Autoregressive Spectral Estimation by Application of the Burg Algorithm to Irregularly Sampled Data

Robert Bos, Stijn de Waele, and Piet M. T. Broersen

***Abstract*—Many methods have been developed for spectral analysis of irregularly sampled data. Currently, popular methods such as Lomb–Scargle and resampling tend to be biased at higher frequencies. Slotting methods fail to consistently produce a spectrum that is positive for all frequencies. In this paper, a new estimator is introduced that applies the Burg algorithm for autoregressive spectral estimation to unevenly spaced data. The new estimator can be perceived as searching for sequences of data that are almost equidistant, and then analyzing those sequences using the Burg algorithm for segments. The estimated spectrum is guaranteed to be positive. If a sufficiently large data set is available, results can be accurate up to relatively high frequencies.**

***Index Terms*—AR spectral estimation, covariance estimation, irregular sampling, turbulence spectra.**

I. Introduction

SPECTRAL analysis of stationary stochastic signals is used in various fields. In some situations, the data to be examined will be irregularly sampled. Examples of irregularly sampled data are astronomical data and turbulence data as observed by Laser–Doppler anemometry.

Many techniques of spectral estimation for unevenly spaced data have been developed. Currently, most popular techniques fall into one of three categories, namely slotting techniques, resampling methods, or variations of the Lomb–Scargle estimator.

Slotting methods estimate an equidistant covariance function from the irregularly sampled data. Many variants of slotting algorithms currently exist, among which are slotting with local normalization and fuzzy slotting [1]. Local normalization reduces the variance of the estimated covariance function. Fuzzy slotting produces a smoother covariance function by distributing products over multiple time slots. Covariance functions as estimated by slotting techniques are usually not positive semi-definite. This results in spectra that can become negative at a large percentage of the estimated frequencies. This effect can sometimes be reduced by using creative windowing schemes such as variable windowing. The variable windowing technique takes a wide covariance window at low frequencies and a smaller one at higher frequencies. The effectiveness of such schemes, however, is dependent on the true characteristics of the data and therefore cannot be used generally [2].

The second class of estimators resamples the data on an equidistant time grid. After resampling, the data can then be analyzed using the periodogram or time series models. While resampling methods can be used to obtain an estimated spectrum that is guaranteed to be positive, results at higher frequencies can be severely biased. An example of a popular resampling method is sample and hold resampling [3]. Results of sample and hold reconstruction can be described as low-pass filtering followed by adding colored noise. These effects can be eliminated using the refined sample and hold estimator [4]. This refined estimator, however, does not guarantee a spectrum that is positive. As a result, the estimated spectra are often negative at higher frequencies.

The third category consists of estimators that use variations of the Lomb–Scargle estimator. Lomb–Scargle directly estimates a spectrum from the irregularly sampled data [5], [6]. The Lomb–Scargle spectrum can be completely alias free and reduces to the periodogram if the data happen to be regularly sampled. It can be perceived as a least squares fit of sines and cosines to the irregularly sampled data. The Lomb–Scargle method is guaranteed to produce a spectrum that is positive. It is able to accurately detect peaks up to very high frequencies in a relatively low noise environment, but fails to accurately describe any slopes in the spectrum.

In this paper, a new estimator is introduced, which attempts to directly fit an autoregressive time series model to the unequally spaced data, using a modified version of the Burg algorithm for segments.

II. Burg for Segments

An autoregressive (AR) model is a possible method of modeling a stationary stochastic process. Given the signal $x(n)$, the autoregressive model can be written as

$$x(n) + a_1 x(n-1) + \cdots + a_L x(n-L) = \varepsilon(n) \qquad (1)$$

where $\varepsilon(n)$ is an independent identically distributed stochastic sequence with zero mean and variance σ_ε^2. The coefficients a_i are called the autoregressive parameters of the model, and L is the model order. It can be shown that almost any stationary stochastic process can be modeled as an unique AR(∞) process, independent of the origin of the process. In practice, finite order AR(L) processes are sufficiently accurate because higher order parameters tend to become small and not significant for estimation. Once such a model of the process is available, its spectrum can be easily computed

$$h(\omega) = \frac{\sigma_\varepsilon^2}{|1 + a_1 e^{-i\omega} + \cdots + a_L e^{-iL\omega}|^2}. \qquad (2)$$

Experience with equidistant data shows that spectra that have been estimated using AR models can be more accurate than the

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best possible windowed periodogram. If this accuracy is not good enough, an estimated AR model can also be used as a basis for estimating the additional moving average (MA) model or even the combined autoregressive moving average (ARMA) models that can increase the accuracy even further.

