical foundations to enhance retention of related ML concepts, while also underscoring the need for further analysis of variables influencing retention.

5.2 Relationships Between Retention, Perceived Difficulty, and Confidence

In addition to prior math knowledge, perceived difficulty and confidence in understanding were analyzed as potential predictors of ML topic retention. Perceived difficulty was expected to correlate negatively with retention, but showed mixed relationships. GD and HC had negative Pearson correlations ($r = -0.29$, $r = -0.23$), suggesting higher difficulty reduced retention, while PCA and BT showed weak positive correlations ($r = 0.13$, $r = 0.18$), indicating that difficulty may have motivated deeper engagement. Confidence positively correlated with retention across topics, especially for GD and HC ($r = 0.64$ for both), while PCA and BT showed weaker correlations ($r = 0.31$, $r = 0.25$), suggesting additional factors might influence retention. Comprehensive heatmaps and scatter plots for each ML topic, provided in Appendix B.2, illustrate these correlations. GD and HC show steep positive confidence-retention slopes and negative difficulty-retention slopes, while PCA and BT exhibit subtler trends. Negative difficulty-retention relations align with expectations that greater difficulty hinders recall and comprehension.

These findings emphasize the importance of confidence-building strategies for improving retention, while the mixed effects of perceived difficulty underscore the need to address challenges without discouraging engagement. Following these, a comparative analysis across the four ML topics explored whether retention differences reveal deeper patterns.

5.3 Comparative Analysis of Retention Across ML Topics

Paired t-tests and one-way ANOVA were conducted to compare retention across ML topics. Paired t-tests revealed no significant differences only among PCA, GD, and BT retention scores ($p - values > 0.05$), but HC retention scores were significantly lower. A one-way ANOVA confirmed these contrasts across topics when HC was included in the analysis ($p = 0.0025$). Descriptive statistics showed consistent mean scores for PCA and GD (0.53), though their standard deviations differed (PCA: 0.23, GD: 0.32), indicating greater variability in GD retention. BT had the highest mean (0.58) and a moderate standard deviation (0.28), reflecting slightly better overall retention. In contrast, HC had the lowest mean (0.31) and a standard deviation of 0.26, signifying consistently lower retention. These differences highlight varying retention patterns across topics, with GD retention showing the widest variation, potentially due to its calculus-based complexity.

Contrary to expectations, HC, a topic with minimal mathematical prerequisites, showed the lowest retention. This may be due to factors such as HC concepts being rarely revisited in subsequent courses, especially for Data track students at TU Delft, who formed the majority of the sample (67.86%). In contrast, PCA and GD are regularly reinforced in advanced courses, aiding retention. Moreover, HC's perceived simplicity may have led to superficial understanding during the course, undermining retention despite lower reported difficulty. Additional visualizations comparing retention across topics are provided in Appendix B.3.

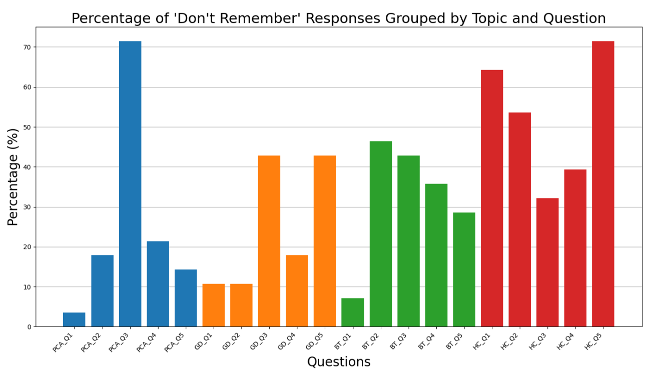

Figure 4: "Don't Remember" Response Percentages by ML Topic

5.4 Math-Linked Question Performance and Retention Implications

HC's surprisingly low retention prompted a deeper analysis of question-specific performance, highlighting the role of mathematical foundations in retaining ML concepts. Question three (Q3), assessing math knowledge for each ML topic, consistently showed the lowest correct response rates for math-intensive topics. This stark contrast underscores the challenges students may face when applying mathematical principles to ML concepts. Similarly, "Don't Remember" responses were highest for Q3 in PCA (71%), GD and BT (43% each), further reflecting the difficulty of applying math principles. In contrast, HC's Q3 had the lowest "Don't Remember" rate (32%). Figure 4 illustrates this pronounced gap for math-intensive topics, with further visualizations reinforcing this trend in Appendix B.4. All these findings suggest foundational math gaps hinder retention and support the need for targeted refreshers to bridge these gaps and improve ML concept understanding.

5.5 Thematic Analysis of Open-Ended Responses

Following the high "Don't Remember" rates for math-intensive questions, students' open-ended responses were thematically analyzed. This provided valuable insights into students' perspectives on the need for mathematical refreshers to enhance ML comprehension, identifying recurring challenges and opportunities for improvement.

For PCA, 78.6% of participants indicated the need for a refresher (Figure 5). Key challenges included eigenvalues, eigenvectors, and covariance matrices, with comments like "Eigenvector and eigenvalue calculation could be a nice refresher". Participants frequently noted the time gap between linear algebra and the ML course, citing difficulty recalling concepts. Suggestions included step-by-step demonstrations and practical examples to improve understanding.

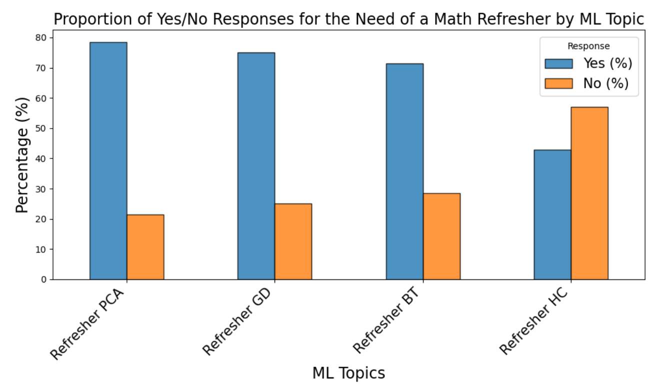

Figure 5: Need for Math Refresher: Yes/No Responses by ML Topic

Similarly, for GD, 75% of participants supported a calculus refresher. Derivatives were a key challenge, as noted in comments like, "Calculating the gradient via a derivative [...] having a refresher on that would be useful". Participants highlighted the lack of mathematical reasoning in the ML course ("The mathematical reasoning behind Gradient Descent was overlooked") and the time gap between calculus and the ML course ("Calculus course was 3 quarters before the ML course"). Respondents emphasized the need for both conceptual clarity and practical refreshers.

For BT, 71.4% of participants supported a refresher. Probability terminology was a common challenge, as one participant noted, "I forgot the meaning of a lot of specialty terms". However, some found the concept straightforward and felt a refresher unnecessary, reflecting diverse prior knowledge levels, with stronger foundations reducing the need for review.

In contrast, only 42.9% opted for the need of a refresher for HC, citing its intuitive and accessible nature. However, a minority highlighted the potential value of reinforcing distance metrics to improve comprehension.

To support these findings, supplementary bar charts in Appendix B.5 illustrate the importance students placed on prior mathematical knowledge and the perceived value of a stronger background, especially for math-intensive ML topics. Additionally, tables with key quotes from the thematic analysis are provided in Appendix B.6. These findings strongly support integrating targeted math refreshers before introducing ML topics, which would enable students to focus on ML concepts rather than struggle with underlying mathematics.

6 Discussion

This study highlights the complex relationship between prior mathematical knowledge, perceived difficulty, and confidence in shaping students' long-term retention of ML concepts. By analyzing four representative ML topics − Principal Component Analysis (PCA), Gradient Descent (GD), Bayes' Theorem (BT), and Hierarchical Clustering (HC) − the findings reveal distinct performance patterns that collectively emphasize the need for targeted educational interventions. This chapter explores the significance of these results, proposes a solution to address identified gaps, and discusses key limitations.

6.1 Interpreting the Results and Implications

A central outcome of this research is the significant correlation between calculus grades and GD retention, suggesting that understanding derivatives and optimization strongly impacts students' ability to recall and apply GD concepts. While PCA and BT also depend on mathematical foundations − linear algebra and probability, respectively − their correlations with math grades were positive but not statistically significant. This discrepancy suggests that retention for PCA and BT may be influenced by additional factors such as repeated exposure in later coursework or practical applications in projects and internships.

A closer look at question-level performance highlights students' struggles with math-intensive ML topics. In particular, question three, which assessed specific mathematical concepts underpinning each ML technique, had the highest "Don't Remember" response rates for PCA, GD, and BT, indicating difficulty when applying foundational math in an ML context. Open-ended responses confirmed this, with students noting challenges in recalling or mastering linear algebra steps for PCA and derivatives for GD. These findings align with studies reporting declines in knowledge retention over time [18].

Hierarchical Clustering and Overall Performance

Contrary to expectations, HC, the least math-intensive topic, had the lowest retention scores. While considered more intuitive, participants reported rarely revisiting it in advanced modules compared to the other topics included in this study. This superficial understanding during the course, or "motivated forgetting" [11], may have hindered long-term recall of HC algorithms and distance metrics. These outcomes highlight that retention declines when topics are perceived as less relevant or are infrequently revisited [60].

Despite the comparatively higher averages for PCA, GD, and BT (around 2.5 out of 5), a 50% correctness rate should not be considered satisfactory, especially given ML's growing importance in industries [2]. This raises concerns that even topics reinforced in later courses can lose rigor if foundational elements are not revisited or properly understood from the beginning. While forgetting is a natural part of the learning process [60], the extent of memory decay highlights the need for significant curricular improvements in ML education.

