Delft University of Technology

Document Version
Final published version

Citation (APA)
Wu, W., Xun, J., Liu, X., & Fu, M. (2025). An Integrated Model of Train Timetable and Speed Profile for Single Bi-directional Track. In J. Liu, W. Li, X. Geng, K. Zhang, H. Ji, & K. Li (Eds.), *The Proceedings of 2024 International Conference on Artificial Intelligence and Autonomous Transportation - Volume VI* (pp. 244-251). (Lecture Notes in Electrical Engineering; Vol. 1394). Springer. https://doi.org/10.1007/978-981-96-3977-9_27

Important note
To cite this publication, please use the final published version (if applicable).
Please check the document version above.

Copyright
In case the licence states "Dutch Copyright Act (Article 25fa)", this publication was made available Green Open Access via the TU Delft Institutional Repository pursuant to Dutch Copyright Act (Article 25fa, the Taverne amendment). This provision does not affect copyright ownership.
Unless copyright is transferred by contract or statute, it remains with the copyright holder.

Sharing and reuse
Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

Takedown policy
Please contact us and provide details if you believe this document breaches copyrights.
We will remove access to the work immediately and investigate your claim.

This work is downloaded from Delft University of Technology.

Green Open Access added to TU Delft Institutional Repository

'You share, we take care!' - Taverne project

https://www.openaccess.nl/en/you-share-we-take-care

Otherwise as indicated in the copyright section: the publisher is the copyright holder of this work and the author uses the Dutch legislation to make this work public.

An Integrated Model of Train Timetable and Speed Profile for Single Bi-directional Track

Wenxing Wu[1], Jing Xun[1](✉), Xiaoyu Liu[2], and Minxue Fu[1]

[1] State Key Laboratory of Advanced Rail Autonomous Operation, Beijing Jiaotong University, Beijing, China
{22120278,Jxun,24120194}@bjtu.edu.cn

[2] Delft Center for Systems and Control, Delft University of Technology, 2628CD Delft, The Netherlands
x.liu-20@tudelft.nl

Abstract. In this paper, we consider the integrated model of train timetable and speed profile for a single bi-directional track. This problem aims to schedule train passes along single track from opposite directions to avoid conflicts. It is assumed that there are different types of stations along the track, and trains can only pass each other simultaneously at stations that have sidings. Focusing on the operation scenario, we propose an integrated model of train timetable and speed profile optimization. To address the nonlinear constraints generated by the meeting conflicts in the single bi-directional operation, the Big-M method is used to transform the constraints into linear constraints incorporating binary integer variables such as meeting and train priority. Thus, a mixed integer linear programming (MILP) model is constructed, which can be solved using the Gurobi solver. The model is formulated with two objective functions: one aims to minimize the time deviation of trains running in pairs considering passenger satisfaction, and the other is to minimize the total train travel time. The proposed model is applied to solve the problem based on a single-line urban railway and the feasibility of the model is verified. The results show that the total train travel time is effectively reduced by 29.8%.

Keywords: Bi-directional Track · Timetable Scheduling · Integrated Model

1 Introduction

With the development of intelligence and automation of urban rail transit, how to further improve the synchronized and coordinated operation of the system is a topical matter of current research [1]. Under the multiple operation modes of urban rail transit, the single bi-directional operation mode is applicable to single-track scenarios and exists at both national and international levels. In urban rail networks, single, double and even multiple tracks are interconnected, which means that the effectiveness of single bi-directional schedules significantly impacts the overall quality of the network schedule. Therefore, under the background of integration, it is of great practical significance and challenge to explore the single bi-directional operation scenario.

© Beijing Paike Culture Commu. Co., Ltd. 2025
J. Liu et al. (Eds.): AIAT 2024, LNEE 1394, pp. 244–251, 2025.
https://doi.org/10.1007/978-981-96-3977-9_27

In order to break the system barriers and improve operational efficiency, the study of integrating train timetable and speed profile has become a popular concern in the field today [2, 3]. On the one hand, the train timetable is the key of scheduling, which determines the train arrival time, running time, and dwell time, on the other hand, the train operation control strategy determines the section running time as Fig. 1. Therefore, to obtain the optimal solution of the whole system, it is necessary to conduct integrated research on the train timetable and speed profile. Many scholars have studied double-track or multi-track lines based on the integration [4], but single tracks are different from double-tracks or multi-track, with strong coupling and complexity of meeting impact factors. Single bi-directional operation mode means that only one track is used for the upstream and downstream, with the opposing trains constrained to each other. When a single track block occurs, the operation mode can optimize running efficiency.