One possible method to estimate the autoregressive coefficients a_i from equidistant data is the Burg algorithm [7]. The Burg algorithm does not directly estimate the parameters a_i, but instead it estimates reflection coefficients k_i. The pth reflection coefficient k_p is a measure of the correlation between $x(n)$ and $x(n-p)$ after the correlation due to the intermediate observations $x(n-1)\cdots x(n-p+1)$ has been filtered out. Reflection coefficients can be transformed into autoregressive parameters by applying the Levinson–Durbin recursion formulas [8].

The Burg algorithm is a recursive algorithm. Specifically, this means in the pth step of the algorithm reflection coefficient k_p is estimated while the previous coefficients $k_1 \ldots k_{p-1}$ remain fixed. The Burg algorithm can be adapted to estimate the reflection coefficients from multiple segments of data. Suppose you have M segments of data, stored as $x^0(n)$ up to $x^{M-1}(n)$, generated by the same stationary stochastic process, but not necessarily of equal length. The first reflection coefficient k_1 can now be estimated using the following equation, with $p=1$ and $f_0^s(n) = b_0^s(s) = x_0^s(n)$

$$k_p = \frac{-2\sum\limits_{s,n} f_{p-1}^s(n) b_{p-1}^s(n-p)}{\sum\limits_{s,n} f_{p-1}^s(n)^2 + \sum\limits_{s,n} b_{p-1}^s(n-p)^2}. \tag{3}$$

New functions $f_p^s(n)$ and $b_p^s(t)$ can then be computed using

$$\begin{aligned} f_p^s(n) &= f_{p-1}^s(n) + k_p b_{p-1}^s(n-p) \\ b_p^s(n) &= b_{p-1}^s(n) + k_p f_{p-1}^s(n+p). \end{aligned} \tag{4}$$

The new functions $f_p^s(n)$ and $b_p^s(n)$, called the forward and backward prediction errors, can then be used to estimate a new reflection coefficient k_{p+1}. After the final parameter k_L has been computed, an AR(L) model can be calculated by applying the Levinson–Durbin formulas. A more complete and detailed description of the Burg algorithm for segments can be found in [9].

This algorithm is particularly useful for estimatng coefficients from segments of unequal length. Suppose you are trying to estimate an AR model of order 20 when you have 10 segments with 10 observations and one longer segment of 100 observations. Using the Burg algorithm for segments, the first reflection coefficients k_1 up to k_9 can be estimated using all segments while reflection coefficients k_{10} up to k_{20} can be estimated from the segment of 100 observations. This example shows that the Burg algorithm for segments uses all information contained in all segments efficiently.

As can be seen from (3) all reflection coefficients are guaranteed to have an absolute value that is smaller than 1. This means that all AR models estimated using the Burg Algorithm are guaranteed to be stationary.

III. Burg for Irregularly Sampled Data

Suppose you are trying to estimate a spectrum up to a maximum frequency of $(2T)^{-1}$. To apply the Burg algorithm of (3) and (4), you would need data sequences whose sampling interval is exactly T. In unequally spaced data, however, the probability of finding two points spaced exactly T apart, is zero. There is, however, a finite probability of finding two points spaced ($T \pm 1/2\Delta\tau$) apart. To exploit this information in the data, (3) can be modified to

$$\begin{aligned} k_p = \frac{-2\sum f_{p-1}(t_1) b_{p-1}(t_2)}{\sum f_{p-1}(t_1)^2 + \sum b_{p-1}(t_2)^2} \\ \forall\, (t_1 - t_2) \in \left\langle pT - \tfrac{1}{2}\,\Delta\tau;\; pT + \tfrac{1}{2}\,\Delta\tau \right\rangle. \end{aligned} \tag{5}$$