Perceived Difficulty, Confidence, and Student Engagement

The interplay between perceived difficulty and retention was more nuanced. For GD and HC, higher difficulty correlated with lower retention, suggesting conceptual barriers. In contrast, PCA and BT showed small positive correlations, indicating that managing challenging content effectively can motivate deeper study and enhance learning outcomes [46, 12]. Confidence positively correlated with retention across all topics, consistent with prior findings on the importance of self-belief in learning outcomes [30, 61]. However, across math-intensive ML concepts, open-ended responses revealed a recurring desire for structured ways to revisit critical math skills, underscoring the need for robust scaffolding to mitigate forgetting and sustain confidence over time.

Collectively, these results emphasize that (1) targeted mathematical preparation is essential for mastering ML algorithms, (2) periodic reinforcement can help mitigate memory decay over time [60], and (3) confidence-building measures, such as practice assignments or interactive tutorials, may further enhance retention [17]. Above all, the thematic analysis and question-level outcomes strongly advocate integrating reinforcement strategies for the mathematical foundations of each ML technique.

6.2 Proposed Solution: A Targeted Mathematical Refresher

To address challenges in long-term ML retention, this study proposes a concise, topic-specific mathematical refresher preceding ML lectures on math-intensive concepts. These refreshers would serve as a scaffold, reinforcing key mathematical concepts tied to the upcoming topic. For example, a module reviewing eigenvalues, eigenvectors, and covariance matrices could precede PCA, while a recap on derivatives and gradient calculations could accompany GD. These materials could be distributed as Jupyter Notebooks or presented as lecture slides.

Structure and Delivery

A practical way to implement this refresher is through modular Jupyter Notebooks or short interactive videos. Each module could include concise concept explanations (e.g., tutorials on targeted math topics), worked examples (e.g., computing derivatives or step-by-step eigen-decomposition), low-stakes practice problems to check understanding and recall, and automated feedback to reinforce correct methods and clarify errors. A more detailed structure for such a refresher is provided in Appendix C.

Evaluation Plan

Evaluating the efficacy of these refreshers could use a mixed-method approach, combining quantitative and qualitative measures. Pre- and post-tests on key math skills and ML outcomes are one option, though introducing control groups that do not receive a refresher raises ethical concerns. Instructors might opt for alternative strategies, including delaying the refresher for a subset of participants or offering different but equivalent intervention formats, similarly to how Grabarnik *et al.* [5] evaluated the effectiveness of a linear algebra refresher by distributing it in two different formats to students taking an ML course. Surveys or interviews could further capture students' perceptions of clarity, motivation, and usefulness, while longer-term follow-ups could assess whether the refresher's benefits persist beyond the immediate learning interval.

By aligning the refreshers' content with the specific mathematical demands of each ML topic, instructors can address foundational gaps, while streamlining their teaching. This adaptable approach allows instructors to tailor the depth and difficulty of the refresher to fit their course goals, making it a valuable addition to existing curricula.

6.3 Limitations

Despite the promising outcomes and the applicability of the proposed refresher, the study is constrained by several limitations that should be acknowledged. First, the small sample size ($N = 28$), as seen in prior studies, may hinder the detection of significant changes and amplify the effects of outliers [53]. Recruitment time constraints and voluntary participation influenced the final sample, limiting the generalizability to larger student populations.

Second, the survey included only five questions to assess retention for each ML topic. While this minimized respondent fatigue, it limited the depth of the assessment. A larger question bank with more detailed questions could provide finer-grained insights. Similarly, only four ML concepts were analyzed. Although PCA, GD, BT, and HC represent a diverse range of mathematical prerequisites, many other techniques in ML remain unexplored.

Another limitation is the use of course grades in foundational mathematics as proxies for prior knowledge. While consistent with prior research [28], these grades may conflate unrelated subtopics. Embedding more targeted math diagnostics in the survey could improve accuracy but might increase completion time, discouraging engagement and reducing data quality due to survey fatigue [49].

Lastly, the constrained timeline prevented collecting retention data at multiple time points, such as immediately after course completion, six months later, and one year later, as done in other longitudinal studies [36, 55]. Such data would have allowed for a more comprehensive analysis of retention decay over time. While an approach using separate student samples, such as those who recently completed the ML course versus those who completed it two years ago, could have been an alternative for comparing retention, it lacks the precision of tracking retention within the same cohort. This limitation underscores the need for external longitudinal studies to better understand knowledge retention in ML education.

Overall, these limitations highlight areas for refinement in measurement and methodology. Nonetheless, the results demonstrate that strengthening mathematical foundations and fostering confidence can meaningfully enhance retention of core ML concepts. The proposed refresher offers a practical solution to address knowledge gaps. Future research can build on this framework by testing the refresher's efficacy at scale and applying similar methodologies to other ML techniques.

7 Responsible Research

This research adheres to the ethical standards outlined by Delft University of Technology and the Netherlands Code of Conduct for Research Integrity. Approval was granted by TU Delft Human Research Ethics Committee (HREC) following a submission of the ethics checklist, data management plan, and informed consent procedure. Key ethical considerations include participant privacy, data protection, and ensuring reproducibility and data integrity throughout the research process.

7.1 Privacy and Data Protection Considerations

To ensure participant privacy and data security, the survey collected only anonymized responses, with no personally identifiable information like names or contact details. Indirectly identifiable data, such as prior academic grades, was carefully handled to prevent re-identification. Clear communication was provided to participants through an opening statement at the start of the survey, outlining the study's purpose, voluntary nature, participant rights, and data protection measures. By clicking "Next" in this introductory section, respondents gave their informed consent to participate in the study.

Participants were informed that their responses would be used solely for this research and could not be withdrawn after anonymization. Data was securely stored on a password-protected device, in compliance with TU Delft's data management policies, with access restricted to the researcher and, if necessary, supervising staff.

7.2 Reproducibility and Data Integrity

The methods and procedures in this study were designed to ensure transparency and reproducibility. Detailed documentation of the survey design, participant recruitment, and data analysis techniques is provided in the Methodology and Experimental Setup sections. Comprehensive descriptions of the survey questions, Likert scales, and MCQ mapping to Bloom's taxonomy enable replication by other researchers.

To ensure data integrity, strict anonymization protocols were followed during processing. The aggregated, anonymized data will be retained temporarily for potential reproducibility checks but not shared publicly to protect participant privacy. The methodology and key findings will be published in the TU Delft Library Repository, ensuring accessibility for academic and educational purposes.

7.3 Reflection on Ethical Research Practices

The principles of honesty, scrupulousness, transparency, independence, and responsibility were upheld throughout the research. Efforts were made to avoid biases, such as recruitment bias from reliance on personal networks, by including a diverse participant pool. Moreover, all participants were treated with respect and fairness.

Furthermore, generative AI tools, including ChatGPT and Grammarly, supported the research process by assisting with grammar, formatting LaTeX data and tables, and generating some Pyhton visualizations. Outputs were modified to ensure accuracy, relevance, and compliance with academic integrity standards. Example prompts used are listed in Appendix D.

By following these ethical guidelines and ensuring methodological rigor, this research contributes to the broader discourse on ML education while maintaining the highest standards of research integrity.

8 Conclusions and Future Work

This research examined the retention of four core Machine Learning concepts − Principal Component Analysis (PCA), Gradient Descent (GD), Bayes' Theorem (BT), and Hierarchical Clustering (HC) − within two years of course completion, focusing on how prior mathematical knowledge influenced retention. The findings revealed a statistically significant positive relationship between calculus grades and retention of GD, while correlations for linear algebra with PCA, and probability with BT were positive but did not reach statistical significance. Perceived difficulty and confidence levels also played key roles, with higher confidence generally correlating with stronger retention.

The study highlights how foundational mathematics, perceived difficulty, and confidence collectively shape retention, emphasizing the importance of periodic reinforcement to sustain knowledge. This underscores the value of concise mathematical refreshers linked directly to each ML topic. The distinct patterns of PCA, GD, BT, and HC, along with the qualitative insights from participants, affirm the need for integrating more tailored approaches into curricula to bridge knowledge gaps and enhance long-term retention.

Possible improvements and questions for further investigation include developing and evaluating mini-modules or tutorials to revisit mathematical concepts prior to presenting advanced ML concepts, as well as testing whether such interventions reduce the frequency of "Don't Remember" responses. Longitudinal studies tracking retention at multiple time intervals could also reveal more precise patterns of learning decay, while expanding the study to assess other ML techniques might provide deeper insights into the interplay between foundational knowledge and retention. Additionally, future research would benefit from larger participant sample sizes and more rigorous math diagnostics to identify which subtopics students find most difficult to recall. Finally, exploring different refresher formats, such as interactive quizzes, short videos, or collaborative workshops, could clarify how to foster continuous engagement and build self-confidence.

In conclusion, combining periodic reviews of foundational mathematics with thoughtfully designed pedagogical tools that emphasize real-world relevance enables educators to foster deeper understanding and transform academic achievements into lasting expertise. This study offers actionable insights to address foundational gaps and equip students to confidently navigate the evolving challenges of Machine Learning.

