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# On-bottom Stability of Pipelines A Safety Assessment

by

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Extract of the introductory part from Sotberg (1990): "Application of Reliability Methods for Safety Assessment of Submarine Pipelines", Dr.ing Thesis, Division of Marine Structures, NTH, Trondheim. The enclosed conference publications B to E covers the remaining topics in the Thesis to some extent.

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## B

Sotberg, T. (1990): "Application of Probabilistic Methods for Calibration of Submarine Pipeline Design Criteria", in Proc. First European Offshore Mechanics Symp., Trondheim.

## C

Sotberg, T., Leira, B.J., Larsen, C.M. and Verley, R.L.P. (1990): "On the Uncertainties related to Stability Design of Submarine Pipelines". Int. Conference on Offshore Mechanics and Arctic Engn., Vol. V, Houston.

## D

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## E

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## INTRODUCTION

Offshore pipelines represent a strategic part of the infrastructure of offshore oil/gas fields. It is important that the design of pipelines is based on an adequate safety margin due to the high cost associated with failure and repair. In general it seems that the pipeline industry has been successful in the sense that most pipelines have had a satisfactory performance during their lifetime. The average safety level for existing offshore pipelines is believed to be adequate considering the low failure rates experienced.

However, recent years of research and project experience has identified limitations in design procedures both related to the physical description as well as due to a large variety and inconsistency in the interpretation of existing codes. There have also been reported some expensive and serious failures (Simpson, 1983 among others). It is thus expected that these problems have lead to a large spread of the actual safety level in which a large proportion of pipeline designs are too conservative and thus cost ineffective and some are non-conservative and unsafe. These observations are related to limitations in design codes in which some technical topics and associated failure modes are not covered or may be represented in a too simple or inaccurate way.

A central part of the pipeline design process is the determination of steel pipe diameter and thickness and the weight of concrete coating. The pipe diameter is determined on the basis of the amount of oil or gas to be transported and pipe thickness is in general calculated based on the internal pipe condition (internal pressure). The amount of concrete cover is related to the necessary weight to secure a satisfactory performance of the as laid pipeline and the limitations due to the installation process. The installation process may limit the total pipe weight so that burial or other intervention work may be necessary immediately after laying. Application of reliability methods in the design process to modify design criteria may, however, reduce the intervention work significantly and then also improve the project economy, (Bruschi and Blaker, 1990).

The recent developments related to on-bottom submarine pipelines have led to a redefinition of the design practice. The revised design philosophy reported by Sotberg et al. (1988 and 1989b), allows for limited movements of the pipeline during extreme environmental conditions. Previously, pipeline stability was based on a simple balance between external hydrodynamic forces and soil reaction

forces.

When considering a pipeline design based on the revised criteria, i.e. allowed movements, the strength limit state needs to be evaluated. Allowing pipeline movements under extreme environmental conditions implies that the stress condition at constrained points along the pipeline has to be checked to verify a satisfactory design. This means that the new design philosophy needs to consider several failure modes such as excessive pipeline movements, yielding, excessive straining and local buckling. In this way, the relaxed design criterion with respect to pipeline stability introduces a need for some additional design controls as compared to the traditional procedure. However, the benefit from this will be a more cost optimal design based on a thorough safety evaluation.

Current design practice for offshore pipelines is mainly based on application of guidelines and codes according to technology developed during the Seventies. These codes are not developed to the same level of completeness as those for fixed offshore structures and do not represent the more recent technological developments.

The main objective for a code writer is to ensure that the design recommendations given represent a satisfactory safety margin with respect to all relevant failure modes for the whole range of pipeline scenarios. A problem concerning pipeline engineering and the application of traditional design codes is, however, the difficulty in quantification of safety levels related to the design. Application of current design rules gives no indication of safety margins against relevant failure modes as the variability in the level of loading, pipeline properties, structural behaviour and pipeline strength are not properly taken into account. The result is an overall conservative and cost ineffective design. The only rational way to improve this situation is to apply structural reliability methods or alternatively use a reliability based design check where design procedures are tailored to different pipeline scenarios.

The main objective of this presentation is to illustrate the main results from recent years of research and project experience into the design process for submarine pipelines in order to improve and refine methods and design procedures. On this technical basis, efficient reliability calculation methods should be developed tailored to different applications. The reliability calculation procedures should be used in the development and calibration of design recommendations with the main aim of obtaining a balanced and uniform safety level taking into account basic uncertainties and failure consequences. (See enclosed papers).

The organization of the document is indicated in the following.

Chapter 2 gives a brief state of the art survey of traditional design procedures and methods for on-bottom stability employed by the industry.

An overall characterization of the physical behaviour of on-bottom pipelines exposed to external wave and current loading is given in Chapter 3. The chapter further describes the main research data basis used for development of numerical models applied in the dynamic analysis. New models are presented related to topics where refinements have been found necessary.

Chapter 4 presents a revised semi-probabilistic design procedure, developed on the basis of the updated technical description.

The enclosed papers covers topics as:

Uncertainty analysis related to parameters and models of significant importance for the pipeline performance as well as development of procedures for sensitivity and reliability calculations tailored to the present application.

Finally, application of the different procedures for safety assessment and calibration of these are indicated together with a thorough discussion and comparison of these alternative methods.

## REVIEW OF TRADITIONAL DESIGN PRACTICE

### 2.1 TRADITIONAL PROCEDURE

The traditional design procedures and acceptance criteria for on-bottom pipeline stability employed by the industry are briefly outlined and reviewed in this section. The traditional engineering practice for stability design consists of giving the pipeline sufficient weight to resist the external forces from waves and current as illustrated in Figure 2.1. Variation of pipe weight is obtained by adjusting the concrete coating thickness. Thus the main objective of the traditional design is to quantify the required concrete coating based on a 2-dimensional static stability analysis.

The required pipe weight is calculated according to the following equation:

$$(W_s - F_z) f_c \geq S F_x \quad (2.1)$$

where

- $W_s$  - submerged pipe weight
- $F_z$  - vertical hydrodynamic lift force
- $f_c$  - Coulomb friction factor for soil resistance
- $F_x$  - total in-line hydrodynamic force (inertia + drag)
- $S$  - safety factor, usually taken as 1.1 (DnV-76)

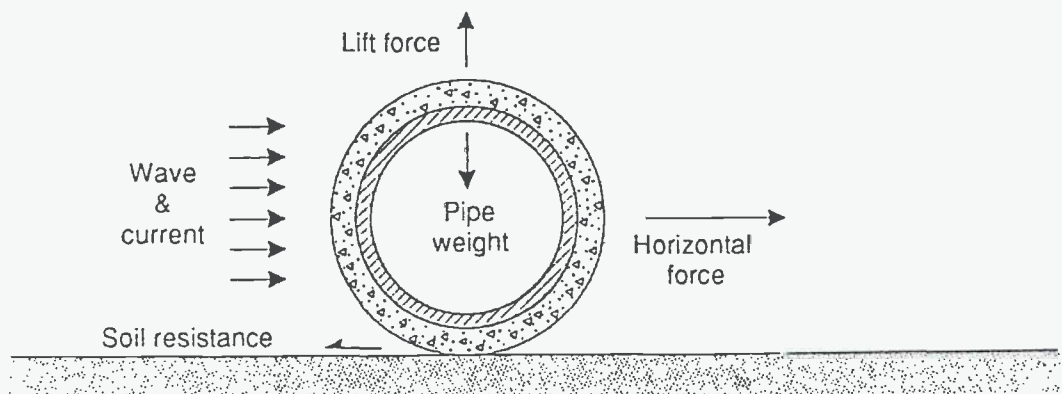


Fig 2.1 - External forces on the pipeline

Most pipeline systems designed to date are based on design equations similar to (2.1). Variation may be found with respect to calculation of design hydrodynamic loads and soil reaction forces as will be briefly outlined below. Equation 2.1 does not contain other physical quantities related to the pipeline than weight and, implicitly, pipe diameter. Stability design according to this procedure can thus be separated from determination of steel wall thickness.

Design wave condition determining the design hydrodynamic loads in Eq. (2.1) is usually conducted through a specification of return period of occurrence, e.g. 100 years. The traditional practice is either to use the maximum wave or, commonly the significant wave related to this condition. In case of application of the maximum wave height, a reduction factor (0.7 in DnV-76) is often used to account for the 3-dimensionality in the wave field.

The flow kinematics to be used as input in the force calculation is found by transformation of a representative regular surface wave condition (as indicated above) down to the sea bottom. The linear Airy wave theory is normally used in this transformation. The steady current is vectorially added to the wave induced velocity. The free stream current is often reduced to include the effect due to the near bottom boundary layer.

Hydrodynamic forces due to wave and current action are calculated from the traditional Morison equation for the horizontal force  $F_x$  and a similar expression for the lift force  $F_z$ :

$$F_x = 0.5 \rho_w D C_D u |u| + 0.25 \rho_w \pi D^2 C_M \dot{u} \quad (2.2)$$

$$F_z = 0.5 \rho_w D C_L u^2 \quad (2.3)$$

where  $u$  and  $\dot{u}$  are the time-dependent total ambient water velocity and acceleration,  $C_D$ ,  $C_M$  and  $C_L$  are drag, inertia and lift coefficients, respectively, and  $\rho_w$  is the water density. The hydrodynamic force coefficients commonly used are those presented in the DnV 1976 pipeline rules which are based on steady flow experiments, (Jones 1971, 1976). The maximum load effect is found by stepping through a wave cycle.

The soil reaction force in equation (2.1) is normally determined based on a Coulomb friction description in which the friction coefficient,  $f_c$ , is usually obtained from sliding pipe tests. The friction coefficient used in the traditional stability design calculation varies typically from 0.3 - 1.0 depending on the soil condition. The coefficient has been linked to the strength of the soil material, increasing strength giving increasing coefficient.

It is quite clear that the selection of force coefficients and soil friction coefficient, as well as the design current and wave condition to be used in the stability check, are fundamental to the design. Large uncertainties are known to be associated with all these aspects of the traditional procedure. The end result is that it is very difficult to quantify the real safety related to this design process. Hence, utilization of a so-called "safety factor",  $S$  in equation (2.1), seems to lack any rational basis ( $S = 1.1$  in DnV 1976).

These shortcomings are discussed in more detail in the following section.



## 2.2 EVALUATION OF THE TRADITIONAL PROCEDURE

### Ocean Environment

The first step in the design process outlined in section 2.2 is to select design data for the wave and current environment. These are found considering a certain return period of occurrence. The basic problem is to choose a long term probability distribution for the environmental parameters. For the North Sea area, scatter diagrams and hindcast models are relatively well developed and can be used directly. For other sites, new measurements may be needed to get a satisfactory prediction of the long term environment. Most pipeline codes refer to an environmental condition with a return period of 100 years to be applied for the operational condition. Generally little or no information about the correlation between waves and current exists. Hence, there is always a question how the wave and current loading should be combined to give the wanted return period of the total load. The most conservative approach of applying a 100 year return period for both wave and current condition seems to be common. This approach should, however, be evaluated on the basis of the target probability level of load or response, considering the correlation between waves and current.

Wave and current directionality and the 3-dimensionality of the waves (short-crestedness) will have a significant effect on the total effective loading on a pipeline system and are thus important to be included. The traditional design check does not take into account the variation of wave and current velocities and directions along the pipeline directly but considers only a section model of the pipeline according to Fig 2.1 and uses an overall reduction factor to account for the 3-dimensionality.

### Design Wave Condition

The next step in the process is to choose a representative regular wave condition to use in the design calculation. Two different approaches are commonly used. (1) Most pipeline codes specify application of the most probable maximum wave height and the associated wave period related to the return period considered and propose a reduction factor to account for wave directionality and spreading, i.e. 0.7 in DnV 1976 rules. (2) The second approach, which is very common in practice, is based on application of the significant wave height for the sea state with the prescribed return period, and the related peak (or significant) wave period. These two approaches will in general give very different design load intensities and thus design weights. It is also noted that the selection of associated wave period is a critical point in the determination of design loads.

To satisfy the basic intention of the traditional design approach, i.e. static stability, it is quite clear that the wave which represents the absolute maximum load should be used in the design equation (2.1). It is further adequate to apply a reduction factor as proposed in DnV-76, to account for wave short-crestedness if present, and other effects that will reduce the effective correlated loading.

To conclude, there is no rational basis of applying the significant wave height in a quasi-static stability calculation. This is not a sound approach if the intention is to reduce any conservatism in the wave loads due to effects from directionality and short-crestedness. This variability with respect to selection



of design wave condition (based on significant or maximum wave), indicates that the resulting pipe design is subjected to a large variation in actual safety level, see numerical case study in Chapter 7.

### Wave Kinematics

The linear Airy wave theory is widely used to calculate the wave kinematics. Wave theories are in general compared by how well they predict the free surface condition. However, with respect to on-bottom pipeline design, the flow kinematics close to the sea bottom is the governing input parameter. As noted in Chapter 5, the prediction of near-bottom velocities based on Airy theory seems to be good, even in cases where prediction of surface velocities is poor.

### Current Boundary Layer

The on-bottom pipeline lies within the fluid boundary layer region, i.e. the area where the steady current velocity is affected by the existence of the bottom surface. The boundary layer normally extends some 3 m to 10 m above the sea bottom. The effective steady current component to be used in the design load calculations needs to be reduced to account for the boundary layer effect. Different models may be found in the literature for calculation of the boundary layer effect on the steady current velocity. The  $1/7$ th power law is commonly used.

### Hydrodynamic Force

As stated above, Eqs. (2.2 and 2.3) have generally been used for hydrodynamic load calculation. Application of these equations with constant force coefficients has been found to give an inaccurate representation of the hydrodynamic forces for combined wave and current loading. This is in particular true in cases where force coefficients based on steady flow conditions have been applied, which seems to represent the most common design practice (DnV-76). It is noted that the problems here are related to the application for a cylinder on a boundary represented by the sea bed and that the Morison equation (Eq. 2.2) in general gives a much better hydrodynamic force prediction for a free cylinder.