In the first step of the redefined algorithm, both $b_0(t)$ and $f_0(t)$ are again set to the irregularly sampled signal $x(t)$. In following steps, only those data points that have contributed to the estimation of k_p can possibly be used in estimating the next reflection coefficient k_{p+1}. The forward and backward prediction errors, $f_p(t)$ and $b_p(t)$, that are required in the next steps of the algorithm, can then be constructed by

$$\begin{aligned} f_p(t_2) &= f_{p-1}(t_2) + k_i b_{p-1}(t_1) \\ &\qquad \forall\, t_2 - t_1 \in \left\langle pT - \tfrac{1}{2}\,\Delta\tau;\; pT + \tfrac{1}{2}\,\Delta\tau \right\rangle \\ b_p(t_1) &= b_{p-1}(t_1) + k_i f_{p-1}(t_2) \\ &\qquad \forall\, t_2 - t_1 \in \left\langle pT - \tfrac{1}{2}\,\Delta\tau;\; pT + \tfrac{1}{2}\,\Delta\tau \right\rangle. \end{aligned} \tag{6}$$

If there are multiple predictions of $f_p(t)$ or $b_p(t)$ at any given point in time, these multiples can be eliminated by choosing that prediction that minimizes $|t_2 - t_1 - pT|$. It can be shown the redefined algorithm of (5) and (6) will reduce to Burg for segments if the data to be examined happen to be regularly sampled.

The redefined algorithm can be perceived as trying to find sequences of irregular data points that are spaced "almost regularly." Whether or not a set of irregular data points is spaced "almost regularly" is determined by the $\Delta\tau$ parameter, which is called the slot width. All of the "almost regularly" spaced sets (or segments) found are then combined, using the same method as is used in Burg's algorithm for segmented data, to estimate the reflection coefficients k_p. One of the properties of the new algorithm is that the estimated reflection coefficients are guaranteed to have an absolute value that is smaller than one. This means that all estimated models are automatically stationary. Also, the estimated power spectra are guaranteed to be positive [8].

Another property of the new algorithm is that the number of products in the numerator of (5) will reduce very fast. This can be explained by realizing that the probability of finding segments that are "almost equidistant" decreases roughly exponentially as the segment length grows. Therefore, to estimate the high-order AR model the number of irregular observations should be very large.

Choosing the value for the slot width can influence the rate at which the number of products decreases. The high value of $\Delta\tau$ will result in a slow decrease, whereas a low value of $\Delta\tau$ will cause a rapid decline in the number of products. An example on how fast the number of products decreases for different values of the slot width, can be found in Fig. 1, where the average number of products available for estimating a parameter k_p is plotted.

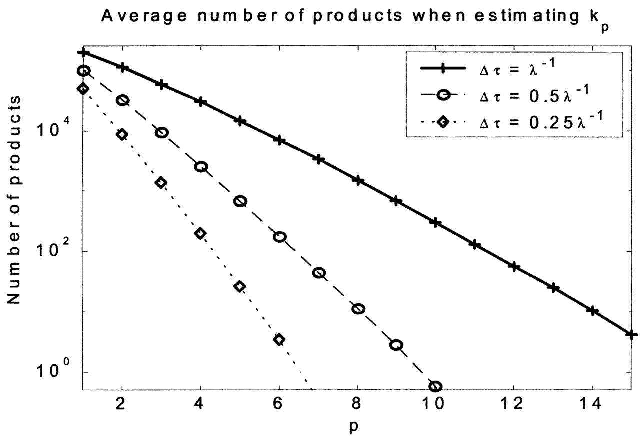

Fig. 1. Average number of products when estimating a parameter k_p from 200,000 irregular observations, using different values of $\Delta\tau$. The average number of observations per time unit is λ.

Fig. 1 has been obtained using 100 runs with 200,000 irregular observations and a constant average number of observations per time unit, λ. The observation instances are uniformly distributed over the observation period. The figure is independent of any other data properties. It can be seen quite clearly that a smaller slot width reduces the number of products much faster than a larger slot width.

In equidistant time series analysis, the variance v_p of estimated parameter k_p, using Burg's method, can be approximated by

$$v_p = \frac{1}{N_p + 1} \tag{7}$$

where N_p is the number of products used to estimate parameter k_p. This empirical result can be found using finite sample theory for AR estimation [10]. When estimating parameters in irregularly sampled data, the variance v_p, is still described accurately using the relation above.

Choosing the value of $\Delta\tau$ not only influences the variance of the estimated parameters, but also the bias. A large value of $\Delta\tau$ will result in parameters that have a low variance, but a high bias. Similarly, choosing a small value for $\Delta\tau$ will result in a model that has a greater variance, but a lower bias.

