Passenger-Oriented Timetable Rescheduling in Railway Disruption Management

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Passenger-Oriented Timetable Rescheduling in Railway Disruption Management

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Delft University of Technology, 2019

Passenger-Oriented Timetable Rescheduling in Railway Disruption Management

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Preface

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Delft, November 2019

Yongqiu Zhu

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Chapter 1

Introduction

1.1 Background

Railway systems are important in passenger transportation. In every day life, many people take the train for commuting or leisure. In the Netherlands for instance, there are around 1.3 million trips by train per day (NS, 2018). Thus, train services are expected to be as reliable as possible to make sure passengers can travel as planned.

Unfortunately, railway systems are vulnerable to various unexpected events, like infrastructure failures, extreme weather, or accidents. These events are called *disruptions*, which usually result in complete or partial track blockages. As a result, the planned timetable that specifies the departure/arrival time of each train at each station will be infeasible, and then has to be rescheduled to become feasible again. This is called *disruption management*. Ghaemi et al. (2017b) divides a disruption into three phases, including the transition phase from the planned timetable to the disruption timetable, the stable phase of performing the disruption timetable, and the recovery phase from the disruption timetable to the planned timetable.

In practice, disruption management relies on contingency plans that are manually designed beforehand. If a disruption occurs, a suitable plan will be chosen, and further adjusted by traffic controllers. This is because a contingency plan only deals with the stable phase of a disruption so that additional adjustments are still needed to this plan to handle the transition and recovery phases. This usually takes a long time and imposes much workload on traffic controllers, while the resulting rescheduled timetable may not be optimal due to the manual adjustments. Furthermore, it is unknown how long a disruption will last at the moment that the disruption starts. Therefore, traffic controllers reschedule the timetable based on a predicted duration, and will repeat doing so if a new duration is predicted.

It happens that a new disruption occurs when a previous disruption is still ongoing, and that there are trains that are to run through both disrupted areas on the basis of their original schedules. Currently, no contingency plans are designed for such multiple-disruption cases, and the contingency plans corresponding to each of these disruptions may conflict with each other and thus are not helpful. Under these circumstances, traffic controllers have to reschedule the timetable based solely on their own experiences, which is extremely time-consuming.

Until now, passengers have been barely considered by traffic controllers when rescheduling a timetable. It is unavoidable that some trains are cancelled in a rescheduled timetable, and the passengers who originally planned to take the trains have to re-plan their journeys. However, based on the current rescheduled timetable, it is usually difficult for these passengers to find preferred alternatives that do not take longer than their planned journeys. Sometimes, even though such alternatives are available, passengers may not be able to board the corresponding trains due to insufficient vehicle capacities and then have to re-plan again, which normally will increase their travel times further.

To improve railway disruption management so that it becomes more efficient, operator-friendly, and passenger-friendly, it is necessary to establish an intelligent decision support system. On the one hand, the support system should be able to handle different disruption scenarios by rapidly generating the corresponding rescheduled timetables that can be implemented in practice and are optimal from the perspectives of operators and passengers. On the other hand, it should be able to predict the distribution of passengers for a given rescheduled timetable to identify potentially crowded trains and give insights into possible solutions. This thesis develops methods for both purposes, particularly focusing on the kind of disruption that results in complete track blockages between two stations.

1.2 Challenges for railway disruption management

1.2.1 Improving the performance of a rescheduled timetable

The dispatching measures that are commonly used to adjust a timetable include rerouting, retiming, reordering, cancelling and short-turning trains. The *performance* of a rescheduled timetable can be improved by applying more flexible dispatching measures than the current ones.

Short-turning means that a train ends its operation at a station before the blocked tracks and the corresponding rolling stock turns at the station to be used by another train in the opposite direction. Usually, a train is only allowed to short-turn at one fixed station (Louwerse and Huisman, 2014; Veelenturf et al., 2015), which then has to be completely cancelled rather than short-turned if the station lacks capacity (e.g. no platform tracks are available to receive the train). To reduce the possibility of cancelling a train completely, Ghaemi et al. (2018a) provide each train with two short-turning station candidates. To decrease the cancelling possibility further, it is better to provide each train with all possible short-turning station candidates, which is called *flexible short-turning*. This measure has not been considered in the literature.

The optimality of a rescheduled timetable can also be improved by *flexible stopping*: for each train the original scheduled stops can be skipped while extra stops can be added. A skipped stop could reduce the delays of the on-board passengers, and an extra stop may provide passengers with more alternative paths for rerouting. Meanwhile, some passengers may be negatively impacted by a skipped stop (the passengers who plan to board a train at a stop that is skipped) or an added stop (the passengers who are already on the train before the added stop). Therefore, it is necessary to consider both the positive and negative impacts on passengers when skipping or adding stops, which is challenging.

1.2.2 Improving the implementability of a rescheduled timetable

Apart from improving the performance of a rescheduled timetable, improving its *implementability* in practice is also important.

On the one hand, the implementability is constrained by *infrastructure capacity* like the number of tracks between two stations, the number of tracks at a station, and the availability of turning facilities at a station. Most literature focuses on either single-track railway lines or double-track railway lines, where different operational regulations should be respected for train separations. At a station level, distinguishing between platform tracks and pass-through tracks is seldom considered in the literature, which however is necessary because a train must be assigned to a platform track at a station where passengers will board or leave the train. Besides, not every station is capable of turning rolling stock, and some stations are only able to turn the rolling stock coming from a specific direction. Whether a station has turning facilities for the rolling stock coming from different directions should be explicitly considered, but is missing in the current literature.

On the other hand, the implementability is constrained by *rolling stock capacity*. For example, a rescheduled timetable cannot be implemented if there is no sufficient rolling stock to operate all scheduled train services. To ensure rolling stock availability, the rolling stock circulations that occur at the terminal stations and the short-turning stations of trains must be dealt with, but few literature studies include both kinds of rolling stock circulations when rescheduling a timetable.

1.2.3 Timetable rescheduling for uncertain disruptions

Most literature assumes that a disruption has a fixed duration that can be anticipated when the disruption starts. In the real world however, the duration of a disruption may vary over time (Zilko et al., 2016). Until now, only Zhan et al. (2016) and Meng and

Zhou (2011) have dealt with timetable rescheduling for uncertain disruptions. Both these studies use rolling horizon approaches but with a deterministic optimization model and a stochastic optimization model, respectively. To improve the robustness of a rescheduled timetable towards possibly longer or shorter disruption durations, it is necessary to take duration uncertainty into account, for which a stochastic model is required. Although Meng and Zhou (2011) propose a stochastic model, its application is restricted to a relatively simple case: a single-track railway line using two dispatching measures: retiming and reordering. No stochastic timetable rescheduling model has ever been investigated in the literature for a more complicated case: a network with both single-track and double-track railway lines using more dispatching measures including retiming, reordering, cancelling, flexible stopping and flexible short-turning.

1.2.4 Timetable rescheduling for multiple connected disruptions

Until now, most literature has focused on one single disruption, with little attention paid to multiple disruptions, particularly when multiple complete track blockages occur at different locations but the corresponding time periods are overlapping and each disruption is connected to another by at least one train line. The main challenge of rescheduling a timetable for multiple connected disruptions is that the train service adjustments for one disruption may influence the ones for another disruption, and vice versa. Such influences mainly exist among short-turning decisions: trains might be short-turned at a station at *each* side of *each* disrupted section (a section refers to the area between two stations), while the short-turning at one station may affect the short-turning at another station. This is not considered in a single-disruption timetable rescheduling model, but should be explicitly formulated in a multiple-disruption timetable rescheduling model.

1.2.5 Passenger-oriented timetable rescheduling

When rescheduling a timetable, it is necessary to estimate the potential impact of different dispatching measures on passengers and then make passenger-friendly dispatching decisions. For example, if one of two train services has to be cancelled due to insufficient rolling stock, then cancelling the train service that carries less passengers might be the best option. However, most literature assumes that the impact of cancelling any two train services are the same if both of them are intercity trains or local trains. Under this circumstance, the train service that carries more passengers could be cancelled instead in the aforementioned situation, as the impact of cancelling either train service is no different. This however may not be a passenger-friendly decision. To make a rescheduled timetable more passenger-friendly, one way is to estimate the individual impact of each dispatching decision on passengers, which is missing in the literature.

Rescheduling a timetable with dynamic passenger demand is another direction, which considers passenger behaviour in a more realistic way. Veelenturf et al. (2017) embed a timetable rescheduling model and a passenger assignment model into an iterative framework where an extra stop is added in each iteration if it reduces passenger inconvenience. Binder et al. (2017b) integrate timetable rescheduling and passenger assignment into one optimization model by retiming, reordering, cancelling, global rerouting, and inserting additional trains. The integrated model is able to generate an optimal solution, but needs more time for the computation, which affects its applicability in practice. With more flexible dispatching measures (i.e. flexible stopping and flexible short-turning) it is even more challenging to formulate a timetable rescheduling model considering dynamic passenger flows, and design an efficient solution approach to obtain high-quality solutions in real time.

1.2.6 Dynamic passenger assignment

During a disruption, it is unavoidable that some trains are completely cancelled or short-turned. Nevertheless, how passengers will respond to such major service variations has not yet been considered in the existing literature. Due to limited vehicle capacity, the path choice of a passenger may be affected by the path choice of another passenger. The information offered to a passenger and the location of this passenger when receiving the information can also affect the path choice of this passenger. It has been barely explored how providing information to passengers on changed services or train congestion at different locations will affect passenger flows and might reduce the total travel time of all passengers.

1.3 Research objectives and questions

The main objectives of this dissertation are to develop optimization models to generate rescheduled timetables for different disruption scenarios, and to propose a passenger assignment model to predict passenger flows under a given rescheduled timetable considering limited vehicle capacity and information interventions during disruptions. Therefore, the main research question is formulated as:

How to support railway disruption management by rescheduled timetables that are operator-friendly and passenger-friendly?

To answer the main question, the following key questions are defined:

- How to predict and affect passenger flows for a given rescheduled timetable? (Chapter 2)
- How to obtain a rescheduled timetable that minimizes the impact on passengers' travel plans and has a high implementability in practice? (Chapter 3)

- How to handle a disruption with uncertain duration by robust rescheduled timetables? (Chapter 4)
- How to deal with multiple connected disruptions in an efficient and operator-friendly way? (Chapter 5)
- How to formulate a timetable rescheduling model considering dynamic passenger flows, and obtain a high-quality solution rapidly? (Chapter 6)

1.4 Thesis contributions

1.4.1 Scientific contributions

- *A dynamic passenger assignment model for disruptions.* A new schedule-based passenger assignment model is proposed for disruptions where cancelling or short-turning trains are necessary. The model formulates the responses of passengers who start travelling before, during and after the disruption considering limited vehicle capacity, and applies different information interventions to affect passenger flows. It helps to evaluate a rescheduled timetable from the perspective of passengers, identifies crowded trains, and gives insights into possible solutions (Chapter 2).
- *A new timetable rescheduling model for railway disruptions.* The model considers station capacity by distinguishing between platform tracks and pass-through tracks, includes rolling stock circulations at both short-turning and terminal stations, and covers all phases of a disruption (Chapters 3, 4, 5 and 6). The dispatching measures of flexible stopping (Chapters 3 and 6) and flexible short-turning (Chapters 3, 4, 5 and 6) are introduced for the first time, and adjusted train running times due to saved/extra decelerations and accelerations when skipping/adding stops are explicitly formulated (Chapter 3). The model improves both the implementability and the performance of a rescheduled timetable.
- *A novel method to estimate the impact of different dispatching decisions on passengers.* According to passengers' travel paths on normal days, a method is proposed to estimate the impact of a decision of cancelling a service (a train run between two adjacent stations), delaying a train arrival, skipping a stop, or adding a stop on passengers. The impact of a decision includes both the number of affected passengers and the resulting lateness/earliness of these passengers, which is used as the weight of this decision in the objective of minimizing passenger delays. The passenger-dependent objective weights help the timetable rescheduling model to efficiently compute a more passenger-friendly rescheduled timetable that can also be preferred by operators (Chapter 3).

- *A new method for handling a disruption with uncertain duration.* A rolling-horizon two-stage stochastic programming model is proposed for generating a robust rescheduled timetable every time the expected durations of a disruption are renewed. The model is more likely to result in fewer train cancellations and delays, compared to a deterministic rolling-horizon approach (Chapter 4).

- *A timetable rescheduling model for multiple connected disruptions.* A multiple-disruption timetable rescheduling model is proposed, where the interactions among short-turning decisions for different disruptions are explicitly formulated. The model results in less train cancellations and/or delays, compared to the approach that uses a single-disruption timetable rescheduling model to solve each disruption sequentially (Chapter 5).

- *A solution approach to the multiple-disruption timetable rescheduling model.* A rolling-horizon approach is developed to the multiple-disruption timetable rescheduling model, which considers the periodic pattern of the rescheduled train services in the second phase of a disruption to speed up the computation. This solution method helps to handle long multiple connected disruptions in a more efficient way (Chapter 5).

- *A passenger-oriented timetable rescheduling model.* A new formulation is proposed to integrate timetable rescheduling with passenger assignment with the objective of minimizing passengers' generalized travel times, which include waiting times at origin/transfer stations, in-vehicle times and the number of transfers. This passenger-oriented timetable rescheduling model considers timetable-dependent passenger behaviour, which is more realistic and helps to reduce passengers' generalized travel times during railway disruptions (Chapter 6).

- *A solution approach to the passenger-oriented timetable rescheduling model.* An iterative solution method is proposed to solve the passenger-oriented timetable rescheduling model with high-quality solutions in an acceptable time. In each iteration, the timetable rescheduling problem is solved for all train services with restricted passenger groups considered (Chapter 6) .

1.4.2 Societal contributions

Operators can apply the developed timetable rescheduling models to deal with different disruption scenarios in a more efficient way and with fewer train cancellations and/or delays. They can relieve rolling stock rescheduling to a certain extent, because rolling stock circulations at both short-turning and terminal stations of trains are handled in all timetable rescheduling models developed in this thesis. The timetable rescheduling models can provide better alternative travel paths (with less generalized travel times) to passengers, which helps operators to keep more passengers staying in the railways after a disruption starts so that revenue loss can be reduced due to this disruption. This is also helpful to prevent revenue loss in the long run because passengers may be less likely to shift from the railways to other transport modes if their travelling experiences during disruptions are improved. The proposed dynamic passenger assignment model can help operators to foresee the potential crowded trains so that some strategies (e.g. allocating more vehicles to specific trains) can be taken in advance to prevent train congestion that may lead to prolonged train running times and possibly more train delays.

Passengers' travelling experiences during disruptions can be improved because of the better alternative train services provided by the timetable rescheduling models and the useful information on service variations and train congestion provided by the dynamic passenger assignment model. Under these circumstances, passengers are more likely to find the train services with acceptable travel times to their destinations and to board the preferred trains successfully. Therefore, the side-effects of a disruption on passengers' societal activities (e.g. working and studying) can be reduced.

The models developed in this thesis improve the resilience of the railway systems towards disruptions. By offering more reliable and punctual train services, the railways can maintain the current passenger demand on the one hand, and attract more passengers to the railways from other transport modes on the other hand. The increase of the market share of the railways is beneficial to the society, because the railways are an environment-friendly transport mode, which consumes less energy than e.g. private cars to serve the same demand.

1.5 Thesis outline

This thesis consists of seven chapters. Chapter 2 is about a passenger assignment model, while Chapters 3 to 6 focus on timetable rescheduling models. Chapter 7 concludes the thesis and points out future research directions. A visual outline is shown in Figure 1.1 followed by a brief descriptions of the main chapters.

During disruptions, some trains can become crowded due to detouring passengers whose planned trains were cancelled. Under this circumstance, some passengers may be denied to board specific trains due to insufficient vehicle capacities. Taking this into account, **Chapter 2** proposes a schedule-based passenger assignment model to predict the passenger flows for a given rescheduled timetable, where different information interventions are applied to see how passenger flows will be affected. This model can also estimate the expected travel plans of passengers in terms of a planned timetable.

Chapter 3 proposes a single-disruption timetable rescheduling model aiming to minimize the impact on passengers' expected travel paths. Passengers' expected travel paths are estimated by the passenger assignment model of Chapter 2 according to the planned timetable. The single-disruption rescheduling model deals with all phases of a disruption, and considers both station capacity and rolling stock circulations. It is further extended in three different directions in the following three chapters.

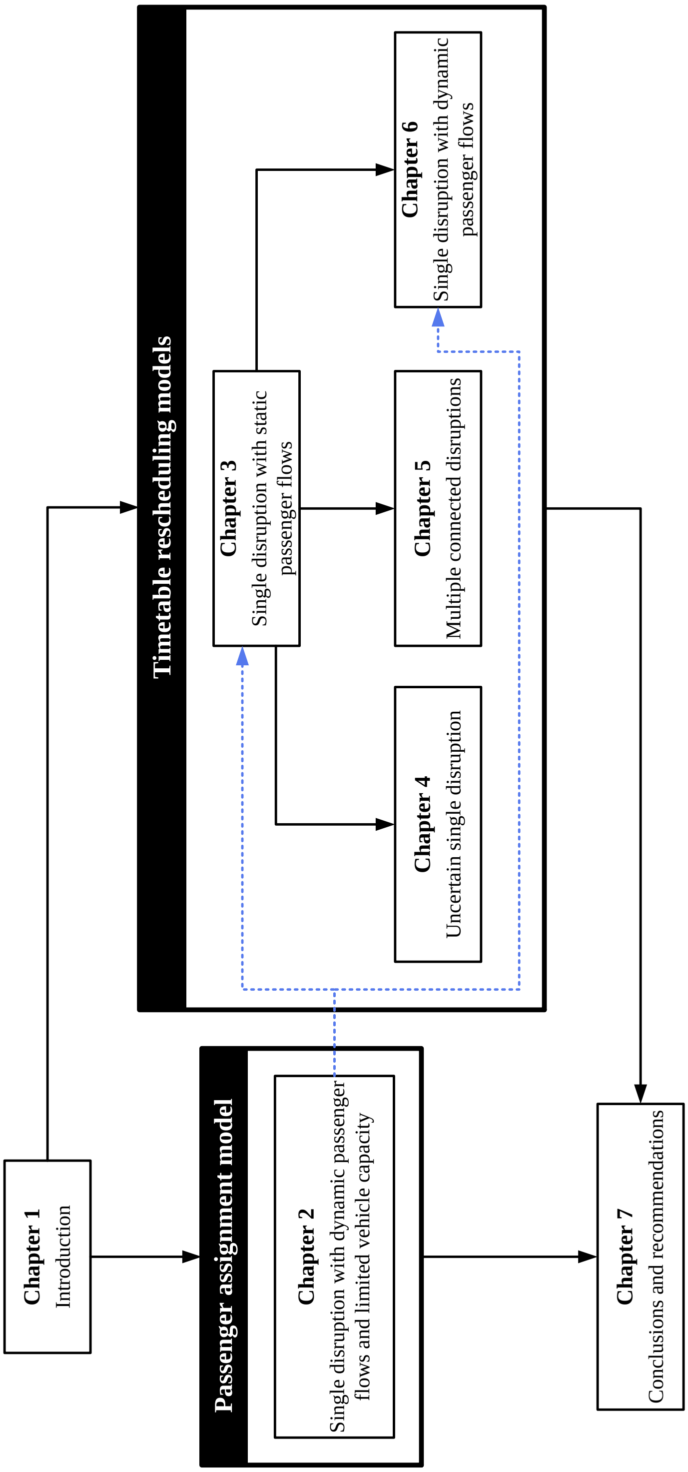

Figure 1.1: Overview of thesis structure

To handle uncertain disruptions, **Chapter 4** realizes a rolling-horizon deterministic approach based on the single-disruption model, and extends the single-disruption model to a rolling-horizon two-stage stochastic programming model that generates a robust rescheduled timetable every time the possible durations of a disruption are renewed. It is found that in most cases, the stochastic approach results in less train cancellations and/or delays than the deterministic approach.

To handle multiple connected disruptions, **Chapter 5** realizes a sequential approach based on the single-disruption model to solve each ongoing disruption in a sequential way, and proposes a combined approach based on a multiple-disruption model. The combined approach solves all ongoing disruptions together considering their combined effects. It is found that the solution computed by the combined approach is more operator friendly than the one by the sequential approach.

To minimize the impact on passengers' realized travel paths in terms of the rescheduled timetable, **Chapter 6** integrates the extended passenger assignment model of Chapter 2 and the single-disruption timetable rescheduling model of Chapter 3 into one optimization model to compute passenger-friendly rescheduled timetables. In this model, passengers are allowed to leave the railways if the alternative travel paths provided by the rescheduled timetable take much longer times than their expected travel paths.

In the end, **Chapter 7** concludes the dissertation and gives recommendations to future research and practice.

Chapter 2

Dynamic passenger assignment for major railway disruptions considering information interventions

Apart from minor updates, this chapter has been published as:

Zhu, Y., Goverde, R.M.P., 2019. Dynamic passenger assignment for major railway disruptions considering information interventions. *Networks and Spatial Economics*, in press.

2.1 Introduction

Unexpected events affect railway operations in everyday life, which are either small service perturbations called *disturbances* or relatively large incidents called *disruptions*. During disturbances, train services will be delayed, but not cancelled/short-turned which however is necessary during disruptions. Due to the complexity of handling disruptions, contingency plans are designed beforehand for different disruption scenarios. When a disruption happens, the corresponding contingency plan is selected, and possibly modified by traffic controllers in terms of the specific condition (Ghaemi et al., 2017b). However, in either the design or modification procedure, passengers who should have been put first, are as yet not incorporated directly, because traffic controllers are unable to anticipate the passenger flows over the network. As a result, many alternatives for passenger reroutings are not considered, and thus passenger travel experiences during disruptions are usually less than satisfactory.

To support passenger-oriented train service adjustments, it is necessary to have a passenger assignment model that can anticipate the distribution of passengers. Based on the model, whether a timetable is passenger-friendly or not can be evaluated, and further how to adjust the timetable in a passenger-friendly way can be guided. Until now, passenger assignment models are mostly proposed for planning purposes or disturbance management (generally regarded as *delay management*), where services are considered to be reliable or with minor perturbations. When major disruptions like complete track blockages occur, multiple dispatching measures, e.g., retiming, reordering, cancelling and short-tuning trains, are commonly applied, which result in delayed trains, changed train orders, completely cancelled trains and short-turned trains (Ghaemi et al., 2017a). As a result, the train services available during disruptions are rather different from the ones on normal days, thus leading to rather different path options to passengers. For passenger assignment models during disruptions, it is necessary to formulate the major service variations properly and model passenger responses to such major service variations accurately. Therefore, this chapter proposes a dynamic passenger assignment model taking major service variations, vehicle capacity, and time-dependent passenger all into account. A preliminary version of the model can be found in Zhu and Goverde (2017a), which is improved by introducing a new network formulation and information interventions for altering passenger behaviour in this chapter.

This chapter considers passengers' en-route travel decisions rather than passengers' pre-trip travel decisions. This means that passengers are assumed to have planned paths in mind before they actually arrive at the origin stations, however, possibly they have to re-plan their paths due to major service variations, denied boardings or train congestion. Such an assumption is justified, since nowadays passengers can rely on various travel-planner applications or the official websites of operator companies to find their preferred paths. This is particularly true for passengers who have a clear travel purpose (e.g. commuters). Thus, once disruptions occur, passengers would make en-route travel decisions by comparing the alternative paths during disruptions with their planned paths.

Passenger attitudes towards path alternatives during disruptions could be different from the ones on normal days. For example, due to reduced operation frequency during disruptions, passengers may be willing to spend more waiting times at origin/transfer stations than usual. Considering this, a new method is proposed to formulate the network with less arcs, which ensures all paths that could be chosen by passengers to be fully covered. The formulated network is a directed acyclic graph (DAG) with passenger perceived times on arcs, based on which the optimal paths perceived by passengers can be searched using efficient shortest path algorithms.

Path alternatives can be different if passengers re-plan paths at different locations and times. This chapter tracks the location of each passenger who starts travelling before, during, or after the disruption, and decides when and where he/she re-plans the path based on the information received. Information interventions are considered by delivering two kinds of information, service variations and train congestion, separately at different locations. Usually, the congestion effect is considered as the increase in travel times perceived by passengers (Cats et al., 2016; Larrain and Muñoz, 2008). Instead, this chapter aims to avoid travel time increase due to denied boarding, by using congestion information to affect passenger behaviour in the following way. Imagine that a train is highly congested when departing from a stop, and there are still many passengers wishing to board this train at its next stop. It is possible that the train is unable to handle all these passengers. Thus, only some of them can board the train successfully, while the others have to be denied. If there must be some passengers being denied for boarding a train, avoiding them to choose the train may help them find better alternative paths compared to the ones they can find after being denied. Considering this situation, if a train is potentially unable to handle all passenger demand at its next stop, part of these passengers are notified with the congestion information in order to encourage them to choose another train, while the other part of these passengers are kept unaware of such information to ensure they will stay with their choice for this train.

The key contributions of this work are summarized as follows:

- Proposing a new schedule-based passenger assignment model during major disruptions.
- Developing a new network formulation to formulate the timetable as a directed acyclic graph (DAG) with passenger perceived times on arcs.
- Taking time-dependent passenger demand, service variations, and vehicle capacity constraints into account.
- Formulating passenger responses towards major service variations, like short-turned or cancelled trains.
- Using information interventions to influence passenger behaviour.
- Dealing with passengers who start travelling before, during and after the disruption.

The remainder of this chapter is organized as follows. Section 2.2 gives an overview of the relevant work. Section 2.3 explains the network modelling approach. In Section 2.4, the proposed dynamic passenger assignment framework is shown, followed by the explanation of the main parts in the framework. Next, the time complexities of the proposed algorithms are analysed in Section 2.5. Finally in Section 2.6, a case study of a complete open track blockage in part of the Dutch railway network is performed.

2.2 Literature review

Passenger assignment models for transit systems are typically classified into schedule-based and frequency-based (Gentile and Noekel, 2016), differing in whether passengers make route choices in terms of the timetable that indicates the departure/arrival time of each train at each station. In general, frequency-based models are suitable for such transit systems where the operations are so frequent that passengers can be assumed to board the first train when waiting at a station. While in railway systems where the operation frequency is relatively low, schedule-based models are commonly used, like Binder et al. (2017a) and Rückert et al. (2015).

Some assignment models are proposed for planning purposes, for example, identifying the phenomenon of macroscopic congestion of a proposed transit system. In these models, services are assumed to be constant or affected by minor perturbations that do not require dispatching measures to be applied. For instance, Khani et al. (2015) propose three path searching algorithms to make the assignment model perform efficiently on large-scale transit networks, by assuming that the operation is reliable and vehicle capacity is infinite. With limited vehicle capacity considered, Poon et al. (2004), Hamdouch and Lawphongpanich (2008), and Binder et al. (2017a) explore the interactions between the supply and the demand over time, which differ in the used priority rules for passenger boardings while share the assumption of trains operating precisely on schedule. In practice, service variations cannot be fully avoided. Thus, Nuzzolo et al. (2001), Hamdouch et al. (2014), and Cats et al. (2016) take service variations into account, and describe the variations as irregularities of train dwell and running times that are thought to be relevant to the passenger loadings of the corresponding trains. The considered train delays do not need timetable rescheduling, which means that train orders remain unchanged and no trains are cancelled or short-turned.

When train delays cannot be absorbed completely by the time supplements reserved in the timetable, timetable rescheduling becomes necessary. A typical question under such a case is that whether a train should wait for a delayed feeder train or better depart on time (*wait-depart decision*). This problem is generally regarded as delay management, where the relevant work mainly focuses on the optimization and thus the formulation corresponding to the passenger assignment is usually simplified by some assumptions. For example, Schöbel (2001) assumes that once passengers miss a transfer connection, they would wait for a complete cycle time to catch the next connection. Kanai et al. (2011), Dollevoet et al. (2012), Sato et al. (2013), and Corman et al. (2016) consider the alternative choices that passengers might have, where the capacities of vehicles are assumed to be infinite. While most papers consider the train delays as known input to the optimization, Rückert et al. (2015) observe the train delays in real time, and predict the passenger flows due to any possible wait-depart decisions to help the dispatchers make informed decisions. In these papers, train orders can be changed, but no trains are delayed significantly or cancelled/short-turned, which however take place during disruptions.

A few papers consider the passenger assignment during disruptions. Cats and Jenelius (2014) focus on disruptions that result in trains delayed significantly. The considered case is that the tracks between two stations are totally blocked for 30 minutes, and trains queue at the station before the blocked tracks during the disruption period. When the disruption ends, all these trains are again allowed to continue the following operations, assuming that all on-board passengers in these delayed trains are unable to alight from the trains at the holding stations. For a long-duration disruption that lasts for one hour or even more, it is unlikely to hold trains at stations, but more likely to short-turn them. In such a case, on-board passengers must alight from the trains, since the trains can no longer reach their expected destinations. Binder et al. (2017b) formulate the passenger assignment as a multi-commodity problem and integrate it with the rescheduling together constituting a passenger-oriented timetable rescheduling model for disruptions. The considered demand is the passengers who start travelling during the disruption. An assumption is implicitly made that all passengers collaborate together to achieve the system optimum. In the real world, passengers may intend to reduce their personal inconvenience without considering and of course incapable of considering the impacts of their choices on the system optimum. Thus, treating passengers as rational actors is necessary, which can be implemented by introducing priority rules for passenger boardings.

The literature does not consider passengers' en-route travel decisions during *major* disruptions for which cancelling/short-turning trains are necessary. This chapter fills the gap by proposing a schedule-based passenger assignment model to formulate the changes of passenger responses from normal situations to during disruptions.

The model is based on three assumptions, which are also used in Cats and Jenelius (2014) and Binder et al. (2017b). First, at the beginning of a disruption, the exact disruption end time is known, which will not be extended or shortened. This assumption can be relaxed by embedding the proposed model into an iterative framework where at each iteration the disruption end time is updated and the model is performed again based on the renewed disruption information and the corresponding disruption timetable. The second assumption is that for the railway operators, the disruption timetable is available directly at the beginning of the disruption. This is possible when applying a real-time optimization model (e.g. Ghaemi et al. (2017a)) to compute the disruption timetable. The third assumption is that the passenger demand during disruptions is the same as on normal days. This assumption is relaxed due to setting the maximum acceptable delay of the re-planned path. In the model, a passenger can drop the railways if the delay due to the re-planned path is not acceptable. Thus, although a passenger is assumed to come to the railway origin station as planned, he/she could immediately leave if the planned path is inapplicable and the minimal delay across the current alternative paths provided by the railways exceeds the maximum acceptable delay. Such an immediate leaving is actually equal to not coming to the railways.

2.3 Event-activity network

A transit assignment model depends on the network formulation that enables travel path generation for passengers. This chapter proposes a new approach to formulate the train services as a weighted DAG based on which the optimal paths perceived by

Table 2.1: Notation of event attributes

Symbol	Description
st_e	The station of event e
tr_e	The train of event e
π_e	The occurrence time of event e

Table 2.2: Attributes of different events

Event	Attributes
Arrival event: $e \in E_{\text{arr}}$	(st_e, tr_e, π_e)
Departure event: $e \in E_{\text{dep}}$	(st_e, tr_e, π_e)
Duplicate departure event: $e \in E_{\text{ddep}}$	(st_e, tr_e, π_e)
Exit event: $e \in E_{\text{exit}}$	st_e

passengers can be quickly searched. The characteristics of railway timetables (e.g. overtakings) and the fact that passengers might choose unusual paths (e.g. the ones with long waiting/transfer times at stations) during disruptions, are all considered in the proposed network formulation. As events are used to represent nodes and activities are used to represent arcs, the formulated network is called an event-activity network. In the following, different kinds of events and activities that are necessary to formulate the network are introduced, as well as the passenger preferred weights on the activities.

2.3.1 Events

There are four types of events in the formulated network. They are arrival events, departure events, duplicate departure events and exit events, which constitute the sets E_{arr}, E_{dep}, E_{ddep} and E_{exit}, respectively. Therefore, the set of events is

$$E = E_{\text{arr}} \cup E_{\text{dep}} \cup E_{\text{ddep}} \cup E_{\text{exit}}. \tag{2.1}$$

For each event $e \in E$, the attribute st_e that indicates the corresponding station of e is assigned. Additionally for each event $e \in E_{\text{arr}}$ or E_{dep}, two more attributes tr_e and π_e are assigned, which refer to the corresponding train and occurrence time of e, respectively. An event $e \in E_{\text{ddep}}$ is the duplicate of a specific departure event with the exactly same attributes that the departure event has. One and only one duplicate is created for each departure event. The reason of creating duplicate departure event is to construct waiting and transfer activities, which is explained in more detail in Section 2.3.2. The notation of event attributes is described in Table 2.1 while the attributes of different events are shown in Table 2.2.

2.3.2 Activities

There are five types of activities in the formulated network. They are running activities, dwell activities, wait activities, transfer activities and exit activities, which constitute the sets A_{run}, A_{dwell}, A_{wait}, A_{trans} and A_{exit}, respectively. In addition, A_{wait} consists of two sub-sets that are A_{ddW} and A_{adW}, which correspond to the wait activities between duplicate departure events and the wait activities between arrival events and duplicate departure events. Namely,

$$A_{\text{wait}} = A_{\text{ddW}} \cup A_{\text{adW}}. \tag{2.2}$$

Therefore, the set of activities is

$$A = A_{\text{run}} \cup A_{\text{dwell}} \cup A_{\text{ddW}} \cup A_{\text{adW}} \cup A_{\text{trans}} \cup A_{\text{exit}}. \tag{2.3}$$

Running activities enable passengers travelling from one station to another:

$$A_{\text{run}} = \left\{ \left(e, e'\right) \middle| e \in E_{\text{dep}}, e' \in E_{\text{arr}}, tr_e = tr_{e'} \text{ and } st_e \text{ is upstream neighbouring to } st_{e'} \right\}. \tag{2.4}$$

Dwell activities enable passengers dwelling at the station in a train:

$$A_{\text{dwell}} = \left\{ \left(e, e'\right) \middle| e \in E_{\text{arr}}, e' \in E_{\text{dep}}, tr_e = tr_{e'} \text{ and } st_e = st_{e'} \right\}. \tag{2.5}$$

Wait activities and transfer activities together enable passengers waiting to board trains at origins or transferring from one train to another at other stations:

$$\begin{aligned} A_{\text{ddW}} = \{(e, e') \,|e \in E_{\text{ddep}} \text{ and } e' = \arg\min \{\pi_{e'} \,|\pi_{e'} > \pi_e : e' \in E_{\text{ddep}}, \\ tr_{e'} \neq tr_e, st_{e'} = st_e\}\}, \end{aligned} \tag{2.6}$$

$$\begin{aligned} A_{\text{adW}} = \{(e, e') \,|e \in E_{\text{arr}} \text{ and } e' = \arg\min \{\pi_{e'} \,|\pi_{e'} > \pi_e : e' \in E_{\text{ddep}}, \\ tr_{e'} \neq tr_e, st_{e'} = st_e\}\}, \end{aligned} \tag{2.7}$$

$$A_{\text{trans}} = \left\{ (e, e') \middle| e \in E_{\text{ddep}}, e' \in E_{\text{dep}}, tr_e = tr_{e'}, st_e = st_{e'} \text{ and } \pi_e = \pi_{e'} \right\}. \tag{2.8}$$

Here, (2.6) means that each duplicate departure event is linked to the next time-adjacent duplicate departure event that is at the same station but for another train. Similarly, (2.7) means that each arrival event is linked to the next time-adjacent duplicate departure event that is at the same station but for another train. Finally, (2.8) means that each duplicate departure event is linked to its original departure event.

Exit activities enable passengers to leave the railway system once arriving at the destinations:

$$A_{\text{exit}} = \left\{ \left(e, e'\right) \middle| e \in E_{\text{arr}}, e' \in E_{\text{exit}}, st_e = st_{e'} \right\}. \tag{2.9}$$

In Figure 2.1, the formulated event-activity network is shown for an example with four stations (i.e. A, B, C and D) and three trains numbered 1 to 3. The attributes corresponding to each event are enclosed in an ellipse, rectangle or circle that refer to an arrival/departure event, duplicate departure event or exit event, respectively. For instance, an ellipse with $(dep,1,B)$ represents the departure event of train 1 at station B. A path is represented by a series of time-ordered events. For example, one of the paths available for a passenger who arrives at station A after time t_1 but before time t_2 and wishes to travel to station D is: $(ddep,2,A) \rightarrow (dep,2,A) \rightarrow (arr,2,B) \rightarrow (ddep,1,B) \rightarrow (dep,1,B) \rightarrow (arr,1,C) \rightarrow (dep,1,C) \rightarrow (arr,1,D)$. This path means that the passenger boards train 2 at station A, but transfers to train 1 at station B and stays in this train until the destination (i.e. station D).

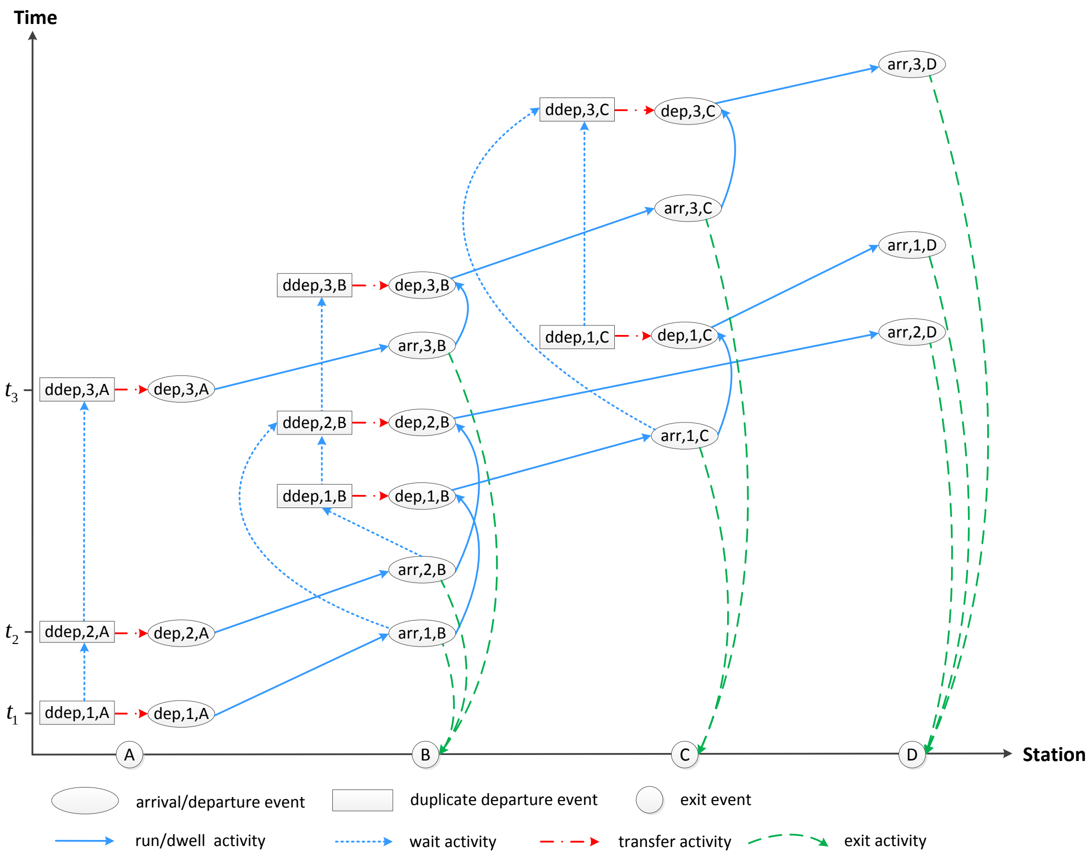

Figure 2.1: Event-activity network

2.3.3 Weights of activities

Usually, paths are perceived differently by passengers due to the path attributes like waiting time at the origin (t_{origin}), in-vehicle time (t_{vehicle}), waiting time at a transfer station (t_{trans}), and number of transfers (n_{trans}). A utility function is used to quantify the utility of each path by giving different weights on the path attributes. In this chapter, the utility of a path r is quantified as:

$$U_r = \beta_1 t^r_{\text{vehicle}} + \beta_2 (t^r_{\text{origin}} + t^r_{\text{trans}}) + \beta_3 n^r_{\text{trans}}, \tag{2.10}$$

where $\beta_1,\ \beta_2,\ \beta_3$ are the weights of the corresponding attributes of path r. Here, the values of β_1, β_2 and β_3 are set as 1, 2 (Wardman, 2004) and 10 (de Keizer et al., 2012) for each minute, respectively.

In this chapter, the path utility calculation is realized in the procedure of path searching. This means that once a path is generated by a path search algorithm (e.g. shortest path algorithm, k-shortest path algorithm, etc.), the *distance* of this path is actually the utility of this path. For this purpose, different weights are assigned to different activities, as follows.

- For each activity $a = (e, e') \in A_{\text{run}} \cup A_{\text{dwell}}$, the weight of a is set as $\beta_1(\pi_{e'} - \pi_e)$.
- For each activity $a = (e, e') \in A_{\text{ddW}} \cup A_{\text{adW}}$, the weight of a is set as $\beta_2(\pi_{e'} - \pi_e)$.
- For each passenger p, the weight of a transfer activity could be different depending on where the passenger started travelling. For an activity $a = (e, e') \in A_{\text{trans}}$, the weight of a is set as zero if $st_e = o_p$ (o_p is the origin of p). Otherwise (i.e. $st_e \neq o_p$), the weight of a is set as a fixed value β_3.
- For each activity $a \in A_{\text{exit}}$, the weight of a is set to the same positive value, since it is not used to distinguish paths.

The utility of a path is the sum of weights of all activities included in this path. The weights of all activities are contained in the set W. Thus, the formulated event-activity network is

$$G = (E, A, W). \tag{2.11}$$

2.3.4 Searching the optimal path perceived by passengers

Let a passenger p have the attributes (o_p, d_p, t_p^o) referring to the origin, destination and actual arrival time at the origin, respectively. To search the optimal path perceived by p for the travel from o_p to d_p, a pair of source and sink nodes should be given. Here, the sink node v is defined as

$$v = \left\{ e \in E_{\text{exit}} \,\middle|\, st_e = d_p \right\}, \tag{2.12}$$

and the source node u is defined as

$$u = \arg\min \left\{ \pi_e \,\middle|\, \pi_e \geq t_p^o : e \in E_{\text{ddep}}, st_e = o_p \right\}, \tag{2.13}$$

which means that the source node u is set as the duplicate departure event e at the origin station o_p, of which the occurring time π_e is closest to the passenger's arrival time at the origin t_p^o. Note that defining the source node this way takes the passenger's choice about the boarding train at the origin into account. For example in Figure 2.1, suppose (*depp*,1,*A*) is chosen as the source node for a passenger who plans to travel from station A to station D. Then, the passenger could take train 1 as the first boarding train, or also could wait a bit longer to take train 2 as the first boarding train.

With the assigned pair of source and sink node, the shortest path in utility can be searched, by performing a shortest path algorithm on the formulated event-activity network G, which by construction is actually a directed acyclic graph (DAG) with positive arc weights. Such a shortest path algorithm topologically sorts the nodes of DAG in passenger perceived times, thus making the predecessor node of an edge always appear before the successor node of the edge in a linear ordering (Cormen et al., 2009). Using the topological order, the shortest path is found in time complexity $O(A+E)$.

Here, the optimal path perceived by a passenger p is represented by r_p, which is first searched by the shortest path algorithm and then processed by excluding the duplicate departure events and the exit event. In other words, r_p only consists of the events that directly serve the path. Based on r_p, the departure/arrival events of p are extracted further, which correspond to the boarding/alighting actions. Here, B_p represents the set of departure events that correspond to the boarding actions of p at origin and transfer stations (if any), and L_p represents the set of arrival events that correspond to the alighting actions at transfer stations (if any) and destination.

Note that the way of deciding the source node in (2.13) is only for the passengers who are at the origins before travelling (and have not been denied for boarding). For the passengers who have already started travelling within trains, at transfer stations, or at the stations where they are forced to get off due to cancelled services, the ways of choosing the source nodes for searching paths are different, which are explained in Section 2.4.3.4.

2.4 Dynamic assignment model

The framework of dynamic passenger assignment during disruptions consists of three parts, as shown in Figure 2.2.

- Part I assigns each passenger to a planned path based on the original timetable.
- Part II decides which passengers are affected under the disruption timetable due to delayed/cancelled services, and also decides when these affected passengers would re-plan the paths considering different locations of publishing service variations.
- Part III simulates passenger loading and unloading procedures and also the path re-plannings of passengers because of service variations, denied boardings, or train congestion.

In what follows, the three parts are introduced successively. The used notation is described in Appendix 2.A.

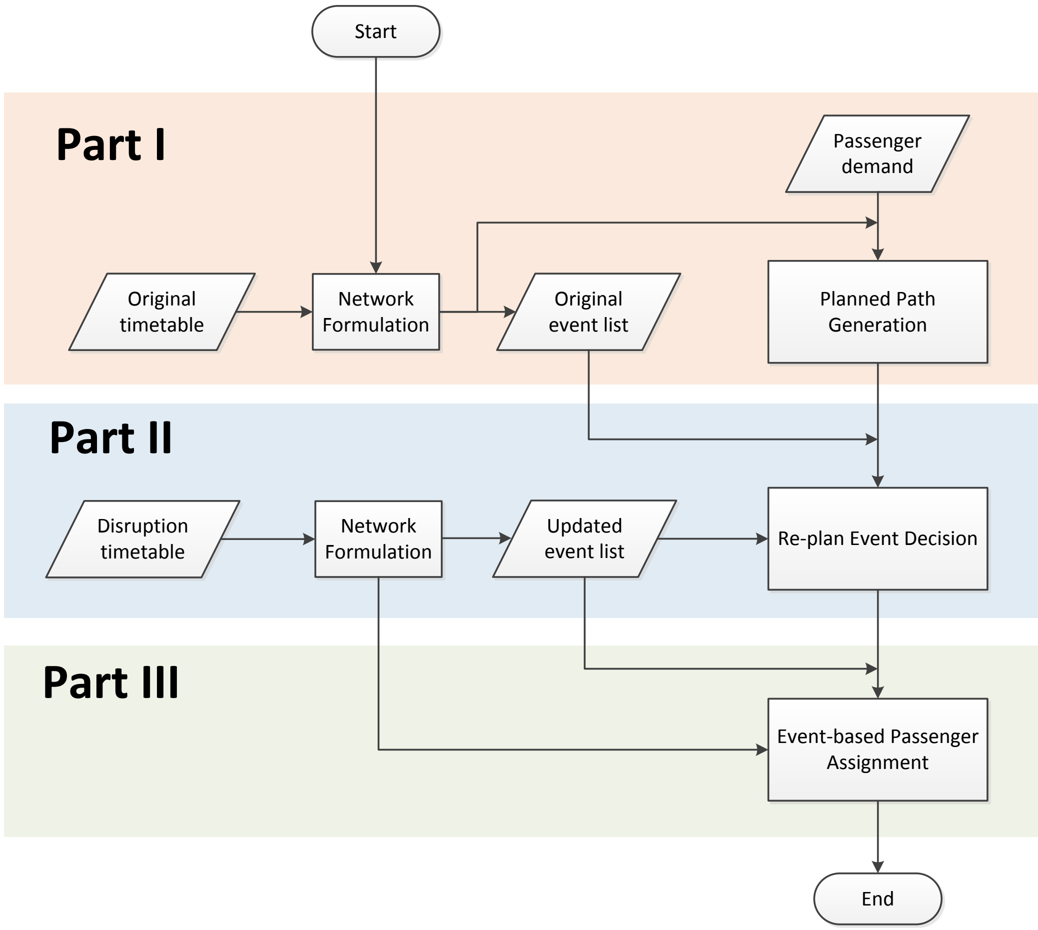

Figure 2.2: Framework of the dynamic passenger assignment model during major disruptions

2.4.1 Passenger planned path assignment (Part I)

In part I, the original timetable is formulated as an event-activity network G_{plan} where the arrival events and departure events form the original ordered event list $E^{\text{plan}}_{\text{train}}$. Passenger demand P is a given input, where each passenger $p \in P$ is described with the attributes (o_p, d_p, t^o_p) that correspond to origin, destination and arrival time at the origin, respectively. The planned path r^{plan}_p of a passenger is searched by performing a shortest path algorithm on G_{plan} assuming that a passenger chooses the path with the minimum utility as shown in (2.10) to be the planned path.

2.4.2 Passenger re-plan event decision (Part II)

In part II, the disruption timetable is formulated as an event-activity network G_{dis} where the arrival and departure events form the updated ordered event list $E^{\text{dis}}_{\text{train}}$. Comparisons are made between $E^{\text{dis}}_{\text{train}}$ and the original event list $E^{\text{plan}}_{\text{train}}$ to define the set $E^{\text{cancel}}_{\text{train}}$ or $E^{\text{delay}}_{\text{train}}$, which contains all events that are cancelled or delayed during the disruption. For each passenger p whose planned path is r^{plan}_p,

- if $r^{\text{plan}}_p \cap E^{\text{cancel}}_{\text{train}} \neq \emptyset$, then r^{plan}_p is a cancelled path either partially or completely;
- if $r^{\text{plan}}_p \cap E^{\text{cancel}}_{\text{train}} = \emptyset$ and $r^{\text{plan}}_p \cap E_{\text{delay}} \neq \emptyset$, then r^{plan}_p is a delayed path.

For the passengers whose planned paths are cancelled, they must reconsider path options. For the passengers whose planned paths are delayed only, they are also given the chance of re-planning paths in the model, while the possibility of staying with the original planned one is still kept in case no better alternative can be found. Here, the passengers whose planned paths are cancelled or delayed are called the affected passengers.

The affected passengers re-plan their paths at different locations and times, which is influenced by two factors: where they are at the moment the disruption occurs and how the information of service variations are delivered to them. The main purpose of Part II is to decide when and where an affected passenger will take the re-plan action, considering his/her location and two ways of publishing service variations, either at stations only or at both stations and trains.

2.4.2.1 Information of service variations is published at stations only

Publishing service variations only at stations means that passengers can only know about the service variations at stations. Under this circumstance, a passenger would consider re-planning either at the planned origin/transfer station or at the station where his/her train is short-turned/cancelled. Figure 2.3 (Figure 2.4) shows how to decide when and where a passenger p with a delayed (cancelled) planned path would re-plan, which is indicated by δ_p.

The basic idea of Figure 2.3 is that:

- for a passenger p whose first planned boarding time at the origin is **after** the disruption start $t^{\text{start}}_{\text{dis}}$, p re-plans at the origin ($\delta_p = \mu$),
- for a passenger p whose first planned boarding time at the origin is **before** $t^{\text{start}}_{\text{dis}}$ but the ith planned boarding ($i \geq 2$ here) at a transfer station happens **after** $t^{\text{start}}_{\text{dis}}$, p re-plans when arriving at the transfer station,
- for a passenger p whose planned boarding time at the origin is **before** $t^{\text{start}}_{\text{dis}}$ while p has no planned transfer or the planned transfers all happen **before** $t^{\text{start}}_{\text{dis}}$, p will not re-plan ($\delta_p = \emptyset$).

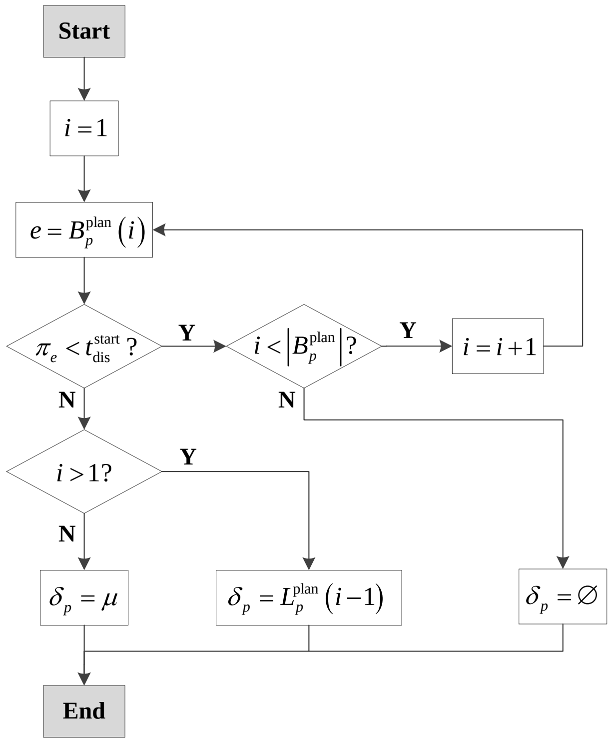

Figure 2.3: Deciding the re-plan events for passengers with delayed planned paths if disruption info is published at stations only

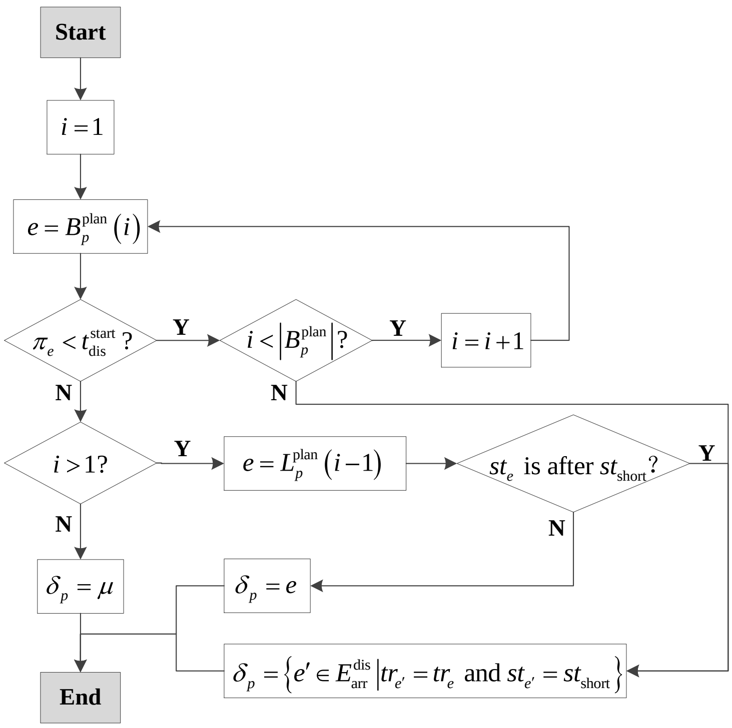

Figure 2.4: Deciding the re-plan events for passengers with cancelled planned paths if disruption info is published at stations only

Figure 2.4 shows how to decide when and where a passenger p with a cancelled planned path would re-plan. The basic idea is that:

- for a passenger p whose planned boarding time at the origin is **after** $t_{\text{dis}}^{\text{start}}$, p re-plans at the origin,
- for a passenger p whose planned boarding time at the origin is **before** $t_{\text{dis}}^{\text{start}}$ but the ith planned boarding ($i \geq 2$ here) at a transfer station happens **after** $t_{\text{dis}}^{\text{start}}$,
  - p re-plans when arriving at the transfer station, if the transfer station is upstream relative to the short-turn station,
  - p re-plans when being forced to get off from the train at the short-turn station, if the transfer station is downstream relative to the short-turn station,
- for a passenger p whose planned boarding times at the origin is **before** $t_{\text{dis}}^{\text{start}}$ while p has no planned transfer or the planned transfers all happen **before** $t_{\text{dis}}^{\text{start}}$, p re-plans when being forced to get off from the train at the short-turn station.

2.4.2.2 Information of service variations is published at both stations and trains

Figure 2.5 shows how to decide when and where a passenger p with delayed/cancelled planned path would re-plan, if service variations are published at both stations and trains. The basic idea of Figure 2.5 is that:

- for a passenger p whose planned boarding time at the origin is **after** the disruption start $t_{\text{dis}}^{\text{start}}$, p re-plans at the origin,
- for a passenger p whose planned boarding time at the origin is **before** $t_{\text{dis}}^{\text{start}}$, the type of the latest occurring event e' of the current train tr when the disruption starts determines δ_p:
  - δ_p is set as e', if e' is an arrival event,
  - δ_p is set as $e''$of which (e', e'') is a run activity, if e' is a departure event.

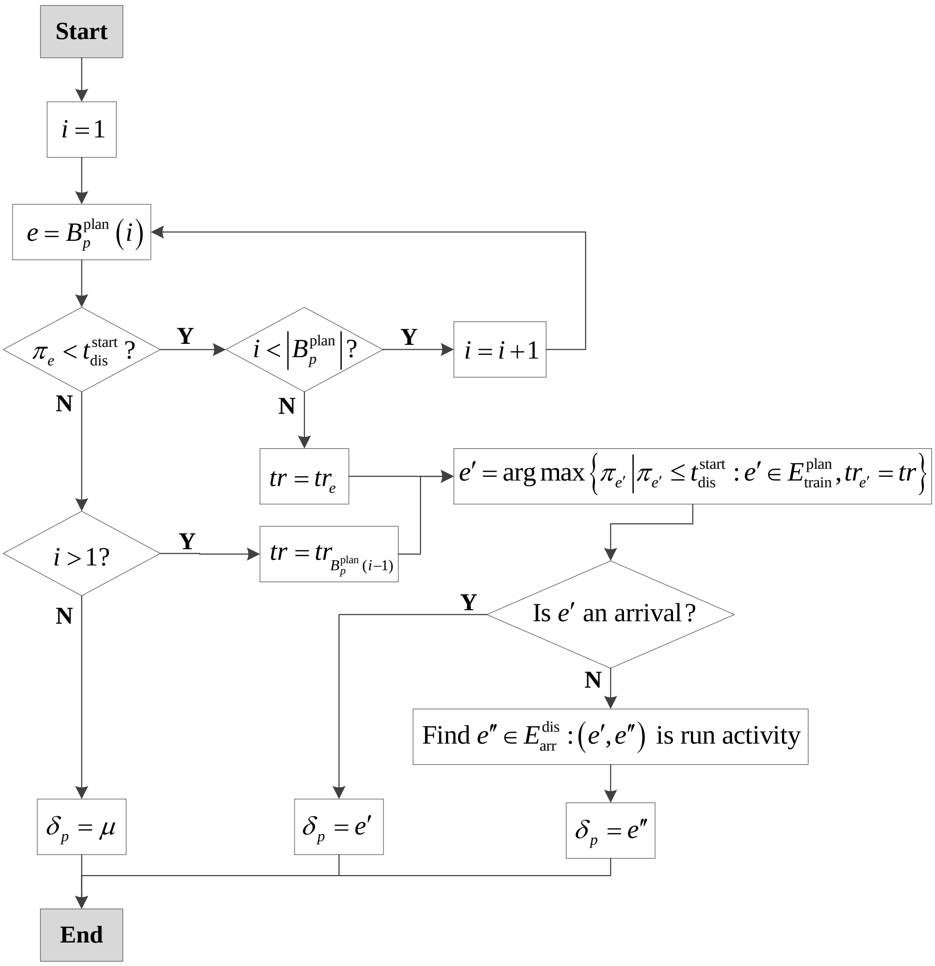

Figure 2.5: Deciding the re-plan events for passengers with delayed/cancelled planned paths if disruption info is published at stations and trains

2.4.3 Passenger realized path confirmation (Part III)

In part III, the passengers' arrivals at the origins, the loading and unloading procedures and the re-plan actions are all implemented by discrete event simulation. Publishing train congestion information on trains or not is considered to constrain some passengers' re-planned path choices. Note that publishing train congestion information at stations makes no sense for limiting passenger awareness of such information (i.e. all passengers can get any information published at stations), while publishing train congestion information at trains can let the passengers who are at the origins be unaware of such information. Therefore, an adaptive event-activity network G^*_{dis} is introduced, which is initialized as G_{dis} and further updated during the assignment by excluding some run activities of which the corresponding train congestions reach a specified level *ratio*. Passengers make re-planned path choices based on either G^*_{dis} or G_{dis} depending on whether they are informed with congestion information. This is explained in detail in Section 2.4.3.4.

In the following, the main algorithm (i.e. Discrete event passenger assignment), together with the three supporting algorithms (i.e. UpdateDep, UpdateArr and RePlan) are introduced successively.

2.4.3.1 Discrete event passenger assignment

Algorithm 2.1: Discrete event passenger assignment

Input: $E_{\text{train}}^{\text{dis}}$, P, $t_{\text{dis}}^{\text{start}}$, G_{dis}, $ratio, FullInfo, \eta$
Output: P_{arr}, P_{drop}
1 $G_{\text{dis}}^{*} = G_{\text{dis}}$;
2 Let $P_{\text{curr}}, P_{\text{arr}}, P_{\text{drop}}$ and E_{congest} all be empty ;
3 $clock_0 \leftarrow 0$;
4 **while** $E_{\text{train}}^{\text{dis}} \neq \emptyset$ **do**
5     $e \leftarrow E_{\text{train}}^{\text{dis}}(1)$;
6     $clock_1 \leftarrow \pi_e$;
7   **foreach** $p \in \left\{p' \in P : clock_0 < t_{p'}^{o} \leq clock_1\right\}$ **do**
8         $P_{\text{curr}} \leftarrow P_{\text{curr}} \cup \{p\}$;
9   **if** $\delta_p = \mu$ **then**
10             $(P_{\text{curr}}, P_{\text{drop}})$=**RePlan**$(p, e, t_{\text{dis}}^{\text{start}}, P_{\text{curr}}, P_{\text{drop}}, G_{\text{dis}}, G_{\text{dis}}^{*}, Z_1, FullInfo, \eta)$;
11         $P \leftarrow P \setminus \{p\}$;
12   **if** *e is a departure event* **then**
13   Find the set $P_{\text{board}} := \left\{p \in P_{\text{curr}} \,|\, B_p(1) = e\right\}$;
14   **if** $P_{\text{board}} \neq \emptyset$ **then**
15             $P_{\text{curr}} \leftarrow P_{\text{curr}} \setminus P_{\text{board}}$;
16             $(P_{\text{board}}, P_{\text{drop}}, E_{\text{congest}}, G_{\text{dis}}^{*})$= **UpdateDep**$\left(e, t_{\text{dis}}^{\text{start}}, P_{\text{board}}, P_{\text{drop}}, G_{\text{dis}}, G_{\text{dis}}^{*}, ratio, FullInfo, E_{\text{congest}}, \eta\right)$;
17             $P_{\text{curr}} \leftarrow P_{\text{curr}} \cup P_{\text{board}}$;
18   **else if** *e is an arrival event* **then**
19   Find the set $P_{\text{alight}} := \left\{p \in P_{\text{curr}} \,|\, L_p(1) = e\right\}$ and the set $P_{\text{replan}} := \left\{p \in P_{\text{curr}} \,|\, \delta_p = e\right\}$;
20   **if** $P_{\text{alight}} \cup P_{\text{replan}} \neq \emptyset$ **then**
21             $P_{\text{curr}} \leftarrow P_{\text{curr}} \setminus \left(P_{\text{alight}} \cup P_{\text{replan}}\right)$;
22             $(P_{\text{alight}}, P_{\text{replan}}, P_{\text{drop}}, P_{\text{arr}})$ =**UpdateArr**$\left(e, t_{\text{dis}}^{\text{start}}, P_{\text{alight}}, P_{\text{replan}}, P_{\text{drop}}, P_{\text{arr}}, G_{\text{dis}}, G_{\text{dis}}^{*}, FullInfo, \eta\right)$;
23             $P_{\text{curr}} \leftarrow P_{\text{curr}} \cup \left(P_{\text{alight}} \cup P_{\text{replan}}\right)$;
24   **if** *FullInfo = Train* **then**
25   **foreach** $p \in P_{\text{curr}} \setminus (P_{\text{alight}} \cup P_{\text{replan}})$ **do**
26   Let r be r_p^{dis} if r_p^{dis} is nonempty. Otherwise, let r be r_p^{plan};
27   **if** $e \in r$ *and* $B_p(1) \in E_{\text{congest}}$ **then**
28                     $(P_{\text{curr}}, P_{\text{drop}})$ = **RePlan**$(p, e, t_{\text{dis}}^{\text{start}}, P_{\text{curr}}, P_{\text{drop}}, G_{\text{dis}}, G_{\text{dis}}^{*}, Z_4, FullInfo, \eta)$;
29     $clock_0 \leftarrow clock_1$;
30     $E_{\text{train}}^{\text{dis}} \leftarrow E_{\text{train}}^{\text{dis}} \setminus \{e\}$;
31 **return** P_{arr}, P_{drop}

In Algorithm 2.1, different sets are initialized, and the previous system clock time is set as 0 (lines 1-3). In line 4, each event in $E_{\text{train}}^{\text{dis}}$ is iterated over to implement a passenger arrival at the origin, and the loading or unloading procedure. $E^{\text{dis}}_{\text{train}}$ is a given input, of which the contained events are previously sorted in time-ascending order and then in alphabetical order regarding the event type (arrival or departure), to ensure that the assignment proceeds with time, and an arrival event occurs before a departure event if their time instants are the same. In lines 5-6, the first element from $E^{\text{dis}}_{\text{train}}$ is chosen as the current event e to be executed, and the current system clock time is set as the occurence time of e. In the loop starting from line 7, each arrival of a passenger at the origin between the previous and the current system clock time is simulated. In line 8, the origin arrival passenger p is included to the set P_{curr} that contains all passengers who are currently staying in the railways. If p needs to re-plan at the origin, the function RePlan will be called to realize the re-plan action (lines 9-10). In line 11, p is excluded from P to avoid being included in P_{curr} again.

If the current event e is a departure (line 12), then the passengers who want to board train tr_e are defined by P_{board} (line 13). If P_{board} is not empty (line 14), it is excluded from the set P_{curr} (line 15) and the loadings of passengers in P_{board} are implemented by calling the function UpdateDep (line 16). In line 17, the updated P_{board} is included to P_{curr} again. The reason of having lines 15 and 17 is that when executing UpdateDep, some passengers could be removed from P_{board} to P_{drop}, because they drop the railways due to denied boardings and no preferred alternatives can be found. Function UpdateDep also outputs E_{congest} and G^*_{dis}, which are the set of departure events that correspond to potential congested run activities, and an adaptive event-activity network that excludes the potential congested run activities from G_{dis}.

If the current event e is an arrival (line 18), the passengers who want to alight from the arriving train are defined by P_{alight} and the passengers who will re-plan paths when e occurs are defined by P_{replan} (line 19). If at least one of the two sets is not empty (line 20), the union $P_{\text{alight}} \cup P_{\text{replan}}$ is excluded from P_{curr} first (line 21) and then the function UpdateArr is called to implement the unloadings of passengers in P_{alight} and the re-plan actions of passengers in P_{replan} (line 22). In line 23, the updated $P_{\text{alight}} \cup P_{\text{replan}}$ is included to P_{curr} again. The reason of having lines 21 and 23 is that when executing UpdateArr, some passengers could be removed from P_{alight} to P_{arr} because they reach the destinations, and some passengers could be removed from P_{replan} to P_{drop} because they cannot find preferred re-planned paths and thus drop the railways. If train congestion information is published on trains (line 24), for each passenger who is dwelling at train tr_e and the next boarding train is highly congested as notified (lines 25-27), the passenger is given the chance of re-planning (line 28).

After finishing executing the current event, the previous system clock time is set to the current system clock time, and the current event is removed from the event list to be executed (lines 29-30).

2.4.3.2 Passenger loading

Algorithm 2.2: UpdateDep

Input: $e, t_{\text{dis}}^{\text{start}}, P_{\text{board}}, P_{\text{drop}}, G_{\text{dis}}, G_{\text{dis}}^{*}, ratio, FullInfo, E_{\text{congest}}, \eta$
Output: $P_{\text{board}}, P_{\text{drop}}, E_{\text{congest}}, G_{\text{dis}}^{*}$
1 **if** $cap_{tr_e} \geq |P_{\text{board}}|$ **then**
2     $cap_{tr_e} \leftarrow cap_{tr_e} - |P_{\text{board}}|$;
3   **foreach** $p \in P_{\text{board}}$ **do**
4         $B_p \leftarrow B_p \backslash \{e\}$;
5 **else if** $cap_{tr_e} < |P_{\text{board}}|$ ***and*** $cap_{tr_e} > 0$ **then**
6     $cap_{tr_e} \leftarrow 0$;
7   Sort P_{board} in ascending order according to $\max\left\{t_p^o, t_p^{\text{alight}}\right\}$;
8   **foreach** $p \in P_{\text{board}}(1 : cap_{tr_e})$ **do**
9         $B_p \leftarrow B_p \backslash \{e\}$;
10   **foreach** $p \in P_{\text{board}}(cap_{tr_e} + 1 : end)$ **do**
11         $\lambda_p \leftarrow \lambda_p + 1$;
12         $(P_{\text{board}}, P_{\text{drop}})$=**RePlan**$(p, e, t_{\text{dis}}^{\text{start}}, P_{\text{board}}, P_{\text{drop}}, G_{\text{dis}}, G_{\text{dis}}^{*}, Z_2, FullInfo, \eta)$;
13 **else if** $cap_{tr_e} = 0$ **then**
14   **foreach** $p \in P_{\text{board}}$ **do**
15         $\lambda_p \leftarrow \lambda_p + 1$;
16         $(P_{\text{board}}, P_{\text{drop}})$=**RePlan**$(p, e, t_{\text{dis}}^{\text{start}}, P_{\text{board}}, P_{\text{drop}}, G_{\text{dis}}, G_{\text{dis}}^{*}, Z_2, FullInfo, \eta)$;
17 **if** *Fulltrain* = *Train* **then**
18   **if** $\left(1 - cap_{tr_e}/cap_{tr_e}^{\max}\right) \geq ratio$ **then**
19   Find the next run activity of train tr_e: $a = (e', e'')$;
20         $E_{\text{congest}} \leftarrow E_{\text{congest}} \cup \{e'\}$;
21         $G_{\text{dis}}^{*} \leftarrow G_{\text{dis}}^{*} \backslash \{a\}$;
22 **return** $P_{\text{board}}, P_{\text{drop}}, G_{\text{dis}}^{*}, E_{\text{congest}}$

In Algorithm 2.2, if the available capacity cap_{tr_e} of a train tr_e is sufficient to cover all passengers P_{board} who want to board the train, cap_{tr_e} is updated accordingly (lines 1-2). Then, for each $p \in P_{\text{board}}$, the current event e is excluded from the set B_p that contains all departure events corresponding to the boarding actions of p (lines 3-4). If the available capacity of a train can only cover part of the passengers in P_{board} (line 5), cap_{tr_e} is updated accordingly (line 6) and then the passengers in P_{board} are sorted in ascending order according to their arriving times at the current stations (line 7). Here, t_p^{alight} refers to the latest alighting time of passenger p, of which the value is initialized with 0 when p arrives at the origin and further be updated when p alights from a train at another station. Line 7 ensures the loading rule of first-come-first-served. The first cap_{tr_e} passengers in P_{board} can board the train (lines 8-9), while the remainders are denied for boarding and RePlan is called for re-planning (line 10-12). If the available capacity of a train is zero, none of the passengers in P_{board} can board the train, but only re-plan paths (lines 13-16).

Furthermore, if train congestion information is published and the congestion ratio of the train tr_e, $\left(1 - cap_{tr_e}/cap_{tr_e}^{\max}\right)$, has reached the specified congestion level *ratio*, then the departure event e' that corresponds to the next run activity a of tr_e is added to E_{congest} (lines 19-20), while a is excluded from the adaptive event activity network G^*_{dis} (line 21). Here, $cap^{\max}_{tr_e}$ represents the maximum number of passengers that train tr_e can hold.

2.4.3.3 Passenger unloading

Algorithm 2.3: UpdateArr

Input: $e, t^{\text{start}}_{\text{dis}}, P_{\text{alight}}, P_{\text{replan}}, P_{\text{drop}}, P_{\text{arr}}, G_{\text{dis}}, G^*_{\text{dis}}, \mathit{FullInfo}, \eta$
Output: $P_{\text{alight}}, P_{\text{replan}}, P_{\text{drop}}, P_{\text{arr}}$

1 **foreach** $p \in P_{\text{alight}}$ **do**
2     $cap_{tr_e} \leftarrow cap_{tr_e} + 1$;
3     $L_p \leftarrow L_p \setminus \{e\}$;
4   **if** $L_p = \emptyset$ **then**
5         $t^d_p = \pi_e$;
6         $P_{\text{alight}} \leftarrow P_{\text{alight}} \setminus \{p\}$;
7         $P_{\text{arr}} \leftarrow P_{\text{arr}} \cup \{p\}$;
8   **else**
9         $t^{\text{alight}}_p = \pi_e$;
10 **foreach** $p \in P_{\text{replan}}$ **do**
11     $(P_{\text{replan}}, P_{\text{drop}})$= **RePlan**$(p, e, t^{\text{start}}_{\text{dis}}, P_{\text{replan}}, P_{\text{alight}}, P_{\text{drop}}, G_{\text{dis}}, G^*_{\text{dis}}, Z_3, \mathit{FullInfo}, \eta)$;
12 **return** $P_{\text{alight}}, P_{\text{replan}}, P_{\text{drop}}, P_{\text{arr}}$

In Algorithm 2.3, for each passenger p who wants to alight from the train tr_e, the available capacity of the train is updated accordingly (lines 1-2), and event e is excluded from the set L_p that contains all arrival events corresponding to the alighting actions of p (line 3). After that, an empty L_p means that passenger p has reached the destination (line 4), thus the actual destination arrival time t^d_p is updated and p is removed from P_{alight} to P_{arr} (lines 5-7). If L_p is not empty, the latest alighting time t^{alight}_p is updated (lines 8-9). For each passenger who will re-plan path when e occurs, RePlan is called (lines 10-11).

2.4.3.4 Passenger re-planning

In Algorithm 2.4, the source and sink nodes (i.e. u and v) are determined for searching the re-planned path of passenger p (line 1). The sink node is always the exit event corresponding to the destination d_p, while the source node u is different under different re-plan situations.

- If $Z_j = Z_1$, u is set as the duplicate departure event that is closest to the passenger's arrival time at the origin: t^o_p.
- If $Z_j = Z_2$, u is set as the duplicate departure event that is closest to the current departure event e at the station st_e.

- If $Z_j = Z_3 \cup Z_4$, u is set as the current arrival event e.

All re-plan situations are listed in Table 2.3.

Algorithm 2.4: RePlan

Input: $p, e, t_{\text{dis}}^{\text{start}}, P_{\text{board}}$ (or P_{replan} and P_{alight}, or P_{curr}), $P_{\text{drop}}, G_{\text{dis}}, G_{\text{dis}}^{*}, Z_j$, *FullInfo*, η

Output: P_{board} (or P_{replan}), P_{drop}

1 Determine the source node u and sink node v of passenger p according to d_p, Z_j, t_p^o and st_e;
2 Determine whether G is set as G_{dis} or G_{dis}^{*} according to Z_j and *FullInfo*;
3 Search the optimal path r from u to v on G;
4 Let t_r^d be the destination arrival time of path r;
5 **if** $\left(t_r^d - \hat{t}_p^d\right) \leq \eta$ **then**
6     $r_p^{\text{dis}} \leftarrow r$;
7   Update B_p and L_p according to r_p^{dis};
8   **if** $Z_j \in \{Z_3, Z_4\}$ ***and*** $p \notin P_{\text{alight}}$ **then**
9   **if** $L_p(1) = e$ **then**
10             $cap_{tr_e} \leftarrow cap_{tr_e} + 1$;
11             $L_p \leftarrow L_p \backslash \{e\}$;
12             $t_p^{\text{alight}} = \pi_e$;
13 **else**
14     $P_{\text{drop}} \leftarrow P_{\text{drop}} \cup \{p\}$;
15   P_{board} (or P_{replan}, or P_{curr}) $\leftarrow P_{\text{board}} \backslash \{p\}$ (or $P_{\text{replan}} \backslash \{p\}$, or $P_{\text{curr}} \backslash \{p\}$);
16 **return** P_{board} (or P_{replan}, or P_{curr}), P_{drop}

Table 2.3: Re-plan situations

Situation	Time and location
Z_1: re-plan before travelling due to service variations	When arriving at the origin station
Z_2: re-plan during travelling due to denied boarding	When planning to board at any possible station
Z_3: re-plan during travelling due to service variations	When arriving at the specified station
Z_4: re-plan during travelling due to train congestion	When arriving at a station

In line 2 of Algorithm 2.4, the event-activity network G that is used to search the re-planned path of passenger p, is determined according to the values of *FullInfo* and Z_j.