In the revised design code (DnV-81) the force coefficients are based on oscillating flow tests (Sarpkaya, 1977 and 1979). These coefficients are considerably higher than those experienced for steady flow and application of these coefficients together with the traditional static design approach leads to unrealistic concrete coating requirements. An increase of pipe weight by a factor of 2 or 3 is typical. However, these higher force coefficients have not in general been applied by the industry, Hildrum et al. (1985). This is due to the fact that modifications to the traditional procedure can only be done by a revision of all aspects entering the design equation (2.1), i.e. also the soil resistance prediction. It is also found that the coefficients in DnV-81 have a conservative bias (Bryndum et al. 1983). It is noted, however, that the traditional Morison equation will have the following shortcomings in spite of the choice of coefficients:

- The detailed time history of the force is not adequately described by adopting the free stream ambient velocity into the Morison equation due to flow separation and wake effects.

- A constant set of force coefficients through one wave cycle does not have the ability to predict the relative difference between the peak forces for the two wave half-cycles.
- Coupling between the different half-cycles is important to include, and this is not possible through the use of the traditional Morison equation.

An illustration of these effects is given in Fig 2.2 (from Verley et al. 1987) where measured hydrodynamic forces from a field measurement program, PFMP (Lambrakos, 1987a), are compared with those predicted from the conventional Morison equation with coefficients from DnV 1981. The figure illustrates a comparison between the peak forces as well as force traces. The selected time series include highly non-Gaussian velocities and a large current component.

The characteristics of the findings are a clear overprediction of peak forces in the first half-cycle, i.e. when current and wave velocity add, and an underprediction in the second half-cycle, when they oppose. The measured forces are not found to exhibit the large difference between the two half-cycles predicted by Eqs. 2.2 - 2.3. This deviation is most significant for the lift forces. The main physical reason for this is found when studying the "effective" near pipe velocity by superimposing the "wake" velocity on the ambient velocity. In an oscillatory flow situation, the wake developed in any half-cycle gives a contribution to the velocity the pipe meets in the next half-cycle. High ambient flow velocity generates a large wake velocity, and contrarily, low ambient flow velocity generates a much smaller wake velocity. The generated wake will thus tend to reduce the differences in maximum effective velocity due to steady current and thus also forces.

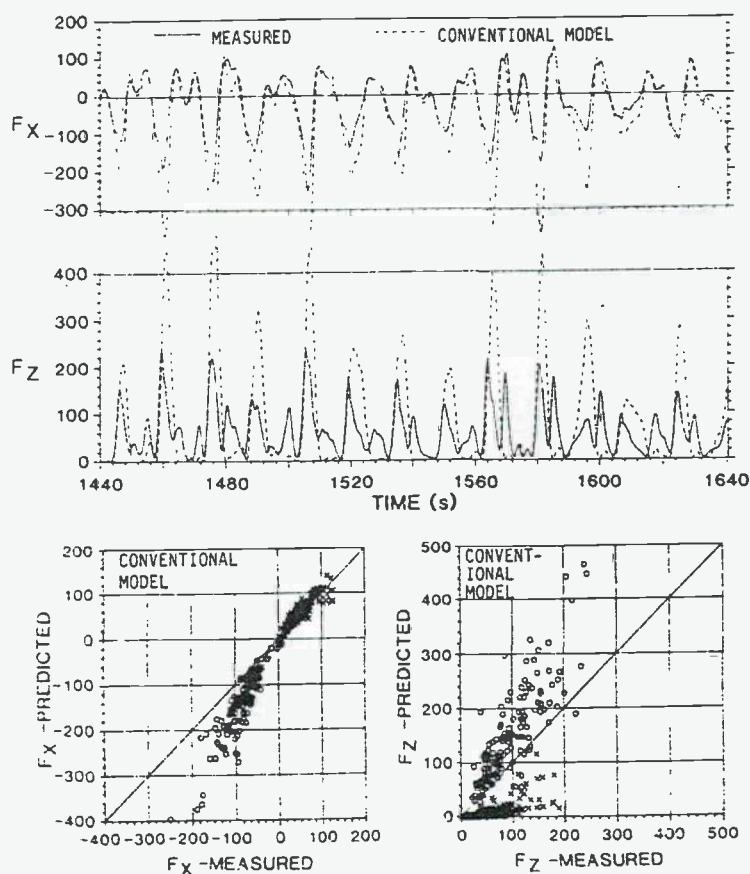


Fig 2.2 - Measured and predicted forces, PFMP data - conventional model

## Soil Resistance

The soil resistance against lateral pipeline movement represents the key strength quantity in the pipeline stability calculations. It is thus of significant importance to have an accurate prediction of this force and the accuracy has to be evaluated relative to other uncertainty sources in the design calculations.

Experimental data have shown that the pipe-soil interaction forces are far more complex than predicted by the traditional Coulomb friction model (Wagner et al. 1987 and Brennodden et al. 1989). There is found to be a clear relationship between the soil capacity against lateral pipe movement and the pipe penetration into the soil and the soil strength. An on-bottom pipeline may penetrate into the soil as a result of small cyclic movements, and the soil resistance is accordingly a function of the external load history. It is also found from experiments that the pipe-soil resistance force has a significant value also for low vertical contact forces, i.e. high lift forces relative to the submerged pipe weight. The measured resistance is considerably larger than predicted by any typical Coulomb friction factor. This observation confirms that the pure Coulomb friction model is not adequate in modelling the resistance, and that a term rather independent of the actual contact force (lift force) but dependent on penetration, load history and soil strength has to be established.

Figure 2.3 from Brennodden et al. (1989) illustrates the characteristics of the pipe-soil interaction forces and pipe penetration as a function of lateral pipe displacement during oscillation. It is seen that the soil response is initially elastic and that the soil capacity and penetration increase gradually as a soil mound is built up in front of the pipe during oscillations. For larger displacements a peak soil resistance level is reached (breakout) and the soil pipe interaction force decreases when the pipe slides on the soil surface with a relatively low penetration. An important observation is that increasing soil strength will decrease the penetration and thus decrease the sliding resistance when "large" displacements are experienced. This effect is particularly pronounced for clay soil and is opposite to the traditional design practice of applying an increasing friction factor for increasing soil strength.

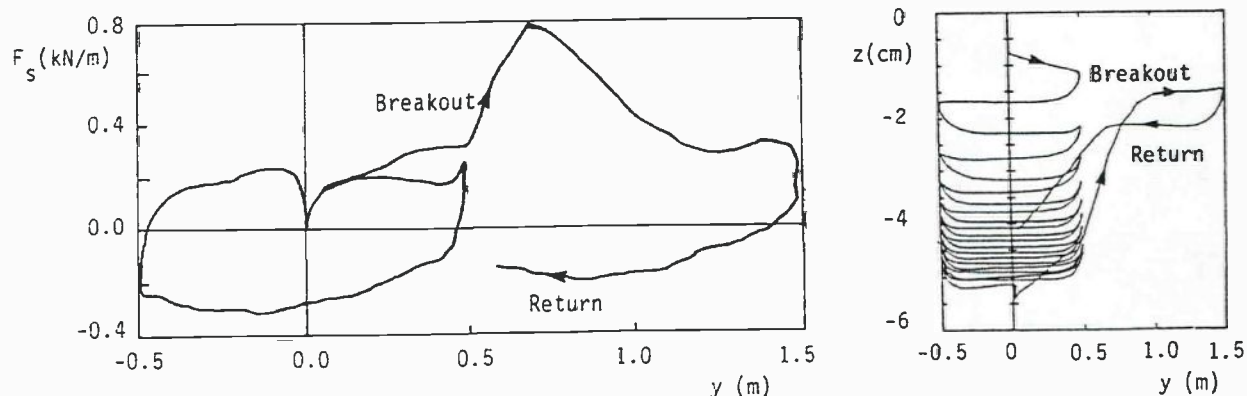


Fig 2.3 - Soil resistance  $F_s$  and penetration  $z$  versus pipe displacement

### Conclusion

The main problems with respect to the traditional design method can be summarized as follows:

- No sound and clear practice exists for selection of design load conditions.
- The Morison equation does not predict the forces for oscillatory flow to a satisfactory level of accuracy whatever the choice of force coefficients might be.
- The pure friction approach for pipe-soil interaction is not adequate in modelling the resistance.
- The pipeline is not modelled as a continuous system, but only by a section model.

To conclude, there is a conflict between the measured hydrodynamic forces on the pipeline and the traditional design forces applied. In addition there is an oversimplification with respect to the pipe-soil reaction forces by applying a friction term. From the above discussion it seems clear that both the hydrodynamic load and the soil resistance used as input in the design equation (2.1) have been inaccurate. It is further noted that different procedures have been used by the industry for design load and resistance prediction, which gives a large spread in possible design results. These basic problems illustrate the need for updated models and procedures and have been the basis for a large amount of research performed on these topics in recent years.

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## DYNAMIC RESPONSE OF SUBMARINE PIPELINES - MODELLING ASPECTS

### 3.1 INTRODUCTION

A general characterization of the physics related to on-bottom pipeline behaviour when exposed to external wave and current loading is given in this chapter. The objective is to give a short presentation to update the physical understanding and to give some insight into the main research data basis used in modelling of the system. Improvements related to modelling of hydrodynamic and soil resistance forces, as well as the design methodology to account for the shortcomings in the traditional procedure are discussed.

### 3.2 MODELLING ASPECTS - DETERMINISTIC MODELS

A number of research projects have been performed during recent years to improve the understanding of pipeline behaviour for different load conditions.

Some of the major activities concerning pipeline stability are the Pipestab project carried out by SINTEF (Wolfram et al. 1987), with the main objective to improve the physical modelling and to develop a technically sound design methodology, and a similar study recently performed for the American Gas Association (AGA), (Allen et al. 1989) as a joint industry project with Brown & Root USA, Danish Hydraulic Institute (DHI) and SINTEF as the main contractors.

Major projects dealing with pipeline free span assessment are the development of Guidelines by the British Department of Energy (Raven 1986), the Pipeline Span Evaluation Manual by DHI (Bryndum et al. 1989) and the Submarine Vortex Shedding Project by Snamprogetti (Bruschi et al. 1988) dealing also with models for free span analysis (Bruschi et al. 1987).

The present study utilizes some of the results both from the Pipestab project and the work performed for the AGA as its main data basis. This study represents a further evaluation with respect to design methodology and related to development and application of reliability methods tailored to the safety assessment of the pipeline design process.



A brief review of this research data basis is given in the following.

### 3.2.1 Modelling of Hydrodynamic Forces

A considerable effort in recent years has been devoted to improve the modelling of hydrodynamic forces on submarine pipelines. Some of these investigations are conducted at the University of Hawaii (e.g. Grace and Nicinski, 1976; Grace and Zee, 1981), the Naval Postgraduate School (Sarpkaya and Rajabi, 1979) the Danish Hydraulic Institute, DHI (e.g. Bryndum et al. 1983; Jacobsen et al. 1984; Bryndum et al. 1988 and Jacobsen et al. 1988) by Exxon Production Research in the Pipeline Field Measurement Program, PFMP (Lambrakos et al. 1987a) and at SINTEF (NHL, 1985; Verley et al. 1987; Fyfe et al. 1987).

Several force models have been developed as a result of the above research work. All of these represent a considerable improvement as compared to the direct application of Morison's equation. Basically, two different approaches are used. The model by Lambrakos et al. (1987a), referred to as the Wake model, is based on a description of the wake velocity behind the cylinder, and a correction of the ambient flow velocity including the wake effect. The second approach is related to the application of Fourier decomposed force data bases, Fyfe et al. (1987); Jacobsen et al. (1988) and Verley and Reed (1989a).

#### Wake Model

The Wake model has similar force expressions to the conventional Morison equation. However, this model includes two important effects; the lift and drag force coefficients are time-dependent and the ambient flow velocity is modified to include the wake effect.

The horizontal and vertical forces are given by:

$$F_x(t) = 0.5 \rho_w D C_D(t) |U_e| U_e + 0.25 \rho \pi_w D^2 (C_M \ddot{u} - C_W \dot{w}) \quad (3.1)$$

$$F_z(t) = 0.5 \rho_w D C_L(t) U_e^2 \quad (3.2)$$

where  $U_e$  is the effective flow velocity corrected for wake effects, and  $w$  is the acceleration of the wake flow.  $C_M$  is the inertia coefficient for the ambient flow and  $C_W$  is the added mass coefficient for the wake flow.  $C_D$  and  $C_L$  are functions of  $s/D$ , where  $s$  is the distance travelled by the fluid particles since the latest flow reversal. This variation in the coefficients is termed a start up effect caused by each flow reversal, and is larger for the lift force than for the drag force. The wake description is based on the classical description of the wake far behind an isolated cylinder in steady motion (e.g. Schlichting, 1979), but it is empirically extended for the wake behind a cylinder at a wall, subjected to a time-dependent flow. The effective flow velocity is determined numerically including pipe encounter with the wake generated in the previous half-cycle. An irregular wave situation is thus handled directly. The Wake model has been checked against field and laboratory data (Lambrakos et al. 1987a; Verley et al. 1987). The model is considered applicable for  $5 \leq K \leq 40$  and  $0 \leq M \leq 0.8$  and can be used for "rough" and "smooth" pipe surface roughnesses.  $K$  is the significant Keulegan-Carpenter number and  $M$  is the current to significant wave velocity ratio. A comparison of measured and predicted forces



is given in Fig 3.1, from Verley et al. (1987). This data is based on the same field measurements as used in Fig 2.2. The model is seen to give a significant improvement as compared to the traditional model.

