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# Turbulence transition in shear flows: chaos in high-dimensional spaces

Bruno Eckhardt

Fachbereich Physik, Philipps-Universität Marburg, 35037 Marburg, Germany J.M. Burgerscentrum, Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands

#### Abstract

The study of the transition to turbulence in parallel shear flows without linear instability of the laminar profile has profited immensely from the application of dynamical systems ideas. Studies of the transition in plane Couette flow and pipe flow, in particular, have shown that the transition is connected with the appearance of 3-d coherent structures that form a chaotic saddle which shows up in a transient turbulent dynamics. It is remarkable that these concepts, initially developed for low-dimensional systems, also work in such a high-dimensional setting. The present note contains a brief summary of key features and a short list of references for further reading.

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### 1. Introduction

Pipe flow, plane Couette flow and boundary layers show turbulent behavior without a linear instability of the underlying laminar profile. Accordingly, the well established routes to chaos and turbulence through sequences of instabilities that give rise to progressively more complex states cannot apply in their original form since the first step, the linear instability of the laminar profile, is missing. Experimentally, one finds that the flow rates above which turbulence can be observed are not well characterized and cover a range of values, that turbulence is transient and shows characteristics of a strange saddle rather than a chaotic attractor, and that there is a transition from localized turbulent patches to a spreading phase with spatio-temporal chaotic dynamics. For recent reviews, see [1-7]. The extension of dynamical systems theories and concepts to high-dimensional spaces has provided the framework in which many of these phenomena can be explained and studied. In the following I will highlight four elements to which we

have contributed: the formation of coherent structures, the transient lifetimes of the turbulent state, the possible transition to persistent turbulence in spatially extended flows, and the identification of edge states intermediate between laminar and turbulent.

## 2. Exact coherent states

Typical chaotic attractors in low-dimensional systems can be analyzed through persistent structures such as fixed points or periodic orbits [8]. Adopting this point of view to turbulent states suggests to search for persistent structures with relatively simple spatial and temporal characteristics. The temporally simplest states are fixed points (as identified in plane Couette flow, [9-11]) or travelling waves (as in pipe flow, [12,13]). These states typically appear in saddle node bifurcations and are dynamically unstable. Nevertheless, they show up transiently during the evolution of the flow [14,15]. The complete bifurcation structure for one family of states in pipe flow has been analyzed in [16] and a similar analysis for plane Couette flow, where the bifurcations are simpler, is under way (T. Kreilos and B. Eckhardt, in preparation).

#### 3. Transient turbulence and lifetimes

Much of the variability in the critical Reynolds numbers that are quoted in the literature [5] can be attributed to the fact that even if a turbulent state is realized, it does not persist forever but can decay [17,18]. Much information is carried in the distribution of lifetimes of localized turbulent patches, which in all cases studied turns out to be exponential, i.e. the probability P (t) to be turbulent at time t varies like  $P(t) \approx exp(-t/\tau(Re))$  [17-22]. This exponential decay is characteristic of the escape from a strange saddle. The mean lifetime  $\tau(Re)$  increases with Reynolds numbers, as is to be expected. According to the most complete studies [21, 22], the lifetimes increase superexponentially, very much like  $\tau(Re) \approx exp(a exp(b Re))$ . While this quickly becomes very large, it does not diverge at a finite Reynolds number, so that these localized perturbations will not show a transition to a persistent chaotic attractor.

#### 4. Spatio-temporal dynamics and percolation transition

In pipe flow, turbulence is localized in the form of puffs (at lower Re) and slugs (at higher Re) [23]. As the Reynolds number is increased, one finds that puffs can split and spread into the neighboring laminar regions [23,24]. The fraction of space covered by turbulence therefore increases with Re [25]. In the limit of an infinite system size the transition from localized to spreading turbulence is connected with a transition in the asymptotic state from one with vanishing turbulence to one with finite coverage. This transition is considered to be in the universality class of directed percolation [26,27].

#### 5. Edge tacking and edge states

The coexistence of laminar and turbulent flows (even if they are only transient) implies the existence of some boundary between small perturbations that relax to the laminar profile and stronger ones that become turbulent. Using the technical tool of edge tracking [28, 29] it is possible to follow trajectories that neither relaminarize nor become turbulent for very long times. Typically, they will converge to an invariant object in this subspace, the so-called edge state. The boundary between laminar and turbulent is then formed by the stable manifold of this co-dimension one relative attractor [30]. In spatially extended systems, these edge states are localized [31, 32], consistent with the expectation that a localized perturbation should be sufficient to initiate turbulence in the system.

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