An example on how the value of $\Delta\tau$ determines the bias of an estimated model can be found in Fig. 2. In Fig. 2, three AR(5) models have been estimated using exactly the same data. It can be seen that the model with the smallest slot width has the smallest bias.

Since there is usually no aliasing in irregularly sampled data, there is no natural limit for the maximum frequency that can be estimated. The maximum frequency of the model to be computed can therefore be selected freely by choosing a value for T. However, describing a spectrum up to higher frequencies could, for example, mean that a declining slope goes down deeper, or extra peaks in the spectrum could be found. In the first example, a lower bias level is required; in the second scenario the higher order AR model is needed. This means that estimating a spectrum up to higher frequencies usually demands more irregularly sampled data.

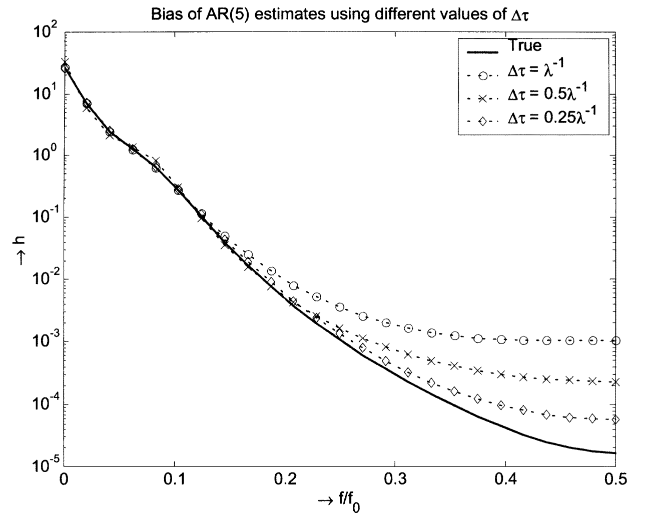

Fig. 2. Bias of AR(5) models using different slot widths, but the same 200,000 irregular observations.

IV. Order Selection

After a number of AR(L) models have been estimated, the user is left with the problem of how to select the best model order. The best order can be estimated by selecting that order which minimizes an order selection criterion. A popular selection criterion for equidistant data is Akaike's information criterion (AIC) [11]. AIC (p) is given by

$$AIC(p) = \ln(RES(p)) + \frac{2p}{N} \tag{8}$$

where RES(p) is equal to the residual variance of the model of order p and N is the number of equidistant observations. The value of 2 in the expression above is often called the penalty factor. RES(p) can be calculated by

$$RES(p) = \sigma_x^2 \prod_{i=1}^{p} (1 - k_i^2) \tag{9}$$

where σ_x^2 is the variance of the irregularly sampled signal $x(t)$. The factor $2p/N$ in the AIC criterion is based on the fact that the variance of each extra selected parameter is asymptotically equal to $1/N$. Using irregularly sampled data, however, the number of products tends to decrease roughly exponentially. To compensate for this, AIC can be modified to

$$AIC_{\text{mod}}(p) = \ln(RES(p)) + 2\sum_{i=0}^{p} \frac{1}{N_i} \tag{10}$$

where N_i is the number of products in the numerator of (5) that were used when estimating parameter k_i.

If this modified AIC criterion is applied to unevenly spaced data, the model order is regularly selected too high. This effect can be partially explained by analyzing the expectation and variance of the AIC criterion for irregularly sampled data [12].

By making two modifications to the order selection criterion, the problem of selecting too high orders can be reduced. The first modification is to raise the penalty factor. By raising the penalty factor, the probability that a model is selected with an order higher than the optimal order reduces, possibly at the cost of some bias.

The second change to the criterion should prevent the selection of AR models that have parameters that have been estimated with less than 15 observations. The reason for this is, that simulations show that parameters that have been estimated with less than 15 observations tend to behave erratically.

Applying the said modifications to (8) results in the following new criterion:

$$\begin{aligned} AIC_{irreg.}(p) &= \ln(RES(p)) + \alpha\sum_{i=0}^{p}\frac{1}{N_i} \quad &\text{if } N_p \geq 15 \\ AIC_{irreg.}(p) &= \infty &\text{if } N_p < 15 \end{aligned} \tag{11}$$

in which α is larger than 2.