- If *FullInfo=Train*, then train congestion information is published at trains only. Thus, G is set as G_{dis}^{*},
  - if $Z_j = Z_2$ and $st_e \neq o_p$, which means passenger p re-plan paths due to denied boarding at the station that is not his/her origin, or
  - if $Z_j \in \{Z_3, Z_4\}$.

  Passenger who satisfies either of the above conditions must have taken a train where he/she is notified with train congestion information.

- If *FullInfo=none*, then train congestion information is published nowhere. Thus, G is always set as G_{dis} whatever Z_j is.

In line 3 of Algorithm 2.4, the optimal path r is searched through G, of which the destination arrival time is t_r^d (line 4). Thus, the resulting delay of r is $(t_r^d - \hat{t}_p^d)$ where $\hat{t}_p^d$ is the planned destination arrival time of p.

If the delay is no longer than passenger's maximum accepted delay η (line 5), r is chosen as the re-planned path (line 6), and the sets of events corresponding to the boardings and alightings are updated accordingly (line 7). Additionally for the re-plan situation of Z_3 or Z_4, a passenger who does not plan to alight from the train tr_e might now want to get off due to the re-planned path (lines 8-9). In such a case, the available train capacity, the set of alighting events, and the latest alighting time are all updated accordingly (lines 10-12).

If the delay of r is longer than η (line 13), the passenger will drop the railway. Thus, the set P_{drop} and the set P_{board} (or P_{replan}) are all updated accordingly (lines 14-15).

2.5 Time complexity

Algorithm 2.1, the main algorithm working for passenger assignment, is based on three sub-algorithms (i.e. Algorithms 2.2, 2.3 and 2.4), while Algorithms 2.2 and 2.3 also need to call Algorithm 2.4. Figure 2.6 shows the relations between the algorithms.

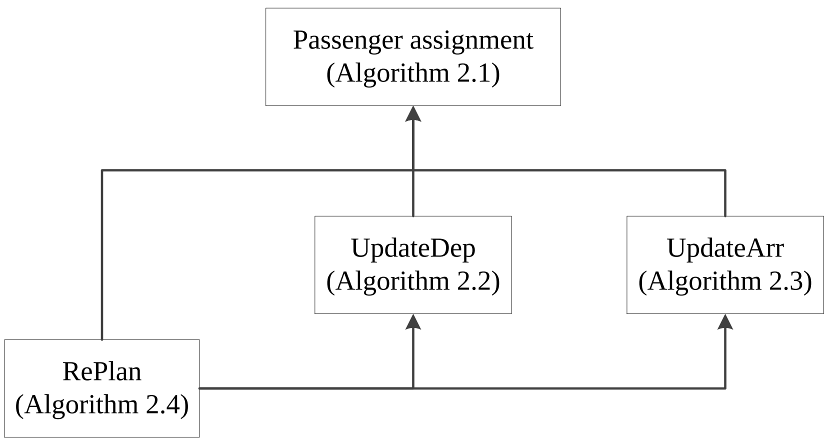

Figure 2.6: The relations between algorithms

In RePlan, lines 1, 3 and 7 require non-constant time. Line 1 takes $O\left(\left|E_{\text{ddep}}\right|\right)$ time, where E_{ddep} represents the set of duplicate departure events. Line 3 takes $O\left(|A|+|E|\right)$ time, where A and E refer to the activities and events contained in the formulated event-activity network, respectively. Line 7 takes $O\left(\left|r_p^{\text{dis}}\right|\right)$ time, where r_p^{dis} refers to the re-planned path of passenger p, which contains all events that p will pass through. Thus, the time complexity of Algorithm 2.4 is

$$\begin{aligned} T_{Alg}^4 &= O\left(\left|E_{\text{ddep}}\right|\right) + O\left(|A|+|E|\right) + O\left(\left|r_p^{\text{dis}}\right|\right), \\ &\leq O\left(|E|\right) + O\left(|A|+|E|\right) + O\left(|E|\right), \\ &= O\left(|A|+|E|\right). \end{aligned}$$

In UpdateArr, the **for** loop from line 1 to line 9 takes $O\left(\left|P_{\text{alight}}\right|\right)$ time, while the **for** loop from line 10 to line 11 takes $O\left(\left|P_{\text{replan}}\right| \cdot (|A|+|E|)\right)$ time. Thus, the time complexity of Algorithm 2.3 is

$$T_{Alg}^3 = O\left(\left|P_{\text{alight}}\right|\right) + O\left(\left|P_{\text{replan}}\right| \cdot (|A|+|E|)\right).$$

In UpdateDep, line 7 takes $O\left(|P_{\text{board}}| \log |P_{\text{board}}|\right)$ time by using heapsort. Thus, lines 1-16 take $O\left(|P_{\text{board}}| \log |P_{\text{board}}|\right) + O\left(|P_{\text{board}}| \cdot (|A|+|E|)\right)$ time. Because $\log |P_{\text{board}}| \ll |A|+|E|$ and lines 17-21 take constant time, the time complexity of Algorithm 2.2 is

$$T_{Alg}^2 = O\left(|P_{\text{board}}| \cdot (|A|+|E|)\right).$$

As for Algorithm 2.1, the **while** loop makes one iteration per event of $E_{\text{train}}^{\text{dis}}$. Let N be the size of $E_{\text{train}}^{\text{dis}}$, thus in all N **while** iterations:

- the **for** loop from line 7 to line 11 takes at most $\sum_{i=1}^{N} \left|P_{\text{ori}}^i\right| \cdot (|A|+|E|)$ operations, where P_{ori}^i represents the set of passengers who arrive at the origins in the ith iteration. This is equal to $O\left(|P| \cdot (|A|+|E|)\right)$ time, where P represents the total passenger demand.

- due to the calls of UpdateDep, lines 12 - 17 take at most $\sum_{i=1}^{N} \left|P_{\text{board}}^i\right| \cdot (|A|+|E|)$ operations, where P_{board}^i represents the set of passengers who wish to board the train at the ith iteration. By defining m as the maximum number of boardings/alightings that a passenger may encounter, there must be $\sum_{i=1}^{N} \left|P_{\text{board}}^i\right| = m|P|$. Because the maximum number of boardings/alightings corresponding to a passenger must be finite and relatively small, the time complexity is $O(|P| \cdot (|A|+|E|))$.

- due to the calls of UpdateArr, lines 18-23 take $\sum_{i=1}^{N} \left|P_{\text{alight}}^i\right| + \left|P_{\text{replan}}^i\right| \cdot (|A|+|E|)$ operations, where P_{alight}^i (P_{replan}^i) represents the set of passengers who will alight from the train (re-plan paths) at the ith iteration. This is not larger than $m|P| + |P| \cdot (|A|+|E|)$. Thus, lines 18-23 totally take $O\left(|P| \cdot (|A|+|E|)\right)$ time.

- lines 5-6 and lines 29-30 totally take $O(N)$ time, where N is smaller than $|E|$.

If train congestion information is not published, the time complexity of Algorithm 2.1 thus is

$$\begin{aligned} T^1_{Alg} &= O\left(|P|\cdot(|A|+|E|)\right)+O(|P|\cdot(|A|+|E|))+O\left(|P|\cdot(|A|+|E|)\right)+O(N), \\ &= O(|P|\cdot(|A|+|E|)). \end{aligned}$$

If train congestion information is published, in each **while** iteration of Algorithm 2.1, the **for** loop from line 25 to line 28, at most take $|P|$ operations in case $P_{\text{curr}} = P$ and $(P_{\text{alight}} \cup P_{\text{replan}}) = \emptyset$. In such a case, T^1_{Alg} becomes $O\left(N\cdot|P|\right)+O\left(|P|\cdot(|A|+|E|)\right)$. As $N < |A|+|E|$, T^1_{Alg} is still $O\left(|P|\cdot(|A|+|E|)\right)$, although train congestion information is published.

The notation used is given in Table 2.4.

Table 2.4: Notations used in proving the time complexity of Algorithm 2.1

Symbol	Description
P^i_{ori}	The set of passengers who arrive at the origins at the ith **while** iteration of Algorithm 2.1
P^i_{board}	The set of passengers who wish to board the train at the ith **while** iteration of Algorithm 2.1
P^i_{alight}	The set of passengers who will alight from the train at the ith **while** iteration of Algorithm 2.1
P^i_{replan}	The set of passengers who re-plan paths upon arrival event due to service variations at the ith **while** iteration of Algorithm 2.1
m	The maximum number of boardings/alightings a passenger could encounter

In summary, the time complexity of the proposed passenger assignment model is relevant to the size of the given passenger demand and the scale of the considered network. To reduce computational burden, one way is to group the passengers who share the same travel characteristics (e.g. the origins, the destinations, the arrival times at the origin, etc.). However, this is at the expense of assignment accuracy, since two passengers who have exactly the same travel characteristics could still be distributed to different trains if vehicle capacities are in short supply.

2.6 Case study

2.6.1 Description

The model is applied to a subnetwork of the Dutch railways, where 17 stations are located and six train lines are operated (see Figure 2.7). The disruption scenario is defined as a complete track blockage between stations Hze and Mz, which occurs from 7:57 to 9:00. Here, only the disruption timetable of the corridor where the disruption happens is shown (see Figure 2.8), as the timetables of the other two corridors remain as planned. The number of nodes (events) and arcs (activities) in the network formulation are 2085 and 3539, respectively. In Figure 2.8, the solid lines represent the services scheduled in the disruption timetable, while the dashed (dotted) lines represent the original scheduled services that are cancelled (delayed) in the disruption timetable.

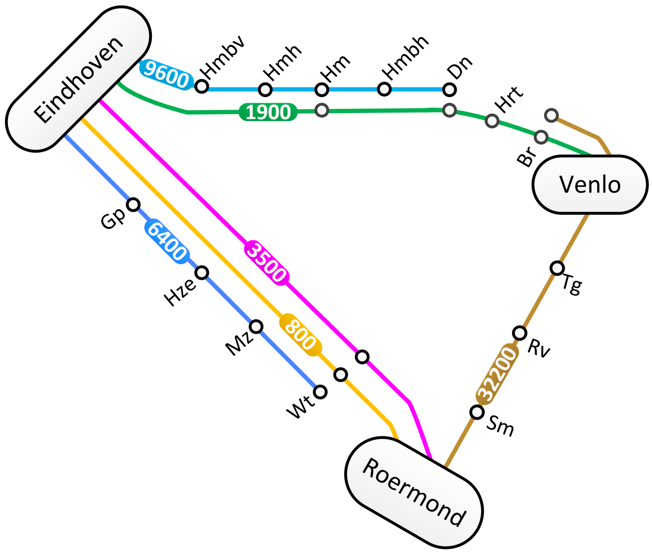

Figure 2.7: The considered network

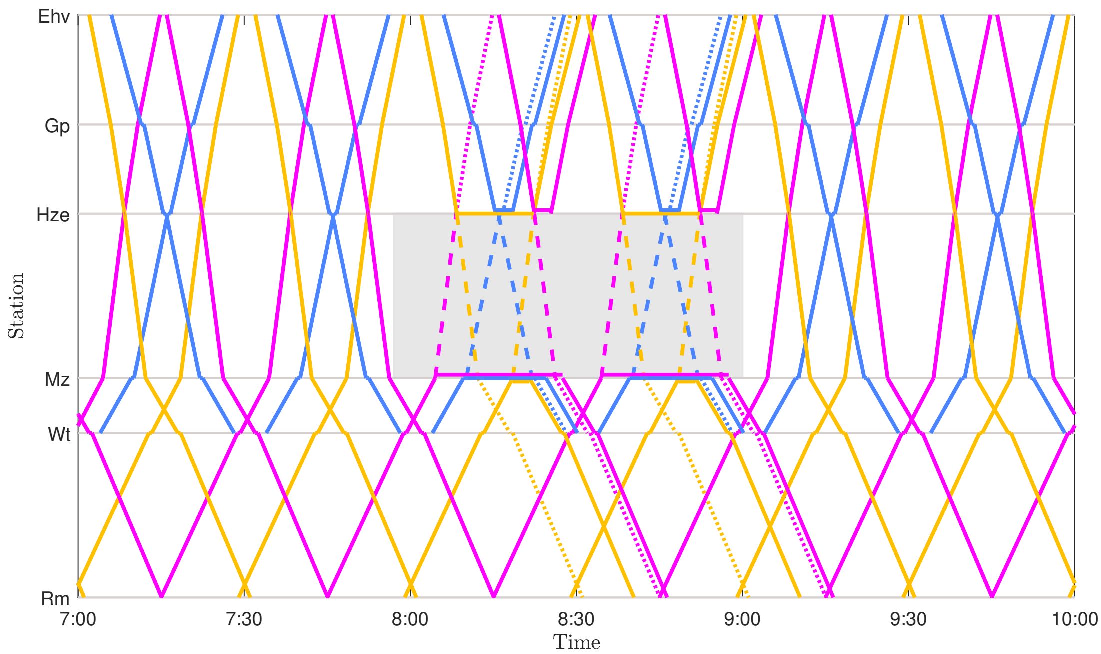

Figure 2.8: Disruption timetable

Passenger demand is generated for the period from 7:00 to 10:00, which contains the time frame before the disruption starts, during the disruption and after the disruption ends, since passengers who start travelling during these hours could be influenced by the disruption. The total number of passengers who travel in the considered network during the considered period is 7515.

To consider information interventions, different schemes of information provision are set, which are listed in Table 2.5. Each row of Table 2.5 indicates the locations where service variations and train congestion information are published (i.e. *ServiceInfo* and *FullInfo*), the specified train congestion level (i.e. *ratio*) that triggers updating the congestion information, and the maximum accepted destination delay of each passenger (i.e. η). For example the current congestion ratio of a train is 0.85; if *ratio* is set as 0.85, then the operators update the congestion information by telling passengers that the next run of this train would be highly congested. However, if *ratio* is set as 0.9, no such information will be given to passengers, because the current congestion ratio of the train, 0.85, does not reach the level of 0.9. A train of which the congestion ratio currently reaches *ratio*, is thought to be potentially unable to satisfy all boarding demands for its next run. Thus, notifying some passengers with the congestion information can avoid them boarding the next run of the train, while some passengers who are not notified with information may still keep their choices. As for η, two kinds of values are set here, which are $(\hat{t}_p^d - t_p^o)$ referring to the planned travel time of passenger p, and 63 min which is the defined disruption duration. The value of η may affect the number of passengers who drop the railways, which further affects the congestion of trains. Also baseline scenarios are created, in which neither service variations nor train congestion is provided. In these scenarios, passengers can only know the service variation when they really experience it themselves. For instance, a passenger knows he/she cannot board a train as planned when the train does not show up at the station due to delay or cancellation.

Table 2.5: Case study settings

ServiceInfo	*FullInfo*	*ratio*	η [min]
Station	Train	0.8,0.9,or 1	$(\hat{t}_p^d - t_p^o)$ or 63
Station	None	–	$(\hat{t}_p^d - t_p^o)$ or 63
Station & Train	Train	0.8,0.9,or 1	$(\hat{t}_p^d - t_p^o)$ or 63
Station & Train	None	–	$(\hat{t}_p^d - t_p^o)$ or 63
None	None	–	$(\hat{t}_p^d - t_p^o)$ or 63

2.6.2 Results

By applying the model on the defined disruption scenarios with the settings of Table 2.5, 18 results are obtained, which are shown in Table 2.6. In each result, three indicators are used, which are the number of dropped passengers, the number of denied boardings, and the travel time deviation. The computation time for scenario 9 is the least, which is 15 seconds, and the computation time for scenario 16 is the most, which is 6 seconds. The computation times for other scenarios are between these two.

Table 2.6: Results of disruption scenarios between stations Hze and Mz

Scenario	η [min]	*ServiceInfo*	*FullInfo*	*ratio*	# drop passengers	# denied boardings	Travel time deviation [min]
1	$\hat{t}_p^d - t_p^o$	Station	Train	0.8	551	0	25,243
2	$\hat{t}_p^d - t_p^o$	Station	Train	0.9	551	0	25,243
3	$\hat{t}_p^d - t_p^o$	Station	Train	1	551	0	25,243
4	$\hat{t}_p^d - t_p^o$	Station	None	–	551	0	25,243
5	$\hat{t}_p^d - t_p^o$	Station & Train	Train	0.8	551	0	25,187
6	$\hat{t}_p^d - t_p^o$	Station & Train	Train	0.9	551	0	25,187
7	$\hat{t}_p^d - t_p^o$	Station & Train	Train	1	551	0	25,187
8	$\hat{t}_p^d - t_p^o$	Station & Train	None	–	551	0	25,187
9	$\hat{t}_p^d - t_p^o$	None	None	–	565	0	28,399
10	63	Station	Train	0.8	118	178	39,758
11	63	Station	Train	0.9	113	193	40,687
12	63	Station	Train	1	113	193	40,693
13	63	Station	None	–	113	193	40,693
14	63	Station & Train	Train	0.8	116	178	39,707
15	63	Station & Train	Train	0.9	110	193	40,590
16	63	Station & Train	Train	1	110	193	40,595
17	63	Station & Train	None	–	110	193	40,595
18	63	None	None	–	88	358	47,234

The number of dropped passengers is calculated as $|P_{\text{drop}}|$, and the number of denied boardings is calculated as $\sum_{p \in P} \lambda_p$ where λ_p represents the number of times a passenger being denied for boarding and $P = P_{\text{arr}} \cup P_{\text{drop}}$. The travel time deviation is calculated as

- $\sum\limits_{p \in P_{\text{arr}}} \left(t_p^d - \hat{t}_p^d\right) + \sum\limits_{p \in P_{\text{drop}}} \left(\hat{t}_p^d - t_p^o\right)$, when η is set as $\hat{t}_p^d - t_p^o$, or
- $\sum\limits_{p \in P_{\text{arr}}} \left(t_p^d - \hat{t}_p^d\right) + 63\left|P_{\text{drop}}\right|$, when η is set as 63,

where $t_p^d(\hat{t}_p^d)$ represents the actual(planned) destination arrival time of passenger p, and t_p^o represents the actual origin arrival time of passenger p. Note that t_p^d could be smaller or larger than $\hat{t}_p^d$, which means the total travel time deviation consists of both negative and positive individual travel time deviations.

2.6.2.1 Influence of maximum accepted delay: η

It is found from Table 2.6 that the total travel time deviations in all cases are positive, which means that the actual travel times increase compared to the planned travel times. When the maximum accepted destination delay (i.e. η) is set as an individual's planned travel time (i.e. $\hat{t}_p^d - t_p^o$), the travel time increases are the smallest, while the numbers of dropped passengers (i.e. 551 or 565) are the largest (scenarios 1-9). When η is set as the disruption length (scenarios 10-18), the travel time increases grow, while the numbers of dropped passengers reduce. These indicate that

- the provided disruption timetable leads to 565 passengers to whom the increased travel time in the railways is at least equal to the planned travel time, and
- the limited vehicle capacities lead to more travel time increase when more passengers remain in the railways to reach the destinations.

If operators aim for a low travel time increase while satisfying passenger demand as much as possible, one way is to design a disruption timetable that provides faster services and ensures less denied boardings by adjusting the schedules to distribute passengers wisely. However, this is rather challenging, since many factors (e.g. passenger behaviour, vehicle capacities, infrastructure restrictions, etc.) need to be considered in the rescheduling. Thus, another way is proposed, information intervention, which is easier to be implemented in practice. Here, information intervention means that operators provide passengers at different locations with different information about service variations or train congestion.

2.6.2.2 Influence of information intervention

When η is set as $\hat{t}_p^d - t_p^o$ or 63 min, compared to not updating passengers with any information (scenario 9 or 18), providing information for them (scenarios 1-8 or 10-17) helps to reduce the number of dropped passengers, the number of denied boardings, and/or the travel time increase. This indicates that it is helpful to update passengers with certain information during disruptions.

When η is set as $\hat{t}_p^d - t_p^o$, providing service variations at both stations and trains (scenarios 5-8) always lead to less travel time increases compared to the cases in which service variations are announced at stations only (scenarios 1-4). However, publishing train congestion information does not make any sense, since no one has been denied for boarding even though train congestion information is not published. This indicates that

- if vehicle capacities are not in short supply, publishing service variations at both stations and trains is able to reduce more travel time increase, compared to publishing service variations at stations only.

This is because by additionally receiving service variations on trains, the on-board passengers at the moment the disruption occurs can re-plan paths just at the next stop of the train rather than several stops later where they get off from the train, and such earlier re-plans help to find better alternatives which result in less travel time increases.

When η is set as 63 min, more passengers remained in the railways and thus some passengers were denied for boarding due to insufficient vehicle capacities. Under this circumstance, additionally publishing train congestion information on trains helps to reduce travel time increase, if $ratio$ is set to an appropriate value. For example when service variations are published at stations only, compared to not publishing train congestion (scenario 13), publishing such information on trains leads to better performance when $ratio$ is set as 0.8 or 0.9 (scenario 10 or 11), or does not change performance when $ratio$ is set as 1 (scenario 12). These phenomena are also found in the cases where service variations are published at both stations and trains (scenarios 14-17). These indicate that

- if vehicle capacities are in short supply, the performance of publishing train congestion on reducing travel time increase is influenced by the value of $ratio$.

For example if $ratio$ is set as 0.8 and the current congestion ratio of train tr_1 is 0.85, then the information that tr_1 is highly congested is published to on-board passengers, which prevent them from boarding the next run of tr_1, while the passengers who wait at their origins to board the next run of tr_1 still keep their choices. In this way, passengers who demand for boarding the next run of tr_1 are distributed, since tr_1 is thought to be highly congested now and may be unable to satisfy all demands later. If there must be some passengers being denied for boarding a train, avoiding them to choose the train may help them find better alternatives compared to the ones they can find after being denied. It is also possible that all subsequent demands are satisfied if these demands are small, or lots of passengers get off before the next run, thus avoiding some passengers boarding the train might not be helpful since the avoided passengers may not be able to find better alternatives. Clearly, the setting of $ratio$ is important, which decides whether publishing train congestion is good or not. One way to ensure the accuracy of $ratio$ is to assign each train with a customized $ratio$ that varies with times and locations according to the estimated boarding demand and alighting amount. However these are also hard to be estimated accurately, as during the period from the departure of the current run to the departure of the next run, there could be extra demand or alightings emerging. In this chapter, a general value of $ratio$ is set to to all trains, while it would be interesting to enable a dynamic customized $ratio$ to each train in future research.

2.6.2.3 Impacts on passengers who start travelling before/during the disruption

In the model, the passengers who start travelling before the disruption, but are still within the railways at the moment the disruption occurs, are also considered explicitly. According to the case study results, these passengers are affected a lot by the disruption, thus overlooking them in the assignment model is unreasonable.

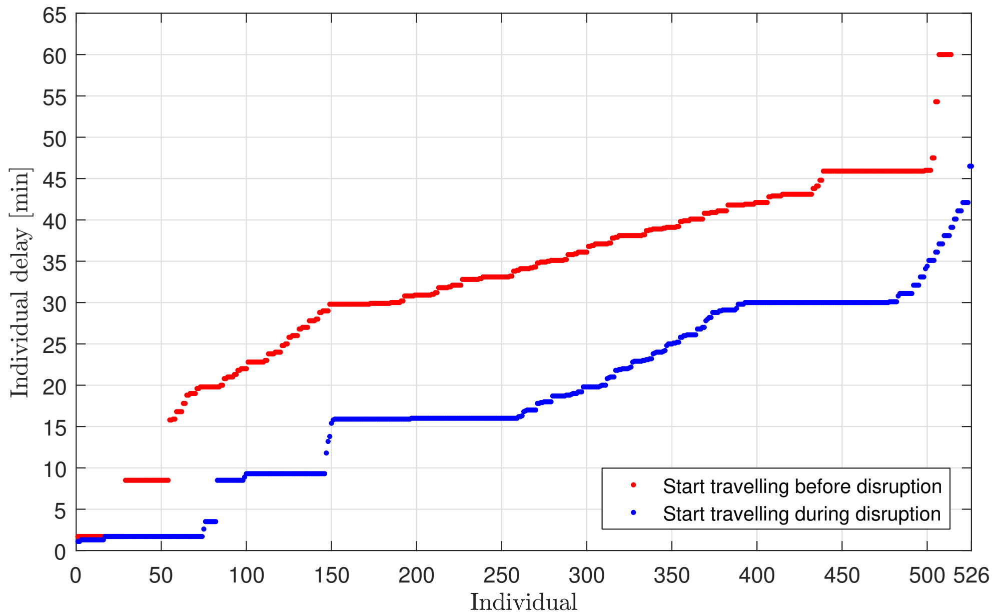

Figure 2.9: Individual delays of scenario 1 (η=individual planned travel time)

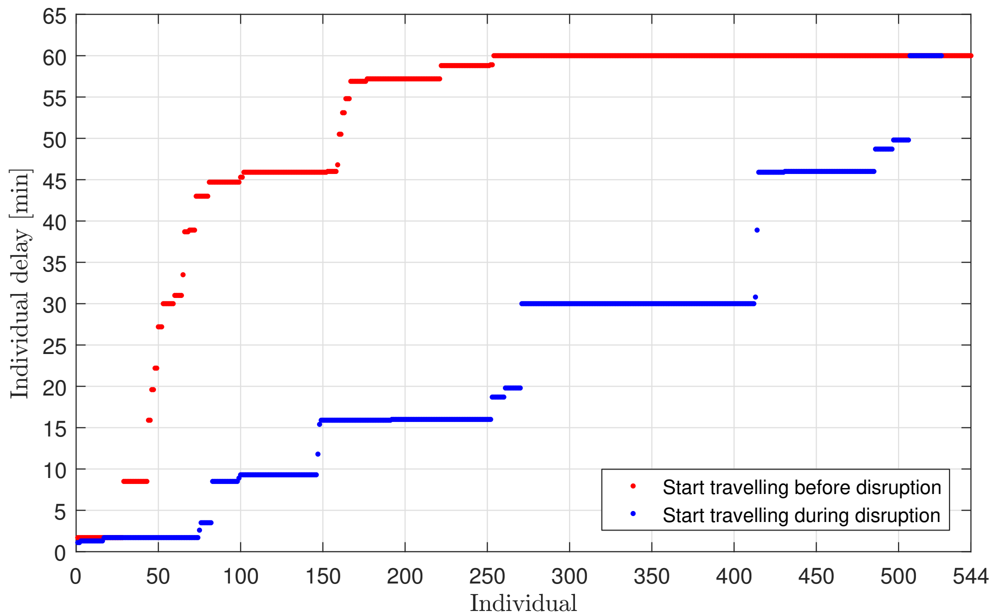

Figure 2.10: Individual delays of scenario 10 (η=disruption length)

In Figures 2.9 and 2.10, the individual delays of scenario 1 and 10, are shown respectively, which are distinguished by passengers' travelling start time. As few passengers who start travelling after the disruption ends (i.e. 9:00) are delayed (9 individuals in scenario 1 or scenario 10), only the individual delays of passengers who start travelling before or during the disruption are shown. It is found that under the same setting of η

(i.e. the maximum acceptable destination delay), individual delays are not significantly different. Thus, scenario 1 is chosen as the representative of setting η to individual planned travel time, and scenario 10 is chosen as the representative of setting η to the defined disruption lenth. Individual delay is only calculated for the passenger who actually arrived at the destination later than planned. Thus, individual delay is calculated as $t_p^d - \hat{t}_p^d$ for $p \in P_{\text{arr}}$, if $t_p^d > \hat{t}_p^d$. For $p \in P_{\text{drop}}$, individual delay is computed as the corresponding value of η. Note that in Figures 2.9 and 2.10, individual delays are shown in ascending order, and an individual numbered in Figure 2.9 does not correspond to the individual numbered the same in Figure 2.10.

Figures 2.9 and 2.10 both show that passengers who start travelling before the disruption are delayed more seriously. Most individual delays are below 50 minutes in Figure 2.9, while in Figure 2.10, there are a lot of individual delays reaching 60 minutes. This indicates that most passengers' planned travel times are below 50 minutes and it is hard for them to find the alternatives of less than 50 minutes delay under the current disruption timetable, while the congestion issue increases passenger delays further. To reduce passenger delays, one way is to improve the disruption timetable by providing passengers with better alternatives (i.e. less resulting delays), which could be done by incorporating passenger responses into timetable rescheduling. Additionally, it is found that there are 21 passengers in scenario 10 being delayed, not because of service variations (i.e. their planned paths are not cancelled/delayed) but due to denied boardings only. Under these circumstances, increasing vehicle capacities or providing alternatives outside the railways (e.g. shuttle buses), would be helpful to reduce passenger delays.

2.7 Conclusions and future research

In this chapter, a dynamic passenger assignment model is proposed considering major disruptions that require trains to be cancelled or short-turned. Information interventions are introduced by delivering the information of service variations and the information of train congestion at different locations. By applying the model on part of the Dutch railway network where a complete track blockage is assumed during one morning peak hour, it is found that when vehicle capacities are always sufficient (i.e. no denied boarding), publishing service variations at both stations and trains helps to reduce the travel time increase due to the disruption, while additionally publishing train congestion does not make any sense. When vehicle capacities are in short supply (i.e. denied boardings exist), additionally publishing train congestion can reduce the travel time increase due to the disruption, of which the performance depends on how a train is defined as highly congested in order to proactively avoid some passengers boarding the next run of the train.

Although only one case is performed in this chapter, more applications could be performed with the proposed model. For example, considering the fluctuation of day-to-day passenger demand and the frequency of disruptions, reasonable vehicle capacity reservations for improving the service resilience during disruptions can be proposed. Besides, the proposed assignment model will be applied on larger networks and combined with rescheduling models in the future, where passengers will be grouped to speed the computation.

Appendix 2.A

Table 2.7: Notation

Symbol	Description
$E_{\text{train}}^{\text{plan}}$	The set of original departure and arrival events (i.e. original event list)
$E_{\text{train}}^{\text{dis}}$	The set of rescheduled departure and arrival events (i.e. updated event list)
$E_{\text{train}}^{\text{cancel}}$	The set of cancelled departure/arrival events: $E_{\text{train}}^{\text{cancel}} \subset E_{\text{train}}^{\text{plan}}$ and $E_{\text{train}}^{\text{cancel}} \cap E_{\text{train}}^{\text{dis}} = \emptyset$.
$E_{\text{train}}^{\text{delay}}$	The set of delayed departure/arrival events: $E_{\text{train}}^{\text{delay}} \cap E_{\text{train}}^{\text{plan}} = E_{\text{train}}^{\text{delay}} \cap E_{\text{train}}^{\text{dis}}$, but they differ in the times of occurrences.
$E_{\text{arr}}^{\text{dis}}$	The set of rescheduled arrival events
E_{congest}	The set of departure events that correspond to potential congested run activities
P	The set of passengers who plan to travel by train
P_{curr}	The set of passengers currently staying in the railways (either at stations or within trains)
P_{drop}	The set of passengers who drop the railways
P_{arr}	The set of passengers who arrive at the destinations by train
P_{board}	The set of passengers who want to board the same train (local variable)
P_{alight}	The set of passengers who want to alight from the same train (local variable)
P_{replan}	The set of passengers who re-plan paths upon the same arrival event (local variable)
o_p	The origin station of passenger p
d_p	The destination station of passenger p
t_p^o	The arrival time of passenger p at the origin station
r_p^{plan}	The planned path of passenger p from o_p to d_p
r_p^{dis}	The re-planned path of passenger p from the station where p re-plans to d_p
t_p^{alight}	The latest alighting time of passenger p, which is initialized with the value 0
$\hat{t}_p^d$	The planned destination arrival time of passenger p
t_p^d	The actual destination arrival time of passenger p
B_p^{plan}	The set of departure events that correspond to the planned boarding actions of passenger p
L_p^{plan}	The set of arrival events that correspond to the planned alighting actions of passenger p

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Symbol	Description
δ_p	The re-plan indication of passenger p, which indicates when and where p would re-plan the path
λ_p	The number of times of passenger p being denied for boarding a train
μ	A unique positive number that is used to indicate that a passenger would re-plan a path at the origin
st_{short}	The station where trains are short-turned or cancelled
$t_{\text{dis}}^{\text{start}}$	Start time of a disruption
FullInfo	The variable indicating the location of publishing train congestion information
	FullInfo=Train: publish train congestion information on trains only
	FullInfo=None: publish train congestion information nowhere
ratio	The specified congestion ratio that triggers updating the corresponding congestion information of a train
η	Maximum passenger accepted destination delay

Chapter 3

Railway timetable rescheduling with flexible stopping and flexible short-turning during disruptions

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3.1 Introduction

In most countries, the railway plays a major role in people's daily travelling. Therefore, reliability of the operations is essential and in particular rapid responses are needed after disruptive events. To this end, efforts are made either during the planning processes to design robust timetables, or during the operations to provide high-quality rescheduled timetables. The latter are considered as real-time rescheduling problems that are further classified into disturbance management and disruption management, depending on the severities of service interruptions.

Usually, disturbance management handles relatively small delays due to, for example, extended running or dwell times of trains, by rescheduling the timetable only. On the contrary, disruption management deals with large incidents, like open-track blockages, station closures, extreme weather conditions, etc., which consists of not only timetable rescheduling, but also rolling stock and crew rescheduling (Jespersen-Groth et al., 2009). An overview of the real-time rescheduling models towards either disturbances or disruptions can be found in Cacchiani et al. (2014).

Until now, many efforts have been put on developing models and algorithms to achieve automatic disturbance management that actually has been realized in Norway since February 2014 (Lamorgese and Mannino, 2015). However, automatic disruption management has not been realized yet. In practice, it is still highly dependent on manual work to deal with disruptions, which usually results in rescheduling solutions of low quality and imposes much work load on the traffic controllers (Ghaemi et al., 2017b). Thus, developing models to generate disruption solutions automatically attracts increasing attention recently, which is exactly the starting point of this chapter.

In this chapter, a Mixed Integer Linear Programming (MILP) model is proposed to handle the railway timetable rescheduling problem during complete track blockages, by retiming, reordering, cancelling, flexible stopping, and flexible short-turning. This is the first time that flexible stopping and flexible short-turning are introduced in one rescheduling model.

Flexible stopping means that for each train the original scheduled stops could be skipped while extra stops could be added, considering that during disruptions a skipped stop could reduce the delays of passengers at their expected destinations, while an added stop could provide passengers with more alternative paths for re-routing. However, a skipped stop will cause inconvenience to some passengers who therefore need to reroute and possibly arrive with some delay at their destinations; while an added stop may increase the total travel times of many passengers. When deciding whether to skip or add stops, the negative and positive impacts on passengers are both considered while the adjusted train running times due to reduced (extra) decelerations and accelerations are explicitly taken into account.

Short-turning means that a train ends its operation at a station before the blocked tracks and the corresponding rolling stock turns at the station to be used by another train in the opposite direction. Usually, a train is short-turned at one station only. As such, the train will be cancelled instead of short-turned if the short-turn station lacks capacity. To reduce the possibility of cancelling trains due to lack of station capacity, we introduce flexible short-turning by giving each train a full choice of short-turn station candidates and the proposed model decides the optimal station and time of short-turning the train. Among all the stations that a train originally serves or passes through, the ones of which the infrastructure layouts allow short-turning are all chosen as the short-turn station candidates for the train.

Compared to most literature, our model is more complete by including realistic characteristics of the infrastructure, disruption, and passengers as much as possible. Regarding to the infrastructure, the model focuses on networks of both double-track railway lines and single-track railway lines (described at a macroscopic level), where multiple types of headways are considered to prevent operational conflicts at stations/sections. The platform tracks and pass-through tracks of a station are distinguished to make sure that each train is assigned to an appropriate track when arriving at the station. The rolling stock circulations at short-turning and terminal stations are both taken into account, and whether a station has turning facilities for the trains arriving from different directions is explicitly considered. The rolling stock circulations at the origins/destinations of trains are called OD turns. To be specific, the model ensures that the rolling stock of a train that reaches its destination turns at the station to operate an opposite train that departs from the same station as the origin. Regarding to a disruption, our model considers all disruption phases, i.e., the transition phase from the original timetable to the disruption timetable, the stable phase where the disruption timetable is performed, and the recovery phase of the disruption timetable resuming to the original timetable (Ghaemi et al., 2017b). As for the passengers, a method is proposed to determine the impacts of dispatching decisions in terms of passengers' planned paths, which are then used as the decision weights in the objective of minimizing passenger delays. The model is tested on real-life instances of a subnetwork of the Dutch railways, which demonstrates fast computations of rescheduling solutions.

The contributions of this chapter are summarized as follows.

- A new rescheduling model is proposed, which includes both flexible stopping and flexible short-turning as well as retiming, reordering, and cancelling trains.
- The model deals with all phases of a disruption.
- Adjusted train running times due to saved (extra) decelerations and accelerations are explicitly considered when skipping (adding) stops.
- Station capacity is considered by ensuring that each train corresponding to passenger boarding/alighting stops at a platform track while the minimum headway times are taken into account.
- Rolling stock circulations at the short-turning and terminal stations of trains are included.
- Dispatching decisions are optimized with the objective of minimizing passenger delays.

In the following, we first give an overview of the literature on timetable rescheduling models in Section 3.2, followed by the mathematical modelling of the problem in Section 3.3. Then, the case study is given in Section 3.4 and finally Section 3.5 concludes this chapter and points out directions for future research.

3.2 Literature review and problem challenge

In this section, we first give an overview of the publications on timetable rescheduling, particularly differentiated by the used dispatching measures. Then, the characteristics of papers relevant to disruptions, including the infrastructure modelling level, the used method, the objective, and whether considering OD turn or station capacity, are discussed and compared to the ones of this chapter. In the end, the challenges of the problem considered in this chapter are explained.

3.2.1 Literature review

During disturbances that cause service perturbations rather than dropped infrastructure capacity, local re-routing and re-timing are commonly adopted to adjust the timetable. For example, D'Ariano et al. (2008) and Corman et al. (2010) sequentially determine train routes and then the arrival and departure times of trains, while Meng and Zhou (2014) specify the routes and schedules of trains simultaneously. These papers describe infrastructure at a microscopic level, and so does Pellegrini et al. (2014) where blocking times are explicitly formulated. Recently, Pellegrini et al. (2019) propose valid inequalities that allow reformulating the model presented in Pellegrini et al. (2014) to boost computation efficiency. A detailed review regarding timetable rescheduling during disturbances and disruptions can be found in Cacchiani et al. (2014). In the following we review the literature on timetable rescheduling during disruptions.

Narayanaswami and Rangaraj (2013) establish an MILP model for a track blockage between two adjacent stations for a single-track railway with the objective of minimizing the delays of trains at the destinations. In their model, the affected trains that will run through the disrupted section during the disruption are forced to be delayed to at least after the disruption ends. Due to this delaying measure, new conflicts may rise up between these delayed trains and the trains that are originally scheduled after the time the disruption ends. Thus, binary precedence variables are introduced to allow re-ordering at stations. Meng and Zhou (2011) also deal with the disruption occurring in a single-track railway, by additionally considering uncertain disruption length and varied running times of trains. A stochastic programming model is established and embedded in a rolling horizon framework, where the measure of delaying trains is applied. Different scenarios are tested in their model with the objective of minimizing the expected secondary delays of trains.

Compared to delaying a train, cancelling usually leads to more passenger inconvenience, if the focus is on the cancelled train only. However, if the disruption is rather long or trains run with high frequencies, cancelling a train might be better than delaying it. Otherwise, more subsequent trains could be delayed, thus resulting in more passenger inconvenience across the whole network. Under these circumstances, train cancellation is necessary. Cadarso et al. (2013) propose an integrated optimization model for rescheduling both timetable and rolling stock. Two dispatching measures are applied, complete train cancellation and emergency train insertion, with the objective of minimizing operational cost, cancellations, denied passengers and service deviations. The departure/arrival times of emergency trains are pre-determined and fixed in the disruption timetable, and the departure/arrival times of planned trains are also fixed. Thus, only binary decision variables are needed to decide which planned trains should be cancelled and which emergency trains should be inserted. An extension on Cadarso et al. (2013) is made by Binder et al. (2017b) who include three additional dispatching measures, partial cancellation, delaying, and global rerouting, into an ILP model with the objective of minimizing operational cost, service deviations and passenger inconveniences. This model depends on a pre-constructed rescheduling graph where delaying and rerouting arcs for planned trains are constructed to make delaying and global rerouting of trains possible. In addition, conflicting arcs are also pre-constructed to prevent any conflicts between trains. As a consequence, binary decision variables are needed in the model to decide which arcs are chosen by trains, to produce a conflict-free disruption timetable. Zhan et al. (2015) propose an MILP model for timetable rescheduling in case of complete track blockage, by including the measures of cancelling, retiming, and reordering trains. Disruption length is assumed to be known and fixed when the disruption starts. Later, they proposed another model to take uncertain disruption lengths into account, and the target case is changed to partial segment blockage (Zhan et al., 2016). In both models, they aim to minimize train delays and cancellations. As seat reservations are necessary in Chinese railways, the measure of short-turning trains is not considered in their models.

Seat reservations are not required in most urban rail transit systems and European railway systems, which makes short-turning trains widely used there during disruptions. Puong and Wilson (2008) declare that when service disruptions are less than 20 minutes, only holding strategies (i.e. increasing dwell times of trains) are used. However for longer disruptions, short-turning trains are usually used together with the holding strategies. The purpose is to keep the headways of both operation directions as regular as possible (Wilson et al., 1992), or to isolate the disrupted area from the whole network (Ghaemi et al., 2018a). Short-turning trains is considered by Louwerse and Huisman (2014) who propose an ILP model to deal with partial or complete blockage on a double-track railway. In their model, the capacities of short-turn stations are considered, while assuming the capacities of other stations to be infinite. By extending the model of Louwerse and Huisman (2014), Veelenturf et al. (2015) take the capacities of all stations into account, while short-turn stations are fixed to trains. This means that for each train, the last scheduled stop approaching the disrupted area is set as the only short-turn station. Instead, Ghaemi et al. (2018a) propose an MILP model where two short-turn station candidates are provided to each train. The model deals with complete track blockages and describes the infrastructure at a macroscopic level. To improve the practicability, another MILP model is proposed by Ghaemi et al. (2017a), which deals with the same problem but describes the infrastructure at a microscopic level. In both models, the objectives are minimizing train delays and cancellations. Van Aken et al. (2017a) establish an MILP model to deal with timetable adjustments for full-day multiple maintenance possessions (i.e. planned disruptions). Each train has one short-turn station to be chosen, and whether a train will be short-turned or not is decided in a preprocessing step. Further, Van Aken et al. (2017b) include short-turning decisions into the model and each train is provided with more short-turn station options. In their model, pre-processing is necessary to identify which services corresponding to a train should be cancelled in case of the train being short-turned or completely cancelled, and the short-turn durations are fixed.

In addition to the dispatching measures mentioned above, changing stopping patterns is also widely adopted when passenger inconvenience (e.g. waiting times, total travel times, etc.) are taken into account. Among the literature on passenger-oriented timetable rescheduling, Sato et al. (2013) allow the types of trains to be changed. This means that the express trains can change to local trains, and vice versa, while the stopping pattern of each train type is fixed. Gao et al. (2016) and Altazin et al. (2017) both allow trains to skip certain stops, while Veelenturf et al. (2017) allow additional stops to be added to trains without considering the extra deceleration and acceleration times.

Most literature is operator-oriented, which usually aims to minimize train delays and cancellations, where the penalties of both measures are determined without clear rules and thus varying across papers. It has been reported in Zhan et al. (2015) and Ghaemi et al. (2017a) that the rescheduling solutions are sensitive to the cancellation and delaying penalties. In other words, different solutions can be obtained when setting different values to the cancellation and/or delaying penalties although the disruption characteristics (the disrupted section and the disruption starting/ending time) remain. For example, increasing the cancellation penalty without changing the delaying penalty may lead to less services cancelled but more services delayed. Zhan et al. (2015) conclude that the penalty choice is a trade-off to be made by railway managers: cancelling less trains by delaying more trains or the other way around; while Ghaemi et al. (2017) suggest to generate multiple solutions by varying penalties, which then will be evaluated by experts from different perspectives to decide an overall best solution.

The passenger-oriented rescheduling models for disruptions are limited, but are increasing over the last years. Cadarso et al. (2013) use a frequency-based passenger assignment model to estimate passengers' geographic travel paths with no timetable known yet, and the rescheduling model aims to accommodate the estimated passenger demand as much as possible. Veelenturf et al. (2017) use a schedule-based passenger assignment model that estimates passengers' exact travel paths with time information. They consider passenger demand in a dynamic way by iteratively adjusting the timetable by adding a stop in each iteration if it is indicated by the passenger assignment model that this stop can reduce passenger inconvenience. The decisions are limited to adding stops. Until now, Binder et al. (2017b) is the only one that integrates passenger rerouting with rescheduling in one single model for disruption cases, where trains can be partially cancelled by neglecting the rolling stock connection between any two of them (i.e. no short-turning).

3.2.2 Summary and contributions of this chapter

In Table 3.1, the dispatching measures used in the publications on timetable rescheduling during disruptions are summarized, as well as the ones used in this chapter. Other characteristics like the infrastructure modelling level, the used method, the objective, and whether considering OD turn or station capacity are also given. The symbol '-' indicates that OD turn or station capacity is neglected in a paper, while the symbol '✓' indicates that it is considered. The publications that consider passengers in the objectives are shown in the lower part.

Table 3.1: Comparisons between publications on railway disruption management and this paper

Publication	Level	Method	Objective	Dispatching decisions	OD turn	Station capacity
Narayanaswami and Rangaraj (2013)	Macro	MILP	Min. train delays at the destinations	D, RO	-	-
Meng and Zhou (2011)	Macro	MILP, RH	Min. the expected secondary delays of trains under different forecasted conditions	D	-	✓
Zhan et al. (2015)	Macro	MILP	Min. train delays and cancellations	D, RO, C	-	✓
Zhan et al. (2016)	Macro	MILP, RH	Min. train deviations and cancellations	D, RO, C	-	✓
Louwerse and Huisman (2014)	Macro	ILP	Min. train delays, cancellations and operation imbalance	D, RO, C, ST (one fixed short-turn station)	✓	(only short-turn stations)
Veelenturf et al. (2015)	Macro	ILP	Min. train delays and cancellations	D, RO, C, RR, ST (one fixed short-turn station)	✓	✓
Ghaemi et al. (2018a)	Macro	MILP	Min. train delays and cancellations	D, C, ST (two short-turn station candidates)	-	✓
Ghaemi et al. (2017a)	Micro	MILP	Min. train delays and cancellations	D, RO, C, ST (two short-turn station candidates)	-	✓
Cadarso et al. (2013)	Macro	MILP, PA	Min. operational costs, cancellations, denied passengers and service deviations	C, E	✓	-
Binder et al. (2017b)	Macro	ILP	Min. operational costs, service deviations and passenger inconveniences	D, RO, C, RR, E	-	✓
Veelenturf et al. (2017)	Macro	ILP, PA	Min. operational costs and passenger delays	A	✓	-
This paper	Macro	MILP, PA	Min. passenger delays	D, RO, C, A, S, FST (full choice of short-turn station candidates)	✓	✓

Method: MILP, ILP, Rolling Horizon (RH), Stochatic Programming (SP), Passenger Assignment (PA)
Dispatching decisions: Delaying (D), Reordering (RO), Cancelling (C), Emergency Train Insertion (E), Rerouting (RR), Short-turning (ST), Adding Stop (A), Skipping Stop (S), Flexible Short-turning (FST).

This chapter describes the infrastructure also at a macroscopic level to allow fast computations, like most of the existing literature. The problem is formulated as an MILP model with the objective of minimizing passenger delays. A schedule-based passenger assignment is adopted to obtain passengers' planned paths, which are used to estimate passenger-dependent weights for decisions included in the objective function. The weight of each decision is individually estimated considering the affected passengers and the impact on them due to the decision. For example, the penalty of skipping a stop is calculated as the number of passengers who originally plan to board or alight from the train at the stop multiplied by an assumed delay per passenger considering that each of these passengers has to reroute. Compared to operator-oriented models, the model generates more passenger-friendly solutions that are also preferred by train operators. This is because the passenger-dependent decision weights are calculated according to passengers' planned paths estimated based on the planned timetable. In that sense, our model aims to reduce the impact on passengers' planned paths, which is, to some extent, in line with reducing the deviations from the planned timetable, while less timetable deviations help to reduce the complexity of rescheduling the rolling stock/crew further. Although passenger demand is considered in a static way, the model ensures fast computations of solutions, which satisfy the real-time requirement. Until now, only a few rescheduling models that consider passenger demand in a dynamic way (i.e. timetable-dependent passenger behaviour) in case of disruptions. These models can reflect passenger behaviour more accurately but are at the expense of more computation time (Binder et al., 2017b) that is usually not acceptable in practice. Different from the existing literature, our model allows more flexibilities for the timetable rescheduling: 1) delaying, reordering, and cancelling are all allowed; 2) short-turning is considered and in a completely flexible way by giving each train a full choice of short-turn station candidates; and 3) adding and skipping stops (i.e. flexible stopping) is innovatively introduced with the deceleration and acceleration times of trains taken into account. To ensure solution feasibility in practice, the rolling stock circulations at the origins/destinations of trains (i.e. OD turns) and the capacity of each station are both considered.

3.2.3 Problem challenge

There are three main challenges of handling the problem considered in this chapter. The first challenge is modelling flexible stopping. In the planned timetable, there are stops and non-stops only. However in a rescheduled timetable, there could be stops, non-stops, skipped stops, added stops, and also cancelled stops and cancelled non-stops due to cancellation measures. These stop types must be recognized by the model individually, as they have different impacts on station capacity, train running times and passengers. The second challenge is modelling flexible short-turning. For fixed short-turning, the station where a train can be short-turned is fixed, thus the decision is only about which opposite train should be served at the station; whereas for flexible short-turning, two decisions are needed, which are where to short-turn a train and which opposite train to be served at the station. If a train is short-turned, its following services that were originally scheduled after the short-turn station must be cancelled. In the literature, this is usually based on a preprocessing step where the cancelled services of a train are included in a set as input. Without pre-processing, the model has to decide which services of a train should be cancelled if the train is short-turned. The third challenge is modelling station capacity under flexible stopping and flexible short-turning, because whether a train needs a platform track at a station can be different than planned. For example, it is unnecessary to assign a platform track to a train that passes through a station. However, a platform track must be assigned: if the train has an additional stop at the station where passengers will board or alight from the train, or if the train is short-turned at a station where passengers will alight from the train. To handle these challenges, a new rescheduling model is proposed.

3.3 Timetable rescheduling model

Timetable rescheduling during complete track blockages adjusts the routes and time-distance train paths to fit the reduced infrastructure capacity without any conflicts between trains. In this section, it is formulated as an MILP model based on an event-activity network that is explained first. Next, the constraints for cancelling, delaying, reordering, flexible stopping, flexible short-turning, rolling stock circulations at terminal stations, and station capacity are introduced successively. Finally, the objective is given including the passenger-dependent weight for each decision considering the affected passengers and the impact on them due to the decision.

3.3.1 Event-activity network

In Chapter 2, the event-activity network is formulated to describe passenger responses towards a given timetable. Hence, the events (e.g. E_{exit}) and activities (e.g. A_{wait}) relevant to passengers are included, but *not* the activities for the adjustments that could be applied to the timetable (e.g. short-turning activities). In contrast, the event-activity network formulated in this chapter includes the activities relevant to timetable adjustments, but *not* the events and activities corresponding to passengers. In the present chapter, each departure/arrival of a train is formulated as a departure/arrival event e with corresponding information: original scheduled time o_e, station st_e, train line tl_e, train number tr_e, and operation direction dr_e. A train line indicates the origin, the destination, all intermediate stops between the origin and the destination, and the operation frequency (e.g. every 30 minutes). For the train that passes through a station, we divide the pass-through action into two events: pass-through departure and pass-through arrival. The benefit of formulating the pass-through action this way is twofold. First, it enables the modelling of the case that a train does not stop at a station but could be short-turned. Second, it makes the modelling of additional stops of a train possible.

Each activity a is a directed arc from one event to another event, i.e., from $tail(a)$ to $head(a)$. The type of activities include running activities, dwell activities, pass-through activities, short-turn activities, OD turn activities and headway activities (arrival headway, departure headway, arrival-departure headway, or departure-arrival headway).

- A running activity a_{run} is defined from a departure event to an arrival event with both events belonging to the same train but occurring at two adjacent stations. The departure event occurs at the upstream station relative to the station where the arrival event occurs.
- A dwell (pass-through) activity a_{dwell} (a_{pass}) is defined from an arrival event to a departure event that belongs to the same train, occurs at the same station, and with the departure event occurring later (at the same time as the arrival event).
- An OD turn activity a_{odturn} describes a turn of a train at its destination where the rolling stock continues to operate an opposite train from the same train line.
- A short-turn activity a_{turn} is defined from an arrival event to a departure event that occurs at the same station but operates in the opposite direction. Both events are with the same train line but different trains.

In general, different train lines may use the same or different rolling stock types, and intercity and local lines use different rolling stock types. Two local lines may use the same rolling stock type but for rolling sock circulations we prefer to keep the rolling stock units on the same train line so that they stay in the same circulations, rather than ending up in complete different areas corresponding to different train line routes. This eases the recovery after the disruption. Therefore, we only allow a short-turn activity to be created between two events from the same train line. More details about creating short-turn activities can be found in Section 3.3.5.

- An arrival-departure headway activity $a_{\text{head}}^{\text{ar,de}}$ is defined from an arrival event to a departure event that occurs at the same station, but belongs to a different train operating in the opposite direction.
- An arrival (departure) headway activity $a_{\text{head}}^{\text{ar}}$ ($a_{\text{head}}^{\text{de}}$) is defined from an arrival (departure) event to another arrival (departure) event that occurs at the same station, operates in the same direction, but belongs to a different train.
- A departure-arrival headway activity $a_{\text{head}}^{\text{de,ar}}$ is defined from a departure event to an arrival event that occurs at the same station but belongs to a different train.

Arrival-departure headway activities are needed for trains operating on single-track railway lines. This is because any two adjacent stations located on single-track railway lines are linked by one track only, which makes it necessary to keep a headway between the arrival of a train and the departure of another train that will enter the open-track section where the arriving train comes from. The area between two adjacent stations is an open-track section. On double-track railway lines, any two adjacent stations are linked by two tracks where each is used by trains operating in the same direction. Arrival (departure) headway activities are needed for following trains operating in either single-track railway lines or double-track railway lines. Departure-arrival headway activities are needed for trains that use the same track at a station, and also both departure-arrival and arrival-departure headway activities are used for trains with crossing routes at stations.

Example: Figure 3.1 shows a timetable where train tr_1 runs from station C to station A with a stop at station B, train tr_2 runs from station C to station A directly, and train tr_3 runs from station A to station C directly. It is assumed that stations A, B and C are located on double-track railway lines, tr_1 continues its operation further after station A, and tr_2 ends its operation at station A and the corresponding rolling stock turns to be used in the operation of tr_3.

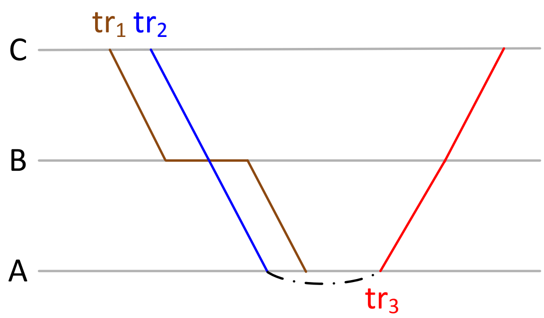


Figure 3.1: A timetable with three trains and three stations located on double-track railway lines

The event-activity network formulation of this example is presented in Figure 3.2, where headway activities are always pairwise created between two events considering that the order between them may change. A rescheduling solution to the event-activity network of Figure 3.2 is shown in Figure 3.3, where a stop is added to tr_2 at station B by delaying the departure of tr_2 at this station. The activities that are not valid in the rescheduling solution are all coloured in grey. For example, if there is no short-turning, the short-turn activity is invalid. Also between two events, one headway activity must be invalid, with the other one being valid, if these two events are not cancelled. Otherwise, both headway activities between them are invalid. A valid activity in a rescheduling solution must satisfy the conditions that 1) the head of the activity occurs no earlier than the tail of the activity and the time difference between them respects the required duration, 2) both the head and tail events are not cancelled, and 3) the activity must be active if it is a short-turn/OD turn activity.

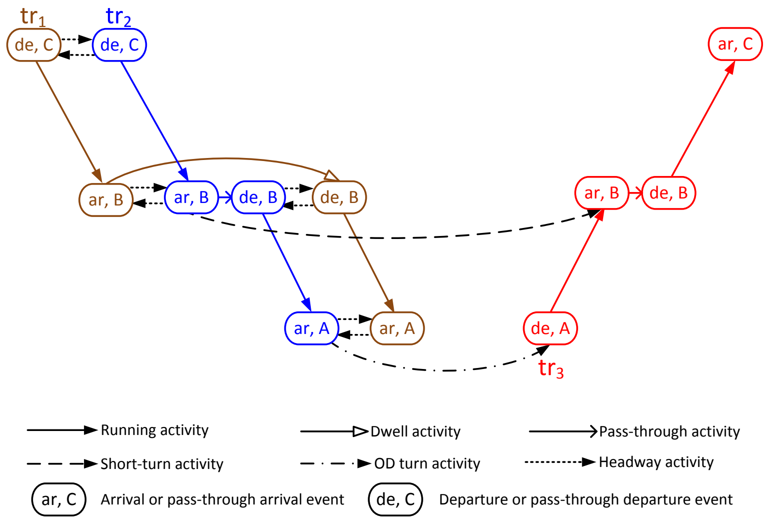


Figure 3.2: Event-activity network formulation of Figure 3.1

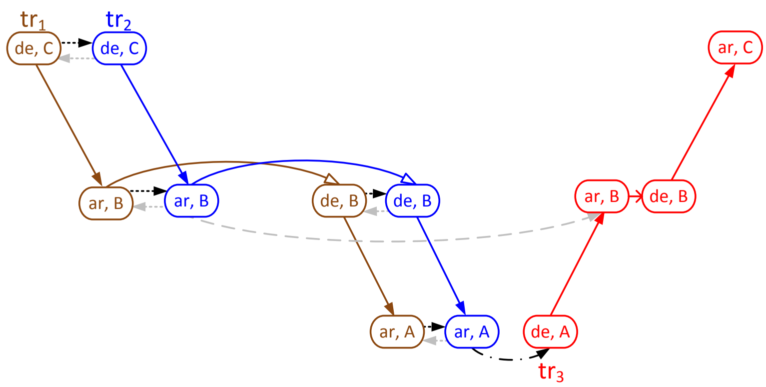


Figure 3.3: A rescheduling solution to Figure 3.1 by adding stop, reordering and delaying

The notation used for sets and parameters are given in Appendix 3.A. The decision variables used in the proposed model are described in Table 3.2.

The proposed model is based on three assumptions. First, it is assumed that the end time of a disruption is given at the moment the disruption starts and will not change. Although uncertain disruption duration is not considered in our model, it can be handled by embedding the proposed model in a rolling horizon framework like Meng and Zhou (2011) and Zhan et al. (2016) did. In the case study, we investigate how the proposed model reacts to different disruption durations. The second assumption is that trains that have already entered the blocked tracks when the disruption starts have passed the blocked points already, and thus can run as planned. The third assumption is that trapped trains that cannot turn at a station before the blocked tracks will dwell at the last possible station until the disruption ends. The passengers may be evacuated using bus services to stations with running trains or stay in the train, depending on the disruption length. Our model considers that the passengers in a trapped

Table 3.2: Decision variables

Notation	Description	Domain
x_e	The rescheduled time of event e	$x_e \geq 0$
d_e	The delay of event e	$d_e \geq 0$
c_e	The binary variable deciding whether event e is cancelled. If yes, $c_e = 1$.	$c_e \in \{0,1\}$
p_e	The binary variable deciding whether train tr_e needs a platform track, $e \in E_{\text{ar}}$. If yes, $p_e = 1$.	$p_e \in \{0,1\}$
$q_{e,e'}$	The binary variable deciding whether event e occurs before e'. If yes, $q_{e,e'} = 1$.	$q_{e,e'} \in \{0,1\}$
$n_{e,e'}$	The binary variable deciding whether train tr_e is occupying a track at station st_e when another train $tr_{e'}$ arrives, $e, e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}$. If yes, $n_{e,e'} = 1$.	$n_{e,e'} \in \{0,1\}$
$n^p_{e,e'}$	The binary variable deciding whether train tr_e is occupying a platform track at station st_e when another train $tr_{e'}$ arrives, $e, e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}$. If yes, $n^p_{e,e'} = 1$.	$n^p_{e,e'} \in \{0,1\}$
s_a	The binary variable deciding whether a scheduled stop $a \in A_{\text{dwell}}$ is skipped or whether an extra stop is added to $a \in A_{\text{pass}}$. If $a \in A_{\text{dwell}}$, then $s_a = 1$ indicates a is skipped. If $a \in A_{\text{pass}}$, then $s_a = 0$ indicates a is added with a stop.	$s_a \in \{0,1\}$
m_a	The binary variable deciding whether a short-turn or an OD turn activity $a \in A_{\text{turn}} \cup A_{\text{odturn}}$ is selected. If yes, $m_a = 1$.	$m_a \in \{0,1\}$
y_e	The binary variable deciding whether station st_e is chosen as the short-turn station for train $tr_e, e \in E^{\text{turn}}_{\text{ar}} \cup E^{\text{turn}}_{\text{de}}$. If yes, $y_e = 1$.	$y_e \in \{0,1\}$

train are delayed at least for the dwelling duration. In the following, the constraints for cancelling, delaying, reordering, flexible stopping, flexible short-turning, rolling stock circulations at terminal stations, and station capacity are introduced successively.

3.3.2 Constraints for cancelling and delaying trains

For an event e, the relation between its rescheduled time x_e, the cancelling decision c_e and the delaying decision d_e is formulated by

$$M_1 c_e \leq x_e - o_e \leq M_1, \qquad e \in E_{\mathrm{ar}} \cup E_{\mathrm{de}}, \tag{3.1}$$

$$x_e - o_e = d_e + M_1 c_e, \qquad e \in E_{\mathrm{ar}} \cup E_{\mathrm{de}}, \tag{3.2}$$

$$d_e \geq 0, \qquad e \in E_{\mathrm{ar}} \cup E_{\mathrm{de}}, \tag{3.3}$$

$$d_e \leq D, \qquad e \in (E_{\mathrm{ar}} \cup E_{\mathrm{de}}) \setminus E^{\mathrm{NMdelay}}, \tag{3.4}$$

where o_e is the original scheduled time of e, E_{ar} (E_{de}) is the set of arrival (departure) events, and E^{NMdelay} is the set of events that do not have an upper limit on their delays. The events in E^{NMdelay} correspond to the trains that are originally scheduled to run through the disrupted section during the disruption but have already departed from the origins before the disruption starts. Thus, these trains can only be short-turned or delayed, but not cancelled. In case these trains are unable to be short-turned due to insufficient station/rolling stock capacity, they have to be delayed at least to the end of the disruption. Considering these situations, no upper limit is imposed on the delays of the events corresponding to these trains. Here, we use E^{NMdelay} to contain such events that do not have an upper limit on their delays. Suppose events e and e' are the departure events of train tr at the origin station and the entry station of the disrupted section, respectively. If event e originally occurs before the disruption starting time t_{start} while event e' originally occurs after t_{start}, then all departure and arrival events that correspond to train tr and originally occur after t_{start} belong to the set E^{NMdelay}. Constraint (3.1) means that for each event e, the rescheduled time x_e is not allowed to occur earlier than the original scheduled time o_e, and it should be removed after the end of the day if it is cancelled (i.e. $c_e = 1$). As we use *minute* as the unit for any x_e or o_e, M_1 is set to 1440 (i.e. one day has 1440 minutes). Thus according to (3.1), the rescheduled time of a cancelled event is the original scheduled time plus 1440. For a cancelled event, its delay d_e is equal to 0, while for an non-cancelled event, its delay is equal to the time difference between the rescheduled time and the original scheduled time (3.2). Constraint (3.3) means that event delay is non-negative. Constraint (3.4) means that an event that does not belong to E^{NMdelay} is allowed to be delayed by at most D minutes, considering that it is not preferred to delay a train too much. Imagine that a train arrives at a station on time but departs from the station much later than planned, then a track of the station will be occupied by the train for a rather long time, which is not good for station capacity utilization. Besides, we consider a cyclic planned timetable to be rescheduled, thus delaying a train longer than the cycle time does not make much sense, since another train with the same origin, destination and stopping patterns will operate later. Considering these, the parameter of maximum allowed delay per event, D, is used. This parameter is also adopted by some rescheduling models for disruptions, like Zhan et al. (2015) and Veelenturf et al. (2015).

During the disruption period, a departure event that is originally scheduled to occur at the entry station of the disrupted section is either cancelled or delayed at least to the end of the disruption,

$$x_e \geq t_{\text{end}}(1-c_e), \qquad e \in E_{\text{de}}, st_e = st_{\text{en}}^{dr_e}, t_{\text{start}} \leq o_e < t_{\text{end}}, \tag{3.5}$$

where $st_{\text{en}}^{dr_e}$ represents the entry station of the disrupted section considering the operation direction of the train corresponding to e (i.e. dr_e). An example of explaining the entry/exit station of a disrupted section is shown in Figure 3.4 where the section between stations B and C is completely blocked from t_{start} to t_{end}, and a short-turning occurs between the blue and red trains at either station B or C. For the blue train that operates in downstream direction, the entry (exit) station of the disrupted section is station C (station B); whereas for the red train that operates in upstream direction, the entry (exit) station of the disrupted section is station B (station C). The dashed lines indicate the cancelled services due to the short-turnings. Here, the departure event of the blue train at station C is the event e of which the original scheduled time o_e satisfies $t_{\text{start}} \leq o_e < t_{\text{end}}$, thus it must be cancelled or delayed after t_{end} according to (3.5). In this case, e is cancelled (i.e. $c_e = 1$). Imagine that e is not cancelled (i.e. $c_e = 0$), then 1) both short-turnings indicated in Figure 3.4 will not occur; and 2) the blue train will depart from station C later than t_{end}. This means that event e will be delayed by at least $t_{\text{end}} - o_e$ minutes, thus the blue train will occupy a track of station C for $t_{\text{end}} - o_e$ minutes at least. However due to the upper limit on delay D, the train will not occupy the track for longer than D minutes. An exception could be that a train is dwelling at the entry station of the disrupted section when the disruption starts but the infrastructure layout of the station is unable for short-turning; thus the train has to remain at the station until the disruption ends and the waiting time can be longer than D minutes. This is defined in the set E^{NMdelay}

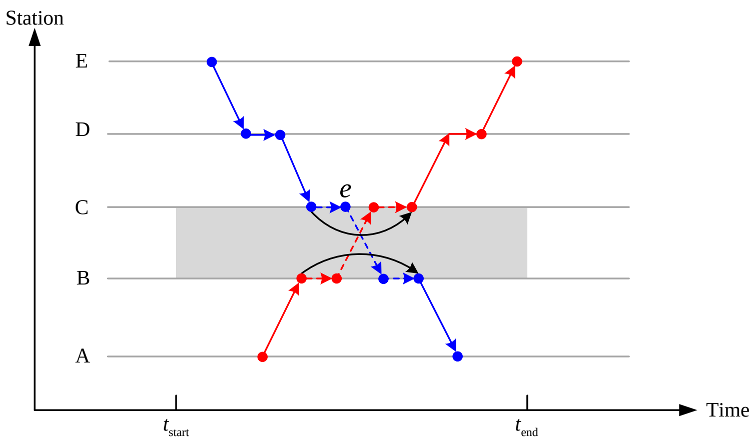


Figure 3.4: An example used for explaining the entry/exit station of the disrupted section for a train

An event that originally occurs before the disruption start t_{start} cannot be cancelled, and should run as planned:

$$c_e = 0, \qquad e \in E_{\text{ar}} \cup E_{\text{de}}, o_e < t_{\text{start}}, \tag{3.6}$$

$$x_e - o_e = 0, \qquad e \in E_{\text{ar}} \cup E_{\text{de}}, o_e < t_{\text{start}}. \tag{3.7}$$

A departure event that originally occurs after $t_{\text{end}} + R$ cannot be cancelled and should run as planned:

$$c_e = 0, \qquad e \in E_{\text{de}}, o_e \geq t_{\text{end}} + R, \tag{3.8}$$

$$x_e - o_e = 0, \qquad e \in E_{\text{de}}, o_e \geq t_{\text{end}} + R. \tag{3.9}$$

Here, R represents the required time length for the disruption timetable resuming to the original timetable after the disruption ends. Setting R helps to avoid the disruption affecting the timetable for the entire day, which is also adopted by Veelenturf et al. (2015). For an arrival event, if its corresponding departure event in the running activity originally occurs after $t_{\text{end}} + R$, this arrival event cannot be cancelled and should run as planned:

$$c_{e'} = 0, \qquad (e, e') \in A_{\text{run}}, o_e \geq t_{\text{end}} + R, \tag{3.10}$$

$$x_{e'} - o_{e'} = 0, \qquad (e, e') \in A_{\text{run}}, o_e \geq t_{\text{end}} + R. \tag{3.11}$$

Note that for an arrival event that originally occurs after $t_{\text{end}} + R$, its corresponding departure event in the running activity could originally occur before $t_{\text{end}} + R$ and possibly be cancelled/delayed, which makes this arrival cancelled or unable to run as planned. Constraints (3.8) - (3.11) require a disruption to be fully recovered after $t_{\text{end}} + R$. This might be impossible if R is set to a very small value like 0, thus resulting in infeasible solution. Considering that an event that originally occurs during the disruption period could be delayed by at most D minutes, it is better to set R at least larger than D to avoid infeasibility.

Any two events that constitute the same running activity are either cancelled or kept simultaneously:

$$c_{e'} - c_e = 0, \qquad (e, e') \in A_{\text{run}}. \tag{3.12}$$

Any two events that constitute the same station activity are either cancelled or kept simultaneously, if neither of these two events corresponds to a short-turn activity:

$$c_{e'} - c_e = 0, \qquad (e, e') \in A_{\text{station}}, e \notin E_{\text{ar}}^{\text{turn}}, e' \notin E_{\text{de}}^{\text{turn}}. \tag{3.13}$$

A station activity can either be a dwell activity or a pass-through activity. Here, $E_{\text{ar}}^{\text{turn}}$ ($E_{\text{de}}^{\text{turn}}$) is the set of the tails (heads) of all short-turn activities contained in A_{turn}. The tail of $a \in A_{\text{turn}}$ must be an arrival event, and the head of $a \in A_{\text{turn}}$ must be a departure event.

Instead of requiring the running time on an open-track section to respect the minimum running time, we constrain it at least to respect the original scheduled running time, in order to keep disruption timetable robustness by the original scheduled time supplements:

$$x_{e'} - x_e \geq o_{e'} - o_e, \qquad a = (e, e') \in A_{\text{run}}. \tag{3.14}$$

During disruptions, small service perturbations could also happen, of which the resulting delays are expected to be mitigated by the time supplements kept in the disruption timetable.

To prevent overlong running in an open-track section, the following constraint is given,

$$x_{e'} - x_e \leq (1+\lambda)(o_{e'} - o_e), \qquad a = (e, e') \in A_{\text{run}}, \tag{3.15}$$

where λ represents the maximum percentage allowed to the running time extension. During disruptions, it happens that a train runs with a slower speed than usual if the station which it is approaching to lacks capacity to receive it. As a result, longer time is needed for the running. However, a train cannot run too slow, thus a maximum percentage allowed to the running time extension, λ, is required here.

3.3.3 Constraints for reordering trains

Minimum arrival/departure headways are required between trains running in the same directions:

$$x_{e'} - x_e \geq L_a + M_2(q_{e,e'} - 1), \qquad a = (e, e') \in A^{\text{ar}}_{\text{head}} \cup A^{\text{de}}_{\text{head}}, \tag{3.16}$$

$$q_{e,e'} + q_{e',e} = 1, \qquad (e, e') \in A^{\text{ar}}_{\text{head}} \cup A^{\text{de}}_{\text{head}}, \tag{3.17}$$

where the order between events e and e' is described by the binary decision variable $q_{e,e'}$ with value 1 indicating that e occurs before e'. Here, L_a represents the minimum duration of activity a, and M_2 is set to two times of M_1.

Train overtaking on an open-track section is forbidden:

$$q_{e_1,e'_1} - q_{e_2,e'_2} = 0, \qquad (e_1, e'_1) \in A^{\text{de}}_{\text{head}}, (e_2, e'_2) \in A^{\text{ar}}_{\text{head}}, (e_1, e_2) \in A_{\text{run}}, (e'_1, e'_2) \in A_{\text{run}}. \tag{3.18}$$

For the stations located on single-track railway lines, minimum headway should be respected between the arrival of a train and the departure of another train that occurs at the same station but operates in opposite direction. An example of such an arrival-departure headway is shown in Figure 3.5.

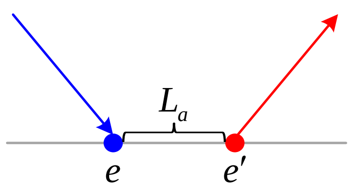


Figure 3.5: Arrival-departure headway on single-track railway lines to ensure at most one train running in an open-track section: $a = (e, e') \in A^{\text{ar,de}}_{\text{head}}$

The constraints for ensuring arrival-departure headways are:

$$x_{e'} - x_e \geq L_a + M_2(q_{e,e'} - 1), \qquad a = (e,e') \in A^{\text{ar,de}}_{\text{head}}, r_{st_e} = 1, \tag{3.19}$$

$$q_{e,e'} + q_{e',e} = 1, \qquad (e,e') \in A^{\text{ar,de}}_{\text{head}}, r_{st_e} = 1, \tag{3.20}$$

where r_{st_e} is a binary parameter with value 1 indicating that station st_e is located on single-track railway lines; and 0 otherwise.

For a station that is located on double-track railway lines and has two tracks, each track of the station is used for trains coming from the same direction. Thus, a minimum headway is required between the departure of a train and the arrival of another train that uses the same track at the station. Likewise, such a headway is needed when a train turns at the station and departs towards the other track. In Figure 3.6, two situations that require such departure-arrival headways are shown.