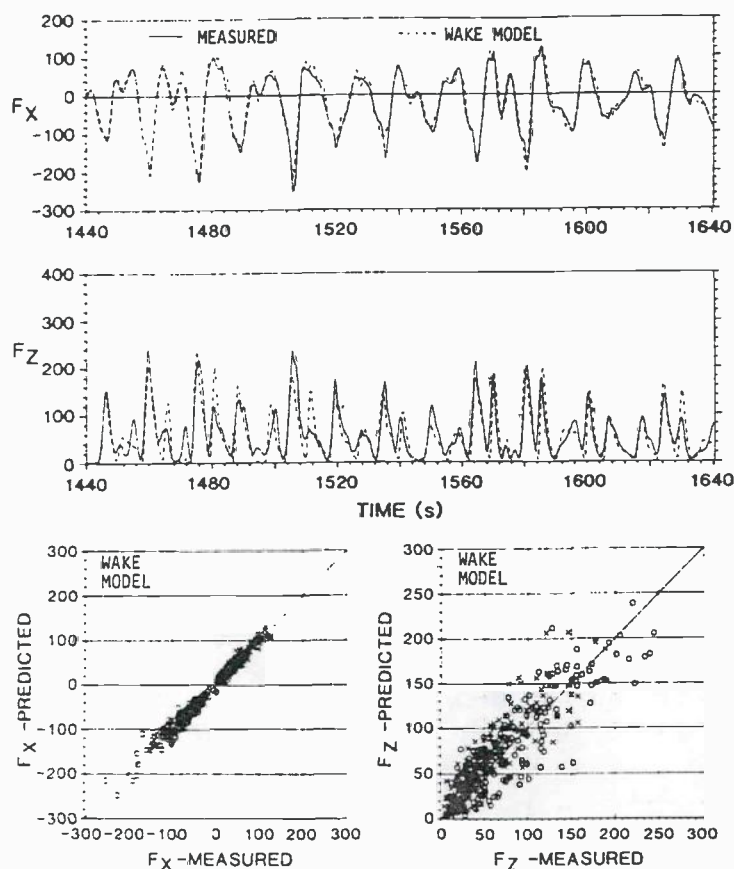


Fig 3.1 - Measured and predicted forces, PFMP data - Wake model

#### Fourier Models

The Fourier models of Fyfe et al. (1987), referred to as the Pipestab model and Jacobsen et al. (1988), referred to as the AGA model are very similar. They are both based on a Fourier decomposition of forces obtained from laboratory tests with combined regular oscillatory and steady velocities, defined through the parameters  $K$  and  $M$  (given here by the maximum flow velocity amplitude). These regular velocity data are then applied to individual velocity half-cycles of irregular waves, in a slightly different manner for the two models. The data bases for the models are qualitatively and quantitatively very similar, as indicated by Bryndum et al. (1988). However, the AGA data are more extensive, covering  $2.5 \leq K \leq 160$  and  $0 \leq M \leq 1.6$  and three pipe surface roughnesses ( $k/D=10^{-3}$ ,  $10^{-2}$ ,  $5 \times 10^{-2}$ ).

The third Fourier based model referred to as the Database model developed by Verley and Reed (1989a), uses the database by Bryndum et al. (1988), but with an improved methodology when applied for irregular waves. All experimental investigations have been conducted for conditions simulating forces on a stationary pipe, whereas application in dynamic analysis calls for use of the models for a moving pipe, with the associated calculation of hydrodynamic damping. The model

by Verley and Reed (1989a), is also based on a consistent approach when applying stationary pipe force data (measured or predicted), to moving pipe situations.

The differences between the AGA model and the Database model are mainly related to how the irregular wave situation is modelled by fitting a combined regular oscillatory and steady velocity to the local irregular velocity cycle. Irregular wave force predictions with the AGA model are based on considering one half-cycle of the near bottom velocity at a time. A considerable improvement is obtained with the Database model by fitting the combined regular oscillatory and steady velocity to a local irregular velocity for the full cycle and calculating forces for only the second half of this cycle. By stepping through a time series one half-cycle at a time, the influence from the previous half-cycle on forces in the present half-cycle is then preserved. The details of this procedure are found in the publication referred to above. It is noted that the AGA and Database models will be identical for regular wave situations.

The Wake, AGA and the Database model have been compared against laboratory data as well as field data by Verley and Reed (1989a). Predicted and measured force time histories have been compared. Predicted peak forces have been plotted against the measured forces and the mean and standard deviation of the ratio between corresponding predicted and measured peak forces have been calculated. The main conclusions from the study are summarized below.

It was found that all three models give a reasonable reproduction of forces as compared to the laboratory data. Predicted and measured peak forces are compared relatively in Fig 3.2 (for  $K = 30$  and  $M = 0$ ) and for a specific time series in Fig 3.3 for AGA and Database, from Verley and Reed (1989a). It is seen that the

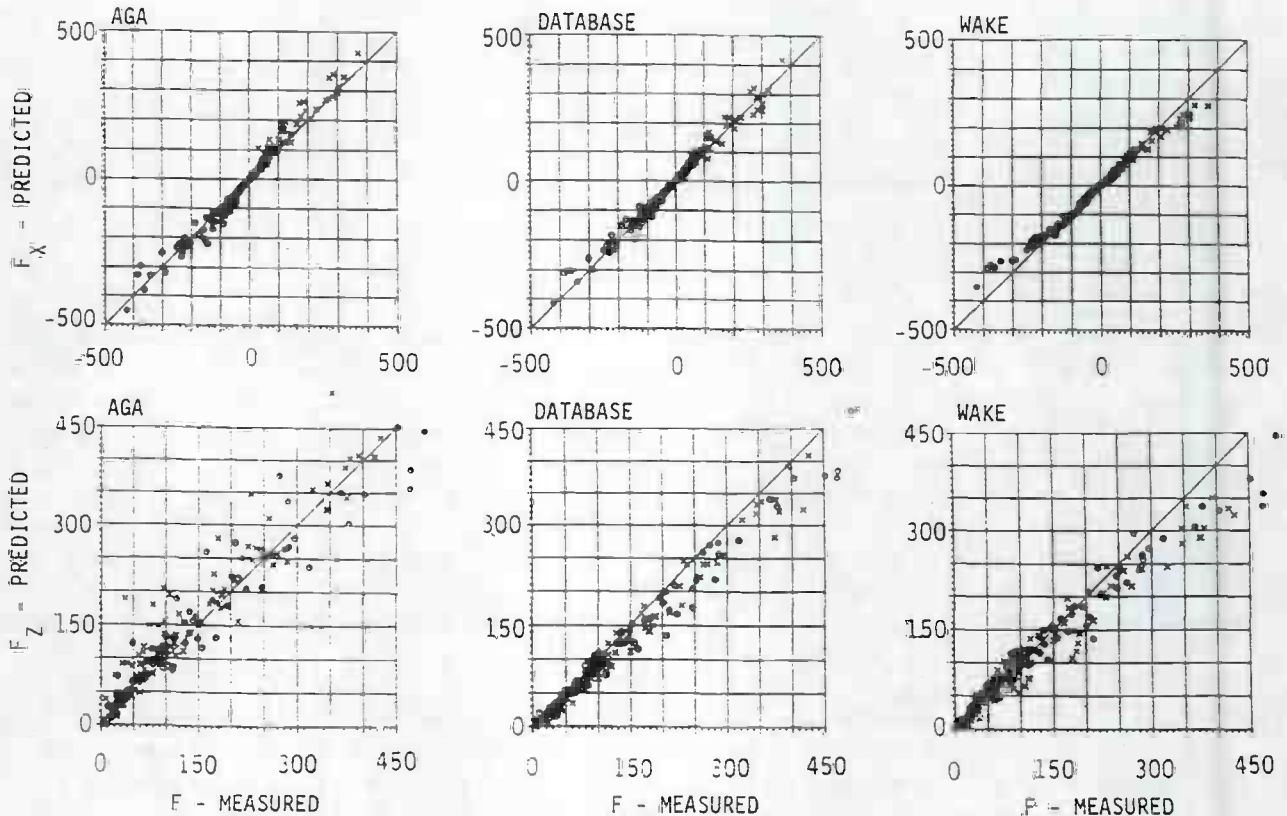


Fig 3.2 - Predicted versus measured peak forces - laboratory data

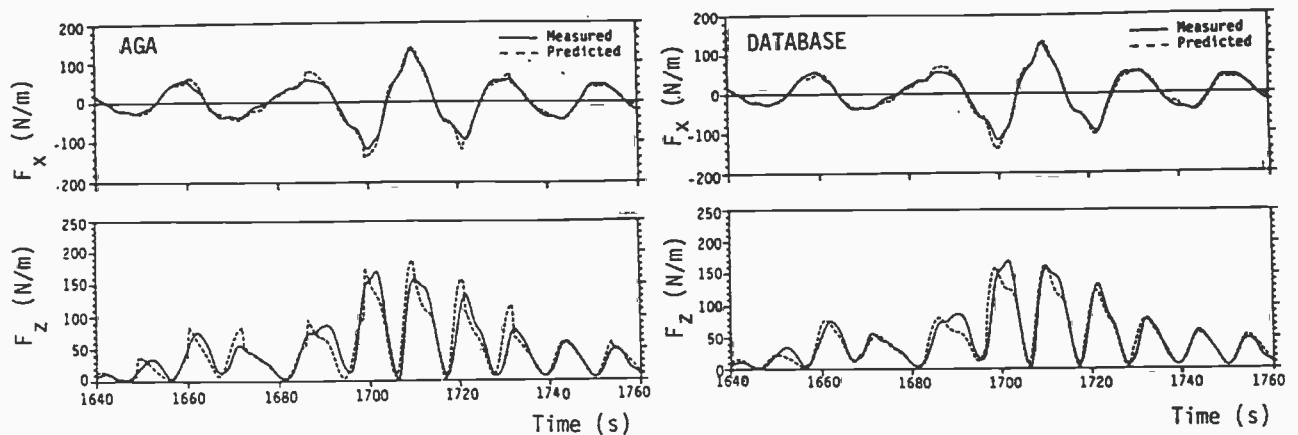


Fig 3.3 - Predicted and measured force traces - laboratory data

horizontal force is well predicted by all models. The AGA model has more scatter in the peak lift forces due to an overprediction of forces in the half-cycle when wave and current velocities oppose. The Database model gives a better representation of lift forces, with a standard deviation of the ratio of predicted to measured forces of about 10 % which is similar to that observed for regular waves. However, for higher  $K$  values ( $K=50$ ) an underprediction of peak lift forces is found, about 15 % when  $M=0$  and 20-25 % when  $M=0.4$ . An even larger underprediction of lift forces is found for the Wake model for the higher  $K$  values and increases for increasing  $M$  values. It is noted that comparison with field data gives in general larger scatter than for laboratory data, in particular when a steady current is superimposed (see Figs 2.2 and 3.1).

The improvements in the Database model related to irregular wave situations are more pronounced when studying field data as e.g. those from PFMP. The Wake model gives less variability of the horizontal force ratio, but a larger spreading for the vertical force ratio than the Database model.

Comparing the above hydrodynamic force models the following is concluded: All four models represent a considerable improvement compared to the classical Morison formulation. Among the Fourier based models, the Database model is definitely the most accurate. The Database model covers a considerably greater range of hydrodynamic conditions than the Wake model, and is concluded to be the best for general applications.

### 3.2.2 Modelling of Soil Resistance Forces

As for the hydrodynamic force modelling, a relatively large volume of research work has been performed during recent years regarding the pipe-soil interaction forces. However, the total amount of work related to pipe-soil interaction is considerably less than that related to hydrodynamic forces. Some of the earlier studies investigating the effect of cyclic pipeline movements on the lateral soil resistance are those reported by Lyons (1973), Karal (1977) and Lambrakos (1985). Recent work within this topic is that performed within the Pipestab project by Brennødden et al. (1986) and Wagner et al. (1987), the work described

by Morris et al. (1988), and Palmer et al. (1988), and finally the most recent work conducted for the AGA (Brennodden et al. 1989).

The numerical models for prediction of pipe-soil interaction forces based on the data from the Pipestab project and the similar work for the AGA will be discussed here. The data for both of these models are obtained from full scale pipe-soil interaction tests performed at SINTEF with test facilities as described by Brennodden et al. (1986).

### Pipestab Soil Model

The laboratory tests executed covered monotonic increasing loading up to failure as well as constant amplitude cyclic loading followed by loading to failure. A two component model was developed consisting of a sliding friction term, proportional to the vertical contact force, and a term describing the additional resistance caused by penetration into the soil. The second term is independent of the vertical contact force but dependent on the displacement history of the pipe, which governs the penetration and thus the failure surface of the soil. The empirical model has the following form:

$$F_s = f_c(W_s - F_z) + F_R \quad (3.3)$$

where

$$F_R = \beta \gamma_s A \quad (\text{sand})$$

$$F_R = \beta S_u A/D \quad (\text{clay})$$

Further,  $f_c$  is the friction factor and  $F_R$  is the additional soil resistance term dependent on pipe penetration.

The friction factor,  $f_c$  is 0.6 for sand and 0.2 for clay soil. The empirical coefficient  $\beta$  is a rather complex function of the lateral pipe displacement and loading history.  $A$  is a measure of the displaced soil area,  $\gamma_s$  is the submerged unit weight of soil and  $S_u$  is the remoulded shear strength of clay. The pipe penetration is a non-linear increasing function of the number and the magnitude of displacement cycles. When the pipe moves a certain distance into the soil mound (approximately half the pipe diameter), a reset of the parameters occurs and the resistance is calculated as if the pipe was placed on a flat sea bed without any additional penetration, i.e. breakout is defined.