V. Performance of the New Estimator

The performance of the new estimator has been tested in simulations. The simulated data had a spectrum consisting of two rapidly declining slopes. The first slope descends at a rate of $\sim f^{-5/3}$ up to 0.1 f_0, and then descends at a rate of $\sim f^{-7}$ from that point onwards, where f_0 is the average data rate. This type of spectrum is representative of turbulence data. To test the ability of the new estimator to detect peaks, a second set of simulations was performed, with peaks added to the simulated spectra.

In equidistant spectral analysis, estimates are usually given for frequencies up to half the mean data rate, due to aliasing constraints. When analyzing unequally spaced data, however, often no aliasing can occur. Therefore, the new estimator was tested not only up to the usual $0.5f_0$, but also up to $0.25f_0$ and f_0. All simulations were done using 200,000 irregular observations.

Test data has been generated using the following procedure. First $100N$ equidistant data points were generated using the high-order AR process. Then, randomly $99N$ data points were discarded. Each data point had a probability of 99/100 to be discarded. This means that effects such as the so-called velocity bias were not simulated. The resulting data was nonequidistant, and time intervals between observations were roughly distributed exponentially.

The Burg method for unequally spaced data was used, employing slot widths of $0.25\lambda^{-1}$, $0.5\lambda^{-1}$ and λ^{-1}, where λ is the average number of irregular observations per time unit. Both AIC$_{\mathrm{mod}}$ and AIC$_{irreg.}$ (using penalty $\alpha = 4$) were used for order selection. As a reference for the performance of the new estimator, results of the sample and hold reconstruction method are given as well. The spectra of the sample and hold data have been estimated using the ARMAsel time series analysis algorithm [13]. Results were not compared with the refined sample and hold estimator, since the refined estimator is not guaranteed to be positive, especially at higher frequencies.

Since in simulations the true properties of the data are known, the quality of estimated results can be established. A quality measure for the fit is the aliased model error, ME$_T$ [14]. ME$_T$ is defined as

$$ME_T = N\left[\frac{PE_T}{\sigma^2_{\varepsilon,T}} - 1\right]. \tag{12}$$

In the equation, above N is the number of irregular observations, PE$_T$ is the expectation of the prediction error of the estimated model using new data at sampling interval T, and $\sigma^2_{\varepsilon,T}$ is the minimal prediction error of the true model at sampling interval T. The model error consists of bias and variance contributions. The bias contribution scales proportional with the number of observations N. The expected variance of the estimated parameters contributes N/N_p per estimated parameter, where N_p is the number of products used in the numerator of (5) to estimate the parameter. The model errors for the estimated spectra were averaged over 100 runs. The results can be found in Tables I and II. Table I presents the averaged model errors of the spectra without a peak; Table II presents the averaged results when the true spectrum contained a peak.

TABLE I

Aliased Model Errors (ME$_T$) of the Estimated Spectra Without a Peak, Up to Frequencies of $f_0/4$, $f_0/2$ and f_0 Using 200 000 Irregular Observations. Results Were Averaged Over 100 Runs

	$f_{max} = 0.25f_0$	$f_{max} = 0.5f_0$	$f_{max} = f_0$
Sample and Hold	$1.2 \cdot 10^5$	$13 \cdot 10^5$	$128 \cdot 10^5$
Burg Irregular, AIC$_{mod}$ (pen 2)			
$\Delta\tau = \lambda^{-1}$	$1.8 \cdot 10^5$	$5.2 \cdot 10^5$	$20 \cdot 10^5$
$\Delta\tau = 0.5\lambda^{-1}$	$1.7 \cdot 10^5$	$1.7 \cdot 10^5$	$16 \cdot 10^5$
$\Delta\tau = 0.25\lambda^{-1}$	$0.60 \cdot 10^5$	$0.60 \cdot 10^5$	$7.4 \cdot 10^5$
Burg Irreg, AIC$_{irreg}$ (pen α=4)			
$\Delta\tau = \lambda^{-1}$	$0.19 \cdot 10^5$	$2.4 \cdot 10^5$	$17 \cdot 10^5$
$\Delta\tau = 0.5\lambda^{-1}$	$0.085 \cdot 10^5$	$0.84 \cdot 10^5$	$12 \cdot 10^5$
$\Delta\tau = 0.25\lambda^{-1}$	$0.083 \cdot 10^5$	$0.30 \cdot 10^5$	$5.3 \cdot 10^5$

TABLE II

Aliased Model Errors (ME$_T$) of the Estimated Spectra Containing a Peak, Up to Frequencies of $f_0/4$, $f_0/2$ and f_0 Using 200 000 Irregular Observations. Results Were Averaged Over 100 Runs