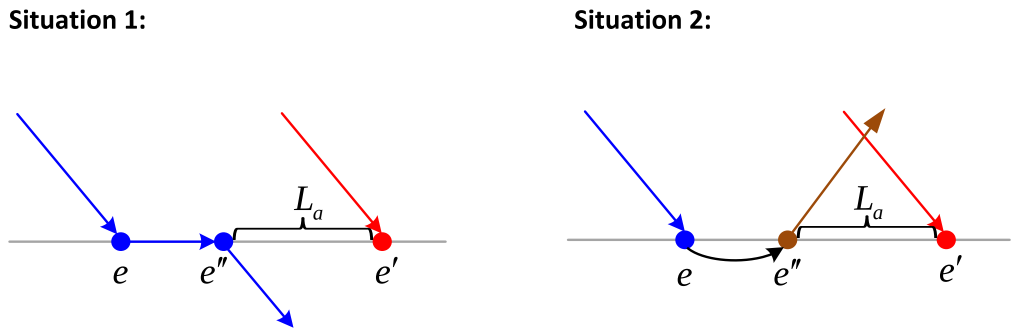


Figure 3.6: Two situations where departure-arrival headways are required in a station that has only two tracks and is located on a double-track railway line: $a = (e'', e') \in A^{\text{de,ar}}_{\text{head}}$

The constraints for ensuring headways shown in Figure 3.6 are:

$$x_{e'} - x_{e''} \geq L_a + M_2(q_{e,e'} - 1 - c_e - c_{e'} - c_{e''}), \qquad (e,e'') \in A_{\text{station}}, \; a = (e'',e') \in A^{\text{de,ar}}_{\text{head}}, N_{st_e} = 2, \tag{3.21}$$

$$x_{e'} - x_{e''} \geq L_a + M_2(q_{e,e'} - 1 - c_e - c_{e'} - (1 - m_{a'})), \; a' = (e,e'') \in A_{\text{turn}} \cup A_{\text{odturn}}, \; a = (e'',e') \in A^{\text{de,ar}}_{\text{head}}, N_{st_e} = 2, \tag{3.22}$$

where N_{st_e} represents the number of tracks in the corresponding station of event e, and $m_{a'}$ is a binary decision variable with value 1 indicating that a short-turn/OD turn activity a' is active, and L_a is the minimum headway duration. Constraints (3.21) and (3.22) are for situation 1 and situation 2, respectively. In (3.21), the time difference between events e' and e'', $x_{e'} - x_{e''}$, does not need to respect the minimum headway L_a, if e, e' or e'' is cancelled, or all of them are kept but e occurs after e' (i.e. $q_{e,e'} = 0$). Also in (3.22), $x_{e'} - x_{e''}$ does not need to respect the minimum headway L_a, if e or e'

is cancelled, both of them are kept but the short-turn/OD turn activity a' relevant to e is not active (i.e. $m_{a'} = 0$), or both e and e' are kept and a' is active but e occurs after e'. The details of short-turn/OD turn activities can be found in Section 3.3.5 and Section 3.3.6.

3.3.4 Constraints for flexible stopping

Recall that flexible stopping means that scheduled stops can be skipped, and extra stops can be added. To realize flexible stopping, a binary variable s_a is introduced. For a scheduled stop $a \in A_{\text{dwell}}$, $s_a = 1$ indicates that the stop is skipped. For a scheduled non-stop $a \in A_{\text{pass}}$, $s_a = 0$ indicates that a stop is added. It happens that a scheduled stop or non-stop is cancelled, which means that the stop type of $a \in A_{\text{dwell}}$ ($a \in A_{\text{pass}}$) in the disruption timetable is also relevant to cancellation decision. Table 6.4 (Table 6.5) shows the values of decision variables $c_e, c_{e'}$ and s_a that indicate the corresponding stop type of $a \in A_{\text{dwell}}$ ($a \in A_{\text{pass}}$) in the disruption timetable.

Table 3.3: The stop type of activity $a = (e,e') \in A_{\text{dwell}}$ in the disruption timetable according to $c_e, c_{e'}$ and s_a

c_e	$c_{e'}$	s_a	Stop type
0	0	0	Stop
0	0	1	Skipped stop
1	0	0	Cancelled stop
0	1	0	Cancelled stop
1	1	0	Cancelled stop

Table 3.4: The stop type of activity $a = (e,e') \in A_{\text{pass}}$ in the disruption timetable according to $c_e, c_{e'}$ and s_a

c_e	$c_{e'}$	s_a	Stop type
0	0	0	Extra stop
0	0	1	Non-stop
1	0	1	Cancelled non-stop
0	1	1	Cancelled non-stop
1	1	1	Cancelled non-stop

The constraints deciding the values of $c_e, c_{e'}$ and s_a are:

$$s_a \leq 1 - c_e, \qquad a = (e,e') \in A_{\text{dwell}}, \tag{3.23}$$

$$s_a \leq 1 - c_{e'}, \qquad a = (e,e') \in A_{\text{dwell}}, \tag{3.24}$$

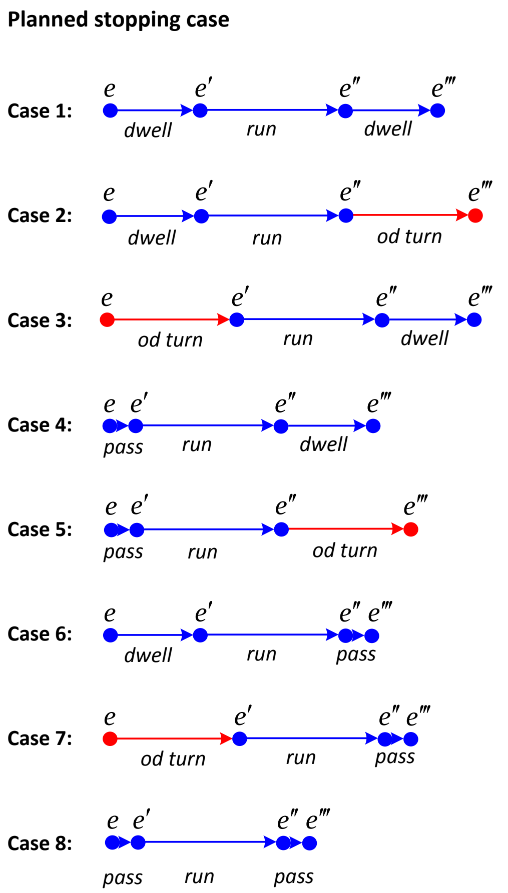
$$s_a \geq c_e, \qquad a = (e,e') \in A_{\text{pass}}, \tag{3.25}$$

$$s_a \geq c_{e'}, \qquad a = (e,e') \in A_{\text{pass}}, \tag{3.26}$$

$$x_{e'} - x_e \geq L_a(1 - s_a - c_e - c_{e'}) - M_1 c_e, \qquad a = (e,e') \in A_{\text{station}}, \tag{3.27}$$

$$x_{e'} - x_e \leq M_2(1 - s_a + c_e + c_{e'}), \qquad a = (e,e') \in A_{\text{station}}, \tag{3.28}$$

where $A_{\text{station}} = A_{\text{dwell}} \cup A_{\text{pass}}$. For $a = (e,e') \in A_{\text{dwell}}$, (3.23) and (3.24) constrain s_a to be 0, if either e or e' is cancelled. For $a = (e,e') \in A_{\text{pass}}$, (3.25) and (3.26) constrain s_a to be 1, if either e or e' is cancelled. According to Table 6.4 and Table 6.5, a *true* stop in the disruption timetable can only be one of which the corresponding c_e, $c_{e'}$ and s_a are all equal to 0. Constraint (3.27) requires a *true* stop to satisfy the minimum dwell duration. Besides, a skipped stop or a non-stop in the disruption timetable can only be one of which the corresponding c_e and $c_{e'}$ are both equal to 0, and $s_a = 1$. Constraints (3.27) and (3.28) together ensure that the duration of a skipped stop or a non-stop is 0. For a cancelled stop (non-stop) that either the corresponding c_e or $c_{e'}$ is equal to 1, (3.27) and (3.28) also remain feasible.



Planned minimum running time

$$L_{a'} = \Delta_{a'}^{\text{acce}} + \tau_{a'} + \Delta_{a'}^{\text{dece}},\ a' = (e', e'')$$

$$L_{a'} = \Delta_{a'}^{\text{acce}} + \tau_{a'} + \Delta_{a'}^{\text{dece}},\ a' = (e', e'')$$

$$L_{a'} = \Delta_{a'}^{\text{acce}} + \tau_{a'} + \Delta_{a'}^{\text{dece}},\ a' = (e', e'')$$

$$L_{a'} = \tau_{a'} + \Delta_{a'}^{\text{dece}},\ a' = (e', e'')$$

$$L_{a'} = \tau_{a'} + \Delta_{a'}^{\text{dece}},\ a' = (e', e'')$$

$$L_{a'} = \Delta_{a'}^{\text{acce}} + \tau_{a'},\ a' = (e', e'')$$

$$L_{a'} = \Delta_{a'}^{\text{acce}} + \tau_{a'},\ a' = (e', e'')$$

$$L_{a'} = \tau_{a'},\ a' = (e', e'')$$

Figure 3.7: Planned stopping patterns of a train at two adjacent stations and the minimum running time between these two stations under different cases where $e, e'' \in E_{\text{ar}}, e', e''' \in E_{\text{de}}, (e', e'') \in A_{\text{run}}$

When changing the stopping pattern of a train, an adjusted train running time due to saved/extra acceleration and deceleration should be considered. Figure 3.7 enumerates all cases of the planned stopping patterns relevant to a train run $a' = (e', e'') \in A_{\text{run}}$, as well as the composition of the planned minimum running time $L_{a'}$ in each case. Here, $\Delta_{a'}^{\text{acce}}$ and $\Delta_{a'}^{\text{dece}}$ represent the acceleration time and the deceleration time needed for a', respectively; while $\tau_{a'}$ represents the pure running time of a'. Note that time supplement is not included in either $L_{a'}$ or $\tau_{a'}$, and it is always satisfied that $L_{a'} \geq \tau_{a'}$. In Figure 3.7, case 1 means that a train stops at two adjacent stations; case 2 means that a train stops at a station after which it reaches the destination where the corresponding rolling stock turns to operate the opposite train (i.e. OD turn); case 3 means that a train starts from the origin and stops at the next adjacent station; case 4 means that a train passes through a station but stops at the next adjacent station; case 5 means that a train passes through a station after which it reaches the destination; case 6 means that a train stops at a station but passes through the next adjacent station; case 7 means that a train starts from the origin and passes through the next adjacent station; and case 8 means that a train passes through two adjacent stations.

In the rescheduled timetable, the minimum running time of a train between two adjacent stations may become longer or shorter than planned, if either of the stopping patterns at these two stations changes. Considering this, the following three constraints are established for cases 1-3 of Figure 3.7, respectively, each requiring that the minimum running time dependent on the changed stopping patterns is respected:

$$x_{e''} - x_{e'} \geq L_{a'} - \Delta_{a'}^{\text{acce}} s_a - \Delta_{a'}^{\text{dece}} s_{a''}, \quad a = (e,e') \in A_{\text{dwell}}, a' = (e',e'') \in A_{\text{run}}, a'' = (e'',e''') \in A_{\text{dwell}},$$

$$x_{e''} - x_{e'} \geq L_{a'} - \Delta_{a'}^{\text{acce}} s_a, \quad a = (e,e') \in A_{\text{dwell}}, a' = (e',e'') \in A_{\text{run}}, a'' = (e'',e''') \in A_{\text{odturn}}^{\text{plan}},$$

$$x_{e''} - x_{e'} \geq L_{a'} - \Delta_{a'}^{\text{dece}} s_{a''}, \quad a = (e,e') \in A_{\text{odturn}}^{\text{plan}}, a' = (e',e'') \in A_{\text{run}}, a'' = (e'',e''') \in A_{\text{dwell}},$$

where $A_{\text{odturn}}^{\text{plan}}$ is the set of all planned turnings of rolling stock at terminal stations. Recall that for $a \in A_{\text{dwell}}$, $s_a = 1$ indicates that a is skipped. According to (3.14), we have $x_{e''} - x_{e'} \geq o_{e''} - o_{e'}$ with $a' = (e',e'') \in A_{\text{run}}$ and $o_{e''} - o_{e'}$ representing the original scheduled running time that includes time supplement. It is always satisfied that $o_{e''} - o_{e'} \geq L_{a'}$, which makes $x_{e''} - x_{e'} \geq L_{a'}$ always respected. As such, the three constraints shown above are always satisfied, because $s_a, s_{a''} \in \{0,1\}$ thus either $-\Delta_{\text{acce}} s_a$ or $-\Delta_{\text{dece}} s_{a''}$ must be non-positive. Considering this, these three constraints are not included in the proposed model.

For cases 4-8 of Figure 3.7, the following constraints are established, respectively. Each of them requires that the minimum running time depending on the changed stopping pattern is respected. Note that these constraints take effect only if a stop is skipped or a passage becomes an added stop.

$$x_{e''} - x_{e'} \geq L_{a'} + \Delta_{a'}^{\text{acce}}(1 - s_a) - \Delta_{a'}^{\text{dece}} s_{a''}, \quad a = (e,e') \in A_{\text{pass}}, a' = (e',e'') \in A_{\text{run}}, a'' = (e'',e''') \in A_{\text{dwell}}, \tag{3.29}$$

$$x_{e''} - x_{e'} \geq L_{a'} + \Delta_{a'}^{\text{acce}}(1 - s_a), \quad a = (e,e') \in A_{\text{pass}}, a' = (e',e'') \in A_{\text{run}}, a'' = (e'',e''') \in A_{\text{odturn}}^{\text{plan}}, \tag{3.30}$$

$$x_{e''} - x_{e'} \geq L_{a'} - \Delta_{a'}^{\text{acce}} s_a + \Delta_{a'}^{\text{dece}}(1 - s_{a''}), \quad a = (e,e') \in A_{\text{dwell}}, a' = (e',e'') \in A_{\text{run}}, a'' = (e'',e''') \in A_{\text{pass}}, \tag{3.31}$$

$$x_{e''} - x_{e'} \geq L_{a'} + \Delta_{a'}^{\text{dece}}(1 - s_{a''}), \quad a = (e,e') \in A_{\text{odturn}}^{\text{plan}}, a' = (e',e'') \in A_{\text{run}}, a'' = (e'',e''') \in A_{\text{pass}}, \tag{3.32}$$

$$x_{e''} - x_{e'} \geq L_{a'} + \Delta_{a'}^{\text{acce}}(1 - s_a) + \Delta_{a'}^{\text{dece}}(1 - s_{a''}), \quad a = (e,e') \in A_{\text{pass}}, a' = (e',e'') \in A_{\text{run}}, a'' = (e'',e''') \in A_{\text{pass}}, \tag{3.33}$$

Recall that for $a \in A_{\text{pass}}$, $s_a = 0$ indicates that a is added with a stop; while for $a \in A_{\text{dwell}}$, $s_a = 1$ indicates that a is skipped.

3.3.5 Constraints for flexible short-turning

Recall that flexible short-turning means that a train is provided with a full choice of short-turn station candidates and the proposed model decides at which station the train will be short-turned and which short-turn activity at the station will be selected. In the following, we first explain how to generate the set of all possible short-turn activities A_{turn} by Algorithm 3.1, and then introduce the constraints that decide which short-turn activities of A_{turn} can be selected in the rescheduled timetable.

One input of Algorithm 3.1, $ST^{tl,dr}_{\text{turn}}$, contains the short-turn station candidates for the trains serving train line tl and operating in direction dr. Note that each station contained in $ST^{tl,dr}_{\text{turn}}$ is the upstream/same station compared to st^{dr}_{en} where $dr \in \{up, down\}$. Recall that st^{dr}_{en} is the entry station of the disrupted section for the trains operating in direction dr. In Figure 3.4, suppose the infrastructure layouts of stations A, B, C, D and E all allow short-turning trains. Then, for the downstream blue train, its short-turn station candidates includes station C that is the entry station of the disrupted section for this train, and also station D that is the upstream station compared to station C. For the upstream red train, station B is the only short-turn station candidate. Another input of Algorithm 3.1 is L_{turn} that contains the minimum short-turn duration required in each station. The set TL_{dis} includes all train lines that can be affected by the disruption, which is also input to Algorithm 3.1. A train line of which the planned operation covers the disrupted section is a train line that could be affected by the disruption.

Algorithm 3.1: Constructing the set of all possible short-turn activities A_{turn}

Input: $ST^{tl,dr}_{\text{turn}}, E_{\text{de}}, E_{\text{ar}}, t_{\text{start}}, t_{\text{end}}, R, D, L_{\text{turn}}\ TL_{\text{dis}}$
Output: A_{turn}

$A^{tl,dr}_{\text{turn}} = \emptyset$;
foreach $st \in ST^{tl,dr}_{\text{turn}}$ **do**
  Define $E^{\text{dis},tl,dr}_{\text{de},st} = \{e' \in E_{\text{de}} \,|\, tl_{e'} = tl, dr_{e'} = dr, st_{e'} = st, t_{\text{start}} \leq o_{e'} < t_{\text{end}} + R\}$;
  Define $E^{\text{tn}}_{\text{ar}} = \left\{ e \in E_{\text{ar}} \,\middle|\, (e,e') \in A_{\text{station}}, e' \in E^{\text{dis},tl,dr}_{\text{de},st} \right\}$;
  Define $E^{\text{tn}}_{\text{de}} = \{e'' \in E_{\text{de}} \,|\, tl_{e''} = tl, dr_{e''} \neq dr, st_{e''} = st, t_{\text{start}} \leq o_{e''} < t_{\text{end}} + R\}$;
  foreach $e \in E^{\text{tn}}_{\text{ar}}$ **do**
    foreach $e'' \in E^{\text{tn}}_{\text{de}}$ **do**
      if $o_{e''} + D - o_e \geq L^{st}_{\text{turn}}$ **then**
        $a_{\text{turn}} = (e, e'')$;
        $A^{tl,dr}_{\text{turn}} = A^{tl,dr}_{\text{turn}} \cup \{a_{\text{turn}}\}$;

$A_{\text{turn}} = \bigcup_{\substack{tl \in TL_{\text{dis}},\\ dr \in \{up,down\}}} A^{tl,dr}_{\text{turn}}$;

In Algorithm 3.1, we first initialize $A_{\text{turn}}^{tl,dr}$ as an empty set (line 1). $A_{\text{turn}}^{tl,dr}$ contains all possible short-turn activities for trains that serve train line tl and operate in direction dr. Then, we iterate over each station st contained in $ST_{\text{turn}}^{tl,dr}$ (line 2) to define the set $E_{\text{de},st}^{\text{dis},tl,dr}$ that contains the departure events corresponding to the trains that serve train line tl, operate in direction dr, and originally occur at station st after the disruption starts but before the disruption end time plus the recovery time (line 3). For example in Figure 3.8 or Figure 3.9, $e' \in E_{\text{de},st}^{\text{dis},tl,dr}$ is a departure event of which the original scheduled time $o_{e'}$ is within the time period $[t_{\text{start}}, t_{\text{end}} + R)$ and is approaching to the disrupted area. Such a departure event could be cancelled, thus its corresponding arrival event e in the station activity could be short-turned, which is included in the set $E_{\text{ar}}^{\text{tn}}$ (line 4). Also, we define the set $E_{\text{de}}^{\text{tn}}$ to contain the departure events that could be served by the arrival events in $E_{\text{ar}}^{\text{tn}}$ for short-turning (line 5), for example the departure event e'' in Figure 3.8 or Figure 3.9. Between any $e \in E_{\text{ar}}^{\text{tn}}$ and $e'' \in E_{\text{de}}^{\text{tn}}$, a short-turn activity a_{turn} is constructed only if the required minimum short-turn duration could be respected in the rescheduled timetable (lines 6-9). Here, $o_{e''} + D$ is the largest rescheduled time which departure event e'' could occur at, and o_e is the smallest rescheduled time which arrival event e could occur at. L_{turn}^{st} refers to the minimum short-turn duration required at station st. For example in Figure 3.9, although e'' originally occurs before e, it could be delayed to occur later than e to make the minimum short-turn duration respected. Thus, a short-turn activity a_{turn} can be constructed from e to e''. Any constructed a_{turn} for a train serving train line tl and operating in direction dr is added to the set $A_{\text{turn}}^{tl,dr}$ (line 10), and the set A_{turn} is constructed by including all $A_{\text{turn}}^{tl,dr}$ with $tl \in TL_{\text{dis}}, dr \in \{up, down\}$ (line 11).

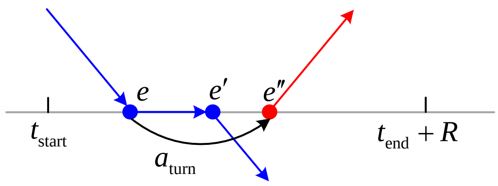


Figure 3.8: Construct a_{turn} from $e \in E_{\text{ar}}$ to $e'' \in E_{\text{de}}$ where $(e, e') \in A_{\text{station}}$, e' and e'' both originally occur after t_{start} but before $t_{\text{end}} + R$, and e'' could occur L_{turn}^{st} minutes later than e after (without) being delayed

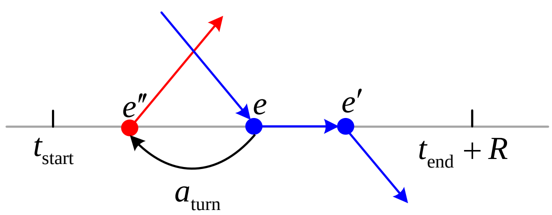


Figure 3.9: Construct a_{turn} from $e \in E_{\text{ar}}$ to $e'' \in E_{\text{de}}$ where $(e, e') \in A_{\text{station}}$, e' and e'' both originally occur after t_{start} but before $t_{\text{end}} + R$, and e'' could occur L_{turn}^{st} minutes later than e after being delayed

A train may have multiple short-turn activities at each short-turn station candidate. An example is given in Figure 3.10 where the tracks between stations A and B are completely blocked from t_{start} to t_{end} and a blue train tr_1 has two short-turn activities at each of stations B, C, D and E. Among all of these short-turn activities, at most one can be selected for the blue train. This means that for each train, the proposed model decides 1) at which station the train will be short-turned; and 2) which short-turn activity at the station is selected for the train. Considering these, we introduce a binary variable y_e with value 1 indicating that station st_e is chosen as the short-turn station for train tr_e; and a binary variable m_a with value 1 indicating that a short-turn activity a is selected. Besides, we construct the set $E_{\text{ar}}^{\text{turn}}$ ($E_{\text{de}}^{\text{turn}}$) that contains all arrival (departure) events that are the tails (heads) of activities included in A_{turn}. For example in Figure 3.10, $e_{1,\text{B}}, e_{1,\text{C}}, e_{1,\text{D}} \in E_{\text{ar}}^{\text{turn}}$, $e'_{2,\text{B}}, e'_{2,\text{C}}, e'_{2,\text{D}}, e'_{3,\text{B}}, e'_{3,\text{C}}, e'_{3,\text{D}} \in E_{\text{de}}^{\text{turn}}$.

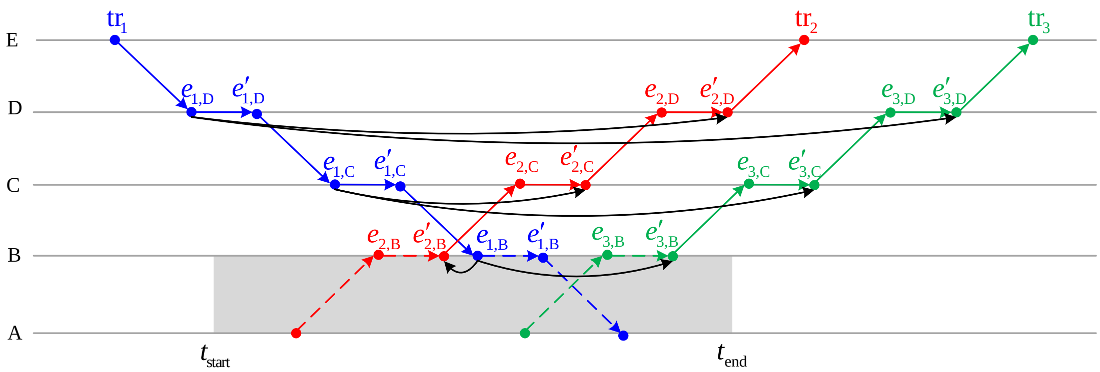


Figure 3.10: The possible short-turn activities (black arcs) at each short-turn station candidate of the blue train

Suppose the blue train in Figure 3.10 is short-turned at station B, then $e_{1,\text{B}}$ must be kept with $e'_{1,\text{B}}$ being cancelled (i.e. $c_{e_{1,\text{B}}} = 0, c_{e'_{1,\text{B}}} = 1$). In that sense, the operation consistency between events $e_{1,\text{B}}$ and $e'_{1,\text{B}}$ is broken. Thus, deciding where to short-turn a train is to decide where to break the operation consistency. The operation consistency can be broken at most one station at each side of the disrupted section for a train. Considering these, the following constraints are established:

$$c_e \leq c_{e'}, \qquad e \in E_{\text{ar}}^{\text{turn}}, (e,e') \in A_{\text{station}}, \tag{3.34}$$

$$c_{e'} \leq c_e + y_e, \qquad e \in E_{\text{ar}}^{\text{turn}}, (e,e') \in A_{\text{station}}, \tag{3.35}$$

$$c_{e'} \geq y_e, \qquad e \in E_{\text{ar}}^{\text{turn}}, (e,e') \in A_{\text{station}}, \tag{3.36}$$

$$c_{e'} \leq c_e, \qquad e' \in E_{\text{de}}^{\text{turn}}, (e,e') \in A_{\text{station}}, \tag{3.37}$$

$$c_e \leq c_{e'} + y_{e'}, \qquad e' \in E_{\text{de}}^{\text{turn}}, (e,e') \in A_{\text{station}}, \tag{3.38}$$

$$c_e \geq y_{e'}, \qquad e' \in E_{\text{de}}^{\text{turn}}, (e,e') \in A_{\text{station}}, \tag{3.39}$$

$$\sum_{e:tr_e=tr} y_e = c_{e'}, \qquad tr \in TR_{\text{turn}}, e \in E^{\text{turn}}_{\text{ar}}, e' \in E_{\text{de}}, tr_{e'} = tr, st_{e'} = s^{dr_{e'}}_{\text{en}}, \tag{3.40}$$

$$\sum_{e':tr_{e'}=tr} y_{e'} = c_e, \qquad tr \in TR_{\text{turn}}, e' \in E^{\text{turn}}_{\text{de}}, e \in E_{\text{ar}}, tr_e = tr, st_e = s^{dr_e}_{\text{ex}}, \tag{3.41}$$

where TR_{turn} is the set of trains that correspond to the events in $E^{\text{ar}}_{\text{turn}} \cup E^{\text{de}}_{\text{turn}}$, and $s^{dr_e}_{\text{en}}$ ($s^{dr_e}_{\text{ex}}$) represents the entry (exit) station of the disrupted section considering the operation direction of event e. Constraints (3.34) and (3.35) mean that if station st_e is not chosen as the short-turn station for train tr_e (i.e. $y_e = 0$), then the operation consistency between events e and e' is kept by requiring them to be cancelled or kept simultaneously. Constraint (3.36) means that if station st_e is chosen as the short-turn station for train tr_e (i.e. $y_e = 1$), then the operation consistency between events e and e' are broken by forcing event e' to be cancelled (i.e. $c_{e'} = 1$) while event e can be either cancelled or kept according to (3.34). Constraints (3.37) and (3.38) mean that if station $st_{e'}$ is not chosen as the short-turn station for train $tr_{e'}$ (i.e. $y_{e'} = 0$), then the operation consistency between events e and e' are kept by requiring them to be cancelled or kept simultaneously. Constraint (3.39) means that if station $st_{e'}$ is chosen as the short-turn station for train $tr_{e'}$ (i.e. $y_{e'} = 1$), then the operation consistency between events e and e' are broken by forcing event e to be cancelled (i.e. $c_e = 1$) while event e' can be either cancelled or kept according to (3.37). Constraints (3.40) and (3.41) mean that if the operation of a train in the disrupted section is cancelled, then at each side of the disrupted section, one station is chosen for the train as the short-turn station.

At the short-turn station, at most one short-turn activity will be selected for the train, which is formulated as

$$\sum_{a \in A_{\text{turn}}, tail(a)=e} m_a = c_{e'} - c_e, \qquad e \in E^{\text{turn}}_{\text{ar}}, (e, e') \in A_{\text{station}}, \tag{3.42}$$

$$\sum_{a \in A_{\text{turn}}, head(a)=e'} m_a = c_e - c_{e'}, \qquad e' \in E^{\text{turn}}_{\text{de}}, (e, e') \in A_{\text{station}}, \tag{3.43}$$

$$M_1 c_e + 2D(1 - m_a) + x_{e'} - x_e \geq m_a L_a, \qquad a = (e, e') \in A_{\text{turn}}. \tag{3.44}$$

Constraint (3.42) means that if event $e \in E^{\text{turn}}_{\text{ar}}$ is kept (i.e. $c_e = 0$) while its corresponding departure event e' in the station activity is cancelled (i.e. $c_{e'} = 1$), one and only one of the short-turn activities corresponding to e will be selected. If e and e' are both cancelled, no short-turn activities will be selected for e. Constraint (3.43) means that if event $e' \in E^{\text{turn}}_{\text{de}}$ is kept (i.e. $c_{e'} = 0$) while its corresponding arrival event e in the station activity is cancelled (i.e. $c_e = 1$), one and only one of the short-turn activities corresponding to e' will be selected. If e and e' are both cancelled, no short-turn activities will be selected for e'. Constraint (3.44) means that if a short-turn activity $a \in A_{\text{turn}}$ is selected (i.e. $m_a = 1$), it has to respect the minimum short-turn duration L_a. For a short-turn activity a that is not selected (i.e. $m_a = 0$), (3.44) also remains feasible.

To summarize, whether a train will be short-turned or not depends on (3.42) - (3.44), while (3.34) - (3.41) together help to decide the values of the variables that are in the right sides of (3.42) and (3.43).

3.3.6 Constraints for rolling stock circulations at terminal stations

When a train arrives at the destination, the corresponding rolling stock turns at the station to operate an opposite train that departs from the station as the origin. We call this an OD turn and take it into account in the proposed model. In the following, we first explain how to generate the set of all possible OD turn activities A_{odturn} by Algorithm 3.2, and then introduce the constraints that decide which OD turn activities of A_{odturn} can be selected in the rescheduled timetable.

Algorithm 3.2 construct the set of OD turn activities: A_{odturn}. We first initialize A_{odturn} as an empty set (line 1). Then, we select any arrival (departure) event that corresponds to a planned OD turn activity but the corresponding departure (arrival) event in this activity originally occurs after t_{start} but before $t_{\text{end}}+R$ (making the planned OD turn possibly impossible in the disruption timetable). These selected arrivals (departures) constitute the set $E_{\text{ar}}^{\text{odturn}}$ ($E_{\text{de}}^{\text{odturn}}$) (lines 2-3). Between any $e \in E_{\text{ar}}^{\text{odturn}}$ and $e' \in E_{\text{de}}^{\text{odturn}}$, an OD turn activity a_{odturn} is created, if e and e' correspond to the same train line, occur at the same station, and the possible largest occurrence time of e' (i.e. $o_{e'}+D$) could be $L_{\text{odturn}}^{st_e}$ minutes later than the possible smallest occurrence time of e (i.e. o_e) (lines 4-7). $L_{\text{odturn}}^{st_e}$ refers to the minimum OD turn duration required at the corresponding station. In line 8, a created a_{odturn} is added to the set A_{odturn}.

Algorithm 3.2: Constructing the set of all possible OD turn activities A_{odturn}

Input: $A_{\text{odturn}}^{\text{plan}}, E_{\text{ar}}, E_{\text{de}}, t_{\text{start}}, t_{\text{end}}, R, D, L_{\text{odturn}}$
Output: A_{odturn}

1 $A_{\text{odturn}} = \emptyset$;
2 Define $E_{\text{ar}}^{\text{odturn}} = \left\{ e \in E_{\text{ar}} \,\middle|\, (e,e') \in A_{\text{odturn}}^{\text{plan}}, t_{\text{start}} \le o_{e'} < t_{\text{end}}+R \right\}$;
3 Define $E_{\text{de}}^{\text{odturn}} = \left\{ e' \in E_{\text{de}} \,\middle|\, (e,e') \in A_{\text{odturn}}^{\text{plan}}, t_{\text{start}} \le o_{e} < t_{\text{end}}+R \right\}$;
4 **foreach** $e \in E_{\text{ar}}^{\text{odturn}}$ **do**
5   **foreach** $e' \in E_{\text{de}}^{\text{odturn}}$ **do**
6   **if** $tl_{e'} = tl_e, st_{e'} = st_e$ *and* $o'_e + D - o_e \ge L_{\text{odturn}}^{st_e}$ **then**
7             $a_{\text{odturn}} = (e,e')$;
8             $A_{\text{odturn}} = A_{\text{odturn}} \cup \{a_{\text{odturn}}\}$;

Based on the constructed $E_{\text{ar}}^{\text{odturn}}, E_{\text{de}}^{\text{odturn}}$ and A_{odturn}, we establish the constraints for rolling stock circulations at terminal stations:

$$\sum_{a \in A_{\text{odturn}}, tail(a)=e} m_a = 1 - c_e, \qquad e \in E_{\text{ar}}^{\text{odturn}}, \tag{3.45}$$

$$\sum_{a \in A_{\text{odturn}}, head(a)=e'} m_a = 1 - c_{e'}, \qquad e' \in E_{\text{de}}^{\text{odturn}}, \tag{3.46}$$

$$M_1 c_e + 2D(1-m_a) + x_{e'} - x_e \ge m_a L_a, \qquad a=(e,e') \in A_{\text{odturn}}. \tag{3.47}$$

Constraint (3.45) means that if an arrival event $e \in E_{\text{ar}}^{\text{odturn}}$ is not cancelled (i.e. $c_e = 0$), then one and only one of the OD turn activities corresponding to e will be selected. Otherwise, no OD turn activities corresponding to e will be selected. Constraint (3.46) means that if a departure event $e' \in E_{\text{de}}^{\text{odturn}}$ is not cancelled (i.e. $c_{e'} = 0$), then one and only one of the OD turn activities corresponding to e' will be selected. Otherwise, no OD turn activities corresponding to e' will be selected. For an selected OD turn activity, the minimum turn duration must be respected (3.47).

3.3.7 Constraints for station capacities

It is necessary to ensure each arrival train to be assigned with a track to dwell or pass through within a station. In other words, there should be at least one station track available for each arrival train. Hence, for each arrival event $e' \in E_{\text{ar}}$, we need to ensure that when e' occurs, the number of trains currently occupying the tracks at station $st_{e'}$ is smaller than the total number of tracks within this station. Here, we introduce a binary variable $n_{e,e'}$ for any two events $e, e' \in E_{\text{ar}}$ that $st_e = st_{e'}, tr_e \neq tr_{e'}$, which indicates whether train tr_e is currently occupying a track at station st_e when another train $tr_{e'}$ is approaching to the same station. If yes, $n_{e,e'} = 1$. In addition, we use N_{st} to represent the total number of tracks at station st. Thus, ensuring $e' \in E_{\text{ar}}$ to be assigned with a station track is actually to make sure that $\sum_{e \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}} n_{e,e'}$ is smaller than N_{st}.

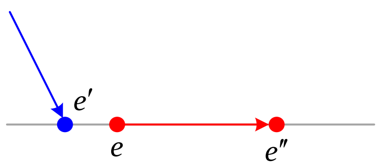


Figure 3.11: If $e' \in E_{\text{ar}}$ occurs before $e \in E_{\text{ar}}$, then $n_{e,e'} = 0$, where (e, e'') is a station activity, a short-turn activity or an OD turn activity

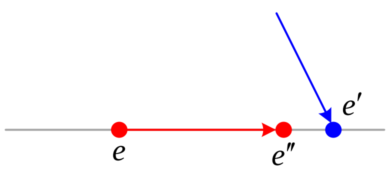


Figure 3.12: If $e' \in E_{\text{ar}}$ occurs after $e'' \in E_{\text{de}}$, then $n_{e,e'} = 0$, where (e, e'') is a station activity, a short-turn activity or an OD turn activity

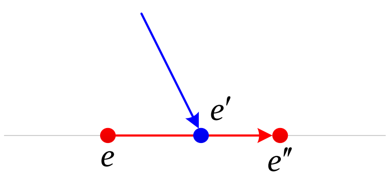


Figure 3.13: If $e' \in E_{\text{ar}}$ occurs after $e \in E_{\text{ar}}$ but before $e'' \in E_{\text{de}}$, then $n_{e,e'} = 1$, where (e, e'') is a station activity, a short-turn activity or an OD turn activity

Figures 3.11 to 3.13 show all cases where the values of $n_{e,e'}$ could be. When e' occurs before e or after e'' (see Figure 3.11 or 3.12), $n_{e,e'}$ should be 0. When e' occurs after e but before e'' (see Figure 3.13), $n_{e,e'}$ should be 1. According to the enumerated cases, we establish the following constraints to determine the value of $n_{e,e'}$:

$$n_{e,e'} \leq q_{e,e'}, \qquad e,e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}, \tag{3.48}$$

$$x_{e'} - x_{e''} \geq M_2(q_{e,e'} - n_{e,e'} - 1), \qquad e,e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}, \; (e,e'') \in A_{\text{station}} \cup A_{\text{turn}} \cup A_{\text{odturn}}, \tag{3.49}$$

$$x_{e'} - x_{e''} \leq M_2(q_{e,e'} - n_{e,e'}), \qquad e,e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}, \; (e,e'') \in A_{\text{station}} \cup A_{\text{turn}} \cup A_{\text{odturn}}, \tag{3.50}$$

where $q_{e,e'}$ is a binary variable indicating whether or not e occurs before e'. If yes, $q_{e,e'} = 1$. Otherwise, $q_{e,e'} = 0$.

If e' occurs before e (i.e. $q_{e,e'} = 0$), $n_{e,e'}$ is forced to be 0 in (3.48). If e' occurs after e (i.e. $q_{e,e'} = 1$), the value of $n_{e,e'}$ is further dependent on the occurrence times of e' and e''. This means that if $q_{e,e'} = 1$ and $x_{e'} - x_{e''} \geq 0$, $n_{e,e'}$ is forced to be 0 by (3.49) and (3.50), and if $q_{e,e'} = 1$ and $x_{e'} - x_{e''} \leq 0$, $n_{e,e'}$ is forced to be 1 by (3.49) and (3.50).

However, it is possible that either e or e' is cancelled, thus $n_{e,e'}$ should be 0:

$$n_{e,e'} \leq 1 - c_e, \qquad e,e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}, \tag{3.51}$$

$$n_{e,e'} \leq 1 - c_{e'}, \qquad e,e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}. \tag{3.52}$$

Considering $e \in E_{\text{ar}}$ may correspond to one station activity only but not any short-turn or OD turn activities (i.e. $e \notin E_{\text{ar}}^{\text{turn}} \cup E_{\text{ar}}^{\text{odturn}}$), $n_{e,e'}$ should be 0, if the corresponding departure event of e in the station activity is cancelled (3.53). Considering $e \in E_{\text{ar}}$ may correspond to one station activity and at least one short-turn activity (i.e. $e \in E_{\text{ar}}^{\text{turn}}$), $n_{e,e'}$ should be 0, if the corresponding departure event of e in the station activity is cancelled and none of the short-turn activities relevant to e is selected (3.54). Moreover, $e \in E_{\text{ar}}$ could correspond to OD turn activities only, if e is a destination arrival. In such a case, $n_{e,e'}$ should be 0, if none of the OD turn activities relevant to e is selected which actually equals to e is cancelled, according to (3.45). Thus, the value of $n_{e,e'}$ in this case can be reflected well by (3.51).

$$n_{e,e'} \leq 1 - c_{e''}, \qquad e \in E_{\text{ar}} \setminus \left(E_{\text{ar}}^{\text{turn}} \cup E_{\text{ar}}^{\text{odturn}}\right), e' \in E_{\text{ar}}, st_e = st_{e'}, \; tr_e \neq tr_{e'}, (e,e'') \in A_{\text{station}}, \tag{3.53}$$

$$n_{e,e'} \leq 1 - (c_{e''} - \sum_{a \in A_{\text{turn}}, tail(a) = e} m_a), \qquad e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}, \; (e,e'') \in A_{\text{station}}. \tag{3.54}$$

Considering these cancellation situations, (3.49) and (3.50) are changed to (3.55) - (3.58) where $L_{a'}$ is the minimum headway required between the departure of a train and the arrival of another train in case they are assigned to the same track at the corresponding station.

$$x_{e'} - (x_{e''} + L_{a'}) \geq M_2 \left(q_{e,e'} - n_{e,e'} - 1 - c_e - c_{e'} - c_{e''}\right), \\ e,e' \in E_{\text{ar}}, (e,e'') \in A_{\text{station}}, a' = (e'',e') \in A^{\text{de,ar}}_{\text{head}}, \tag{3.55}$$

$$x_{e'} - (x_{e''} + L_{a'}) \leq M_2 \left(q_{e,e'} - n_{e,e'} + c_e + c_{e'} + c_{e''}\right), \\ e,e' \in E_{\text{ar}}, (e,e'') \in A_{\text{station}}, a' = (e'',e') \in A^{\text{de,ar}}_{\text{head}}, \tag{3.56}$$

$$x_{e'} - (x_{e''} + L_{a'}) \geq M_2 \left(q_{e,e'} - n_{e,e'} - 1 - c_e - c_{e'} - (1 - m_a)\right), \\ e,e' \in E_{\text{ar}}, a = (e,e'') \in A_{\text{turn}} \cup A_{\text{odturn}}, a' = (e'',e') \in A^{\text{de,ar}}_{\text{head}}, \tag{3.57}$$

$$x_{e'} - (x_{e''} + L_{a'}) \leq M_2 \left(q_{e,e'} - n_{e,e'} + c_e + c_{e'} + 1 - m_a\right), \\ e,e' \in E_{\text{ar}}, a = (e,e'') \in A_{\text{turn}} \cup A_{\text{odturn}}, a' = (e'',e') \in A^{\text{de,ar}}_{\text{head}}. \tag{3.58}$$

To summarize, (3.48) and (3.51) - (3.58) together decide the value of $n_{e,e'}, \forall e,e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}$. In these constraints, the element $q_{e,e'}$ that indicates the sequence of e and e' is necessary. Recall that constraints (3.16) and (3.17) decide the value of $q_{e,e'}$ but only for such e and e' that correspond to the same operation directions. Thus, additional constraint is needed for determining the $q_{e,e'}$ that e and e' correspond to different operation directions:

$$M_2(q_{e,e'} - 1) \leq x_{e'} - x_e \leq M_2 q_{e,e'}, \qquad e,e' \in E_{\text{ar}}, st_e = st_{e'}, dr_e \neq dr_{e'}, \tag{3.59}$$

$$q_{e,e'} + q_{e',e} = 1, \qquad e,e' \in E_{\text{ar}}, st_e = st_{e'}, dr_e \neq dr_{e'}. \tag{3.60}$$

Based on $n_{e,e'}$, we ensure that each arrival train has at least one station track to dwell or pass through by

$$\sum_e n_{e,e'} \leq N_{st} - 1, \qquad e,e' \in E_{\text{ar}}, st_e = st_{e'}, st = st_{e'}, tr_e \neq tr_{e'}, \tag{3.61}$$

where N_{st} represents the total number of tracks at station st. Here, N_{st} is the sum of N^{p}_{st} and N^{th}_{st} that refer to the number of platform tracks and the number of pass-through tracks at station st, respectively. To ensure that each arrival that corresponds to passenger boarding/alighting to be assigned with a platform track, additional constraints need to be added, which are based on two kinds of decision variables. One is the binary variable p_e, for all $e \in E_{\text{ar}}$ with value 1 indicating that e needs a platform track. The other one is the binary variable $n^p_{e,e'}$ for any two events $e,e' \in E_{\text{ar}}$ such that $st_e = st_{e'}, tr_e \neq tr_{e'}$. $n^p_{e,e'} = 1$ indicates that train tr_e is occupying a platform track at station st_e at the moment that another train $tr_{e'}$ arrives at the same station. In the following, how to decide the values of p_e and $n^p_{e,e'}$ are explained successively.

If event $e \in E_{\text{ar}}$ is cancelled, it does not need a platform track:

$$p_e \leq 1 - c_e, \qquad e \in E_{\text{ar}}. \tag{3.62}$$

Otherwise, e needs a platform track, if e corresponds to a true stop (i.e. $s_a = 0, c_e = 0, c_{e'} = 0, a = (e,e') \in A_{\text{station}}$, see Table 6.4 and Table 6.5):

$$p_e \geq 1 - s_a - c_e - c_{e'}, \qquad e \in E_{\text{ar}}, a = (e,e') \in A_{\text{station}}, \tag{3.63}$$

or e corresponds to an selected short-turning/OD turning (i.e. $\sum_{a \in A_{\text{turn}} \cup A_{\text{odturn}}, tail(a)=e} m_a = 1$):

$$p_e \geq \sum_{a \in A_{\text{turn}} \cup A_{\text{odturn}}, tail(a)=e} m_a, \qquad e \in E_{\text{ar}}^{\text{turn}} \cup E_{\text{ar}}^{\text{odturn}}. \tag{3.64}$$

For arrival event e that is not relevant to any short-turn/OD turn activities, no platform track is needed by e, if e corresponds to a skipped stop or a non-stop (i.e. $c_e = 0, c_{e'} = 0, s_a = 1$, see Table 6.4 and Table 6.5):

$$p_e \leq 1 - s_a, \qquad e \in E_{\text{ar}} \setminus \left(E_{\text{ar}}^{\text{turn}} \cup E_{\text{ar}}^{\text{odturn}} \right), a = (e,e') \in A_{\text{station}}. \tag{3.65}$$

For an arrival event e that is relevant to short-turn activities, no platform track is needed by e, if e does not correspond to a *true* stop and no short-turn activities relevant to e are selected:

$$p_e \leq 1 - s_a + \sum_{a \in A_{\text{turn}}, tail(a)=e} m_a, \qquad e \in E_{\text{ar}}^{\text{turn}}, a = (e,e') \in A_{\text{station}}. \tag{3.66}$$

Based on p_e and $n_{e,e'}$, we can determine $n^p_{e,e'}$, of which the value 1 indicating that train tr_e is occupying a platform track at the moment that another train $tr_{e'}$ arrives at the same station. $n^p_{e,e'} = 1$ happens only if $n_{e,e'} = 1$ and $p_e = 1$, which is formulated by

$$n^p_{e,e'} \leq n_{e,e'}, \qquad e,e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}, \tag{3.67}$$

$$n^p_{e,e'} \leq p_e, \qquad e,e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}, \tag{3.68}$$

$$n^p_{e,e'} \geq n_{e,e'} + p_e - 1, \qquad e,e' \in E_{\text{ar}}, st_e = st_{e'}, tr_e \neq tr_{e'}. \tag{3.69}$$

When $e' \in E_{\text{ar}}$ needs a platform track (i.e. $p_{e'} = 1$), the constraint below ensures that e' is assigned with a platform track:

$$\sum_e n^p_{e,e'} \leq (N_{st} - 1)(1 - p_{e'}) + p_{e'}(N^p_{st} - 1), \qquad e,e' \in E_{\text{ar}}, st_e = st_{e'}, st = st_{e'}, tr_e \neq tr_{e'}. \tag{3.70}$$

3.3.8 Objective

The proposed model is based on constraints (3.1) - (3.48) and (3.51) - (3.70), with the objective

$$\min \sum_{e \in E_{\text{ar}}} w_e^{\text{delay}} d_e + \sum_{e \in E_{\text{ar}}} w_e^{\text{cancel}} c_e + \sum_{a \in A_{\text{dwell}}} w_a^{\text{skip}} s_a - \sum_{a \in A_{\text{pass}}} w_a^{\text{add}} (1 - s_a). \tag{3.71}$$

This objective considers the potential impacts of different dispatching measures on passengers, which include the impacts of delaying and cancelling trains, the negative impact of skipping stops, and the positive impact of adding stops. Although the positive impact of skipping stops and the negative impact of adding stops are not directly included in the objective, they are actually accounted for by the first term of the objective. For example, a skipped stop can help a passenger who should have been delayed to arrive on time (i.e. zero delay) or to be delayed less, while an added stop can delay a passenger who should have arrived on time. The weight of each decision variable is passenger-dependent, which considers the influenced passengers and the impact on these passengers. Each weight is individually estimated, based on the data of the passengers' planned paths that are obtained by the schedule-based passenger assignment model of Chapter 2. In the following, we first introduce how to obtain passengers' planned paths and then elaborate how they will be used to estimate passenger-dependent weights.

The schedule-based passenger assignment model proposed by Chapter 2 is able to estimate passenger path choices when given a timetable and passenger information regarding the origins, the destinations and the arrival times at the origins. In Chapter 2, the path with the shortest generalized travel time is chosen for each passenger. Generalized travel time is the weighted travel time considering passenger's preferences on waiting time, in-vehicle time, transfer time and the number of transfers. A path is constituted by a series of time-ordered departure and arrival events corresponding to the trains that the passenger wishes to take. The departure (arrival) event that corresponds to the boarding (alighting) of the passenger is indicated in the path. This means that from the path of a passenger, it is able to tell when and where the passenger will board or alight from which train. Also, the events that are in the path of a passenger but do not correspond the boarding/alighting of the passenger indicate that when the passenger will pass through which station by which train. In this chapter, we input the planned timetable to the schedule-based passenger assignment model to obtain the planned path of each passenger, namely the path that a passenger wishes to take on normal days. If a disruption occurs, the planned path of a passenger could be influenced due to different dispatching decisions applied. For example, if a train skips a stop, then the passengers who plan to board or alight from the train at the stop will be affected, and thus have to reroute and arrive possibly with delay at their destinations. In that sense, the weight of a decision is constituted by two parts: 1) the affected passengers, and 2) the passenger delays due to the decision. How to estimate these two parts for each decision is elaborated in the following.

- When delaying an arrival event $e \in E_{\text{ar}}$, the affected passengers include 1) the passengers z_e^{alight} who plan to alight from train tr_e at station st_e; and 2) the passengers z_e^{pass} who plan to pass through station st_e in train tr_e. The delay to each of these passengers is d_e minutes, where d_e is a decision variable representing the delay of event e. The weight of delaying an arrival event is:

$$w_e^{\text{delay}} = z_e^{\text{alight}} + z_e^{\text{pass}}, \qquad e \in E_{\text{ar}}.$$

The resulting passenger delays due to delaying event $e \in E_{\text{ar}}$ is $w_e^{\text{delay}} d_e$.

- When cancelling an arrival event $e \in E_{\text{ar}}$, the affected passengers include :1) the passengers z_e^{alight} who plan to alight from train tr_e at station st_e; and 2) the passengers z_e^{pass} who plan to pass through station st_e in train tr_e. Because of the cancellation, these passengers cannot stick to their planned paths but have to reroute, and the delay due to the rerouting is assumed as α minutes for each of them. Thus, the weight of cancelling an arrival event is:

$$w_e^{\text{cancel}} = \alpha(z_e^{\text{alight}} + z_e^{\text{pass}}), \qquad e \in E_{\text{ar}},$$

which represents the resulting passenger delays when e is cancelled (i.e. $c_e = 1$).

- When skipping a stop $a = (e, e') \in A_{\text{dwell}}$ where $e \in E_{\text{ar}}, e' \in E_{\text{de}}$, the affected passengers include 1) the passengers z_e^{alight} who plan to alight from train tr_e at station st_e; and 2) the passengers $z_{e'}^{\text{board}}$ who plan to board train $tr_{e'}$ at station $st_{e'}$. Here, tr_e and $tr_{e'}$ must be the same train and st_e and $st_{e'}$ must be the same station, since $(e, e') \in A_{\text{dwell}}$. Because of the skipped stop, these passengers cannot stick to their planned paths but have to reroute, and the delay due to the rerouting is assumed as β minutes for each of them. Thus, the negative impact of skipping a stop is:

$$w_a^{\text{skip}} = \beta(z_e^{\text{alight}} + z_{e'}^{\text{board}}), \qquad a = (e, e') \in A_{\text{dwell}},$$

which represents the resulting passenger delays when $a \in A_{\text{dwell}}$ is skipped (i.e. $s_a = 1$).

- When adding a stop to $a = (e, e') \in A_{\text{pass}}$ where $e \in E_{\text{ar}}, e' \in E_{\text{de}}$, passengers can benefit from the added stop by earlier boarding or earlier alighting. Figure 3.14 shows an example of a passenger boarding earlier due to an added stop. In Figure 3.14, a passenger who arrives at station B earlier than time $o_{e'}$ and plans to board train tr_2 that departs later than tr_1, may board train tr_1 instead if a stop is added to tr_1 at the station. We calculate z_a^{Eboard} as the number of passengers who can benefit from such earlier boarding due to an added stop to $a \in A_{\text{pass}}$. Figure 3.15 shows an example of a passenger alighting earlier due to an added stop. In Figure 3.15, a passenger plans to pass through station B by train tr_1 and then transfer to train tr_2 at station C to reach his/her destination station B. However, if train tr_1 is added with a stop at station B, this passenger will alight from tr_1 at station B. We calculate z_a^{Eoff} as the number of such passengers who can benefit from earlier alighting due to an added stop to $a \in A_{\text{pass}}$. The saved time for each passenger who benefits from earlier boarding/alighting is assumed as γ minutes. Thus, the positive impact of adding a stop is

$$w_a^{\text{add}} = \gamma(z_a^{\text{Eboard}} + z_a^{\text{Eoff}}), \qquad a \in A_{\text{pass}},$$

which represents the resulting passenger saved times when $a \in A_{\text{pass}}$ is added with a stop (i.e. $s_a = 0$).

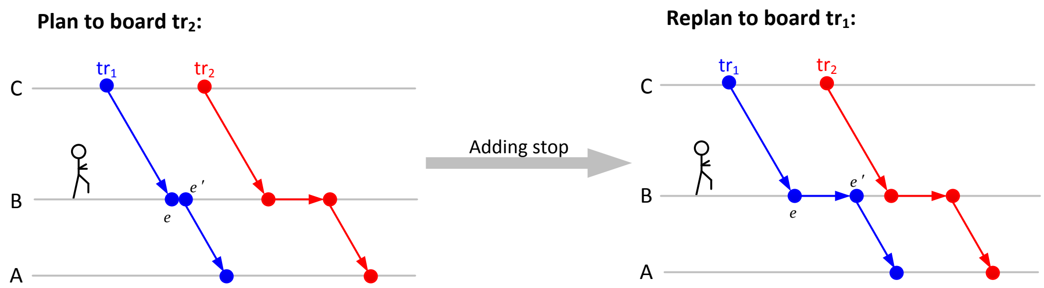


Figure 3.14: Illustration of earlier boarding when adding a stop to $a = (e, e') \in A_{\text{pass}}$

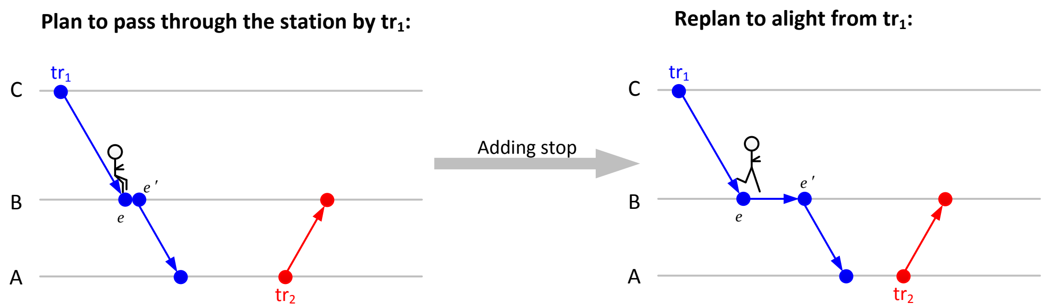


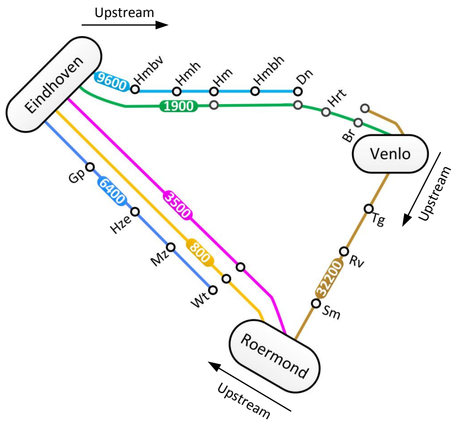
Figure 3.15: Illustration of earlier alighting when adding a stop to $a = (e, e') \in A_{\text{pass}}$

Note that the values of α, β and γ depend on many factors, which makes it difficult to estimate them. For example, α could be affected by the disruption locations, the disruption durations, the travel times of re-routing paths, the frequencies of external alternatives (e.g. shuttle buses), etc. Therefore, we have to make some assumptions here to simplify the estimation of these values in our case study. The delay of a passenger whose planned path is cancelled and the delay of a passenger whose planned boarding/alighting is skipped, are both assumed to be the disruption length (i.e. $\alpha = \beta = t_{\text{end}} - t_{\text{start}}$). In the case study, we applied the rescheduling model in a limited network that does not contain the areas that are far beyond the disruption sections, while adding stops could lead to delay propagation to the considered network beyond and further increase the passenger inconvenience there. Thus, we only assume 1 minute earliness (i.e. $\gamma = 1$) to each passenger who can benefit from adding stops, in order to offset the underestimation of the negative effects of adding stops.

The formulas of weights and the passenger groups and parameters that are necessary to determine the weights can be found in Appendix 3.B.

3.4 Case study

In this section, two experiments are carried out. In the first experiment, the proposed model is performed for 408 disruption scenarios, in order to explore 1) the effect of the recovery duration setting; and 2) the effect of the setting for the maximum allowed delay per event, when applying flexible stopping or applying flexible short-turning. In this experiment, the disruption duration is fixed for each scenario. However in the second experiment, different disruption durations are tested to investigate the influence of the disruption duration on the optimal rescheduling solution. In the end, the computation efficiency of the proposed model is analysed. All computational scenarios were solved to optimality (with a gap less than 0.00001%) by using the optimization software GUROBI release 7.0.1 on a desktop with Intel Xeon CPU E5-1620 v3 at 3.50 GHz and 16 GB RAM.



Train line	Type	Terminal
800	Intercity	—
1900	Intercity	Venlo (Vl)
3500	Intercity	—
6400	Sprinter	Wt and Eindhoven (Ehv)
9600	Sprinter	Dn
32200	Sprinter	Roermond (Rm)

Figure 3.16: The train lines operating in the considered network

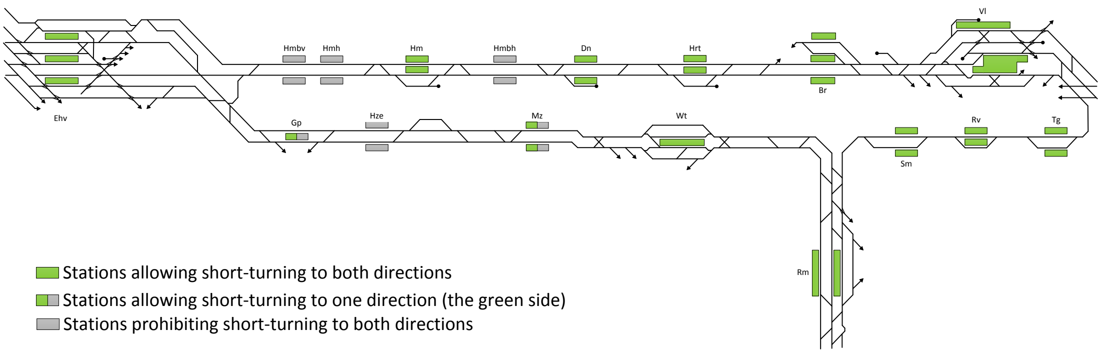


Figure 3.17: The schematic track layout in the considered network

The network considered is shown in Figure 3.16 where six train lines operate every 30 minutes in either upstream or downstream direction. The black arrows indicate the upstream direction, which is the clockwise direction starting from Roermond (Rm) and back to itself; while the downstream direction is the anticlockwise one. The rolling stock circulations are only taken into account for the trains of which the terminal stations are located in the considered network. Such terminal stations are indicated in Figure 3.16, as well as the type of each train line.

The schematic track layout in the considered network is shown in Figure 3.17 where stations Tg, Rv and Sm are located on single-track railway lines while the others are located on double-track railway lines. Due to the infrastructure layouts, some stations are unable for short-turning the trains that operate in a specific direction or event both directions: 1) stations Hze, Hmbv, Hmh and Hmbh are unable for short-turning the trains operating in both directions (colored in full grey); 2) stations Mz and Gp are unable for short-turning the trains operating in upstream direction (colored in half grey); while the others are able for short-turning the trains operating in both directions (colored in full green).

The parameter settings are detailed here: the minimum short-turn or OD turn duration is 300 s; the minimum arrival/departure headway on open-track section is 180 s; the minimum headway of following trains at a station is 180 s; and the minimum dwell time at a station is 30 s. The maximum percentage allowed to running time extensions is set to 67%, considering that added stops increase the running times. Note that excessive running time extensions are unlikely to happen when no stops are added to a train, because that will lead to train delays that are not preferred by the model. The acceleration (deceleration) time needed for a train that serves train line 800, 1900 or 3500 is set to 62 s (34 s); the acceleration (deceleration) time needed for a train that serves train line 6400 or 9600 is set to 39 s (28 s); and the acceleration (deceleration) time needed for a train that serves train line 32200 is set to 74 s (26 s).

3.4.1 Experiment 1: disruptions with fixed duration

Table 3.5: Disruption scenarios for experiment 1

Scenario No.	Disrupted section	Stopping	Short-turning	Maximum allowed delay per event [min]
1-6	Rm - Wt	Fixed	Fixed	5,10,15,20,25 and 30
7-12	Rm - Wt	Flexible	Fixed	5,10,15,20,25 and 30
13-18	Rm - Wt	Fixed	Flexible	5,10,15,20,25 and 30
19-24	Rm - Wt	Flexible	Flexible	5,10,15,20,25 and 30
25-30	Wt - Mz	Fixed	Fixed	5,10,15,20,25 and 30
31-36	Wt - Mz	Flexible	Fixed	5,10,15,20,25 and 30
37-42	Wt - Mz	Fixed	Flexible	5,10,15,20,25 and 30
43-48	Wt - Mz	Flexible	Flexible	5,10,15,20,25 and 30
⋮	⋮	⋮	⋮	⋮
385-390	Sm - Rm	Fixed	Fixed	5,10,15,20,25 and 30
391-396	Sm - Rm	Flexible	Fixed	5,10,15,20,25 and 30
397-402	Sm - Rm	Fixed	Flexible	5,10,15,20,25 and 30
403-408	Sm - Rm	Flexible	Flexible	5,10,15,20,25 and 30

In this experiment, we construct 408 disruption scenarios that differ in 1) the disrupted section; 2) whether flexible stopping is applied; 3) whether flexible short-turning is applied; and 4) the maximum allowed delay per event. The characteristics of these disruptions scenarios are shown in Table 3.5.

In each scenario, the disrupted section is assumed with a complete track blockage that starts at 8:00 and ends at 11:00, thus the passengers who start travelling during the period of 8:00 - 11:00 can be affected by the disruption. The passengers who start travelling before 8:00 or after 11:00 could also be affected. For example, a passenger who boards a train at 7:45 could be forced to alight from the train if the train is short-turned at a station during the disruption, or a passenger who plans to board a train at 11:15 may be unable to board the train as planned considering that the train could be delayed during the recovery period. Thus in each scenario, the passengers who start travelling during the period of 7:00 - 12:00 are all considered by using their planned paths to estimate the passenger-dependent weight of each decision in the objective. In other words, we consider the rescheduling impact on these passengers. The method of estimating passenger-dependent weights based on passengers' planned paths is introduced in Section 3.3.8. Recall that the planned path of a passenger is a series of time-ordered departure/arrival events that correspond to the train(s) the passenger plans to take. The dynamic passenger assignment proposed by Chapter 2 is adopted to estimate the planned path of each passenger, which uses the planned timetable and the information of each passenger (i.e. the origin, the destination, the time he/she arrives at the origin) as the input. This passenger information is obtained from a full-day OD matrix of the whole Dutch railways by applying the hourly distribution considering the time period concerned. The OD matrix is an artificial data representing the reality. The used full-day OD matrix and the hourly distribution are the same ones as adopted in Ghaemi et al. (2018b).

For each scenario, the rescheduled timetable is generated and the analysis of the result is described as follows.

3.4.1.1 The effect of recovery duration

To avoid the disruption affecting the timetable for the whole day, we set the recovery duration R to ensure that trains run as planned again after R minutes of the disruption ending time. In each disruption scenario, R is set with the same value as the maximum allowed delay per event.

The value of R affects solution feasibility. When it is set to 5 or 10 minutes in the scenarios where section Rm-Wt or Dn-Hrt is disrupted, no solutions can be found unless increasing R to 15 minutes. In other scenarios where the disrupted section is neither Rm-Wt nor Dn-Hrt, optimal solutions can be obtained even though R is set to 5 minutes. This indicates that the location of the disruption affects the required recovery duration. Thus a proper setting of R is necessary. Recall that we set R to the same value as the maximum allowed delay per event in each scenario. This setting of R ensures optimal solutions for 392 of the 408 scenarios.

The value of R affects the number of cancelled train services and the total train delay. When it is set to a smaller value, more train services are cancelled but with less train delays. This indicates that shortening the recovery duration aggravates the consequence during the disruption but mitigates the post-disruption consequence due to less delay propagation. In that sense, the value of R defines the trade-off between the consequence during the disruption and the post-disruption consequence. This setting deserves attention particularly when focusing on a large-scale network where a longer recovery duration may worsen the problem of delay propagation across the network. Optimizing this parameter is out of the scope for this chapter, but is interesting to be investigated in future research.

3.4.1.2 The effect of maximum allowed delay per event, flexible stopping or flexible short-turning

To explore the effect of the maximum allowed delay per event, flexible stopping or flexible short-turning, three performance indicators are used: the total number of cancelled train services, total train arrival delay, and total passenger delay. The total number of cancelled train services is the number of the train services that are cancelled. A train service represents the running of a train between two adjacent stations. Total train arrival delay is the sum of arrival delays of the train services that are not cancelled. Total passenger delay refers to the objective value. These three indicators are calculated for the rescheduled timetable of each disruption scenario. The minimal, average, and maximal values of these indicators over the scenarios that have the same settings about stopping, short-turning and maximum allowed delay per event are calculated and shown in Table 3.6.

Setting the maximum allowed delay with a larger value results in less cancelled train services and less passenger delays, but sometimes more train delays. This is because when more train services are kept instead of cancelled, more conflicts could emerge between them, which are resolved at the expense of introducing more train delays.

When applying flexible stopping or flexible short-turning, the average value of total number of cancelled services decreases, while the corresponding maximal or minimal value remains. This is because applying flexible stopping or flexible short-turning helps to reduce the number of cancelled train services in most scenarios, but not in a few scenarios where the disruption occurs in the area where only one train line operates (e.g. section Tg-Rv) or the disrupted section is Hmbv-Hmh or Hmh-Hm and the maximum allowed delay per event is set to 5 minutes. This indicates that flexible stopping or flexible short-turning is more likely to bring benefits in the situations that 1) the operation frequency is relatively high; and 2) a train departure/arrival is allowed to be delayed for a relatively long time. This is because such a situation provides a wider search space for the flexible dispatching measures to explore.

When applying flexible stopping and flexible short-turning, the average total passenger delay is the smallest, compared to the one when either or neither of flexible stopping

Table 3.6: Results on total number of cancelled services, total train arrival delay, and total passenger delay in experiment 1

Stopping	Short-turning	D	Total number of cancelled train services			Total train arrival delay [min]			Total passenger delay (i.e. the objective value) [min]		
			Min	Avg	Max	Min	Avg	Max	Min	Avg	Max
Fixed	Fixed	5	12	60	138	0	90	367	72,321	456,361	1,141,471
		10	12	58	138	0	114	367	72,321	439,018	1,141,471
		15	12	56	135	0	145	402	72,321	424,651	1,080,987
		20	12	51	130	0	241	999	72,321	398,095	1,025,639
		25	10	49	130	0	277	1,075	72,321	389,929	1,022,740
		30	10	49	122	0	274	671	72,321	386,815	1,022,740
Flexible	Fixed	5	12	59	138	0	107	530	72,321	452,857	1,141,471
		10	12	57	138	0	147	530	72,321	435,240	1,141,471
		15	12	54	135	0	211	658	72,321	411,151	1,021,524
		20	12	50	130	0	251	775	72,321	390,513	1,008,233
		25	10	49	130	0	283	816	72,321	385,841	1,004,318
		30	10	48	122	0	295	655	72,321	382,415	989,612
Fixed	Flexible	5	12	59	138	0	90	367	72,321	444,690	1,141,471
		10	12	56	138	0	127	367	72,321	427,536	1,137,755
		15	12	53	135	0	157	431	72,321	408,345	1,051,481
		20	12	50	130	0	244	1,010	72,321	390,583	1,017,767
		25	10	48	130	6	291	1,134	72,321	383,980	1,004,484
		30	10	48	122	6	287	749	72,321	380,866	1,004,484
Flexible	Flexible	5	12	58	138	0	108	530	72,321	441,066	1,141,471
		10	12	55	138	0	158	530	72,321	423,981	1,137,755
		15	12	51	135	0	219	721	72,321	398,397	1,004,790
		20	12	49	130	0	251	775	72,321	386,729	1,000,334
		25	10	48	130	6	297	875	72,321	379,970	984,648
		30	10	47	122	6	297	720	72,321	376,544	971,251

and flexible short-turning is applied. To have a deeper insight on the impact of these two measures, Table 3.7 shows the details of the rescheduled timetables by applying flexible stopping and flexible short-turning with the maximum allowed delay per event set to 30 minutes.

Table 3.7: The results of applying flexible stopping and flexible short-turning with the maximum allowed delay per event set to 30 minutes

Disrupted section	# Skipped stops	# Added stops	Total number of cancelled train services	Total train arrival delay [min]	Total passenger delay [min]
Rm-Wt	6	12	20	240	128,175
Wt-Mz	1	5	48	308	195,580
Mz-Hze	1	5	95	504	528,350
Hze-Gp	0	3	95	465	513,271
Gp-Ehv	0	6	122	633	694,103
Ehv-Hmbv	5	1	76	704	971,251
Hmbv-Hmh	6	1	70	720	958,273
Hmh-Hm	7	1	70	510	966,616
Hm-Hmbh	6	0	44	22	383,259
Hmbh-Dn	6	0	44	278	398,090
Dn-Hrt	1	0	12	69	98,366
Hrt-Br	0	1	10	96	77,066
Br-Vl	1	1	20	330	86,132
Vl-Tg	3	0	24	58	140,020
Tg-Rv	3	0	12	58	75,220
Rv-Sm	0	0	12	52	72,321
Sm-Rm	0	0	20	6	115,154

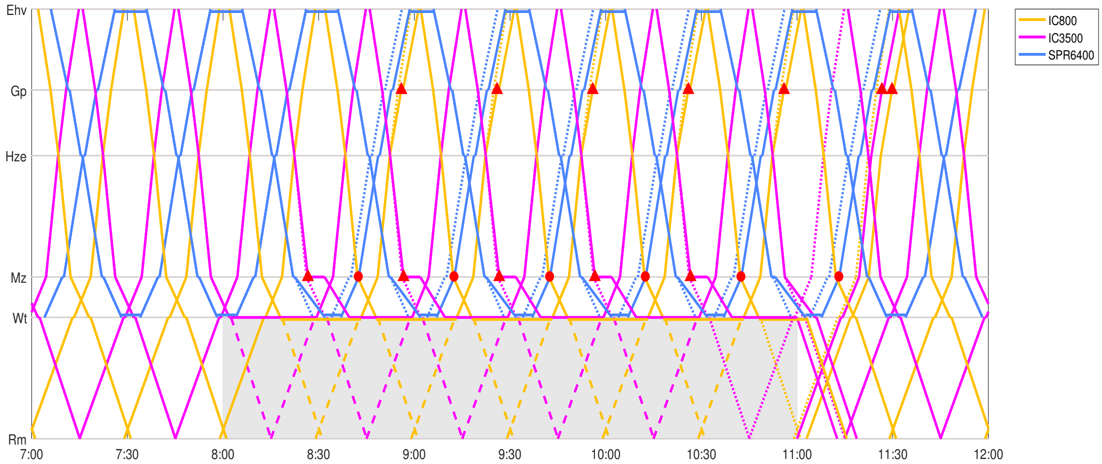


Figure 3.18: The disruption timetable with flexible stopping and flexible short-turning and maximum allowed delay per event set to 30 minutes for disrupted section Rm-Wt

Table 3.7 indicates that when Rm-Wt is disrupted, the number of added stops is the largest. The disruption timetable for this case is shown in Figure 3.18 where the dotted (dashed) lines represent the planned services that are delayed (cancelled) in the disruption timetable; the solid lines represent the services that are scheduled in the disruption timetable; and each red triangle (circle) represents an added (skipped) stop. Stops are added to five trains from line IC3500 (in pink color) at station Mz. These trains additionally stop at station Mz to wait for the trains from line SPR6400 (in blue color) to leave from station Wt, which is assumed to only provide platform tracks for two trains at the same time. To respect the minimum short-turn duration, six trains from line SPR6400 (in blue color) depart from station Wt with delays, and these delays continue to station Gp. As such, six trains from line IC800 (in yellow color) and one train from line IC3500 (in pink color) have to be delayed at station Gp to respect the minimum departure headway. These trains are all added with stops at station Gp.

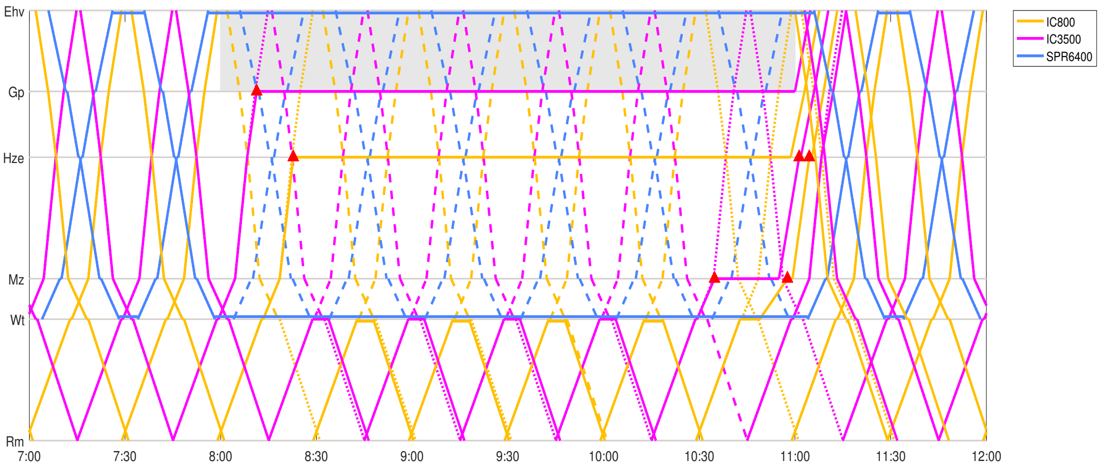


Figure 3.19: The disruption timetable with flexible stopping and flexible short-turning and maximum allowed delay per event set to 30 minutes for disrupted section Gp-Ehv

Table 3.7 indicates that when Gp-Ehv is disrupted, the number of cancelled train services is the largest. The disruption timetable for this case is shown in Figure 3.19. Although only section Gp-Ehv is disrupted, lots of train services between stations Wt and Gp are cancelled. Recall that due to the infrastructure layouts, stations Gp and Mz are unable for short-turning the trains operating in upstream direction and station Hze is unable for short-turning the trains operating in both directions. As such, an upstream train from line IC3500 (in pink color) cannot be short-turned at station Gp to serve the opposite operation, thus can only dwell at station Gp until the disruption ends. This is why lots of train services from line IC3500 (in pink color) are cancelled. The same reason explains the cancelled train services from line IC800 (in yellow color) or SPR6400 (in blue color). When a train has to stop at a station where it originally passes through, the required extra acceleration/deceleration time may cause an infeasible solution, if the resulting extension on the train running times is larger than allowed. For example, the upstream train from line IC3500 (in pink color) has to stop at station Gp, and thus extra deceleration time should be added to the train when running from station Hze to station Gp. The required deceleration time is 34 seconds that accounts for 22.7% extension on the scheduled running time of 150 seconds for the train from station Hze to station Gp. Recall that our model avoids overlong running in an open-track section by constraint (3.15) where a maximum percentage λ allowed to a running time extension is imposed. If λ is set to 20%, it would cause infeasibility of the adjusted timetable in the example. However in our case study, no infeasible rescheduling solutions were found due to adding stops, because we set λ to 67%, which is large enough to avoid the infeasibility caused by adding stops (to short station distances) in the disruption scenarios considered. We can take a relatively high percentage because when no stops are added to a train, excessive running time extensions are unlikely to happen, because that will lead to train delays that are not preferred by the model.