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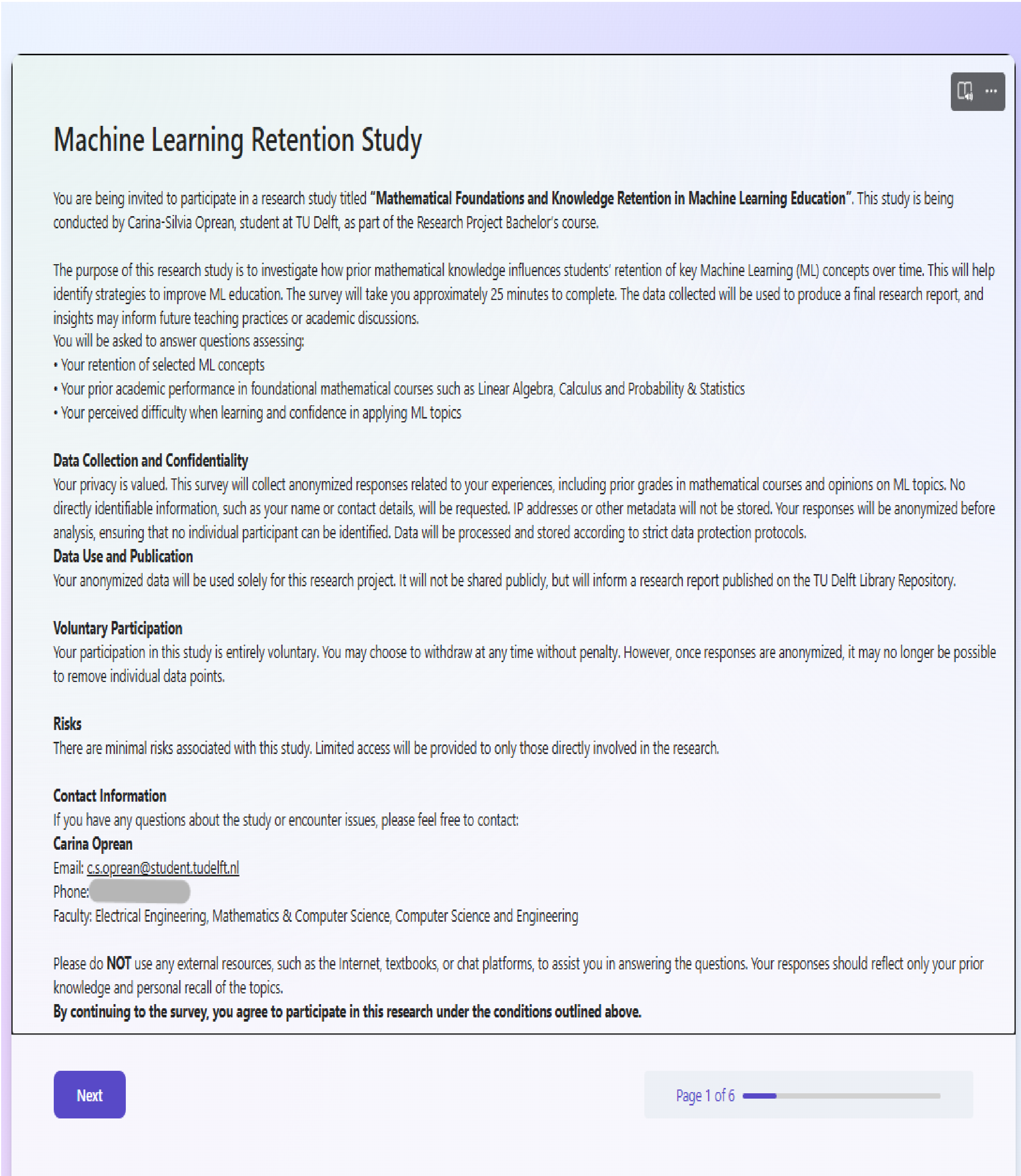
Appendix A: Final Survey Instrument



Machine Learning Retention Study

You are being invited to participate in a research study titled **"Mathematical Foundations and Knowledge Retention in Machine Learning Education"**. This study is being conducted by Carina-Silvia Oprean, student at TU Delft, as part of the Research Project Bachelor's course.

The purpose of this research study is to investigate how prior mathematical knowledge influences students' retention of key Machine Learning (ML) concepts over time. This will help identify strategies to improve ML education. The survey will take you approximately 25 minutes to complete. The data collected will be used to produce a final research report, and insights may inform future teaching practices or academic discussions.
You will be asked to answer questions assessing:
• Your retention of selected ML concepts
• Your prior academic performance in foundational mathematical courses such as Linear Algebra, Calculus and Probability & Statistics
• Your perceived difficulty when learning and confidence in applying ML topics

Data Collection and Confidentiality
Your privacy is valued. This survey will collect anonymized responses related to your experiences, including prior grades in mathematical courses and opinions on ML topics. No directly identifiable information, such as your name or contact details, will be requested. IP addresses or other metadata will not be stored. Your responses will be anonymized before analysis, ensuring that no individual participant can be identified. Data will be processed and stored according to strict data protection protocols.
Data Use and Publication
Your anonymized data will be used solely for this research project. It will not be shared publicly, but will inform a research report published on the TU Delft Library Repository.

Voluntary Participation
Your participation in this study is entirely voluntary. You may choose to withdraw at any time without penalty. However, once responses are anonymized, it may no longer be possible to remove individual data points.

Risks
There are minimal risks associated with this study. Limited access will be provided to only those directly involved in the research.

Contact Information
If you have any questions about the study or encounter issues, please feel free to contact:
Carina Oprean
Email: c.s.oprean@student.tudelft.nl
Phone: [redacted]
Faculty: Electrical Engineering, Mathematics & Computer Science, Computer Science and Engineering

Please do **NOT** use any external resources, such as the Internet, textbooks, or chat platforms, to assist you in answering the questions. Your responses should reflect only your prior knowledge and personal recall of the topics.
By continuing to the survey, you agree to participate in this research under the conditions outlined above.

Next

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Figure 6: Opening Statement of Survey

Demographic Information

All your information is highly confidential and for internal use only.

In what academic year did you take the Machine Learning course (CSE2510)? *
If you took the course multiple times, only the year in which you passed the course should be stated.

- ○ 2021-2022
- ○ 2022-2023
- ○ 2023-2024
- ○ 2024-2025

What was your final grade (not rounded) for the **Machine Learning** (CSE2510) course? *

Number must be between 1 ~ 10

What was your final grade (not rounded) for the **Linear Algebra** (CSE1205) course? *

Number must be between 1 ~ 10

What was your final grade (not rounded) for the **Calculus** (CSE1200) course? *

Number must be between 1 ~ 10

What was your final grade (not rounded) for the **Probability and Statistics** (CSE1210) course? *

Number must be between 1 ~ 10

How many hours of mathematical lectures per week did you have in **High School**? *

The value must be a number

What variant did you choose during your Bachelor? *

- ○ Data
- ○ Multimedia
- ○ Systems
- ○ I have not chosen a variant yet

What Minor did you do during your Bachelor? *
If you are still pursuing your Minor, you can still mention it.

To what extent have you engaged with or encountered ML concepts or applications in your studies or career since completing the course? *

Not at all	Rarely	Occasionally	Frequently	Very Frequently
○	○	○	○	○

Pick a number for a customized experience. *

- ○ 1
- ○ 2
- ○ 3
- ○ 4

Principal Component Analysis

The following questions will assess your retention of PCA concepts, as well as you perceived difficulty during the course and your perceived importance of **Linear Algebra** in learning PCA.

If you are unsure of the answer to a question, please choose the "*Don't remember*" option instead of guessing. For the purpose of this research, it is better to acknowledge uncertainty than to select an answer that might only be correct by chance.

Which of the following best describes how PCA reduces dimensionality? * (1 Point)

- ○ It clusters data points and eliminates outliers.
- ○ It projects data onto new orthogonal axes ordered by variance.
- ○ Don't remember.
- ○ It eliminates correlated features from the dataset.
- ○ It compresses high-variance features while discarding low-variance ones.

Suppose you apply PCA to reduce a dataset to 2 dimensions for visualization. The first two components explain 95% of the variance. What is the primary trade-off of using PCA in this case? * (1 Point)

- ○ Don't remember.
- ○ Reduced interpretability of the original features.
- ○ Dependence on feature scaling for effective results.
- ○ Increased computational complexity due to eigenvalue calculation.
- ○ Loss of 5% variance, which may exclude some important information.

\# The first step in performing dimensionality reduction with PCA is to calculate the covariance matrix of the data. Select the correct covariance matrix calculated by the maximum likelihood estimator for the given dataset (rows are samples, columns are features): * (1 Point)

$$\begin{bmatrix} 4 & 10 \\ 8 & 6 \end{bmatrix}$$

- ○ $\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$
- ○ Don't remember.
- ○ $\begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$
- ○ $\begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}$
- ○ $\begin{bmatrix} 4 & 8 \\ 8 & 4 \end{bmatrix}$

A dataset contains 10 highly correlated features. After applying PCA, the first principal component explains 70% of the variance, while the second explains 20%. What does this imply about the dataset's dimensionality? * (1 Point)

- ○ The first two principal components eliminate all redundancy in the dataset.
- ○ The dataset contains significant variance in all 10 dimensions.
- ○ Don't remember.
- ○ Most of the dataset's information can be represented in two dimensions.
- ○ The dataset cannot be reduced without significant information loss.

& When should you use PCA as a preprocessing step in a machine learning pipeline? * (1 Point)

- ○ When you want to remove outliers.
- ○ When you suspect multicollinearity among features.
- ○ When you want to increase the number of features.
- ○ Don't remember.
- ○ PCA should never be used as a preprocessing step.

Understanding of PCA **during the ML course** *

For the following statements, please try to recall and answer as accurately as possible. Think of all the methods, concepts, and mathematical formulas taught during the course.

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
PCA was very difficult to understand during the course.					
My prior knowledge of Linear Algebra helped me understand PCA better.					
A stronger background in Linear Algebra would have made PCA easier to understand.					
I am confident I understood the core principles and concepts underlying PCA.					
I am confident I can apply PCA in practical scenarios.					

Do you think a refresher of Linear Algebra concepts at the time of learning PCA would have made it easier to understand? *

- Yes
- No

Why? / Why not?

Please provide a short motivation for your answer to the previous question.