The models are found to predict the experimental maximum soil resistance within about 20 % standard deviation for sand, and 30 % for clay with almost no bias, see Fig 3.4 from Wagner et al. (1987). However, it is noted that this is related to the laboratory test program, and extrapolation of the data from the test program, in particular pipe penetration, is associated with uncertainty.

A laboratory test program was performed, Verley and Reed (1989b), to check the above model under realistic conditions. It was found, as noted by Sotberg and Remseth (1986b), that the breakout mechanism was not properly handled as the model is not mathematically continuous when predicting breakout. The model predicted the response up to the first breakout reasonably well. However, total displacement was overpredicted and penetration underpredicted caused by too frequent resets of the model compared to the laboratory tests, i.e. modelled soil resistance is too small giving a conservative bias.

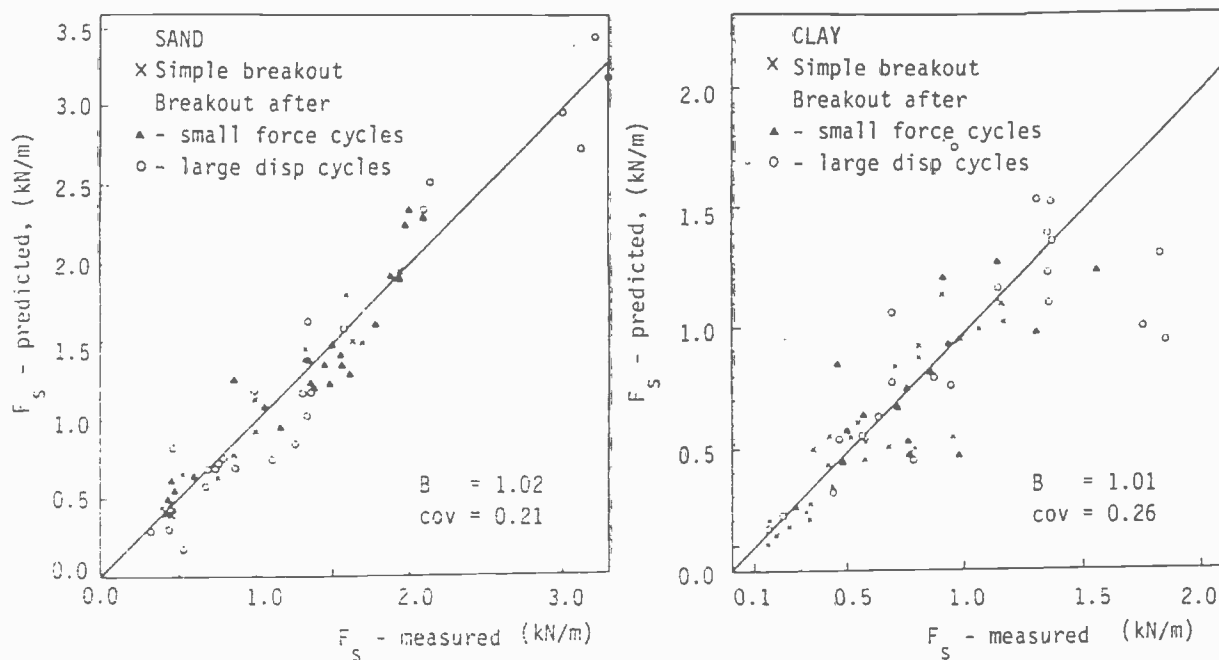


Fig 3.4 - Predicted versus measured peak soil resistance - Pipestab

#### AGA Soil Model

Further full scale tests were performed at SINTEF for the AGA using the same test facilities as for the Pipestab project. The test data generated from the study should be the basis for development of an empirical model, being applicable in numerical simulation of the pipe-soil interaction. It was thus important to define a model concept, prior to the test program. The basis for the model concept was developed by Sotberg and Brennodden (1987), and is briefly outlined below. Further details related to the model are given by Brennodden et al. (1989).

The model concept is based on a separation of the total soil resistance into two terms as for the Pipestab soil model above:

$$F_s = f_c(W_s - F_r) + F_r \quad (3.4)$$

where

$$F_r = \alpha F_{r_0}$$

The first term is a sliding resistance term as in Eq. (3.3) (Coulomb friction), and the second,  $F_r$ , is a term which takes into account the effect from pipe penetration on the total soil resistance.  $F_{r_0}$  is the residual resistance experienced by the pipeline in virgin soil areas, before additional penetration due to pipe movements. The values of  $f_c$  are identical to those given above and independent of the soil strength.

The basic idea behind the model is simple. The penetration development governs the soil resistance term  $F_r$ . Penetration is physically dependent on parameters such as submerged pipe weight,  $W_s$ , outer pipe diameter,  $D$ , relative soil density,  $\rho_r$ , for sandy soil, or remoulded shear strength,  $S_u$ , for clay, in addition to the response history of the pipeline section studied through accumulated energy.



The energy dissipation when oscillating the pipe in the soil will partly be used to cause the pipe to dig in (increased penetration), and partly to push soil material to each side of the pipeline.

A relation between the accumulated energy in the soil, caused by the additional resistance term  $F_R$  and the relative penetration, is the basis for including the effect on penetration due to pipe oscillation. Only the work done by the additional force term  $F_R$  is included in the accumulated energy calculation. A basic assumption for the model is that the friction force does not cause any penetration.

The different steps in the calculation of  $F_R$  are illustrated as follows. The first step comprises energy calculation. The energy dissipated in the soil from the term  $F_R$  (no friction contribution) is then applied to update the penetration of the pipe into the soil as the second step. And finally, the total soil resistance is then calculated according to the updated penetration and a relation between pipe penetration and soil capacity based on the experimental data.

A comparison of the model prediction and the measured data is illustrated in Fig 3.5 for the regular cyclic tests in which maximum soil resistance forces are given. Pipeline penetration is predicted by the model with a standard deviation of about 15 %, whereas standard deviation of resistance prediction is 17 % for the sand model and 7 % for clay soil. These results are concluded to be very promising and this level of accuracy is in this context considered satisfactory.

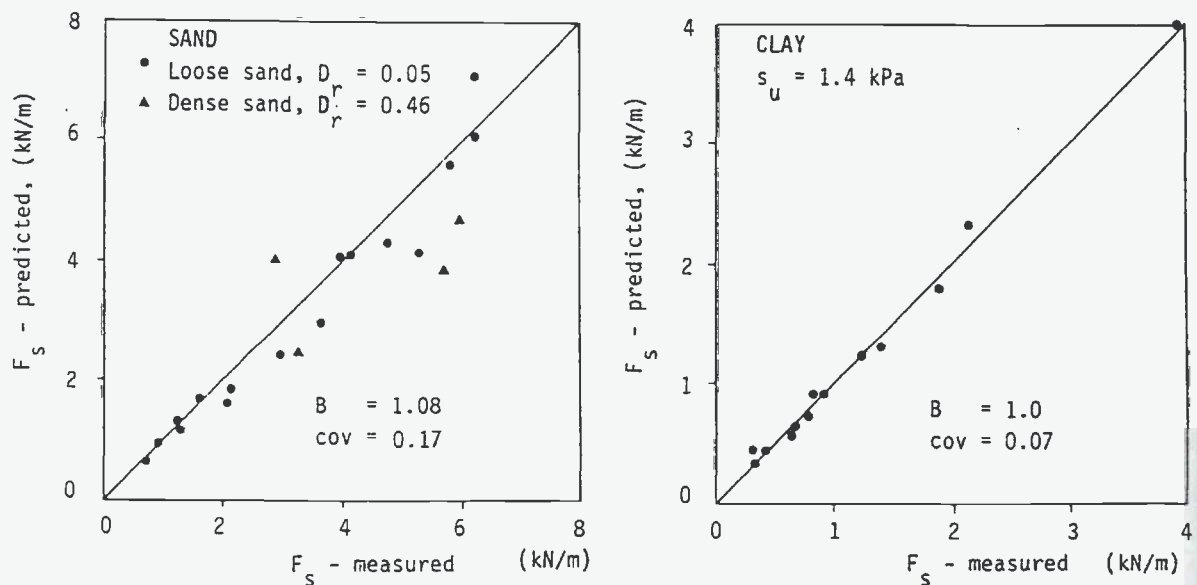


Fig 3.5 - Predicted versus measured peak soil resistance - AGA

The present model is entirely empirical, but has included the most significant and characteristic effects by modelling the embedment and lateral soil resistance as a function of the pipe movement history, pipe diameter and the contact force between the pipeline and the soil. The basic principle behind the empirical modelling is that some part of the work caused by the pipe-soil interaction force makes the pipe penetrate into the soil and the remaining is used to push the soil mound. The experimental data give the answer with respect



to how these effects are distributed, and illustrate the typical trend with respect to a variation in penetration level. Typically, for increasing penetration, an increasing part of the energy generated from the pipe-soil interaction force goes to move the build-up mound and a smaller part to further increase penetration.

The major improvements with this model are that the prediction of the pipe penetration is modelled with a satisfactory degree of accuracy and the effect of pipe penetration on the soil-pipe resistance force is properly taken into account as well as that the numerical formulation is continuous.

A check of the model based on realistic irregular force data was performed by Hale (1989). The conclusions from this study were that the model behaviour was satisfactory also for irregular wave conditions as the calculated penetration and pipe displacement were close to the measured. A comprehensive study was performed by Verley and Sotberg (1991), which included some modifications in the way the lift forces was handled in the model. This study gave an even better comparison than from Hale (1989) in particular for sandy soils.

### 3.2.3 Pipeline Response Model

An accurate prediction of the pipeline response due to environmental and functional loading is important with respect to evaluation of the safety of the pipeline system. When dynamic pipeline response is essential in the design process, high accuracy is demanded from the pipeline response model.

Pipeline response is here related to the lateral movement of a pipe section due to external wave and current loading and also bending stress in cross sections along a pipeline with some length and end restrictions.

The submarine pipeline response behaviour is a rather complex function of the parameters affecting the pipe-soil interaction as well as hydrodynamic forces. This problem is quite different as compared to the traditional design, (maximum force balance check), in which the main problem is related to estimating the maximum external loading and the static soil capacity. When allowing for pipeline movements it is necessary to take into account the time dependence and spatial variation of hydrodynamic forces as well as soil resistance. The pipeline response characteristics are highly non-linear, mainly due to the non-linearities in the pipe-soil interaction forces and hydrodynamic loading. As a consequence, the response calculation has to be performed in the time domain.

The end products from both the Pipestab project and the similar work for the AGA were the development of computer program systems for response calculation with updated models based on the experimental data basis generated during the projects. A FEM time-domain program system, PONDUS (Holthe et al. 1987) was developed during the Pipestab project and a very similar program, PIPEDYN (Lammert, Hale and Jacobsen, 1989 and Michalopoulos, 1986a) was one of the end results from the projects conducted by the AGA.

#### PONDUS Program

A brief description of the PONDUS program is given below, and a more detailed theoretical basis is given by Holthe and Sotberg, 1986.

The development of the PONDUS program was based on the need to have an efficient special purpose program tool which included the improvements in the hydrodynamic and soil resistance force models developed during the project.

The numerical modelling approach in PONDUS is based on 2-dimensional beam elements including the translational and rotational degrees of freedom in the horizontal plane. Small deflection theory is used, but accounting for the important geometric stiffening effect from the increase in axial force due to lateral displacement.

In matrix notation the equation of motion for the on-bottom pipeline may be written as:

$$M_p \ddot{r} + C_p \dot{r} + K_p r = R_h - R_s \quad (3.5)$$

where

- $r, \dot{r}, \ddot{r}$  - nodal displacement, velocity and acceleration vector, respectively
- $M_p$  - pipe mass matrix (lumped mass)
- $C_p$  - pipe damping matrix (Rayleigh damping)
- $K_p$  - pipe stiffness matrix for linear material including the geometric stiffness matrix from the effective tension
- $R_h$  - hydrodynamic force vector including both drag and inertia forces
- $R_s$  - pipe-soil interaction force vector

Hydrodynamic force modelling: Several hydrodynamic force models have been implemented into the program. The hydrodynamic forces are generally calculated using a representative relative velocity and acceleration between the moving pipeline and the water. The effective external forces are thus reduced when the pipe moves with the water flow compared to forces on a fixed pipe.

Among the models implemented during the execution of the Pipestab project are the traditional Morison's equation, a Fourier component based model (Fyfe et al. 1987) and the Wake model (Lambrakos et al. 1987a). From these models the Wake model was regarded as the most accurate one. However, it was noted that the range of applicability of the Wake force model was rather limited and during some later work, the Database model (Verley and Reed 1989a), which covers a larger hydrodynamic parameter range was implemented. The Database model has an improved methodology when applied to irregular waves compared to other Fourier based models.

Soil resistance modelling: Different models are included in the program for prediction of the pipe soil interaction forces. The simple Coulomb friction model, which is traditionally used for offshore pipeline design calculations, (Eq. 2.1) is included. The more complex empirical model developed during the Pipestab project is also included in the program. The more recent model based on soil-pipe interaction experiments performed for the AGA (Brennodden et al. 1989) with a modification by Sotberg et al. (1989a) is also implemented in the program.

A direct comparison of the pipeline response predicted by PONDUS utilizing the modified AGA soil model, with the response data from an experimental test program was conducted (Verley and Sotberg 1991, to appear). The findings and overall conclusion from this comparison are that the model gives a good reproduction of the soil resistance forces and further gives a less conservative prediction than the Pipestab soil model.