Table 3.7 indicates that when Ehv-Hmbv is disrupted, the total passenger delay is the largest. The disruption timetable for this case is shown in Figure 3.20. Compared to Figure 3.19, less train services are cancelled in Figure 3.20; however the services that are cancelled here correspond to more passenger demand, thus cancelling them results in more passenger delays. Here, five short-turnings of the IC1900 occur at station Hm, while one short-turning occurs at an earlier station Dn. This early short-turning is due to the required recovery duration, which is 30 minutes in this case. If this short-turning does not occur at station Dn but at station Hm instead, more delays will happen to trains, which cannot be completely absorbed within the recovery duration.

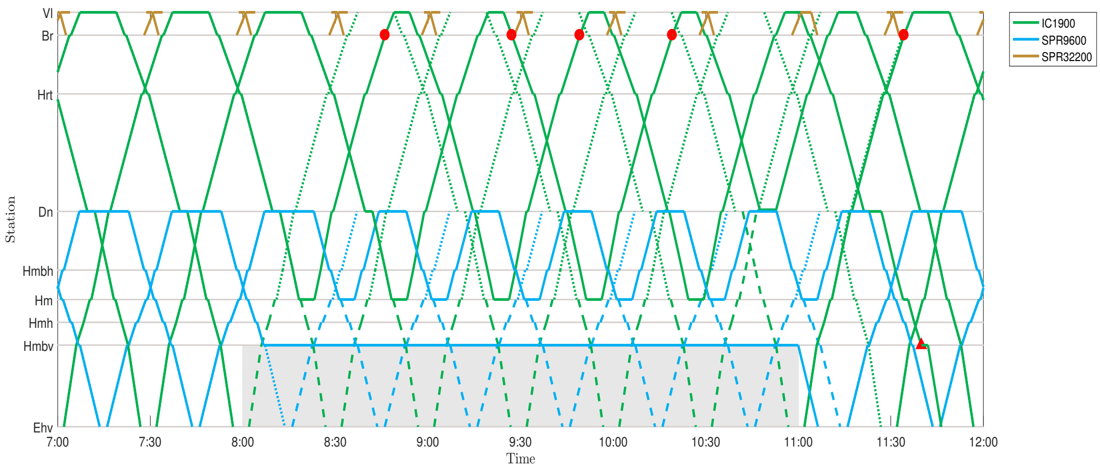


Figure 3.20: The disruption timetable with flexible stopping and flexible short-turning and setting allowed delay per event set to 30 minutes for disrupted section Ehv-Hmbv

From experiment 1 it is concluded that 1) although shortening the recovery duration is at the expense of more train services being cancelled, it can mitigate the problem of delay propagation; 2); better solutions can be found when setting the maximum allowed delay per event to a larger value; and 3) applying flexible stopping and flexible short-turning helps to reduce the number of cancelled train services and the total passenger delay, especially when the disruption occurs in the area where the train operation frequency is relatively high.

3.4.2 Experiment 2: disruptions with different durations

To explore the influence of the disruption duration on the optimal rescheduling solution, we construct 85 scenarios that differ in 1) the disrupted section and 2) the disruption ending time. These scenarios are shown in Table 3.8, where the maximum allowed delay per event *D* and the required recovery duration *R* are both set to 30 minutes.

Table 3.8: Disruption scenarios for experiment 2

Scenario No.	Disrupted section	Start time	End time	Stopping	Short-turning	*D* [min]	*R* [min]
1	Rm - Wt	8:00	10:00	Flexible	Flexible	30	30
2	Rm - Wt	8:00	10:15	Flexible	Flexible	30	30
3	Rm - Wt	8:00	10:30	Flexible	Flexible	30	30
4	Rm - Wt	8:00	10:45	Flexible	Flexible	30	30
5	Rm - Wt	8:00	11:00	Flexible	Flexible	30	30
6	Wt - Mz	8:00	10:00	Flexible	Flexible	30	30
7	Wt - Mz	8:00	10:15	Flexible	Flexible	30	30
8	Wt - Mz	8:00	10:30	Flexible	Flexible	30	30
9	Wt - Mz	8:00	10:45	Flexible	Flexible	30	30
10	Wt - Mz	8:00	11:00	Flexible	Flexible	30	30
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
81	Sm - Rm	8:00	10:00	Flexible	Flexible	30	30
82	Sm - Rm	8:00	10:15	Flexible	Flexible	30	30
83	Sm - Rm	8:00	10:30	Flexible	Flexible	30	30
84	Sm - Rm	8:00	10:45	Flexible	Flexible	30	30
85	Sm - Rm	8:00	11:00	Flexible	Flexible	30	30

For each scenario, the rescheduled timetable is generated and three performance indicators are calculated: the total passenger delay (i.e. the objective value), the number of cancelled train services, and the total train arrival delay, of which the values are shown in Figures 3.21-3.23, respectively. The y-axis represents the indicator value, the x-axis represents the disrupted section, and the legend indicates the disruption ending time corresponding to each point.

Figure 3.21 indicates that in each disrupted section, the total passenger delay increases gradually with the extension of disruption duration. From Figures 3.22 and 3.23, we found that similar patterns exist among disruption ends with an interval of 30 minutes apart, indicated with lines with the same colors. The green lines correspond to the scenarios where the disruption ends at 10:00, 10:30 or 11:00, while the pink lines correspond to the scenarios where the disruption ends at 10:15 or 10:45. Recall that in our case study, the trains from each train line operate every 30 minutes in each direction. This reveals that the optimal rescheduling solution is sensitive to the disruption duration, but keeps some regularities if the disruption duration extends periodically. In the following, an example is given to show how the optimal rescheduling solution changes when the disruption duration extends with different time lengths.

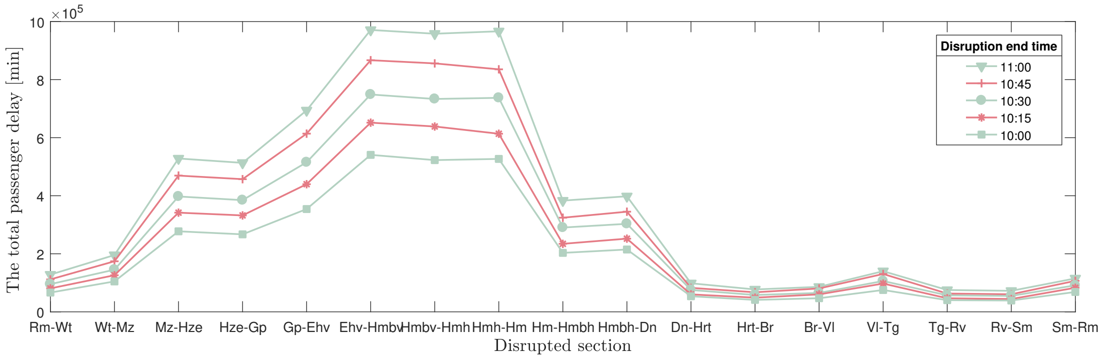


Figure 3.21: The total passenger delay in each scenario of Table 3.8

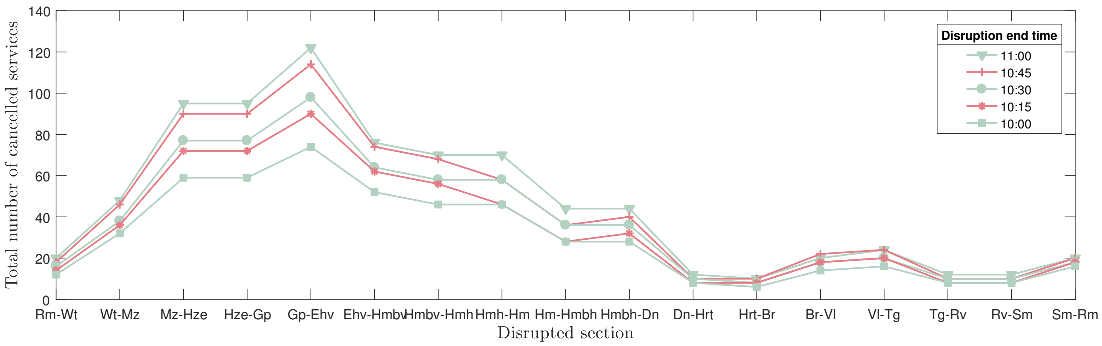


Figure 3.22: The number of cancelled train services in each scenario of Table 3.8

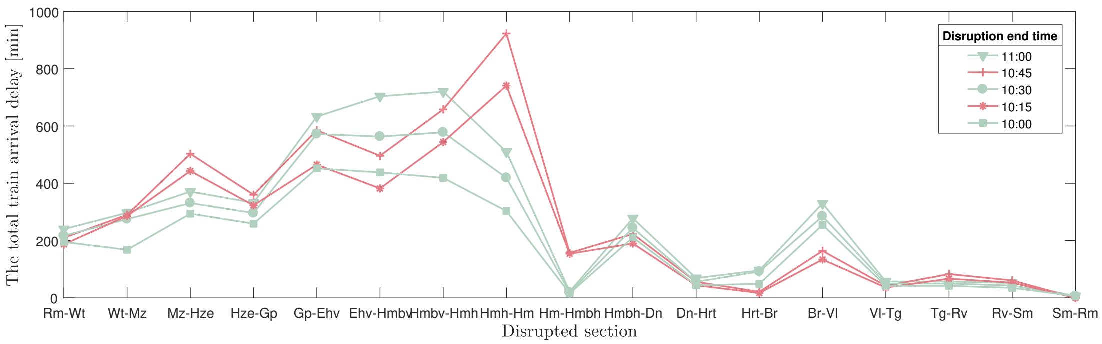


Figure 3.23: The total train arrival delay in each scenario of Table 3.8

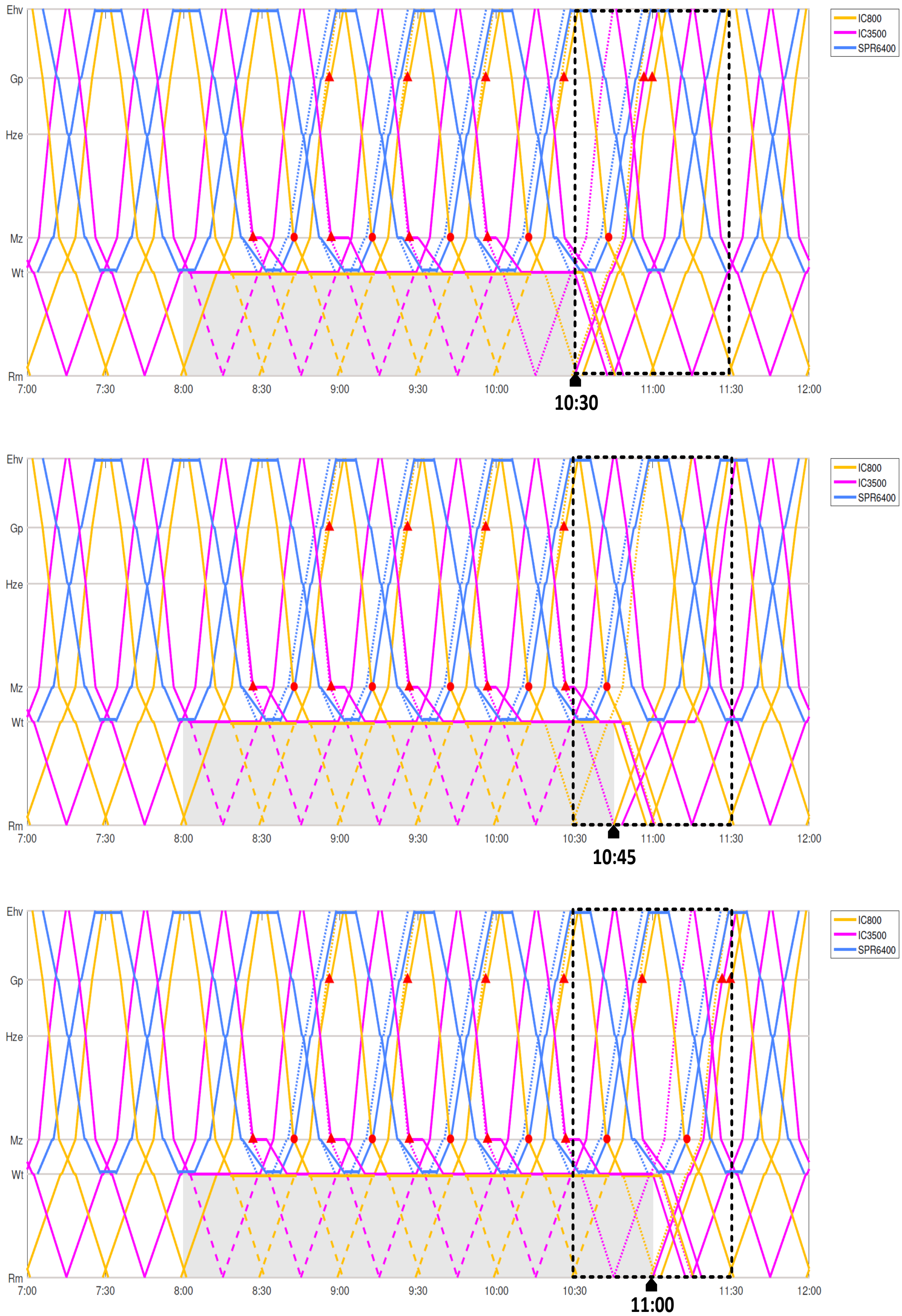


Figure 3.24: The rescheduled timetables of three scenarios that only differ in the disruption ending times

In Figure 3.24, three rescheduled timetables are shown for the scenarios where the disruptions all start at 8:00, but end at 10:30, 10:45 and 11:00, respectively. The disruption ending time is highlighted on the bottom of each timetable and three dotted black rectangles are used to highlight the parts that are different in these timetables. When the disruption ends at 10:30, a train from line IC3500 (in pink color) is delayed at station Wt; while when the disruption ends at 10:45, the train is suggested to short-turn at station Wt where a following train from line IC3500 (in pink color) is delayed instead. When the disruption ends at 11:00, more delays are introduced to trains, but the short-turning patterns and the stopping patterns both look similar to the case where the disruption ends at 10:30. This indicates that the optimal rescheduling solution is sensitive to the disruption ending time that affects the decisions of short-turning or delaying the last trains that approach the disrupted section before the disruption ends and these decisions will further affect the stopping patterns of trains during the recovery period. However, some regularities can be kept in the rescheduled timetables corresponding to the periodic pattern.

In real life, the disruption ending time is uncertain, which means that the first predicted ending time may be extended to another new one that could also be extended further (Zilko et al., 2016). Under these circumstances, the rescheduled timetable has to be updated every time a new disruption ending time is renewed. A direct solution to this problem is to apply the model at the time when the disruption ending time is renewed, where the current time (the renewed ending time) is regarded as the disruption starting (ending) time and the train arrivals/departures that have already been realized are respected with the previous rescheduled timetable as the reference. In this way, a rescheduled timetable can be obtained for the extended disruption. Including the uncertainty of disruption duration during the rescheduling helps to generate a robust solution. This is out of scope of this chapter but is interesting to be investigated further.

3.4.3 Computation efficiency analysis

Among the 408 scenarios of experiment 1 (see Table 3.5), only 7 take more than 15 seconds (but less than 80 seconds). These scenarios are the ones where both flexible stopping and flexible short-turning are applied and the maximum event per delay is set to 30 minutes. Figure 3.25 illustrates the impact of dispatching measures and parameter settings on computation time, based on the results of the scenarios in Table 3.5 where the disruptions have the same duration (3 hours). Here, each circle represents the average computation time over the scenarios that only differ in the disrupted sections. The average computation time grows with the increase of maximum allowed delay per event. With the same setting of maximum allowed delay per event, longer computation times are needed when more flexible dispatching measures are applied. This is because more binary variables about stopping (short-turning) decisions are needed when applying flexible stopping (short-turning), thus increasing the computation complexity. Compared to applying flexible short-turning, more binary variables are needed when applying flexible stopping, which is why the average computation times due to flexible stopping and fixed short-turning are longer than the ones due to fixed stopping and flexible short-turning.

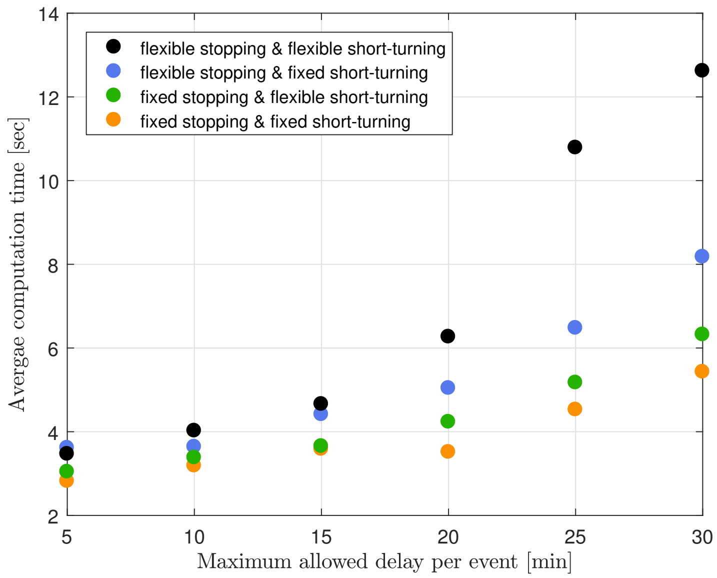


Figure 3.25: The average computation times over the scenarios that use the same dispatching measures and the same setting of maximum allowed delay per event

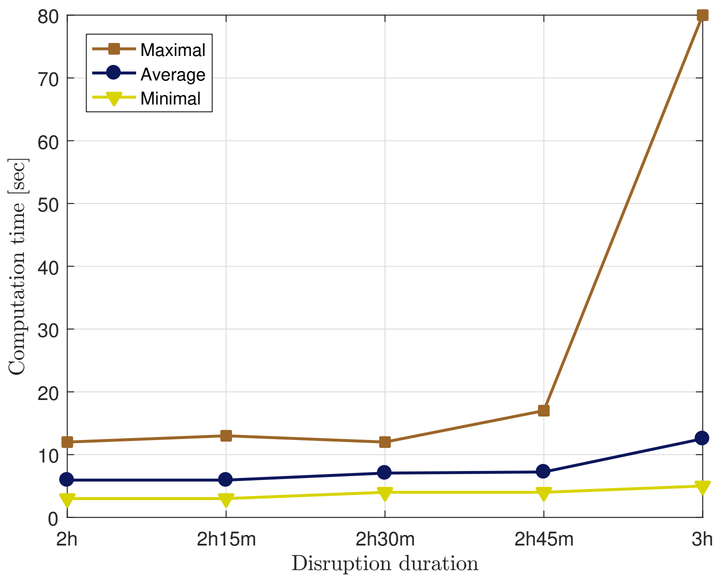


Figure 3.26: The maximal, average, and minimal computation times over the scenarios that have the same disruption durations

Figure 3.26 shows the impact of disruption duration on computation time, based on the results of the scenarios in Table 3.8. Recall that in these scenarios, flexible stopping and flexible short-turning are both applied, and the maximum event per delay and the required recovery duration are both set to 30 minutes. In Figure 3.26, each square, circle and triangle represent the maximal, average, and minimal computation time over the scenarios that only differ in the disrupted sections, respectively. The average and minimal computation time both grow gradually with the extension of disruption duration; whereas a steep growth is observed for the maximal computation time when the disruption duration extends from 2 hours and 45 minutes to 3 hours. Nevertheless, the maximal value of the computation time is 80 seconds, which is still acceptable in practice.

In the Dutch railways, the average disruption duration was 2 hours and 40 minutes over the time period from Jan, 2011 to Sep 14, 2018 (data source: *www.rijdendetreinen.nl*). Once a disruption occurs, it is expected to be handled as soon as possible. For an up to three-hour disruption, the proposed model is able to generate an optimal rescheduling solution in an average of 13 seconds approximately, and thus can be applied for real time dispatching.

3.5 Conclusions and future directions

In this chapter, an MILP model is proposed for rescheduling a timetable during railway disruptions, where flexible stopping and flexible short-turning are innovatively integrated with delaying, cancelling and reordering. The deceleration and acceleration times are considered when changing the stopping patterns, and each train that corresponds to passenger boarding/alighting at a station is ensured with a platform track. To make the disruption timetable passenger-friendly, each decision in the objective is assigned with an individual weight that is estimated from time-dependent passenger demand. In the case study, hundreds of disruption scenarios are established on a subnetwork of the Dutch railways. By the proposed model, the optimal rescheduling solutions to these scenarios were generated mostly within 13 seconds, and the worst case cost no longer than 80 seconds. The results indicate that flexible stopping and flexible short-turning are more likely to work in the situations where the operation frequency is relatively high and trains are allowed to be delayed with a relatively long time, because such situations provide a wider search space for the flexible dispatching measures to explore. It is found that applying flexible stopping and flexible short-turning results in less passenger delays, compared to applying either or neither of them. Moreover, shortening the recovery duration is good for mitigating the post-disruption consequence by less delay propagation, but is at the expense of more cancelled train services during the disruption. It will be interesting to explore how to make the trade-off between the consequence during the disruption and the post-disruption consequence, particularity when focusing on a large-scale network where a longer recovery duration may worsen the problem of delay propagation across the network. Also, it is found that the optimal rescheduling solution is sensitive to the disruption duration, but keeps some regularities when the disruption duration extends periodically.

In this chapter, passenger demand is handled in a static way. In other words, the dynamic interaction between passengers and the disruption timetable is neglected. To consider such dynamic interaction, one way is to embed the timetable rescheduling model and the passenger assignment model in an iterative framework where in each iteration a disruption timetable is generated and the resulting passenger inconvenience is evaluated and then included to the rescheduling model in the next iteration as feedback from the passengers. Another way is to consider passenger reactions towards the disruption timetable during the rescheduling process (i.e. integrating passenger routing and timetable rescheduling in one single model). We will further explore both ways by considering the trade-off between the solution quality and the computational efficiency in future work. In real life, the duration of a disruption is uncertain, thus another future direction is extending the model to deal with uncertain disruption duration.

Appendix 3.A. Sets and parameters

Table 3.9: Sets

Notation	Description
E_{de}	Set of departure and pass-through departure events
E_{ar}	Set of arrival and pass-through arrival events
$E_{\text{ar}}^{\text{turn}}$	The subset of E_{ar}, which includes all tails of activities in A_{turn} $E_{\text{ar}}^{\text{turn}} = \bigcup_{a \in A_{\text{turn}}} \{tail\,(a)\}$
$E_{\text{de}}^{\text{turn}}$	The subset of E_{de}, which includes all heads of activities in A_{turn} $E_{\text{de}}^{\text{turn}} = \bigcup_{a \in A_{\text{turn}}} \{head\,(a)\}$
$E_{\text{ar}}^{\text{odturn}}$	The subset of E_{ar}, which includes all tails of activities in A_{odturn} $E_{\text{ar}}^{\text{odturn}} = \bigcup_{a \in A_{\text{odturn}}} \{tail\,(a)\}$
$E_{\text{de}}^{\text{odturn}}$	The subset of E_{de}, which includes all heads of activities in A_{odturn} $E_{\text{de}}^{\text{odturn}} = \bigcup_{a \in A_{\text{odturn}}} \{head\,(a)\}$
E^{NMdelay}	Set of events that are not given the upper limit on their delays
TL	Set of train lines
TL_{dis}	Set of train lines that are affected by the disruption: $TL_{\text{dis}} \subseteq TL$
TR_{turn}	Set of trains that correspond to the events contained in $E_{\text{ar}}^{\text{turn}} \cup E_{\text{de}}^{\text{turn}}$
A_{run}	Set of running activities
A_{dwell}	Set of dwell activities
A_{pass}	Set of pass-through activities
A_{station}	Set of station activities: $A_{\text{station}} = A_{\text{dwell}} \cup A_{\text{pass}}$
$A_{\text{head}}^{\text{ar}}$	Set of arrival headway activities for following trains
$A_{\text{head}}^{\text{de}}$	Set of departure headway activities for following trains
$A_{\text{head}}^{\text{ar,de}}$	Set of arrival-departure headway activities for crossing trains and trains operating on single-track railways
$A_{\text{head}}^{\text{de,ar}}$	Set of departure-arrival headway activities for crossing trains and trains using the same track at a station
$A_{\text{turn}}^{tl,dr}$	Set of short-turn activities for trains serving train line tl and operating in direction dr, $A_{\text{turn}}^{tl,dr} \subset A_{\text{turn}}$
A_{turn}	Set of short-turn activities
A_{odturn}	Set of OD turn activities

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Notation	Description
$A_{\text{odturn}}^{\text{plan}}$	Set of planned OD turn activities: $A_{\text{odturn}}^{\text{plan}} \subset A_{\text{odturn}}$. The activities in $A_{\text{odturn}}^{\text{plan}}$ are the planned turnings of rolling stock at terminal stations
$ST_{\text{turn}}^{tl,dr}$	Set of short-turn station candidates for the trains serving train line tl and operating in direction dr. Each station contained in $ST_{\text{turn}}^{tl,dr}$ must be the upstream/same station compared to st_{en}^{dr}
L_{turn}	Set of minimum short-turn times at stations
L_{odturn}	Set of minimum OD turn times at stations
$E_{\text{de},st}^{\text{dis},tl,dr}$	A local set used in Algorithm 1, which contains the departure events of the trains that serve train line tl, operate in direction dr, and occur at station st after t_{start} but before $t_{\text{end}}+R$
$E_{\text{ar}}^{\text{tn}}$	A local set used in Algorithm 1, which contains the corresponding arrival events of $E_{\text{de},st}^{\text{dis},tl,dr}$ in station activities
$E_{\text{de}}^{\text{tn}}$	A local set used in Algorithm 1, which contains the departure events that could be served by the arrival events of $E_{\text{ar}}^{\text{tn}}$ for short-turning

Table 3.10: Parameters

Notation	Description
e_{de}	Departure or pass-through departure event: $e_{\text{de}} \in E_{\text{de}}$
e_{ar}	Arrival or pass-through arrival event: $e_{\text{ar}} \in E_{\text{ar}}$
o_e	The original time of event e
st_e	The corresponding station of event e
tr_e	The corresponding train of event e
tl_e	The corresponding train line of event e
dr_e	The operation direction of event e, which is either *upstream* or *downstream*: $dr_e \in \{up, down\}$
r_{st}	A binary parameter indicating whether or not station st is located on single-track railway lines. If yes, $r_{st}=1$.
a_{run}	Running activity: $a_{\text{run}} \in A_{\text{run}}$ $a_{\text{run}} = (e,e'),\ e \in E_{\text{de}}, e' \in E_{\text{ar}}, tr_e = tr_{e'}, dr_e = dr_{e'}, st_e$ is upstream neighbouring to $st_{e'}$
a_{dwell}	Dwell activity: $a_{\text{dwell}} \in A_{\text{dwell}}$ $a_{\text{dwell}} = (e,e'),\ e \in E_{\text{ar}}, e' \in E_{\text{de}}, tr_e = tr_{e'}, dr_e = dr_{e'}, st_e = st_{e'}, o_e < o_{e'}$
a_{pass}	Pass-through activity: $a_{\text{pass}} \in A_{\text{pass}}$ $a_{\text{pass}} = (e,e'),\ e \in E_{\text{ar}}, e' \in E_{\text{de}}, tr_e = tr_{e'}, dr_e = dr_{e'}, st_e = st_{e'}, o_e = o_{e'}$
$a_{\text{head}}^{\text{ar}}$	Arrival headway activity: $a_{\text{head}}^{\text{ar}} \in A_{\text{head}}^{\text{ar}}$ $a_{\text{head}}^{\text{ar}} = (e,e'),\ e,e' \in E_{\text{ar}}, tr_e \neq tr_{e'}, dr_e = dr_{e'}, st_e = st_{e'}$
$a_{\text{head}}^{\text{de}}$	Departure headway activity: $a_{\text{head}}^{\text{de}} \in A_{\text{head}}^{\text{de}}$ $a_{\text{head}}^{\text{de}} = (e,e'),\ e,e' \in E_{\text{de}}, tr_e \neq tr_{e'}, dr_e = dr_{e'}, st_e = st_{e'}$

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Notation	Description
$a^{\text{ar,de}}_{\text{head}}$	Arrival-departure headway activity: $a^{\text{ar,de}}_{\text{head}} \in A^{\text{ar,de}}_{\text{head}}$ $a^{\text{ar,de}}_{\text{head}} = (e,e'),\ e \in E_{\text{ar}}, e' \in E_{\text{de}}, tr_e \neq tr_{e'}, dr_e \neq dr_{e'}, st_e = st_{e'}$
$a^{\text{de,ar}}_{\text{head}}$	Departure-arrival headway activity: $a^{\text{de,ar}}_{\text{head}} \in A^{\text{de,ar}}_{\text{head}}$ $a^{\text{de,ar}}_{\text{head}} = (e,e'),\ e \in E_{\text{de}}, e' \in E_{\text{ar}}, tr_e \neq tr_{e'}, st_e = st_{e'}$
a_{turn}	Short-turn activity: $a_{\text{turn}} \in A_{\text{turn}}$ $a_{\text{turn}} = (e,e'),\ e \in E_{\text{ar}},\ e' \in E_{\text{de}}, tr_e \neq tr_{e'}, dr_e \neq dr_{e'}, st_e = st_{e'}, tl_e = tl_{e'},$ $st_e \in ST^{tl_e,dr_e}_{\text{turn}}$
a_{odturn}	OD turn activity that refers to the rolling stock of one train turning at a terminal station to operate an opposite train from the same train line: $a_{\text{odturn}} \in A_{\text{odturn}}$, $a_{\text{odturn}} = (e,e'), e \in E_{\text{ar}}, e' \in E_{\text{de}}, tr_e \neq tr_{e'}, dr_e \neq dr_{e'}, st_e = st_{e'}, tl_e = tl_{e'}$
st^{dr}_{en}	The entry station of the disrupted section for trains operating in direction $dr \in \{up, down\}$
st^{dr}_{ex}	The exit station of the disrupted section for trains operating in direction $dr \in \{up, down\}$
$tail(a)$	The tail of activity a, which is the event that a starts from
$head(a)$	The head of activity a, which is the event that a points to
t_{start}	The start time of the disruption
t_{end}	The end time of the disruption
L_a	The minimum duration of activity a
L^{st}_{turn}	The minimum short-turn time at station st
λ	The maximum percentage allowed to running time extension
D	The maximum allowed delay per event
R	The required recovery duration after the disruption end time.
M_1	A positive large number that is set to 1440
M_2	A positive large number that is set to twice of M_1: $M_2 = 2M_1$
N^{p}_{st}	The number of platform tracks at station st
N^{th}_{st}	The number of pass-through tracks at station st
N_{st}	The number of tracks at station st: $N_{st} = N^{\text{p}}_{st} + N^{\text{th}}_{st}$
τ_a	Pure running time that does not include acceleration time, deceleration time, and time supplement: $a \in A_{\text{run}}$
Δ^{acce}_a	Acceleration time needed for a train run: $a \in A_{\text{run}}$
Δ^{dece}_a	Deceleration time needed for a train run: $a \in A_{\text{run}}$

Appendix 3.B. Weights of decisions, and the passenger groups and parameters used for estimating the weights

Table 3.11: The descriptions of weights

Weight	Description
w_e^{delay}	The weight of delaying an arrival event e
w_e^{cancel}	The weight of cancelling an arrival event e
w_a^{skip}	The negative impact of skipping a stop
w_a^{add}	The positive impact of adding a stop

Table 3.12: The passenger groups and parameters used for estimating the weights

Symbol	Description
z_e^{alight}	The number of passengers who plan to alight from train tr_e at station st_e, $e \in E_{\text{ar}}$
z_e^{pass}	The number of passengers who plan to pass through station st_e in train $tr_e, e \in E_{\text{ar}}$
z_e^{board}	The number of passengers who plan to board train tr_e at station $st_e, e \in E_{\text{de}}$
z_a^{Eboard}	The number of passengers who may benefit from earlier boarding, $a = (e, e') \in A_{\text{pass}}$ It is calculated as the number of passengers who arrive at station st_e before time o_e and plan to board another train that departs later than train $tr_{e'}, (e, e') \in A_{\text{pass}}$
z_a^{Eoff}	The number of passengers who may benefit from earlier alighting, $a = (e, e') \in A_{\text{pass}}$ It is calculated as the number of passengers who plan to pass through station st_e by train tr_e while their destinations are $st_e, (e, e') \in A_{\text{pass}}$
α	The delay of a passenger whose planned path is unavailable due to partial/complete train cancellation
β	The delay of a passenger whose planned alighting/boarding is impossible due to a skipped stop
γ	The saved time of a passenger who has earlier alighting/boarding option compared to his/her planned path due to an added stop

Appendix 3.C. Standard abbreviations of stations

Table 3.13: The standard abbreviation of each station considered in the case study

Station	Abbreviation	Station	Abbreviation
Blerick	Br	Horst-Sevenum	Hrt
Deurne	Dn	Maarheeze	Mz
Eindhoven	Ehv	Reuver	Rv
Geldrop	Gp	Roermond	Rm
Heeze	Hze	Swalmen	Sm
Helmond	Hm	Tegelen	Tl
Helmond Brandevoort	Hmbv	Venlo	Vl
Helmond Brouwhuis	Hmbh	Weert	Wt
Helmond't Hout	Hmh		

Chapter 4

Dynamic and robust timetable rescheduling for uncertain railway disruptions

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Apart from minor updates, this chapter has been submitted as:

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4.1 Introduction

Railway systems are vulnerable to unexpected disruptions caused by for instance incidents, infrastructure failures, and extreme weather. A typical consequence of a disruption is that the tracks between two stations are completely blocked for a few hours. Under this circumstance, trains are forbidden to enter the blocked tracks, and therefore the planned timetable is no longer feasible. Thus, traffic controllers have to reschedule the timetable for which they usually apply a pre-designed contingency plan specific to the disruption. Since the contingency plan is manually designed, its optimality cannot be guaranteed, and sometimes cannot even meet all operational constraints (Ghaemi et al., 2017b). For this reason, increasing attention is being paid to developing optimization models for computing rescheduling solutions. A detailed review can be found in Cacchiani et al. (2014).

Until now, many timetable rescheduling models have been proposed to deal with disruptions, which differ in e.g. the complexity of the network, the infrastructure modelling, the used dispatching measures, the objective, and the number of disruptions considered. For instance, Zhan et al. (2015) propose a Mixed Integer Linear Programming (MILP) model to reschedule the timetable in case of a complete track blockage by delaying, reordering and cancelling trains. They focus on a Chinese high-speed railway corridor where seat reservations are necessary for passengers, and therefore the measure of short-turning trains is not applicable. Veelenturf et al. (2015) propose an ILP model to handle partial or complete track blockages focusing on a part of the Dutch railway network where short-turning trains is commonly used during disruptions. They assign each train with the last scheduled stop before the blocked track as the only short-turn station. If the short-turn station lacks capacity to short-turn a train then it has to be cancelled completely. To reduce complete train cancellations, Ghaemi et al. (2018a) propose an MILP model to decide the optimal time and station of short-turning a train by assigning two short-turn station candidates. This has also been implemented in Ghaemi et al. (2017a) where the infrastructure is modelled at a microscopic level to improve solution feasibility in practice. The aforementioned papers aim to minimize train cancellations and delays. To reduce passenger inconveniences during disruptions, Chapter 3 proposes an MILP model where more short-turn station candidates are given for each train and also the stopping patterns of trains can be changed flexibly (i.e. skipping stops and adding stops). Binder et al. (2017b) integrate passenger rerouting and timetable rescheduling into one ILP model where limited vehicle capacity is taken into account. While most literature focus on a single disruption, Zhu and Goverde (2019) propose an MILP model to deal with multiple disruptions that have overlapping periods and are pairwise connected by at least one train line. Most literature share the assumption that the disruption duration is known and will not change over time. However in practice, a disruption may become shorter or longer than predicted (Zilko et al., 2016), thus dynamic adjustments are required.

To deal with the uncertainty of the disruption duration, Zhan et al. (2016) embed their rescheduling model into a rolling horizon framework where the timetable is adjusted gradually with renewed disruption durations taken into account. Ghaemi et al. (2018b) develop an iterative approach to reschedule the timetable in each iteration when a new disruption duration is updated. In both cases, deterministic models are used for the rescheduling. To obtain a robust solution, Meng and Zhou (2011) propose a stochastic programming model that takes the uncertainty of the disruption duration into account. The model reschedules the timetable dynamically by a rolling horizon approach for single-track railway lines using two dispatching measures: delaying and reordering. Quaglietta et al. (2013) also propose a rolling horizon approach to manage stochastic disturbances (small train delays) using retiming and reordering, where at regular rescheduling intervals the current delays are measured and the associated conflicts are predicted over a prediction horizon of fixed length. Then rescheduling solutions are generated for the entire prediction horizon but only the first part is implemented in the next rescheduling interval.

This chapter deals with uncertain disruptions using two methods. We implemented a deterministic rolling-horizon approach based on the deterministic timetable rescheduling model of Chapter 3. Also, we propose a stochastic rolling-horizon approach based on a two-stage stochastic timetable rescheduling model. Different from the existing literature, both methods are devoted to more complicated conditions, where 1) single-track and double-tack railway lines both exist; 2) a wide range of dispatching measures is allowed: delaying, reordering, cancelling, adding stops and flexible short-turning; 3) rolling stock circulations at terminal stations are considered, and 4) station capacity is taken into account. The rescheduling solution is computed until the normal schedule has been recovered.

The main contributions of this chapter are summarized as follows:

- A rolling horizon two-stage stochastic timetable rescheduling model is proposed to handle uncertain disruptions by robust solutions.
- The proposed model allows delaying, reordering, cancelling, adding stops and flexible short-turning, and considers station capacity and rolling stock circulations at terminal stations.
- We test the stochastic method on a part of the Dutch railways, and compare it to a deterministic rolling-horizon method.

The remainder of the chapter is organized as follows. Section 4.2 introduces the deterministic and stochastic methods. Both methods are tested with real-life instances in Section 4.3. Finally, Section 4.4 concludes the chapter.

4.2 Methodology

A brief introduction is given to the basics considered in the deterministic and stochastic methods. After that, both methods are explained.

4.2.1 Basics

4.2.1.1 Event-activity network

The rescheduling model is based on an event-activity network formulated by the method introduced in Chapter 3. An *event* e is either a train departure or arrival that is associated with the original scheduled time o_e, station st_e, train line tl_e, train number tr_e, and operation direction dr_e. All departure (arrival) events constitute the set E_{de} (E_{ar}). An *activity* is a directed arc from an event to another. Multiple kinds of activities are established, including running activities A_{run}, dwell activities A_{dwell}, pass-through activities A_{pass}, headway activities A_{head}, short-turn activities A_{turn}, and OD turn activities A_{odturn}. We refer to Chapter 3 for the details.

4.2.1.2 Decision variables

Any event $e \in E_{\text{de}} \cup E_{\text{ar}}$ corresponds to the following decision variables: 1) the rescheduled time x_e, 2) the delay d_e, 3) and the binary decision c_e with value 1 indicating that e is cancelled. Particularly for an event $e \in E_{\text{de}}^{\text{turn}} \cup E_{\text{ar}}^{\text{turn}}$, a binary decision y_e is needed, of which value 1 indicates that train tr_e is short-turned at station st_e. Here, $E_{\text{de}}^{\text{turn}}$ ($E_{\text{ar}}^{\text{turn}}$) is the set of departure (arrival) events that have short-turning possibilities. To deal with station capacity, for any arrival event $e \in E_{\text{ar}}$, two binary decision variables are needed: 1) $u_{e,i}$ with value 1 indicating that train tr_e occupies the ith platform of station st_e, 2) and $v_{e,j}$ with value 1 indicating that train tr_e occupies the jth pass-through track of station st_e.

A short-turn (OD-turn) activity $a \in A_{\text{turn}}$ ($a \in A_{\text{odturn}}$) corresponds to a binary decision variable m_a with value 1 indicating that a is selected. A pass-through activity $a \in A_{\text{pass}}$ corresponds to a binary decision variable s_a with value 1 indicating that a is added with a stop. For any two different events $e, e' \in E_{\text{de}} \cup E_{\text{ar}}$, we have a binary decision variable $q_{e,e'}$ with value 1 indicating that e occurs before e'.

Note that due to our formulation, once the decisions regarding x_e, d_e, c_e and y_e are determined, the other decisions are also determined.

4.2.1.3 Disruptions

This chapter considers a disruption that occurs at t_{start} and is predicted to end within the period $\left[t_{\text{end}}^{\min}, t_{\text{end}}^{\max}\right]$. The disruption duration is a random input that is assumed to have a finite number of possible realizations, called scenarios, $1, \ldots, W$, with corresponding probabilities, $p_1, \ldots, p_W$, satisfying $\sum_{w=1}^{W} p_w = 1$. Each scenario w has a unique disruption duration $\left[t_{\text{start}}, t_{\text{end}}^{w}\right]$ where $t_{\text{end}}^{\min} \leq t_{\text{end}}^{w} \leq t_{\text{end}}^{\max}$.

During a disruption, the range of the disruption end time may change when new information is received from the disruption site. Therefore, we define the concept of *stages* at which the estimated range of the disruption end time is updated, which triggers a rescheduling model to compute a new solution based on the updated range. The range of the disruption end time updated at stage k is defined as $\left[t_{\text{end}}^{k,\min}, t_{\text{end}}^{k,\max}\right]$, where $t_{\text{end}}^{k,\min}$ ($t_{\text{end}}^{k,\max}$) refers to the minimal (maximal) disruption end time predicted at stage k with $t_{\text{end}}^{k,\max} \geq t_{\text{end}}^{k,\min}$. It is assumed that $t_{\text{end}}^{k,\min} \geq t_{\text{end}}^{k-1,\min}$, while $t_{\text{end}}^{k,\max}$ is allowed to be equivalent to, smaller, or larger than $t_{\text{end}}^{k-1,\max}$. This paper is also based on the following assumptions:

- At stage $k = 1$, the range of the disruption end time $\left[t_{\text{end}}^{k,\min}, t_{\text{end}}^{k,\max}\right]$ is obtained at the disruption start time t_{start}
- At stage $k \in [2, K-1]$, the range of the disruption end time $\left[t_{\text{end}}^{k,\min}, t_{\text{end}}^{k,\max}\right]$ is updated before time $t_{\text{end}}^{k-1,\min} - \ell$

- At final stage K, the exact disruption end time t_{end} is received at time $t_{\text{end}}^{K-1,\min} - \ell$, and $t_{\text{end}} \geq t_{\text{end}}^{K-1,\min}$

Here, ℓ is a given parameter relevant to the update time, which must ensure a timely implementation of a new rescheduling solution based on the updated information. The value of ℓ is relevant to the traffic density of the considered network and the extent of the deviation from the planned timetable. A network that has a denser traffic and the corresponding rescheduled timetable has more deviations than the planned one may need longer time for implementing the rescheduled timetable.

The notation of sets and parameters is described in Table 4.1.

Table 4.1: Sets and parameters

Notation	Description
E_{ar}	Set of arrival events
E_{de}	Set of departure events
E	Set of events: $E = E_{\text{ar}} \cup E_{\text{de}}$
$E_{\text{ar}}^{\text{turn}}$	Set of arrival events that have short-turning possibilities
$E_{\text{de}}^{\text{turn}}$	Set of departure events that have short-turning possibilities
E^{turn}	Set of events that have short-turning possibilities: $E^{\text{turn}} = E_{\text{ar}}^{\text{turn}} \cup E_{\text{de}}^{\text{turn}}$
o_e	The original scheduled time of event $e \in E_{\text{ar}} \cup E_{\text{de}}$
p_w	The occurrence probability of scenario $w \in \{1, \ldots, W\}$
$p_{w_{k,n}}$	The occurrence probability of scenario $w_{k,n}, n \in \{1, \ldots, W_k\}$
r_e^{k-1}	The rescheduled time of event e determined at stage $k-1$, which is a known value at stage k
R_k	The recovery time length at stage $k \in \{1, \ldots, K\}$
$R_k^{w_{k,n}}$	The recovery time length of scenario $w_{k,n}, n \in \{1, \ldots, W_k\}$ at stage $k \in \{1, \ldots, K\}$
st_e	The station corresponding to event $e \in E_{\text{ar}} \cup E_{\text{de}}$
tr_e	The train corresponding to event $e \in E_{\text{ar}} \cup E_{\text{de}}$
t_{start}	The actual disruption starting time
t_{end}	The actual disruption ending time
$t_{\text{end}}^{\min}$	The predicted minimal disruption ending time
$t_{\text{end}}^{\max}$	The predicted maximal disruption ending time
t_{end}^{w}	The predicted disruption ending time of scenario $w \in \{1, \ldots, W\}$: $t_{\text{end}}^{\min} \leq t_{\text{end}}^{w} \leq t_{\text{end}}^{\max}$
$t_{\text{end}}^{k,\min}$	The predicted minimal disruption ending time at stage $k \in \{1, \ldots, K\}$
$t_{\text{end}}^{k,\max}$	The predicted maximal disruption ending time at stage $k \in \{1, \ldots, K\}$
$t_{\text{end}}^{w_{k,n}}$	The predicted disruption ending time of scenario $w_{k,n}, n \in \{1, \ldots, W_k\}$: $t_{\text{end}}^{k,\min} \leq t_{\text{end}}^{w_{k,n}} \leq t_{\text{end}}^{k,\max}$

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Notation	Description
$w_{k,n}$	The nth scenario defined at stage k, where $n \in \{1, \cdots, W_k\}, k \in \{1, \cdots, K\}$
W_k	The total number of scenarios defined at stage k
X	Set of the 1st-stage decisions in the two-stage stochastic model
X_k	Set of the 1st-stage decisions in the two-stage stochastic model formulated at update stage $k \in \{1, \cdots, K\}$
$Y(w)$	Set of the 2nd-stage decisions of scenario $w \in \{1, \cdots, W\}$ in the two-stage stochastic model
$Y_k(w_{k,n})$	Set of the 2nd-stage decisions of scenario $w_{k,n}, n \in \{1, \cdots, W_k\}$ in the two-stage stochastic model formulated at update stage $k \in \{1, \cdots, K\}$
Z^{I}	Set of constraints for the 1st-stage decisions X
$Z^{\mathrm{II}}(X, w)$	Set of constraints for the 2nd-stage decisions given X in scenario $w \in \{1, \cdots, W\}$
ℓ	A given time period ensuring a timely implementation of a new rescheduling solution
β_c	The penalty of cancelling a train run between two adjacent stations

4.2.2 Deterministic rolling-horizon method

A deterministic rescheduling model can only consider *one* possible disruption duration $\left[t_{\text{start}}, t_{\text{end}}^{w_{k,n}}\right]$ at stage k, where $t_{\text{end}}^{k,\min} \leq t_{\text{end}}^{w_{k,n}} \leq t_{\text{end}}^{k,\max}, w_{k,n} \in \left\{w_{k,1}, \cdots, w_{k,W_k}\right\}$. Here, $w_{k,n}$ refers to the nth scenario defined in stage k, and $1 \leq n \leq W_k$, where W_k is the total number of scenarios defined in stage k. The choice of $t_{\text{end}}^{w_{k,n}}$ depends on the adopted strategy. For example, the value of $t_{\text{end}}^{w_{k,n}}$ is chosen as 1) $t_{\text{end}}^{k,\min}$ in an optimistic strategy, 2) $t_{\text{end}}^{k,\max}$ in a pessimistic strategy, 3) or $\sum_{n=1}^{W_k} p_{w_{k,n}} t_{\text{end}}^{w_{k,n}}$ in an expected-value strategy.

In the remainder of this section, we give an example of a rolling horizon approach for a deterministic rescheduling model with a pessimistic strategy, see Figure 4.1. Note that a new stage starts when a new prediction about the range of the disruption ending time is updated.

At stage $k \in [1, K-1]$, the prediction $\left[t_{\text{end}}^{k,\min}, t_{\text{end}}^{k,\max}\right]$ is updated. Using a pessimistic strategy, a control horizon is then defined as $\left[t_{\text{start}} + \ell, t_{\text{end}}^{k,\max}\right]$ if $k = 1$, where ℓ is a time period ensuring the decisions determined for the control horizon at stage 1 to be successfully implemented. It is assumed that the planned timetable is applied for the period $[t_{\text{start}}, t_{\text{start}} + \ell)$ during which some trains may have to wait at the last stations before the blocked tracks. A recovery horizon is defined as $\left(t_{\text{end}}^{k,\max}, t_{\text{end}}^{k,\max} + R_k\right]$ if $k = 1$. Here, R_k represents the recovery time length after $t_{\text{end}}^{k,\max}$, which is not a given input to the rescheduling model but an output that can only be known after the rescheduling solution has been computed. The deterministic rescheduling model computes a rescheduling solution over the combined control and recovery horizons. When $k \geq 2$, the rescheduling solution respects the previous disruption management decisions up to (1) $t_{\text{end}}^{k-1,\max}$ if $t_{\text{end}}^{k,\max} \geq t_{\text{end}}^{k-1,\max}$ or (2) $t_{\text{end}}^{k,\max}$ if $t_{\text{end}}^{k,\max} < t_{\text{end}}^{k-1,\max}$, and thus $\left[t_{\text{start}} + \ell, t_{\text{end}}^{k-1,\max}\right]$ or $\left[t_{\text{start}} + \ell, t_{\text{end}}^{k,\max}\right]$ is regarded as the rescheduled timetable horizon. Figure 4.1 is an example of case (1). The proposed rolling-horizon approach also applies to case (2) in which the current time point (the update time) is ensured to be before $t_{\text{end}}^{k,\max}$ because it is assumed that the update at stage k occurs before $t_{\text{end}}^{k-1,\min} - \ell$ that holds for $t_{\text{end}}^{k-1,\min} - \ell \leq t_{\text{end}}^{k,\min} \leq t_{\text{end}}^{k,\max}$. A rescheduling solution is constituted by a set of disruption management decisions (e.g. cancelling trains and short-turning trains) that were introduced in Section 4.2.1.

At the final stage K, an exact disruption end time t_{end} is assumed to be known. If $t_{\text{end}} = t_{\text{end}}^{K-1,\max}$, the rescheduling solution obtained at stage $K-1$ is used without any further adjustments. If $t_{\text{end}} \neq t_{\text{end}}^{K-1,\max}$, the rescheduling model is solved again by respecting the previous disruption management decisions up to 1) $t_{\text{end}}^{K-1,\max}$ if $t_{\text{end}} \geq t_{\text{end}}^{K-1,\max}$, or 2) t_{end} if $t_{\text{end}} < t_{\text{end}}^{K-1,\max}$. In case 1) the control horizon is $\left[t_{\text{end}}^{K-1,\max}, t_{\text{end}}\right]$, while in case 2) the control horizon is zero. In both cases, the recovery horizons are $(t_{\text{end}}, t_{\text{end}} + R_K]$

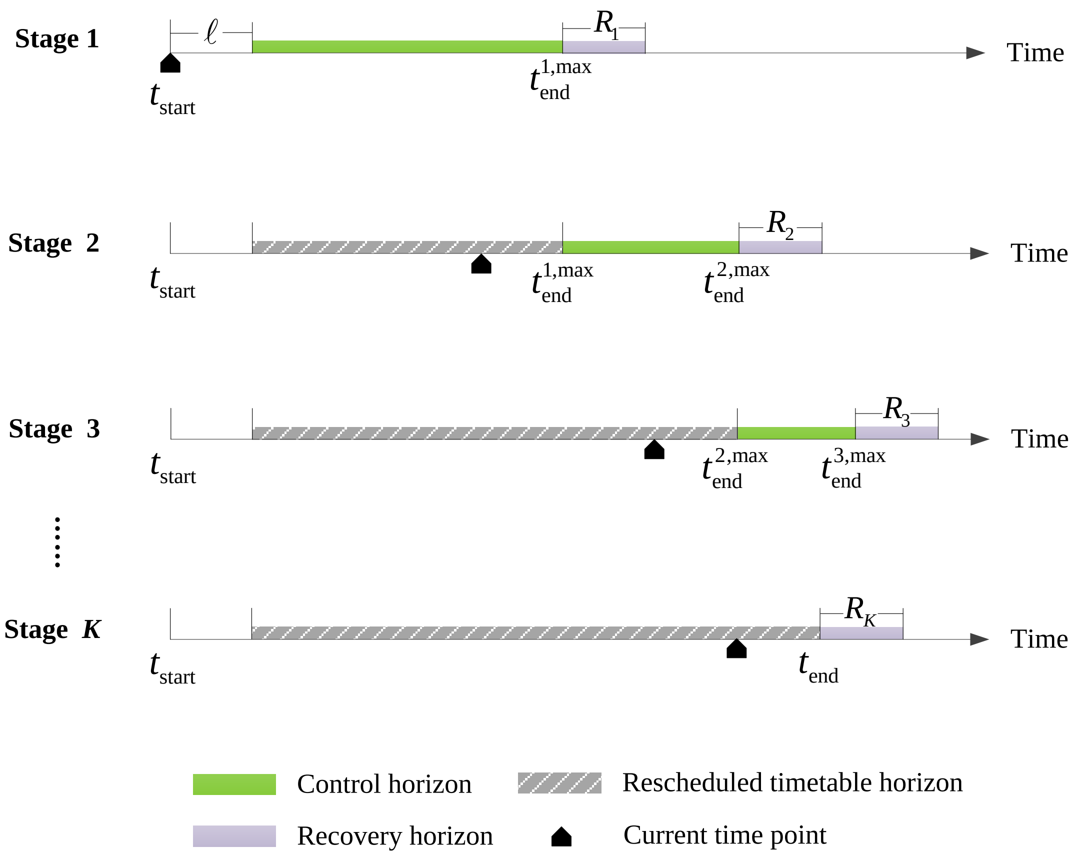

Figure 4.1: The rolling horizon approach based on a deterministic rescheduling model using a pessimistic strategy

This chapter uses the rescheduling model of Chapter 3 for the deterministic rolling-horizon method, where the dispatching measure of skipping stops is removed due to the new objective of minimizing train cancellation and delay, and the station capacity part is reformulated as in Zhu and Goverde (2019) for faster computation.

4.2.3 Stochastic rolling-horizon method

The timetable rescheduling problem taking into account the uncertainty of the disruption duration is formulated as a rolling horizon two-stage stochastic program in deterministic equivalent form (Birge and Louveaux, 2011). For clarity, the stochastic timetable rescheduling model is introduced first without considering different update stages of the disruption durations, which are explicitly included later when describing the corresponding rolling horizon approach.

4.2.3.1 Stochastic timetable rescheduling model

The stochastic rescheduling model considers multiple possible disruption durations at each computation as follows. The set of disruption management decisions are divided into two groups: 1) the 1st-stage decisions that have to be taken before the exact scenario with a given disruption duration is known are called *control decisions* and the horizon when these decisions are applied is called *control horizon*, and 2) the 2nd-stage decisions that could be taken after the exact scenario with a given disruption duration is known are called *look-ahead decisions* with corresponding *look-ahead horizon*. Recall that we have an estimated range of disruption end time $\left[t_{\text{end}}^{\min}, t_{\text{end}}^{\max}\right]$ to represent the stochastic part of disruption duration, and each scenario $w \in \{1, \cdots, W\}$ is defined with a unique disruption duration $\left[t_{\text{start}}, t_{\text{end}}^{w}\right]$ where $t_{\text{end}}^{\min} \leq t_{\text{end}}^{w} \leq t_{\text{end}}^{\max}$.

In each scenario w, $\left[t_{\text{start}} + \ell, t_{\text{end}}^{\min}\right]$ is defined as the control horizon, while $\left(t_{\text{end}}^{\min}, t_{\text{end}}^{w} + R^{w}\right]$ is defined as the look-ahead horizon, where ℓ refers to a time period ensuring the control decisions to be timely implemented, and R^{w} represents the recovery time to the planned timetable. The planned timetable is applied for the period $[t_{\text{start}}, t_{\text{start}} + \ell)$ where some trains might be forced to wait at the last stations before the blocked tracks. Recall that R^{w} can only be known after the disruption management decisions for scenario w are determined, and so the value may vary across scenarios. A look-ahead horizon consists of a *disruption horizon* $\left(t_{\text{end}}^{\min}, t_{\text{end}}^{w}\right]$ in which the disruption is ongoing, and a *recovery horizon* $\left(t_{\text{end}}^{w}, t_{\text{end}}^{w} + R^{w}\right]$ that goes from the end of the disruption until completely resuming to the planned timetable. The 1st-stage control decisions are scenario independent and are thus the same over all scenarios. The 2nd-stage look-ahead decisions are scenario dependent, which can be different among scenarios. As shown in Figure 4.2, determining the control decisions up to $t_{\text{end}}^{\min}$ is the first stage of the stochastic timetable rescheduling model, and determining the look-ahead decisions within the period $\left(t_{\text{end}}^{\min}, t_{\text{end}}^{w} + R^{w}\right]$ for any scenario w is the second stage. The control decisions determined at the first stage affect the look-ahead decisions determined at the second stage.

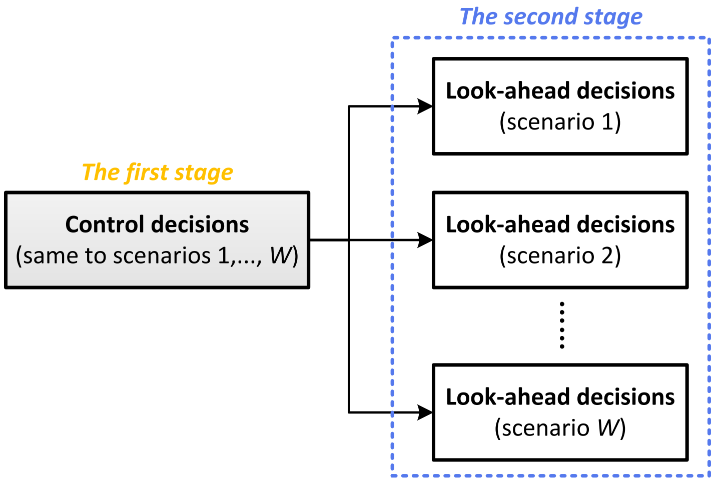

Figure 4.2: Illustration of the two stages in the stochastic timetable rescheduling model

The two-stage stochastic timetable rescheduling model can be formulated in a more *compact* form as

$$\min \; Q^{\mathrm{I}}(X) + E_w \left[\min \; Q^{\mathrm{II}}\left(Y(w)\right)\right], \tag{4.1}$$

$$\text{s.t.} \quad X \in Z^{\mathrm{I}}, \tag{4.2}$$

$$Y(w) \in Z^{\mathrm{II}}(X, w), \qquad w \in \{1, \dots, W\} \tag{4.3}$$

where X are the **1st-stage decisions** defined as the scenario-independent disruption management decisions associated with the train arrival and departure events e of which the original scheduled times o_e are in the control horizon $\left[t_{\text{start}} + \ell, t_{\text{end}}^{\min}\right]$,

$$X = \left\{\{c_e, d_e, x_e\} : o_e \in [t_{\text{start}} + \ell, t_{\text{end}}^{\min}], e \in E\right\} \cup \left\{y_e : o_e \in [t_{\text{start}} + \ell, t_{\text{end}}^{\min}], e \in E^{\text{turn}}\right\},$$

and $Y(w)$ are the **2nd-stage decisions** of scenario w, which are defined as the disruption management decisions associated with the train arrival and departure events e of which the original scheduled times o_e are in the look-ahead horizon $\left(t_{\text{end}}^{\min}, t_{\text{end}}^{w} + R^{w}\right]$ of scenario w,

$$Y(w) = \left\{\{c_e^w, d_e^w, x_e^w\} : o_e \in (t_{\text{end}}^{\min}, t_{\text{end}}^{w} + R^{w}], e \in E\right\} \cup \left\{y_e^w : o_e \in (t_{\text{end}}^{\min}, t_{\text{end}}^{w} + R^{w}], e \in E^{\text{turn}}\right\}, w \in \{1, \dots, W\}.$$

$Y(w)$ is dependent on X since X and $Y(w)$ are jointly optimized in (4.1)-(4.3). Here, c_e represents the decision to cancel event $e \in E$, d_e represents the delay of event $e \in E$, x_e represents the rescheduled time of event $e \in E$, and y_e represents the decision to short-turn train tr_e at station st_e considering event $e \in E^{\text{turn}}$. Recall that E is the set of arrival and departure events, and E^{turn} is the set of arrival and departure events that have short-turning possibilities. In the 2nd stage the same notation is used with a superscript w to indicate the scenario. The developed two-stage stochastic timetable rescheduling model includes more decision variables (see Section 4.2.1.2) than those shown in the formulation of (4.1)-(4.3). We only show the event-related decision variables with respect to cancelling, delaying, re-timing, and short-turning in the compact formulation because once these decisions are determined the other decisions will be determined implicitly as well. $Q^{\mathrm{I}}(\cdot)$ is the cost function for X, and $Q^{\mathrm{II}}(\cdot)$ is the cost function for $Y(w)$, which are formulated respectively as follows:

$$Q^{\mathrm{I}}(X) = \beta_c \sum_{e \in E_{\mathrm{ar}}:\ c_e \in X} c_e + \sum_{e \in E_{\mathrm{ar}}:\ d_e \in X} d_e,$$
$$Q^{\mathrm{II}}(Y(w)) = \beta_c \sum_{e \in E_{\mathrm{ar}}:\ c_e^w \in Y(w)} c_e^w + \sum_{e \in E_{\mathrm{ar}}:\ d_e^w \in Y(w)} d_e^w, \qquad w \in \{1, \ldots, W\},$$

where parameter β_c refers to the cost of cancelling a train run between two adjacent stations. The cost function $Q^{\mathrm{I}}(\cdot)$ ($Q^{\mathrm{II}}(\cdot)$) measures the train cancellations and arrival delays within the control horizon (look-ahead horizon) relevant to the first stage (the second stage) of the stochastic timetable rescheduling model. The objective (4.1) is to minimize the train cancellations and arrival delays within the control horizon plus the expectation of the train cancellations and arrival delays within the look-ahead horizons of all scenarios. The expectation $E_w[\cdot]$ is defined as $\sum_{w=1}^{W} p_w \cdot Q^{\mathrm{II}}(Y(w))$, where p_w represents the occurrence probability of scenario w. In (4.2), Z^{I} refers to the constraint set for X. In (4.3), $Z^{\mathrm{II}}(X, w)$ refers to the constraint set for $Y(w)$ given X under scenario w. $Y(w)$ is required to be consistent with X. For any scenario $w \in \{1, \ldots, W\}$, the decisions X and $Y(w)$ together constitute a feasible rescheduling solution satisfying the constraints in $Z^{\mathrm{I}} \cup Z^{\mathrm{II}}(X, w)$ for the time horizon $\left[t_{\mathrm{start}} + \ell, t_{\mathrm{end}}^w + R^w\right]$.

The two-stage stochastic timetable rescheduling model of (4.1)-(4.3) is based on a compact representation of scenarios as shown in the left part of Figure 4.3, where each root-to-leaf path refers to a specific scenario w. For simplicity, we used a splitting variable representation (Escudero et al., 2013) as shown in the right part of Figure 4.3. In this way, the first-stage decisions X is duplicated for each scenario $w \in \{1, \ldots, W\}$ as $X(w)$.

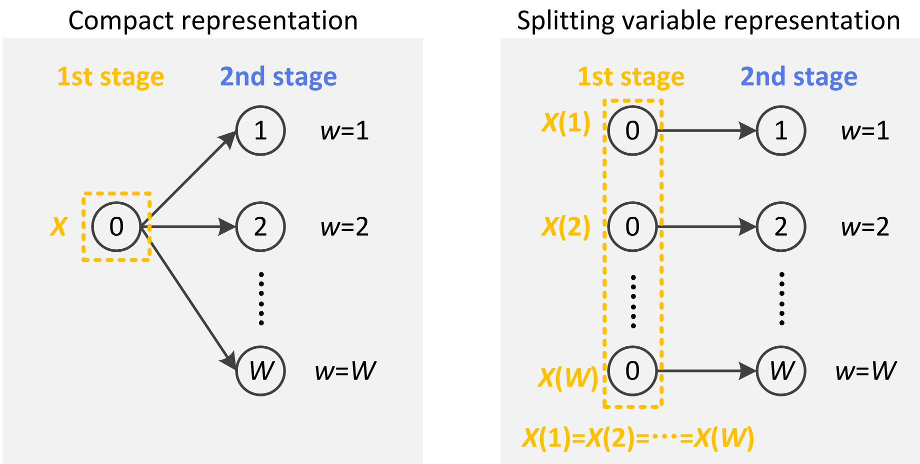

Figure 4.3: Illustration of scenarios

Based on the splitting variable representation,, we reformulated the two-stage stochastic timetable rescheduling model of (4.1)-(4.3) with explicit *nonanticipativity* constraints considering stage $k = 1$ (the range of the disruption end time is updated for the first time),

$$\min \sum_{n=1}^{W_1} p_{w_{1,n}} \left(\left(\beta_c \sum_{e \in E_{\text{ar}}:\, c_e^{w_{1,n}} \in X_1(w_{1,n})} c_e^{w_{1,n}} + \sum_{e \in E_{\text{ar}}:\, d_e^{w_{1,n}} \in X_1(w_{1,n})} d_e^{w_{1,n}} \right) + \left(\beta_c \sum_{e \in E_{\text{ar}}:\, c_e^{w_{1,n}} \in Y_1(w_{1,n})} c_e^{w_{1,n}} + \sum_{e \in E_{\text{ar}}:\, d_e^{w_{1,n}} \in Y_1(w_{1,n})} d_e^{w_{1,n}} \right) \right) \tag{4.4}$$

$$\text{s.t. } X_1(w_{1,n}) \in Z_1^{\text{I}}(w_{1,n}), \qquad n \in \{1, \dots, W_1\}, \tag{4.5}$$

$$Y_1(w_{1,n}) \in Z_1^{\text{II}}(X_1(w_{1,n}), w_{1,n}), \quad n \in \{1, \dots, W_1\}, \tag{4.6}$$

$$X_1(w_{1,n}) = X_1(w_{1,m}), \qquad n, m \in \{1, \dots, W_1\} : n \neq m, \tag{4.7}$$

where the first-stage decisions $X_1(w_{1,n})$ of scenario $w_{1,n}$ is

$$X_1(w_{1,n}) = \left\{ \{c_e^{w_{1,n}}, d_e^{w_{1,n}}, x_e^{w_{1,n}}\} : o_e \in [t_{\text{start}} + \ell, t_{\text{end}}^{1,\min}], e \in E \right\} \cup \left\{ y_e^{w_{1,n}} : o_e \in [t_{\text{start}} + \ell, t_{\text{end}}^{1,\min}], e \in E^{\text{turn}} \right\}, n \in \{1, \dots, W_1\},$$

and the second-stage decisions $Y_1(w_{1,n})$ of scenario $w_{1,n}$ is

$$Y_1(w_{1,n}) = \left\{ \{c_e^{w_{1,n}}, d_e^{w_{1,n}}, x_e^{w_{1,n}}\} : o_e \in (t_{\text{end}}^{1,\min}, t_{\text{end}}^{w_{1,n}} + R_1^{w_{1,n}}], e \in E \right\} \cup \left\{ y_e^{w_{1,n}} : o_e \in (t_{\text{end}}^{1,\min}, t_{\text{end}}^{w_{1,n}} + R_1^{w_{1,n}}], e \in E^{\text{turn}} \right\}, n \in \{1, \dots, W_1\}.$$

Here, $w_{1,n}$ represents the nth scenario defined at stage 1, W_1 refers to the number of scenarios defined at stage 1, and $t_{\text{end}}^{1,\min}$ is the minimal disruption end time update at stage 1. Note that $X_1(w_{1,n}) = X_1$ for some optimally determined X_1 for all $w_{1,n}, n \in \{1, \dots, W_1\}$. The formulation of (4.4)-(4.7) can be seen as W_1 separate deterministic Mixed-Integer Linear Programming (MILP) timetable rescheduling models linked together by the so-called nonanticipativity constraints (4.7) (Escudero et al., 2010), which force the 1st-stage decisions $X_1(w_{1,n})$ to be the same in any scenario $w_{1,n}, n \in \{1, \dots W_1\}$. To be more specific, (4.7) requires each decision of $X_1(w_{1,n})$ to be equivalent to the same type of decision corresponding to the same event in $X_1(w_{1,m})$ considering two different scenarios $w_{1,n}$ and $w_{1,m}$. For example, $c_e^{w_{1,n}} = c_e^{w_{1,m}}$, where $c_e^{w_{1,n}} \in X(w_{1,n}), c_e^{w_{1,m}} \in X(w_{1,m}), n \neq m$. In (4.5), $Z_1^{\text{I}}(w_{1,n})$ refers to the constraint set for $X_1(w_{1,n})$. In (4.6), $Z_1^{\text{II}}(X_1(w_{1,n}), w_{1,n})$ refers to the constraint set for $Y_1(w_{1,n})$ given $X_1(w_{1,n})$ under scenario $w_{1,n}$. The objective (4.4) is to minimize the expected consequences measured in train cancellations and arrival delays both in the 1st stages and 2nd stages of all scenarios.

To establish (4.4)-(4.7), we construct, for each scenario $w_{1,n}, n \in \{1, \dots, W_1\}$, an independent deterministic MILP timetable rescheduling model by the method of Chapter 3, of which the variables are $\{X_1(w_{1,n}), Y_1(w_{1,n})\}$, and the constraints are $\{Z_1^{\mathrm{I}}(w_{1,n}), Z_1^{\mathrm{II}}(X_1(w_{1,n}), w_{1,n})\}$ that ensure feasible rescheduling solutions adjusted by delaying, reordering, cancelling, adding stops and flexible short-turning trains. For a detailed MILP constraint formulation we refer to Chapter 3. The variables $\bigcup_{n\in\{1,\ldots,W_1\}}\{X_1(w_{1,n}), Y_1(w_{1,n})\}$ and constraints $\bigcup_{n\in\{1,\ldots,W_1\}}\{Z_1^{\mathrm{I}}(w_{1,n}), Z_1^{\mathrm{II}}(X_1(w_{1,n}), w_{1,n})\}$ are established for all scenarios that are used in the stochastic timetable rescheduling model with also nonanticipativity constraints (4.7).

The notation of the decision variables are described in Table 4.2.

Table 4.2: Part of decision variables

Notation	Description
c_e^w	Binary variable with value 1 indicating that event e is cancelled in scenario w, and 0 otherwise
d_e^w	Delay of event e in scenario w
x_e^w	Rescheduled time of event e in scenario w
y_e^w	Binary variable with value 1 indicating that train tr_e is short-turned at station st_e in scenario w, and 0 otherwise

The rescheduling solution formed by X_1 will be delivered to the traffic controllers directly. As for the scenario-dependent 2nd-stage decisions $Y_1(w_{1,n}), n \in \{1,\cdots,W_1\}$, only one of them will be delivered at time $t_{\text{end}}^{1,\min} - \ell$ when the exact scenario is foreseen to be a specific scenario $w_{1,n}$. ℓ is set to an appropriate value (e.g. 10 minutes) to ensure that the 2nd-stage decisions can be implemented in time. If none of the defined scenarios correspond to the exact scenario, the rescheduling model computes a new solution considering one single scenario with disruption duration $\left[t_{\text{end}}^{1,\min}, t_{\text{end}}\right]$, which should be consistent with the 1st-stage decisions up to $t_{\text{end}}^{1,\min}$. Here, t_{end} represents the exact disruption end time. Note that in this case, nonanticipativity constraints are not needed.

4.2.3.2 Rolling horizon approach based on stochastic model

During the disruption, the range of the disruption end time $\left[t_{\text{end}}^{\min}, t_{\text{end}}^{\max}\right]$ may change several times. Under this circumstance, we have a multiple-stage stochastic timetable rescheduling problem. We solve this problem by a rolling horizon approach with successive application of the two-stage stochastic timetable rescheduling model every time an estimated range of the disruption end time is updated in a new stage. The rolling horizon approach is based on the assumptions given in Section 4.2.1.3. An example of the rolling-horizon stochastic method is shown in Figure 4.4.

At stage $k \in [1, K-1]$, the prediction $\left[t_{\text{end}}^{k,\min}, t_{\text{end}}^{k,\max}\right]$ is updated. Thus, W_k scenarios are defined where each has a unique disruption duration $\left[t_{\text{start}} + \ell, t_{\text{end}}^{w_{k,n}}\right]$, and $t_{\text{end}}^{k,\min} \leq$

$t_{\text{end}}^{w_{k,n}} \le t_{\text{end}}^{k,\max}, w_{k,n} \in \{w_{k,1}, \ldots, w_{k,W_k}\}$. Recall that $w_{k,n}$ refers to the nth scenario defined at stage k, and the planned timetable is applied for the period $[t_{\text{start}}, t_{\text{start}} + \ell)$. Based on the new scenarios defined at stage k, the two-stage stochastic optimization is performed, and the 1st-stage decisions X_k from the optimization are delivered to the traffic controllers directly. The 1st-stage decisions X_k are for the period $\left[t_{\text{start}} + \ell, t_{\text{end}}^{k,\min}\right]$ if $k = 1$ or the period $\left[t_{\text{end}}^{k-1,\min}, t_{\text{end}}^{k,\min}\right]$ if $k \ge 2$, which will no longer change at later stages. This is why the period $\left[t_{\text{start}} + \ell, t_{\text{end}}^{k-1,\min}\right]$ is regarded as the *rescheduled timetable horizon* when $k \ge 2$. The 2nd-stage decisions $Y_k(w_{k,n})$ of scenario $w_{k,n}$ is for the period $\left(t_{\text{end}}^{k,\min}, t_{\text{end}}^{w_{k,n}} + R_k^{w_{k,n}}\right]$ that consists of the disruption horizon $\left(t_{\text{end}}^{k,\min}, t_{\text{end}}^{w_{k,n}}\right]$ and the recovery horizon $\left(t_{\text{end}}^{w_{k,n}}, t_{\text{end}}^{w_{k,n}} + R_k^{w_{k,n}}\right]$.