	$f_{max} = 0.25f_0$	$f_{max} = 0.5f_0$	$f_{max} = f_0$
Sample and Hold	$0.32 \cdot 10^5$	$3.0 \cdot 10^5$	$19 \cdot 10^5$
Burg Irregular, AIC$_{mod}$ (pen 2)			
$\Delta\tau = \lambda^{-1}$	$1.54 \cdot 10^5$	$2.3 \cdot 10^5$	$6.8 \cdot 10^5$
$\Delta\tau = 0.5\lambda^{-1}$	$0.86 \cdot 10^5$	$0.96 \cdot 10^5$	$3.4 \cdot 10^5$
$\Delta\tau = 0.25\lambda^{-1}$	$0.89 \cdot 10^5$	$0.87 \cdot 10^5$	$2.1 \cdot 10^5$
Burg Irreg, AIC$_{irreg}$ (pen α=4)			
$\Delta\tau = \lambda^{-1}$	$0.22 \cdot 10^5$	$0.69 \cdot 10^5$	$6.2 \cdot 10^5$
$\Delta\tau = 0.5\lambda^{-1}$	$0.081 \cdot 10^5$	$0.28 \cdot 10^5$	$2.3 \cdot 10^5$
$\Delta\tau = 0.25\lambda^{-1}$	$0.47 \cdot 10^5$	$0.51 \cdot 10^5$	$1.7 \cdot 10^5$

When estimating up to a frequency of $1/4f_0$ (Figs. 3 and 4), the Burg irregular method already performs better than sample and hold. In spectra containing only slopes, it can be seen that the bias of the Burg method is not significant, even at the largest slot width λ^{-1}. Therefore, the results using different slot widths are similar. When a peak is added to the spectrum at $0.18f_0$, the performance of Burg irregular with slot width $0.25\lambda^{-1}$ is relatively poor. This can be understood by realizing that using

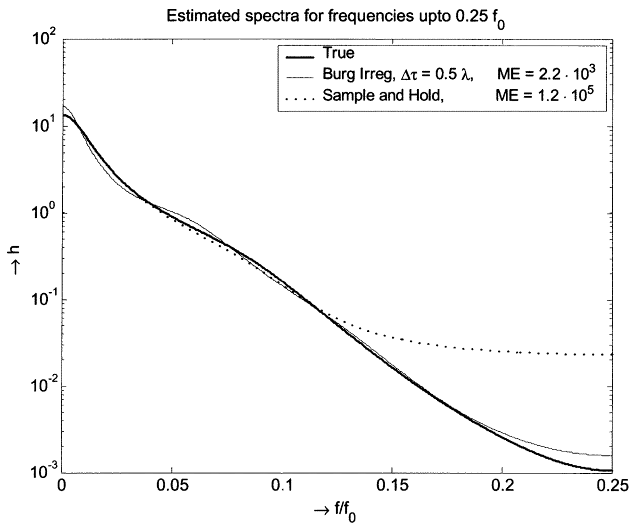

Fig. 3. True Spectrum and the selected spectra up to 0.25 f_0 using 200,000 irregular observations, using sample and hold reconstruction and Burg irregular model ($\Delta\tau = 0.5\lambda^{-1}$).

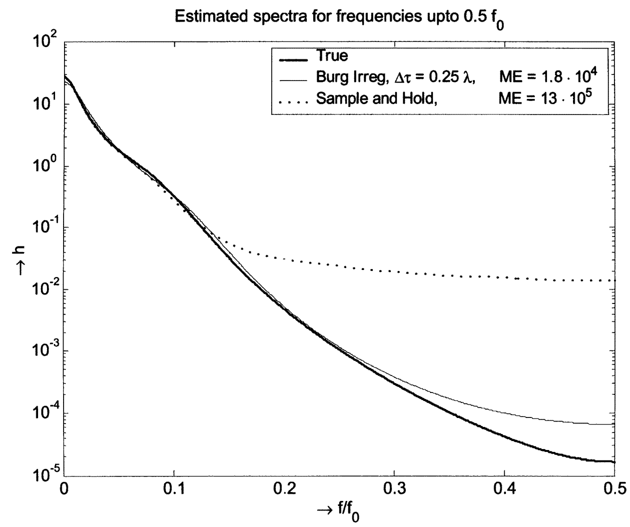

Fig. 5. True spectrum and the selected spectra up to $0.5f_0$ using 200,000 irregularly sampled observations, using sample and hold reconstruction and Burg irregular ($\Delta\tau = 0.25\lambda^{-1}$).