Wave environment description: A preprocessor to the analysis module allows input of wave velocities from either a 3-dimensional wave simulation module (Stansberg, 1986a) or from measured data. Through the input data different model wave spectra can be specified. The sea state is modelled as stationary for a time period of 1 to 3 hours. The method is, however, not limited to 3 hours. By using an inverse FFT (Fast Fourier Transformation) algorithm random waves are generated. A cosine wave directional spreading function,  $\cos^n(\theta - \theta_w)$ , where  $\theta_w$  is the mean wave propagation direction, can be applied. The random nature of the waves is described through a random phase angle for each individual wave frequency or alternatively both random phase and amplitude. In this context it is noted that using a random phase angle to model the stochastic nature of the waves generally gives an underprediction of the statistical variation compared to the alternative approach with random phase and random wave amplitude. This effect is also illustrated by Larsen and Passano (1990) considering marine risers.

Surface waves are transformed to sea bottom level using linear Airy wave theory. Wave velocity and acceleration time series are generated at specified points (grid points) along the actual pipeline section modelled. A rectangular grid is used to describe short-crested wave conditions. Only one grid point is used (one time series) for a complete description of the ocean wave environment when a long-crested sea state propagates normal to the pipeline.

Solution procedure: An incremental form of the total equilibrium equation from time  $t_1$  to time  $t_2$  is used to solve the dynamic problem defined by Eq (3.5):

$$M_p \Delta \ddot{r} + C_p \Delta \dot{r} + K_p \Delta r = \Delta R_h - \Delta R_s \quad (3.6)$$

The incremental hydrodynamic force vector for all force models may be written as

$$\Delta R_h = \Delta P_h - C_h \Delta \dot{r} - M_h \Delta \ddot{r} \quad (3.7)$$

where:

- $\Delta P_h$  - incremental force vector depending on change in water velocity and acceleration from time  $t_1$  to time  $t_2$
- $C_h$  - hydrodynamic damping matrix
- $M_h$  - hydrodynamic mass matrix (added mass)

Since the lumped form of the force is used, both  $C_h$  and  $M_h$  are pure diagonal matrices with only translational terms. Similarly, the pipe soil interaction force vector may be expressed as:

$$\Delta R_s = \Delta P_s - C_s \Delta \dot{r} - K_s \Delta r \quad (3.8)$$

where:

- $\Delta P_s$  - incremental force vector caused by change in the lift force due to change in water velocity from time  $t_1$  to time  $t_2$
- $C_s$  - soil damping matrix
- $K_s$  - soil stiffness matrix

Again, only translational diagonal terms are present in  $K_s$  and  $C_s$ . The expression for the terms in  $\Delta P_s$ ,  $K_s$  and  $C_s$  are dependent upon whether the nodal soil forces are in the elastic or plastic state. The dynamic equilibrium equation may now be rewritten as:

$$M_t \ddot{\Delta r} + C_t \dot{\Delta r} + K_t \Delta r = \Delta P_h + \Delta P_s \quad (3.9)$$

where

$$\begin{aligned} M_t &= M_p + M_h && \text{total mass matrix} \\ C_t &= C_p + C_h + C_s && \text{total damping matrix} \\ K_t &= K_p + K_s && \text{total stiffness matrix} \end{aligned}$$

Since the hydrodynamic forces and the soil forces are highly non-linear, modified Newton equilibrium iterations are performed to ensure equilibrium at time  $t_2$ .

The well known Newmark  $\beta$  method with constant time steps and constant average acceleration is used to integrate the incremental stiffness relation. To ensure a safe time-stepping procedure, a time step is automatically subdivided into a number of smaller time steps if equilibrium is not obtained after a certain number of iterations for the initial time step. This is particularly efficient when considering the high non-linearities in the transition zone between elastic and plastic pipe-soil interaction conditions, and generally makes it possible to use a larger mean timestep than if no subdivision is employed.

Convergence is assumed when a scaled norm of the translational components of the incremental displacement within an iteration becomes less than a predefined small value. The solution accuracy during a time history simulation is kept approximately constant.

A program system overview is given in Fig 3.6 where the four modules are illustrated. WAVESIM generates the ambient water velocities at specified grid points along the pipeline section based on a specified input model wave spectrum. After some pre-processing of the wave time series in PREPONDUS, the dynamic response calculation in the time domain is performed by the PONDUS analysis module. A post-processing of data is done by the PLOTPO module. The program system has the capability of simulating a full 3 hour sea state response for a realistic pipeline model.

The program has been verified against the SPAN program (Michalopoulos, 1984 and 1986b), a fully 3-dimensional formulation of the structural response. Results have been shown to be very close, however with a reduction in computing time by a factor of about 10. This confirms that the formulation in PONDUS is very efficient for the present problem and has included the most significant effects with respect to the structural behaviour. Parts of the program have also been verified against the ABAQUS (Hibbitt et al. 1984) general purpose computer program.

The PONDUS program includes the most recently developed models for pipe-soil interaction and hydrodynamic forces due to wave and current loading. It is a special purpose program, developed to be most efficient for the analysis of a submarine pipeline, and has shown a satisfactory accuracy in the response prediction. It is thus concluded that the program represents a suitable tool to be applied for refined pipeline design calculations and for evaluation of structural safety.

Application of the program is illustrated later in this thesis during discussion of design methods and application of reliability methods.

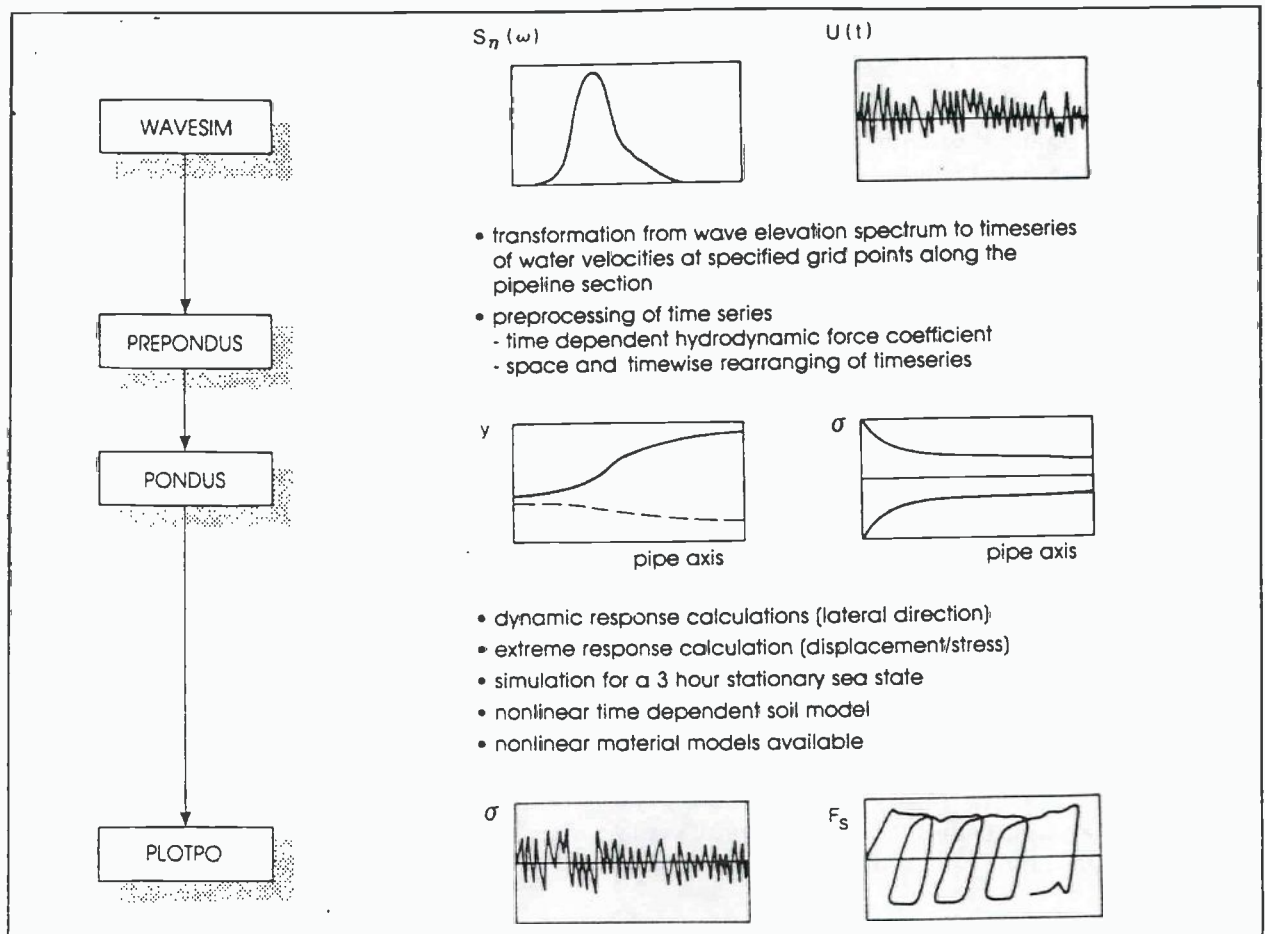


Fig 3.6 - PONDUS program system

### 3.3 DIMENSIONAL ANALYSIS

#### 3.3.1 General

Accurate predictions of the dynamic response of submarine pipelines can be performed utilizing computer programs such as the ones described above. The PONDUS program is efficient for this application, but may still require time consuming simulations for repeated design application. Generally, there is a large number of basic design parameters which are random quantities and should be varied in the design process. In such cases it is often beneficial to reduce the dimension of the problem by utilizing dimensional analysis. This approach is commonly used in fluid mechanics and other disciplines, and can be used here to scale the physical behaviour of a submarine pipeline exposed to wave and current loading in terms of a set of non-dimensional parameters.

A generalized description of the physical problem has several benefits. The pipeline response can be expressed in terms of a few non-dimensional parameters representing combinations of a larger number of physical quantities describing the pipeline, soil and the ocean environment. A generalized response data base given in terms of the non-dimensional scaling groups can be generated and used for easy transformation from basic load parameters to response quantities relevant for design purposes.



### 3.3.2 Scaling of Pipeline Response

Development of non-dimensional scaling groups and a response data base in terms of these scaling parameters was conducted for the models applied during the Pipestab project. The response data base was derived by repeated simulations with the PONDUS program, and simulations were also used to verify the appropriateness of the non-dimensional parameters. A brief outline of the dimensional analysis is given below, from Lambrakos et al. (1987b).

Scaling of the response is performed by employing the equation of motion for the pipeline. A single degree of freedom system, SDOF, is first assumed followed a clamped end model that generates stress in the pipe wall.

#### SDOF Model

The equation of motion in the horizontal direction for pipeline sections with no boundary disturbances from end constraints, or possibly a free-free section is:

$$m \frac{\partial^2 y}{\partial t^2} = F_x - F_s \quad (3.10)$$

where  $m$  is the mass of the pipeline per unit length;  $y$  and  $t$  denote pipeline displacement and time, respectively;  $F_x$  is the hydrodynamic force per unit length, and  $F_s$  is the soil resistance force per unit length.

The various quantities in the equation of motion are scaled as follows:

$$y' = y/D, \quad t' = t/T_u, \quad u' = u/U_s, \quad s' = s/D \quad (3.11)$$

where  $U_s$  is the significant particle velocity normal to the pipe and  $T_u$  is the wave velocity zero up-crossing period. Substituting the scaled quantities and collecting terms leads to the following dimensionless equation of motion, see Eqs. (3.1 - 3.3):

$$\left[ \frac{2LKN}{\pi} + C_M \right] \frac{\partial^2 y'}{\partial t'^2} = \frac{2C_D (s')}{\pi} \left[ Ku' - \frac{\partial y'}{\partial t'} \right] \left[ Ku' - \frac{\partial y'}{\partial t'} \right] + \frac{2f_c C_L (s')}{\pi} \left[ Ku' - \frac{\partial y'}{\partial t'} \right]^2 + K \cdot C_M \frac{\partial u'}{\partial t'} - f_c \frac{2LK^2}{\pi} - F_R' \quad (3.12)$$

This dimensionless equation illustrates that the relative pipeline displacement ( $y'$ ) depends on the quantities  $K$ ,  $L$ ,  $N$  (defined below), and  $u'$ ,  $s'$  and  $t'$ . Analysis of the Wake model equations and response simulations has shown that  $u'$  and  $s'$  scale with the parameters  $K$  and  $M$ . This is confirmed by independent research which has shown that the forces are well predicted by using the Keulegan-Carpenter number, the velocity ratio and a representative roughness parameter. Although other forms of the dimensionless equation are possible, equation (3.12) is quite convenient since the influence from the parameter  $N$  is greatly reduced by the term  $2LKN/\pi$  being small compared to  $C_M$  for most cases of interest.



Thus, for a given sea state and without considering the dimensionless  $F_R$  term (see below), the five dimensionless groups governing the pipeline displacement are:

$$K = \frac{U_s T_u}{D}, \quad L = \frac{W_s}{1/2 \rho_w D U_s^2}, \quad M = \frac{V_c}{U_s}, \quad N = \frac{U_s}{g T_u}, \quad T = \frac{T_1}{T_u} \quad (3.13)$$

where  $T_u$  and  $V_c$  are zero up-crossing velocity period and steady current velocity for the sea state, respectively;  $D$  and  $W_s$  are the pipeline outer diameter and submerged weight per unit length, respectively;  $g$  and  $\rho_w$  are the acceleration of gravity and mass density of water, respectively.  $T_1$  is the sea state duration in seconds. The wave velocity  $U_s$  and the current velocity  $V_c$  refer to the components normal to the pipeline.

The scaling parameters  $K$ ,  $L$ ,  $M$ ,  $N$  and  $T$  can be interpreted as follows:  $K$  is a Keulegan-Carpenter number (loading parameter),  $L$  is a ratio between pipe weight and hydrodynamic forces (pipe weight parameter),  $M$  is a current to wave velocity ratio,  $N$  is a representative acceleration for the sea state and  $T$  represents the number of waves in the sea state.