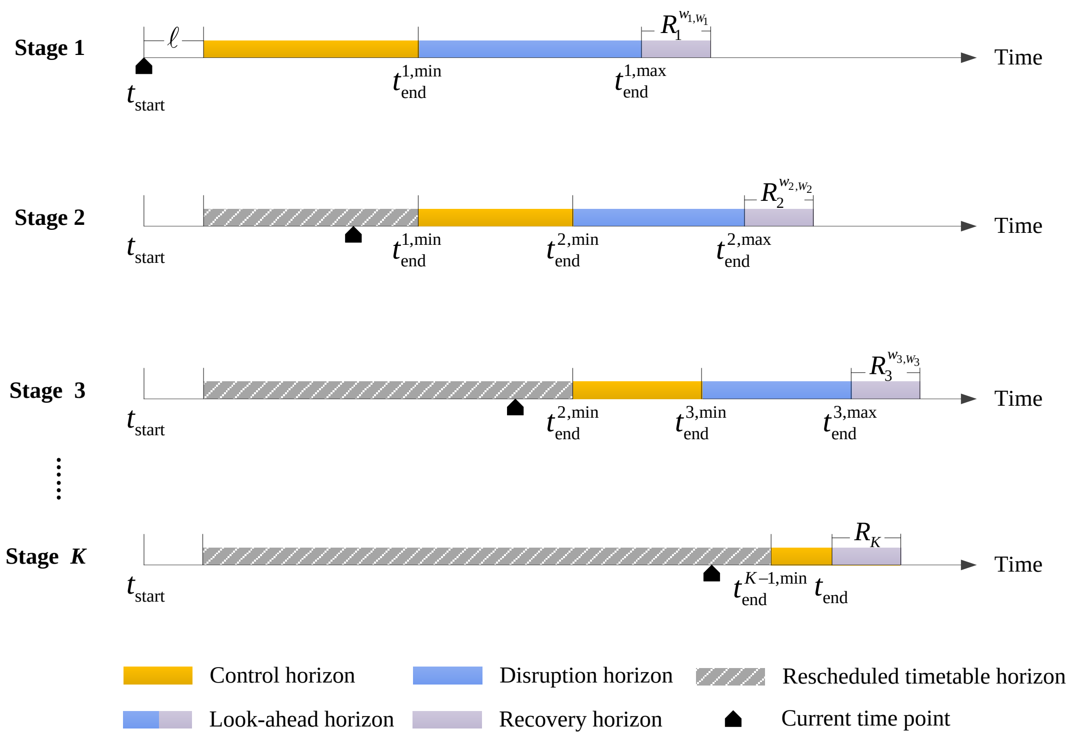

Figure 4.4: The rolling-horizon two-stage stochastic timetable rescheduling model to solve the multiple-stage stochastic timetable rescheduling problem

The two-stage stochastic timetable rescheduling model is then used for each following stage where new scenarios are defined according to the updated range of disruption end time. The two-stage stochastic timetable rescheduling model with *nonanticipativity* constraints for stage $1 \leq k \leq K-1$ is

$$\min \sum_{n=1}^{W_k} p_{w_{k,n}} \left(\left(\beta_c \sum_{e \in E_{\mathrm{ar}}:\ c_e^{w_{k,n}} \in X_k(w_{k,n})} c_e^{w_{k,n}} + \sum_{e \in E_{\mathrm{ar}}: d_e^{w_{k,n}} \in X_k(w_{k,n})} d_e^{w_{k,n}} \right) + \left(\beta_c \sum_{e \in E_{\mathrm{ar}}:\ c_e^{w_{k,n}} \in Y_k(w_{k,n})} c_e^{w_{k,n}} + \sum_{e \in E_{\mathrm{ar}}:\ d_e^{w_{k,n}} \in Y_k(w_{k,n})} d_e^{w_{k,n}} \right) \right) \tag{4.8}$$

$$\text{s.t. } X_k(w_{k,n}) \in Z_k^{\mathrm{I}}(w_{k,n}), \qquad n \in \{1,\ldots,W_k\}, \tag{4.9}$$

$$Y_k(w_{k,n}) \in Z_k^{\mathrm{II}}(X_k(w_{k,n}), w_{k,n}), \qquad n \in \{1,\ldots,W_k\}, \tag{4.10}$$

$$X_k(w_{k,n}) = X_k(w_{k,m}), \qquad n,m \in \{1,\ldots,W_k\} : n \neq m, \tag{4.11}$$

where the first-stage decisions

$$\begin{aligned} X_k(w_{k,n}) = & \left\{ \{c_e^{w_{k,n}}, d_e^{w_{k,n}}, x_e^{w_{k,n}}\} : o_e \in [t_{\mathrm{start}} + \ell, t_{\mathrm{end}}^{k,\min}], e \in E \right\} \cup \\ & \left\{ y_e^{w_{k,n}} : o_e \in [t_{\mathrm{start}} + \ell, t_{\mathrm{end}}^{k,\min}], e \in E^{\mathrm{turn}}, \right\}, n \in \{1,\ldots,W_k\}, \text{if } k=1, \\ X_k(w_{k,n}) = & \left\{ \{c_e^{w_{k,n}}, d_e^{w_{k,n}}, x_e^{w_{k,n}}\} : r_e^{k-1} \in [t_{\mathrm{end}}^{k-1,\min}, t_{\mathrm{end}}^{k,\min}], e \in E \right\} \cup \\ & \left\{ y_e^{w_{k,n}} : r_e^{k-1} \in [t_{\mathrm{end}}^{k-1,\min}, t_{\mathrm{end}}^{k,\min}], e \in E^{\mathrm{turn}} \right\}, n \in \{1,\ldots,W_k\}, \\ & \text{if } 2 \leq k \leq K-1, \end{aligned} \tag{4.12}$$

and the second-stage decisions

$$\begin{aligned} Y_k(w_{k,n}) = & \left\{ \{c_e^{w_{k,n}}, d_e^{w_{k,n}}, x_e^{w_{k,n}}\} : o_e \in (t_{\mathrm{end}}^{k,\min}, t_{\mathrm{end}}^{w_{k,n}} + R_k^{w_{k,n}}], e \in E \right\} \cup \\ & \left\{ y_e^{w_{k,n}} : o_e \in (t_{\mathrm{end}}^{k,\min}, t_{\mathrm{end}}^{w_{k,n}} + R_k^{w_{k,n}}], e \in E^{\mathrm{turn}} \right\}, n \in \{1,\ldots,W_k\}, \text{if } k=1, \\ Y_k(w_{k,n}) = & \left\{ \{c_e^{w_{k,n}}, d_e^{w_{k,n}}, x_e^{w_{k,n}}\} : r_e^{k-1} \in \left(t_{\mathrm{end}}^{k,\min}, t_{\mathrm{end}}^{w_{k,n}} + R_k^{w_{k,n}}\right], e \in E \right\} \cup \\ & \left\{ y_e^{w_{k,n}} : r_e^{k-1} \in \left(t_{\mathrm{end}}^{k,\min}, t_{\mathrm{end}}^{w_{k,n}} + R_k^{w_{k,n}}\right], e \in E^{\mathrm{turn}} \right\}, n \in \{1,\ldots,W_k\}, \\ & \text{if } 2 \leq k \leq K-1, \end{aligned} \tag{4.13}$$

in which o_e is the original scheduled time of event e, r_e^{k-1} is a known value representing the rescheduled time of event e determined at the previous stage $k-1$, and $w_{k,n}$ refers to the nth scenario defined at stage k. Note that $X_k = X_k(w_{k,n}), n \in \{1,\ldots,W_k\}, 1 \leq k \leq K-1$. In (4.9), $Z_k^{\mathrm{I}}(w_{k,n})$ refers to the constraint set for $X_k(w_{k,n})$. In (4.10), $Z_k^{\mathrm{II}}\left(X_k(w_{k,n}), w_{k,n}\right)$ refers to the constraint set for $Y_k(w_{k,n})$ given $X_k(w_{k,n})$ under scenario $w_{k,n}$.

For the final stage K, the exact disruption end time t_{end} is received. If a disruption end time of a scenario $w_{K-1,n}$ defined at the previous stage is equal to t_{end} (i.e. $t_{\mathrm{end}}^{w_{K-1,n}} = t_{\mathrm{end}}$), then the corresponding 2nd-stage decisions $Y_{K-1}(w_{K-1,n})$ will be delivered to the traffic controllers directly. If none of the previous scenarios corresponds to the exact scenario, the rescheduling model can simply compute a new solution considering the single scenario with the disruption duration $\left[t_{\text{end}}^{K-1,\min}, t_{\text{end}}\right]$, which should be consistent with the previous control decisions up to $t_{\text{end}}^{K-1,\min}$. In this case, nonanticipativity constraints are not needed in the rescheduling model.

4.3 Case study

The deterministic and stochastic methods are tested on a part of the Dutch railway network. Section 4.3.1 investigates the impact of the range of the disruption end time, and Section 4.3.2 analyses the computation performances of both methods.

Figure 4.5 shows the schematic track layout of the considered network with 38 stations and both single-track and double-track railway lines.

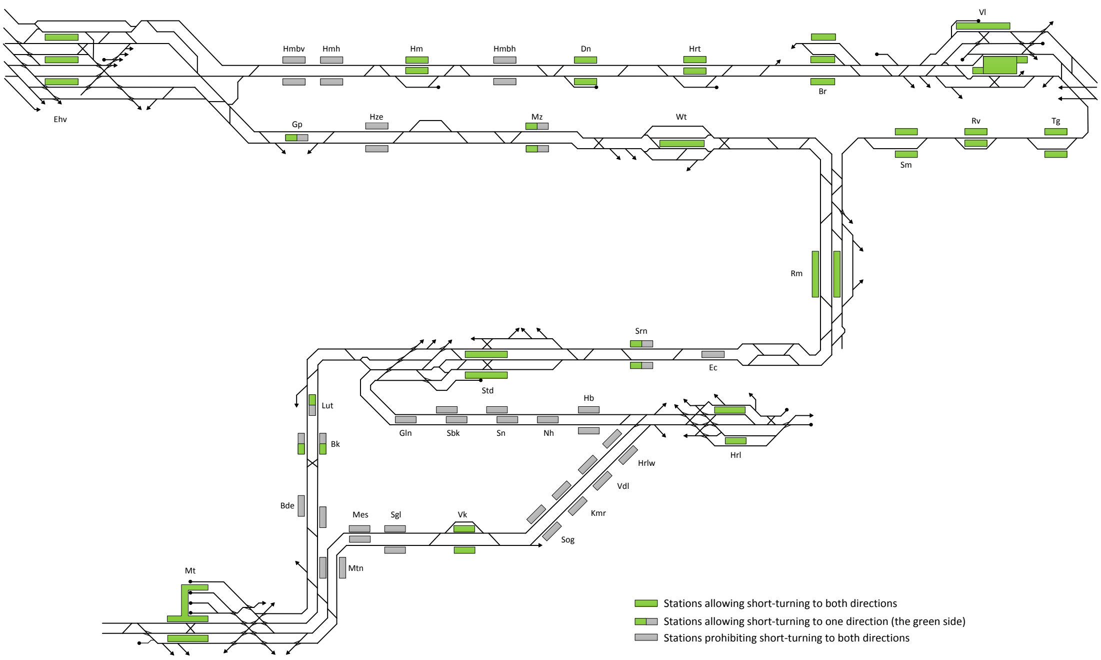

Figure 4.5: The schematic track layout of the considered network

In the considered network, 10 train lines operate half-hourly in each direction. Figure 4.6 shows the scheduled stopping pattern of each train line. Table 4.3 lists the terminals of the train lines that are located in the considered network, while the terminals outside the considered network are neglected. The deterministic and stochastic rescheduling models both consider trains turning at the terminals to operate the return direction (i.e. OD turnings). We distinguish between intercity (IC) and local (called sprinter (SPR) in Dutch) train lines. Both rescheduling models were developed in MATLAB and solved using GUROBI release 7.0.1 on a desktop with Intel Xeon CPU E5-1620 v3 at 3.50 GHz and 16 GB RAM.

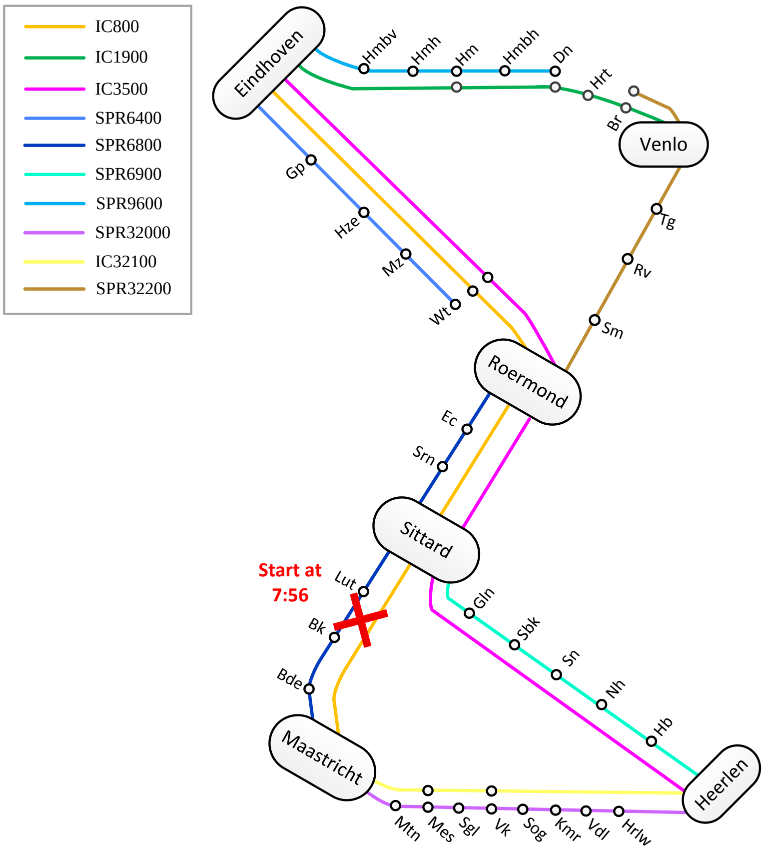

Figure 4.6: The train lines operating in the considered network

Table 4.3: Train lines in the considered network

Train line	Terminals in the considered network
IC800	Maastricht (Mt)
IC1900	Venlo (Vl)
IC3500	Heerlen (Hrl)
SPR6400	Eindhoven (Ehv) and Wt
SPR6800	Roermond (Rm)
SPR6900	Sittard (Std) and Hrl
SPR9600	Ehv and Dn
SPR32000	—
IC32100	Mt and Hrl
SPR32200	Rm

The penalty β_c of cancelling a train run between two neighbouring stations is set to 100 min, and the time period ℓ that ensures a new rescheduling solution to be implemented is set to 10 min. Besides, we set the minimum duration required for short-turning or OD turning to 300 s, the minimum duration required for each headway to 180 s, the maximum delay allowed for each train departure/arrival to 15 min, and the minimum dwell time of an extra stop to 30 s.

We consider a complete track blockage between station Bk and station Lut starting at 7:56 (see Figure 4.6). The range of the disruption end time update at each stage is indicated by Table 4.4, which is uniformly distributed to 7 scenarios with the same probabilities: $1/7$. Three cases are considered: cases I and II differ in the range of the disruption end time update at the 1st stage, and cases II and III differ in the range of the disruption end time update at the 2nd stage. At each stage, the stochastic method considers 7 disruption scenarios simultaneously, whereas the deterministic method considers one single disruption scenario of which the corresponding end time using optimistic, expected-value, and pessimistic strategies are colored in green, blue and red, respectively. Recall that the optimistic strategy considers the minimum disruption end time $t_{\text{end}}^{k,\min}$, the pessimistic strategy considers the maximum disruption end time $t_{\text{end}}^{k,\max}$, and the expected-value strategy considers the expected disruption end time $\sum_{n=1}^{W_k} p_{w_{k,n}} t_{\text{end}}^{w_{k,n}}$ at update stage k.

Table 4.4: The predicted disruption end times at each stage of three cases

Case	Stage	Disruption end time						
	k	$t_{\text{end}}^{w_{k,1}}$	$t_{\text{end}}^{w_{k,2}}$	$t_{\text{end}}^{w_{k,3}}$	$t_{\text{end}}^{w_{k,4}}$	$t_{\text{end}}^{w_{k,5}}$	$t_{\text{end}}^{w_{k,6}}$	$t_{\text{end}}^{w_{k,7}}$
I	1	9:51	9:56	10:01	10:06	10:11	10:16	10:21
	2	10:36	10:41	10:46	10:51	10:56	11:01	11:06
II	1	10:06	10:11	10:16	10:21	10:26	10:31	10:36
	2	10:36	10:41	10:46	10:51	10:56	11:01	11:06
III	1	10:06	10:11	10:16	10:21	10:26	10:31	10:36
	2	10:51	10:56	11:01	11:06	11:11	11:16	11:21

Optimistic; Expected-value; Pessimistic

4.3.1 The influence of the range of the disruption end time

Table 4.5 shows the results of the deterministic method at stage 1, including the objective values, the numbers of cancelled services, and the total train delays. Cases II and III have the same result since the range of the disruption times are the same to both cases at stage 1. No matter which case, the optimistic strategy generated the best solution, the pessimistic strategy generated the worst solution, and the expected-value strategy was in between. It is obvious that for the deterministic method the optimal solution considering one disruption duration satisfies the shorter the better.

Table 4.5: Results of the rescheduled timetables by the deterministic method at stage 1

Approach	Case I				Case II or III			
	Predicted end time	Obj [min]	# Cancelled services	Total train delay [min]	Predicted end time	Obj [min]	# Cancelled services	Total train delay [min]
Optimistic	9:51	2,967	26	367	10:06	3,078	28	278
Expected-value	10:06	3,078	28	278	10:21	3,641	32	351
Pessimistic	10:21	3,641	32	441	10:36	3,751	34	351

Table 4.6 shows the results of the stochastic method at stage 1. In each case, 7 rescheduled timetables are obtained, where the services rescheduled up to 9:51 are forced

Table 4.6: Results of the rescheduled timetables by the stochastic method at stage 1

	Case I				Case II or III			
Approach	Predicted end time	Obj [min]	# Cancelled services	Total train delay [min]	Predicted end time	Obj [min]	# Cancelled services	Total train delay [min]
Stochastic	9:51	3,078	28	278	10:06	3,394	30	394
	9:56	3,078	28	278	10:11	3,394	30	394
	10:01	3,078	28	278	10:16	3,399	30	399
	10:06	3,078	28	278	10:21	3,751	34	351
	10:11	3,122	28	322	10:26	3,751	34	351
	10:16	3,192	28	392	10:31	3,751	34	351
	10:21	3,641	32	441	10:36	3,751	34	351

to be the same in case I, and the services rescheduled up to 10:06 are forced to be the same in case II and III. In case I, the first 4 scenarios have the same result, although the corresponding disruption end times are different. The reason is that no further train services were affected when the disruption end time was extended from 9:51 up to 10:06, due to the service pattern of the planned timetable. In this chapter, we use a cyclic planned timetable that has a cycle time of 30 minutes, which is why we observed a similar phenomenon in case II and III that no changes happened to the results when the disruption end time was extended from 10:21 up to 10:36.

At stage 1, the stochastic method generated solutions that were no better than the deterministic method, due to the anticipation towards longer disruptions that was considered. Just because of the anticipation, at later stages when the ranges of the disruption end times are updated, better solutions can be obtained by the stochastic method compared to the deterministic method. The results of both methods at the final stage are shown in Table 4.7, Table 4.8 and Table 4.9 for cases I, II, and III, respectively, including the average performances.

We consider 7 different actual disruption end times, 10:36, 10:41, 10:46, 10:51, 10:56, 11:01, 11:06, in cases I and II that have the same range of the disruption end time at stage 2. As for case III which has a different range of the disruption end time at stage 2, the considered actual disruption end times are: 10:51, 10:56, 11:01, 11:06, 11:11, 11:16, 11:21. Recall that the actual end time t_{end} updated at the final stage K is not smaller than the minimum end time $t_{\text{end}}^{K-1,\min}$ updated at the previous stage. Under such settings of actual end times, the stochastic method obtained the final rescheduled timetables at stage 2, while in most situations the deterministic method needed to recompute new solutions based on the solutions from stage 2 and thus the final stage were stage 3 (see Tables 4.7 to 4.9). In Tables 4.7 to 4.9, also the value of the stochastic solution (*VSS*) is shown, which quantifies the cost of ignoring uncertainty in decision making. It is calculated as *VSS=EEV-RP*, where *EEV* is the expected result of using the expected-value solution and *RP* is the optimal solution of the two-stage stochastic model (Birge and Louveaux, 2011). In our case (a minimization problem), the higher the *VSS* is, the better the stochastic solution will be. The improvement percentages with respect to *VSS* were also calculated, which were between 6.1% and 10.2% in our cases, demonstrating the benefit of the stochastic formulation. The relevant results can be found in Tables 7 to 9.. The relevant results can be found in Tables 4.7 to 4.9.

Table 4.7: Results of the final rescheduled timetables in Case I

Actual end time	Approach	Obj [min]	# Cancelled services	Total train delay [min]	Final stage
10:36	**Stochastic**	4,452	40	451	2
	Optimistic	4,135	38	335	2
	Expected-value	4,135	38	335	3
	Pessimistic	4,452	40	451	3
10:41	**Stochastic**	4,452	40	451	2
	Optimistic	4,180	38	380	3
	Expected-value	4,667	42	467	3
	Pessimistic	4,808	44	408	3
10:46	**Stochastic**	4,457	40	457	2
	Optimistic	4,250	38	450	3
	Expected-value	4,685	42	485	3
	Pessimistic	4,808	44	408	3
10:51	**Stochastic**	4,808	44	408	2
	Optimistic	4,698	42	498	3
	Expected-value	4,698	42	498	2
	Pessimistic	4,808	44	408	3
10:56	**Stochastic**	4,808	44	408	2
	Optimistic	5,193	48	393	3
	Expected-value	5,509	50	509	3
	Pessimistic	4,808	44	408	3
11:01	**Stochastic**	4,808	44	408	2
	Optimistic	5,193	48	393	3
	Expected-value	5,509	50	509	3
	Pessimistic	4,808	44	408	3
11:06	**Stochastic**	4,808	44	408	2
	Optimistic	5,193	48	393	3
	Expected-value	5,509	50	509	3
	Pessimistic	4,808	44	408	2
Average	**Stochastic**	**4,656**	**42**	**428**	–
	Optimistic	4,691	43	406	–
	Expected-value	4,959	45	473	–
	Pessimistic	4,757	43	414	–
VSS	$4{,}959 - 4{,}656 = 303$				
Improvement	$303/4{,}959 = 6.1\%$				

In case I (Table 4.7), the optimistic strategy performed better than the stochastic method when the actual disruption end time was from 10:36 up to 10:51, whereas the stochastic method performed no worse than any deterministic strategy when the actual disruption end time was from 10:56 up to 11:06. On average, the stochastic method is the best, which is slightly better than the optimistic strategy which is the best among all deterministic strategies.

Compared to case I (Table 4.7), in case II (Table 4.8) the stochastic method performed much better than the deterministic method: for each considered actual disruption end time (except 10:36), the stochastic method was better than any deterministic strategy. This is because the ranges of the disruption end times update at stage 1 are different in cases I and II, and thus result in different robust solutions by the stochastic method at stage 1, which further affect the robust solutions at stage 2. The pessimistic strategy resulted in the best solution when the actual end time was 10:36, because it was the optimal solution obtained at stage 1 where 10:36 is the considered disruption end time for the pessimistic strategy (see Table 4.4).

Table 4.8: Results of the final rescheduled timetables in Case II

Actual end time	Approach	Obj [min]	# Cancelled services	Total train delay [min]	Final stage
10:36	**Stochastic**	4,067	36	467	2
	Optimistic	4,135	38	335	2
	Expected-value	4,452	40	452	3
	Pessimistic	3,751	34	351	3
10:41	**Stochastic**	4,067	36	467	2
	Optimistic	4,180	38	380	3
	Expected-value	4,808	44	408	3
	Pessimistic	4,808	44	408	3
10:46	**Stochastic**	4,073	36	473	2
	Optimistic	4,250	38	450	3
	Expected-value	4,808	44	408	3
	Pessimistic	4,808	44	408	3
10:51	**Stochastic**	4,424	40	424	2
	Optimistic	4,698	42	498	3
	Expected-value	4,808	44	408	2
	Pessimistic	4,808	44	408	3
10:56	**Stochastic**	4,424	40	424	2
	Optimistic	5,193	48	393	3
	Expected-value	4,808	44	408	3
	Pessimistic	4,808	44	408	3
11:01	**Stochastic**	4,424	40	424	2
	Optimistic	5,193	48	393	3
	Expected-value	4,808	44	408	3
	Pessimistic	4,808	44	408	3
11:06	**Stochastic**	4,424	40	424	2
	Optimistic	5,193	48	393	3
	Expected-value	4,808	44	408	3
	Pessimistic	4,808	44	408	2
Average	**Stochastic**	**4,272**	**38**	**443**	–
	Optimistic	4,691	43	406	–
	Expected-value	4,757	43	415	–
	Pessimistic	4,657	43	400	–
VSS	$4{,}757 - 4{,}272 = 485$				
Improvement	$485/4{,}757 = 10.2\%$				

The stochastic method also performed much better than any deterministic strategy for each considered actual disruption end time in case III (Table 4.9), which has the same range of the disruption end time at stage 1 as in case II. The average performance of the stochastic method in case III is even better than the one in case I (Table 4.7), although case III considers longer actual disruption end times. The reason is related to the robust solution obtained at stage 1, which is affected by the corresponding range of the disruption end time. In case III (Table 4.9) the result of the stochastic method is all the same when the actual end time is 10:51 up to 11:06, and the result of any deterministic strategy is all the same when the actual end time is 10:56 up to 11:06. These also happen in case I (Table 4.7) or case II (Table 4.8). The reason is that no further train services were affected when the disruption end time was extended from 10:51 up to 11:06 for the stochastic method, or from 10:56 up to 11:06 for the deterministic method. Recall that this is due to to the service pattern of the timetable.

Table 4.9: Results of the final rescheduled timetables in Case III

Actual end time	Approach	Obj [min]	# Cancelled services	Total train delay [min]	Final stage
10:51	**Stochastic**	4,424	40	424	2
	Optimistic	4,698	42	498	2
	Expected-value	4,808	44	408	3
	Pessimistic	4,808	44	408	3
10:56	**Stochastic**	4,424	40	424	2
	Optimistic	5,509	50	509	3
	Expected-value	4,808	44	408	3
	Pessimistic	4,808	44	408	3
11:01	**Stochastic**	4,424	40	424	2
	Optimistic	5,509	50	509	3
	Expected-value	4,808	44	408	3
	Pessimistic	4,808	44	408	3
11:06	**Stochastic**	4,424	40	424	2
	Optimistic	5,509	50	509	3
	Expected-value	4,808	44	408	2
	Pessimistic	4,808	44	408	3
11:11	**Stochastic**	4,469	40	469	2
	Optimistic	5,509	50	509	3
	Expected-value	4,853	44	453	3
	Pessimistic	5,340	48	540	3
11:16	**Stochastic**	4,539	40	539	2
	Optimistic	5,514	50	514	3
	Expected-value	4,923	44	523	3
	Pessimistic	5,358	48	558	3
11:21	**Stochastic**	4,987	44	587	2
	Optimistic	5,866	54	466	3
	Expected-value	5,371	48	571	3
	Pessimistic	5,371	48	571	2
Average	**Stochastic**	**4,527**	**41**	**470**	–
	Optimistic	5,445	49	502	–
	Expected-value	4,912	45	454	–
	Pessimistic	5,043	46	472	–
VSS	$4{,}912-4{,}527=385$				
Improvement	$385/4{,}912=7.8\%$				

Tables 4.7 to 4.9 indicate that compared to the deterministic method, the stochastic method is more likely to generate better rescheduling solutions for uncertain disruptions by less cancelled train services and/or train delays. This is mainly because the

stochastic method generates solutions that are flexible to the short-turning patterns under different disruption durations. We explain this by the example of the actual disruption end time of 10:36 in case II as follows.

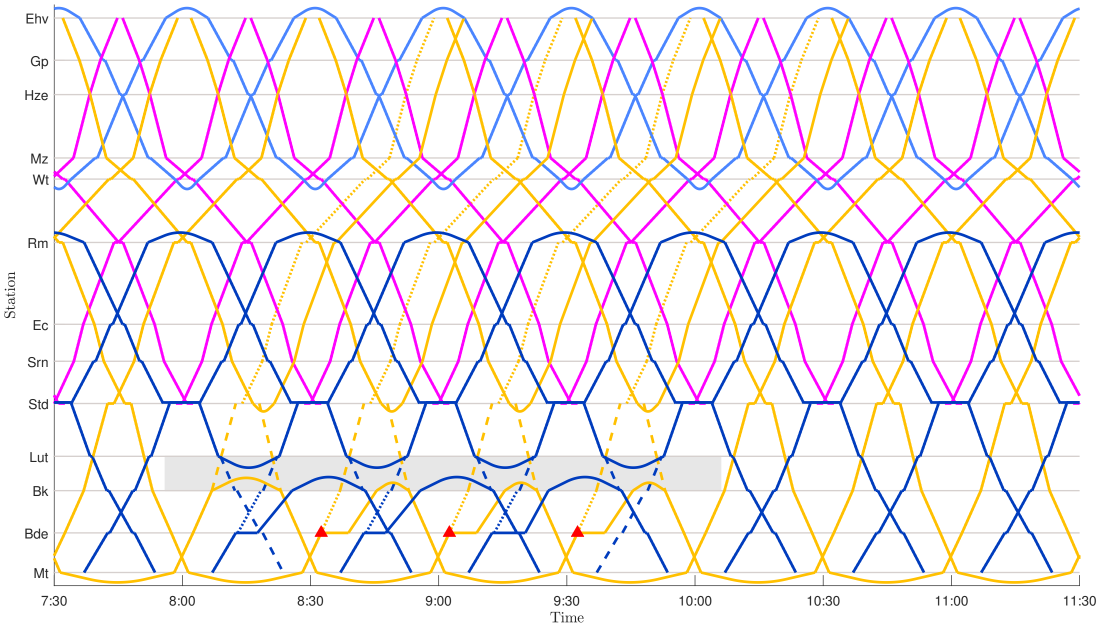

Figure 4.7: The rescheduled timetable by the optimistic strategy at stage 1 in case II (disruption end time: 10:06)

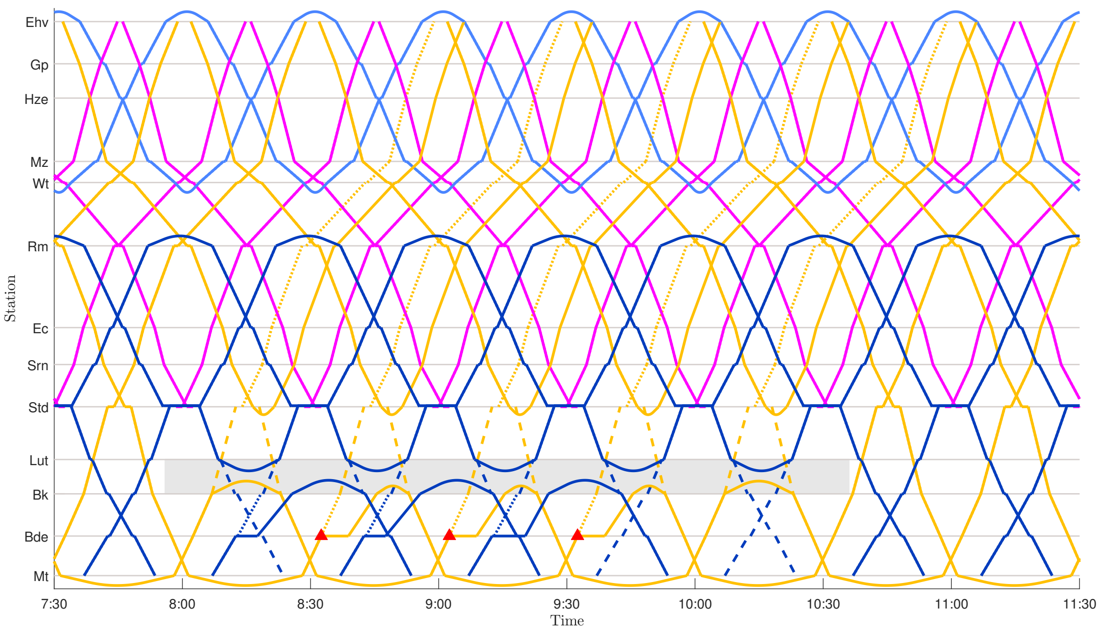

Figure 4.8: The rescheduled timetable by the optimistic strategy at stage 2 in case II (disruption end time: 10:36)

Figures 4.7 and 4.8 show the time-distance diagrams of the rescheduled timetables obtained by the deterministic method for the optimistic strategy at stages 1 and 2 in case

II, respectively. The dashed (dotted) lines represent the original scheduled services that are cancelled (delayed) in the rescheduled timetables, while the solid lines represent the services scheduled in the rescheduled timetables. The red triangles indicate extra stops. Compared to stage 1 (Figure 4.7), more services were cancelled at stage 2 (Figure 4.8) due to the extended disruption. At stage 1, the operation of a dark blue train from stations Mt to Bk is cancelled (Figure 4.7), which is why the operation of another dark blue train from stations Bk to Mt has to be cancelled at stage 2 (Figure 4.8) to keep consistent control decisions.

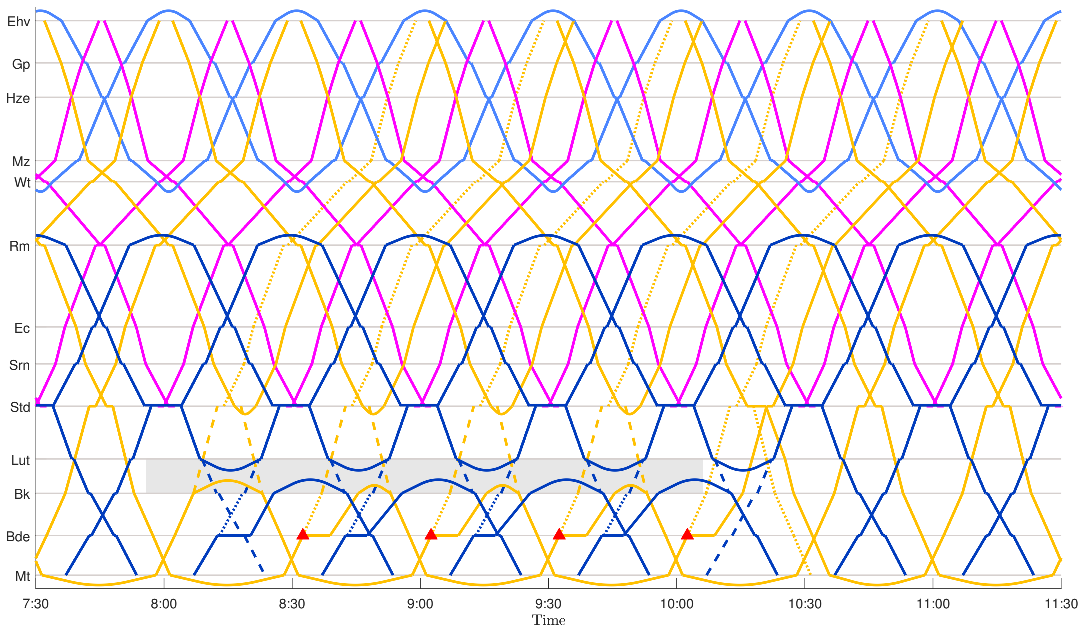

Figure 4.9: The rescheduled timetable by the stochastic approach at stage 1 in case II (disruption end time: 10:06)

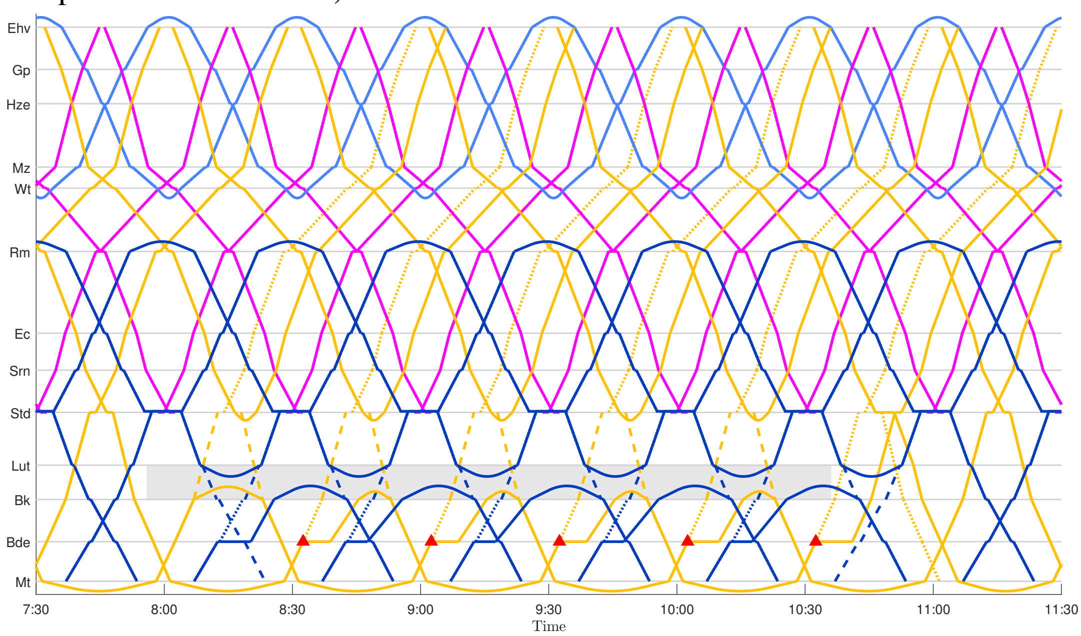

Figure 4.10: The rescheduled timetable by the stochastic approach at stage 2 in case II (disruption end time: 10:36)

Figures 4.9 and 4.10 show the time-distance diagrams of the rescheduled timetables obtained by the stochastic method at stages 1 and 2 in case II, respectively. Compared to the solution of the optimistic strategy at stage 1 (Figure 4.7), more services were cancelled/delayed in the solution of the stochastic method at stage 1 (Figure 4.9) due to the anticipation towards longer disruption durations in consideration. Just because of the anticipation, at stage 2, the solution of the stochastic approach resulted in less cancelled services and train delays, compared to the solution of the optimistic strategy (Figure 4.10).

It is found that the robustness of the solution by the stochastic method can be affected by the range of the disruption end time update. An example is given as follows. Figures 4.11 and 4.12 show the time-distance diagrams of the rescheduled timetables obtained by the stochastic method at stages 1 and 2 in case I, respectively. Recall that cases I and II have different ranges of the disruption end times at stage 1, but the same range of the disruption end times at stage 2 (see Table 4.4).

At stage 1, compared to the solution of case II (Figure 4.9) that considered the end time range of [10:06,10:36], the solution of case I (Figure 4.11) resulted in less cancelled services and train delays due to an earlier end time range of [9:51,10:21] considered. In case II (Figure 4.9) the cancelled operation of a dark blue train from stations Mt to Bk was after the minimum end time of stage 1, 10:01, and thus this cancellation decision was a look-ahead decision at stage 1, which did not need to be respected at stage 2 (see Figure 4.10); while in case I (Figure 4.11) the cancelled operation of a dark blue train from stations Mt to Bk was before the minimum end time of stage 1, 9:51, and thus this cancellation decision was a control decision at stage 1, which had to be respected at stage 2 (see Figure 4.12) causing the operation of another dark blue train from stations Bk to Mt cancelled at stage 2.

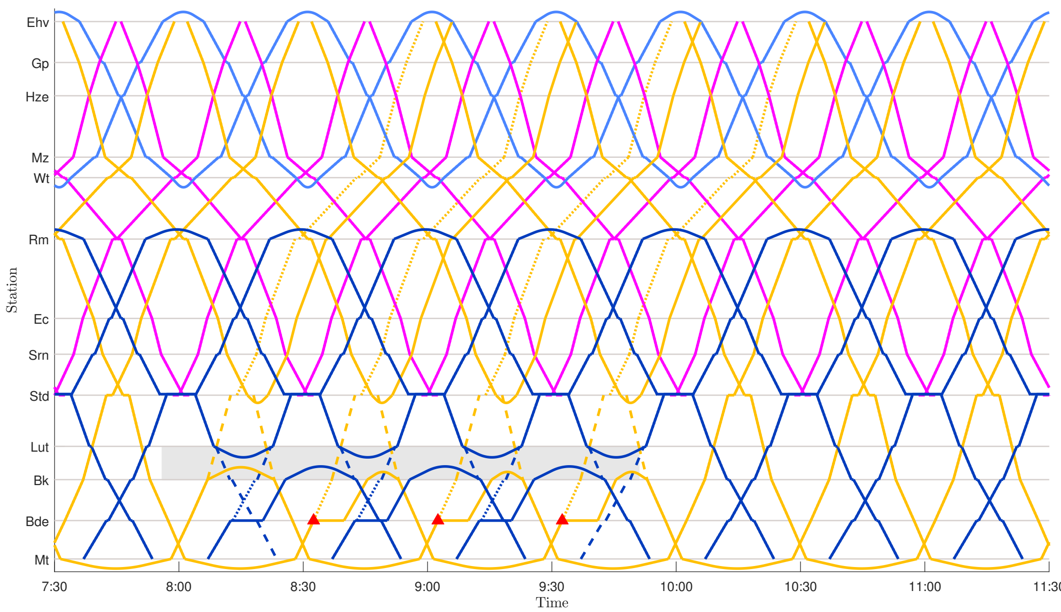

Figure 4.11: The rescheduled timetable by the stochastic approach at stage 1 in case I (disruption end time: 9:51)

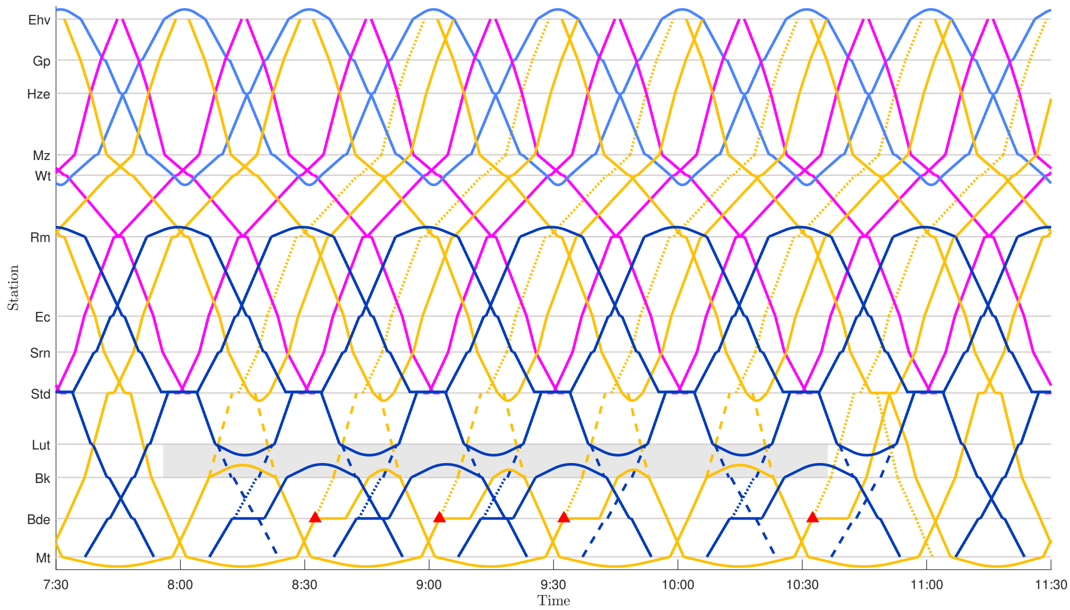

Figure 4.12: The rescheduled timetable by the stochastic approach at stage 2 in case I (disruption end time: 10:36)

This shows that the range of the disruption end time affects the flexibility of a solution, which is relevant to short-turning patterns. Smooth short-turning patterns for possible longer disruptions like in case II (Figures 4.9 and 4.10) help to reduce cancelled train services. Case II has an later range of the disruption end time at stage 1 than case I, while both cases have the same range of the disruption end time at stage 2. In that sense, compared to case I, case II considers that longer disruption durations are more likely to happen at stage 1, which turns to be true due to another range update at stage 2. From the results of both cases, we infer that in the situations where longer disruption durations are more likely to happen, short-turning the last train services approaching to the predicted minimum disruption end time (e.g. Figure 4.9 corresponding to case II) rather than cancelling them (e.g. Figure 4.11 corresponding to case I) might be helpful to improve solution flexibility.

4.3.2 Computation analysis

Table 4.10: Computation times [sec] at each update stage

Approach	Case I		Case II		Case III	
	Stage 1	Stage 2	Stage 1	Stage 2	Stage 1	Stage 2
Stochastic	234	66	244	51	244	51
Optimistic	10	3	9	3	9	3
Expected-value	10	3	11	3	11	3
Pessimistic	11	3	10	2	11	3

Table 4.10 shows the computation times for the stochastic method and the deterministic method for different strategies at stage 1 and 2 for all cases. In each case, the computa-

tion time of each approach to stage 1 is longer than the one to stage 2. This is because at a later stage only the dispatching decisions for the new control and look-ahead horizons (for the extended duration) need to be made. The deterministic method for each strategy costs much less computation time than the stochastic method, as it considers a single disruption scenario at each computation. Although the stochastic method is relatively time-consuming, the rescheduling solutions are robust to uncertain disruption durations. Table 4.11 shows the numbers of variables, binary variables and constraints required respectively by the stochastic method and the deterministic method using a pessimistic strategy. We only show the pessimistic strategy in Table 4.11, because it needs more variables and constraints compared to the optimistic or expected-value strategy due to longer disruption duration considered. Because the stochastic method handled 7 scenarios at a stage, the required variables and constraints (see Table 4.11) were longer than the ones of the deterministic method using a pessimistic strategy, which handled only 1 scenario at a stage.

Among all cases, the longest computation time of a stochastic solution was around 4 min. This shows the applicability of applying the proposed stochastic approach assuming that the range of the disruption end time prediction update is provided at least 10 min before the current minimal end time prediction ($\ell = 10$ min).

4.4 Conclusions

This chapter proposed a rolling horizon two-stage stochastic timetable rescheduling model to manage uncertain disruptions with better solutions. It was tested on a part of the Dutch railways and compared to a deterministic rolling horizon timetable rescheduling model. The results showed that compared to the deterministic method, the stochastic method is more likely to generate better rescheduling solutions for uncertain disruptions by less train cancellations and/or delays, due to the flexibility towards the short-turning patterns under different disruption durations. The flexibility of a solution by the stochastic method can be impacted by the range of the disruption end time. From the results we infer that in the situations where longer disruption durations are more likely to happen, short-turning the last train services approaching to the predicted minimum disruption end time rather than cancelling them might be helpful to improve solution flexibility. This will be examined in near future. The stochastic programming model considers several scenarios simultaneously, is therefore larger and thus takes longer computation time. The computation time might be reduced without affecting the solution quality by optimizing the number of scenarios, the size of the network, the length of the look-ahead horizon, or exploiting the periodic structure of the (rescheduled) timetable.

This chapter used a discrete uniform distribution over the range of the estimated disruption end to define scenarios with the same occurrence probabilities. From the case study results we found that although some scenarios had different disruption durations

Table 4.11: The problem sizes of the stochastic and deterministic models

Approach	Indicators	Case I		Case II		Case III	
		Stage 1	Stage 2	Stage 1	Stage 2	Stage 1	Stage 2
Stochastic	# variables	476,105	476,994	476,378	476,994	476,378	477,337
	# binary variables	423,017	423,906	423,290	423,906	423,290	424,249
	# constraints	2,200,666	2,323,312	2,229,771	2,325,510	2,229,771	2,361,890
Pessimistic	# variables	68,015	68,142	68,054	68,142	68,054	68,191
	# binary variables	60,431	60,558	60,470	60,558	60,470	60,607
	# constraints	293,467	306,538	296,046	306,853	296,046	310,299

the rescheduling solutions to these scenarios were the same. The scenario estimation method can be improved by identifying various different scenarios with essentially different outcomes to find a rescheduling solution. As we rely on a periodic planned timetable there should be a finite number of discrete scenarios that lead to essentially different outcomes. It is beneficial to identify these representative scenarios, of which the probabilities can be assigned based on the relative sub-range that they would occur. This will be part of future work.

Chapter 5

Dynamic railway timetable rescheduling for multiple connected disruptions

Apart from minor updates, this chapter has been submitted as:

Zhu, Y., Goverde, R.M.P., 2019. Dynamic railway timetable rescheduling for multiple connected disruptions. *Transportation Research Part C: Emerging Technologies*, *under review*.

5.1 Introduction

Railways play a significant role in passenger transportation. For example, there are approximately 1.3 million trips by train every day in the Netherlands (NS, 2018). Thus, reliable train services are important. However, railway operations are often disturbed by unexpected events like extreme weather, accidents and infrastructure failures, which are getting worse in recent years. On the Dutch railways, the number of unplanned disruptions occurring each year was 1846 in 2011 and increased to 4085 in 2017 (RijdendeTreinen, 2018). Such disruptions usually last for a few hours, causing considerable negative impact on passengers and imposing much extra workload on personnel. In practice, disruptions are still handled manually by traffic controllers who make dispatching decisions (e.g. cancelling, delaying and short-turning) with predesigned contingency plans as guidelines, while each contingency plan corresponds to one disruption at a specific location. When disruptions occur simultaneously at different locations, the contingency plans corresponding to them may conflict with each other. Under these circumstances, traffic controllers have to adjust the timetable based on their own experiences without any guidelines, which leads to time-consuming and suboptimal solutions (Ghaemi et al., 2017b). Therefore, it is necessary to develop a more efficient way of handling multiple disruptions, which has not been dealt with in the literature so far.

In this chapter, we are concerned with unplanned disruptions that cause complete track blockages between stations for hours. Our focus is on rescheduling the timetable in case of multiple complete track blockages where each is connected to another by at least one train line. In these cases, train services should be adapted to multiple time-space disruption windows that are located in different locations and may start/end at different time instants. The main challenge is that the service adjustments towards one disruption window may influence the one towards another disruption window, and vice versa.

To solve this challenge, we put forward a multiple-disruption rescheduling model based on the single-disruption rescheduling model of Chapter 3. The single-disruption rescheduling model applies delaying, reodering, cancelling, flexible short-turning and flexible stopping, and considers station capacity as well as trains turning at terminal stations. Short-turning means that a train ends its operation at a station before the blocked tracks and the corresponding rolling stock turns to operate another train in the opposite direction. Flexible short-turning means that for each train a full choice of short-turn station candidates is given, and the model decides the optimal station and time of short-turning the train. Flexible stopping means that for each train the scheduled stops can be skipped and extra stops can be added. Except skipping stops, the other characteristics are all kept in the multiple-disruption rescheduling model that aims to minimize service cancellations and deviations from the planned timetable.

To deal with multiple connected disruptions in a dynamic environment, two approaches are proposed, which are a sequential approach based on a single-disruption rescheduling model, and a combined approach based on a multiple-disruption rescheduling model.

The contributions of this chapter are summarized as follows:

- We develop a multiple-disruption timetable rescheduling model for multiple complete track blockages that are pairwise connected by at least one train line.
- We propose two approaches, the sequential approach and the combined approach, to deal with multiple connected disruptions in a dynamic environment.
- We propose a new rolling horizon solution method to generate high-quality solutions for long multiple connected disruptions in an acceptable time.
- The sequential and combined approaches are tested on real-life instances on a subnetwork of the Dutch railways.
- It is shown that the combined approach is able to handle more kinds of multiple disruption scenarios and generate better solutions than the sequential approach.

The remainder of this chapter is organized as follows. Section 5.2 gives a literature review on timetable rescheduling models for railway disruptions. Section 5.3 introduces the sequential approach and the combined approach, and the differences between the two rescheduling models used in these approaches, the single-disruption model and the multiple-disruption model. Section 5.4 gives the detailed mathematical formulation of the multiple-disruption model. In Section 5.6, a case study is carried out to explore the performance of the sequential or combined approach. Finally, Section 5.7 concludes the chapter.

5.2 Literature review

A typical consequence of a disruption is that the tracks between two stations are partially or completely blocked. In case of partial blockages, some trains can still use the remainder track as in Zhan et al. (2016) where a partial blockage is considered in a double-track railway line. In case of complete blockages, the consequence becomes more serious that no trains can run through the blocked area at all during the disruption period. This problem has been dealt with more widely in the literature compared to partial blockages, see Meng and Zhou (2011), Narayanaswami and Rangaraj (2013), Zhan et al. (2015), Ghaemi et al. (2017a), Ghaemi et al. (2018a) and Chapter 3. There are also models focusing on both partial and complete blockages, including Cadarso et al. (2013), Louwerse and Huisman (2014), Veelenturf et al. (2015) and Binder et al. (2017b).

Different dispatching measures are used to reschedule the timetable during disruptions. Meng and Zhou (2011) allow retiming trains, while Narayanaswami and Rangaraj (2013) allow both retiming and reordering trains. In both papers, the considered disruption durations are at most 1 hour. For longer disruptions that last for a few hours, cancelling trains is necessary, because it helps to reduce train delays that may propagate to the network beyond the disrupted area. Zhan et al. (2015) use retiming, reordering and cancelling trains focusing on Chinese railways where seat reservations are needed. Under this circumstance, short-turning trains is not applied in their models, which however is a common strategy used in the systems without seat reservations, e.g., metro systems and some European railway systems. The models allowing short-turning trains include Louwerse and Huisman (2014), Veelenturf et al. (2015), Ghaemi et al. (2017a), Ghaemi et al. (2018a) and Chapter 3. In general, the last stop of a train before the blocked tracks is fixed as the station where the train can short-turn, as in Louwerse and Huisman (2014) and Veelenturf et al. (2015). However, a train may be completely cancelled rather than short-turned, if the short-turn station lacks capacity. To reduce the possibility of a train being completely cancelled, both Ghaemi et al. (2017a) and Ghaemi et al. (2018a) allow a train to short-turn at either of the last two stations before the blocked tracks, while Chapter 3 introduces more flexibility by allowing a train to short-turn at one of all possible short-turn stations that are before the blocked tracks. Another way of reducing cancelled trains is rerouting trains. The trains that originally plan to run through the blocked tracks can be rerouted through another corridors to reach the destinations, while part/all of the intermediate stations in the planned paths may change. This strategy is applied in both Veelenturf et al. (2015) and Binder et al. (2017b). When passengers are taken into account, particular strategies are used to mitigate the negative impact on passengers, such as adding additional trains, adding extra stops and skipping stops. Both Cadarso et al. (2013) and Binder et al. (2017b) allow adding additional trains. Veelenturf et al. (2017) allow adding stops, while Chapter 3 allows both adding stops and skipping stops.

Most papers assume that the disruption duration is known at the beginning of the disruption and will not change over time. However in practice, a disruption could either end earlier or extend than expected (Zilko et al., 2016). A few papers deal with uncertain disruptions. Zhan et al. (2016) propose a rolling horizon framework where the timetable is rescheduled gradually with renewed disruption durations taken into account. Meng and Zhou (2011) propose a stochastic programming model that takes the uncertainty of the disruption duration into account. The model reschedules the timetable dynamically by a rolling horizon approach.

In the real world, multiple disruptions occur on a daily basis, while how to deal with them is rarely considered in the existing literature. Veelenturf et al. (2015) proposed a model said to be applicable to multiple track blockages, but no results were given. Van Aken et al. (2017a) designed alternative timetables for handling multiple planned disruptions (i.e. infrastructure maintenance possessions). They focus on periodic timetables for full-day possessions, and as such do not consider the transitions between the original timetable and the rescheduled timetable, and vice versa. For shorter disruptions, such transitions have to be taken into account. According to Ghaemi et al. (2017b), a disruption consists of three phases: the transition phase from the planned timetable to the disruption timetable, the stable phase where the disruption timetable is implemented, and the recovery phase from the disruption timetable to the planned timetable. Veelenturf et al. (2015) and Chapter 3 consider all phases of one single disruption.

This chapter proposes a Mixed Integer Linear Programming (MILP) model to reschedule the timetable in case of multiple disruptions that are pairwise connected by at least one train line. The multiple-disruption rescheduling model considers all phases of each disruption that causes complete track blockage for hours. Retiming, reordering, cancelling, adding stops and flexible short-turning are all formulated into the model that also takes into account rolling stock circulations and station capacity. Two approaches are developed to deal with multiple disruptions in a dynamic environment. A sequential approach applies a single-disruption rescheduling model to handle each new disruption with the previous rescheduled timetable as the reference. A combined approach applies a multiple-disruption rescheduling model to handle each new disruption considering the combined effects of all ongoing disruptions.

5.3 Problem description

In this chapter, multiple connected disruptions are defined as two or more disruptions that

- have overlapping periods,
- occur at different geographic locations,
- may start/end at different time instants, and
- are pairwise connected by at least one train line.

Two disruptions having overlapping periods means that a disruption occurs when another disruption is ongoing. To be more specific, suppose a disruption i starts at time t^i_{start} and will end at t^i_{end} ($t^i_{\text{start}} < t^i_{\text{end}}$), and another disruption j starts at time t^j_{start} and will end at t^j_{end} ($t^j_{\text{start}} < t^j_{\text{end}}$). Then, the durations of these two disruptions are overlapping, if $t^i_{\text{start}} \leq t^j_{\text{start}} < t^i_{\text{end}}$ or $t^j_{\text{start}} \leq t^i_{\text{start}} < t^j_{\text{end}}$. It is possible that the disruption periods of two disruptions are *not* overlapping, but a train is influenced by a first disruption, and then later will be affected by a second disruption that started after the other disruption already ended. In this situation, during the first disruption it is unknown that there will be a second disruption occurring later. Therefore, these two disruptions can only be seen as two separate disruptions, and they can still be handled by either the sequential approach or the combined approach proposed in this chapter. We require overlapping periods as one of the criteria to define multiple connected disruptions of which the combined effects can be actually taken into account during timetable rescheduling.

Figure 5.1 illustrates different kinds of multiple disruptions.

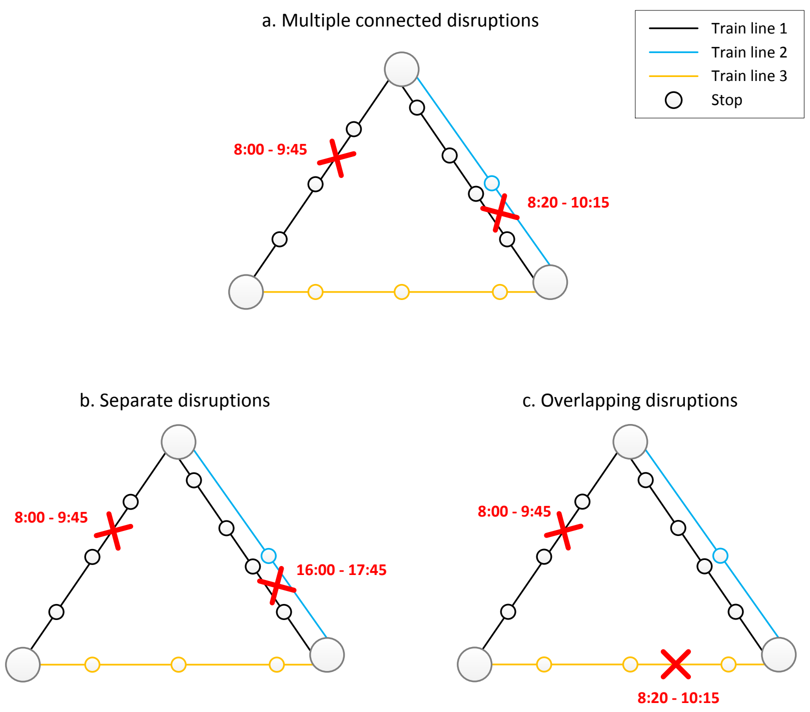

Figure 5.1: Examples of multiple disruptions

In each case of Figure 5.1, three train lines are operated in a triangle network where the stops served by each train line are indicated by circles. In case a, the first disruption occurs between 8:00 and 9:45 and affects train line 1, while the second disruption occurs between 8:20 and 10:15 at a different location and affects both train line 1 and train line 2. These two disruptions occur at different locations, have overlapping period, and are connected by train line 1, which are thus regarded as multiple connected disruptions. If the second disruption occurs between 16:00 and 17:45 as in case b, then these two disruptions are separate disruptions, since they do not have overlapping period. If the second disruption occurs at a different location as in case c, although these two disruptions have an overlapping period, they are not regarded as multiple connected disruptions, because they are not connected by any train line. Compared to case a, cases b and c are more easily handled using the method of Chapter 3, because there are no/few interactions among the timetable adjustments towards each disruption.

In this chapter, our focus is on handling multiple connected disruptions. To this end, two approaches are proposed. One is the sequential approach that uses the single-disruption rescheduling model to solve each disruption sequentially. Another is the combined approach that applies the multiple-disruption model to handle each extra disruption with all ongoing disruptions taken into account. The introductions to these two approaches are given as follows.

5.3.1 The sequential approach

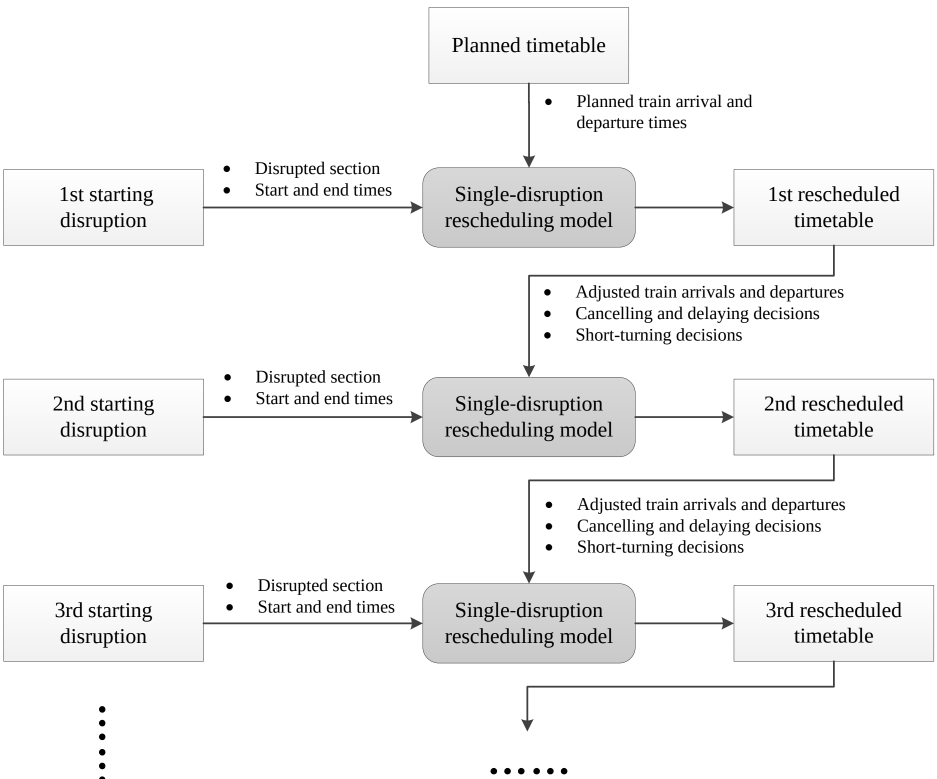

Figure 5.2: Dealing with multiple connected disruptions by the sequential approach

The schematic layout of the sequential approach is shown in Figure 5.2, where the single-disruption rescheduling model is applied every time a new disruption emerges. This can be considered as the straightforward extension to multiple disruptions that traffic controllers would apply to keep the complexity manageable for manual decision making. As this model can deal with one disruption at a time only, it uses the previous solution as reference when handling the disruption that starts later. This means that 1) the train services that are previously decided to be cancelled will remain cancelled; 2) the train departures/arrivals that are previously decided to be delayed can no longer occur before those time instants, as early departures/arrivals are not allowed; and 3) the short-turnings of the trains that do not run through the new track blockage will remain.

5.3.2 The combined approach

The schematic layout of the combined approach is shown in Figure 5.3. Here, the single-disruption rescheduling model is applied for the 1st disruption only and the multiple-disruption rescheduling model is applied every time an extra disruption emerges. When handling later disruptions, the multiple-disruption rescheduling model makes service adjustments by taking all ongoing disruptions into account and respecting the train arrivals and departures that have already been realized according to the previous rescheduled timetable up to the starting time of the emerging disruption.

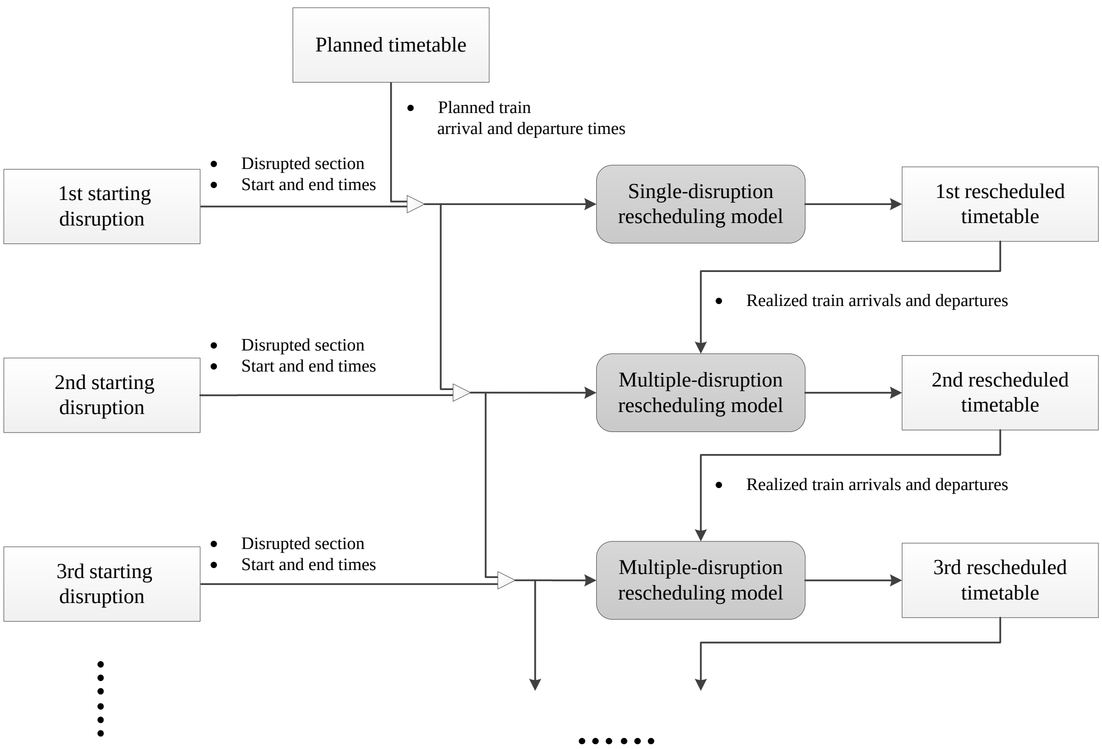

Figure 5.3: Dealing with multiple connected disruptions by the combined approach

In this chapter, the sequential approach is based on the single-disruption rescheduling model of Chapter 3 by removing the measure of skipping stops and replacing the objective with the one of the multiple-disruption model that is introduced in Section 5.4.

5.3.3 Differences between the single-disruption model and the multiple-disruption model

Compared to the single-disruption rescheduling model, the multiple-disruption rescheduling model additionally considers the interactions among the dispatching decisions towards different disruptions. These interactions mainly occur among short-turning decisions. Recall that short-turning means that a train ends its operation at a station before the blocked tracks and the corresponding rolling stock turns to be used by another train in the opposite direction. With the following example, we explain the differences between the single-disruption model and the multiple-disruption model.

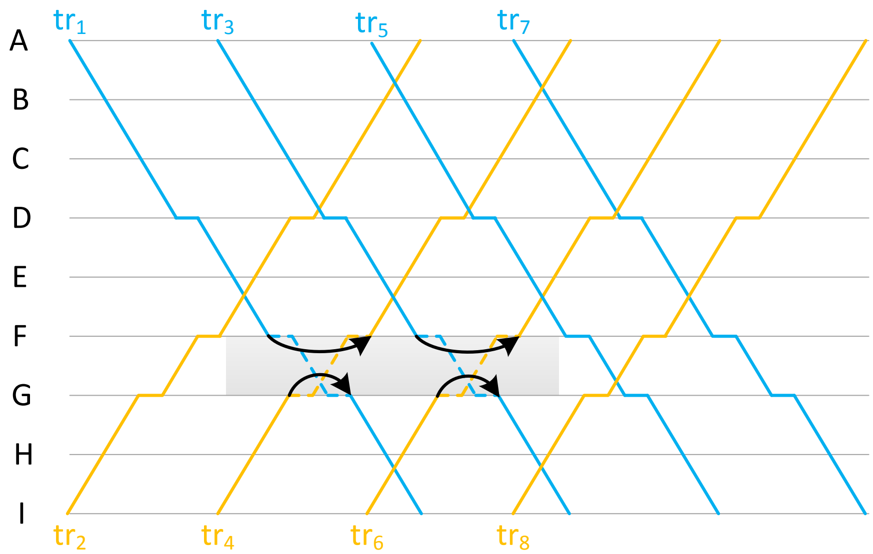

Figure 5.4: Example of single-disruption rescheduling solution

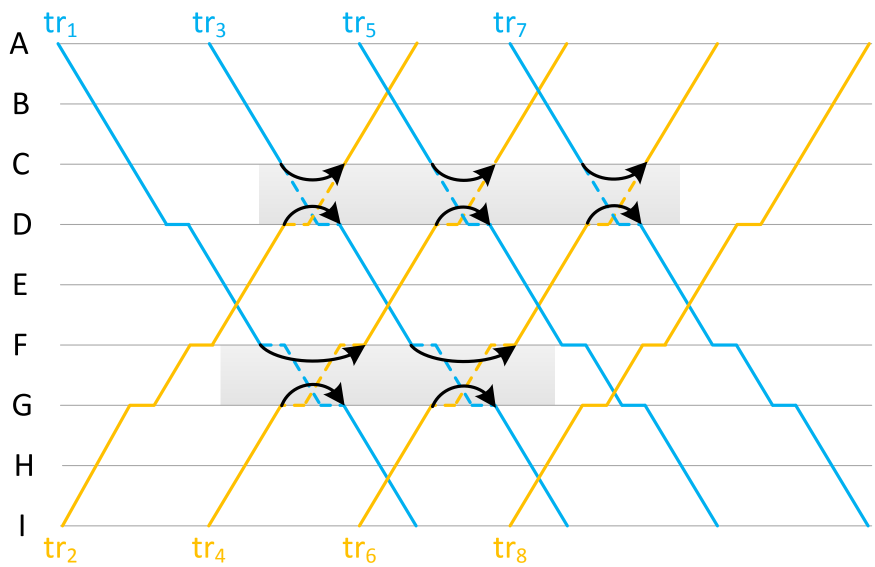

Figure 5.5: Example of multiple-disruption rescheduling solution

In Figure 5.4, four blue trains, $\mathrm{tr}_1, \mathrm{tr}_3, \mathrm{tr}_5$, and tr_7, operate from station A to station I; and four yellow trains, $\mathrm{tr}_2, \mathrm{tr}_4, \mathrm{tr}_6$, and tr_8, operate from station I to station A. Between stations F and G, a disruption occurs for a certain period, which is illustrated by a grey rectangle. Due to the disruption, two blue trains, tr_1 and tr_3, are short-turned at station F to take over the operations of two yellow trains, tr_4 and tr_6, from station F to station A; while these yellow trains are short-turned at station G to take over the operations of these blue trains from station G to station I.

Suppose a little bit later another disruption occurs between stations C and D as Figure 5.5 shows. Then more short-turnings will happen even to the same train, and some short-turnings are interdependent. For example, train tr_3 is now short-turned at both stations C and F. Moreover, the short-turning between trains tr_2 and tr_3 at station D enables the short-turning between trains tr_3 and tr_6 at station F, which in turn enables the short-turning between trains tr_6 and tr_7 at station D. This indicates that during such multiple connected disruptions, a train might be short-turned at a station at each side of *each* disrupted section, and the short-turning at one station may affect the short-turning at another station. These are not considered in the single-disruption model, but should be formulated in the multiple-disruption model.

5.4 The multiple-disruption rescheduling model

5.4.1 Definitions

The multiple-disruption rescheduling model is based on an event-activity network formulated by the method introduced in Chapter 3. Train departures (arrivals) are formulated as departure (arrival) *events*, which are contained in the set E_{de} (E_{ar}). Each event $e \in E_{\text{de}} \cup E_{\text{ar}}$ is associated with the original scheduled time o_e, station st_e, train line tl_e, train number tr_e, and operation direction dr_e. A train line indicates the origin, the destination, all intermediate stops between the origin and the destination, and the operation frequency (e.g. every 30 minutes).

Directed arcs between events are called *activities*. Running activities $(e,e') \in A_{\text{run}}$ describe train running between adjacent stations:

$$A_{\text{run}} = \left\{ \left(e,e'\right) \in E_{\text{de}} \times E_{\text{ar}} : tr_e = tr_{e'}, \text{ and } tr_e \text{ goes directly from station } st_e \text{ to } st_{e'} \right\}.$$

Dwell (pass-through) activities $(e,e') \in A_{\text{dwell}}$ (A_{pass}) describe trains dwelling at (passing through) stations:

$$\begin{aligned} A_{\text{dwell}} &= \left\{ \left(e,e'\right) \in E_{\text{ar}} \times E_{\text{de}} : tr_e = tr_{e'}, st_e = st_{e'}, \text{ and } o_e < o_{e'} \right\}, \\ A_{\text{pass}} &= \left\{ \left(e,e'\right) \in E_{\text{ar}} \times E_{\text{de}} : tr_e = tr_{e'}, st_e = st_{e'}, \text{ and } o_e = o_{e'} \right\}. \end{aligned}$$

Short-turn activities $(e,e') \in A_{\text{turn}}$ describe trains turning at stations before blocked tracks to operate the trains in the opposite directions from the same train line:

$$\begin{aligned} A_{\text{turn}} = \{ a = (e,e') \in E_{\text{ar}}^{\text{turn}} \times E_{\text{de}}^{\text{turn}} : tl_e = tl_{e'}, tr_e \neq tr_{e'}, dr_e \neq dr_{e'}, st_e = st_{e'}, \\ \text{and } o_{e'} + D - o_e \geq L_a \} \end{aligned} \tag{5.1}$$

in which the arrival (departure) events are defined by the set $E_{\text{ar}}^{\text{turn}}$ ($E_{\text{de}}^{\text{turn}}$), D is the maximum delay allowed to an event, and L_a represents the minimum duration required for a short-turn activity a. We allow a short-turn activity a to be created from an arrival event e to a departure event e' that was originally planned to occur earlier than e considering that the rescheduled time of e' may be later than the rescheduled time of e so that the short-turning between them could be possible.