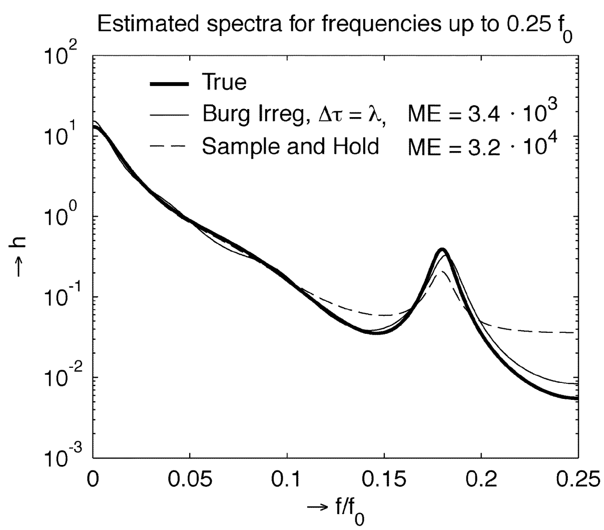

Fig. 4. True spectrum and the selected spectra up to 0.25 f_0 using 200,000 irregular observations, using sample and hold reconstruction and Burg irregular model ($\Delta\tau = \lambda^{-1}$).

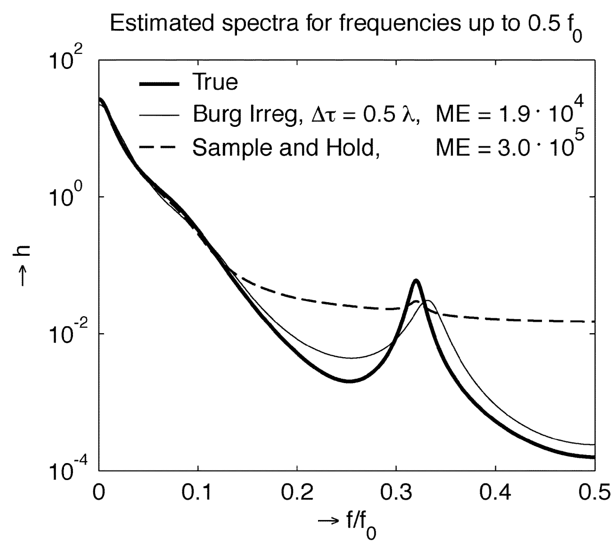

Fig. 6. True spectrum and the selected spectra up to $0.5f_0$ using 200,000 irregularly sampled observations, using sample and hold reconstruction and Burg irregular ($\Delta\tau = 0.5\lambda^{-1}$).

this slot width means that on average only four parameters can be reliably estimated. This is simply not enough to describe both peak and slopes; thus bias is introduced. Using $\Delta\tau = 0.5\lambda^{-1}$ or $\Delta\tau = \lambda^{-1}$, results remain good because enough parameters can be estimated to describe both the peak and the slopes.

When estimating up to $0.5f_0$ (the usual frequency in the equidistant case), the new estimator performs much better than sample and hold reconstruction (Figs. 5 and 6). This result is caused by the fact that, even at a slot width of λ^{-1}, the bias of the Burg irregular method is much smaller than the bias in the spectrum of the sample and hold technique. The optimum slot width for Burg irregular is $\Delta\tau = 0.25\lambda^{-1}$ for the spectrum without peak. The reason for this is that only a few parameters are required to describe such a spectrum. A small slot width will then have the advantage because of its lower bias. When a peak at $0.32f_0$ is added, the optimum slot width is $0.5\lambda^{-1}$. A slot width of $0.25\lambda^{-1}$ performs relatively poorly for the same reasons as mentioned before. A slot with of λ^{-1} allows enough parameters to be estimated to describe the peak accurately, but the larger bias of the larger slot width causes the result to be somewhat less accurate.

Results from estimates up to the mean data rate (Figs. 7 and 8), again show that Burg irregular outperforms sample and hold reconstruction. Both estimators, however, fail to describe the slope properly at higher frequencies. Results of the Burg estimator could have been better if more unequally spaced observations had been available. This would have allowed the usage of smaller slot widths that have a lower bias.