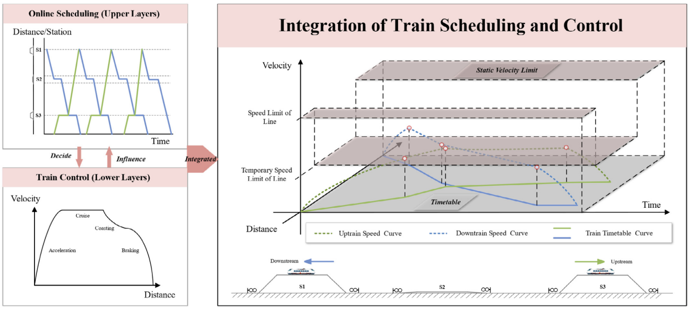

Fig. 1. Integration of train scheduling and control.

As early as 1970s, for the single-track railway scheduling problem, Szpigel first applied operations research to generate train timetable [5]. Further research has led to the development of various programming models for the description of single bi-directional scheduling problem. Li et al. [6] constructed a nonlinear mixed integer programming model and proposed a heuristic algorithm based on the prediction of global conflict distribution. In order to further study the complexity and the lower bound of single track train scheduling, Harbering et al. [7] confirmed that for any fixed number of blocks and different cases, the problem can be solved polynomially in the number of trains and the maximum block length by dynamic programming. Considering both strict and relaxed cases, Xu et al. [8] establish two MILP models for integrated optimization, which realizes simultaneous locomotive allocation and train scheduling. Yang et al. [9] constructed a MILP model by using the integer variable of train occupancy order, which aims to minimize the total travel time and operation cost. It is proved by simulation that the train operation mode of running in pairs has higher overall efficiency and shorter computation time than the hybrid mode. However, it is not considered that the deviation of the running time of opposite trains when running in pairs, which may affect the passenger satisfaction with the efficiency of bi-directional operation.

The main contributions of this paper are as follows. Under the integrated development of urban rail transit, a MILP model is constructed to express the meeting-crossing constraints using integer variables such as train priority, considering the minimization of the train travel time and the deviation of opposite trains running time. Based on the data of a certain urban rail single line, the proposed integrated model is utilized to calculate and verify the validity of the method.

2 Problem Statement

Under the single bi-directional operation mode, there are different types of stations affecting the train operation. To simplify the problem, these stations are considered as a group of sites, which can be classified into three types.

1) The first type of site, denoted as **S1**, represents a common station that including sidings.
2) The second type of site, referred to as **S2**, has only a single track and does not allow trains to meet.
3) The third type of site, **S3**, known as a passing loop, is similar to **S1** but serves only as a meeting site for opposing trains and typically does not allow for stopping.

Similar to **S2**, it is not possible to have a meeting within a track section, which is also a challenging issue to consider when optimizing the train timetable. We will limit the operational constraints of stations according to the type of sites, which will be described in the next chapter. Furthermore, this problem also takes into account the arrival priority of opposing trains during meeting, which contributes to providing factors of greater interest to operators [10]. If trains in opposite directions occupy the corresponding section, preventing upstream or downstream trains from reaching the next section, this is known as a deadlock situation.

3 An Integrated Model of Train Timetable and Speed Profile for Single Bi-directional Line

3.1 Train Timetable Constraints

The departure and arrival time constraints for upstream trains are shown in (1)–(2), and the same applies for downstream.

$$\tau_{i,s}^{\min} \leq t_{i,s}^{d} - t_{i,s}^{a} \leq \tau_{i,s}^{\max} \tag{1}$$

$$r_{i,s-1}^{\min} \leq t_{i,s}^{a} - t_{i,s-1}^{d} \leq r_{i,s-1}^{\max} \tag{2}$$

The headway constraints for upstream or downstream trains traveling in the same direction as (3)–(4). These constraints can be linearized using the Big-M method [11].

$$h_{\min}^{s} \leq |t_{i,s}^{d} - t_{i-1,s}^{d}| \leq h_{\max}^{s} \tag{3}$$

$$h_{\min}^{s} \leq |t_{i,s}^{a} - t_{i-1,s}^{a}| \leq h_{\max}^{s} \tag{4}$$

Due to the simultaneous operation of trains in opposite directions in the single bi-directional operation mode, the trains in opposite directions must satisfy the minimum headway interval constraints.