We also use the original OD turn activities A_{odturn} to describe trains turning at the destinations to the opposite trains from the same train line, and headway activities A_{head} to describe the headways of following or crossing trains. Chapter 3 already describes the sets A_{odturn}, A_{head}, and the constraints or decision variables corresponding to them (i.e. the constraints or decision variables about OD turns and reordering trains), which are used also in the multiple-disruption model exactly the same. Hence, for details we refer to Chapter 3. For the multiple-disruption model, we introduce constraints of cancelling, delaying, flexible short-turning trains and station capacity, as well as the decision variables used in these constraints. Recall that flexible short-turning means that each train is given a full choice of short-turning station candidates, and the model decides the optimal station and time of short-turning the train. In particular, a train may short-turn a station earlier if the capacity of a later short-turning station is insufficient.

We will use the following decision variables:

- x_e : continuous variable deciding the rescheduled time of event e,
- d_e : continuous variable deciding the delay of event e,
- c_e : binary variable with value 1 indicating that e is cancelled, and 0 otherwise,
- y_e : binary variable with value 1 indicating that station st_e is a short-turn station of train tr_e, and 0 otherwise,
- m_a : binary variable with value 1 indicating that a short-turn activity $a \in A_{\text{turn}}$ is selected, and 0 otherwise.
- $u_{e,i}$: binary variable with value 1 indicating that train tr_e occupies the ith platform of station $st_e, e \in E_{\text{ar}}$, and 0 otherwise.
- $v_{e,j}$: binary variable with value 1 indicating that train tr_e occupies the jth pass-through track of station $st_e, e \in E_{\text{ar}}$, and 0 otherwise.

For the notation of parameters and sets we refer to Appendix 5.A.

5.4.2 Objective

The objective is minimizing train service cancellations and deviations from the planned timetable,

$$\text{minimize} \sum_{e \in E_{\text{ar}}} wc_e + \sum_{e \in E_{\text{ar}} \cup E_{\text{de}}} d_e, \tag{5.2}$$

where E_{ar} (E_{de}) is the set of arrival (departure) events, and w is a fixed penalty for each cancelled service. A service refers to a train run between two adjacent stations. This objective aims to minimize the impact of the disruption to the rest of the network.

5.4.3 Constraints

5.4.3.1 Cancelling and delaying trains

For each train departure or arrival event e, the rescheduled time x_e is relevant to the delaying decision d_e and the cancelling decision c_e as follows:

$$M_1 c_e \le x_e - o_e \le M_1, \qquad e \in E_{\text{ar}} \cup E_{\text{de}}, \tag{5.3}$$

$$x_e - o_e = d_e + M_1 c_e, \qquad e \in E_{\text{ar}} \cup E_{\text{de}}, \tag{5.4}$$

$$d_e \ge 0, \qquad e \in E_{\text{ar}} \cup E_{\text{de}}, \tag{5.5}$$

$$d_e \le D, \qquad e \in (E_{\text{ar}} \cup E_{\text{de}}) \backslash E^{\text{NMdelay}}. \tag{5.6}$$

Constraint (5.3) states that the rescheduled time of a cancelled event e (i.e. $c_e = 1$) is the original scheduled time o_e plus M_1 that is set to 1440 minutes here. Constraint (5.4) states that the rescheduled time of an non-cancelled event is the original scheduled time plus the delayed time. Constraints (5.5) and (5.6) require that the delay of an event is non-negative, and should be no larger than D minutes if the event does not belong to E^{NMdelay}. The set E^{NMdelay} contains all events that are not imposed with the upper delay limit. These events correspond to the trains that have already started from the origins before a disruption starts. These trains can no longer be cancelled and short-turning them could also be impossible due to rolling stock or station capacity shortage, and thus they have to dwell at the last possible stations before the blocked tracks until the disruption ends.

5.4.3.2 Avoiding trains entering any disrupted section

Suppose the current emerging disruption is the nth disruption ($n \ge 2$), then trains are forbidden to enter the blocked tracks due to any disruption i ($1 \le i \le n$) during the corresponding disruption period that starts at time t^i_{start} and ends at time t^i_{end}:

$$x_e \ge t^i_{\text{end}}(1 - c_e), \qquad e \in E_{\text{de}}, st_e = st^{i,tr_e}_{\text{en}}, t^i_{\text{start}} \le o_e < t^i_{\text{end}}, 1 \le i \le n, \tag{5.7}$$

where st^{i,dr_e}_{en} represents the entry station of the ith disrupted section in direction dr_e that is either upstream or downstream. For instance in Figure 5.5, for the downstream blue train tr_3: the entry station of the 1st disrupted section (i.e. section F-G) is F and the entry station of the 2nd disrupted section (i.e. section C-D) is C, while for the upstream yellow train tr_4: the entry station of the 1st disrupted section is G and the entry station of the 2nd disrupted section is D. It is assumed that the duration of a disruption is known at the beginning of the disruption and will not change over time.

5.4.3.3 Operation consistency for trains without short-turning possibilities

For each train, the operation consistency of the two events that constitute the same running activity is always kept (i.e. both events are cancelled/kept simultaneously):

$$c_{e'} - c_e = 0, \qquad (e, e') \in A_{\text{run}}, \tag{5.8}$$

and the operation consistency of the two events that constitute the same dwell/pass-through activity is always kept if neither of them is relevant to any short-turn activity (i.e. no short-turning possibility):

$$c_e - c_{e'} = 0, \qquad (e, e') \in A_{\text{station}}, e \in E_{\text{ar}} \backslash E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}} \backslash E_{\text{de}}^{\text{turn}}, \tag{5.9}$$

where the set of station activities $A_{\text{station}} = A_{\text{dwell}} \cup A_{\text{pass}}$. Recall that $E_{\text{ar}}^{\text{turn}}$ ($E_{\text{de}}^{\text{turn}}$) is the set of arrival (departure) events that are the tails (heads) of short-turn activities. The tail (head) of an activity a is the event that a starts from (points to).

5.4.3.4 Breaking operation consistency for trains with short-turning possibilities

If a train is short-turned at a station, the operation consistency of its arrival and departure events at the station must be broken. For example in Figure 5.6 where section E-F is completely blocked and five possible short-turn activities (grey arcs) are created between trains tr_1 and tr_2. If possible the short-turn activity at station D is selected, then for train tr_1 the arrival event e_1 must be kept while the departure event e'_1 must be cancelled, and for train tr_2 the arrival event e_2 must be cancelled while the departure event e'_2 must be kept. In this case, $e_1 \in E_{\text{ar}}^{\text{turn}}, e'_1 \in E_{\text{de}} \backslash E_{\text{de}}^{\text{turn}}$, and $e_2 \in E_{\text{ar}} \backslash E_{\text{ar}}^{\text{turn}}, e'_2 \in E_{\text{de}}^{\text{turn}}$.

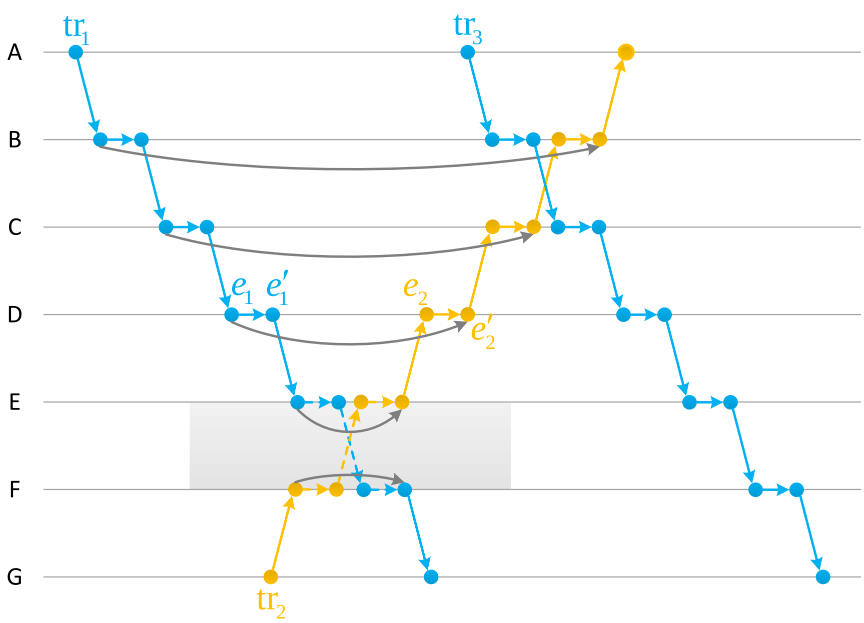

Figure 5.6: Example of short-turning options under one single disruption

To decide whether to break the operation consistency of such two events e and e' that form a station activity and only *one* of them has a short-turning possibility, the following constraints are established:

$$c_e \leq c_{e'}, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}} \backslash E_{\text{de}}^{\text{turn}}, \tag{5.10}$$

$$c_{e'} \leq c_e + y_e, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}} \backslash E_{\text{de}}^{\text{turn}}, \tag{5.11}$$

$$c_{e'} \geq y_e, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}} \backslash E_{\text{de}}^{\text{turn}}, \tag{5.12}$$

$$c_{e'} \leq c_e, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}} \backslash E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{5.13}$$

$$c_e \leq c_{e'} + y_{e'}, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}} \backslash E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{5.14}$$

$$c_e \geq y_{e'}, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}} \backslash E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{5.15}$$

where y_e is a binary decision variable with value 1 indicating that station st_e is chosen as the short-turn station of train tr_e, and 0 otherwise. If station st_e is *not* chosen as the short-turn station of arriving train tr_e (i.e. $y_e = 0$), then constraints (5.10) and (5.11) ensure that the operation consistency of events e and e' are kept; otherwise, constraint (5.12) requires event e' that does not have a short-turning possibility to be cancelled. Constraints (5.13) - (5.15) are similar but consider the different case where the departure event e' has a short-turning possibility while the arrival event e does not.

A train could be affected by two or more disruptions such as train tr_2 shown in Figure 5.7 where another section B-C is also disrupted. In this case, more short-turning activities are created due to the extra disruption, and in particular events e_2 and e'_2 both have short-turning possibilities, $e_2 \in E_{\text{ar}}^{\text{turn}}, e'_2 \in E_{\text{de}}^{\text{turn}}$, but at most one of them will make it.

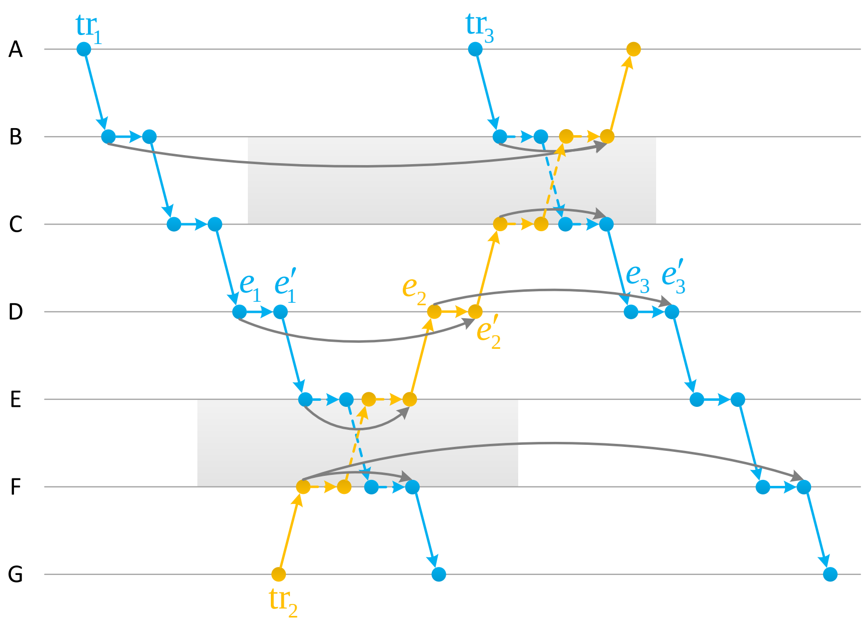

Figure 5.7: Example of short-turning options under two connected disruptions

To decide whether to break the operation consistency of such two events e and e' that form a station activity and *both* of them have short-turning possibilities, the following constraints are established:

$$c_e - c_{e'} = y_{e'} - y_e, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{5.16}$$

$$y_e + y_{e'} \leq 1, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{5.17}$$

$$c_{e'} \geq y_e, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{5.18}$$

$$c_e \leq 1 - y_e, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{5.19}$$

$$c_e \geq y_{e'}, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}}, \tag{5.20}$$

$$c_{e'} \leq 1 - y_{e'}, \qquad (e,e') \in A_{\text{station}}, e \in E_{\text{ar}}^{\text{turn}}, e' \in E_{\text{de}}^{\text{turn}},. \tag{5.21}$$

Constraint (5.16) ensures that if y_e and $y_{e'}$ are both equal to 0, then the operation consistency of events e and e' must be kept. Constraint (5.17) requires that at most one of y_e and $y_{e'}$ can be 1. Constraints (5.18) and (5.19) ensure that if $y_e = 1$ and $y_{e'} = 0$, then event e' must be cancelled, whereas event e must be kept due to the short-turning. Constraints (5.20) and (5.21) ensure that if $y_e = 0$ and $y_{e'} = 1$, then event e must be cancelled, whereas event e' must be kept due to the short-turning.

5.4.3.5 Limiting the short-turning stations for each train

At each side of the ith disrupted section, at least one short-turn station is chosen for a train if its operation in the disrupted section is cancelled:

$$\sum_{e:tr_e=tr} y_e \geq c_{e'}, \;\; tr \in TR_{\text{turn}}^i, e \in E_{\text{ar}}^{i,\text{turn}}, e' \in E_{\text{de}}, tr_{e'} = tr, st_{e'} = st_{\text{en}}^{i,dr_{e'}}, 1 \leq i \leq n, \tag{5.22}$$

$$\sum_{e':tr_{e'}=tr} y_{e'} \geq c_e, tr \in TR_{\text{turn}}^i, e' \in E_{\text{de}}^{i,\text{turn}}, e \in E_{\text{ar}}, tr_e = tr, st_e = st_{\text{ex}}^{i,dr_e}, 1 \leq i \leq n, \tag{5.23}$$

where $E_{\text{ar}}^{i,\text{turn}} \subset E_{\text{ar}}^{\text{turn}}$ ($E_{\text{de}}^{i,\text{turn}} \subset E_{\text{de}}^{\text{turn}}$) is the set of arrival (departure) events relevant to the short-turn activities corresponding to the ith disruption, TR_{turn}^i is the set of trains corresponding to the events in $E_{\text{ar}}^{i,\text{turn}} \cup E_{\text{de}}^{i,\text{turn}}$, and $st_{\text{en}}^{i,dr_{e'}}$ (st_{ex}^{i,dr_e}) represents the entry (exit) station of the ith disrupted section in direction $dr_{e'}$ (dr_e). In (5.22) and (5.23), we use "$\geq$" instead of "$=$" because the short-turn activities relevant to one train could correspond to different disruptions. In other words, it is possible that an event $e \in E_{\text{ar}}^{i,\text{turn}} \cap E_{\text{ar}}^{j,\text{turn}}$ (or $e' \in E_{\text{de}}^{i,\text{turn}} \cap E_{\text{de}}^{j,\text{turn}}$), while $i \neq j, 1 \leq i, j \leq n$. For example in Figure 5.7, the short-turn activity from train tr_2 to train tr_1 at station F corresponds to the first disruption and also corresponds to the second disruption as an early short-turning. In this case, the arrival event of train tr_2 at station F must belong to both $E_{\text{ar}}^{1,\text{turn}}$

and $E_{\mathrm{ar}}^{2,\mathrm{turn}}$, and the departure event of train tr_1 at station F must belong to both $E_{\mathrm{de}}^{1,\mathrm{turn}}$ and $E_{\mathrm{de}}^{2,\mathrm{turn}}$.

At one side of all disrupted sections, the number of short-turn stations chosen for a train cannot be larger than the number of its departure (arrival) events that originally occur at the entry (exit) stations of these disrupted sections but were cancelled.

$$\sum_{e:tr_e=tr} y_e \leq \sum_{e'} c_{e'},\quad tr \in TR_{\mathrm{turn}}, e \in E_{\mathrm{ar}}^{\mathrm{turn}}, e' \in E_{\mathrm{de}}, tr_{e'} = tr, st_{e'} \in ST_{\mathrm{en}}^{dr_{e'}}, 1 \leq i \leq n, \tag{5.24}$$

$$\sum_{e':tr_{e'}=tr} y_{e'} \leq \sum_{e} c_e, tr \in TR_{\mathrm{turn}}, e' \in E_{\mathrm{de}}^{\mathrm{turn}}, e \in E_{\mathrm{ar}}, tr_e = tr, st_e \in ST_{\mathrm{ex}}^{dr_e}, 1 \leq i \leq n, \tag{5.25}$$

where $ST_{\mathrm{en}}^{dr_{e'}} = \bigcup_{i=1}^{n} st_{\mathrm{en}}^{i,dr_{e'}}$ and $ST_{\mathrm{ex}}^{dr_e} = \bigcup_{i=1}^{n} st_{\mathrm{ex}}^{i,dr_e}$. Constraint (5.24) ensures that at one side of all disrupted sections, the number of short-turn stations chosen for train tr is not larger than the number of its departure events that originally occurred at the entry stations of the disrupted sections but were cancelled. Constraint (5.25) ensures that at the other end of all disrupted sections, the number of short-turn stations chosen for train tr is not larger than the number of its arrival events that originally occurred at the exit stations of these disrupted sections but were cancelled.

5.4.3.6 Selecting short-turn activities

For each train, at most one short-turn activity will be selected at a short-turn station. This is formulated by

$$\sum_{\substack{a \in A_{\mathrm{turn}},\\ tail(a)=e}} m_a = c_{e'} - c_e, \qquad (e,e') \in A_{\mathrm{station}}, e \in E_{\mathrm{ar}}^{\mathrm{turn}}, e' \in E_{\mathrm{de}} \backslash E_{\mathrm{de}}^{\mathrm{turn}}, \tag{5.26}$$

$$\sum_{\substack{a \in A_{\mathrm{turn}},\\ head(a)=e'}} m_a = c_e - c_{e'}, \qquad (e,e') \in A_{\mathrm{station}}, e \in E_{\mathrm{ar}} \backslash E_{\mathrm{ar}}^{\mathrm{turn}}, e' \in E_{\mathrm{de}}^{\mathrm{turn}}, \tag{5.27}$$

$$\sum_{\substack{a \in A_{\mathrm{turn}},\\ tail(a)=e}} m_a = c_{e'} - c_e + y_{e'}, \qquad (e,e') \in A_{\mathrm{station}}, e \in E_{\mathrm{ar}}^{\mathrm{turn}}, e' \in E_{\mathrm{de}}^{\mathrm{turn}}, \tag{5.28}$$

$$\sum_{\substack{a \in A_{\mathrm{turn}},\\ head(a)=e'}} m_a = c_e - c_{e'} + y_e, \qquad (e,e') \in A_{\mathrm{station}}, e \in E_{\mathrm{ar}}^{\mathrm{turn}}, e' \in E_{\mathrm{de}}^{\mathrm{turn}}. \tag{5.29}$$

where m_a is a binary decision with value 1 indicating that the short-turn activity a is selected. Constraints (5.26) and (5.27) are for the cases where a train is affected by one disruption only, while constraints (5.28) and (5.29) are for the cases where a train is affected by two or more disruptions. In (5.28), it may happen that $c_{e'} = 0$ and $c_e = 1$, which makes $c_{e'} - c_e = -1$ while the left term of this equality must be non-negative. Considering this, $y_{e'}$ is added to the right side, of which the value must be 1 in this case due to constraints (5.16) and (5.18). A similar reasoning is applied for adding y_e to the right side of (5.29).

If a short-turn activity is selected, the minimum short-turn duration must be respected, which is formulated by

$$M_1 c_e + 2D(1 - m_a) + x_{e'} - x_e \geq m_a L_a, \qquad a = (e, e') \in A_{\text{turn}}, \tag{5.30}$$

where A_{turn} is the set of all possible short-turn activities, and L_a represents the minimum duration required for short-turn activity a. If a short-turn activity $a \in A_{\text{turn}}$ is not selected (i.e. $m_a = 0$) while event e is not cancelled (i.e. $c_e = 0$), $2D$ is added to $x_{e'}$ to make constraint (5.30) still feasible, as $x_{e'}$ could be smaller than x_e. In this case, $2D$ is sufficient enough to make (5.30) feasible according to the definition of a short-turn activity given in Section 5.4.1.

5.4.3.7 Respecting realized train services

Recall that the current emerging disruption is the nth disruption ($n \geq 2$) starting at time t^n_{start}. Then, each departure or arrival event e that has occurred before t^n_{start} must be respected:

$$c_e = 0, \qquad e \in E_{\text{ar}} \cup E_{\text{de}}, r_e < t^n_{\text{start}}, n \geq 2, \tag{5.31}$$

$$x_e - r_e = 0, \qquad e \in E_{\text{ar}} \cup E_{\text{de}}, r_e < t^n_{\text{start}}, n \geq 2, \tag{5.32}$$

where r_e is a known value that refers to the previous rescheduled time of event e. Besides, each departure or arrival event e of which the previous rescheduled time r_e was after t^n_{start} cannot be rescheduled to before t^n_{start} in the current rescheduling procedure:

$$x_e \geq t^n_{\text{start}}, \qquad e \in E_{\text{ar}} \cup E_{\text{de}}, r_e \geq t^n_{\text{start}}, n \geq 2. \tag{5.33}$$

5.4.3.8 Station capacity

Each arriving train must be assigned with a track to stop at or pass through a station, and the track has to be a platform track if the train stops at the station. These are formulated by

$$\sum_{i=1}^{N^p_{ste}} u_{e,i} + \sum_{j=1}^{N^{th}_{ste}} v_{e,j} = 1 - c_e, \qquad e \in E_{\text{ar}}, \tag{5.34}$$

$$\sum_{i=1}^{N^p_{ste}} u_{e,i} \geq 1 - s_a - c_e - c_{e'} \qquad e \in E_{\text{ar}}, a = (e, e') \in A_{\text{station}}, \tag{5.35}$$

$$\sum_{i=1}^{N^p_{ste}} u_{e,i} \geq \sum_{\substack{a \in A_{\text{turn}}, \\ tail(a) = e}} m_a \qquad e \in E^{\text{turn}}_{\text{ar}}, \tag{5.36}$$

$$\sum_{i=1}^{N^p_{ste}} u_{e,i} = 1 - c_e, \qquad e \in E^{\text{odturn}}_{\text{ar}}, \tag{5.37}$$

where $u_{e,i}$ ($v_{e,j}$) is a binary variable with value 1 indicating that train tr_e occupies the ith (jth) platform (pass-through) track at station st_e and 0 otherwise, $N^p_{st_e}$ ($N^{th}_{st_e}$) represents the number of platform (pass-through) tracks at station st_e, and s_a is a binary variable to realize adding stops. A station activity $a = (e, e') \in A_{\text{station}}$ corresponds to a *true* stop (either a kept scheduled stop or an added stop), if and only if $s_a = 0, c_e = 0$ and $c_{e'} = 0$. For the constraints of determining s_a we refer to Chapter 3. Constraint (5.34) requires one station track to be assigned to an arriving train tr_e if event e is not cancelled. A platform track must be assigned to an arriving train tr_e if 1) it stops at the station ((5.34) and (5.35)); 2) it short-turns at the station ((5.34) and (5.36)); or 3) it reaches the destination ((5.34) and (5.37)). $E^{\text{odturn}}_{\text{ar}}$ is the set of arrival events that occur at the destinations and thus the corresponding rolling stock turns to operate the trains in the opposite directions. For the details of $E^{\text{odturn}}_{\text{ar}}$ we refer to Chapter 3.

If two trains occupy the same track of a station, there must be a minimum time interval to be respected between their occupations. In other words, the arrival of a train has to be a certain time later than the departure of another train that uses the same station track earlier. This is formulated by

$$\begin{aligned} x_{e'} - x_{e''} \geq h_{e,e'} + M_2 \left(q_{e,e'} - c_e - c_{e'} - c_{e''} + u_{e,i} + u_{e',i} - 3 \right), \\ e, e' \in E_{\text{ar}}, st_{e'} = st_e, (e, e'') \in A_{\text{station}}, \end{aligned} \tag{5.38}$$

$$\begin{aligned} x_{e'} - x_{e''} \geq h_{e,e'} + M_2 \left(q_{e,e'} - c_e - c_{e'} - (1 - m_a) + u_{e,i} + u_{e',i} - 3 \right), \\ e, e' \in E_{\text{ar}}, st_{e'} = st_e, a = (e, e'') \in A_{\text{turn}} \cup A_{\text{odturn}}, \end{aligned} \tag{5.39}$$

$$\begin{aligned} x_{e'} - x_{e''} \geq h_{e,e'} + M_2 \left(q_{e,e'} - c_e - c_{e'} - c_{e''} + v_{e,j} + v_{e',j} - 3 \right), \\ e, e' \in E_{\text{ar}}, st_{e'} = st_e, (e, e'') \in A_{\text{station}}, \end{aligned} \tag{5.40}$$

$$\begin{aligned} x_{e'} - x_{e''} \geq h_{e,e'} + M_2 \left(q_{e,e'} - c_e - c_{e'} - (1 - m_a) + v_{e,j} + v_{e',j} - 3 \right), \\ e, e' \in E_{\text{ar}}, st_{e'} = st_e, a = (e, e'') \in A_{\text{turn}} \cup A_{\text{odturn}}, \end{aligned} \tag{5.41}$$

where M_2 is a large positive number set to twice of M_1, $h_{e,e'}$ is a given parameter representing the minimum time interval required between the occurring times of e and e' if corresponding to trains occupying the same station track, and $q_{e,e'}$ is a binary variable with value 1 indicating that event e occurs before event e' and 0 otherwise. For the constraints of determining $q_{e,e'}$ we refer to Chapter 3, as well as the set A_{odturn} that contains all OD turn activities. Constraint (5.38) means that if arrival event e occurs before arrival event e' (i.e. $q_{e,e'} = 1$), events e, e' and e'' are all not cancelled (i.e. $c_e = 0, c_{e'} = 0$ and $c_{e''} = 0$) and both events e and e' occupy the same platform track (i.e. $u_{e,i} = 1$, and $u_{e',i} = 1$), then event e' must occur at least $h_{e,e'}$ later than the departure event e'' in the station activity corresponding to e. Constraint (5.39) means that if arrival event e occurs before arrival event e' (i.e. $q_{e,e'} = 1$), events e and e' are both not cancelled (i.e. $c_e = 0$ and $c_{e'} = 0$), the short-turn (OD turn) activity a relevant to e is selected (i.e. $m_a = 1$), and both events e and e' occupy the same platform track (i.e. $u_{e,i} = 1$ and $u_{e',i} = 1$), then event e' must occur at least $h_{e,e'}$ later than the departure event e'' in the short-turn (OD turn) activity corresponding to e. Constraint (5.40) ((5.41)) is similar to (5.38) ((5.39)), but considers a pass-through track.

5.5 Rolling horizon solution method

The multiple-disruption rescheduling model can be solved to optimality or near-optimality, if the disruption durations are not long (e.g. 2-hour disruptions). However in some disruption scenarios, a solver may not find high-quality solutions in an acceptable time if the disruption durations become rather long (e.g. 6-hour disruptions). Therefore, we propose a rolling-horizon solution method to the multiple-disruption model, which considers the periodic pattern of the rescheduled train services in the second phase of a disruption to speed up the computation.

A disruption consists of three phases: the 1st phase from the planned timetable transiting to the disruption timetable, the 2nd phase where the disruption timetable is in use, and the 3rd phase from the disruption timetable recovering to the planned timetable (Ghaemi et al., 2017b). A periodic short-turning/cancelling pattern exists among the rescheduled train services in the 2nd phase, due to the periodicity of the planned timetable (Chapter 3). That means, for example, if a train is short-turned at station A then another train that serves the same train line in a later period will be short-turned at the same station as well. Taking such a periodic pattern into account is helpful to release the computational burden by first obtaining the pattern considering a relatively short time horizon and then applying this pattern to the following train services gradually over time. How often the pattern will repeat varies with train lines. It is observed that for the train lines that are only affected by one disruption the length of the period is equal to the cycle time of the planned timetable, while for the train lines that are affected by at least two disruptions the period of the disruption solution may take longer than the planned cycle time and varies with disruption scenarios. An example is given in Figure 5.8, where (a) shows that a train line is planned to operate between station A and station F periodically, (b) shows that the rescheduled train services due to one disruption have a periodic pattern and the length of the period is the same as the planned cycle time, and (c) shows that the rescheduled train services due to two disruptions also have a periodic pattern but the length of the period could be much longer than the planned cycle time. Note that the length of the period may be further increased by increasing the short-turning durations to reflect the possible lower passenger demand. The dotted (dashed) lines in Figure 5.8(b) and (c) represent the original train services that are delayed (cancelled) in the rescheduled timetable, and the red arcs refer to the short-turning activities. In Figure 5.8(b) and (c), train services are delayed to respect the minimum short-turning durations, and all train services that were originally planned to operate in the disrupted sections are cancelled. Also in Figure 5.8(c), some train services from station C to station E are cancelled (the thick dashed lines), because they can only be kept if delayed by one planned cycle time to be operated by the rolling stock of the previous short-turned train, but another train service belonging to the same train line will already operate at that time.

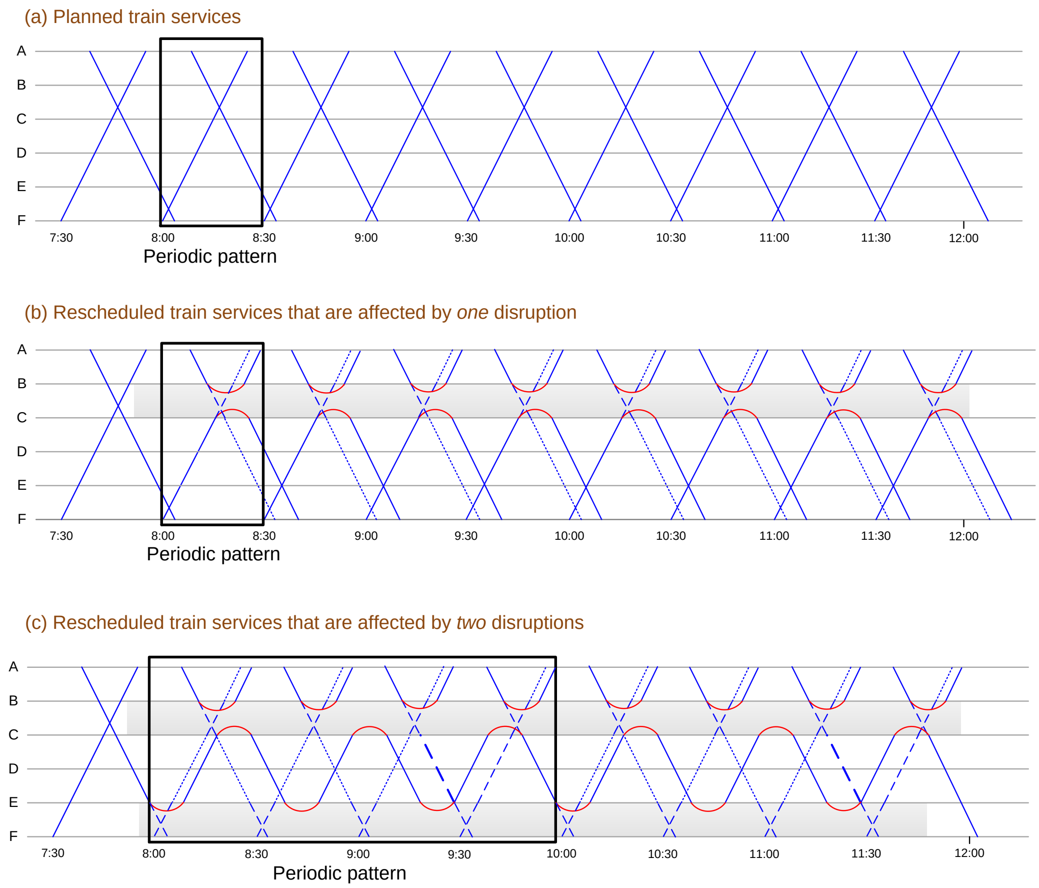

Figure 5.8: Illustration of the periodic pattern of train services

The rolling-horizon method divides the time horizon, from the starting time of a new connected disruption until the latest ending time among all connected disruptions, into successive stages. For the train lines that are only affected by one disruption, the periodic pattern is computed at stage 1, which is applied to the corresponding train services in the following stages. For the train lines that are affected by at least two disruptions, no periodic pattern will be computed at stage 1, because as explained before the length of the corresponding period varies with disruption scenarios and thus determining the pattern with an assumed length may affect the solution quality. Therefore, the train services corresponding to these train lines are rescheduled at each stage from scratch.

An illustration of the rolling-horizon method is given in Figure 5.9, while the details are given in Algorithm 5.1. The notation used in the algorithm is listed in Appendix 5.B. Algorithm 5.1 needs the following inputs: the set of ongoing disruptions $DIS = \{1, \cdots, n\}$, the starting (ending) time t^i_{start} (t^i_{end}) of the ith disruption, the set $TL^i_{\text{dis},1}$ containing the train lines that are only affected by the ith disruption, the set ST_{tl} containing the planned stopping and passage stations of train line $tl \in TL^i_{\text{dis},1}$, the length of a disruption h_r considered at a stage, and the maximum allowed delay per event D. All ongoing disruptions in DIS are sorted in ascending order according to their starting times, and the nth disruption is the emerging disruption. The setting of h_r affects the solution quality as well as the computation time. The value of h_r is set to at least bigger than D. A larger h_r may lead to a better solution but meanwhile could cost longer computation time. The influence of h_r on a solution is investigated in Section 5.6.3.

Algorithm 5.1: A rolling-horizon solution method to the multiple-disruption timetable rescheduling model

Input: $DIS = \{1, \cdots, n\}, \left\{\left(t^i_{\text{start}}, t^i_{\text{end}}, TL^i_{\text{dis},1}\right)\right\}_{i \in DIS}, \{ST_{tl}\}_{tl \in TL^i_{\text{dis},1}}, h_r, D$

Output: The rescheduled timetable for n connected disruptions

1 $k = 1$; `// Stage 1`

2 $DIS^k = \{1, \cdots, n_k\}, n_k = n$;

3 $\tilde{t}^k_{\text{start}} = t^{n_k}_{\text{start}}, i \in DIS^k$;

4 $\tilde{t}^{i,k}_{\text{end}} = \min\{\tilde{t}^k_{\text{start}} + h_r, t^i_{\text{end}}\}, i \in DIS^k$;

5 Solve the multiple-disruption model considering the ith disruption with duration $\left[\tilde{t}^k_{\text{start}}, \tilde{t}^{i,k}_{\text{end}}\right]$, $i \in DIS^k$ to obtain the set of the cancelled events E^k_{cancel};

6 $E^{\text{ar}}_{\text{cancel}} = \emptyset, E^{\text{ar}}_{\text{keep}} = \emptyset$; `// Extract the periodic pattern (lines 6-20)`

7 **for** $i = 1 : n_k$ **do**

8   **foreach** $tl \in TL^i_{\text{dis},1}$ **do**

9   Define $E^{tl,i}_{\text{ar}} = \left\{e \middle| e \in E_{\text{ar}}, tl_e = tl, \tilde{t}^k_{\text{start}} \le o_e \le \tilde{t}^{i,k}_{\text{end}}, \tilde{t}^{i,k}_{\text{end}} < t^i_{\text{end}}\right\}$;

10   **foreach** $st \in ST_{tl}$ **do**

11   Define $E^{st,tl,i}_{\text{ar}} = \left\{e \middle| e \in E^{tl,i}_{\text{ar}}, st_e = st\right\}$;

12   Find $e' = \arg\min\left\{o_{e'} : e' \in E^{st,tl,i}_{\text{ar}}\right\}$;

13   **if** $e' \in E^k_{\text{cancel}}$ **then**

14                 $E^{\text{ar}}_{\text{cancel}} = E^{\text{ar}}_{\text{cancel}} \bigcup e'$;

15   **else**

16   Find $e'' = \arg\min\left\{o_{e''} : e'' \in E^{st,tl,i}_{\text{ar}} \backslash e'\right\}$;

17   **if** $e'' \in E^k_{\text{cancel}}$ **then**

18                     $E^{\text{ar}}_{\text{cancel}} = E^{\text{ar}}_{\text{cancel}} \bigcup e''$;

19   **else**

20                     $E^{\text{ar}}_{\text{keep}} = E^{\text{ar}}_{\text{keep}} \bigcup e''$;

21 Remove the ith disruption from DIS^k if $\tilde{t}^{i,k}_{\text{end}} = t^i_{\text{end}}, i \in DIS^k$, and then update the number of the remainder disruptions as n_{k+1} and define $DIS^{k+1} = \{1, \cdots, n_{k+1}\}$;

22 **while** $n_{k+1} \ge 1$ **do**

23     $k = k + 1$; `// Stage 2 and onwards`

24     $\tilde{t}^k_{\text{start}} = \tilde{t}^{j,k-1}_{\text{end}} - D, i \in DIS^k$, j corresponds to the sequence of the current ith disruption at the previous stage;

25     $\tilde{t}^{i,k}_{\text{end}} = \min\{\tilde{t}^k_{\text{start}} + h_r, t^i_{\text{end}}\}, i \in DIS^k$;

26   **for** $i = 1 : n_k$ **do** `// Determine the events that will follow the pattern (lines 26-30)`

27   **foreach** $tl \in TL^i_{\text{dis},1}$ **do**

28   Define $E^{tl,i,k}_{\text{fix}} = \left\{e \middle| e \in E_{\text{ar}}, tl_e = tl, \tilde{t}^k_{\text{start}} \le o_e \le \tilde{t}^{i,k}_{\text{end}} - D\right\}$;

29   Define $E^{tl,i,k}_{\text{cancel}} = \left\{e \middle| e \in E^{tl,i,k}_{\text{fix}}, e' \in E^{\text{ar}}_{\text{cancel}}, tl_e = tl_{e'}, st_e = st_{e'}, dr_e = dr_{e'},\right\}$;

30   Define $E^{tl,i,k}_{\text{keep}} = \left\{e \middle| e \in E^{tl,i,k}_{\text{fix}}, e' \in E^{\text{ar}}_{\text{keep}}, tl_e = tl_{e'}, st_e = st_{e'}, dr_e = dr_{e'},\right\}$;

31   Add constraints $\left\{c_e = 1, e \in \bigcup_{i \in DIS^k} E^{tl,i,k}_{\text{cancel}}\right\}$ to the multiple-disruption model; `// Apply the pattern`

32   Add constraints $\left\{c_e = 0, e \in \bigcup_{i \in DIS^k} E^{tl,i,k}_{\text{keep}}\right\}$ to the multiple-disruption model; `// Apply the pattern`

33   Solve the multiple-disruption model considering the ith disruption with duration $\left[\tilde{t}^k_{\text{start}}, \tilde{t}^{i,k}_{\text{end}}\right], i \in DIS^k$;

34   Remove the ith disruption from DIS^k if $\tilde{t}^{i,k}_{\text{end}} = t^i_{\text{end}}, i \in DIS^k$, and then update the number of the remainder disruptions as n_{k+1} and define $DIS^{k+1} = \{1, \cdots, n_{k+1}\}$;

35 **Return** the rescheduled timetable obtained at final stage k; `// Terminate`

Algorithm 5.1 is called every time a new connected disruption occurs. The algorithm starts in stage 1 by defining the set of ongoing disruptions at the current stage as DIS^1 where the number of ongoing disruptions n_1 is set to the value n (lines 1-2). For each disruption $i \in DIS^1$, the starting time $\tilde{t}^1_{\text{start}}$ considered at stage 1 is set to equal to the starting time of the emerging disruption (line 3), while the ending time $\tilde{t}^{i,1}_{\text{end}}$ considered at stage 1 is set to the minimal value among $\tilde{t}^1_{\text{start}} + h_r$ and t^i_{end}, where in the latter case $\tilde{t}^1_{\text{start}} + h_r$ is larger than the ending time of the disruption t^i_{end} (line 4). The multiple-disruption model is solved considering that each disruption i lasts from $\tilde{t}^1_{\text{start}}$ to $\tilde{t}^{i,1}_{\text{end}}$ to obtain the set E^1_{cancel} containing all cancelled events at stage 1 (line 5). Note that at stage 1 (and at each following stage), the rescheduling solution is computed until the normal schedule has been recovered. Based on E^1_{cancel}, the periodic pattern of the rescheduled train services is obtained into two sets, $E^{\text{ar}}_{\text{cancel}}$ and $E^{\text{ar}}_{\text{keep}}$, which include the representative arrival event e at each stopping/passage station $st \in ST_{tl}$ of train line $tl \in TL^i_{\text{dis},1}$, of which the determined cancellation decision c_e should be followed by the same kind of event in the following periods (lines 6-20). Recall that any arrival and departure events that constitute the same running activity are cancelled or kept simultaneously due to constraint (5.8), which is why only $E^{\text{ar}}_{\text{cancel}}$ and $E^{\text{ar}}_{\text{keep}}$ are defined.

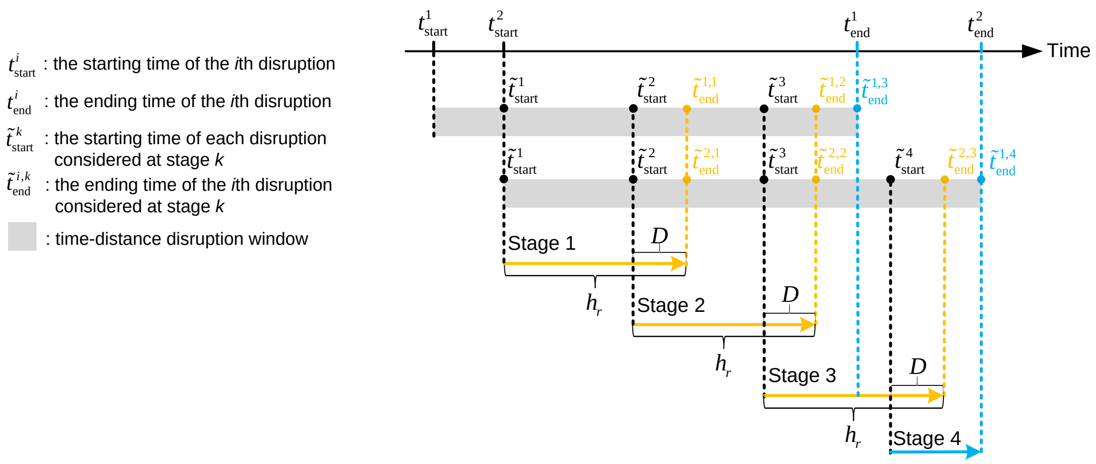

Figure 5.9: Illustration of the rolling-horizon solution method with two connected disruptions: the method is called when the 2nd disruption occurs

Before Algorithm 5.1 proceeds to the next stage $k+1$, the disruption of which the total duration has been considered completely in the current stage will be excluded from the ongoing disruptions of which the number is then updated as n_{k+1} (line 21). If there is at least one disruption remaining (line 22), then the algorithm will proceed to the next stage (line 23). For each disruption $i \in DIS^k$ at the current stage, the considered starting time $\tilde{t}^k_{\text{start}}$ is set to its previous considered ending time minus the maximum allowed delay per event D (line 24). Recall that the previous considered ending time $\tilde{t}^{j,k-1}_{\text{end}}$ is the previous considered starting time $\tilde{t}^{k-1}_{\text{start}}$ plus h_r (see line 4) while h_r is set larger than D. In that sense, $\tilde{t}^k_{\text{start}} = \tilde{t}^{j,k-1}_{\text{end}} - D$ is equivalent to $\tilde{t}^k_{\text{start}} = \tilde{t}^{k-1}_{\text{start}} + h_r - D$, in which $h_r - D$ is always positive. Note that a disruption of which the previous considered duration is smaller than h_r has already been removed from the ongoing disruptions before proceeding to the current stage (see line 21), and is not considered at the current stage, nor any following stages. Setting the considered starting time at the current stage as in line 24 avoids unnecessary train delays/cancellations due to the recovery phase at the previous stage. This is explained in Figure 5.10 where case (a) is the example of setting the starting time of a disruption considered at stage k to its ending time considered at stage $k-1$, and case (b) is the example of setting the starting time of a disruption considered at stage k to its ending time considered at stage $k-1$ minus D. As the train departures/arrivals to be rescheduled at the current stage cannot occur before the start time of this stage (the ones outside the blue shadow), two train services (the thick lines) are delayed longer in case (a) than in case (b).

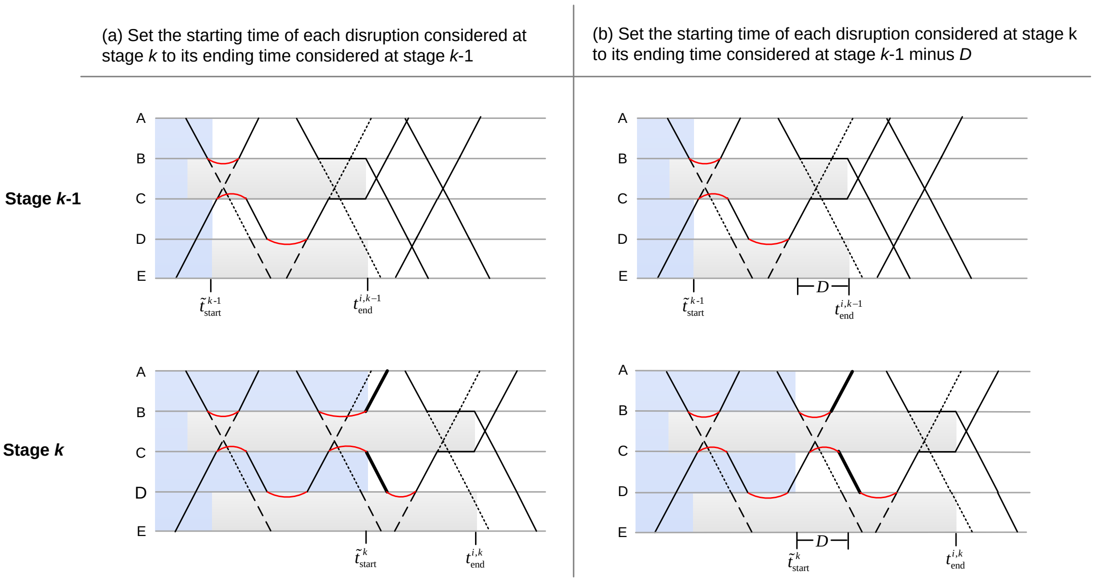

Figure 5.10: Two examples of setting the starting time of a disruption considered at a stage (case (b) is used by the rolling-horizon approach)

The ending time of a disruption considered at the current stage is set in the same way as introduced before (line 25). Recall that only the periodic pattern of the train lines that are affected by one disruption is computed. Thus for each disruption i, we iterate over the train line $tl \in TL^{i}_{\text{dis},1}$ that is only affected by the ith disruption to define the set $E^{tl,i,k}_{\text{fix}}$, which includes the events that should follow the determined periodic pattern of train line tl at the current stage k. The set $E^{tl,i,k}_{\text{cancel}} \subseteq E^{tl,i,k}_{\text{fix}}$ ($E^{tl,i,k}_{\text{keep}} \subseteq E^{tl,i,k}_{\text{fix}}$) that includes the events that should be cancelled (kept) at the current stage k is defined according to $E^{\text{ar}}_{\text{cancel}}$ ($E^{\text{ar}}_{\text{keep}}$) (lines 26-30). $E^{tl,i,k}_{\text{fix}}$ does not contain the events that were originally planned to occur during the recovery phase of a disruption, in which the periodic pattern may not be applicable. A recovery phase may start at D minutes before the disruption ending time due to constraints (5.6) and (5.7), in which a train can be delayed to the end of a disruption rather than short-turned at a station before the blocked tracks like the similar trains in the previous periods (as Figure 5.10 shows). The constraints that demand the events in $\bigcup_{i\in DIS^k} E^{tl,i,k}_{\text{cancel}}$ ($\bigcup_{i\in DIS^k} E^{tl,i,k}_{\text{keep}}$) to be cancelled (kept) are added to the multiple-disruption model, which is then solved considering that each disruption i lasts from $\tilde{t}^{k}_{\text{start}}$ to $\tilde{t}^{i,k}_{\text{end}}$ (lines 31-33). Next, the disruption of which the total duration has been completely considered at the current stage will be excluded (line 34). If there is at least one disruption remaining, the algorithm proceeds to the next stage. Otherwise, the algorithm terminates by returning the rescheduled timetable obtained at the final stage (line 35).

5.6 Case study

We tested the model on a subnetwork of the Dutch railways. There are 38 stations located in this network with 10 train lines operating half-hourly in each direction. The train lines operating in the network are shown in Figure 5.11. We distinguish between intercity (IC) and local (called sprinter (SPR) in Dutch) train lines. In the model, trains turning at the terminals to operate the opposite operations (i.e. OD turnings) are taken into account. Table 5.1 lists the terminals of the train lines that are located in the considered network, while the terminals outside the considered network are neglected. The model was developed in MATLAB on a desktop with Intel Xeon CPU E5-1620 v3 at 3.50 GHz and 16 GB RAM. The solver GUROBI release 7.0.1 was used either to solve the model directly or called by the rolling-horizon method to solve the model gradually over time.

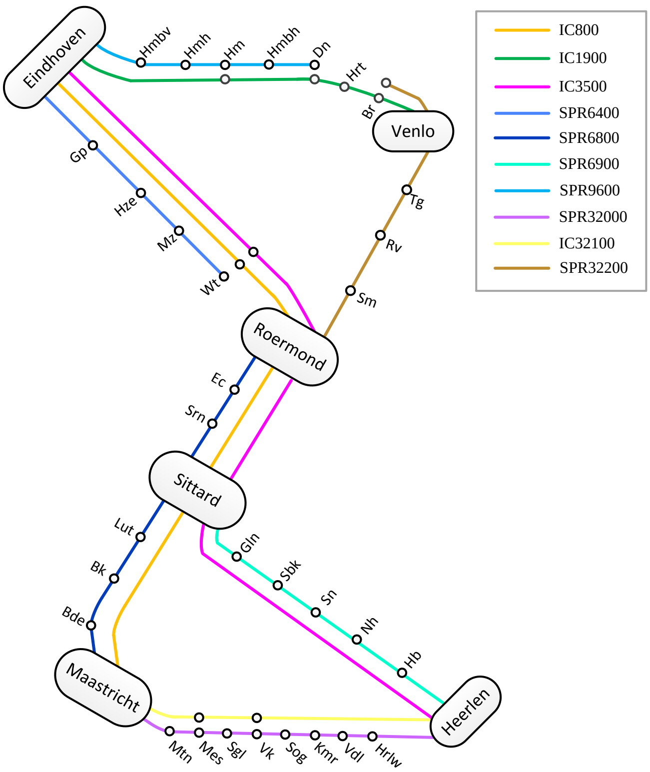

Figure 5.11: The train lines operating in the considered network

Table 5.1: Train lines in the considered network

Train line	Terminals in the considered network
IC800	Maastricht (Mt)
IC1900	Venlo (Vl)
IC3500	Heerlen (Hrl)
SPR6400	Eindhoven (Ehv) and Wt
SPR6800	Roermond (Rm)
SPR6900	Sittard (Std) and Hrl
SPR9600	Ehv and Dn
SPR32000	—
IC32100	Mt and Hrl
SPR32200	Rm

The schematic track layout of the considered network is shown in Figure 5.12 where stations Tg, Rv and Sm are located on single-track railway lines while the others are located on double-track railway lines. Due to the infrastructure layouts, some stations do not allow short-turning trains that operate in a specific direction or even both directions. In Figure 5.12, the stations that prohibit short-turning trains to both sides are colored in full grey, the stations that allow short-turning trains to both sides are colored in full green, and the stations that allow (prohibit) short-turning trains to one side are colored in half green (grey).

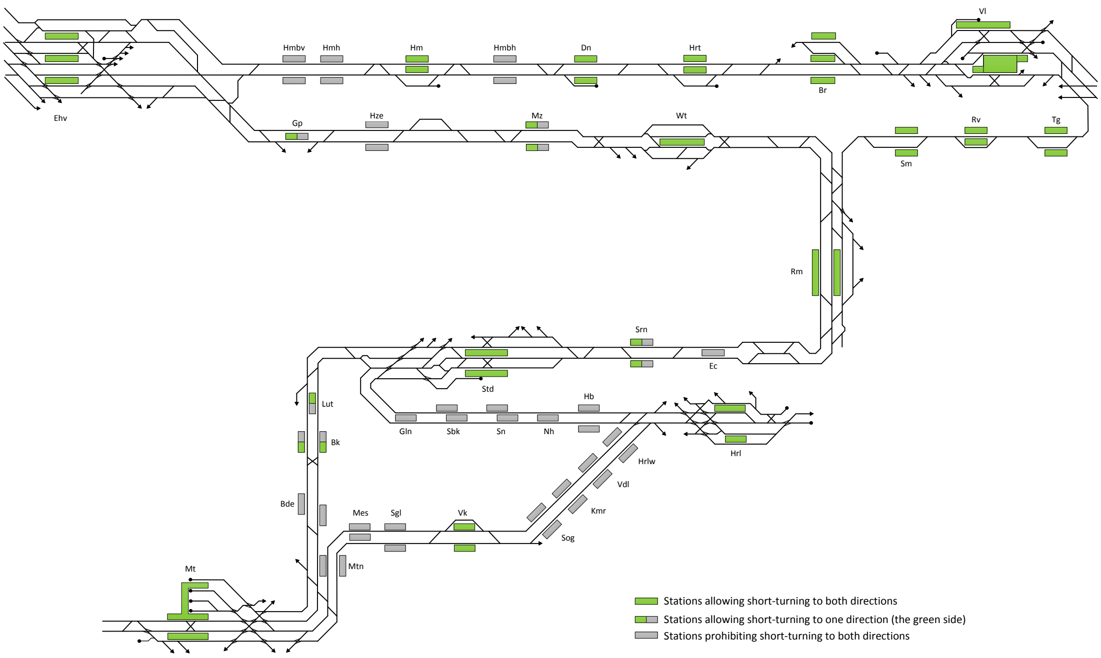

Figure 5.12: The schematic track layout in the considered network

We set the minimum duration required for short-turning or OD turning to 300 s, the minimum duration required for each headway to 180 s, and the penalty of cancelling a service to 100 min. Recall that a service refers to a train run between two adjacent stations. The maximum delay allowed for a train departure or arrival event $e \in E_{\text{ar}} \cup E_{\text{de}} \backslash E^{\text{NMdelay}}$ is set to 25 min. This is because we use a periodic planned timetable that has a cycle time of 30 min. Under this circumstance, delaying a train arrival/departure by 30 min might be unnecessary since at that time there will be a same kind of train departure/arrival originally scheduled. We allow extra stops to be added, considering that a train may dwell at a station where it originally passes through to wait for the platform capacity to be released in a downstream station where it will be short-turned. The minimum dwell time of an extra stop is set to 30 s.

In the following, Section 5.6.1 explores the performance of the sequential and combined approaches on two connected disruptions occurring in different locations. Each of the two disruptions is considered to last for 2 hours approximately, and their durations are almost fully overlapped. Section 5.6.2 investigates whether and how the length of the overlapping duration would affect the performance of the sequential and combined approaches. Section 5.6.3 analyzes the performance of the proposed rolling-horizon solution method when dealing with two connected disruptions with longer durations.

5.6.1 Multiple connected disruptions occurring in different sections

Table 5.2: Characteristics of scenarios 1-32

Scenario	First disruption	Second disruption	Scenario	First disruption	Second disruption
1	Bk - Lut	Rm - Wt	17	Hze - Gp	Bk - Lut
2	Bk - Lut	Wt - Mz	18	Hze - Gp	Lut - Std
3	Bk - Lut	Gp - Ehv	19	Hze - Gp	Mt - Bde
4	Bk - Lut	Hze - Gp	20	Hze - Gp	Srn -Ec
5	Lut - Std	Rm - Wt	21	Rm - Wt	Bk - Lut
6	Lut - Std	Wt - Mz	22	Rm - Wt	Lut - Std
7	Lut - Std	Gp - Ehv	23	Rm - Wt	Mt - Bde
8	Lut - Std	Hze - Gp	24	Rm - Wt	Srn -Ec
9	Mt - Bde	Rm - Wt	25	Wt - Mz	Bk - Lut
10	Mt - Bde	Wt - Mz	26	Wt - Mz	Lut - Std
11	Mt - Bde	Gp - Ehv	27	Wt - Mz	Mt - Bde
12	Mt - Bde	Hze - Gp	28	Wt - Mz	Srn -Ec
13	Srn -Ec	Rm - Wt	29	Gp - Ehv	Bk - Lut
14	Srn -Ec	Wt - Mz	30	Gp - Ehv	Lut - Std
15	Srn -Ec	Gp - Ehv	31	Gp - Ehv	Mt - Bde
16	Srn -Ec	Hze - Gp	32	Gp - Ehv	Srn -Ec

We establish 32 scenarios where each has two complete blockages occurring in different sections as shown in Table 5.2. For each scenario, we consider that the first disruption starts at 8:06 and ends at 10:06, while the second disruption starts at 8:12 and ends at 10:16. Both disruptions are connected by at least one train line. Considering the real-time requirement for computation, we set 300 s as the upper time limit to get a solution from either the sequential approach or the combined approach by a solver. Table 5.3 shows the results of handling scenarios 1-32 by both sequential and combined approaches.

In Table 5.3, the objective value, the number of cancelled services, the total train delay, the computation time, and the optimality gap are indicated for each solution obtained by either approach for each scenario. Recall that a service refers to a train run between two adjacent stations. For each solution, the optimality gap is the difference between the current best integer objective (i.e. the upper bound) and the current lower objective bound of the solution divided by the upper bound. We use "↓" to highlight the cases where smaller values were obtained in the objectives, the numbers of cancelled services, and the total train delays by the combined approach (compared to the sequential approach), while using "↑" to highlight the cases where larger values were obtained. In terms of objective values, the combined approach generated the solutions that were at least as good as the sequential approach. In 20 of 32 scenarios, the combined approach generated better solutions that resulted in less cancelled services and/or less train delays. For example in scenarios 1 and 5, the combined approach reduced both cancelled services and train delays. In some scenarios, it cancelled less services at the expense of introducing more train delays (e.g. scenarios 3); while in one scenario (i.e. scenario 4), it resulted in less train delays at the expense of cancelling more services.

Under the computation time limit of 300 seconds, the combined approach found optimal solutions for 30 of 32 scenarios, and high-quality solutions with an optimality gap of less than 0.60% for the other two scenarios. Scenarios 1 and 5 are the hardest to solve, which are the two cases resulting in less cancelled services and less train delays at the same time. This is due to the wider search spaces in both scenarios, helping to find better solutions but costing more computation times. The size of the search space is relevant to the location of each disruption. Compared to the combined approach, the sequential approach took less times to compute optimal solutions, which however cannot find feasible solutions for scenarios 13 and 24 where the disrupted sections are the same though the sequence of the occurrence is the other way around. In both scenarios, some services that were required to be cancelled when handling the first disruption cannot be cancelled when handling the second disruption, due to the starting times and locations of both disruptions. This is in conflict with that the sequential approach relies on the previous cancellation decisions, and thus leads to infeasible solutions.

Using scenario 5 as an example, we show the time-distance diagrams of rescheduling solutions obtained by the sequential and combined approaches. The 1st rescheduled timetable corresponding to the 1st disruption obtained by the sequential or combined approach is the same, which is shown in Figure 5.13. The solid lines represent the res-

Table 5.3: Results of scenarios 1-32 with 1st disruption in [8:06,10:06] and 2nd disruption in [8:12,10:16]

Scenario	Sequential (solver)					Combined (solver)				
	Obj [min]	# Cancelled services	Total train delay [min]	Time [sec]	Gap [%]	Obj [min]	# Cancelled services	Total train delay [min]	Time [sec]	Gap [%]
1	6,286	50	1,286	39	0.00	5,621 ↓	44↓	1,221 ↓	300	0.56
2	6,802	52	1,602	37	0.00	6,260 ↓	52	1,060 ↓	166	0.00
3	13,003	120	1,003	24	0.00	12,850 ↓	114↓	1,450 ↑	223	0.00
4	11,930	104	1,530	17	0.00	11,843 ↓	108↑	1,043 ↓	139	0.00
5	6,568	54	1,168	30	0.00	6,121 ↓	50↓	1,121 ↓	300	0.25
6	7,035	56	1,435	32	0.00	6,916 ↓	56	1,316 ↓	125	0.00
7	13,357	124	957	24	0.00	13,290 ↓	116↓	1,690 ↑	58	0.00
8	12,274	108	1,474	22	0.00	12,274	108	1,474	55	0.00
9	6,618	58	818	18	0.00	6,618	58	818	25	0.00
10	7,670	68	870	28	0.00	7,571 ↓	66↓	971 ↑	50	0.00
11	14,485	138	685	23	0.00	14,403 ↓	130↓	1,403 ↑	50	0.00
12	13,410	116	1,810	27	0.00	13,410	116	1,810	85	0.00
13	infeasible	–	–	–	–	13,705	78	5,905	183	0.00
14	11,507	80	3,507	52	0.00	11,093 ↓	80	3,093 ↓	166	0.00
15	18,044	150	3,044	31	0.00	17,676 ↓	142↓	3,476 ↑	95	0.00
16	16,955	134	3,555	26	0.00	16,679 ↓	134	3,279 ↓	170	0.00
17	11,572	90	2,572	30	0.00	11,572	90	2,572	55	0.00
18	12,372	98	2,572	28	0.00	12,372	98	2,572	80	0.00
19	13,138	104	2,738	20	0.00	13,138	104	2,738	50	0.00
20	15,345	114	3,845	19	0.00	15,334 ↓	114	3,934 ↓	70	0.00
21	5,220	42	1,020	13	0.00	5,214 ↓	42	1,014 ↓	125	0.00
22	6,014	50	1,014	13	0.00	6,008 ↓	50	1,008 ↓	91	0.00
23	6,788	56	1,188	11	0.00	6,774 ↓	56	1,174 ↓	40	0.00
24	infeasible	–	–	–	–	12,769	68	5,969	160	0.00
25	6,075	52	875	17	0.00	6,075	52	875	69	0.00
26	6,875	60	875	20	0.00	6,875	60	875	71	0.00
27	7,655	66	1,055	11	0.00	7,641 ↓	66	1,041 ↓	83	0.00
28	10,010	78	2,210	7	0.00	9,999 ↓	78	2,199 ↓	76	0.00
29	12,968	116	1,368	19	0.00	12,968	116	1,368	53	0.00
30	13,768	124	1,368	16	0.00	13,768	124	1,368	38	0.00
31	14,548	130	1,548	11	0.00	14,534 ↓	130	1,534 ↓	45	0.00
32	16,748	140	2,748	19	0.00	16,737 ↓	140	2,737 ↓	37	0.00

cheduled services, the dotted (dashed) lines represent the original scheduled services that are delayed (cancelled) in the rescheduled timetable, and the red triangles indicate extra stops. Due to the infrastructure layout, station Lut prohibits short-turning the trains coming from station Bk, which is why these trains short-turn earlier at station Bk. As station Bk has two tracks only, a minimum headway has to be respected between the arrival of a train and the departure of another train that previously arrives at station Bk from the same direction. Thus, three trains from SPR6800 (in dark blue) have to be delayed at station Bde to respect the minimum headway between their arrivals and the departures of previous arriving trains from IC800 (in orange) at station Bk. The similar reasoning is applied for the extra stops and delays happening to three trains from IC800 (in orange) at station Bde.

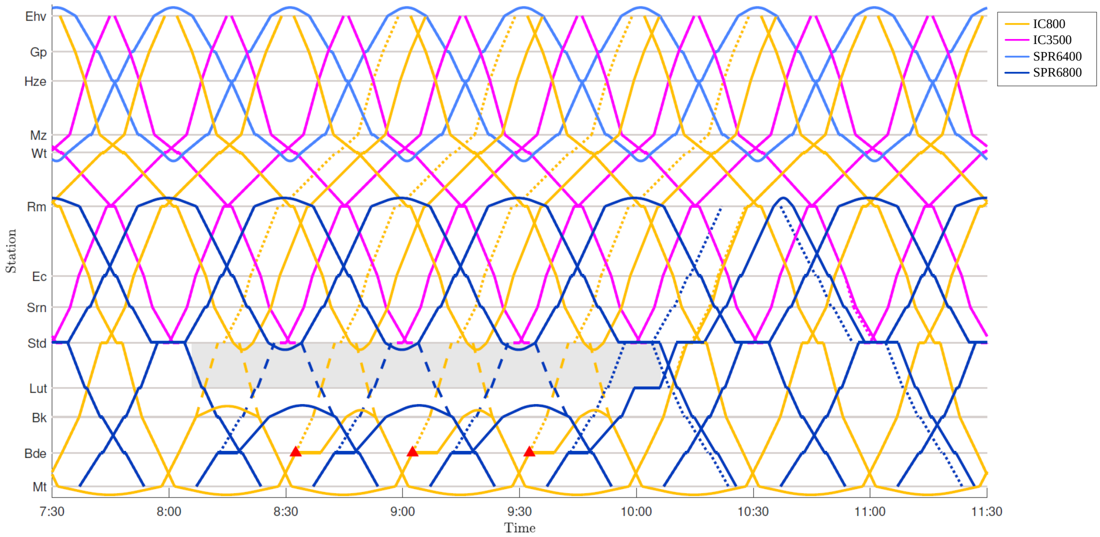

Figure 5.13: The 1st rescheduled timetable obtained by the sequential/combined approach for scenario 5: from Eindhoven (Ehv) to Maastricht (Mt)

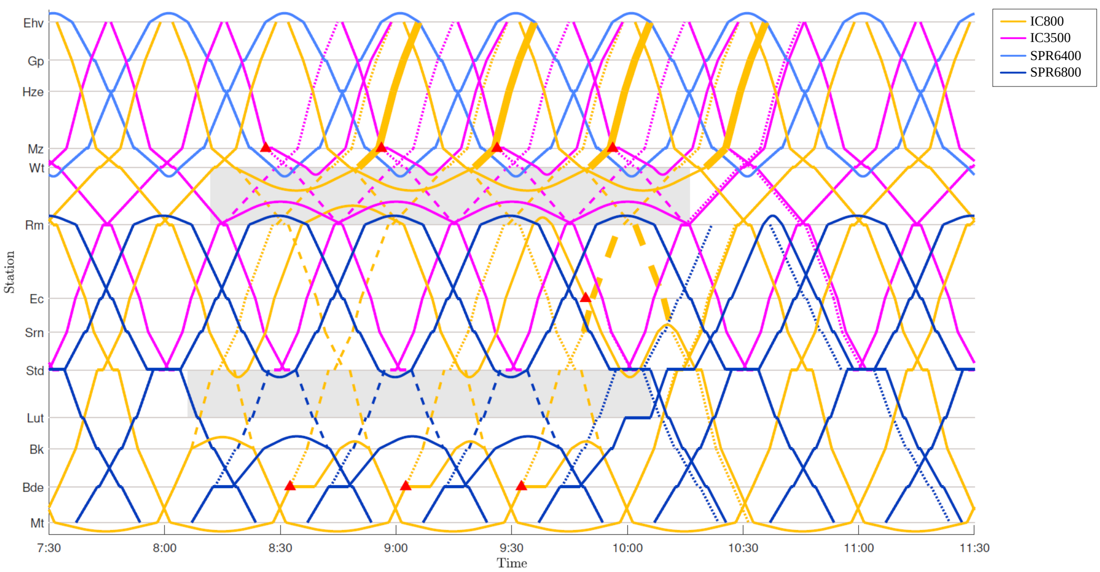

Figure 5.14: The 2nd rescheduled timetable obtained by the sequential approach for scenario 5: from Eindhoven (Ehv) to Maastricht (Mt)

The 2nd rescheduled timetable obtained by the sequential approach is shown in Figure 5.14. Compared to Figure 5.13, there are more train services from IC800 (in orange) cancelled between stations Std and Rm in Figure 5.14. This is because trains from IC800 (in orange) have to be short-turned at station Rm due to the emerging disruption (disrupted section Rm-Wt), which however may be inoperable due to their short-turnings at station Std. An earlier short-turning is observed at station Srn between trains from IC800 (in orange). This is because if this short-turning occurs at station Rm instead, although there would be four services cancelled less, the resulting train delays are more than the penalty on cancelling four services. At the top of the disrupted section Rm-Wt, four trains from IC3500 (in pink) additionally dwell at station Mz. This is because station Wt has four tracks while only two of them are alongside platforms. Thus, each of these four trains from IC3500 (in pink) has to wait at station Mz to ensure the headway between its arrival and the departure of a short-turned train from SPR6400 (in light blue) at station Wt where a train from IC800 (in orange) is still occupying another platform at that time. At station Wt, the departures of four upstream trains from IC800 (in orange) are delayed more than necessary. This is because in the sequential approach, the delaying decisions made for the previous disruption are kept. Hence, the adjusted arrival and departure times from the previous step are now the reference timetable, while early arrivals/departures are not allowed, which now is with respect to this timetable.

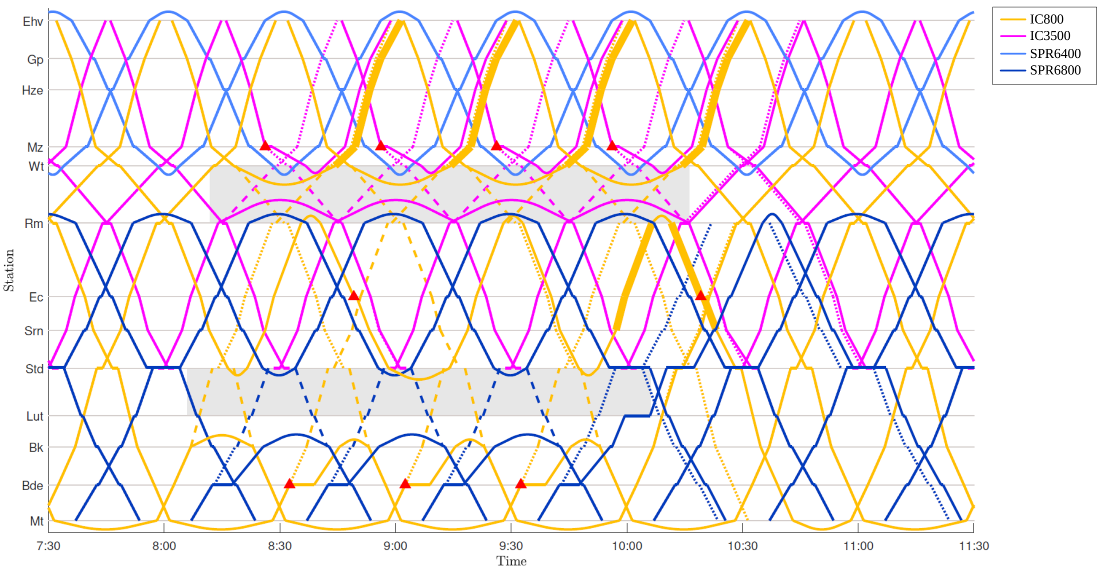

Figure 5.15: The 2nd rescheduled timetable obtained by the combined approach for scenario 5: from Eindhoven (Ehv) to Maastricht (Mt)

The 2nd rescheduled timetable obtained by the combined approach is shown in Figure 5.15 where different short-turning patterns of trains from IC800 (in orange) are observed. For example in Figure 5.15 trains from IC800 (thick solid lines in orange) were short-turned at station Rm around 10:10 instead of at station Srn around 10:15 as in the sequential approach (Figure 5.14) in which four more services were cancelled (thick dashed lines in orange). With the combined approach (Figure 5.15), four upstream trains from IC800 between Wt and Ehv (thick solid lines in orange) were less delayed than when using the sequential approach (Figure 5.14).

From these results it is concluded that the combined approach is able to handle more kinds of multiple-disruption scenarios and find better solutions than the sequential approach in some cases. This is because the combined approach does not rely on previously taken decisions, thus having a wider search space that helps to find a better solution but also costs longer computation time.

5.6.2 Multiple connected disruptions with different overlapping durations

Section 5.6.1 considers two disruptions that last for around 2 hours, respectively, and the overlapping duration is 1 hour and 54 minutes (almost fully overlapping). To explore whether the length of the overlapping duration affects the performance of the combined approach and the corresponding computation time, this sections considers two disruptions that have the same durations as in Section 5.6.1 but are overlapping to different extents. Several instances differing in the overlapping durations are established as shown in Table 5.4, in which instance * represents the duration setting used in Section 5.6.1.

Table 5.4: Two disruptions with different lengths of overlapping durations

Instance	First disruption period	Second disruption period	Total disruption duration (in HH:MM format)	Overlapping duration (in HH:MM format)
*	[8:06,10:06]	[8:12,10:16]	02:00 + 02:04	01:54
a	[8:06,10:06]	[8:32,10:36]	02:00 + 02:04	01:34
b	[8:06,10:06]	[8:52,10:56]	02:00 + 02:04	01:14
c	[8:06,10:06]	[9:12,11:16]	02:00 + 02:04	00:54
d	[8:06,10:06]	[9:32,11:36]	02:00 + 02:04	00:34
e	[8:06,10:06]	[9:52,11:56]	02:00 + 02:04	00:14
f	[8:06,10:06]	[10:12,12:16]	02:00 + 02:04	00:00

From Table 5.3 we know that compared to the sequential approach, the combined approach performed much better in scenario 1 , slightly better in scenario 6, and the same in scenario 9 when considering overlapping duration instance *. Hence, we take scenarios 1, 6 and 9 as examples to test whether the performance of the combined approach would be different when considering different overlapping duration instances in the same scenario. We implemented instances a-f in these scenarios, and displayed the results in Table 5.5. The result of implementing instance * on each of these scenarios has already been shown in Table 5.3, which is also displayed in Table 5.5.