Gradient Descent

The following questions will assess your retention of Gradient Descent concepts, as well as you perceived difficulty during the course and your perceived importance of **Calculus** in learning Gradient Descent.

If you are unsure of the answer to a question, please choose the "*Don't remember*" option instead of guessing. For the purpose of this research, it is better to acknowledge uncertainty than to select an answer that might only be correct by chance.

Which of these claims is true for Gradient Descent? * (1 Point)

- Given enough steps, gradient descent converges to the global minimum.
- For linear regression, we typically do not need gradient descent, because there is a closed-form (analytic) solution to the empirical risk minimization problem.
- Stochastic gradient descent uses fewer computations per step, so it will always converge faster than batch (non-stochastic) gradient descent.
- Don't remember.
- Larger step sizes lead to faster convergence.

Gradient Descent is applied to optimize a quadratic loss function. The learning rate is adjusted dynamically during training. Which of the following best describes the advantage of this approach? * (1 Point)

- A constant learning rate ensures steady convergence.
- An increasing learning rate improves gradient accuracy.
- A decreasing learning rate allows for finer convergence near the minimum.
- Don't remember.
- A dynamic learning rate prevents local minima.

The loss function for a model is given by $L(w) = (w - 3)^2$, where w is the weight. Using Gradient Descent with a learning rate of η=0.1, what will the value of w be after one iteration starting from $w0 = 0$? * (1 Point)

- 0.3
- 1.2
- 0.6
- Don't remember.
- 0.9

A Machine Learning model using Gradient Descent fails to converge during training. Upon investigation, the loss function oscillates instead of reducing steadily. What could be the most likely cause? * (1 Point)

- The dataset is not normalized, affecting gradient calculations.
- Don't remember.
- The regularization parameter is too large, altering the loss function.
- The learning rate is too high, causing the algorithm to overshoot the minimum.
- The learning rate is too low, leading to slow convergence.

You are training a linear regression model using Gradient Descent. How does adding regularization terms (e.g., L1 or L2) affect the optimization? * (1 Point)

- It makes Gradient Descent slower.
- It changes the loss function to discourage large coefficients.
- Don't remember.
- It prevents the algorithm from reaching a minimum.
- It forces the learning rate to decrease.

Understanding of Gradient Descent **during the ML course** *

For the following statements, please try to recall and answer as accurately as possible. Think of all the methods, concepts, and mathematical formulas taught during the course.

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
Gradient Descent was very difficult to understand during the course.					
My prior knowledge of Calculus helped me understand Gradient Descent better.					
A stronger background in Calculus would have made Gradient Descent easier to understand.					
I am confident I understood the core principles and concepts underlying Gradient Descent.					
I am confident I can apply Gradient Descent in practical scenarios.					

Do you think a refresher of Calculus concepts at the time of learning Gradient Descent would have made it easier to understand? *

- Yes
- No

Why? / Why not?

Please provide a short motivation for your answer to the previous question.

Bayes' Theorem

The following questions will assess your retention of Bayes' Theorem concepts, as well as you perceived difficulty during the course and your perceived importance of **Probability and Statistics** in learning Bayes' Theorem.

If you are unsure of the answer to a question, please choose the "*Don't remember*" option instead of guessing. For the purpose of this research, it is better to acknowledge uncertainty than to select an answer that might only be correct by chance.

Which statement best describes Bayes' Theorem? * (1 Point)

- ◯ It calculates the probability of a hypothesis given new evidence.
- ◯ It minimizes classification errors using posterior probabilities.
- ◯ It maximizes the likelihood of observed data.
- ◯ Don't remember.
- ◯ It predicts future outcomes based on historical data.

How does the likelihood ratio interact with prior probabilities to determine the posterior ratios and posterior probabilities? * (1 Point)

- ◯ The likelihood ratio replaces the prior probabilities in determining the posterior probabilities.
- ◯ The likelihood ratio is added to the prior probabilities to produce the posterior probabilities.
- ◯ Don't remember.
- ◯ The likelihood ratio has no influence if the prior probabilities are equal.
- ◯ The likelihood ratio adjusts the prior probabilities to produce the posterior probabilities.

& A virus has infected 1.8% of a population. A test detects this virus 95% of the time when it is actually present, but it returns a false positive 3% of the time when the virus is not present. If a person selected at random from this population tests positive for the virus, what is the probability that this person is actually infected? You can round the answer to the nearest percent. * (1 Point)

- ◯ 34%
- ◯ Don't remember.
- ◯ 63%
- ◯ 66%
- ◯ 37%

Suppose a classifier has high confidence in a prediction based on an incorrect prior probability. How will this affect the posterior probability automatically? * (1 Point)

- ◯ The posterior probability will adjust the prior probability automatically.
- ◯ The posterior probability will be uniformly distributed.
- ◯ The posterior probability will reflect the incorrect prior's influence, leading to biased results.
- ◯ Don't remember.
- ◯ The posterior probability will remain unaffected as likelihood dominates.

If the prior probabilities are equal, and likelihood values differ, which statement best explain the posterior? * (1 Point)

- ◯ Don't remember.
- ◯ The posterior depends only on the prior.
- ◯ The posterior is equally split between hypotheses.
- ◯ Likelihoods cancel out when priors are equal.
- ◯ The posterior is higher for the hypothesis with greater likelihood.

Understanding of Bayes' Theorem **during the ML course** *

For the following statements, please try to recall and answer as accurately as possible. Think of all the methods, concepts, and mathematical formulas taught during the course.

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
Bayes' Theorem was very difficult to understand during the course.	◯	◯	◯	◯	◯
My prior knowledge of Probability & Statistics helped me understand Bayes' Theorem better.	◯	◯	◯	◯	◯
A stronger background in Probability & Statistics can make Bayes' Theorem easier to understand.	◯	◯	◯	◯	◯
I am confident I understood the core principles and concepts underlying Bayes' Theorem.	◯	◯	◯	◯	◯
I am confident I can apply Bayes' Theorem in practical scenarios.	◯	◯	◯	◯	◯

Do you think a refresher of Probability & Statistics concepts at the time of learning Bayes' Theorem would have made it easier to understand? *

- ◯ Yes
- ◯ No

Why? / Why not?

Please provide a short motivation for your answer to the previous question.

Hierarchical Clustering

The following questions will assess your retention of Hierarchical Clustering concepts, as well as you perceived difficulty during the course and your perceived importance of **general mathematics** in learning Hierarchical Clustering.

If you are unsure of the answer to a question, please choose the "*Don't remember*" option instead of guessing. For the purpose of this research, it is better to acknowledge uncertainty than to select an answer that might only be correct by chance.

How does switching from single-linkage to complete-linkage affect the resulting clusters in a dataset with overlapping points? * (1 Point)

- ◯ Complete-linkage creates more compact clusters, reducing overlap.
- ◯ Don't remember.
- ◯ Single linkage creates more compact clusters, reducing overlap.
- ◯ Single linkage and complete-linkage yield identical results.
- ◯ Complete-linkage creates elongated clusters to preserve data separation.

A dendogram produced by hierarchical clustering shows several distinct branches at a specific height. What does this indicate about the number of clusters at that level? * (1 Point)

- ◯ The number of clusters equals the height of the tallest branch.
- ◯ The number of clusters depends on the linkage method used.
- ◯ The number of clusters is determined by the total number of data points.
- ◯ The number of clusters equals the number of distinct branches at that height.
- ◯ Don't remember.

Two points *A* = *(1, 2)* and *B* = *(4, 6)* belong to different clusters. A third point *C* = *(2, 4)* is being merged into one of the clusters. Using single-linkage clustering, which cluster will *C* join first? * (1 Point)

- Don't remember.
- Cluster of *B*.
- Forms its own cluster.
- Randomly assigned to either cluster.
- Cluster of *A*.

When using single-linkage in agglomerative clustering, how is the distance between two clusters determined? * (1 Point)

- Don't remember.
- By the average distance between all pairs of points.
- By the minimum distance between any two points in the clusters.
- By the maximum distance between any two points in the clusters.
- By the centroid distance of the clusters.

Agglomerative hierarchical clustering is used for clustering a dataset of non overlapping points. The following parameters were used in the setup:
Merging rule: single-linkage
Distance measure: Euclidean
In each iteration of the hierarchical clustering, a new cluster is created. If the merging changes to complete linkage, how would that affect the creation of a cluster after the very first iteration of the algorithm? * (1 Point)

- None of the answers are correct.
- All of the answers are correct.
- Complete linkage seeks more internal cohesion, so the sum of distances between points of the first cluster using complete linkage will be smaller.
- Single linkage seeks more isolated groups, so the sum of distances measured between points of the first cluster using complete linkage will be larger.
- Don't remember.
- The first cluster created might not be the same when using different merging rules.

Understanding of Hierarchical Clustering **during the ML course** *

For the following 3 statements, please try to recall and answer as accurately as possible. Think of all the methods, concepts, and mathematical formulas taught during the course.