When a peak is added, the Burg irregular estimate is not able to describe the peak accurately; high slot widths are too biased, and low slot widths do not allow for enough parameters to be estimated for an accurate description of the extra peak.

In all frequency ranges, for spectra with or without peak, it can be seen that order selection using $\mathrm{AIC}_{irreg.}$ performs better than $\mathrm{AIC}_{\mathrm{mod}}$, see Tables I and II. This is especially true for

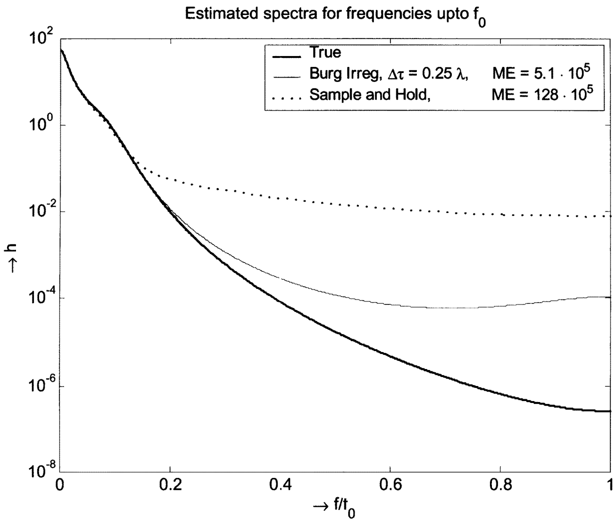

Fig. 7. True spectrum and the selected spectra up to f_0 using 200,000 irregularly sampled observations, using sample and hold reconstruction and Burg irregular ($\Delta\tau = 0.25\lambda^{-1}$).

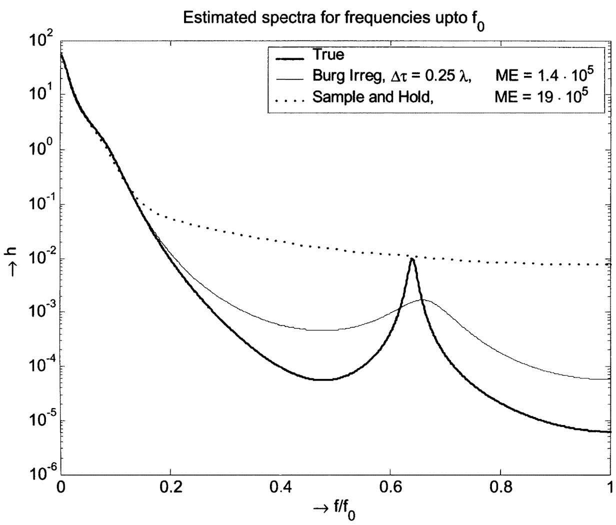

Fig. 8. True spectrum and the selected spectra up to f_0 using 200,000 irregularly sampled observations, using sample and hold reconstruction and Burg irregular ($\Delta\tau = 0.25\lambda^{-1}$).

those situations where the average model error is relatively small (average $\mathrm{ME}_T < N$). This can be explained by realizing that order selection only helps to reduce the variance contribution to the model error. If the model error is mainly caused by bias, the reduction in model error will be less significant. Although AIC_{irreg} already performs better than $\mathrm{AIC}_{\mathrm{mod}}$, the best model order is often not selected. This means that the performance could be improved by using a better selection criterion.

VI. Conclusion

A new estimator is introduced that fits an AR model directly to irregularly sampled data. The new estimator combines a spectrum that is guaranteed to be positive with accurate results at higher frequencies. Results are often better than those that can be obtained by existing techniques.

Order selection for the Burg irregular algorithm requires new order selection criteria. A modified AIC criterion has been presented that performs better than the standard AIC criterion.

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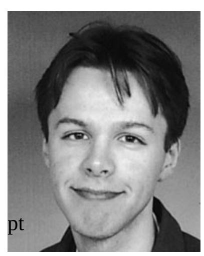

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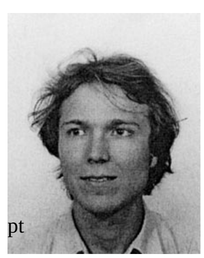

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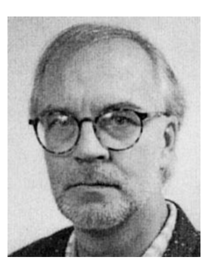

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