Table 5.5: Results of considering the duration instances of Table 5.4

		Sequential (solver)			Combined (solver)			
	Instance	Obj [min]	Gap [%]	Time [sec]	Obj [min]	Gap [%]	Time [sec]	Obj decrease
Scenario 1	*	6,286	0.00	39	5,621 ↓	0.56	*300*	*665*
	a	5,172	0.00	10	5,038 ↓	0.00	30	134
	b	5,102	0.00	9	5,015 ↓	0.00	14	87
	c	5,778	0.00	40	5,518 ↓	0.00	45	260
	d	5,312	0.00	12	5,265 ↓	0.00	18	47
	e	5,242	0.00	12	5,242	0.00	11	0
	f	5,350	0.00	15	5,350	0.00	16	0
Scenario 6	*	7,035	0.00	32	6,916 ↓	0.00	125	119
	a	7,987	0.00	67	7,675 ↓	0.00	*165*	*312*
	b	6,974	0.00	40	6,902 ↓	0.00	50	72
	c	7,021	0.00	45	6,934 ↓	0.00	60	87
	d	7,658	0.00	40	7,452 ↓	0.00	50	206
	e	6,671	0.00	42	6,670 ↓	0.00	38	1
	f	6,680	0.00	28	6,680	0.00	28	0
Scenario 9	*	6,618	0.00	18	6,618	0.00	25	0
	a	6,931	0.00	22	6,872 ↓	0.00	*141*	*59*
	b	6,530	0.00	10	6,530	0.00	11	0
	c	6,770	0.00	35	6,721 ↓	0.00	40	49
	d	6,947	0.00	71	6,888 ↓	0.00	50	59
	e	6,530	0.00	11	6,529 ↓	0.00	11	1
	f	6,778	0.00	25	6,730 ↓	0.00	30	48

Table 5.5 indicates that the performance of the combined approach can change with the overlapping duration between disruptions, and that the change is scenario dependent. Recall that a scenario is different from another scenario in terms of the disrupted sections (see Table 5.2). In scenario 1 (scenario 6), the combined approach performed the best in terms of the objective under instance * (instance a) in which disruptions were time overlapping to a large extent. In these scenarios longer overlapping duration means more interactions between disruptions, and therefore more interdependent decisions relevant to multiple disruptions need to be decided. These interdependent decisions do not exist in the sequential approach that is unable to consider the combined effects of multiple disruptions. Thus with the increase of interdependent decisions, the solution space of the combined approach becomes larger so that it is more likely to generate a better solution than the sequential approach but meanwhile requires longer computation time. For example either in scenario 1 or scenario 6, the longest computation time happened in the instance where the largest objective decrease was obtained by the combined approach. When considering a much shorter or even zero overlapping duration (instance e or f), the computation times of the combined approach were shorter, and there were few or no differences between the performances of the combined and the sequential approaches in scenarios 1 and 6. Compared to scenario 1

or 6, scenario 9 showed less objective decrease from the combined approach in most instances. The performance of the combined approach in scenario 9 was not relevant to the length of the overlapping disruption duration as in scenario 1 or 6. This is because in scenario 1 or 6 the number of trains that were less delayed or of which less services were cancelled due to the combined approach increased with the overlapping duration, whereas in scenario 9 only one train was delayed less due to the combined approach, which occurred in specific instances depending on the starting/ending times of the disruptions but not on the length of the overlapping duration.

5.6.3 Multiple connected disruptions with longer (overlapping) durations

In Sections 5.6.1 and 5.6.2, the duration considered for each disruption is 2 hours approximately. For two connected disruptions with such durations, the combined approach outperforms the sequential approach in terms of solution quality by up to 300 seconds computation. For longer disruptions, whether this still holds should be investigated. This is for the consideration that the combined approach needs longer computation time than the sequential approach and thus may generate sub-optimal solutions under the required time limit, which then could be worse than the solutions obtained by the sequential approach. This section tests both approaches on longer disruptions using 300 seconds as the computation time limit still. Particularly in the combined approach, the multiple-disruption model is solved by the rolling-horizon approach proposed in Section 5.5, as well as an optimization solver for comparison.

According to Table 5.3, scenario 1 is chosen as an example, because it is the most difficult scenario to be solved by the combined approach. Twelve cases of disruption durations are considered for this scenario, which are shown in Table 5.6.

Table 5.6: Two connected disruptions with longer (overlapping) durations

Case	First disruption period	Second disruption period	Total disruption duration (in HH:MM format)	Overlapping duration (in HH:MM format)
*	[8:06,10:06]	[8:12,10:16]	02:00 + 02:04	01:54
I	[8:06,10:26]	[8:12,10:36]	02:20 + 02:24	02:14
II	[8:06,10:46]	[8:12,10:56]	02:40 + 02:44	02:34
III	[8:06,11:06]	[8:12,11:16]	03:00 + 03:04	02:54
IV	[8:06,11:26]	[8:12,11:36]	03:20 + 03:24	03:14
V	[8:06,11:46]	[8:12,11:56]	03:40 + 03:44	03:34
VI	[8:06,12:06]	[8:12,12:16]	04:00 + 04:04	03:54
VII	[8:06,12:26]	[8:12,12:36]	04:20 + 04:24	04:14
VIII	[8:06,12:46]	[8:12,12:56]	04:40 + 04:44	04:34
IX	[8:06,13:06]	[8:12,13:16]	05:00 + 05:04	04:54
X	[8:06,13:26]	[8:12,13:36]	05:20 + 05:24	05:14
XI	[8:06,13:56]	[8:12,13:56]	05:40 + 05:44	05:34
XII	[8:06,14:06]	[8:12,14:16]	06:00 + 06:04	05:54

The results of applying the sequential/combined approach to deal with duration cases I-XII in scenario 1 are indicated in Table 5.7. These results are obtained by the optimization solver GUROBI. We use O-gap to indicate the percentage difference between the obtained solution and the optimal solution. If no optimal solution was obtained by the solver up to 24h computation, we calculated U-gap and L-gap to represent the percentage difference between the obtained solution and the best found upper bound, and the percentage difference between the obtained solution and the best found lower bound, respectively. The sequential approach found optimal solutions within 300 s for most cases, except case XII (the longest disruption case) for which it took 572 s to find the optimal solution. Although the combined approach computed sub-optimal solutions within 300 s, these solutions were still better than the optimal solutions by the sequential approach. By up to 24h computation, the combined approach obtained optimal solutions for cases I-III, and near-optimal solutions for cases IV-XII.

The proposed rolling-horizon solution method was also applied for the combined approach to solve cases I-XII in scenario 1. The computation time at each stage of the rolling-horizon method is restricted to 300 s. The results under different settings of h_r are shown in Table 5.8. Recall that h_r represents the length of a disruption considered at each stage (except the final stage). By comparing Table 5.8 with Table 5.7, we found that the solutions obtained by the rolling-horizon method under whichever setting of h_r were better than the ones obtained by the solver up to 300 s computation. When increasing h_r from 1h to 1.5h, optimal solutions were found for cases I-III, and solutions with improved U-gaps and L-gaps were obtained for cases IV-XII. When increasing h_r from 1.5h to 2h further, the solutions obtained for cases I-VI were the same, but the solutions found for cases VII-XII mostly became worse (U-gaps and L-gaps both increased). This is due to the computation limit of 300 s required at a stage of the rolling-horizon method. When h_r was set to 1h or 1.5h, an optimal solution was always obtained at each stage within the required time limit. When h_r was set to 2h, sub-optimal solutions were obtained at specific stages due to the time limit, which affected the overall solution optimality.

Figure 5.16 shows the stage computation times of cases I-XII under different settings of h_r. Each circle indicates the computation time at a specific stage that is distinguished by color. The circles on the same vertical line correspond to the same case. Because the disruption durations are different among cases, the number of stages needed at a case can be different from one to another case although both cases were under the same setting of h_r. When h_r=1h, the stage computation times were mostly below 25 s with 7 exceptions that ranged from 35 s to 210 s and all corresponded to the final stages of the relevant cases. When h_r=1.5h, stage computation times increased due to longer disruption durations considered, and the most time-consuming stages took 225 s, which were the first stages in all cases. A stage computation time is very sensitive to the starting and ending times of disruptions considered at the stage, which is why it varied with stages although under the same setting of h_r. When h_r increased to 2h, most stage computation times reached the limit of 300 s, and in some cases only the

Table 5.7: Results of scenario 1 by using a solver

Case	Sequential (solver)			Combined (solver)					Combined (solver up to 24h)			
	Obj [min]	O-gap [%]	Time [sec]	Obj [min]	O-gap [%]	U-gap [%]	L-gap [%]	Time [sec]	Optimal	Upper bound	Lower bound	L-gap [%]
I	7,787	0.00	21	7,084 ↓	2.67			300	6,895			0.00
II	8,456	0.00	48	7,653 ↓	1.74			300	7,520			0.00
III	10,096	0.00	300	8,765 ↓	4.48			300	8,372			0.00
IV	11,663	0.00	300	10,027 ↓		3.42	3.55	300		9,684	9,671	0.13
V	12,229	0.00	300	11,111 ↓		7.61	8.16	300		10,266	10,204	0.61
VI	13,196	0.00	300	11,982 ↓		7.26	7.65	300		11,112	11,065	0.43
VII	14,578	0.00	300	12,635 ↓		2.12	2.94	300		12,368	12,263	0.85
VIII	15,115	0.00	300	13,671 ↓		5.60	6.48	300		12,905	12,785	0.93
IX	16,036	0.00	300	15,654 ↓		12.04	12.79	300		13,769	13,652	0.85
X	17,417	0.00	300	16,505 ↓		8.86	10.10	300		15,043	14,838	1.36
XI	17,954	0.00	300	16,362 ↓		4.58	5.65	300		15,612	15,438	1.11
XII	21,338	11.50	300	17,537 ↓		6.44	7.79	300		16,407	16,170	1.44
	(18,875	0.00	572)									

Table 5.8: Results of scenario 1 by using the rolling-horizon solution method for the combined approach (up to 300 sec computation at each stage)

Case	$h_r = 1$h				$h_r = 1.5$h				$h_r = 2$h			
	Obj [min]	O-gap [%]	U-gap [%]	L-gap [%]	Obj [min]	O-gap [%]	U-gap [%]	L-gap [%]	Obj [min]	O-gap [%]	U-gap [%]	L-gap [%]
I	6,991	1.37			6,895	0.00			6,895	0.00		
II	7,595	0.99			7,520	0.00			7,520	0.00		
III	8,498	1.48			8,372	0.00			8,372	0.00		
IV	9,810		1.28	1.42	9,684		0.00	0.13	9,684		0.00	0.13
V	10,415		1.43	2.03	10,289		0.22	0.83	10,289		0.22	0.83
VI	11,318		1.82	2.24	11,191		0.71	1.13	11,191		0.71	1.13
VII	12,630		2.08	2.91	12,452		0.68	1.52	12,504		1.10	1.93
VIII	13,235		2.49	3.40	13,041		1.04	1.96	13,109		1.56	2.47
IX	14,318		2.61	3.44	13,961		1.38	2.21	14,011		1.73	2.56
X	15,450		2.63	3.96	15,235		1.26	2.61	15,272		1.50	2.84
XI	16,055		2.76	3.84	15,861		1.57	2.67	15,861		1.57	2.67
XII	16,957		3.24	4.64	16,712		1.83	3.24	16,781		2.23	3.64

circles that indicated the final stage computation times are visible, because the ones that represented the previous stages were overlapping due to the same computation times. The total computation time of the rolling-horizon method is the sum of the computation times required at all stages. Table 5.9 shows the minimum, average and maximum total computation times across cases under the same setting of h_r in scenario 1. These values all increase with the growth of h_r. For example when h_r=1h the maximum total computation time was below 300 sec, while when h_r=2h the minimum total computation time was over 300 sec. Although the total computation times were mostly (all) longer than 300 sec when setting h_r to 1.5h (2h), the corresponding stage computation times were all below 300 sec as shown in Figure 5.16. Therefore in practice, a rescheduling solution can be rapidly obtained at a stage and immediately delivered to traffic controllers, and then updated gradually over time for the following stages. Although we assume that the disruption durations will not change over time, with minor changes the proposed rolling-horizon method can be used to deal with the dynamic variations regarding the disruption durations.

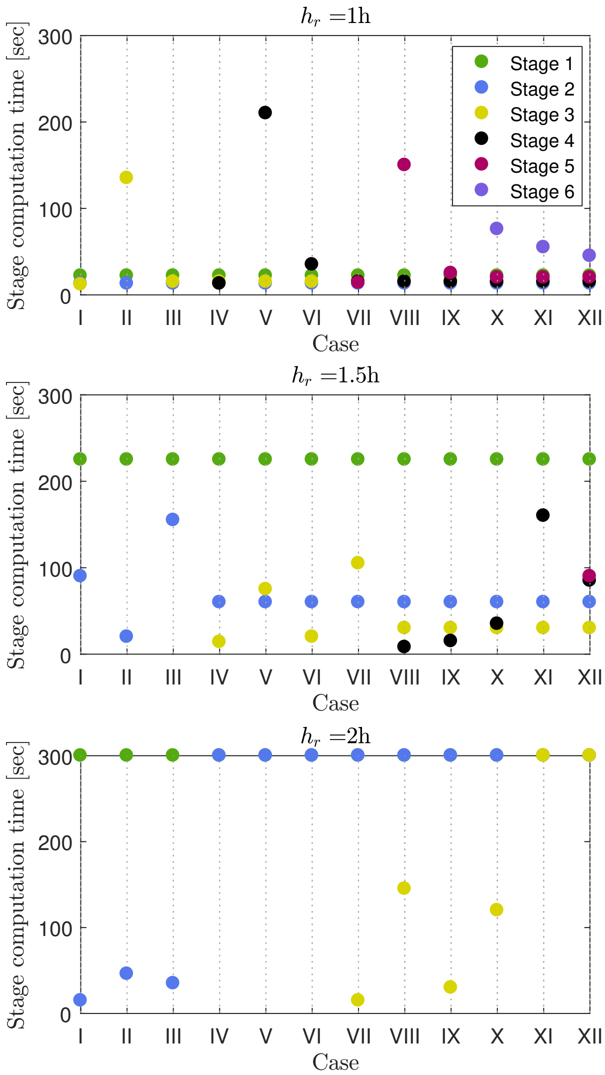

Figure 5.16: Stage computation times [sec] under different settings of h_r in the rolling-horizon method

Table 5.9: The minimum, average and maximum total computation times [sec] of the rolling-horizon method for the combined approach across cases in scenario 1

h_r	Min	Avg	Max
1h	47	124	260
1.5h	245	355	490
2h	315	609	900

In scenario 1, the combined approach performs much better than the sequential approach, and thus the sub-optimal solutions by the combined approach can still be better than the optimal solutions by the sequential approach. For the scenarios where the combined approach performs at least as good as the sequential approach, it is able to generate solutions below the required computation time limit. According to Table 5.3, scenarios 6 and 9 are chosen as two more example instances, and the corresponding results are shown in Tables 5.10 and 5.11, respectively. In these two scenarios, we set h_r to 2h, under which optimal solutions were always obtained at stage level under the required time limit in all cases.

Table 5.10 shows that for scenario 6, the combined approach found better solutions than the sequential approach in all cases when using the rolling-horizon solution method, which however was not achieved when using a solver. The optimality gaps of the solutions by the rolling-horizon method were all below 0.40%. Table 5.11 shows that for scenario 9, the combined approach found the solutions that were at least as good as the ones obtained by the sequential approach in all cases when using either a solver or the rolling-horizon method. In both scenarios, the maximum stage computation time of the rolling-horizon method was below the required time limit of 300 sec. These results indicate that the computational complexity of the combined approach is scenario dependent, and that the proposed rolling-horizon method is able to generate high-quality solutions in an acceptable time.

Table 5.10: Results of scenario 6

Case	Sequential (solver)			Combined (solver)			Combined (rolling-horizon)		
	Obj [min]	O-gap [%]	Time [sec]	Obj [min]	O-gap [%]	Time [sec]	Obj [min]	O-gap [%]	Max stage time [sec]
I	8,569	0.00	162	8,462 $\downarrow$	0.00	93	8,462 $\downarrow$	0.00	170
II	9,588	0.00	188	9,366 $\downarrow$	0.00	100	9,366 $\downarrow$	0.00	170
III	10,568	0.00	145	10,425 $\downarrow$	0.00	69	10,425 $\downarrow$	0.00	170
IV	12,165	0.00	282	12,064 $\downarrow$	0.64	300	11,987 $\downarrow$	0.00	170
V	13,185	0.00	241	12,967 $\downarrow$	0.59	300	12,904 $\downarrow$	0.10	170
VI	14,204	0.00	300	14,305 $\uparrow$	2.49	300	13,965 $\downarrow$	0.11	170
VII	15,785	0.00	300	18,314 $\uparrow$	15.25	300	15,580 $\downarrow$	0.38	170
VIII	16,804	0.00	300	16,764 $\downarrow$	2.09	300	16,451 $\downarrow$	0.22	170
IX	17,888	0.58	300	17,564 $\downarrow$	0.52	300	17,539 $\downarrow$	0.38	70
X	19,825	2.24	300	19,755 $\downarrow$	3.59	300	19,063 $\downarrow$	0.09	170
XI	20,490	0.43	300	23,552 $\uparrow$	15.34	300	19,980 $\downarrow$	0.21	170
XII	21,451	0.15	300	21,205 $\downarrow$	0.99	300	21,039 $\downarrow$	0.21	170

Table 5.11: Results of scenario 9

Case	Sequential (solver)			Combined (solver)			Combined (rolling-horizon)		
	Obj [min]	O-gap [%]	Time [sec]	Obj [min]	O-gap [%]	Time [sec]	Obj [min]	O-gap [%]	Max stage time [sec]
I	8,090	0.00	15	8,090	0.00	93	8,090	0.00	45
II	9,170	0.00	21	9,170	0.00	100	9,170	0.00	45
III	10,089	0.00	45	10,089	0.00	69	10,089	0.00	45
IV	11,561	0.00	73	11,561	0.00	300	11,561	0.00	45
V	12,641	0.00	105	12,641	0.00	300	12,641	0.00	45
VI	13,560	0.00	231	13,560	0.00	260	13,560	0.00	45
VII	15,032	0.00	300	15,032	0.00	300	15,032	0.00	45
VIII	16,112	0.00	300	16,112	0.00	300	16,112	0.00	45
IX	17,031	0.00	300	17,031	0.00	300	17,031	0.00	45
X	18,503	0.00	300	18,503	0.00	300	18,503	0.00	45
XI	19,583	0.00	300	19,583	0.00	300	19,583	0.00	90
XII	20,518	0.08	300	20,506 ↓	0.02	300	20,502 ↓	0.00	45

From these results we conclude that the proposed rolling-horizon method is helpful to solve longer multiple connected disruptions by high-quality rescheduling solutions in an acceptable time. The value of h_r used in the rolling-horizon method affects the overall solution optimality due to the time limit of 300 s required for a stage computation. In different scenarios the appropriate setting of h_r can be different, but under whichever setting of h_r (1h, 1.5h, or 2h), the rolling-horizon solution method performs well regarding the solution quality.

5.7 Conclusions and future research

To deal with multiple connected disruptions that occur unexpectedly, this chapter proposed two approaches, the sequential approach and the combined approach. The sequential approach is based on the single-disruption rescheduling model proposed by Chapter 3, which solves disruptions one by one with the previous rescheduling decisions as reference. The combined approach is based on the multiple-disruption rescheduling model developed in this chapter, which reschedules all train services together each time an extra disruption occurs. Both approaches were applied to a subnetwork of the Dutch railways with 38 stations and 10 train lines operating half-hourly in each direction. Numerous experiments revealed that the combined approach resulted in less cancelled train services and/or train delays than the sequential approach. The outperformance of the combined approach may change with the overlapping duration between disruptions, and the change is relevant to the disruption locations. To deal with long multiple connected disruptions in a more efficient way, we proposed a new rolling-horizon method that is able to generate high-quality rescheduling solutions in an acceptable time. The case study applied both approaches to deal with two connected disruptions. In future work, we will test larger railway networks where three or more connected disruptions are more likely to happen. This may need the technique of decomposing the large-scale network into several coordinated local rescheduling zones to release the potential computational burden considering that the network scale and the number of disruptions both increase. In addition, it is important to take into account the uncertainty of disruption durations, for which both the technique of stochastic programming and a rolling-horizon method need to be employed. This is better to be explored from single-disruption cases first and then extended to multiple-disruption cases due to its complexity.

Appendix 5.A

Table 5.12: Notation

Symbol	Description
o_e	The original scheduled time of event e
tl_e	The corresponding train line of event e
tr_e	The corresponding train of event e
st_e	The corresponding station of event e
dr_e	The operation direction of event e
w	Cancellation penalty
n	The nth disruption that currently emerges
r_e	The previous rescheduled time of event e
t^i_{start}	The start time of the ith disruption, $1 \leq i \leq n$
t^i_{end}	The end time of the ith disruption, $1 \leq i \leq n$
$st^{i,dr}_{\text{en}}$	The entry station of the ith disrupted section in direction $dr_e \in \{up, down\}$
$st^{i,dr}_{\text{ex}}$	The exit station of the ith disrupted section in direction $dr_e \in \{up, down\}$
$tail(a)$	The tail of activity a: which is the event a that starts from
$head(a)$	The head of activity a: which is the event a that points to
D	The maximum allowed delay per event
L_a	The minimum duration of an activity a
M_1	A positive large number that is set to 1440
M_2	A positive large number that is set to twice of M_1: $M_2 = 2M_1$
$h_{e,e'}$	A minimum interval between the occurring times of events e and e' if corresponding to trains occupying the same station track
A_{run}	Set of running activities
A_{dwell}	Set of dwell activities
A_{pass}	Set of pass-through activities
A_{station}	Set of station activities: $A_{\text{station}} = A_{\text{dwell}} \cup A_{\text{pass}}$
A_{turn}	Set of short-turn activities
A^i_{turn}	Set of short-turn activities for the ith disruption: $A^i_{\text{turn}} \subset A_{\text{turn}}$

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Symbol	Description
A_{odturn}	Set of OD turn activities
E_{ar}	Set of arrival events
E_{de}	Set of departure events
E^{NMdelay}	Set of events that do not have upper limit on their delays
$E_{\text{ar}}^{\text{turn}}$	The subset of E_{ar}, which includes all tails of activities in A_{turn}: $E_{\text{ar}}^{\text{turn}} = \bigcup_{a \in A_{\text{turn}}} tail\,(a)$
$E_{\text{ar}}^{i,\text{turn}}$	The subset of $E_{\text{ar}}^{\text{turn}}$, which includes all tails of activities in A_{turn}^{i}: $E_{\text{ar}}^{i,\text{turn}} \subset E_{\text{ar}}^{\text{turn}}$
$E_{\text{ar}}^{\text{odturn}}$	The subset of E_{ar}, which includes all tails of activities in A_{odturn}: $E_{\text{ar}}^{\text{odturn}} = \bigcup_{a \in A_{\text{odturn}}} tail\,(a)$
$E_{\text{de}}^{\text{turn}}$	The subset of E_{de}, which includes all heads of activities in A_{turn}: $E_{\text{de}}^{\text{turn}} = \bigcup_{a \in A_{\text{turn}}} head\,(a)$
$E_{\text{de}}^{i,\text{turn}}$	The subset of $E_{\text{de}}^{\text{turn}}$, which includes all heads of activities in A_{turn}^{i}: $E_{\text{de}}^{i,\text{turn}} \subset E_{\text{de}}^{\text{turn}}$
$ST_{\text{en}}^{dr_e}$	Set of entry stations of all disrupted sections in direction $dr_e \in \{up, down\}$
$ST_{\text{ex}}^{dr_e}$	Set of exit stations of all disrupted sections in direction $dr_e \in \{up, down\}$
TR_{turn}	Set of trains that correspond to the events contained in $E_{\text{ar}}^{\text{turn}} \cup E_{\text{de}}^{\text{turn}}$
TR_{turn}^{i}	Set of trains that correspond to the events contained in $E_{\text{ar}}^{i,\text{turn}} \cup E_{\text{de}}^{i,\text{turn}}$

Appendix 5.B

Table 5.13: Notation used in Algorithm 5.1

Symbol	Description
$\tilde{t}_{\text{start}}^{k}$	The considered starting time of a disruption at stage k
$\tilde{t}_{\text{end}}^{i,k}$	The considered ending time of the ith disruption at stage k
h_r	The duration of a disruption considered at a stage (except the final stage)
n_k	The number of ongoing disruptions at stage k
DIS^{k}	The list of ongoing disruptions at stage k
$TL_{\text{dis},1}^{i}$	The set of train lines that is only affected by the ith disruption
ST_{tl}	The set of planned stopping and passage stations of train line tl
E_{cancel}^{k}	The set of events that are cancelled at stage k
$E_{\text{cancel}}^{\text{ar}}$	The set of cancelled arrival events
$E_{\text{keep}}^{\text{ar}}$	The set of kept arrival events
$E_{\text{ar}}^{tl,i}$	The set of arrival events from train line tl that is affected by the ith disruption
$E_{\text{ar}}^{st,tl,i}$	The set of arrival events occurring at station st and belonging to train line tl that is affected by the ith disruption

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Symbol	Description
$E_{\text{fix}}^{tl,i,k}$	The set of events that should follow the determined periodic pattern of train line tl at stage $k \geq 2$
$E_{\text{cancel}}^{tl,i,k}$	The set of events from train line tl, which should be cancelled at stage $k \geq 2$
$E_{\text{keep}}^{tl,i,k}$	The set of events from train line tl, which should be kept at stage $k \geq 2$

Chapter 6

Integrated timetable rescheduling and passenger reassignment during railway disruptions

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Apart from minor updates, this chapter has been submitted as:

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6.1 Introduction

Railway systems play an important role in people's daily travelling so that the operations are required as reliable as possible to ensure passenger punctuality. Unfortunately, unexpected disruptions occur in the railways on a daily basis (Zhu and Goverde, 2017b), during which many train services are delayed and cancelled that disturb passenger planned journeys significantly. When rescheduling a timetable in case of a disruption, traffic controllers decide which services have to be delayed or cancelled in terms of pre-designed contingency plans, where the impact on passengers is considered to a very limited extent (Ghaemi et al., 2017b). As a result, the rescheduled train services may not be passenger-friendly. For example, passengers may hardly find alternative train services to reach the expected destinations in reasonable travel times. To provide passengers with better alternatives during disruptions, it is necessary to reschedule a timetable in a more passenger-oriented way.

Passenger-oriented timetable rescheduling started from the field of *delay management* that decides whether a train should wait for a delayed feeder train to guarantee the transfer connection of some passengers. Schöbel (2001) is the first one dealing with this problem based on the assumption that if passengers missed the transfer connections, they would wait for a complete cycle time to catch the next connection considering that the planned timetable is periodic. Dollevoet et al. (2012) make an extension by introducing the possibility of rerouting passengers who are assumed to take the shortest paths for their following travels in case of transfer missing. Both papers describe the infrastructure at a macroscopic level neglecting signals and block sections. To improve solution feasibility in practice, Corman et al. (2016) propose a delay management model in which the infrastructure is described at a microscopic level. Albert et al. (2017) formulate passenger behaviours in stations (e.g. queueing in boarding trains) at a microscopic level to describe passenger influences on train delays rather than considering the impact of train delays on passenger behaviours only.

Delay management deals with the interaction between timetable and passengers, but not the interaction between timetable and reduced infrastructure availability, which however must be taken into account by disruption management. Operator-oriented disruption management considers only the latter kind of interaction, while passenger-oriented disruption management considers both kinds of interactions.

Most literature on disruption management is operator-oriented, including Meng and Zhou (2011); Louwerse and Huisman (2014); Veelenturf et al. (2015); Zhan et al. (2015,0); Ghaemi et al. (2017a,0); Zhu and Goverde (2019). The differences among these papers lie in the considered railway lines (single-track lines or double-track lines), the adopted dispatching measures, whether considering the transition from the planned timetable to the disruption timetable and vice versa, the extent of infrastructure description (macroscopic or microscopic level), the number of considered disruptions (single disruption or multiple disruptions), and/or the characteristic of disruption length (deterministic or uncertain). The similarity among these papers is that they all use operator-oriented objectives: e.g., minimizing train delays and/or cancellations, in which a constant cancellation penalty is used to represent the delay of cancelling each train. However, train delays are not equal to passenger delays that also depend on the amount of passengers and the route choice of passengers.

A few works focus on passenger-oriented disruption management. Cadarso et al. (2013) propose a two-step approach in which a frequency-based passenger assignment model is performed first to estimate the passenger demand and then a rescheduling model (for timetable and rolling stock) is solved to accommodate the passenger demand as much as possible. The adopted dispatching measures are limited to cancelling original trains and inserting additional trains. Chapter 3 adopts a schedule-based passenger assignment model to obtain the travel path of each passenger in terms of the planned timetable. With this information, the potential impact of each dispatching decision on passenger planned travels is estimated, which is used as weight in the objective to minimizing passenger delays. The adopted dispatching measures include re-timing, re-ordering, cancelling, flexible stopping (i.e. adding extra stops and skipping scheduled stops), and flexible short-turning (i.e. each train is given a full choice of short-turning station candidates). Both Cadarso et al. (2013) and Chapter 3 consider static passenger demand, which neglect that passengers may choose other travel paths rather than the planned ones due to the rescheduled train services. To formulate passenger behaviour in a more realistic way, it is necessary to take into account passenger responses towards the rescheduled train services. Veelenturf et al. (2017) propose an iterative approach that embeds a timetable rescheduling model and a passenger assignment model into an iterative framework where at each iteration an adjustment will be applied on the timetable if it reduces the total passenger inconvenience as evaluated by the passenger assignment model. The adjustments are restricted to adding stops. Binder et al. (2017b) propose an integrated approach of formulating the timetable rescheduling and the passenger assignment into one single model that computes a rescheduled timetable by an optimization solver directly. The applied dispatching measures include re-timing, re-ordering, cancelling, global re-routing and inserting additional trains. The rolling stock circulations at the short-turning and terminal stations of trains are neglected. Gao et al. (2016) also propose a timetable rescheduling model considering dynamic passenger flows, while focusing on the recovery phase of a disruption. As the target case is a metro corridor, all passengers are assumed to choose direct trains (i.e. no transfers). The dispatching measures of stop-skipping and re-timing are used to adjust the timetable to reduce passenger waiting times at stations. Due to the computational complexity, the master problem of generating a rescheduled timetable is decomposed into a series of sub-problems that each reschedules one train only. When solving a sub-problem for one train, the stopping patterns and time schedules of the previous considered trains are all fixed.

Compared to the literature, this chapter extends Chapter 3 by a new formulation to integrate dynamic passenger behaviour with timetable rescheduling. The main challenges lie in two aspects. First, it is difficult to formulate the dynamic interactions between passengers and rescheduled timetables: a rescheduled timetable affects passenger path choices and vice versa. A path refers to a sequence of train services from his/her origin to the destination, which has multiple attributes like in-vehicle times, the number of transfers, and waiting times at stations. A rescheduled timetable determines whether a path is available, and also the attributes of the path. The attributes of the paths affect passenger path choices which in turn affect how the timetable should be rescheduled to provide paths with better attributes that can mitigate passenger inconveniences. The difficulty of formulating the dynamic interactions between passengers and rescheduled timetables increases further when allowing both flexible stopping and flexible short-turning trains, which can lead to more changes to planned services and thereby more path options to passengers whose responses towards these options might be different and have to be modelled properly. The second challenge is designing an efficient algorithm to solve the integrated passenger-oriented timetable rescheduling model with high-quality solutions in an acceptable time. This has been reported as a challenging task in the literature so far (Corman et al., 2016; Binder et al., 2017b).

The key contributions of this chapter are summarized as follows:

- Passenger re-routing and timetable rescheduling are integrated into a new Mixed

Integer Linear Programming (MILP) model, which applies re-timing, re-ordering, cancelling, flexible stopping and flexible short-turning trains to obtain a rescheduled timetable with the objective of minimizing generalized travel times (i.e. weighted travel times considering passenger preferences on waiting times at origin/transfer stations, in-vehicle times, and the number of transfers).

- An iterative algorithm is designed to solve the passenger-oriented timetable rescheduling model with high-quality solutions in an acceptable time.
- The passenger-oriented timetable rescheduling model is tested on real-life instances on a subnetwork of the Dutch railways, and compared to an operator-oriented timetable rescheduling model.
- It is shown that the passenger-oriented model is able to generate rescheduling solutions with shorter generalized travel times than an operator-oriented model.

In this chapter, each train is assumed to have unlimited capacity, which means that a passenger is able to board any train if he/she decides to board this train. This is because we focus on providing better alternative train services to passengers so that the possible impact of vehicle capacity on passengers is neglected. In this way, we can get the optimal rescheduled timetable in terms of generalized travel times. This optimal rescheduled timetable can then be used as an input to rolling stock rescheduling that aims to accommodate the passenger demand as much as possible. For example, Kroon et al. (2014) and Van der Hurk et al. (2018) both deal with passenger-oriented rolling stock rescheduling with a rescheduled timetable given as input. We also assume that the duration of a disruption is known at the beginning of the disruption, and will not change over time.

The remainder of the chapter is organized as follows. Section 6.2 introduces the general framework of establishing the passenger-oriented timetable rescheduling model. Section 6.3 explains how to formulate a timetable into an event-activity network to describe passenger path choices. The planned timetable can be formulated into an event-activity network Ω_{plan}, which is then extended to a transition network Ω^* that enables the dynamic formulation of event-activity networks during timetable rescheduling. The method of constructing a transition network is introduced in Section 6.4. Based on a transition network, the passenger-oriented timetable rescheduling model is proposed in Section 6.5 followed by Section 6.6 that introduces the methods of reducing the computational complexity of the model. In Section 6.7, numerous experiments were carried out to a part of the Dutch railways. Finally, Section 6.8 concludes the chapter and points out future research directions.

6.2 General framework

This chapter integrates timetable rescheduling with passenger re-routing into an MILP model, for which two processing steps are needed. Figure 6.1 gives an overview of the model.

The first preprocessing step transforms the planned timetable into an event-activity network Ω_{plan}, which is a directed acyclic graph used to describe passenger path choices. The method of constructing an event-activity network from a timetable is introduced in Section 6.3. In case of a disruption, the planned timetable will become infeasible, and so does the corresponding event-activity network Ω_{plan} that now is unable to reflect the paths currently available in the railways. Under this circumstance, the timetable has to be rescheduled, and during rescheduling the corresponding event-activity networks have to be updated as well to consider timetable-dependent passenger behaviours. To enable a dynamic event-activity network formulation during timetable rescheduling, we perform the second preprocessing step to construct a transition network Ω^*. A transition network is extended from the event-activity network Ω_{plan} by adding all events and activities that could exist in any event-activity network Ω_{dis} corresponding to a feasible rescheduled timetable obtained for a specific disruption. In other words, $\Omega^* = \bigcup_i \Omega^i_{\text{dis}} \cup \Omega_{\text{plan}}$, where Ω^i_{dis} refers to the event-activity network corresponding to the ith feasible rescheduled timetable. For one specific disruption there are usually multiple feasible rescheduled timetables. Note that Ω^* varies with the disruption characteristics (i.e. location and starting/ending time) and the dispatching measures allowed. A transition network Ω^* is *not* a directed acyclic graph as it includes the possibility of changing the order of trains. The method of constructing a transition network is introduced in Section 6.4.

The constructed transition network, the planned timetable, the disruption characteristics, and the allowed dispatching measures are all necessary inputs to establish the passenger-oriented timetable rescheduling model, which is formulated as an MILP in this chapter. This model consists of the constraints for three purposes: 1) timetable rescheduling, 2) dynamic event-activity network formulation, and 3) passenger reassignment. The timetable rescheduling constraints ensure a rescheduled timetable does not violate any infrastructure and operational restrictions. The constraints relevant to the dynamic event-activity network formulation decide which activities and events of Ω^* should be selected to construct an event-activity network Ω_{dis} in terms of a rescheduled timetable. The passenger reassignment constraints decide the weight of each activity of Ω_{dis} from the perspectives of passengers, and assign each passenger to one path only. A path is described by a sequence of connected activities. The total activity weight of a path is the generalized travel time of this path. The objective of the model is minimizing the generalized travel times of all passengers. By this model, a rescheduled timetable that leads to the shortest generalized travel times of all passengers can be obtained, as well as the path chosen by each passenger under the rescheduled timetable.

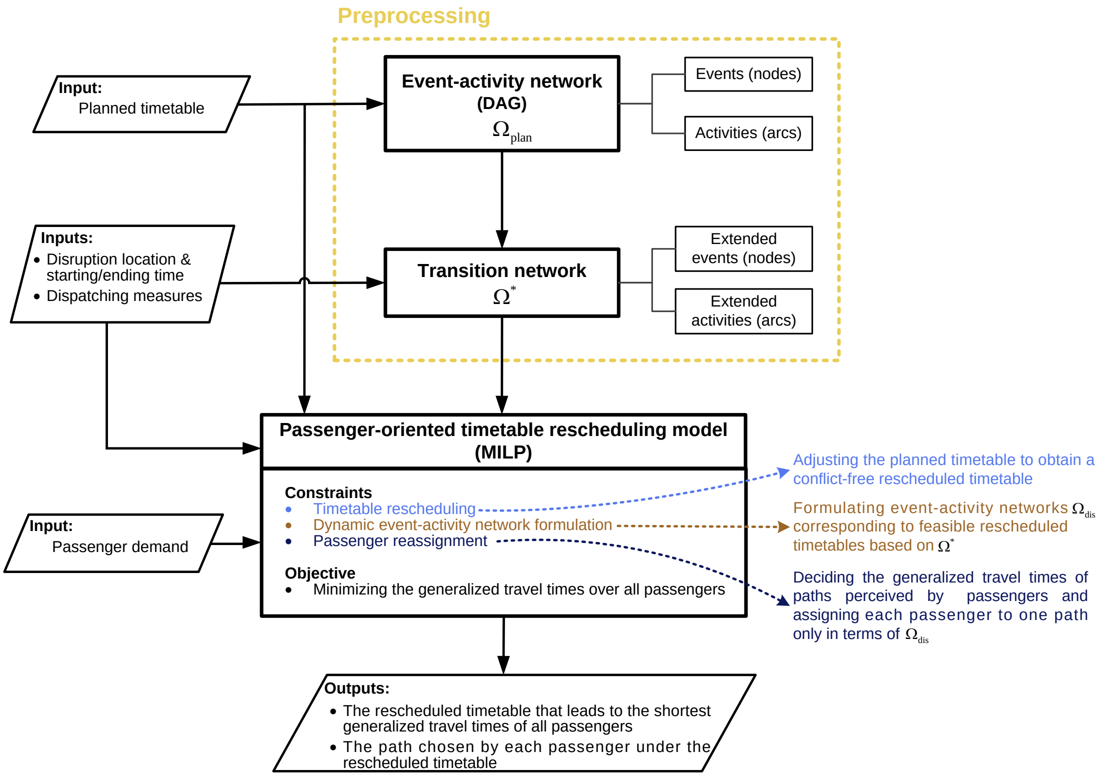


Figure 6.1: An overview of the passenger-oriented timetable rescheduling model

6.3 Event-activity network

An event-activity network is a representation of a timetable, based on which passenger path choices can be described. Chapter 2 introduce a method of constructing such an event-activity network from a timetable, which is improved in this chapter by including more kinds of passenger-related events/activities and redefining certain types of activities in a more reasonable way while still ensuring the constructed event-activity network is a directed acyclic graph. For example in this chapter, whether a train arrival/departure event corresponds to passenger boarding/alighting is explicitly formulated, penalty events/activities are introduced to formulate passenger leaving the railways, and the transfer activities defined in Chapter 2 are redefined as boarding activities, for which the weights are also calculated differently here. More details about the improved event-activity network formulation method are introduced as follows.

6.3.1 Events

Six types of events are created in an event-activity network. They are arrival events, departure events, duplicate departure events, entry events, exit events and a penalty event, which constitute the sets E_{ar}, E_{de}, E_{dde}, E_{entry}, E_{exit} and E_{penal}, respectively. Therefore, the set of events is

$$E = E_{\text{ar}} \cup E_{\text{de}} \cup E_{\text{dde}} \cup E_{\text{entry}} \cup E_{\text{exit}} \cup E_{\text{penal}}.$$

In particular,

$$E_{\mathrm{ar}} = E_{\mathrm{ar}}^{\mathrm{alight}} \cup E_{\mathrm{ar}}^{\mathrm{pass}}, \text{ and } E_{\mathrm{de}} = E_{\mathrm{de}}^{\mathrm{board}} \cup E_{\mathrm{de}}^{\mathrm{pass}},$$

where $E_{\mathrm{ar}}^{\mathrm{alight}}$ is the set of arrival events that correspond to passenger alighting, and $E_{\mathrm{de}}^{\mathrm{board}}$ is the set of departure events that correspond to passenger boarding. For example, if a train stops at a station, then its arrival (departure) event at this station corresponds to passenger alighting (boarding). The arrival (departure) events associated to a through train that do *not* correspond to passenger alighting (boarding) constitute the set of $E_{\mathrm{ar}}^{\mathrm{pass}}$ ($E_{\mathrm{de}}^{\mathrm{pass}}$).

Each event $e \in E \backslash E_{\mathrm{penal}}$ is assigned with an attribute st_e to indicate the corresponding station of e. For event $e \in E_{\mathrm{ar}} \cup E_{\mathrm{de}}$, the attributes of tr_e, tl_e and o_e are assigned, which refer to the corresponding train, train line and scheduled time of e, respectively. A train line is served by a series of trains that are operated with the same stopping pattern between the same origin and destination under certain frequency. An event $e \in E_{\mathrm{dde}}$ is the duplicate of a departure event $e' \in E_{\mathrm{de}}^{\mathrm{board}}$ with exactly the same attributes which e' has, and with an extra attribute λ_e to indicate the departure event e' corresponding to e:

$$E_{\mathrm{dde}} = \left\{ e \middle| \lambda_e = e', e' \in E_{\mathrm{de}}^{\mathrm{board}} \right\}.$$

One and only one duplicate is created for a departure event $e' \in E_{\mathrm{de}}^{\mathrm{board}}$. Duplicate departure events are used for constructing wait, boarding and transfer activities, which are explained in more detail in Section 6.3.2. Note that this chapter defines these activities differently than Chapter 2. As for E_{penal}, it contains only one penalty event for constructing the penalty arcs that enable passengers who cannot find preferred paths to leave the railways. Table 6.1 shows the notation of event attributes.

Table 6.1: Event attributes

Symbol	Description
st_e	The corresponding station of event $e \in E \backslash E_{\mathrm{penal}}$
tr_e	The corresponding train of event $e \in E_{\mathrm{ar}} \cup E_{\mathrm{de}} \cup E_{\mathrm{dde}}$
tl_e	The corresponding train line of event $e \in E_{\mathrm{ar}} \cup E_{\mathrm{de}} \cup E_{\mathrm{dde}}$
λ_e	The corresponding departure event of $e \in E_{\mathrm{dde}}$
o_e	The scheduled time of event $e \in E_{\mathrm{ar}} \cup E_{\mathrm{de}} \cup E_{\mathrm{dde}}$

6.3.2 Activities

An activity is a directed arc between two different events. Ten types of activities are created in an event-activity network. They are running activities, dwell activities, pass-through activities, wait activities, transfer activities, boarding activities, entry activities, exit activities, entry penalty activities and exit penalty activities, which constitute the sets A_{run}, A_{dwell}, A_{pass}, A_{wait}, A_{trans}, A_{board}, A_{entry}, A_{exit}, A_{enpenal} and A_{expenal}, respectively. Therefore, the set of activities is

$$A = A_{\mathrm{run}} \cup A_{\mathrm{dwell}} \cup A_{\mathrm{pass}} \cup A_{\mathrm{wait}} \cup A_{\mathrm{trans}} \cup A_{\mathrm{board}} \cup A_{\mathrm{entry}} \cup A_{\mathrm{exit}} \cup A_{\mathrm{enpenal}} \cup A_{\mathrm{expenal}}.$$

Different activities are established between different events depending on specific rules that are explained as follows.

Entry activities enable passengers to enter the railways when arriving at the origins,

$$A_{\text{entry}} = \left\{ \left(e, e'\right) \left| e \in E_{\text{entry}}, e' \in E_{\text{dde}}, st_e = st_{e'} \right. \right\}.$$

Exit activities enable passengers to leave the railways when arriving at the destinations,

$$A_{\text{exit}} = \left\{ \left(e, e'\right) \left| e \in E_{\text{ar}}^{\text{alight}}, e' \in E_{\text{exit}}, st_e = st_{e'} \right. \right\}.$$

Entry penalty activities and *exit penalty activities* together enable passengers to drop the railways in case no preferred paths can be found,

$$A_{\text{enpenal}} = \left\{ \left(e, e'\right) \left| e \in E_{\text{entry}}, e' \in E_{\text{penal}} \right. \right\},$$
$$A_{\text{expenal}} = \left\{ \left(e' e\right) \left| e' \in E_{\text{penal}}, e \in E_{\text{exit}} \right. \right\}.$$

Boarding activities enable passengers to board a train,

$$A_{\text{board}} = \left\{ \left(e, e'\right) \left| e \in E_{\text{dde}}, e' \in E_{\text{de}}^{\text{board}}, e' = \lambda_e \right. \right\},$$

where each duplicate departure event is linked to its corresponding departure event.

Running activities enable passengers to travel from one station to another in a train,

$$A_{\text{run}} = \left\{ \left(e, e'\right) \left| e \in E_{\text{de}}, e' \in E_{\text{ar}}, tr_e = tr_{e'}, st_e \text{ is the upstream station adjacent to } st_{e'} \right. \right\}.$$

Dwell activities enable passengers to wait at a station in a train,

$$A_{\text{dwell}} = \left\{ \left(e, e'\right) \left| e \in E_{\text{ar}}^{\text{alight}}, e' \in E_{\text{de}}^{\text{board}}, tr_e = tr_{e'}, st_e = st_{e'}, o_{e'} - o_e > 0 \right. \right\}.$$

Pass-through activities enable passengers to pass through a station in a train,

$$A_{\text{pass}} = \left\{ \left(e, e'\right) \left| e \in E_{\text{ar}}^{\text{pass}}, e' \in E_{\text{de}}^{\text{pass}}, tr_e = tr_{e'}, st_e = st_{e'}, o_{e'} - o_e = 0 \right. \right\}.$$

Wait activities enable passengers to wait at a station,

$$A_{\text{wait}} = \left\{ \left(e, e'\right) \left| e \in E_{\text{dde}}, e' = \arg\min \left\{ o_{e'} | o_{e'} \geq o_e, e' \in E_{\text{dde}}, tr_{e'} \neq tr_e, st_{e'} = st_e \right\} \right. \right\},$$

where each duplicate departure event is linked to the next time-closest duplicate departure event that is at the same station but corresponds to another train.

Transfer activities enable passengers to transfer from one train to another,

$$A_{\text{trans}} = \left\{ \left(e, e'\right) \left| e \in E_{\text{ar}}^{\text{alight}}, e' = \arg\min \left\{ o_{e'} \left| o_{e'} \geq o_e + \ell_{e,e'}^{\text{trans}}, e' \in E_{\text{dde}}, tr_{e'} \neq tr_e, \right. \right. \right. \right.$$
$$\left. \left. st_{e'} = st_e \right\} \right\},$$

where each arrival event is linked to the next time-closest duplicate departure event that occurs at least $\ell_{e,e'}^{\text{trans}}$ later at the same station but corresponds to another train. Here, $\ell_{e,e'}^{\text{trans}}$ represents the minimum transfer time required from the arrival train tr_e to another departure train $tr_{e'}$, which are alongside the same platform or different platforms affecting the value of $\ell_{e,e'}^{\text{trans}}$.

An event-activity network is $\Omega = E \cup A$, which is a directed acyclic graph (DAG). An example of formulating an event-activity network from a timetable is given below. **Example 1**: Figure 6.2 shows a timetable with three stations A, B and C, and four trains $\text{tr}_1, \text{tr}_2, \text{tr}_3$ and tr_4. Train tr_1 runs from station A to station C with a stop at B. Train tr_2 runs from station A to station C directly. Trains tr_3 and tr_4 both run from station B to station C. The event-activity network formulated from this timetable is shown in Figure 6.3.

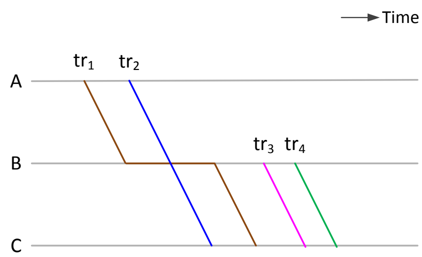


Figure 6.2: A timetable with three stations and four trains

Suppose a passenger plans to travel from station A to station C, and he/she arrives at station A before train tr_1 departs from station A. Then, the passenger has five alternative paths.

- Path 1 is taking train tr_1 from station A to station C directly:
  (entry, A) $\overset{\text{entry}}{\to}$ (dde, tr_1, A) $\overset{\text{board}}{\to}$ (de, tr_1, A) $\overset{\text{run}}{\to}$ (ar, tr_1, B) $\overset{\text{dwell}}{\to}$ (de, tr_1, B) $\overset{\text{run}}{\to}$ (ar, tr_1, C) $\overset{\text{exit}}{\to}$ (exit ,C);

- Path 2 is taking train tr_2 from station A to station C directly:
  (entry, A) $\overset{\text{entry}}{\to}$ (dde, tr_1, A) $\overset{\text{wait}}{\to}$ (dde, tr_2, A) $\overset{\text{board}}{\to}$ (de, tr_2, A) $\overset{\text{run}}{\to}$ (ar, tr_2, B) $\overset{\text{pass}}{\to}$ (de, tr_2, B) $\overset{\text{run}}{\to}$ (ar, tr_2, C) $\overset{\text{exit}}{\to}$ (exit ,C);

- Path 3 is taking train tr_1 at station A and then transferring to train tr_3 at station B to reach station C:
  (entry, A) $\overset{\text{entry}}{\to}$ (dde, tr_1, A) $\overset{\text{board}}{\to}$ (de, tr_1, A) $\overset{\text{run}}{\to}$ (ar, tr_1, B) $\overset{\text{transfer}}{\to}$ (dde, tr_3, B) $\overset{\text{board}}{\to}$ (de, tr_3, B) $\overset{\text{run}}{\to}$ (ar, tr_3, C) $\overset{\text{exit}}{\to}$ (exit ,C);

- Path 4 is taking train tr_1 at station A and then transferring to train tr_4 at station B to reach station C:
  (entry, A) $\overset{\text{entry}}{\to}$ (dde, tr_1, A) $\overset{\text{board}}{\to}$ (de, tr_1, A) $\overset{\text{run}}{\to}$ (ar, tr_1, B) $\overset{\text{transfer}}{\to}$ (dde, tr_3, B) $\overset{\text{wait}}{\to}$ (dde, tr_4, B) $\overset{\text{board}}{\to}$ (de, tr_4, B) $\overset{\text{run}}{\to}$ (ar, tr_4, C) $\overset{\text{exit}}{\to}$ (exit ,C);

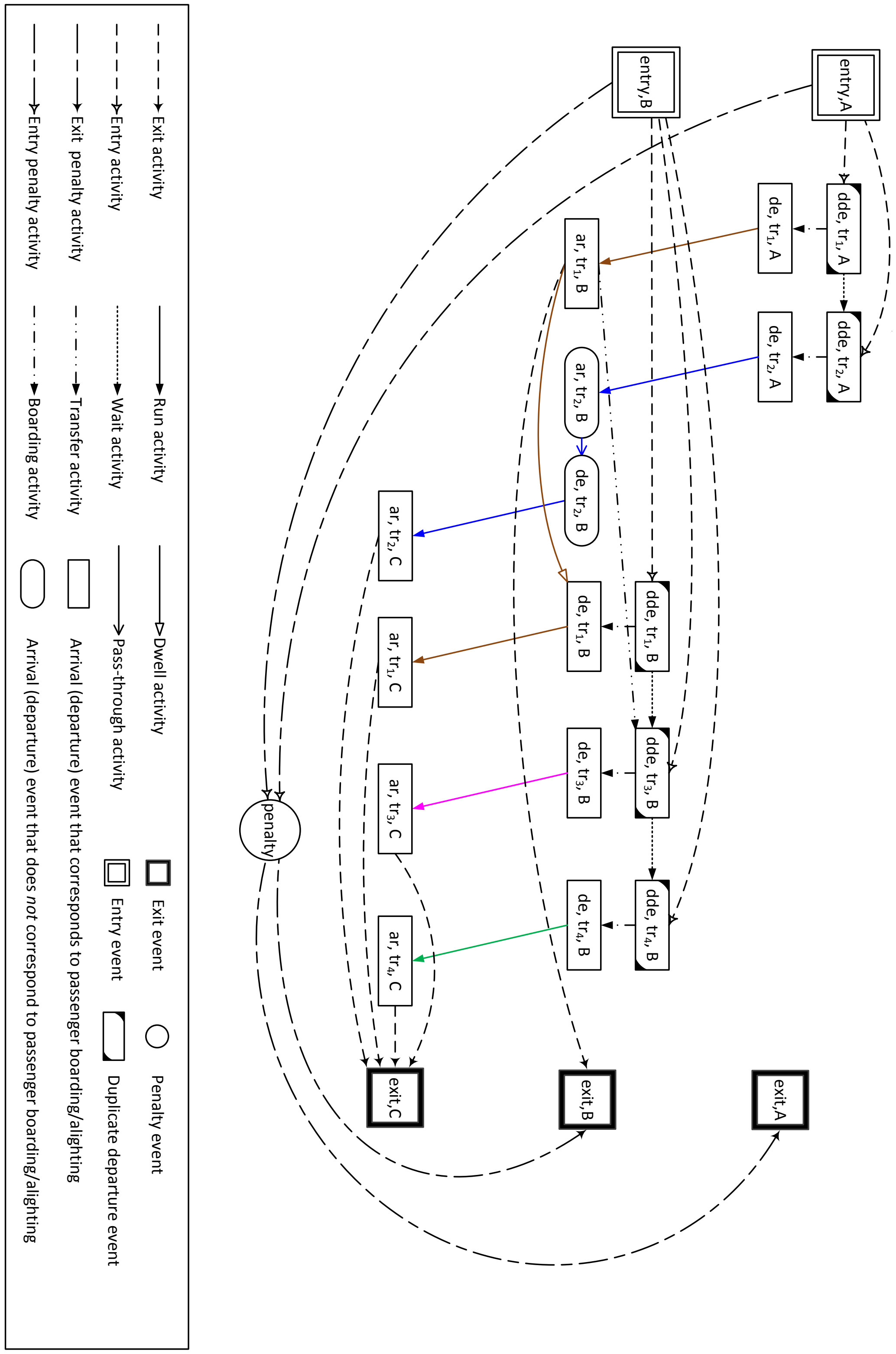


Figure 6.3: The event-activity network constructed from the timetable of Figure 6.2

- Path 5 is leaving the railways:
  (entry, A) $\overset{\text{entry}}{\to}$ (penalty) $\overset{\text{exit}}{\to}$ (exit, A).

Which of these five paths will be chosen by the passenger depends on the generalized travel times of these paths. The generalized travel time of a path is obtained by summing up the weights of activities in the path.

6.3.3 Weights of activities

For running, dwell and pass-through activities, the weights are set to the time differences between the corresponding events multiplied by β_{vehicle} that represents passenger preference on in-vehicle times. For wait activities, the weights are set to the time differences between the corresponding events multiplied by β_{wait} that represents passenger preference on waiting times at stations. For each transfer activity, the weight is set to the sum of the time difference between the corresponding events multiplied by β_{wait} and a fixed value β_{trans} representing the time penalty of one transfer.

The weight of an entry activity is passenger-dependent. Suppose a passenger g arrives at the origin at time t_g^{ori}. Then the passenger can only choose an entry activity (e, e') that corresponds to his/her origin and of which the time of the corresponding duplicate departure event e' occurs no earlier than t_g^{ori}. For example in Figure 6.3, if station A is the origin of passenger g and t_g^{ori} is between the time of event (dde, tr_1, A) and the time of (dde, tr_2, A), then g can only choose the entry activity from (entry, A) to (dde, tr_2, A), of which the weight is set to the multiplied difference between the time of (dde, tr_2, A) and t_g^{ori} by β_{wait}. The weight of an entry penalty activity is also passenger-dependent. If the maximum generalized travel time accepted by passenger g is $T_g^{\max}$, the weight of an entry penalty activity perceived by this passenger will be set to $T_g^{\max}$.

The weights of boarding, exit and exit penalty activities are all set to 0, as these activities are not used to distinguish paths.

6.4 Transition network

According to the approach of Section 6.3, the planned timetable can be formulated as an event-activity network Ω_{plan} to describe passenger path choices on normal days. When a disruption occurs, the planned timetable has to be rescheduled, while during the rescheduling the corresponding event-activity networks Ω_{dis} have to be formulated to describe the alternative paths currently available to passengers. To enable a dynamic event-activity network formulation during timetable rescheduling, a transition network is introduced.

A transition network Ω^* is extended from the event-activity network Ω_{plan} by adding all events and activities that could exist in any event-activity network Ω_{dis} corresponding to a feasible rescheduled timetable obtained for a specific disruption. A transition network varies with the characteristics of the disruption, as well as the applied dispatching measures. In this chapter, we consider the disruption of complete track blockages between two stations, and apply the dispatching measures of re-timing, re-ordering, cancelling, flexible stopping, and flexible short-turning. Before giving the details of constructing a transition network, an example on a simple case is given below to explain the basic idea.

Example 2: Figure 6.4 shows a planned timetable with three stations A, B and C, and two trains tr_1 and tr_2. Train tr_1 runs from station A to station C by intermediately stopping at station B, while train tr_2 runs from station A to station C directly. The constructed transition network Ω^* from the timetable is located in the blue box.

In Figure 6.4, Ω^* is extended from Ω_{plan} by adding a new event and new activities (both are colored in orange) that do not exist in the planned timetable but could exist in a rescheduled timetable. Due to the dispatching measure of re-ordering, train tr_1 could depart later than train tr_2 at station A, although train tr_1 was originally planned to depart earlier than train tr_2. Considering this possible train order change, an extra wait activity is added from event (dde, tr_2, A) to event (dde, tr_1, A). Due to the dispatching measure of flexible stopping, an extra stop could be added to train tr_2 at station B. Thus, a new event (dde, tr_2, B) is added as well as six new activities: 1) an entry activity from event (entry, B) to event (dde, tr_2, B); 2) a boarding activity from event (dde, tr_2, B) to event (de, tr_2, B); 3) a wait activity from event (dde, tr_2, B) to event (dde, tr_1, B) in case train tr_2 departs before train tr_1 at station B; 4) a wait activity from event (dde, tr_1, B) to event (dde, tr_2, B) in case train tr_1 departs before train tr_2 at station B; 5) a transfer activity from event (ar, tr_1, B) to event (dde, tr_2, B) considering passengers in train tr_1 may transfer to train tr_2; and 6) an exit activity from event (ar, tr_2, B) to event (exit, B). As can be seen entry/exit penalty activities always remain the same no matter what changes made to the planned timetable.

In the following, we introduce how to construct a transition network by extending the event-activity network Ω_{plan} corresponding to a planned timetable. The set notation with the superscript of *plan* represents the events/activities sets in Ω_{plan}. Table 6.2 shows the notation of sets relevant to a transition/event-activity network.

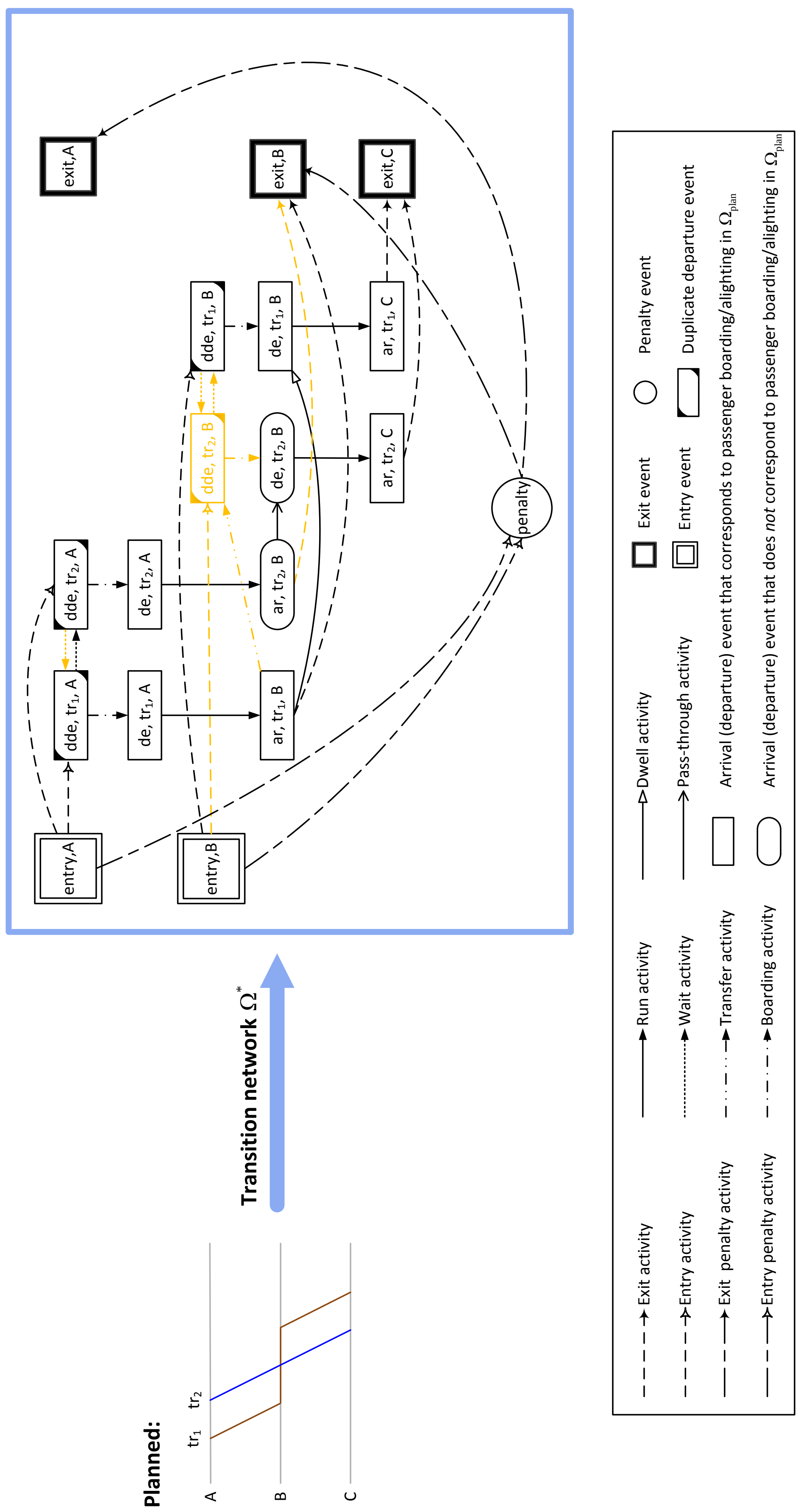


Figure 6.4: A planned timetable with the constructed event-activity network and transition network

Table 6.2: Sets relevant to a transition/event-activity network

Notation	Description
Ω^*	Transition network: $\Omega^* = E^* \cup A^*$
Ω_{plan}	Event-activity network formulated from the planned timetable: $\Omega_{\text{plan}} = E^{\text{plan}} \cup A^{\text{plan}}$ and $\Omega_{\text{plan}} \subset \Omega^*$
Ω_{dis}	Event-activity network formulated from any possible disruption timetable by adjusting the planned timetable: $\Omega_{\text{dis}} \subset \Omega^*$
E^*	Set of events in Ω^*
E^{plan}	Set of events in Ω_{plan}: $E^{\text{plan}} \subset E^*$
E_i^{plan}	Set of i events in Ω_{plan}, $i \in \{\text{ar}, \text{de}, \text{dde}, \text{entry}, \text{exit}, \text{penal}\}$: $E_i^{\text{plan}} \subset E^{\text{plan}}$
$E_{\text{ar}}^{\text{alight,plan}}$	Set of arrival events that correspond to passenger alighting in Ω_{plan}: $E_{\text{ar}}^{\text{alight,plan}} \subseteq E_{\text{ar}}^{\text{plan}}$
$E_{\text{ar}}^{\text{pass,plan}}$	Set of arrival events that do *not* correspond to passenger alighting in Ω_{plan}: $E_{\text{ar}}^{\text{pass,plan}} = E_{\text{ar}}^{\text{plan}} \setminus E_{\text{ar}}^{\text{alight,plan}}$
$E_{\text{de}}^{\text{board,plan}}$	Set of departure events that correspond to passenger boarding in Ω_{plan}: $E_{\text{de}}^{\text{board,plan}} \subseteq E_{\text{de}}^{\text{plan}}$
$E_{\text{de}}^{\text{pass,plan}}$	Set of departure events that do *not* correspond to passenger boarding in Ω_{plan}: $E_{\text{de}}^{\text{pass,plan}} = E_{\text{de}}^{\text{plan}} \setminus E_{\text{de}}^{\text{board,plan}}$
A^*	Set of activities in Ω^*
A_i^*	Set of i activities in Ω^*: $A_i^* \subset A^*$, $i \in \{\text{wait}, \text{trans}, \text{board}, \text{entry}, \text{exit}\}$
A^{plan}	Set of activities in Ω_{plan}: $A^{\text{plan}} \subset A^*$
A_i^{plan}	Set of i activities in Ω_{plan}: $A_i^{\text{plan}} \subset A^{\text{plan}}$, $i \in \{\text{run}, \text{dwell}, \text{pass}, \text{wait}, \text{trans}, \text{board}, \text{entry}, \text{exit}, \text{enpenal}, \text{expenal}\}$
A_k^{undis}	Set of undisrupted k activities in Ω^*: $A_k^{\text{undis}} \subset A_k^{\text{plan}}$, $k \in \{\text{run}, \text{dwell}, \text{pass}, \text{wait}, \text{trans}, \text{board}, \text{entry}, \text{exit}\}$
$A_{k_1}^{\text{dis}}$	Set of disrupted k_1 activities in Ω^*: $A_{k_1}^{\text{dis}} = A_{k_1}^{\text{plan}} \setminus A_{k_1}^{\text{undis}}$, $k_1 \in \{\text{run}, \text{dwell}, \text{pass}\}$
$A_{k_2}^{\text{dis}}$	Set of disrupted k_2 activities in Ω^*: $A_{k_2}^{\text{dis}} = A_{k_2}^* \setminus A_{k_2}^{\text{undis}}$, $k_2 \in \{\text{wait}, \text{trans}, \text{board}, \text{entry}, \text{exit}\}$

6.4.1 Extended events

In a transition network Ω^*, the set of events E^* is defined as

$$E^* = E_{\text{ar}}^{\text{plan}} \cup E_{\text{de}}^{\text{plan}} \cup E_{\text{dde}}^* \cup E_{\text{entry}}^{\text{plan}} \cup E_{\text{exit}}^{\text{plan}} \cup E_{\text{penal}}^{\text{plan}},$$

where only the set of duplicate departure events E_{dde}^* is extended: $E_{\text{dde}}^* \supseteq E_{\text{dde}}^{\text{plan}}$. Recall that in an event-activity network, duplicates are only created for departure events that correspond to passenger boarding. Instead in a transition network, a duplicate is created for each departure event $e \in E_{\text{de}}$ no matter e corresponds to passenger boarding or not in a planned timetable. In other words,

$$E^*_{\text{dde}} = \left\{ e \middle| \lambda_e = e', e' \in E^{\text{plan}}_{\text{de}} \right\},$$

where $E^{\text{plan}}_{\text{de}} = E^{\text{board,plan}}_{\text{de}} \cup E^{\text{pass,plan}}_{\text{de}}$. Here, $E^{\text{board,plan}}_{\text{de}}$ refers to the set of departure events that correspond to passenger boarding in the planned timetable, and $E^{\text{pass,plan}}_{\text{de}}$ represents the set of departure events that do *not* correspond to passenger boarding in the planned timetable.

6.4.2 Extended activities

In a transition network Ω^*, the set of activities A^* is defined as

$$A^* = A^{\text{plan}}_{\text{run}} \cup A^{\text{plan}}_{\text{dwell}} \cup A^{\text{plan}}_{\text{pass}} \cup A^*_{\text{wait}} \cup A^*_{\text{trans}} \cup A^*_{\text{board}} \cup A^*_{\text{entry}} \cup A^*_{\text{exit}} \cup A^{\text{plan}}_{\text{enpenal}} \cup A^{\text{plan}}_{\text{expenal}},$$

where five types of activities are extended: $A^*_{\text{wait}}, A^*_{\text{trans}}, A^*_{\text{board}}, A^*_{\text{entry}}$ and A^*_{exit}. In other words, we have $A^*_k \supseteq A^{\text{plan}}_k, k \in \{\text{wait}, \text{trans}, \text{board}, \text{entry}, \text{exit}\}$.

To distinguish whether an activity could be affected by the disruption, each type of activities is further classified into two categories: undisrupted or disrupted, except entry/exit penalty activities. Thus, we have

$$\begin{aligned} A^{\text{plan}}_i &= A^{\text{undis}}_i \cup A^{\text{dis}}_i, \qquad && i \in \{\text{run}, \text{dwell}, \text{pass}\}, \\ A^*_k &= A^{\text{undis}}_k \cup A^{\text{dis}}_k, && k \in \{\text{wait}, \text{trans}, \text{board}, \text{entry}, \text{exit}\}, \end{aligned}$$

An activity is defined as undisrupted, if *both* of the corresponding events will *not* be delayed/cancelled due to the considered disruption. Otherwise, the activity will be defined as disrupted. In this chapter, we ensure an arrival (departure) event that was originally scheduled to occur before the disruption starting time t_{start} or at least R minutes later than the disruption ending time t_{end} will not be delayed/cancelled, in which R is the time length required for the normal schedule to be recovered after the disruption ends. This also applies to duplicate departure events, which are always with the same occurrence times as their corresponding departure events. In other words, event $e \in E^{\text{plan}}_{\text{ar}} \cup E^{\text{plan}}_{\text{de}} \cup E^*_{\text{dde}}$ will not be delayed/cancelled in any feasible rescheduled timetables, if the original scheduled time o_e is *not* within the time period $[t_{\text{start}}, t_{\text{end}} + R)$. Otherwise, this event could be delayed/cancelled in a rescheduled timetable. Based on these, we decide whether an activity is undisrupted or disrupted as follows.

6.4.2.1 Running, dwell, and pass-through activities

Disrupted running, dwell, or pass-through activities are defined as:

$$A^{\text{dis}}_i = \left\{ (e, e') \in A^{\text{plan}}_i | t_{\text{start}} \le o_e < t_{\text{end}} + R \text{ or } t_{\text{start}} \le o_{e'} < t_{\text{end}} + R \right\}, \\ i \in \{\text{run}, \text{dwell}, \text{pass}\},$$

where o_e refers to the original scheduled time of e, t_{start} (t_{end}) represents the start (end) time of the disruption, and R represents the duration required for the disruption timetable resuming to the planned timetable after the disruption ends. Undisrupted running, dwell, or pass-through activities are defined as $A_i^{\text{undis}} = A_i^{\text{plan}} \backslash A_i^{\text{dis}}, i \in \{\text{run}, \text{dwell}, \text{pass}\}$.

6.4.2.2 Entry activities

The planned entry activities that could become inapplicable due to the disruption are defined as

$$A_{\text{entry}}^{\text{dis},1} = \left\{ \left(e, e'\right) \in A_{\text{entry}}^{\text{plan}} | t_{\text{start}} \leq o_{e'} < t_{\text{end}} + R \right\}.$$

The entry activities that are not in Ω_{plan} but might be needed due to extra stops added in a rescheduled timetable are defined as

$$A_{\text{entry}}^{\text{dis},2} = \left\{ \left(e, e'\right) \middle| e \in E_{\text{entry}}^{\text{plan}}, e' \in E_{\text{dde}}^{*} \backslash E_{\text{dde}}^{\text{plan}}, st_e = st_{e'}, t_{\text{start}} \leq o_{e'} < t_{\text{end}} + R \right\}.$$

Then, the disrupted entry activities are defined as $A_{\text{entry}}^{\text{dis}} = A_{\text{entry}}^{\text{dis},1} \cup A_{\text{entry}}^{\text{dis},2}$, while the undisrupted entry activities are defined as $A_{\text{entry}}^{\text{undis}} = A_{\text{entry}}^{\text{plan}} \backslash A_{\text{entry}}^{\text{dis},1}$.

6.4.2.3 Exit activities

The planned exit activities that could become inapplicable due to the disruption are defined as:

$$A_{\text{exit}}^{\text{dis},1} = \left\{ \left(e, e'\right) \in A_{\text{exit}}^{\text{plan}} | t_{\text{start}} \leq o_{e} < t_{\text{end}} + R \right\},$$

The exit activities that are not in Ω_{plan} but might be needed due to extra stops added in a rescheduled timetable are defined as

$$A_{\text{exit}}^{\text{dis},2} = \left\{ \left(e, e'\right) \middle| e \in E_{\text{ar}}^{\text{pass,plan}}, e' \in E_{\text{exit}}^{\text{plan}}, st_e = st_{e'}, t_{\text{start}} \leq o_{e} < t_{\text{end}} + R \right\}.$$

Then, the disrupted exit activities are defined as $A_{\text{exit}}^{\text{dis}} = A_{\text{exit}}^{\text{dis},1} \cup A_{\text{exit}}^{\text{dis},2}$, while the undisrupted exit activities are defined as $A_{\text{exit}}^{\text{undis}} = A_{\text{exit}}^{\text{plan}} \backslash A_{\text{exit}}^{\text{dis},1}$.

6.4.2.4 Boarding activities

The planned boarding activities that could become inapplicable due to the disruption are defined as:

$$A_{\text{board}}^{\text{dis},1} = \left\{ \left(e, e'\right) \in A_{\text{board}}^{\text{plan}} | t_{\text{start}} \leq o_{e'} < t_{\text{end}} + R \right\}.$$

The boarding activities that are not in Ω_{plan} but might be needed due to extra stops added in a rescheduled timetable are defined as:

$$A_{\text{board}}^{\text{dis},2} = \left\{ (e, e') \middle| e \in E_{\text{dde}}^{*} \backslash E_{\text{dde}}^{\text{plan}}, e' \in E_{\text{de}}^{\text{pass,plan}}, e' = \lambda_e, t_{\text{start}} \le o_{e'} < t_{\text{end}} + R \right\}.$$

Then, the disrupted boarding activities are defined as $A_{\text{board}}^{\text{dis}} = A_{\text{board}}^{\text{dis},1} \cup A_{\text{board}}^{\text{dis},2}$, while the undisrupted boarding activities are defined as $A_{\text{board}}^{\text{undis}} = A_{\text{board}}^{\text{plan}} \backslash A_{\text{board}}^{\text{dis},1}$.

6.4.2.5 Wait activities

To construct disrupted wait activities, we first define three event sets,

$$E_{\text{dde}}^{\max} = \left\{ \arg\max \left\{ o_e | e \in E_{\text{dde}}^{\text{plan}}, o_e < t_{\text{start}}, st_e = st \right\} \right\}_{st \in ST},$$

$$E_{\text{dde}}^{\min} = \left\{ \arg\min \left\{ o_e | e \in E_{\text{dde}}^{\text{plan}}, o_e \ge t_{\text{end}} + R, st_e = st \right\} \right\}_{st \in ST},$$

$$E_{\text{dde}}^{\text{dis}} = \left\{ e \in E_{\text{dde}}^{*} | t_{\text{start}} \le o_e < t_{\text{end}} + R \right\},$$

in which ST is the set of stations. Set $E_{\text{dde}}^{\max}$ includes at each station $st \in ST$ the duplicate departure event of which the original scheduled time is nearest to the disruption starting time t_{start} among all duplicate departures that were originally planned to occur before t_{start} at this station. Set $E_{\text{dde}}^{\min}$ includes at each station $st \in ST$ the duplicate departure event of which the original scheduled time is nearest to the disruption ending time t_{end} plus the recovery length R among all duplicate departures that were originally planned to occur after $t_{\text{end}} + R$ at this station. The events in $E_{\text{dde}}^{\max}$ and $E_{\text{dde}}^{\min}$ will not be affected by the disruption, while set $E_{\text{dde}}^{\text{dis}}$ includes all duplicate departure events that could be affected by the disruption. Based on $E_{\text{dde}}^{\max}$, $E_{\text{dde}}^{\min}$ and $E_{\text{dde}}^{\text{dis}}$, we construct the disrupted wait activities:

$$A_{\text{wait}}^{\text{dis},1} = \left\{ (e, e') \middle| e \in E_{\text{dde}}^{\max}, e' \in E_{\text{dde}}^{\text{dis}}, st_{e'} = st_e, o_{e'} - o_e \le \ell_{\text{wait}}^{\max} \right\},$$

$$A_{\text{wait}}^{\text{dis},2} = \left\{ (e, e') \middle| e \in E_{\text{dde}}^{\text{dis}}, e' \in E_{\text{dde}}^{\min}, st_{e'} = st_e, o_{e'} - o_e \le \ell_{\text{wait}}^{\max} + D \right\},$$

$$A_{\text{wait}}^{\text{dis},3} = \left\{ (e, e') \middle| e, e' \in E_{\text{dde}}^{\text{dis}}, e \ne e', st_e = st_{e'}, 0 \le o_{e'} - o_e \le \ell_{\text{wait}}^{\max} + D \right\},$$

$$A_{\text{wait}}^{\text{dis},4} = \left\{ (e, e') \middle| e, e' \in E_{\text{dde}}^{\text{dis}}, e \ne e', st_e = st_{e'}, -D \le o_{e'} - o_e < 0 \right\},$$

where D represents the maximum allowed delay per event, and $\ell_{\text{wait}}^{\max}$ represents the maximum waiting time that a passenger would like to spend at a station. We assume that $\ell_{\text{wait}}^{\max} \ge D$.