	Strongly Disagree	Disagree	Neutral	Agree	Strongly agree
Hierarchical Clustering was very difficult to understand during the course	○	○	○	○	○
My prior knowledge of general mathematics helped me understand Hierarchical Clustering better	○	○	○	○	○
A stronger background in mathematics would have made Hierarchical Clustering easier to understand.	○	○	○	○	○
I am confident I understood the core principles and concepts underlying Hierarchical Clustering.	○	○	○	○	○
I am confident I can apply Hierarchical Clustering in practical scenarios.	○	○	○	○	○

Do you think a refresher of general mathematical concepts at the time of learning Hierarchical Clustering would have made it easier to understand? *

- Yes
- No

Appendix B: Additional Plots and Visualizations for Result Analysis

B.1: Correlation Analysis Between Prior Math Knowledge and ML Topic Retention

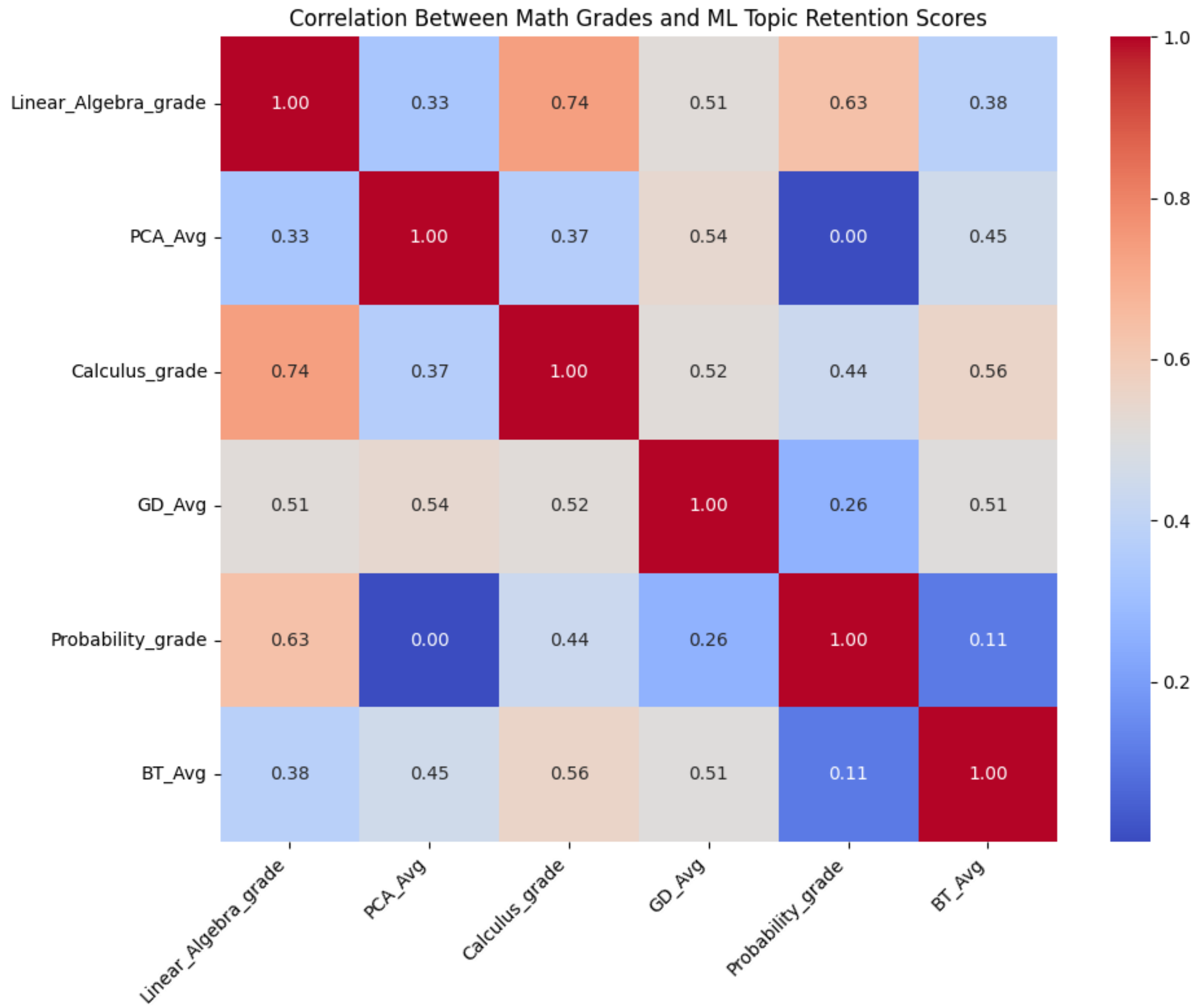


Figure 7: Correlation Heatmap Between Foundational Math Grades and ML Topic Retention Scores

B.2: Relationships Between Retention, Perceived Difficulty, and Confidence

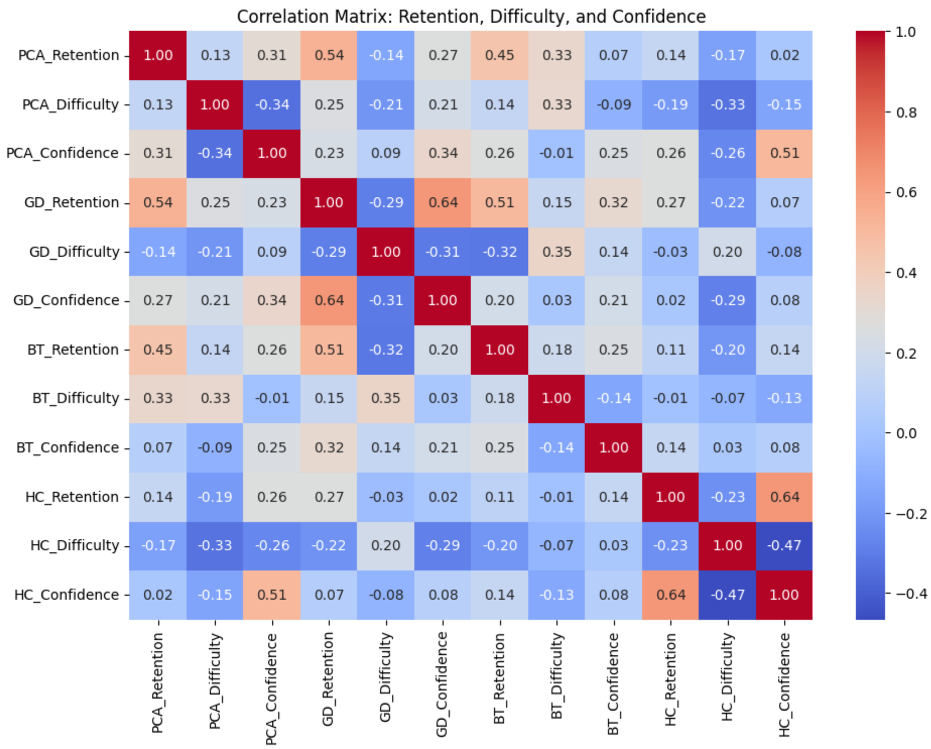


Figure 8: Correlation Matrix Between Retention, Difficulty, and Confidence Across All ML Topics

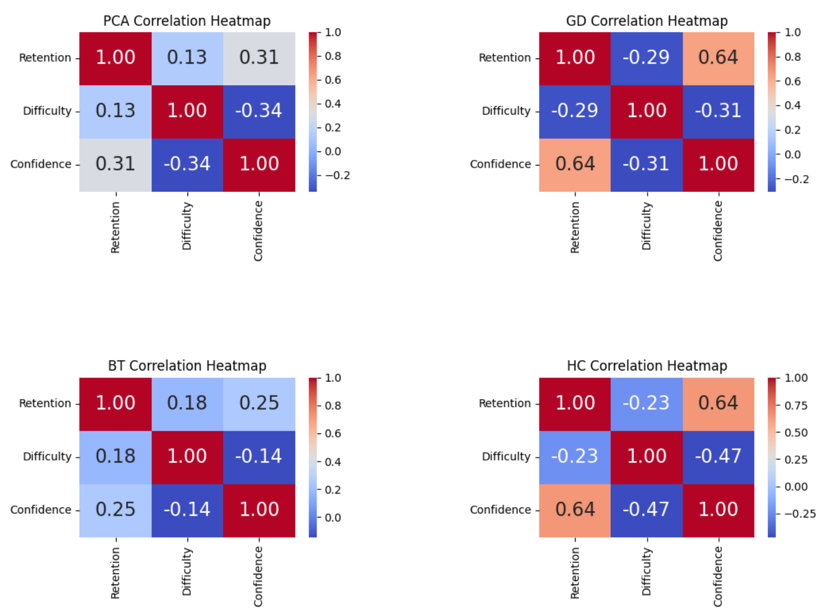
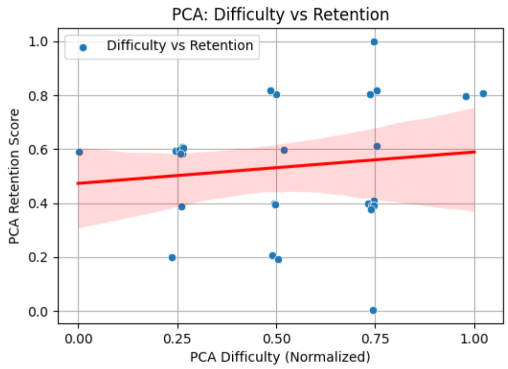
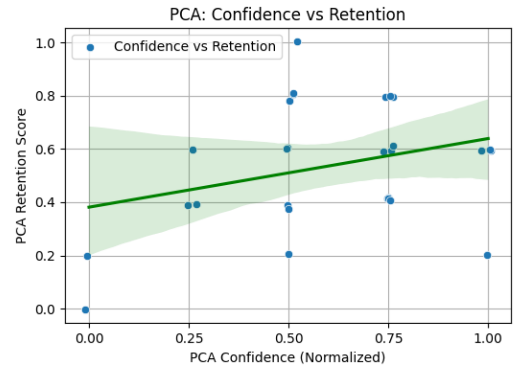


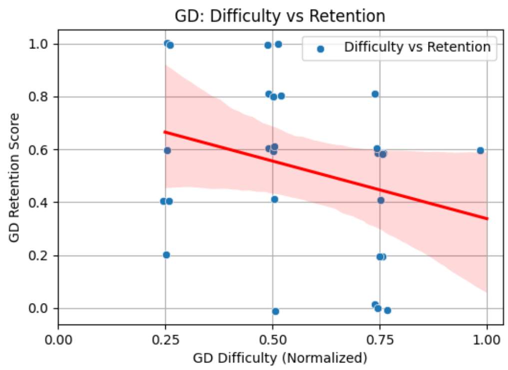
Figure 9: Correlation Heatmaps for Retention, Difficulty, and Confidence Across Individual ML Topics