The non-linear soil resistance term,  $F_R$ , which is a rather complex function of the response history and soil properties can be expressed by the above groups plus the following additional dimensionless parameters (Sotberg and Remseth, 1987b).

$$I_s = \frac{k_s}{\gamma_w D}, \quad J_s = \frac{\alpha_s}{T_p}, \quad G = \frac{\gamma_s'}{\gamma_w}, \quad S = \frac{W_s}{D S_u} \quad (3.14)$$

where  $I_s$  is a non-dimensional elastic soil stiffness parameter, and  $J_s$  is a non-dimensional soil damping parameter.  $G$  is the relative soil weight and  $S$  is the shear strength parameter classifying the clay soil. The various quantities in the above scaling parameters are:

- $k_s$ ,  $\alpha_s$  are the elastic and damping constants for the soil, respectively.
- $\gamma_w$  is the specific gravity of sea water.
- $\gamma_s'$  is the submerged soil gravity, i.e.  $\gamma_s' = \gamma_s - \gamma_w$  where  $\gamma_s$  is the specific gravity of the soil.
- $S_u$  is the remoulded shear strength for clay.

#### MDOF Model

The equation of motion for a pipeline near a fixed constraint is, (Fig 3.9):

$$\frac{\partial^2}{\partial x^2} (EI \frac{\partial^2 y}{\partial x^2}) - \frac{\partial}{\partial x} (P \frac{\partial y}{\partial x}) + (m + C_M - 1) \frac{\partial^2 y}{\partial t^2} = F \quad (3.15)$$

where  $F$  is the total external environmental force per unit length on the pipeline, and  $EI$  and  $P$  are the pipeline stiffness and axial tension, respectively.

The scaling scheme used above may be applied to equation (3.15), with the distance  $x$  scaled by the length  $l$ , i.e.  $x' = x/l$ , where  $l$  may be taken as representative of the distance from the end restraint beyond which the pipeline stiffness does not significantly affect the displacement (see Fig 3.9 below). The equation reduces to the following dimensionless equation of motion for the pipeline:

$$\frac{EID}{l^4 W_S} \cdot \frac{2K^2 \cdot L}{\pi} \cdot \left[ \frac{\partial^4 y'}{\partial x'^4} - \frac{Pl^2}{EI} \cdot \frac{\partial^2 y'}{\partial x'^2} \right] + \left( \frac{2LKN}{\pi} + C_M \right) \frac{\partial^2 y'}{\partial t'^2} = F' \quad (3.16)$$

where  $F'$  is the scaled total external force.

Two additional dimensionless groups,

$$I_1 = \frac{EID}{l^4 W_S}, \quad \text{and} \quad P_1 = \frac{Pl^2}{EI}, \quad (3.17)$$

are thus introduced for bending and tension effects, respectively.

The maximum bending strain,  $\epsilon$ , in the pipeline may be approximated by:

$$\epsilon = \frac{D_S}{2} \frac{\partial^2 y}{\partial x^2}, \quad \text{or} \quad (3.18a)$$

$$\epsilon = 1/2 \frac{DD_S}{l^2} \frac{\partial^2 y'}{\partial x'^2} \quad (.18b)$$

where  $D_S$  is the outer steel diameter of the pipeline. From (3.18b) a scaled maximum strain,  $\epsilon'$ , is

$$\epsilon' = \epsilon \cdot \frac{l^2}{DD_S} \quad (3.19a)$$

or, by preserving the group  $I_1$

$$\epsilon' = \frac{\epsilon}{D_S} \cdot \left( \frac{EI}{DW_S} \right)^{1/2} \quad (3.19b)$$

Similarly the scaled tension is:

$$P' = P \cdot \left( \frac{D}{EIW_S} \right)^{1/2} \quad (3.20)$$

The invariant quantity for maximum bending stress is similar to that for strain.

### Governing Parameters

Simulation results (Sothberg and Remseth 1987b) verified that the importance of the parameter  $N$  over a reasonable range of values was small and that it could be neglected. Further, the response was found not to be sensitive to reasonable variations in the parameters  $I_S$ ,  $J_S$ , and for clay soils,  $G$ . Therefore, the most significant parameters for response scaling were found to be  $(K, L, M, S)$  for clay soils and  $(K, L, M, G)$  for sand soils.

Fig 3.7, from Lambrakos et al. (1987b) illustrates that the lateral displacement response is scaled nearly exactly when the  $N$  parameter is included.

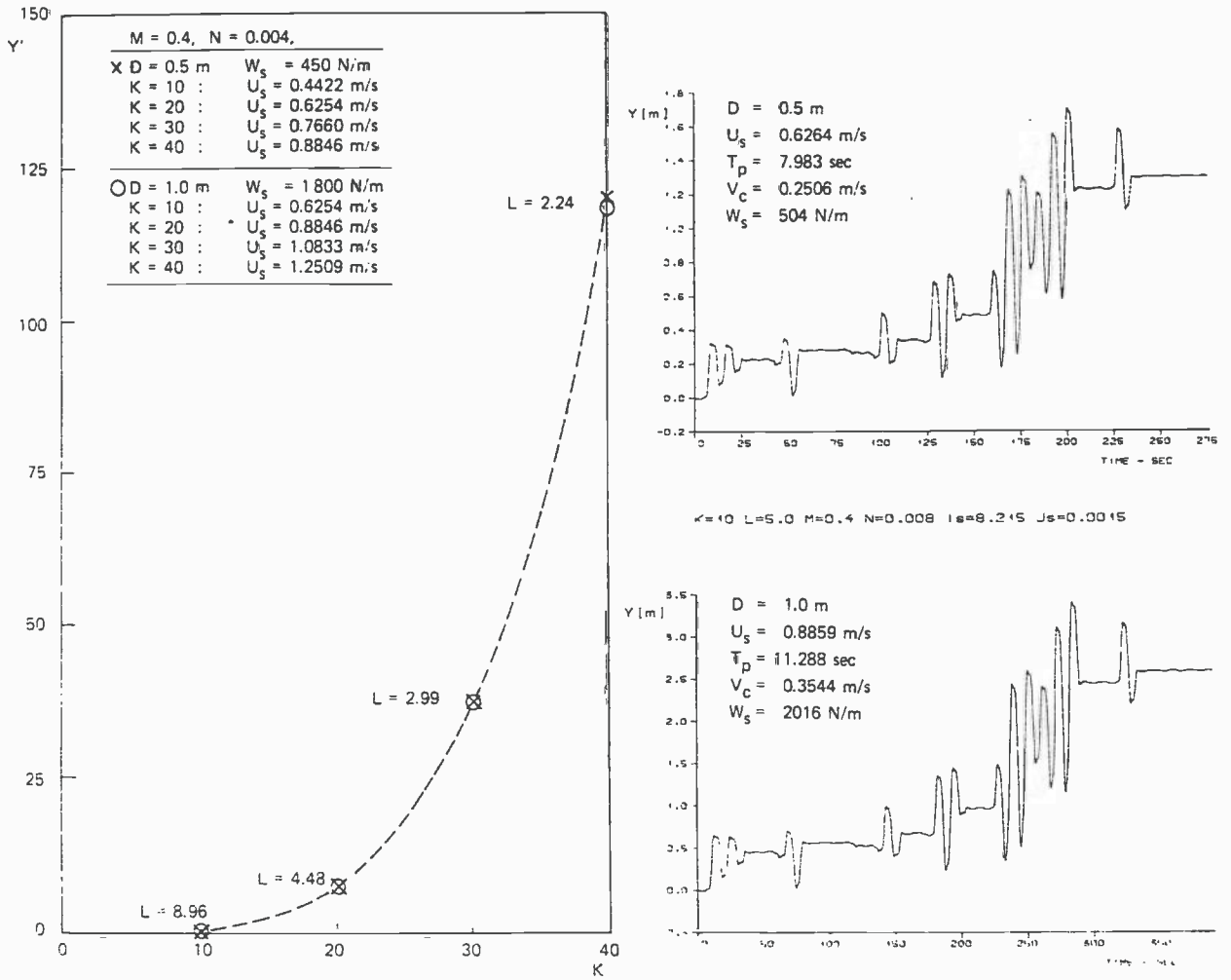


Fig 3.7 - Scaling of pipeline displacement

The low sensitivity to the N parameter is demonstrated in Fig 3.8 for sand ( $G=0.82$ ), where scaled displacement results are shown for three values of the parameter N corresponding to a four-fold variation. The variations in the displacements for the cases with the same K, L, M values are within 10 %. The same holds true for clay. Reasonable variations in the parameters  $I_s$  and  $J_s$  have an even smaller effect on the scaled displacement.

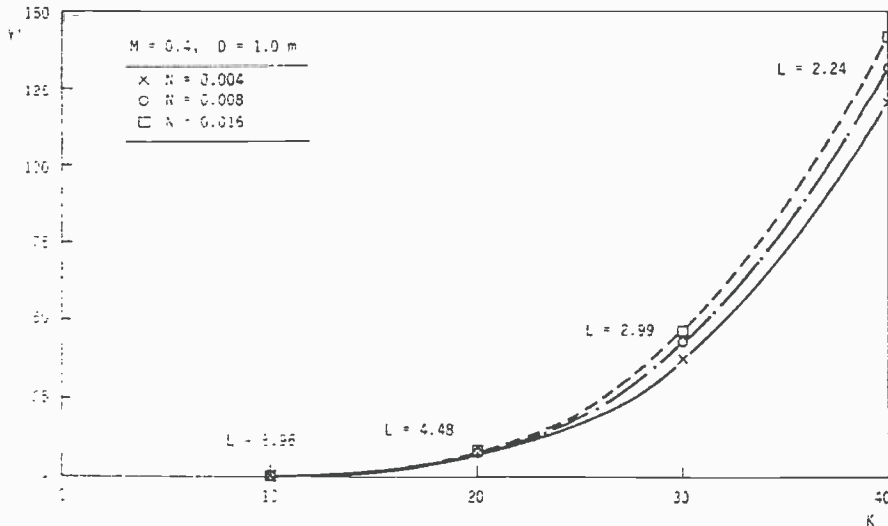


Fig 3.8 - Displacement variation with K and N

The generalization of strain response is expected to be subjected to a larger variability than for displacement as several scaling groups have to be approximated. However, the adequacy of the above scaling for strain is illustrated on Table 3.1. The invariant maximum strains ( $\epsilon'$ ) for each of the two sets of cases (three cases in each set) are within 1.5 % for a given set of the parameter K, M and L, even though the pipe-line steel diameter and wall thickness were varied in each set. The unscaled maximum strains,  $\epsilon_{max}$ , exhibit a variation of about 25 %.

Table 3.1 - Strains for different pipeline cross sections

K = 30    M=0.4    N=0.008    D=1.12 m $\gamma_s = 8.2 \text{ kN/m}^3$						
Case no	L	$W_s$	$D_s$	$t_s$	$\epsilon_{max} \%$	$\epsilon'_{max}$
1	3.15	4630	.9144	.0222	.218	1.24
2	3.15	4630	.800	.017	.262	1.22
3	3.15	4630	.9144	.014	.270	1.22
-----						
4	2.57	3780	.9144	.0222	.286	1.80
5	2.57	3780	.800	.017	.350	1.80
6	2.57	3780	.9144	.014	.356	1.78

#### Generalized Response Data Base

A generalized response data base has been established (Sotberg and Remseth 1987b) in terms of the above non-dimensional parameters. A large number of response simulations with the PONDUS program were performed for a set of tabulated values of the scaling parameters. Examples of such generalized response data and the main findings from these are presented in the following.

The response data base covers both the strain response and the lateral displacement given in terms of the non-dimensional parameters. Strain is related to a structural pipeline model shown in Fig 3.9 with a fully clamped end. This model represents a conservative formulation of the boundary condition for a real pipe section as some relaxation of the rotational and translation degrees of freedom will always be present and will reduce the extreme bending strain. Limitations in this model are discussed further in section 5.5.1 for different application.

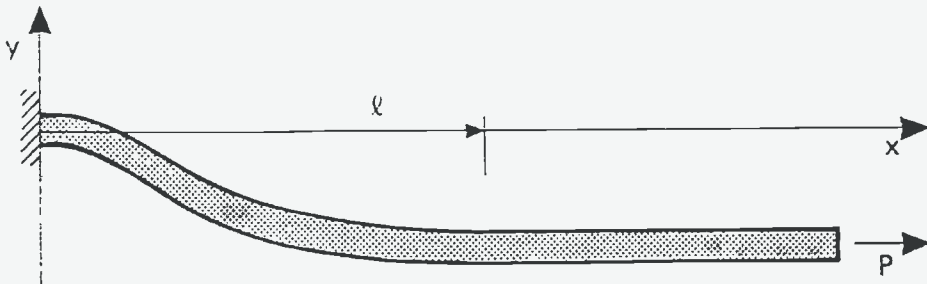


Fig 3.9 - Pipeline structural model for strain calculations

The displacement response simulations are based on a short pipeline segment with both ends free. This model gives accurate estimates for displacement of the free end of the above model, if the free end is far enough from the restraint (about 1000 m for a 1 m diameter pipeline) so that the displacement is not affected by the pipe stiffness. This has been verified through simulations. The presence of tension will in general decrease the displacement, however, for relative small displacements the tension effect will be small and is not included in this model.

Illustration of typical generalized response data is given in Fig 3.10 for the pipe displacement as a function of  $K$  and  $L$ , for  $M = 0.2$  for medium sand soil ( $\gamma'_s = 8.2 \text{ kN/m}^3$ ), based on the Pipestab soil resistance and hydrodynamic models. Displacement refers to the position of the pipeline after a 3 hour simulation.