The headway intervals of opposing trains are constrained as follows:

$$h^o_{\min} \le |t^a_{i,s} - t^d_{j,\varepsilon}| \le h^o_{\max} \tag{5}$$

$$h^o_{\min} \le |t^d_{i,s} - t^a_{j,\varepsilon}| \le h^o_{\max} \tag{6}$$

3.2 Meeting-Crossing Constraints

In the train timetable of single bi-directional operation, there is a nonlinear constraint between opposing trains, which is the deadlock situation mentioned above. The relationship refers to the meeting-crossing constraint of opposing trains, which exists not only in **S2**, as in (7), but also in the track section of opposing trains as (8).

$$(t^d_{i,s-1} - t^a_{j,\varepsilon+1}) \cdot (t^a_{i,s} - t^d_{j,\varepsilon}) > 0 \tag{7}$$

$$(t^d_{i,s-1} - t^a_{j,\varepsilon+1}) \cdot (t^a_{i,s+1} - t^d_{j,\varepsilon-1}) > 0 \tag{8}$$

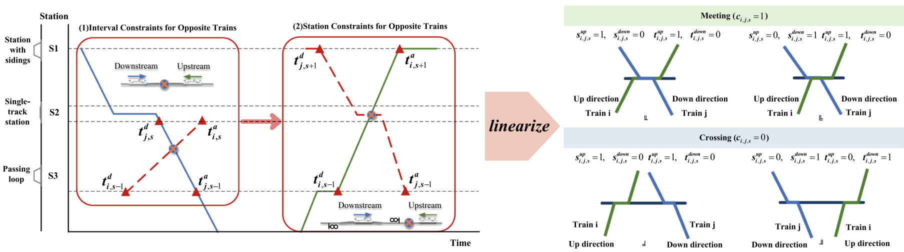

Fig. 2. Illustration of nonlinear constraint for meeting of opposite trains

In order to facilitate the representation of the same station being served by opposing trains, we set the platform order for the upward direction to s and the opposite direction to ε. Thus, $\varepsilon = \mathbf{S} - s + 1$, and **S** is the number of sites. To linearize the nonlinear constraints (7)–(8) generated by the single bi-directional operation, the Big-M method is used to associate continuous dynamic variables with discrete logistic variables, which requires the establishment of corresponding binary variables as described below. To summarize the linearization process, we plot how intersecting train running lines are linearized into four types of scenarios using integer variables in Fig. 2. The first two types correspond to situations where opposite trains meet, while the last two correspond to scenarios where opposite trains cross.

A meeting binary variable $c_{i,j,s}$ is set for the sites that may generate a meeting-avoidance at a single-track platform or passing loop. The relevant constraints on $c_{i,j,s}$ are described by:

$$\sum_{j \in \mathrm{J}} c_{i,j,s} \leq 1, \ \forall i \in \mathrm{I}, s \in \mathrm{S} \tag{9}$$

$$\sum_{i \in \mathrm{I}} c_{i,j,s} \leq 1, \ \forall j \in \mathrm{J}, s \in \mathrm{S} \tag{10}$$

$$t_{i,1}^{d} - t_{j,\mathrm{S}}^{a} \leq M \cdot (1 - \sum_{s \in \mathrm{S}} c_{i,j,s}) \tag{11}$$

$$t_{i,\mathrm{S}}^{a} - t_{j,1}^{d} \geq -M \cdot (1 - \sum_{s \in \mathrm{S}} c_{i,j,s}) \tag{12}$$

Constraints (9)–(10) respectively mean that any pair of opposing trains i and j can only meet with the reverse train j or i at station s at most once. Constraints (11)–(12) imply that for all station s, any pair of opposing trains i and j can meet at most once at the station where a meeting may occur if certain conditions are met.