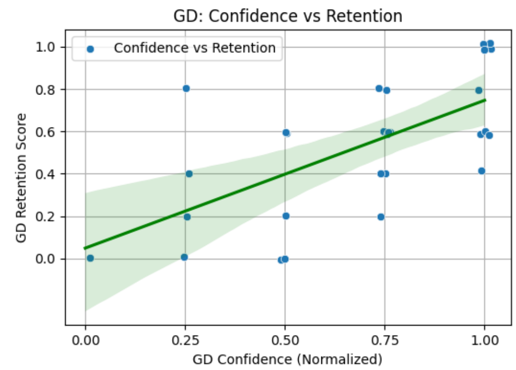
(a) PCA: Relationship Between Difficulty and Retention



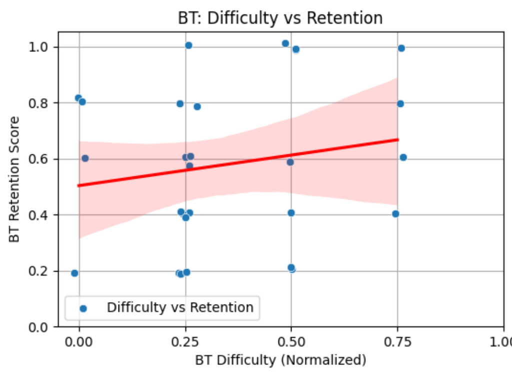
(b) PCA: Relationship Between Confidence and Retention



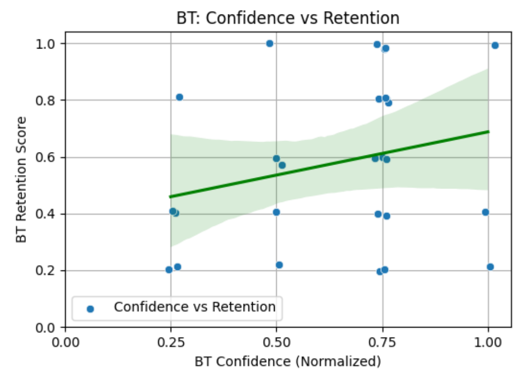
(c) GD: Relationship Between Difficulty and Retention



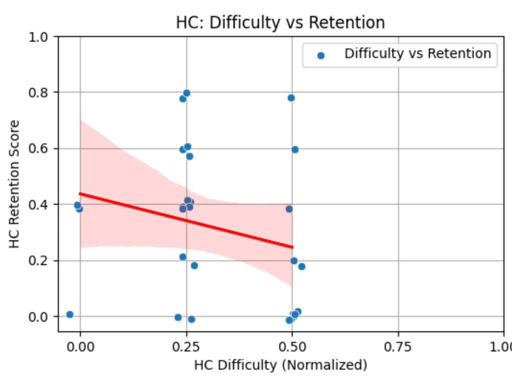
(d) GD: Relationship Between Confidence and Retention



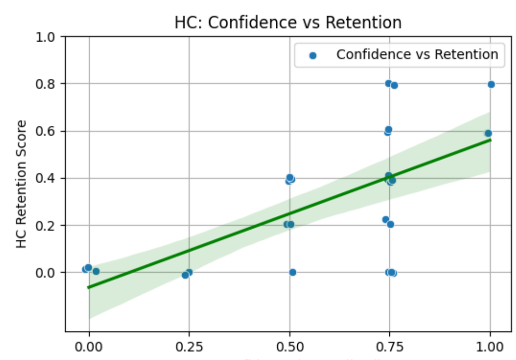
(e) BT: Relationship Between Difficulty and Retention



(f) BT: Relationship Between Confidence and Retention



(g) HC: Relationship Between Difficulty and Retention



(h) HC: Relationship Between Confidence and Retention

Figure 10: Scatter Plots Showing the Relationships Between Retention, Difficulty, and Confidence for Each ML Topic (PCA, GD, BT, HC)