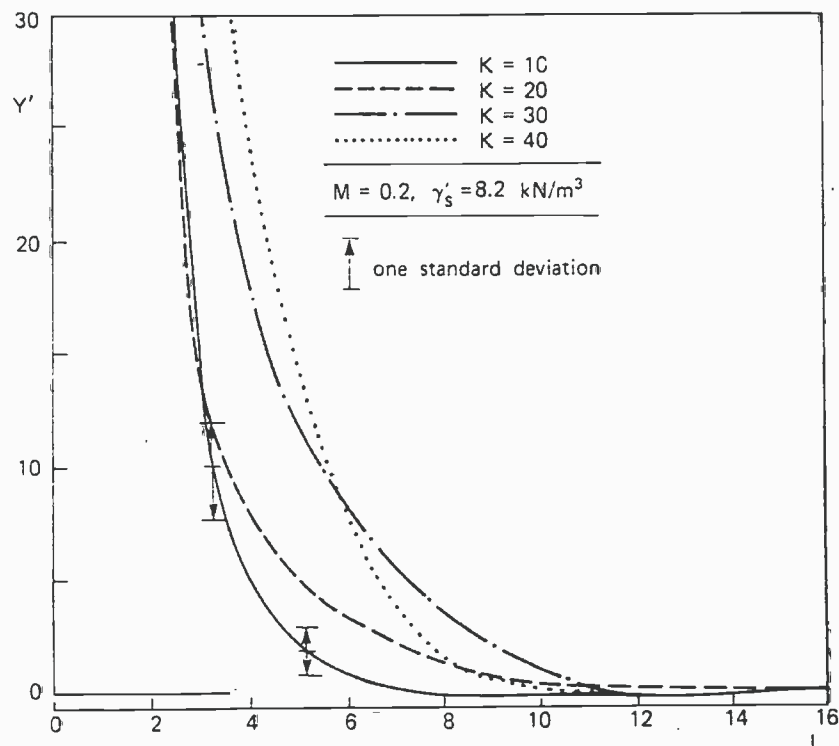


Fig 3.10 - Generalized displacement for sandy soil

The curves for both types of soil material show a rather sharp increase of lateral displacement at low levels of the weight parameter,  $L$ . The net displacement remains small as the  $L$  parameter decreases from the static stability (no movement) value. This holds true for  $L$  values as small as one third of the static stability value (or submerged pipe weight for a given sea state). For further decrease in  $L$ , the displacement increases sharply.

The displacement curves for clay display a much sharper rise than those for sand. This is related to the tendency of the pipe to dig into soft and medium clay soils during its motion. Increased penetration increases the soil resistance and makes the pipe "stable" for a greater range of  $L$  values than that for sand. For very low  $L$ -values (light pipes), however, the pipe will not dig in, and will experience extremely large displacements. The steep part of the



clay displacement curves suggests the existence of a critical weight parameter, above which the pipe will be stable due to digging in.

Simulation indicated that the total strain can be partitioned into a mean component,  $\epsilon_0$ , related to the mean lateral displacement, and a fluctuating (dynamic) component,  $\epsilon_1$ , dependent on the dynamic loading (wave) only. Fig 3.11a shows that the mean stress (indicated by the solid line) is correlated to the level of lateral displacement, and similarly Fig 3.11b illustrates that the dynamic component is dependent on the intensity of wave loading only. Dynamic stress ranges relative to yield stress are given in Fig 3.11b versus peak-to-peak wave velocity normal to the pipe. The individual points (x) correspond to waves occurring at arbitrary time points during the sea state when the mean displacements differ; yet the points fall on a straight line, i.e. dynamic stress is a linear function of peak wave velocity and independent of mean lateral displacement.

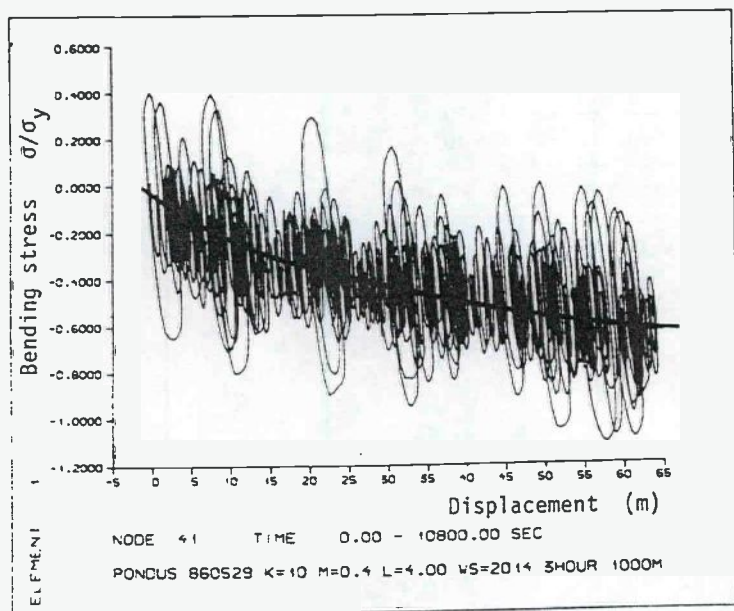
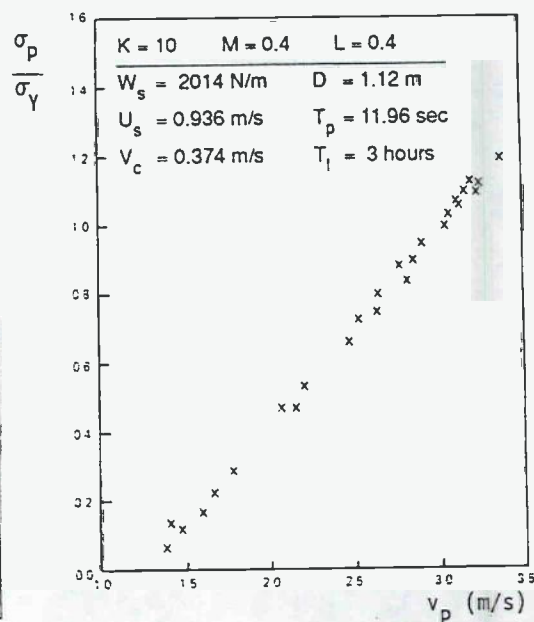


Fig 3.11 - a) Stress versus pipe displacement



b) Dynamic stress versus wave velocity

A further discussion of generalization of displacement and strain response is given by Lambarakos et al. (1987b) and Sotberg et al. (1988). The most significant observations are given below.

The non-dimensional mean bending strain component,  $\epsilon_0'$ , varies with  $K$  and  $M$  but not with  $L$  for a given lateral displacement level. There is a rather strong dependency on the current to wave velocity ratio,  $M$ , which reflects the dependency on the mean value of the loading. Fig 3.12a gives mean bending strain for medium sand for various values of  $K$  and  $\gamma'$ , and  $M = 0.2$ .

Fig 3.12b shows fluctuating strain,  $\epsilon_1'$ , dependent on the  $L$ -parameter for specific values of the parameter  $K$ , for  $M = 0.2$  and a medium sand soil,  $\gamma_s' = 8.2 \text{ kN/m}^3$ . Generally there is a relatively weak  $M$ -dependence in the fluctuating strain component. The strain increases sharply as the value of  $L$  becomes small.



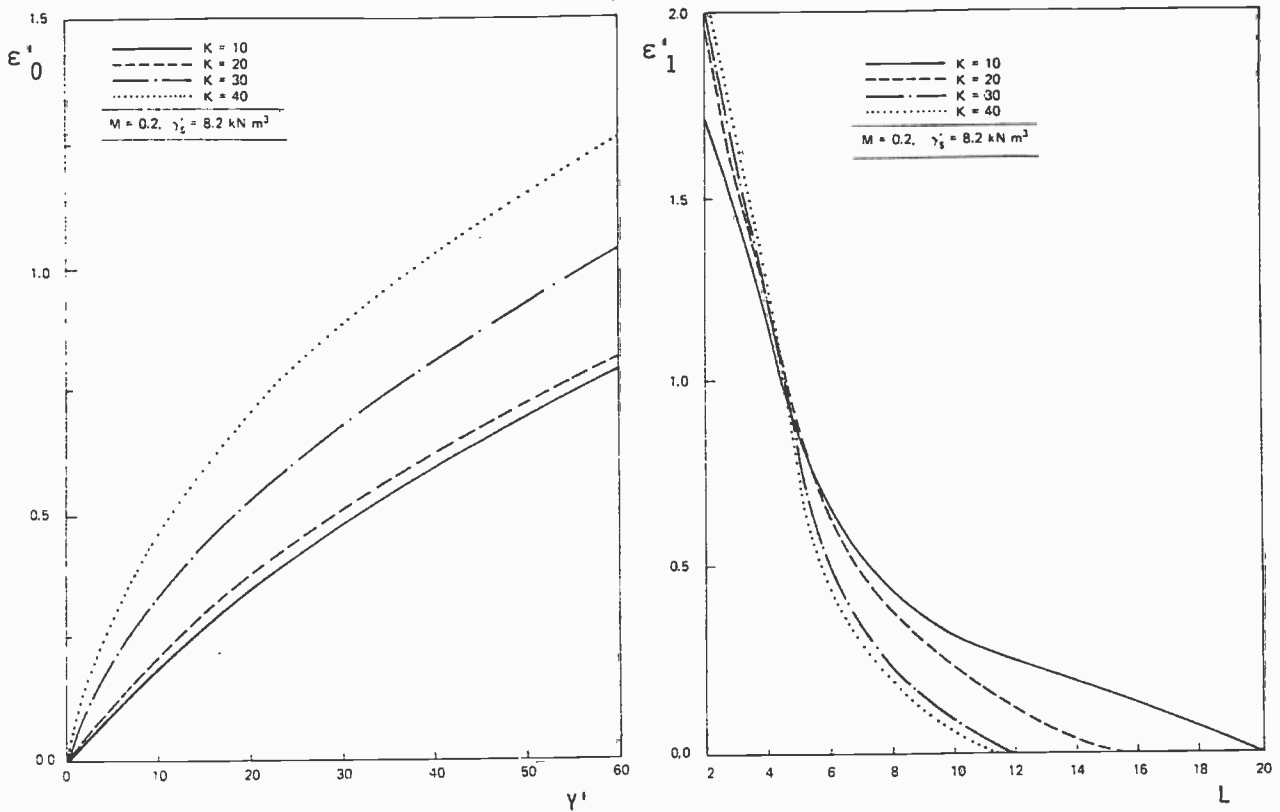


Fig 3.12 - a) Generalized mean strain

b) Generalized dynamic strain

The effect on pipe response from wave spreading and directionality has also been investigated in terms of the non-dimensional parameters  $K$ ,  $M$  and  $L$ . It has been found, Sotberg and Remseth (1987b), that the effect on pipeline response can be taken into account by using the normal component of the significant wave velocity in calculations of the non-dimensional parameters  $K$ ,  $M$  and  $L$ .

A response data base of this type has not been developed utilizing the more recently developed AGA soil resistance model and the Database hydrodynamic force model. The Database hydrodynamic force model is based on a Fourier representation of the forces, and it is principally different from the Wake force model applied above. However, it is shown (Bryndum et al. 1988 and Jacobsen et al. 1988) that the forces may be described by the parameters  $K$  and  $M$  for a given pipe roughness. The scaling parameters arising from the AGA soil model are similar to those from the Pipestab model except for the formulation of the  $F_R$  term which has a dependence on the relative soil density  $D_r$ . This parameter is however related to the parameter  $G$  for sand. For clay soil, the shear strength parameter,  $S$ , dominates.

Application of the response data base is illustrated in the next chapter in relation to a semi-probabilistic design approach and in Chapter 7 for a more complete reliability calculation procedure.

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## REVISED DESIGN PROCEDURE

### 4.1 INTRODUCTION

The shortcomings in the traditional design procedure have been pointed out in Chapter 2. The main problems are related to the inaccurate models applied for hydrodynamic and soil resistance forces and in determining the design environmental load condition.

Application of the improved models for pipe-soil interaction and hydrodynamic force described in Chapter 3, together with the traditional design criterion which requires an absolutely stable pipeline, would lead to considerably heavier pipelines than used at present. However, a general increase of design pipe weight is not realistic considering the satisfactory operational experience of existing pipelines.

A relaxed design criterion thus needs to be introduced and applied together with the improved mathematical models and computational tools. Limited pipeline movements can be allowed under extreme environmental conditions provided that the pipeline has a sufficient degree of safety against damage. Any discussion of a movement criterion should address the potential for yielding and pipe collapse due to bending, progressive ovalization and fatigue failure.

The semi-probabilistic design procedure developed by Sotberg et al. (1988), is presented in this chapter. The procedure is based on the improved mathematical modelling as developed during the Pipestab project and a relaxed design criterion with respect to on-bottom stability. Further, the method is based on utilization of generalized pipeline response data through the non-dimensional parameters described in the preceding chapter. The design load condition is defined for different phases such as installation and operation. Relevant design acceptance criteria are subsequently specified and checked.

The key features in the design procedure are outlined below:

- Specification of the long term variability of the wave and current environment.
- Transformation of the long term surface wave condition to water particle kinematics at the sea bottom.

- Specification of the design load condition for wave and current, given in terms of the return period of occurrence.
- Specification of other design parameters describing the pipeline and soil properties.
- Calculation of the design loading parameters and utilization of a generalized response data base to estimate the necessary pipe weight to meet the specified criteria for allowed pipe displacement and strain.

The design procedure is an iterative process as a variation of pipe diameter will change the design load and vice versa. The computer program PIPE, Sotberg and Remseth (1987a), was developed to conduct design according to the above procedure more efficiently.

The design method considers a deterministic modelling of hydrodynamic forces, soil resistances forces and the structural model. The models are further assumed to give correct expectation values, i.e. no bias. The long term variability in the wave and current environment is taken into account by applying model distributions. Also the response uncertainty, due to the statistical variability in the wave realization, may be estimated through a representation of this variability based on repeated simulations. The return period for design load is selected implying a semi-probabilistic measure of violation of design criteria.