The constraints on two pairs of train sequence binary variables are described as:

$$s_{i,j,s}^{up} + s_{i,j,s}^{down} = 1 \tag{13}$$

$$t_{i,j,s}^{up} + n_{i,j,s}^{down} = 1 \tag{14}$$

The two pairs of train sequence binary variables are related to the arrival sequence of opposite trains, thus they can be calculated by the priority of trains. When the train i and j meet at station s, that is $c_{i,j,s} = 1$, the train sequence variables is determined by the priority of the opposite trains. Taking $s_{i,j,s}^{up}$ and $t_{i,j,s}^{up}$ as the example, $s_{i,j,s}^{down}$ and $t_{i,j,s}^{down}$ can be calculated by (13)–(14).

$$s_{i,j,s}^{up} = \begin{cases} 1, & \text{if } p_i \leq p_j \\ 0, & \text{if } p_i > p_j \end{cases} \tag{15}$$

$$t_{i,j,s}^{up} = 1, \ \forall p_i, p_j \tag{16}$$

The constraint (7) for **S2** can be transformed into (17), with the single-track section constraints (8) similarly transformed. h_{saf} is the minimum safety interval between meeting.

$$\begin{cases} t_{i,s-1}^{d} - t_{j,\varepsilon+1}^{a} \geq h_{saf} - M \cdot s_{i,j,s}^{up} \\ t_{j,\varepsilon+1}^{a} - t_{i,s-1}^{d} \geq h_{saf} - M \cdot s_{i,j,s}^{up} \\ t_{i,s+1}^{a} - t_{j,\varepsilon-1}^{d} \geq h_{saf} - M \cdot s_{i,j,s}^{down} \\ t_{j,\varepsilon-1}^{d} - t_{i,s+1}^{a} \geq h_{saf} - M \cdot s_{i,j,s}^{down} \end{cases} \tag{17}$$

3.3 Objectives

The objective function consists of two parts: minimizing the total train running time and reducing the deviation in single trip duration for bi-directional train services.

$$J_1 = \sum_{i \in \mathrm{I}} T_i^{up} + \sum_{j \in \mathrm{J}} T_j^{down} \quad (18)$$

$$J_2 = \sum_{i \le N, j \le N, i=j} |T_i^{up} - T_j^{down}|, N = \min(|\mathrm{I}|, |\mathrm{J}|) \quad (19)$$

where T_i represents the total travel time of train i along the entire route, and T_j similarly, which can be calculated using the decision variables related to train operation time. N denotes the smaller value between **I** and **J**. It is used to compare the travel time deviations of pairs of opposing trains.

4 Case Study

In this chapter, we propose two train timetables for comparative validation. One is an empirically based manually prepared timetable, and the other is the integrated solution proposed in this paper. It is worth noting that the integrated model is solved using the commercial solvers Gurobi 11.0.1.

4.1 Parameters Setup

The line utilized in this case contains 14 stations, and the specific parameters are shown in Table 1, including the minimum/maximum dwell time, distance between stations and speed limit. In particular, the minimum dwell time at terminal stations can be regarded as the minimum turnaround time. To ensure that the single bi-directional operation retains flexibility for operational adjustments, the train departure interval is set at 560 s.

According to previous studies [12, 13], it is known that the train inter-site running time can be set within a suitable range based on the minimum train inter-site running time. Therefore, we calculate the minimum inter-site running time through train dynamics and line parameters, and set the maximum inter-site running time to 1.2 times this value.

4.2 Experimental Results

We consider dispatching 9 train services in each direction. Figure 3 illustrates the difference between the manually prepared timetable and the timetable generated by the integrated model. The total travel duration in the manual timetable is 30,390 s, while the integrated timetable achieves a duration of 21,331 s, resulting in approximately 29.8% time savings.

The integrated model can generate the optimal timetable in a short time while maintaining flexibility for train operation adjustments. Additionally, it effectively meets the fundamental requirements of manually prepared timetables.

Table 1. Simulation line parameter

Sites	Type	$\tau^{\min}(s)$	$\tau^{\max}(s)$	Inter-site distance(m)	Speed limit(km/h)
1	**S1**	30	90	–	–
2	**S3**	30	45	276	20
3	**S2**	30	90	251	25
4	**S3**	0	45	154	20
5	**S2**	30	90	238	25
6	**S3**	30	45	380	20
7	**S2**	30	90	311	20
8	**S3**	0	45	315	15
9	**S2**	30	90	115	45
10	**S1**	30	90	236	20
11	**S1**	30	90	547	20
12	**S3**	0	45	651	40
13	**S2**	30	90	290	25
14	**S1**	30	90	461	25