B.3: Comparative Analysis of Retention Across ML Topics

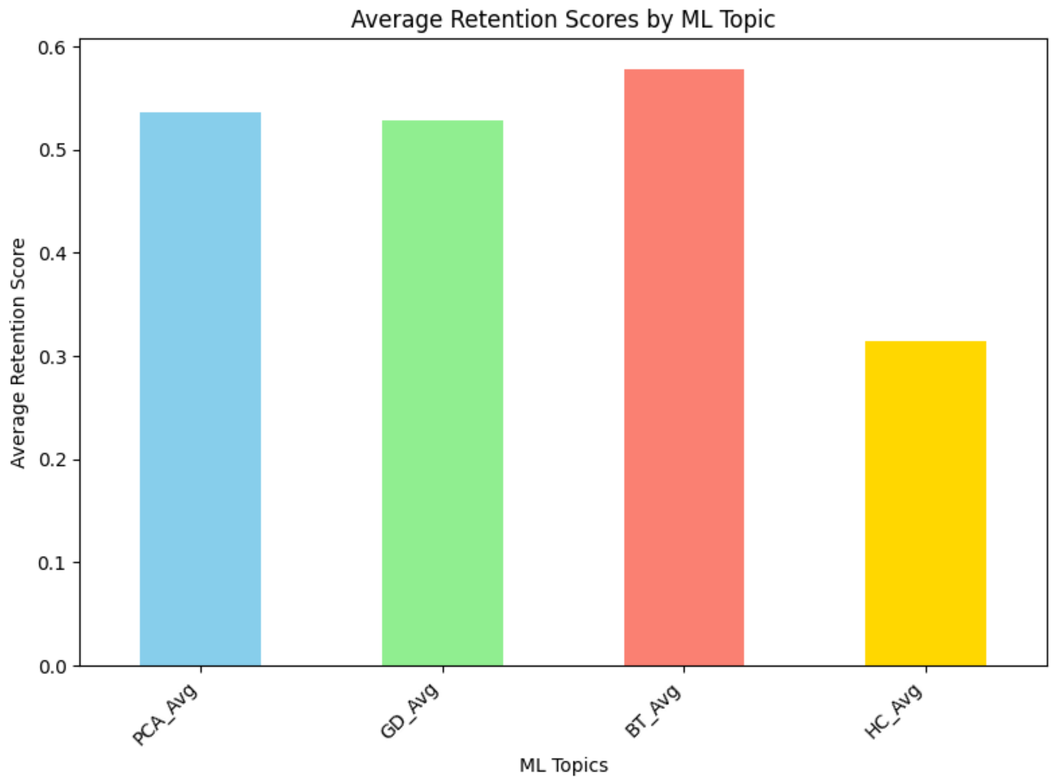


Figure 11: Average Retention Scores Across ML Topics

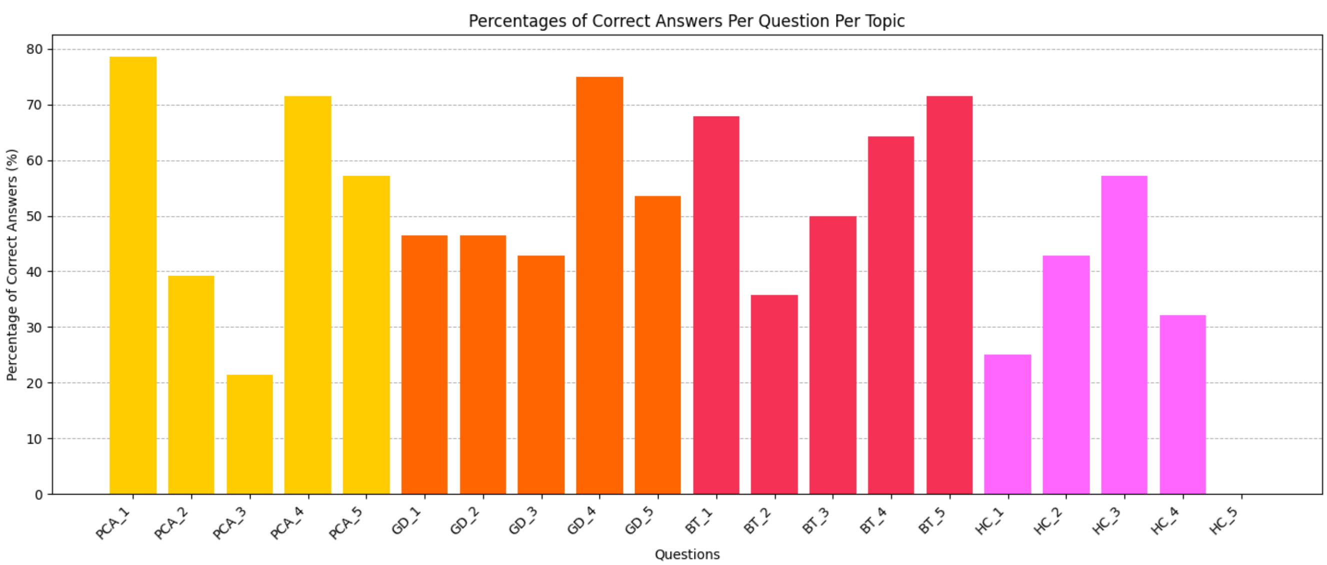


Figure 12: Percentages of Correct Answers Per Question Across ML Topics

B.4: Math-Linked Question Performance and Retention Implications

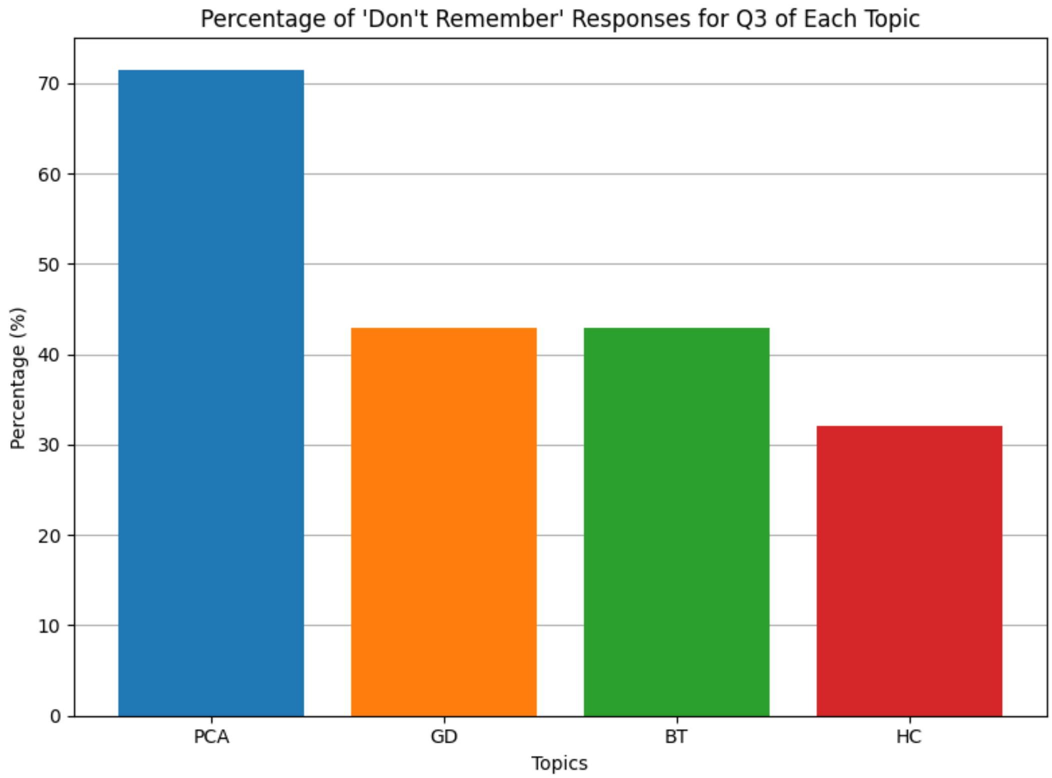


Figure 13: Percentage of “Don’t Remember” Responses for Question 3 Across ML Topics

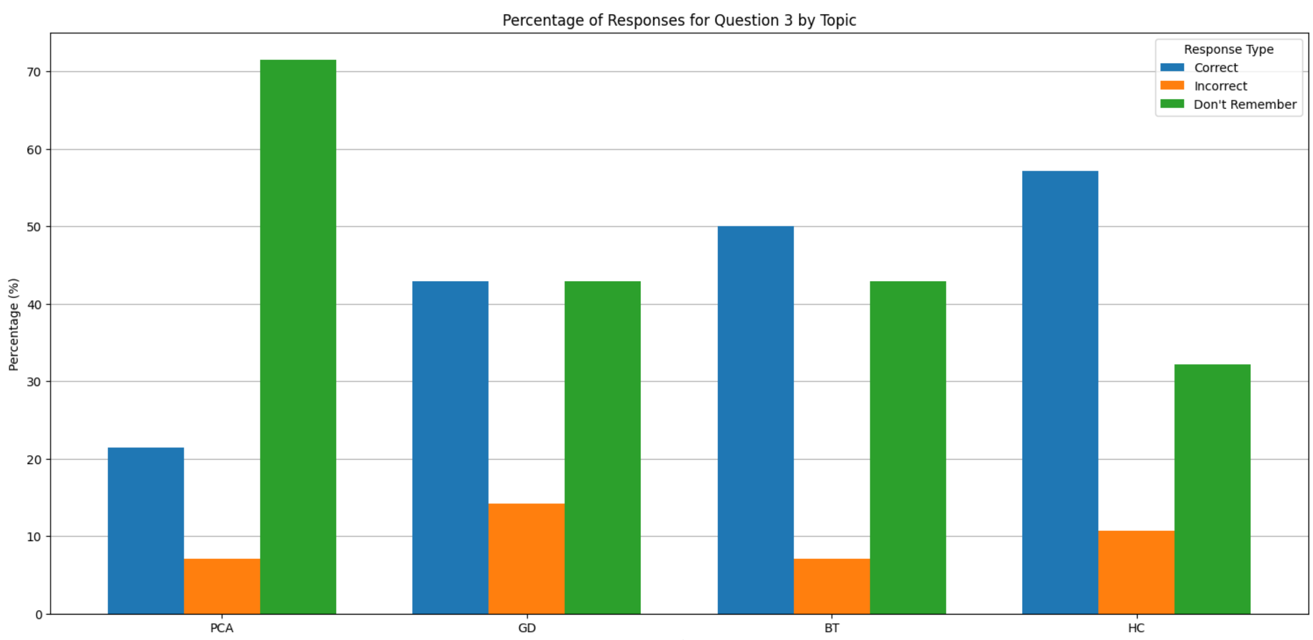


Figure 14: Percentage of Response Types (Correct, Incorrect, Don’t Remember) for Question 3 by ML Topic

B.5: Thematic Analysis of Open-Ended Responses – Additional Plots

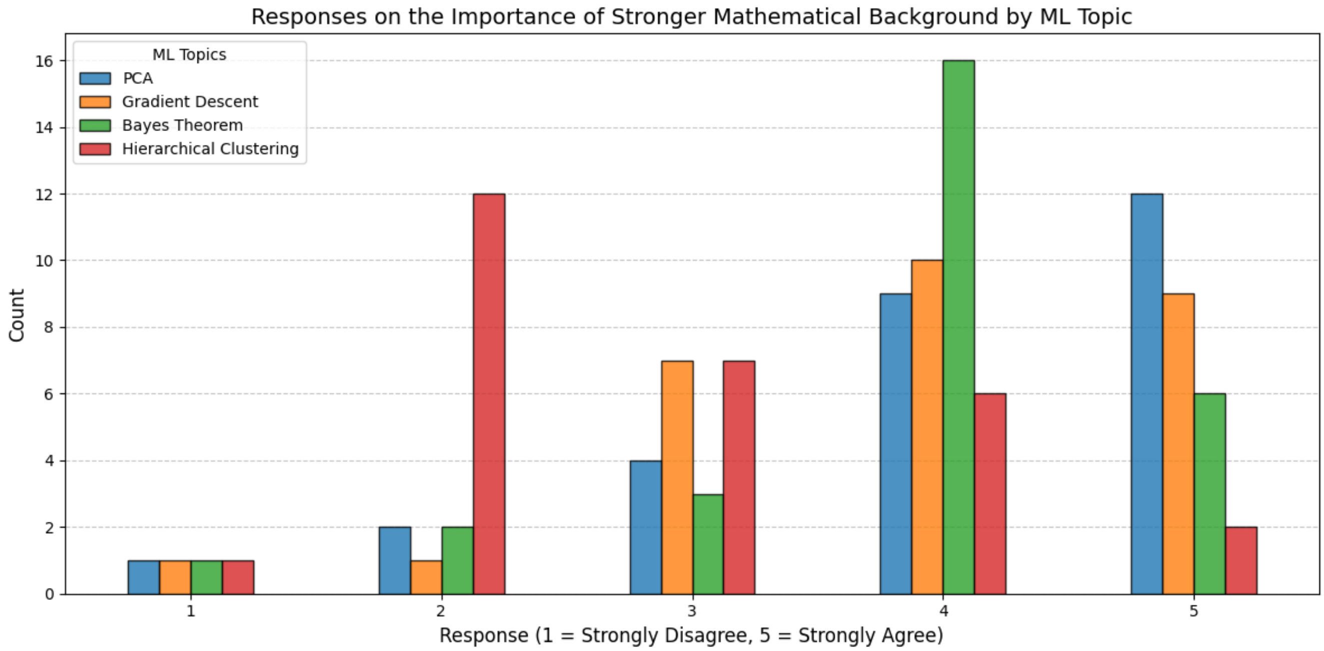


Figure 15: Responses on the Importance of a Stronger Mathematical Background by ML Topic