This chapter describes each step of the method, including first a discussion of the response data base itself, the treatment of the necessary environmental data, design acceptance criteria and finally computer program implementation of the procedure.

## 4.2 RESPONSE DATA BASE

The response program PONDUS has been used to develop a response data base through numerous individual simulations. The following range of non-dimensional parameter variations is covered:

$$\begin{aligned} K &= 5 - 40, & M &= 0 - 0.8, & L &= 1 - 20 \\ G &= 0.7 - 1.0 & & \text{(for sand)} \\ S &= 0.05 - 8.0 & & \text{(for clay)} \end{aligned}$$

In general only one simulation was conducted for each combination of the above parameters. However, a statistical uncertainty is present in the prediction of pipeline response due to the variability of the wave realization in the time domain. This variability in response was investigated briefly for certain specific combinations of the parameters. Series of 20 simulations were conducted using different time series realizations of the same wave process (3 hour time series with random phase angles). Results from these simulations are included in the data base to give estimates of the uncertainty in response due to variability in wave realizations.

Both resulting lateral pipeline displacement and maximum strain response in the sea state have been included in the data base for application by the design program PIPE.

The Wake hydrodynamic force model and the Pipestab soil resistance model have been used in the development of the response data base. The models employ empirical parameters that have been experimentally verified over the range of parameters given above. The generalized response results included in the response data base will thus have corresponding limitations.

### 4.3 DESIGN PROCEDURE - PIPE PROGRAM

The design process utilizing the PIPE program starts with the definition of a long term wave environmental description. One method for providing such data is described below. Other less sophisticated techniques may also be used if the data basis for a given location is more limited.

#### Environmental Data

To access the response data base, environmental parameters normal to the pipe at the sea bottom of the significant orbital velocity,  $U_s$ , the zero upcrossing period,  $T_u$ , and current velocity,  $V_c$ , are needed. The return period associated with a given significant velocity,  $U_s$ , should be related to the long term velocity process at the bottom, rather than the long term surface wave process. A sea state with lower wave height but longer period may give higher near bottom velocities than, say, the 100 year surface wave sea state due to the frequency dependence of the depth attenuation. The determination of the long term velocity process at the sea bottom is properly achieved by transformation of the surface wave process, described by the joint distribution of  $H_s$  and  $T_p$ , into an equivalent joint distribution of  $U_s$  and  $T_u$  at the bottom.

The joint distribution of  $H_s$  and  $T_p$  can for example be given in terms of a measured scatter diagram or an analytical joint distribution model. For the single sea states thus defined, the velocity spectrum at the bottom is obtained by transformation of the wave elevation spectrum using linear wave theory. The spectrum of velocity perpendicular to the pipe is calculated by applying a reduction factor to account for wave directionality and short-crestedness of the waves. This calculation is given as follows:

$$S_u(\omega) = \left[ \omega / \sinh(kd) \right]^2 \cdot S_{\eta}(\omega) \quad (4.1)$$

where  $S_u(\omega)$  and  $S_{\eta}(\omega)$  are the bottom velocity spectrum and the wave elevation spectrum, respectively, (long-crested);  $k$  is the wave number and  $\omega$  circular frequency. The environmental parameters  $U_s$  and  $T_u$  at the sea bottom are then:

$$U_s = 2\sqrt{m_0} \cdot R \quad (4.2)$$

$$T_u = 2\pi \{m_0/m_2\}^{1/2} \quad (4.3)$$

where the spectral moments  $m_n$  and the reduction factor  $R$  are found from:

$$m_n = \int_0^{\infty} \omega^n S_u(\omega) d\omega \quad (4.4)$$



$$R = \left[ \int_{\theta_w - \pi/2}^{\theta_w + \pi/2} \psi(\theta, \theta_w) \cos^2(\theta_p - \theta_w) d\theta \right]^{1/2} \quad (4.5)$$

where  $\psi(\theta, \theta_w)$  is the wave spreading function,  $\theta_w$  is the mean wave propagation direction and  $\theta_p$  is the direction perpendicular to the pipe axis.

In this manner the joint distribution of  $U_s$  and  $T_u$  is established, from which values of  $U_s$  and mean  $T_u$  corresponding to specified probabilities of occurrence (return periods,  $R_p$ ) can be found.

In addition to the wave parameters  $U_s$  and  $T_u$ , a representative steady current velocity needs to be accounted for in the design process. Since the current bottom boundary layer normally is of the order of 5-10 meters thick, the pipe will be within this region of reduced velocity. It is then necessary to determine the average velocity over the pipe in the boundary layer. This may for example be done by using a logarithmic boundary layer model as described by Slaattelid et al. (1987). This model accounts for the apparent roughness caused by wave action as well as the physical bottom roughness. The model is supported by field data as discussed by Tryggestad et al. (1987) and Myrhaug (1984). The magnitude of the current velocity reduction can be considerable if wave velocities are high or pipe diameter is small. The calculated effective current is specified as input to the program.

#### Design Acceptance Criteria

For sand soil it is seen that, to avoid unnecessary conservatism, the design method should allow limited pipeline movements in extreme environmental conditions, see Chapter 7. Pipeline movements may be considered provided that the displacement induced strains are evaluated during the design check.

The displacement criteria will in general be site specific and depend on several factors, such as national regulations, distance to neighbouring structures and sea bed obstructions, width of surveyed corridor, etc.

When evaluating the strain criteria, consideration should be given to the buckling capacity and allowable ovalization. Limiting criteria may be found in relevant codes.

A more thorough discussion of pipeline capacity and design criteria is given in Chapter 5. The design method as described in the present section calculates the necessary pipe weight to satisfy a prescribed design criterion for displacement or strain for an environmental condition of specified return period.

#### Computer Implementation

The computer program PIPE was developed to conduct the stability design process according to the method described above.

A flow chart of the program is illustrated in Fig 4.1. The program requires as input long term environmental (wave) data which may be given either in terms of a specific analytical joint probability model for  $H_s$  and  $T_p$ , or as a scatter diagram of  $H_s$  and  $T_p$ . When using a scatter diagram, extrapolation beyond the limits of the diagram is obtained by fitting a two parameter Weibull distribution of  $U_s$  to the upper part of the diagram, and using a linear extrapolation



for the mean value of  $T_U$ . If neither a scatter diagram nor data for an analytical distribution are available, a Weibull distribution of  $H_s$  can alternatively be applied with a linear variation of  $T_p$  with  $H_s$ , defined by specifying  $H_s$  and  $T_p$  values for two specific return periods. Wave directionality and spreading may also be included if such information is available. The current velocity is given by the component acting perpendicular to the pipe,  $V_{c\theta}$ , multiplied by a boundary layer reduction factor according to a procedure such as the one described above. Dimensions and material density specifications of the steel pipe, corrosion coating and pipe content are given as input, together with the density of concrete coating to be used.

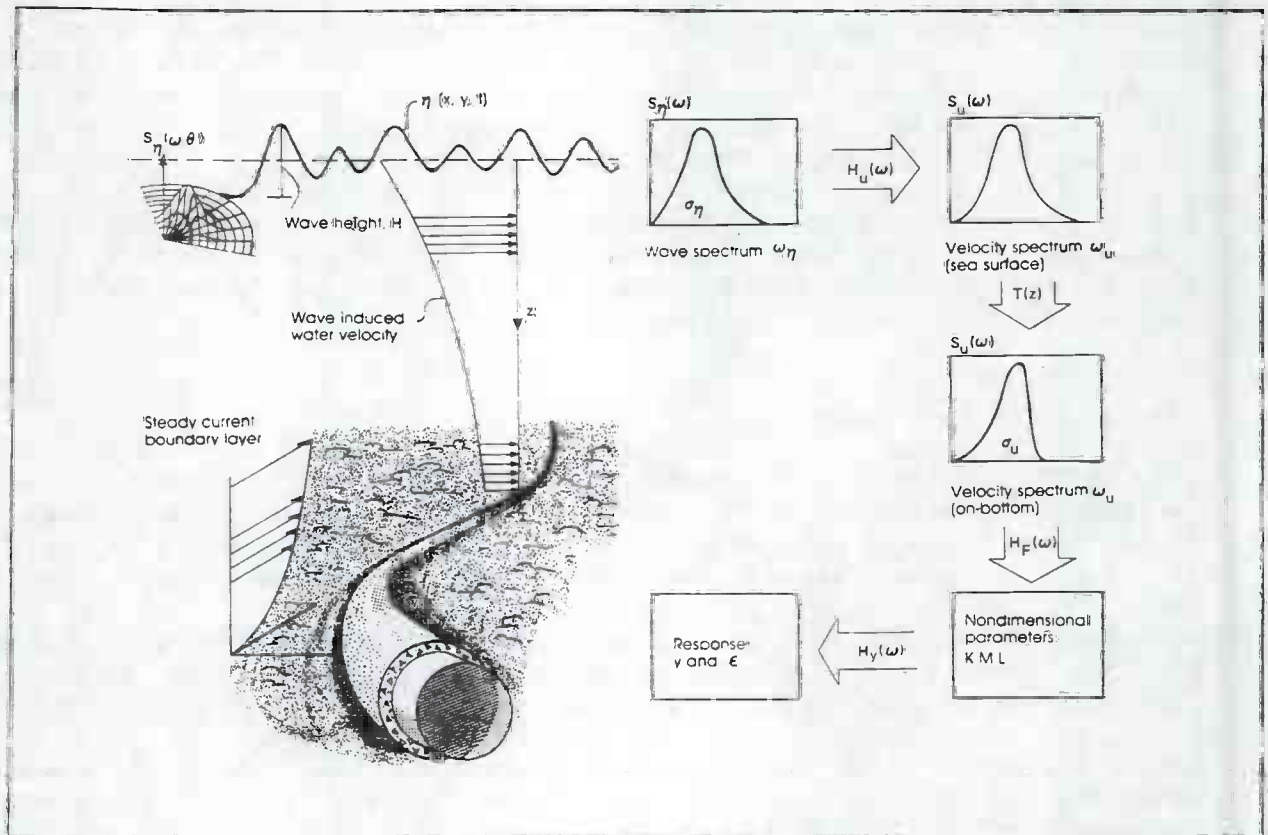


Fig 4.1 - Flowchart of the design program PIPE

For sand soil, the user further specifies the soil density, the target level of displacement or strain (i.e. design criteria) and the return period of bottom velocity for which it applies.

The program transforms the long term wave data to equivalent data for near bottom oscillatory velocity perpendicular to the pipe,  $U_s$ , and zero upcrossing period,  $T_U$ , Eqs. (4.1 - 4.5). An iterative procedure relating this environmental and pipe data to dimensionless parameters and generalized response gives the necessary submerged pipe weight and outside diameter to meet the specified criteria. The program may also calculate long term accumulated response over the specified design lifetime. This is done by a straight forward probability weighted summation of response values for all single sea states in the long term distribution. Furthermore, response can be calculated for any specified return period.

The above procedure is limited to sand soil. In the case of clay soil, a critical pipe weight, above which the pipe will be completely stable, is calculated for a specified return period of the near bottom oscillatory velocity. This weight is provided by the program from an iterative procedure similar to the one used for displacement and strain criteria for sand soil.

It should be noted that the data for strain in the response data base are obtained with a linearly elastic material model for the pipe, and should thus only be used up to the proportionality limit,  $\epsilon_p$  (appr. 0.2 %). Repeated cyclic straining at levels above  $\epsilon_p$  may lead to accumulation of strain considerably in excess of that predicted from a linearly elastic model. In order to address strain effects above the proportionality limit, the computer program PONDUS can optionally be used with a non-linear material element included. A Ramberg-Osgood model is applied for the non-linear stress-strain relationship from Murphy and Langner (1985). Plastic strain in the pipeline has been considered in the numerical study in Chapter 7.

#### 4.4 SAFETY ASSESSMENT

The above design method is based on determination of a design sea state according to a representative long term distribution of the wave environment. The pipeline response is then evaluated for that sea state by direct application of the generalized response data base. The response data base is further developed as described above, based on state of the art models which are found to give a good prediction of the physics of pipeline behaviour.

The design procedure illustrated above has been the basis for the development of the Recommended Practice RP E305, (Dnv-1988). The design program PIPE is a computerized version of the generalized design method described in RP E 305.

When considering design recommendations and codes, it is essential to evaluate the safety level implied using these. Development of the generalized pipeline response data base in PIPE is based on deterministic models for calculation of hydrodynamic forces, soil resistance forces and for the structural modelling. These models have been assumed to give correct expectation values for the recommended ranges of application, i.e. no bias in modelling is assumed. The long term wave environment may be properly described through a long term distribution model related to the amount of data available. Finally, the response uncertainty, caused by statistical variability of the wave realizations, may be accounted for by repetitive simulations for specific cases.

Selection of return period for the design wave condition implies a semi-probabilistic measure of limit state violation. If wave environment was the only random process, application of a specific return period of occurrence for the sea state, say 100 years, would then indicate directly the safety level in the design, or the probability of exceeding the design criteria applied. However, a number of parameters essential for determination of the pipeline response are random quantities, and should be properly represented. The method as described above gives no information about the relative effect (importance) of different quantities to the total uncertainty, or the real level of safety in the design, as only a few sources of uncertainties are included in the analysis.

It is hence concluded that it is still difficult to quantify the real safety level implied by using the above design method as well as the relative effect of different sources of uncertainty in parameters and models describing the pipeline system. More refined procedures are needed, which include the main sources of uncertainties, in order to evaluate the safety level and the relative importance of different factors.

Note: This is a revised extract of the introductory part of the Dr.ing thesis by Sotberg (1990). The main chapters concerning the uncertainty analysis, development of procedures for reliability analysis, and application of these methods for safety assessment of the pipeline are left out. However, the enclosed conference publications cover these topics to some extent.

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