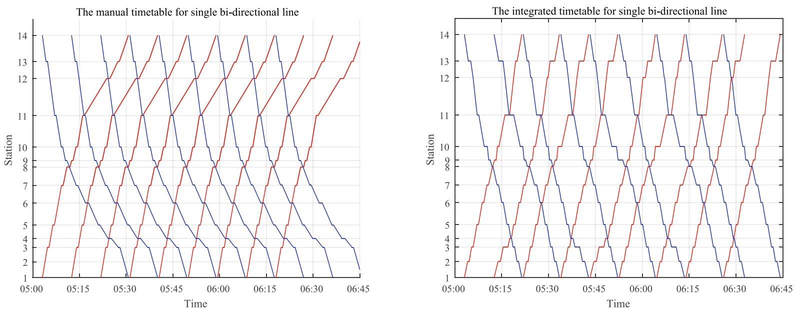

Fig. 3. Comparison of the train timetable

5 Conclusions

This paper introduces an integrated model for single bi-directional operations in urban rail transit. The model optimizes train scheduling by linearizing conflict constraints for single-track platforms using binary integer variables related to meetings and train priority. It minimizes total train travel time while accounting for deviations in paired train running times. Through a case study, it is confirmed that the model can effectively enhance the line capacity of the single bi-directional operation mode. This improvement provides valuable decision support for urban rail transport scheduling.

Acknowledgments. This research is supported by National Key R&D Program of China (2024YFE0104400) and Beijing Natural Science Foundation (L231028).

References

1. Zhuo, S., Miao, J., Meng, L., Yang, L., Shang, P.: Demand-driven integrated train timetabling and rolling stock scheduling on urban rail transit line. Transportmetrica A: Transport Sci. **20**(3), 2181024 (2024)
2. Yin, J., Pu, F., Yang, L., D'ariano, A., Wang, Z.: Integrated optimization of rolling stock allocation and train timetables for urban rail transit networks: a benders decomposition approach. Transp. Res. Part B: Methodol. **176**, 102815 (2023)
3. Li, S., Liu, R., Gao, Z., Yang, L.: Integrated train dwell time regulation and train speed profile generation for automatic train operations on high-density metro lines: a distributed optimal control method. Transp. Res. Part B: Methodol. **148**, 82–105 (2021)
4. Liu, X., Dabiri, A., Xun, J., De Schutter, B.: Bi-level model predictive control for metro networks: Integration of timetables, passenger flows, and train speed profiles. Transp. Res. Part E: Logistics Transp. Rev. **180**, 103339 (2023)
5. Szpigel, B.: Optimal train scheduling on a single line railway. Oper. Res. Int. J. **7**, 244–351 (1973)
6. Li, F., Sheu, J.B., Gao, Z.Y.: Deadlock analysis, prevention and train optimal travel mechanism in single-track railway system. Transp. Res. Part B: Methodol. **68**, 385–414 (2014)
7. Harbering, J., Ranade, A., Schmidt, M., Sinnen, O.: Complexity, bounds and dynamic programming algorithms for single track train scheduling. Ann. Oper. Res. **273**, 479–500 (2019)
8. Xu, X., Li, K., Lu, X.: Simultaneous locomotive assignment and train scheduling on a single-track railway line: a simulation-based optimization approach. Comput. Ind. Eng. **127**, 1336–1351 (2019)
9. Yang, L., Ji, Y., Li, D.: Optimization of train schedule with simultaneous station track allocation on a bi-directional single-track suburban railway. In: 2021 International Intelligent Transportation Systems Conference (ITSC), pp. 2380–2386. IEEE (2021)
10. Yang, F., et al.: Single-track railway scheduling with a novel gridworld model and scalable deep reinforcement learning. Transp. Res. Part C: Emerg. Technol. **154**, 104237 (2023)
11. Williams, H.P.: Model building in mathematical programming. John Wiley & Sons (2013)
12. Yin, J., Yang, L., Tang, T., Gao, Z., Ran, B.: Dynamic passenger demand oriented metro train scheduling with energy-efficiency and waiting time minimization: mixed-integer linear programming approaches. Transp. Res. Part B: Methodol. **97**, 182–213 (2017)
13. Shi, J., Sun, Y., Schonfeld, P., Qi, J.: Joint optimization of tram timetables and signal timing adjustments at intersections. Transp. Res. Part C: Emerg. Technol. **83**, 104–119 (2017)