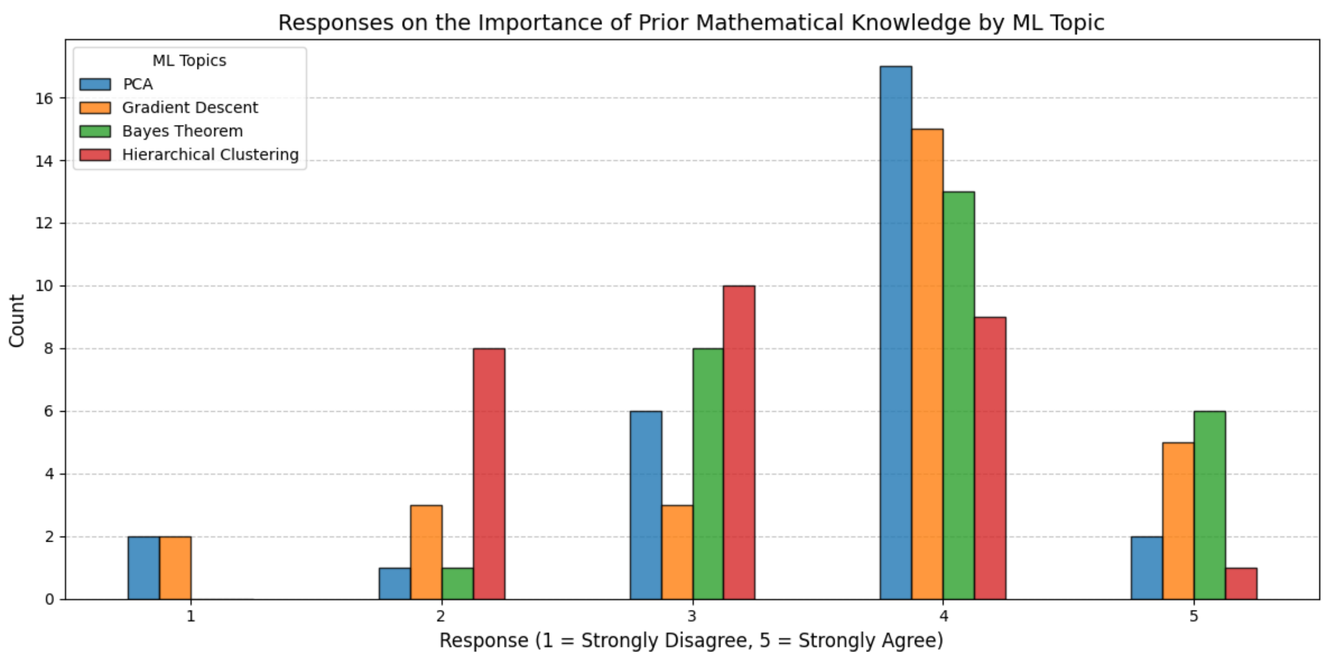


Figure 16: Responses on the Importance of Prior Mathematical Knowledge by ML Topic

B.6: Thematic Analysis of Open-Ended Responses – Additional Participant Responses for Identified Themes

Table 2: Key Themes and Representative Responses for Principal Component Analysis

Theme	Example Response
Need for Linear Algebra Refresher	"Eigenvector and eigenvalue calculation could be a nice refresher." "Would be nice to recap eigenvalues and their properties before this lecture." "As I was first learning it, I for sure needed a recap of how eigenvalues are calculated to understand how PCA works" "The calculation of eigenvalues and other linear algebra concepts used in the PCA algorithm are important to understand when learning about PCA. A refresher would be crucial." "A refresher of how to calculate covariance matrices, eigenvectors, and eigenvalues is needed."
Conceptual Understanding	"Since matrix multiplication is at the core of PCA, properly understanding what the math is actually doing helps a lot in conceptually understanding PCA." "I had to recap Linear Algebra to really understand the concept."
Practical Applications	"Maybe a practical rehearsal would be needed in order for an easier focus on proper concepts rather than the difficulty of calculations." "It is difficult to understand how to apply PCA in practical scenarios."

Table 3: Key Themes and Representative Responses for Gradient Descent

Theme	Example Response
Need for Calculus Refresher	"Calculating the gradient (via a derivative) and understanding that it is the slope was an essential part of understanding gradient descent, so having a refresher on that would be useful." "I believe GD is on the side of the harder topics we learned in the ML course, and I believe a Calculus refresher of partial derivatives would have made it easier to understand the calculations required." "Distance between Calculus and ML is 1 year, so a lot is forgotten." "Recapping the derivatives would have been useful." "A refresher in multi-variable/vector calculus would have been nice."
Insufficient Course Coverage	"The mathematical reasoning behind Gradient Descent was overlooked [...] I think a refresher on Calculus would have been useful at that point."
Practical Demonstrations	"An example of how it works helps students understand the concept." "A brief demonstration of the formula, showing how parameters are updated in the direction of the steepest decrease, can help clarify the intuition behind gradient descent and how the formula works."

Table 4: Key Themes and Representative Responses for Bayes' Theorem

Theme	Example Response
Need for Probability Refresher	"Bayes' equation is one of the most important equations when it comes to understanding the way ML works. If this is not understood by a student, it will definitely be reflected in the future understandings of ML concepts, leading to a snowball effect and lack in understanding. I believe that, although not a complex concept, it is important to mention it again." "Definitely a refresher is needed." "A small recap with a more emphasis on things required specifically in the context of machine learning would have been great." "I think a refresher would have helped since Bayes' Theorem is strongly tied to Probability and Statistics"
Forgetfulness of Concepts	"Probabilities can sometimes get a bit foggy as a concept, and a small refresher helps indeed." "I forgot the meaning of a lot of terms like likelihood and prior probabilities."
Practical and Conceptual Challenges	"A practical refresh would be a key aspect to get used to the calculations." "Also, more practical exercises would be more helpful than just stating the equation again. For me, even though I had a good understanding of it, in the beginning I struggled to see the applicability of it."
Terminology Confusion	"Getting a better grasp on the influence of posterior, prior probabilities in relation with likelihood." "What could be refreshed is the terminology around it: "posterior", "likelihood ratio" etc."
Simplicity of Formula	"Bayes' rule doesn't deal with any advanced statistical topics, so no refresher needed." "It is a simple formula - a refresher seems kinda useless." "The amount of probability theory used in Bayes' Theorem is very small, thus only basic concepts are needed. "

Table 5: Key Themes and Representative Responses for Hierarchical Clustering

Theme	Example Response
Simplicity of Mathematics	"The math behind hierarchical clustering was one of the easiest to understand." "The mathematical concepts used in clustering are not advanced, thus I see no refresher is needed." "Mathematically it's not complicated. For these clustering methods it is important to get the intuition right."
No Need for Refresher	"No advanced mathematical concepts are used in hierarchical clustering, so no recap needed." "A refresher will not improve the learning experience because no complex mathematical concepts are needed for this topic." "I do not think a refresher would have made it even easier to grasp."
Visualization and Logic	"A good spatial and logical thinking that can be achieved by having a good mathematical foundation helps visualize the concept better." "Being familiar with geometry helps in creating a visual image of the clustering process, making it easier to understand."
Subtler Mathematical Topics Needing a Refresher	"Placing more focus on the mathematical concepts used in hierarchical clustering [...] would help clarify linkage methods." "Yes, because the actual meaning behind measurements for within cluster points would have been more intuitive."

Appendix C: Template for a Topic-Specific Mathematical Refresher

The following structure outlines a flexible refresher module that instructors can adapt to their specific ML-related needs. It provides a comprehensive road-map for reinforcing essential mathematical concepts prior to introducing a given ML technique. While instructors should tailor the depth, examples, and exercises to suit their course objectives, the overall framework remains broadly applicable to any math-intensive ML topic.

1. Overview and Learning Objectives

- Title: Clearly indicate the specific topic (e.g., "Refresher on Eigenvalues and Eigenvectors").
- Objective:
  - Explain the purpose of the refresher.
  - Briefly state why this refresher is necessary and how it ties into the upcoming ML topic (e.g., "Eigenvalue and eigenvector concepts are integral to PCA. Understanding these will ease your grasp of dimensionality reduction.")
- Learning Goals: List a short set of specific, measurable objectives for the refresher.

2. Conceptual Overview

- Provide a concise explanation of the topic.
- Include:
  - Key Terminology: List the essential terms that students must understand (e.g., covariance matrix, eigenvalues, eigenvectors).
  - Definitions (e.g., eigenvalues and eigenvectors, their properties).
  - Intuition and Relevance (e.g., how eigenvalues relate to PCA in ML).
- If applicable, use diagrams or visual aids for better comprehension.

3. Step-by-Step Guide

- Breakdown of Core Techniques:
  - Divide the topic into manageable subtopics.
  - Example for Linear Algebra with PCA:
    * Step 1: Setting up the eigenvalue equation ($Ax = \lambda x$).
    * Step 2: Solving the characteristic equation ($|A - \lambda I| = 0$).
    * Step 3: Computing eigenvectors.
- Include worked examples with detailed explanations.
- Highlight common mistakes or tricky steps (e.g., forgetting to check matrix dimensions).

4. Practical Applications

- Demonstration of Use in ML:
  - Provide a real-world application of the topic in ML.
  - Example: "Applying eigenvalues to reduce dimensionality in PCA".
- Encourage platforms like Jupyter Notebooks to provide immediate feedback or hints.
- Include a simple Python implementation to illustrate concepts (e.g., numpy for matrix decomposition).

5. Practice Problems

- Objective: Allow students to reinforce learning.
- Include:
  - Basic Problems: Focused on fundamental calculations.
  - Intermediate Problems: Introduce context or combine subtopics.
  - Applied Problems: Tie the math back to an ML scenario.
- Provide solutions with detailed steps.

6. Summary and Key Takeaways

- Highlight:
  - Key formulas and properties (e.g., the eigenvalue equation).
  - Core insights (e.g., "Eigenvalues indicate the magnitude of principal directions").
  - Reinforce why a grasp of this math is pivotal for deeper understanding and better performance in the ML topic.

7. Final Assessment

- Objective: Measure improvement and readiness to apply concepts in ML.
- Methods could include:
  - A quiz containing a combination of multiple-choice, fill-in-the-blank, and short essay questions.
  - Coding exercises that incorporate both the math and its ML application.

8. Supplementary Resources

- Suggested readings, videos, or tools for further learning (e.g., links to tutorials on matrix algebra).

Appendix D: Generative AI Prompts

- Grammar and Style:
  - Can you please check this paragraph for grammar mistakes?
- Formatting in LaTeX:
  - Please format this table in LaTeX: $\langle data \rangle$.
  - How can I improve the layout of $\langle data \rangle$ in LaTeX?
  - Please help me format the position of these 2 images in LaTeX so that they appear next to each other: $\langle images \rangle$.
  - How can I fix this $\langle error \rangle$ in LaTeX?
- Visualizations in Python:
  - I want to visualize this $\langle data \rangle$ with a heatmap. Can you please provide the necessary code in Python to create the visualization?
  - How can I make the font of the labels on the x and y axes of my plots bigger?