Examples on constructing $A_{\text{wait}}^{\text{dis},1}$, $A_{\text{wait}}^{\text{dis},2}$, $A_{\text{wait}}^{\text{dis},3}$, and $A_{\text{wait}}^{\text{dis},4}$ are described by Figures 6.5 to 6.8, respectively, where the dashed arcs represent the constructed disrupted wait activities, the black solid circles represent the duplicate departure events that could be delayed/cancelled, and the blue solid circles refer to the duplicate departure events that will *not* be delayed/cancelled. In Figure 6.5, we construct a disrupted wait activity

$a \in A^{\text{dis},1}_{\text{wait}}$ from the event $e_2 \in E^{\max}_{\text{dde}}$ to an event $e'_i \in E^{\text{dis}}_{\text{dde}}, i = \{1,2,3,4\}$ if the time difference between them is smaller than the maximum acceptable waiting time $\ell^{\max}_{\text{wait}}$. Otherwise, there is no need to construct such a wait activity since passengers would not wait for so long. Note that in Figures 6.5 to 6.8 the time difference between two events is calculated based on the original scheduled times of these events. In Figure 6.6, we construct a disrupted wait activity $a \in A^{\text{dis},2}_{\text{wait}}$ from an event $e_i \in E^{\text{dis}}_{\text{dde}}, i = \{1,2,3,4\}$ to the event $e'_1 \in E^{\min}_{\text{dde}}$ if the time difference between them is smaller than $\ell^{\max}_{\text{wait}} + D$. Compared to constructing $A^{\text{dis},1}_{\text{wait}}$ (Figure 6.5), here we increase the required time difference by D minutes. This is because an event $e \in E^{\text{dis}}_{\text{dde}}$ could be delayed by at most D minutes. The same reasoning applies to the situation in Figure 6.7, where a disrupted wait activity $a \in A^{\text{dis},3}_{\text{wait}}$ is constructed between two different events from $\{e_1, e_2, e_3, e_4\} \subseteq E^{\text{dis}}_{\text{dde}}$ if the time difference between these two events is smaller than $\ell^{\max}_{\text{wait}} + D$. Considering train orders could change, a disrupted wait activity $a \in A^{\text{dis},4}_{\text{wait}}$ will be constructed from an event $e \in E^{\text{dis}}_{\text{dde}}$ to another event $e' \in E^{\text{dis}}_{\text{dde}}$ that was originally planned to occur before e, if the *absolute* time difference between them is smaller than D. This is illustrated by Figure 6.8. We use the disrupted wait activity $(e_2, e_1) \in A^{\text{dis},4}_{\text{wait}}$ as an example: e_1 could be delayed after e_2 in a rescheduled timetable so that (e_2, e_1) could be effective in the corresponding event-activity network.

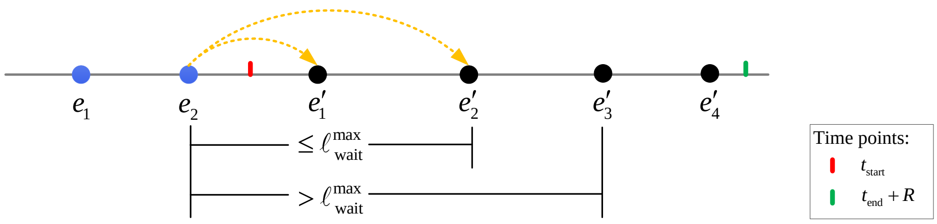


Figure 6.5: Example on constructing $A^{\text{dis},1}_{\text{wait}}$ (the dashed arcs): $e_1 \in E^{\text{plan}}_{\text{dde}} \setminus E^{\max}_{\text{dde}}, e_2 \in E^{\max}_{\text{dde}}, e'_i \in E^{\text{dis}}_{\text{dde}}, i = \{1,2,3,4\}$

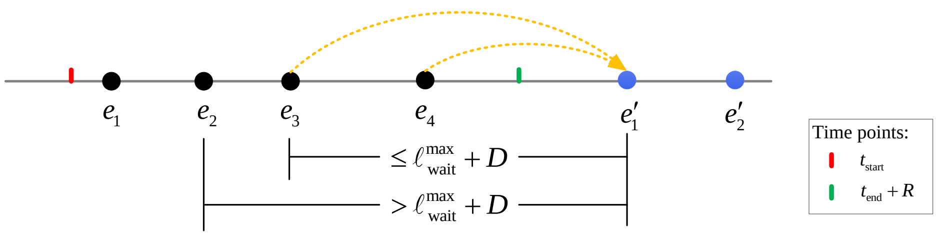


Figure 6.6: Example on constructing $A^{\text{dis},2}_{\text{wait}}$ (the dashed arcs): $e_i \in E^{\text{dis}}_{\text{dde}}, i = \{1,2,3,4\}, e'_1 \in E^{\min}_{\text{dde}}, e'_2 \in E^{\text{plan}}_{\text{dde}} \setminus E^{\min}_{\text{dde}}$

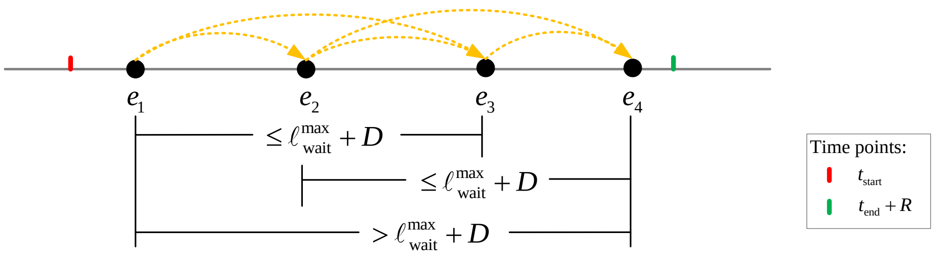


Figure 6.7: Example on constructing $A_{\text{wait}}^{\text{dis},3}$ (the dashed arcs): $e_i \in E_{\text{dde}}^{\text{dis}}, i = \{1,2,3,4\}$

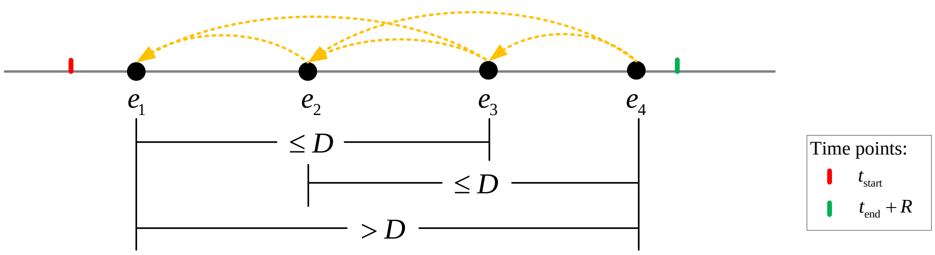


Figure 6.8: Example on constructing $A_{\text{wait}}^{\text{dis},4}$ (the dashed arcs): $e_i \in E_{\text{dde}}^{\text{dis}}, i = \{1,2,3,4\}$

The set of disrupted wait activities is defined as $A_{\text{wait}}^{\text{dis}} = \bigcup_{j \in \{1,\ldots,4\}} A_{\text{wait}}^{\text{dis},j}$. Then, undisrupted wait activities are defined as $A_{\text{wait}}^{\text{undis}} = A_{\text{wait}}^{\text{plan}} \setminus \left(A_{\text{wait}}^{\text{plan}} \cap A_{\text{wait}}^{\text{dis}}\right)$.

6.4.2.6 Transfer activities

To construct disrupted transfer activities, we first establish two event sets:

$$E_{\text{ar}}^{\text{trans}} = \left\{ e \middle| e \in E_{\text{ar}}^{\text{plan}}, o_e < t_{\text{start}}, (e,e') \in A_{\text{trans}}^{\text{plan}}, t_{\text{start}} \leq o_{e'} < t_{\text{end}} + R \right\},$$
$$E_{\text{ar}}^{\text{dis}} = \left\{ e \middle| e \in E_{\text{ar}}^{\text{plan}}, t_{\text{start}} \leq o_e < t_{\text{end}} + R \right\}.$$

Set $E_{\text{ar}}^{\text{trans}}$ contains the arrival events that will not be delayed/cancelled by the disruption but the corresponding planned transfer activities could become inapplicable due to the disruption. Set $E_{\text{ar}}^{\text{dis}}$ includes the arrival events that could be delayed/cancelled due to the disruption. Then, we construct the disrupted transfer activities:

$$A_{\text{trans}}^{\text{dis},1} = \left\{ (e,e') \middle| e \in E_{\text{ar}}^{\text{trans}}, e' \in E_{\text{dde}}^{\min}, tr_{e'} \neq tr_e, st_{e'} = st_e, \ell_{e,e'}^{\text{trans}} \leq o_{e'} - o_e \leq \ell_{\text{trans}}^{\max} \right\},$$
$$A_{\text{trans}}^{\text{dis},2} = \left\{ (e,e') \middle| e \in E_{\text{ar}}^{\text{trans}}, e' \in E_{\text{dde}}^{\text{dis}}, tr_{e'} \neq tr_e, st_{e'} = st_e, o_{e'} - o_e \leq \ell_{\text{trans}}^{\max} \right\},$$
$$A_{\text{trans}}^{\text{dis},3} = \left\{ (e,e') \middle| e \in E_{\text{ar}}^{\text{dis}}, e' \in E_{\text{dde}}^{\min}, tr_{e'} \neq tr_e, st_{e'} = st_e, \ell_{e,e'}^{\text{trans}} \leq o_{e'} - o_e \leq \ell_{\text{trans}}^{\max} + D \right\},$$
$$A_{\text{trans}}^{\text{dis},4} = \left\{ (e,e') \middle| e \in E_{\text{ar}}^{\text{dis}}, e' \in E_{\text{dde}}^{\text{dis}}, tr_{e'} \neq tr_e, st_{e'} = st_e, 0 \leq o_{e'} - o_e \leq \ell_{\text{trans}}^{\max} + D \right\},$$
$$A_{\text{trans}}^{\text{dis},5} = \left\{ (e,e') \middle| e \in E_{\text{ar}}^{\text{dis}}, e' \in E_{\text{dde}}^{\text{dis}}, tr_{e'} \neq tr_e, st_{e'} = st_e, \ell_{e,e'}^{\text{trans}} - D \leq o_{e'} - o_e < 0 \right\},$$

where $\ell_{e,e'}^{\text{trans}}$ represents the minimum transfer time, and $\ell_{\text{trans}}^{\max}$ represents the maximum transfer time that a passenger would like to spend at a station. We assume that $\ell_{\text{trans}}^{\max} \geq$

$D > \ell^{\text{trans}}_{e,e'}$. $A^{\text{dis},1}_{\text{trans}}$ and $A^{\text{dis},2}_{\text{trans}}$ are both relevant to $E^{\text{trans}}_{\text{ar}}$, while $A^{\text{dis},3}_{\text{trans}}$, $A^{\text{dis},4}_{\text{trans}}$ and $A^{\text{dis},5}_{\text{trans}}$ are all relevant to $E^{\text{dis}}_{\text{ar}}$.

Examples on constructing $A^{\text{dis},1}_{\text{trans}}$, $A^{\text{dis},2}_{\text{trans}}$, $A^{\text{dis},3}_{\text{trans}}$, $A^{\text{dis},4}_{\text{trans}}$ and $A^{\text{dis},5}_{\text{trans}}$ are described in Figures 6.9 to 6.13, respectively, where the dash double dotted arcs represent the constructed disrupted transfer activities, and the hollow (solid) circles refer to the arrival (duplicate departure) events. The events that will *not* be delayed/cancelled due to the disruption are colored in blue, while the events that will be delayed/cancelled due to the disruption are colored in black.

In Figure 6.9, we construct a disrupted transfer activity $a \in A^{\text{dis},1}_{\text{trans}}$ from an arrival event $e_i \in E^{\text{trans}}_{\text{ar}}, i = \{1,2\}$ to the duplicate departure event $e'_1 \in E^{\min}_{\text{dde}}$ if the time difference between these two events is larger than the minimum transfer time $\ell^{\text{trans}}_{e_i,e'_1}$ and smaller than the maximum transfer time $\ell^{\max}_{\text{trans}}$. Note that the time differences are calculated based on the original scheduled times of events in Figures 6.9 to 6.13. In Figure 6.10, we construct a disrupted transfer activity $a \in A^{\text{dis},2}_{\text{trans}}$ from an arrival event $e_i \in E^{\text{trans}}_{\text{ar}}, i = \{1,2\}$ to a duplicate departure event $e'_j \in E^{\text{dis}}_{\text{dde}}, j = \{1,2\}$ if the time difference between these two events is smaller than $\ell^{\max}_{\text{trans}}$. In this case, it is unnecessary to require the time difference to be larger than the minimum transfer time, because event $e' \in E^{\text{dis}}_{\text{dde}}$ was originally planned to occur after event $e \in E^{\text{trans}}_{\text{ar}}$ and could be delayed by at most D minutes ($D > \ell^{\text{trans}}_{e,e'}$), in which case the minimum transfer time must be satisfied. In Figure 6.11, we construct a disrupted transfer activity $a \in A^{\text{dis},3}_{\text{trans}}$ from an event $e_i \in E^{\text{dis}}_{\text{ar}}, i = \{1,2,3,4\}$ to the event $e'_1 \in E^{\min}_{\text{dde}}$ if the time difference between these two events is larger than $\ell^{\text{trans}}_{e_i,e'_1}$ and smaller than $\ell^{\max}_{\text{trans}} + D$. Compared to constructing $A^{\text{dis},1}_{\text{trans}}$ and $A^{\text{dis},2}_{\text{trans}}$, the upper limit on the time difference is increased by D when constructing $A^{\text{dis},3}_{\text{trans}}$ because the tail of an activity $a \in A^{\text{dis},3}_{\text{trans}}$ is an event $e \in E^{\text{dis}}_{\text{ar}}$, which could be delayed by at most D minutes. The same reasoning applies to the situation in Figure 6.12, where a disrupted transfer activity $a \in A^{\text{dis},4}_{\text{trans}}$ is constructed from an arrival event $e_i \in E^{\text{dis}}_{\text{ar}}, i = \{1,2\}$ to a duplicate event $e'_j \in E^{\text{dis}}_{\text{dde}}, j = \{1,2\}$ that was originally planned to occur later than e_i if the time difference between e_i and e'_j is smaller than $\ell^{\max}_{\text{trans}} + D$. A lower limit on the time difference is not needed similar to the construction of $A^{\text{dis},2}_{\text{trans}}$. In Figure 6.13, we construct a disrupted transfer $a \in A^{\text{dis},5}_{\text{trans}}$ from an arrival event $e_i \in E^{\text{dis}}_{\text{ar}}, i = \{1,2\}$ to a duplicate event $e'_j \in E^{\text{dis}}_{\text{dde}}, j = \{1,2\}$ that was originally planned to occur earlier than e_i if the *absolute* time difference between between e_i and e'_j is larger than $D - \ell^{\max}_{\text{trans}}$. We use the disrupted transfer activity $(e'_2, e_2) \in A^{\text{dis},5}_{\text{trans}}$ as an example: e'_2 could be delayed by at most D minutes in a rescheduled timetable so that (e'_2, e_2) can satisfy the minimum transfer time.

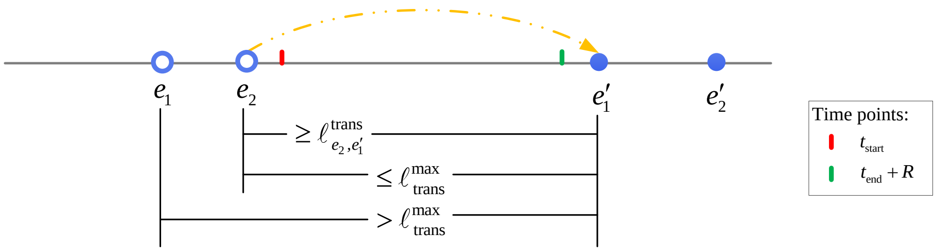


Figure 6.9: Example on constructing $A^{\text{dis},1}_{\text{trans}}$ (the dash double dotted arcs): $e_i \in E^{\text{trans}}_{\text{ar}}, i = \{1,2\}, e'_1 \in E^{\min}_{\text{dde}}, e'_2 \in E^{\text{plan}}_{\text{dde}} \backslash E^{\min}_{\text{dde}}$

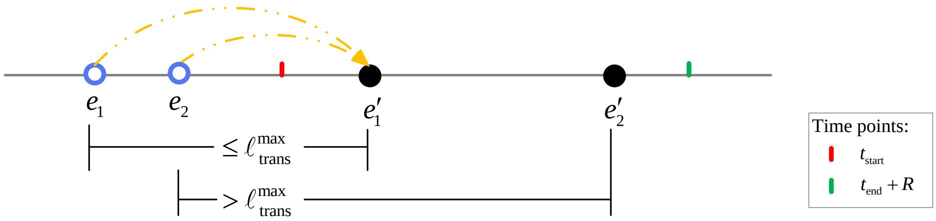


Figure 6.10: Example on constructing $A^{\text{dis},2}_{\text{trans}}$ (the dash double dotted arcs): $e_i \in E^{\text{trans}}_{\text{ar}}, i = \{1,2\}, e'_j \in E^{\text{dis}}_{\text{dde}}, j = \{1,2\},$

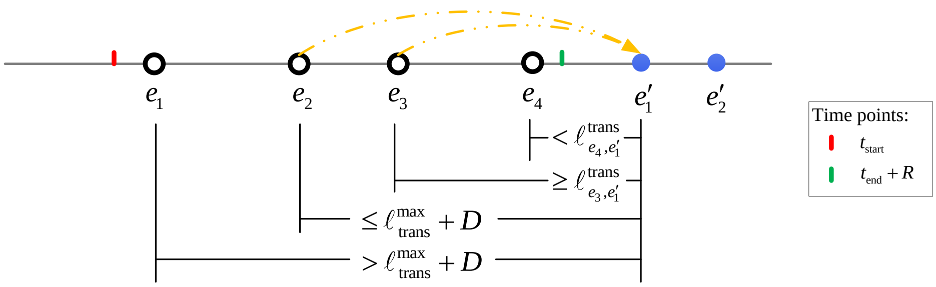


Figure 6.11: Example on constructing $A^{\text{dis},3}_{\text{trans}}$ (the dash double dotted arcs): $e_i \in E^{\text{dis}}_{\text{ar}}, i = \{1,2,3,4\}, e'_1 \in E^{\min}_{\text{dde}}, e'_2 \in E^{\text{plan}}_{\text{dde}} \backslash E^{\min}_{\text{dde}}$

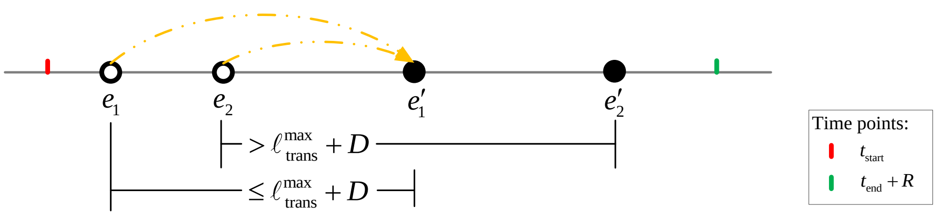


Figure 6.12: Example on constructing $A^{\text{dis},4}_{\text{trans}}$ (the dash double dotted arcs): $e_i \in E^{\text{dis}}_{\text{ar}}, i = \{1,2\}, e'_j \in E^{\text{dis}}_{\text{dde}}, j = \{1,2\},$

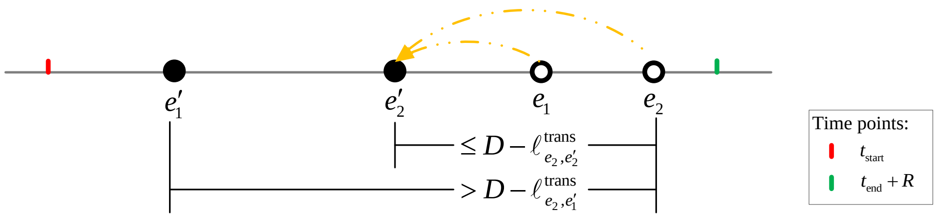


Figure 6.13: Example on constructing $A^{\text{dis},5}_{\text{trans}}$ (the dash double dotted arcs): $e_i \in E^{\text{dis}}_{\text{ar}}, i = \{1,2\}, e'_j \in E^{\text{dis}}_{\text{dde}}, j = \{1,2\},$

The set of disrupted transfer activities is defined as $A^{\text{dis}}_{\text{trans}} = \bigcup_{j\in\{1,\ldots,5\}} A^{\text{dis},j}_{\text{trans}}$. Undisrupted transfer activities are then defined as $A^{\text{undis}}_{\text{trans}} = A^{\text{plan}}_{\text{trans}} \setminus \left(A^{\text{plan}}_{\text{trans}} \cap A^{\text{dis}}_{\text{trans}}\right)$.

In summary, this section introduced the method of constructing a transition network Ω^* to include all possible events and activities. In other words, $\Omega^* = \bigcup_i \Omega^i_{\text{dis}} \cup \Omega_{\text{plan}}$, where Ω^i_{dis} refers to the event-activity network corresponding to the ith feasible rescheduled timetable for a specific disruption, and Ω_{plan} represents the event-activity network corresponding to the planned timetable.

6.5 Passenger-oriented timetable rescheduling model

In this section, we formulate the passenger-oriented timetable rescheduling problem as an MILP model, with as objective minimizing generalized travel times and which consists of three constraint modules: 1) timetable rescheduling, 2) dynamic event-activity network formulation, and 3) passenger reassignment.

The timetable rescheduling module adjusts the planned timetable by delaying, re-ordering, cancelling, flexible stopping and flexible short-turning trains, and considers station capacities and rolling stock circulations at both short-turning and terminal stations of trains. It computes a rescheduled timetable from the starting of a disruption until the normal schedule is recovered. The dynamic event-activity network formulation module formulates an event-activity network Ω_{dis} corresponding to a rescheduled timetable based on the constructed transition network Ω^*. To be more specific, the dynamic event-activity network formulation module decides which events and activities of Ω^* should be selected to formulate Ω_{dis} in terms of a rescheduled timetable, by respecting the rules of constructing an event-activity network introduced in Section 6.3. The passenger reassignment module decides the weight of each activity $a \in \Omega^*$ perceived by each passenger, and assigns each passenger to one path only. A path is a sequence of connected activities that all belong to the formulated Ω_{dis}. The total activity weight of a path is the generalized travel time of this path. Under the objective of minimizing generalized travel times, the passenger reassignment module assigns each passenger to the path with the shortest generalized travel time perceived by this passenger.

The constraints used in the timetable rescheduling module are all from Chapter 3 so that we do not present them in this chapter, neither the decision variables that are only used in this module. We refer to Chapter 3 for details. In this chapter, we present the constraints in the modules of the dynamic event-activity network formulation and the passenger reassignment, as well as the corresponding decision variables. Table 6.3 lists these decision variables and the modules in which they are used. The notation of parameters/sets can be found in the Appendix 6.A.

Due to flexible stopping, scheduled stops could be skipped and extra stops could be added. The scheduled stops (non-stops) can also be cancelled, due to short-turning or complete train cancellation. Tables 6.4 and 6.5 show all possible stop types in a rescheduled timetable, and the corresponding values of the relevant decision variables. There are specific constraints in the timetable rescheduling module to limit the value combinations of $c_e, c_{e'}$ and s_a. We refer to Chapter 3 for details.

Table 6.4: The stop type of activity $a = (e, e') \in A^{\text{plan}}_{\text{dwell}}$ in a rescheduled timetable according to $c_e, c_{e'}$ and s_a

c_e	$c_{e'}$	s_a	Stop type
0	0	0	Stop
0	0	1	Skipped stop
1	0	0	Cancelled stop
0	1	0	Cancelled stop
1	1	0	Cancelled stop

Table 6.5: The stop type of activity $a = (e, e') \in A^{\text{plan}}_{\text{pass}}$ in a rescheduled timetable according to $c_e, c_{e'}$ and s_a

c_e	$c_{e'}$	s_a	Stop type
0	0	0	Extra stop
0	0	1	Non-stop
1	0	1	Cancelled non-stop
0	1	1	Cancelled non-stop
1	1	1	Cancelled non-stop

6.5.1 Dynamic event-activity network formulation

The dynamic event-activity network formulation module decides which events and activities of the transition network Ω^* are effective in an event-activity network Ω_{dis} corresponding to a rescheduled timetable by respecting the rules of constructing an event-activity network introduced in Section 6.3. Recall that $\Omega^* = E^* \cup A^*$, where $E^* = E^{\text{plan}}_{\text{ar}} \cup E^{\text{plan}}_{\text{de}} \cup E^*_{\text{dde}} \cup E^{\text{plan}}_{\text{entry}} \cup E^{\text{plan}}_{\text{exit}} \cup E^{\text{plan}}_{\text{penal}}$, and $A^* = A^{\text{plan}}_{\text{run}} \cup A^{\text{plan}}_{\text{dwell}} \cup A^{\text{plan}}_{\text{pass}} \cup A^*_{\text{wait}} \cup A^*_{\text{trans}} \cup A^*_{\text{board}} \cup A^*_{\text{entry}} \cup A^*_{\text{exit}} \cup A^{\text{plan}}_{\text{enpenal}} \cup A^{\text{plan}}_{\text{expenal}}$. In particular, $A^{\text{plan}}_i = A^{\text{undis}}_i \cup A^{\text{dis}}_i, i \in \{\text{run}, \text{dwell}, \text{pass}\}$, and $A^*_j = A^{\text{undis}}_j \cup A^{\text{dis}}_j, j \in \{\text{wait}, \text{trans}, \text{board}, \text{entry}, \text{exit}\}$, which means that in the transition network Ω^*, each kind of activity set consists of two

Table 6.3: Decision variables

Symbol	Description	Module
x_e	Continuous variable deciding the rescheduled time of an event $e \in E_{\text{ar}}^{\text{plan}} \cup E_{\text{de}}^{\text{plan}} \cup E_{\text{dde}}^{*}$.	1, 2, 3
c_e	Binary variable with value 1 deciding event $e \in E_{\text{ar}}^{\text{plan}} \cup E_{\text{de}}^{\text{plan}} \cup E_{\text{dde}}^{*}$ is cancelled, and 0 otherwise.	1, 2
s_a	Binary variable deciding whether a scheduled stop $a \in A_{\text{dwell}}^{\text{plan}}$ is skipped or whether an extra stop is added to $a \in A_{\text{pass}}^{\text{plan}}$. If $a \in A_{\text{dwell}}^{\text{plan}}$, then $s_a = 1$ indicates a is skipped. If $a \in A_{\text{pass}}^{\text{plan}}$, then $s_a = 1$ indicates a is added with a stop.	1, 2
y_a	Binary variable with value 1 deciding activity $a \in \Omega^*$ is effective in Ω_{dis}, and 0 otherwise.	2, 3
u_a^g	Binary variable with value 1 deciding activity $a \in \Omega^*$ is chosen by passenger group g, and 0 otherwise.	3
w_a^g	Continuous variable deciding the weight of activity $a \in \Omega^*$ perceived by each passenger in group g	3

Module 1: timetable rescheduling; Module 2: dynamic event-activity network formulation; Module 3: passenger reassignment

subsets: an undisrupted activity set, and a disrupted activity set. For an undisrupted activity, both of the corresponding events will not be delayed/cancelled by the disruption; while for a disruption activity, at least one of the corresponding events could be delayed/cancelled by the disruption.

6.5.1.1 Deciding which events are effective in Ω_{dis}

The binary cancellation decision c_e of an event $e \in E_{\text{ar}}^{\text{plan}} \cup E_{\text{de}}^{\text{plan}} \cup E_{\text{dde}}^{*}$ is equivalent to deciding whether this event is effective in Ω_{dis}. An event $e \in E_{\text{ar}}^{\text{plan}} \cup E_{\text{de}}^{\text{plan}} \cup E_{\text{dde}}^{*}$ is effective in Ω_{dis} if it is not cancelled, $c_e = 0$. The cancellation decision c_e and the rescheduled time x_e of an arrival (departure) event $e \in E_{\text{ar}}^{\text{plan}} (e \in E_{\text{de}}^{\text{plan}})$ are determined in the timetable rescheduling module. A duplicate departure event $e' \in E_{\text{dde}}^{*}$ is required to be cancelled/kept simultaneously as its corresponding departure event $e \in E_{\text{de}}^{\text{plan}}$, and the rescheduled times of both events are forced to be the same:

$$c_{e'} = c_e, \qquad e' \in E_{\text{dde}}^{*}, e \in E_{\text{de}}^{\text{plan}}, \lambda_{e'} = e, \tag{6.1}$$

$$x_{e'} = x_e, \qquad e' \in E_{\text{dde}}^{*}, e \in E_{\text{de}}^{\text{plan}}, \lambda_{e'} = e, \tag{6.2}$$

where $\lambda_{e'}$ is a given attribute indicating the departure event corresponding to duplicate departure event e'.

An event $e \in E_{\text{entry}}^{\text{plan}} \cup E_{\text{exit}}^{\text{plan}} \cup E_{\text{penal}}^{\text{plan}}$ is always effective in any Ω_{dis}.

6.5.1.2 Deciding which activities are always effective in any Ω_{dis}

Entry/exit penalty activities, and undisrupted activities are effective in any Ω_{dis}:

$$y_a = 1, \qquad a \in A_{\text{enpenal}}^{\text{plan}} \cup A_{\text{expenal}}^{\text{plan}}, \tag{6.3}$$

$$y_a = 1, \qquad a \in \left\{A_k^{\text{undis}}\right\}_{k \in K}, K = \{\text{run}, \text{dwell}, \text{pass}, \text{wait}, \text{trans}, \text{board}, \text{entry}, \text{exit}\}, \tag{6.4}$$

where y_a is a binary variable with value 1 indicating that activity a is effective in Ω_{dis}, and 0 otherwise. Recall that both of the events corresponding to an undisrupted activity will not be delayed/cancelled due to the disruption.

6.5.1.3 Deciding which disrupted run activities are effective in Ω_{dis}

Recall that a running activity is from a departure event e to an arrival event e', which correspond to the same train at neighbouring stations. A disrupted running activity in the transition network Ω^* will be effective in an event-activity network Ω_{dis} if neither of the corresponding events is cancelled:

$$y_a = 1 - c_e, \qquad a = (e, e') \in A_{\text{run}}^{\text{dis}}, \tag{6.5}$$

$$y_a = 1 - c_{e'}, \qquad a = (e, e') \in A_{\text{run}}^{\text{dis}}. \tag{6.6}$$

Note that in the timetable rescheduling module (Chapter 3), the departure event e and the arrival event e' in the same running activity are forced to be cancelled/kept simultaneously: c_e=$c_{e'}$, which is why we use equalities for (6.5) and (6.6).

6.5.1.4 Deciding which disrupted dwell/pass-through activities are effective in Ω_{dis}

Recall that a dwell (pass-through) activity is from an arrival event e to a departure event e', which correspond to the same train at the same station. We decide whether a disrupted dwell (pass-through) activity of Ω^* will be effective in Ω_{dis} by:

$$y_a \leq 1 - c_e, \qquad a = (e,e') \in A^{\text{dis}}_{\text{dwell}} \cup A^{\text{dis}}_{\text{pass}}, \tag{6.7}$$

$$y_a \leq 1 - c_{e'}, \qquad a = (e,e') \in A^{\text{dis}}_{\text{dwell}} \cup A^{\text{dis}}_{\text{pass}}, \tag{6.8}$$

$$y_a \geq 1 - c_e - c_{e'}, \qquad a = (e,e') \in A^{\text{dis}}_{\text{dwell}} \cup A^{\text{dis}}_{\text{pass}}. \tag{6.9}$$

Constraints (6.7) and (6.8) mean that a disrupted dwell (pass-through) activity will not be effective in Ω_{dis} if at least one of the corresponding events is cancelled; otherwise, it must be effective ((6.9)). Recall that $A^{\text{dis}}_{\text{dwell}} \subseteq A^{\text{plan}}_{\text{dwell}}$ and $A^{\text{dis}}_{\text{pass}} \subseteq A^{\text{plan}}_{\text{pass}}$.

6.5.1.5 Deciding which disrupted entry activities are effective in Ω_{dis}

Recall that an entry activity is from an entry event e to a duplicate departure event e', which both correspond to the same station. We use a binary parameter $r_{e'}$ with value 1 to indicate that a (duplicate) departure event e' corresponds to a train origin departure, and 0 otherwise. For a disrupted entry activity $a = (e,e')$ of which the duplicate departure event e' corresponds to a train origin departure, a will be effective in Ω_{dis} if e' is not cancelled:

$$y_a = 1 - c_{e'}, \qquad a = (e,e') \in A^{\text{dis}}_{\text{entry}}, r_{e'} = 1. \tag{6.10}$$

For a disrupted entry activity $a = (e,e')$ of which the duplicate departure event e' does *not* correspond to a train origin departure, we established the following constraints to decide whether a is effective in Ω_{dis}:

$$y_a \leq 1 - c_{e'}, \qquad a = (e,e') \in A^{\text{dis}}_{\text{entry}}, r_{e'} = 0, \tag{6.11}$$

$$y_a \leq 1 - s_{a'} + c_{e''} + c_{e'''}, \qquad a = (e,e') \in A^{\text{dis}}_{\text{entry}}, r_{e'} = 0, e''' = \lambda_{e'}, \; a' = (e'',e''') \in A^{\text{dis}}_{\text{dwell}} \cup A^{\text{dis}}_{\text{pass}}, \tag{6.12}$$

$$y_a \geq 1 - s_{a'} - c_{e''} - c_{e'''}, \qquad a = (e,e') \in A^{\text{dis}}_{\text{entry}}, r_{e'} = 0, e''' = \lambda_{e'}, \; a' = (e'',e''') \in A^{\text{dis}}_{\text{dwell}} \cup A^{\text{dis}}_{\text{pass}}. \tag{6.13}$$

Constraint (6.11) means that a disrupted entry activity $a = (e,e')$ will not be effective in Ω_{dis} if its corresponding duplicate departure event e' is cancelled. Otherwise, a will be effective only if its corresponding duplicate departure event e' is associated with a *real* stop $a' = (e'', e''') \in A^{\text{dis}}_{\text{dwell}} \cup A^{\text{dis}}_{\text{pass}}$ that has $c_{e''} = 0, c_{e'''} = 0$ and $s_{a'} = 0$ (see Table 6.4 and Table 6.5), in which e''' is the departure event corresponding to e': $e''' = \lambda_{e'}$. This is represented by (6.12) and (6.13).

6.5.1.6 Deciding which disrupted boarding activities are effective in Ω_{dis}

Recall that a boarding activity is from a duplicate departure event e to the corresponding departure event e'. For a disrupted boarding activity $a = (e, e')$ of which the duplicate departure event e corresponds to a train origin departure, a will be effective in Ω_{dis} if e is not cancelled.:

$$y_a = 1 - c_e, \qquad a = (e, e') \in A^{\text{dis}}_{\text{board}}, r_e = 1. \tag{6.14}$$

For a disrupted boarding activity $a = (e, e')$ of which the duplicate departure event e does *not* correspond to a train origin departure, we decide whether a is effective in Ω_{dis} by

$$y_a \leq 1 - c_e, \qquad a = (e, e') \in A^{\text{dis}}_{\text{board}}, r_e = 0, \tag{6.15}$$

$$y_a \leq 1 - s_{a'} + c_e + c_{e'}, \qquad a = (e, e') \in A^{\text{dis}}_{\text{board}}, r_e = 0, a' = (e'', e') \in A^{\text{dis}}_{\text{dwell}} \cup A^{\text{dis}}_{\text{pass}}, \tag{6.16}$$

$$y_a \geq 1 - s_{a'} - c_{e''} - c_{e'}, \qquad a = (e, e') \in A^{\text{dis}}_{\text{board}}, r_e = 0, a' = (e'', e') \in A^{\text{dis}}_{\text{dwell}} \cup A^{\text{dis}}_{\text{pass}}. \tag{6.17}$$

Constraint (6.15) means that a disrupted boarding activity a will not be effective in Ω_{dis} if its corresponding duplicate departure event e is cancelled. Otherwise, a will be effective only if its corresponding departure event e' is associated with a *real* stop $a' = (e'', e') \in A^{\text{dis}}_{\text{dwell}} \cup A^{\text{dis}}_{\text{pass}}$ that has $c_{e''} = 0, c_{e'} = 0$ and $s_{a'} = 0$. This is represented by (6.16) and (6.17).

6.5.1.7 Deciding which disrupted exit activities are effective in Ω_{dis}

Recall that an exit activity is from an arrival event e to an exit event e', which both correspond to the same station. We use a binary parameter f_e with value 1 to indicate that an arrival event e corresponds to a train destination arrival, and 0 otherwise. For a disrupted exit activity $a = (e, e')$ of which the arrival event e corresponds to a train destination arrival, a will be effective in Ω_{dis} if e is not cancelled:

$$y_a = 1 - c_e, \qquad a = (e, e') \in A^{\text{dis}}_{\text{exit}}, f_e = 1. \tag{6.18}$$

For a disrupted exit activity $a = (e, e')$ of which the arrival event e' does *not* correspond to a train destination arrival, we established the following constraints to decide whether

a is effective in Ω_{dis}:

$$y_a \le 1 - c_e, \qquad a = (e,e') \in A^{\text{dis}}_{\text{exit}}, f_e = 0, \tag{6.19}$$

$$y_a \le 1 - s_{a'} + c_e + c_{e''}, \qquad a = (e,e') \in A^{\text{dis}}_{\text{exit}}, f_e = 0, a' = (e,e'') \in A^{\text{dis}}_{\text{dwell}} \cup A^{\text{dis}}_{\text{pass}}, \tag{6.20}$$

$$y_a \ge 1 - s_{a'} - c_e - c_{e''}, \qquad a = (e,e') \in A^{\text{dis}}_{\text{exit}}, f_e = 0, a' = (e,e'') \in A^{\text{dis}}_{\text{dwell}} \cup A^{\text{dis}}_{\text{pass}}, \tag{6.21}$$

Constraint (6.19) means that a disrupted exit activity $a = (e,e')$ will not be effective in Ω_{dis} if its corresponding arrival event e is cancelled. Otherwise, a will be effective only if its corresponding arrival event e is associated with a *real* stop $a' = (e,e'') \in A^{\text{dis}}_{\text{dwell}} \cup A^{\text{dis}}_{\text{pass}}$ that has $c_e = 0, c_{e''} = 0$ and $s_{a'} = 0$. This is stated by (6.20) and (6.21).

6.5.1.8 Deciding which disrupted wait activities are effective in Ω_{dis}

Recall that a wait activity is from a duplicate departure event e to the next time-closest duplicate departure event e' that occurs at the same station but corresponds to a different train. We decide whether a disrupted wait activity $a = (e,e')$ is effective in Ω_{dis} by

$$y_a \le 1 - c_e, \qquad a = (e,e') \in A^{\text{dis}}_{\text{wait}}, \tag{6.22}$$

$$y_a \le 1 - c_{e'}, \qquad a = (e,e') \in A^{\text{dis}}_{\text{wait}}, \tag{6.23}$$

$$y_a \le 1 - s_{a'} + c_{e''} + c_{e'''}, \qquad a = (e,e') \in A^{\text{dis}}_{\text{wait}}, r_e = 0, e''' = \lambda_e, \quad a' = (e'',e''') \in A^{\text{dis}}_{\text{dwell}} \cup A^{\text{dis}}_{\text{pass}}, \tag{6.24}$$

$$y_a \le 1 - s_{a'} + c_{e''} + c_{e'''}, \qquad a = (e,e') \in A^{\text{dis}}_{\text{wait}}, r_{e'} = 0, e''' = \lambda_{e'}, \quad a' = (e'',e''') \in A^{\text{dis}}_{\text{dwell}} \cup A^{\text{dis}}_{\text{pass}}, \tag{6.25}$$

$$y_a + y_{a'} \le 1, \qquad a = (e,e') \in A^{\text{dis}}_{\text{wait}}, a' = (e',e) \in A^{\text{dis}}_{\text{wait}} \tag{6.26}$$

$$\sum_{\substack{a \in A^{\text{dis}}_{\text{wait}},\\ tail(a)=e}} y_a \le 1, \qquad e \in E^{\text{dis}}_{\text{dde}}, \tag{6.27}$$

$$\sum_{\substack{a \in A^{\text{dis}}_{\text{wait}},\\ head(a)=e'}} y_a \le 1, \qquad e' \in E^{\text{dis}}_{\text{dde}}, \tag{6.28}$$

$$x_{e'} - x_{e''} \le M(1 - y_a), \qquad a = (e,e') \in A^{\text{dis}}_{\text{wait}}, a' = (e,e'') \in A^{\text{dis}}_{\text{wait}}, \tag{6.29}$$

$$x_{e''} - x_{e'} \le M(1 - y_{a'}), \qquad a = (e,e') \in A^{\text{dis}}_{\text{wait}}, a' = (e,e'') \in A^{\text{dis}}_{\text{wait}}, \tag{6.30}$$

$$x_{e'} - x_e \ge -M(1 - y_a), \qquad a = (e,e') \in A^{\text{dis}}_{\text{wait}}, \tag{6.31}$$

where $tail(a)$ refers to the tail event of an activity: the event which an activity starts from, $head(a)$ refers to the head event of an activity: the event which an activity directs to, and M is a sufficiently large number of which the value is set to 2880. Constraints (6.22) and (6.23) mean that a disrupted wait activity will not be effective in Ω_{dis} if at least one of the corresponding events is cancelled. Constraint (6.24) ((6.25)) requires a disrupted wait activity $a = (e, e')$ to be *ineffective* if the corresponding duplicate departure event e (e') does not correspond to a train origin departure and is not associated with a real stop. A duplicate departure event could be relevant to multiple disrupted wait activities in a transition network (see Figure 6.8 as an example), while at most one of these activities can be effective in an event-activity network Ω_{dis} ((6.26)-(6.28)). Constraints (6.29)-(6.31) together ensure that a duplicate departure event e can only be linked to the next time-closest duplicate departure event to construct an effective wait activity in Ω_{dis}.

6.5.1.9 Deciding which disrupted transfer activities are effective in Ω_{dis}

Recall that a transfer activity is from an arrival event e to the next time-closest duplicate departure event e' that occurs at the same station as e but corresponds to a different train. We decide whether a disrupted transfer activity $a = (e, e')$ is effective in Ω_{dis} by

$$y_a \leq 1 - c_e, \qquad a = (e, e') \in A^{\text{dis}}_{\text{trans}}, \tag{6.32}$$

$$y_a \leq 1 - c_{e'}, \qquad a = (e, e') \in A^{\text{dis}}_{\text{trans}}, \tag{6.33}$$

$$y_a \leq 1 - s_{a'} + c_e + c_{e''}, \qquad a = (e, e') \in A^{\text{dis}}_{\text{trans}}, f_e = 0, \quad a' = (e, e'') \in A^{\text{dis}}_{\text{dwell}} \cup A^{\text{dis}}_{\text{pass}}, \tag{6.34}$$

$$y_a \leq 1 - s_{a'} + c_{e''} + c_{e'''}, \qquad a = (e, e') \in A^{\text{dis}}_{\text{trans}}, r_{e'} = 0, e''' = \lambda_{e'}, \quad a' = (e'', e''') \in A^{\text{dis}}_{\text{dwell}} \cup A^{\text{dis}}_{\text{pass}}, \tag{6.35}$$

$$\sum_{\substack{a \in A^{\text{dis}}_{\text{trans}},\\ tail(a) = e}} y_a \leq 1, \qquad e \in E^{\text{trans}}_{\text{ar}} \cup E^{\text{dis}}_{\text{ar}}, \tag{6.36}$$

$$x_{e'} - x_{e''} \leq M(1 - y_a), \qquad a = (e, e') \in A^{\text{dis}}_{\text{trans}}, a' = (e, e'') \in A^{\text{dis}}_{\text{trans}}, \tag{6.37}$$

$$x_{e''} - x_{e'} \leq M(1 - y_{a'}), \qquad a = (e, e') \in A^{\text{dis}}_{\text{trans}}, a' = (e, e'') \in A^{\text{dis}}_{\text{trans}}, \tag{6.38}$$

$$x_{e'} - x_e \geq -M(1 - y_a) + \ell^{\text{trans}}_{e,e'}, \qquad a = (e, e') \in A^{\text{dis}}_{\text{trans}}, \tag{6.39}$$

where $\ell^{\text{trans}}_{e,e'}$ refers to the minimum transfer time. Constraints (6.32) and (6.33) means that a disrupted transfer activity will not be effective in Ω_{dis} if at least one of the corresponding events is cancelled. Constraint (6.34) requires a disrupted transfer activity $a = (e, e')$ to be *ineffective* if the corresponding arrival event e does not correspond to a train destination arrival and is not associated with a real stop. Constraint (6.35) requires a disrupted transfer activity $a = (e, e')$ to be *ineffective* if the corresponding duplicate departure event e' does not correspond to a train origin departure and is not associated with a real stop. Constraint (6.36) means that for an arrival event $e \in E^{\text{trans}}_{\text{ar}} \cup E^{\text{dis}}_{\text{ar}}$, which has multiple disrupted transfer activities starting from it (see Figure 6.13 as an example), at most one of these activities will be effective in an event-activity network Ω_{dis}. Constraints (6.37)-(6.39) together ensure that an arrival event e can only be linked to the next time-closest duplicate departure event to construct an effective transfer activity of which the minimum transfer time must be respected.

6.5.2 Passenger reassignment

There could be multiple passengers who share exactly the same journeys in terms of the planned timetable: the same origin station, the same arrival time at the origin station, the same expected destination, and the same expected generalized travel time from the origin to the destination. These passengers form a same group $g \in G$, which is assumed to be inseparable in case of a disruption. G represents the set of passenger groups, possibly consisting of a single passenger. Recall that a path is a sequence of connected activities. Deciding which path will be chosen by a passenger group is equivalent to deciding which activities will be chosen by this group, while each group g is associated with the same activity choice set A^*. The passenger reassignment module decides which activity $a \in A^*$ will be chosen by a passenger group g and the weight of each activity $a \in A^*$ perceived by g.

6.5.2.1 Assigning each passenger group to one path only

An activity $a \in A^*$ cannot be chosen by a passenger group if a is *not* effective in Ω_{dis} ($y_a = 0$):

$$u_a^g \leq y_a, \qquad a \in A^*, g \in G, \tag{6.40}$$

where u_a^g is a binary decision with value 1 indicating that activity $a \in A^*$ is chosen by passenger group $g \in G$, and 0 otherwise.

A path that could be chosen by a passenger group g must start from an entry (entry penalty) event corresponding to his/her origin O_g, end in an exit (exit penalty) event corresponding to his/her destination D_g, and include at least one intermediate event to connect them:

$$\sum_{\substack{a \in A_{\text{entry}}^{\text{undis}} \cup A_{\text{entry}}^{\text{dis}} \cup A_{\text{enpenal}}^{\text{plan}}, \\ st_{tail(a)} = O_g}} u_a^g = 1, \qquad g \in G, \tag{6.41}$$

$$\sum_{\substack{a \in A_{\text{entry}}^{\text{undis}} \cup A_{\text{entry}}^{\text{dis}} \cup A_{\text{enpenal}}^{\text{plan}}, \\ st_{tail(a)} \neq O_g}} u_a^g = 0, \qquad g \in G. \tag{6.42}$$

$$\sum_{\substack{a \in A_{\text{exit}}^{\text{undis}} \cup A_{\text{exit}}^{\text{dis}} \cup A_{\text{expenal}}^{\text{plan}}, \\ st_{tail(a)} = D_g}} u_a^g = 1, \qquad g \in G, \tag{6.43}$$

$$\sum_{\substack{a \in A_{\text{exit}}^{\text{undis}} \cup A_{\text{exit}}^{\text{dis}} \cup A_{\text{expenal}}^{\text{plan}}, \\ st_{tail(a)} \neq D_g}} u_a^g = 0, \qquad g \in G; \tag{6.44}$$

$$\sum_{a \in In_e} u_a^g = \sum_{a' \in Out_e} u_{a'}^g, \qquad e \in E^* \backslash \left\{ E_{\text{entry}}^{\text{plan}}, E_{\text{exit}}^{\text{plan}} \right\}, g \in G, \tag{6.45}$$

$$M(1 - u_a^g) + x_{e'} \geq t_g^{\text{ori}}, \qquad a = (e, e') \in A_{\text{entry}}^{\text{undis}} \cup A_{\text{entry}}^{\text{dis}}, st_e = O_g, g \in G, \tag{6.46}$$

where $\left\{A^{\text{undis}}_{\text{entry}}, A^{\text{dis}}_{\text{entry}}, A^{\text{plan}}_{\text{enpenal}}\right\}$ contains all entry and entry penality activities in Ω^*, and $\left\{A^{\text{undis}}_{\text{exit}}, A^{\text{dis}}_{\text{exit}}, A^{\text{plan}}_{\text{expenal}}\right\}$ contains all exit and exit penalty activities in Ω^*. Recall that an entry (entry penalty activity) is from an entry event to a duplicate departure (penalty) event, while an exit (exit penalty activity) is from an arrival (penalty) event to an exit event. $st_{tail(a)}$ refers to the corresponding station of the tail event of an activity, In_e (Out_e) is the set of activities going in (going out) event e, and t^{ori}_g represents the time of passenger group g arriving at origin station O_g. Constraint (6.41) means that among the entry and entry penalty activities relevant to the origin of a passenger group, one and only one of them will be chosen by this group. Constraint (6.42) means that among the entry and entry penalty activities that do *not* correspond to the origin station of a passenger group, *none* of them will be chosen by this group. Constraint (6.43) means that among the exit and exit penalty activities relevant to the destination of a passenger group, one and only one of them will be chosen by this group. Constraint (6.44) means that among the exit and exit penalty activities that do *not* correspond to the destination station of a passenger group, *none* of them will be chosen by this group. Constraint (6.45) is for flow balance at intermediate events (i.e. excluding entry and exit events). It means that if an activity $a = (e', e)$, which goes into an intermediate event e, is chosen by a passenger group g (u^g_a=1), then another activity $a' = (e, e'')$, which goes out from event e should also be chosen by this group ($u^g_{a'}$=1). Constraint (6.46) means that an entry activity $a = (e, e')$ that corresponds to the origin station of a passenger group could be chosen by this group only if the rescheduled time $x_{e'}$ of the duplicate departure event e' in a is later than the time of this group arriving at the origin station t^{ori}_g .

6.5.3 Deciding the weight of each activity perceived by a passenger group

Suppose we use w_a to represent the decision on the weight of an activity a. Then, the generalized travel time of a passenger in group g can be described as $\sum_{a\in A^*} w_a \cdot u^g_a$, which is a nonlinear formulation because w_a and u^g_a are both decision variables. To formulate the generalized travel time of a passenger in a linear way, we use w^g_a instead, which is a continuous variable indicating the weight of an activity a perceived by each passenger in group g. The generalized travel time of each passenger in group g is then formulated as $\sum_{a\in A^*} w^g_a$. The value of w^g_a is forced to be 0 if activity a is not chosen by group g. Otherwise, the value of w^g_a is determined according to the time cost of activity a and passenger preference on the type of a. In the following, we introduce the constraints of deciding w^g_a for each kind of activity.

If an activity a is not chosen by group g ($u_a^g = 0$), the weight of this activity will be 0:

$$w_a^g \leq M^* u_a^g,\ a = (e,e') \in A_j^{\text{undis}} \cup A_j^{\text{dis}},\ j \in \{\text{entry}, \text{wait}, \text{run}, \text{dwell}, \text{pass}, \text{trans}\}, g \in G, \tag{6.47}$$

$$w_a^g \geq 0, \qquad a = (e,e') \in A_j^{\text{undis}} \cup A_j^{\text{dis}},,\ j \in \{\text{entry}, \text{wait}, \text{run}, \text{dwell}, \text{pass}, \text{trans}\}, g \in G, \tag{6.48}$$

where M^* is a sufficiently larger number, of which the value is set to $\beta_{\text{wait}} M$. Here, β_{wait} is the multiplier of waiting time perceived by passengers at stations.

If an entry activity a is chosen by group g ($u_a^g = 1$), the weight of this entry activity perceived by each passenger in group g is determined by

$$w_a^g \leq \beta_{\text{wait}} \left(x_{e'} - t_g^{\text{ori}} \right) + M^* \left(1 - u_a^g\right), \qquad a = (e,e') \in A_{\text{entry}}^{\text{undis}} \cup A_{\text{entry}}^{\text{dis}}, g \in G, \tag{6.49}$$

$$w_a^g \geq \beta_{\text{wait}} \left(x_{e'} - t_g^{\text{ori}} \right) - M^* \left(1 - u_a^g\right), \qquad a = (e,e') \in A_{\text{entry}}^{\text{undis}} \cup A_{\text{entry}}^{\text{dis}}, g \in G, \tag{6.50}$$

where w_a^g is forced to be $\beta_{\text{wait}} \left(x_{e'} - t_g^{\text{ori}} \right)$ if $u_a^g = 1$, in which case $x_{e'}$ must be larger than t_g^{ori} due to (6.46).

If a wait activity a is chosen by group g ($u_a^g = 1$), the weight of this wait activity perceived by each passenger in group g is determined by

$$w_a^g \leq \beta_{\text{wait}} \left(x_{e'} - x_e \right) + M^* \left(1 - u_a^g\right), \qquad a = (e,e') \in A_{\text{wait}}^{\text{undis}} \cup A_{\text{wait}}^{\text{dis}}, g \in G, \tag{6.51}$$

$$w_a^g \geq \beta_{\text{wait}} \left(x_{e'} - x_e \right) - M^* \left(1 - u_a^g\right), \qquad a = (e,e') \in A_{\text{wait}}^{\text{undis}} \cup A_{\text{wait}}^{\text{dis}}, g \in G, \tag{6.52}$$

where w_a^g is forced to be $\beta_{\text{wait}} \left(x_{e'} - x_e \right)$ if $u_a^g = 1$, in which case $x_{e'}$ must be larger than x_e (otherwise a would not be effective and then would not be chosen by g).

If a run, dwell or pass-through a is chosen by group g ($u_a^g = 1$), the weight of this activity perceived by each passenger in group g is determined by

$$w_a^g \leq \beta_{\text{vehicle}} \left(x_{e'} - x_e \right) + M^* \left(1 - u_a^g\right), \qquad a = (e,e') \in A_j^{\text{undis}} \cup A_j^{\text{dis}}, \quad j \in \{\text{run}, \text{dwell}, \text{pass}\}, g \in G, \tag{6.53}$$

$$w_a^g \geq \beta_{\text{vehicle}} \left(x_{e'} - x_e \right) - M^* \left(1 - u_a^g\right), \qquad a = (e,e') \in A_j^{\text{undis}} \cup A_j^{\text{dis}}, \quad j \in \{\text{run}, \text{dwell}, \text{pass}\}, g \in G, \tag{6.54}$$

where β_{vehicle} is the multiplier of in-vehicle time perceived by passengers. Here, w_a^g is forced to be $\beta_{\text{vehicle}} \left(x_{e'} - x_e \right)$ if u_a^g=1, in which case $x_{e'}$ must be larger than x_e (otherwise a would not be effective and then would not be chosen by g).

If a transfer activity a is chosen by group g ($u_a^g = 1$), the weight of this transfer activity perceived by each passenger in group g is determined by

$$w_a^g \leq \beta_{\text{trans}} + \beta_{\text{wait}} \left(x_{e'} - x_e \right) + M^* \left(1 - u_a^g\right), \qquad a = (e,e') \in A_{\text{trans}}^{\text{undis}} \cup A_{\text{trans}}^{\text{dis}}, g \in G, \tag{6.55}$$

$$w_a^g \geq \beta_{\text{trans}} + \beta_{\text{wait}} \left(x_{e'} - x_e \right) - M^* \left(1 - u_a^g\right), \qquad a = (e,e') \in A_{\text{trans}}^{\text{undis}} \cup A_{\text{trans}}^{\text{dis}}, g \in G, \tag{6.56}$$

where β_{trans} is the fixed time penalty of one transfer. Here, w_a^g is forced to be $\beta_{\text{trans}} + \beta_{\text{wait}}(x_{e'} - x_e)$ if u_a^g=1, in which case $x_{e'}$ must be larger than x_e (otherwise a would not be effective and then would not be chosen by g).

The weight of an entry penalty activity is determined by

$$w_a^g = \mu \cdot T_g^{\text{plan}} \cdot u_a^g, \qquad a = (e,e') \in A_{\text{enpenal}}^{\text{plan}}, g \in G, \mu \geq 1, \tag{6.57}$$

where T_{plan}^g refers to the expected generalized travel time of passenger group g in terms of the planned timetable, and $\mu \cdot T_{\text{plan}}^g$ refers to the maximum generalized travel time which passenger group g would accept during a disruption. Both T_{plan}^g and μ are given parameters.

The weight of a boarding activity, an exit activity, or an exit penalty activity is set to 0:

$$w_a^g = 0, \qquad a = (e,e') \in \left\{A_{\text{expenal}}^{\text{plan}}, A_{\text{board}}^{\text{undis}}, A_{\text{board}}^{\text{dis}}, A_{\text{exit}}^{\text{undis}}, A_{\text{exit}}^{\text{dis}}\right\}, g \in G. \tag{6.58}$$

6.5.4 Objective

The objective is to minimize the generalized travel times of all passengers, which is

$$z_p = \sum_{g \in G} \sum_{a \in A^*} n_g w_a^g, \tag{6.59}$$

where n_g represents the number of passengers in group g.

To summarize, the proposed passenger-oriented timetable rescheduling model (POTR) is given by constraints (6.1) - (6.58) presented in this chapter, as well as the constraints presented in Chapter 3, with objective (6.59).

6.6 Reducing the computational complexity of the passenger-oriented timetable rescheduling model

When dealing with a large railway network and/or considering numerous passengers, the proposed passenger-oriented timetable rescheduling model (POTR) may not be able to find a high-quality solution in an acceptable time, because a binary variable u_a^g is created for each activity $a \in A^*$ associated with each passenger group $g \in G$, of which the total number is $|A^*| \times |G|$. To reduce the computational complexity, we propose 1) a pre-processing method to shrink the activity choice set for each passenger group in a reasonable way, and 2) an iterative solution method to solve the model with limited passenger groups considered in each iteration, which also restricts the solution space to avoid excessive operational deviations that are not preferred by the railway operators. We introduce both methods in this section.

6.6.1 Shrinking the activity choice set of a passenger group

The passenger-oriented timetable rescheduling model proposed in Section 6.5 considers A^* as the activity choice set for each passenger group $g \in G$, while A^* contains some activities that will never be chosen by g. Thus, we introduce a method of constructing an improved activity choice set A^*_g for passenger group $g \in G$ by excluding the activities that will never be chosen by g. In other words, $A^*_g \subset A^*$. Recall that $A^* = A^{\text{plan}}_{\text{run}} \cup A^{\text{plan}}_{\text{dwell}} \cup A^{\text{plan}}_{\text{pass}} \cup A^*_{\text{wait}} \cup A^*_{\text{trans}} \cup A^*_{\text{board}} \cup A^*_{\text{entry}} \cup A^*_{\text{exit}} \cup A^{\text{plan}}_{\text{enpenal}} \cup A^{\text{plan}}_{\text{expenal}}$.

To decide which activities of A^* should be selected to construct A^*_g, we first define that

- an arrival event *e could be reachable* by passenger group g if $t^{\text{ori}}_g + \ell^{\min}_{O_g,st_e} - D \leq o_e \leq t^{\text{ori}}_g + \mu T^{\text{plan}}_g - \ell^{\min}_{st_e,D_g}$ and $st_e \neq O_g$, and
- a (duplicate) departure event *e could be reachable* by passenger group g if $t^{\text{ori}}_g + \ell^{\min}_{O_g,st_e} - D \leq o_e \leq t^{\text{ori}}_g + \mu T^{\text{plan}}_g - \ell^{\min}_{st_e,D_g}$ and $st_e \neq D_g$.

in which o_e is the original scheduled time of event e, D is the maximum allowed delay per event, O_g (D_g) represents the origin (destination) of passenger group g, t^{ori}_g is the time of passenger group g arriving at the origin, μT^{plan}_g is the maximum acceptable generalized travel time of passenger group g, $\ell^{\min}_{O_g,st_e}$ represents the minimum train running time from the origin of group g to the corresponding station of event e, and $\ell^{\min}_{st_e,D_g}$ represents the minimum train running time from the corresponding station of event e to the destination of group g.

Then, we add

- an activity $a = (e,e') \in A^{\text{plan}}_i$ to $A^{\text{plan}}_{i,g}$ for any $i \in \{\text{run}, \text{dwell}, \text{pass}\}$, if *both* events e and e' could be reachable by passenger group g,
- an activity $a = (e,e') \in A^*_i$ to $A^*_{j,g}$ for any $j \in \{\text{wait}, \text{trans}, \text{board}\}$, if *both* events e and e' could be reachable by passenger group g,
- an activity $a = (e,e') \in A^*_{\text{entry}}$ to $A^*_{\text{entry},g}$ if duplicate departure event e' could be reachable by passenger group g and $st_{e'} = O_g$,
- an activity $a = (e,e') \in A^*_{\text{exit}}$ to $A^*_{\text{exit},g}$ if arrival event e could be reachable by passenger group g and $st_{e'} = D_g$, and
- each activity $a \in A^{\text{plan}}_{\text{enpenal}} \cup A^{\text{plan}}_{\text{expenal}}$ to A^*_g.

Thus, $A^*_g = \left\{A^{\text{plan}}_{i,g}\right\}_{i \in I} \cup \left\{A^*_{k,g}\right\}_{k \in K} \cup A^{\text{plan}}_{\text{enpenal}} \cup A^{\text{plan}}_{\text{expenal}}$, in which $I = \{\text{run}, \text{dwell}, \text{pass}\}$, $K = \{\text{wait}, \text{trans}, \text{board}, \text{entry}, \text{exit}\}$. The constructed A^*_g reduces the number of binary variables u^g_a and continuous variables w^g_a, as well as the corresponding constraints in the passenger-oriented timetable rescheduling model. For example, due to the improved entry and exit activity sets, $A^*_{\text{entry},g}$ and $A^*_{\text{exit},g}$, constraints (6.42) and (6.44) are not needed.

Table 6.6: Activity sets relevant to passenger group g

Notation	Description
A^*_g	The activity choice set associated with passenger group g: $A^*_g \subset A^*$
$A^{\text{plan}}_{i,g}$	Set of i activities associated with passenger group g: $A^{\text{plan}}_{i,g} \subset A^{\text{plan}}_i, i \in \{\text{run}, \text{dwell}, \text{pass}\}$
$A^*_{k,g}$	Set of k activities associated with passenger group g:: $A^*_{k,g} \subset A^*_k, k \in \{\text{wait}, \text{trans}, \text{board}, \text{entry.exit}\}$

6.6.2 An iterative solution method

We propose a method (Algorithm 6.1) to solve the passenger-oriented timetable rescheduling model iteratively by considering limited passenger groups in each iteration, where the timetable rescheduling problem is solved for all train services although restricted passenger groups are considered. Each iteration determines the activities for the next set of additional considered groups while keeping the activities of the previous considered groups as fixed.

The number of new passenger groups n_{new} considered in an iteration determines the number of required iterations $I = \left\lceil \frac{|G|}{n_{\text{new}}} \right\rceil$, where G represents the set of all passenger groups, which is sorted in a descending order regarding the total expected generalized travel times of all passengers contained in a group: $n_g T^{\text{plan}}_g, g \in G$. The impacts of n_{new} on the solution quality and the computation time of the iterative solution method are investigated in Section 6.7.3. The iterative solution method terminates until either all passenger groups in G are considered or the total running time limit T^{stop} is reached, while in the latter case one more process will be needed to evaluate the responses of all passengers towards the rescheduled timetable finally obtained. At each iteration a computation time limit t^{stop} is set to avoid excessive searching for the optimal solution, while a longer computation than t^{stop} will be allowed to find a feasible solution in case no feasible solution can be obtained within t^{stop}.

Algorithm 6.1 needs the following inputs: the operator-oriented timetable rescheduling model (OOTR), the passenger-oriented timetable rescheduling model (POTR), the set of passenger groups G, the set of events E^*, the activity choice set A^*_g of each passenger group $g \in G$, the number of new passengers n_{new} considered in each iteration, the computation time limit t^{stop} of each iteration, the total computation time limit T^{stop}, and the maximum allowed deviation Δ from the optimal operator-oriented objective value in terms of train cancellations and delays. The OOTR model is the timetable rescheduling module from Chapter 3, which in this chapter adopts the objective of minimizing train cancellations and delays: $z_o = \sum_{e \in E^{\text{plan}}_{\text{ar}}} w_c c_e + d_e$, where c_e is a binary cancellation decision, d_e is a continuous decision representing the delay of event e, and w_c is the penalty of cancelling a train service between two neighbouring stations. Solving the OOTR model to optimality gets the optimal operator-oriented objective z_o^*. The POTR model consists of the timetable rescheduling module, the dynamic event-activity network formulation module, and the passenger reassignment module, which aims to minimize the generalized travel times of passengers: $z_p = \sum_{g \in G} \sum_{a \in A_g^*} n_g w_a^g$.

Algorithm 6.1: Iterative solution method

Input: $\text{OOTR}, \text{POTR}, G, E^*, \{A_g^*\}_{g \in G}, n_{\text{new}}, t^{\text{stop}}, T^{\text{stop}}, \Delta$

1 Solve the OOTR model to get z_o^*;

2 Add constraint $\sum_{e \in E_{\text{ar}}^{\text{plan}}} w_c c_e + d_e \le z_o^* + \Delta$ to the POTR model;

3 $I = \left\lceil \frac{|G|}{n_{\text{new}}} \right\rceil$;

4 $G' = \emptyset$;

5 $\text{POTR}^1 = \text{POTR}$;

6 $i = 1$;

7 **while** $i \le I$ *and* T^{stop} *is not reached* **do**

8   **if** $i < I$ **then**

9         $G_{\text{new}} = \{g_{j+1}, \cdots, g_{j+n_{\text{new}}}\} \subset G, j = (i-1)n_{\text{new}}$;

10         $G' = G' \cup G_{\text{new}}$;

11         $\{\tilde{x}_e, \tilde{u}_a^g, \tilde{z}_p\} \leftarrow$ solve POTR^i for all $g \in G'$ within t^{stop} or until a feasible solution is found, using the solution of OOTR model as an initial solution if $i = 1$ or the solution of POTR^{i-1} as an initial solution if $i \ge 2$;

12   Construct POTR^{i+1} by adding constraints: $u_a^g = \tilde{u}_a^g, a \in A_g^*, g \in G_{\text{new}}$, into POTR^i ;

13         $i = i + 1$;

14   **else**

15         $G_{\text{new}} = \{g_{(i-1)n_{\text{new}}+1}, \cdots, g_{|G|}\} \subset G$;

16         $G' = G' \cup G_{\text{new}} = G$;

17         $\{\tilde{x}_e, \tilde{u}_a^g, \tilde{z}_p\} \leftarrow$ solve POTR^i for all $g \in G$ within t^{stop} or until a feasible solution is found, using the solution of POTR^{i-1} as an initial solution;

18 **if** $G' \neq G$ **then**

19   Construct POTR' by adding constraints: $x_e = \tilde{x}_e, e \in E_{\text{ar}}^{\text{plan}} \cup E_{\text{de}}^{\text{plan}} \cup E_{\text{dde}}^*$, into POTR ;

20     $\{\tilde{x}_e, \tilde{u}_a^g, \tilde{z}_p\} \leftarrow$ solve POTR' for all $g \in G$;

21 **Return** $\{\tilde{x}_e, \tilde{u}_a^g, \tilde{z}_p\}$ finally obtained;

In Algorithm 6.1, the OOTR model is solved first to obtain the optimal operator-oriented rescheduled timetable that has the operator-oriented objective value of z_o^* (line 1). To find a passenger-friendly rescheduled timetable that can also be preferred by railway operators, we add a constraint to the POTR model to require that the passenger-oriented rescheduled timetable obtained by the POTR model will not deviate from the optimal operator-oriented rescheduled timetable by Δ in terms of train cancellations and delays (line 2). Line 3 initializes the number of iterations needed to solve the POTR model, and line 4 initializes the set of passenger groups considered at each iteration as an empty set. In line 5, we define the passenger-oriented timetable rescheduling model to be solved in the 1st iteration as POTR^1. The iteration is initialized in line 6. If the required iterations are not completely performed and the total computation time until the current iteration is shorter than T^{stop}, then the while-loop starting from line 7 continues.

If the current iteration is not the final iteration (line 8), we select n_{new} passenger groups from G as the passenger groups that are *newly* considered in the current iteration (line 9), which are then added to G' (line 10). Considering all passenger groups in G', the current POTR^i model is solved within the required time limit t^{stop} or until a feasible solution is found (line 11). To speed up the computation time, we give an initial feasible solution to the solver. When the current iteration is the first iteration, the initial solution is chosen as the solution obtained earlier from the OOTR model. When the current iteration is the second or a later iteration, the initial solution is chosen as the solution from the POTR^{i-1} model in the previous iteration. The outputs of the current model POTR^i include the rescheduled time $\tilde{x}_e$ of event $e \in E_{\text{ar}}^{\text{plan}} \cup E_{\text{de}}^{\text{plan}} \cup E_{\text{dde}}^{*}$, the choice $\tilde{u}_a^g$ of passenger group $g \in G'$ on activity $a \in A_g^*$, and the passenger-oriented objective value $\tilde{z}_p$ that represents the generalized travel times over all passenger groups in G'. In line 12, we construct the passenger-oriented timetable rescheduling model POTR^{i+1} to be solved in the next iteration by adding constraints to the current POTR^i. These constraints require the activity choices of the passenger groups that are newly considered in the current iteration to be fixed in all following iterations. In line 13, we proceed to the next iteration.

If the current iteration is the final iteration (line 14), we add the passenger groups that have not be considered yet (line 15) to the G' (line 16), which now includes all passenger groups of G. Considering all passenger groups in G, the current POTR^i model is solved within the required time limit t^{stop} or until the first feasible solution is found by giving the solution from the previous iteration as an initial feasible solution to the solver (line 17). In this case, the algorithm terminates by returning the results from POTR^i (lines 21).

The while-loop could end before all required iterations are performed due to the total computation limit of T^{stop}. Under this circumstance, the passenger groups in G have not been completely considered (line 18), which means that the $\tilde{z}_p$ finally obtained in the while-loop does not represent the generalized travel times of all passenger groups in G. Therefore, we construct POTR$'$ by adding constraints to the original POTR model, which require the rescheduled timetable finally obtained in the while-loop to be fixed (line 19). In that sense, solving POTR$'$ is *not* to compute a new rescheduled timetable but to evaluate the generalized travel times of all passenger groups in G under a given rescheduled timetable that is finally obtained in the while-loop (line 20). Hence, the computation time of solving POTR$'$ is not counted in T^{stop}.

6.7 Case study

The case study aims to investigate the performance of the passenger-oriented timetable rescheduling model on shortening generalized travel times during railway disruptions, and to analyse the computational efficiency of the proposed iterative solution method to the passenger-oriented timetable rescheduling model. Section 6.7.1 describes the case study, while Section 6.7.2 and Section 6.7.3 report the performance of the passenger-oriented model and the computational efficiency of the iterative method, respectively.

6.7.1 Setup

The case study is performed to a part of the Dutch railways, of which the schematic track layout is shown in Figure 6.14. The considered network is totally around 128 km long, which has both single-track (23.5 km) and double-track (104.5 km) railway lines with in total 17 stations. The stations that allow short-turning to both directions are colored in full green, the stations that prohibit short-turning to both directions are colored in full grey, and the stations that allow (prohibit) short-turning to one direction are colored in half green (grey). Six train lines operate half-hourly in each direction in the considered network, of which the scheduled stopping patterns are indicated in Figure 6.15, as well as the terminal stations of these train lines in the considered network. The rolling stock circulations at the short-turning and terminal stations of trains are both dealt with. We distinguish between intercity (IC) and local (called sprinter (SPR) in Dutch) train lines. All experiments were carried out in MATLAB on a desktop with Intel Xeon CPU E5-1620 v3 at 3.50 GHz and 16 GB RAM. The solver GUROBI release 7.0.1 was used either to solve the passenger-oriented timetable rescheduling model directly or called by the iterative solution method to solve the model iteratively.

The parameters used to construct an event-activity/transition network $\ell^{\text{trans}}_{e,e'}$, $\ell^{\max}_{\text{trans}}$ and $\ell^{\max}_{\text{wait}}$ are set to 5 min, 30 min and 30 min, respectively. Recall that $\ell^{\text{trans}}_{e,e'}$ represents the minimum transfer time, and $\ell^{\max}_{\text{trans}}$ ($\ell^{\max}_{\text{wait}}$) represents the maximum transfer (waiting) time which a passenger is willing to spend at a station. The maximum delay allowed to a train departure/arrival D is set to 25 min. The disruption timetable is required to recover to the planned timetable no later than 25 min after the disruption ends: $R = 25$. Passengers are assumed to leave the railways if they cannot find paths with less than two times of their expected generalized travel times within the railways: $\mu = 2$. The coefficient of waiting time at an origin/transfer station β_{wait} is set to 2.5 and the coefficient of in-vehicle time β_{vehicle} is set to 1 (Wardman, 2004). The penalty of one transfer β_{trans} is set to 10 min (de Keizer et al., 2012). For the iterative solution method to the passenger-oriented timetable rescheduling model, we set the total computation time limit T^{stop} to 300 s, and the computation time limit of each iteration t^{stop} to 30 s. The passenger group set G is sorted in a descending order regarding the total expected generalized travel times of all passengers contained in a group: $n_g T_g^{\text{plan}}, g \in G$. In that sense, a group g with a larger value of $n_g T_g^{\text{plan}}$ will be handled at an earlier iteration. Table 6.7 lists the parameter values.

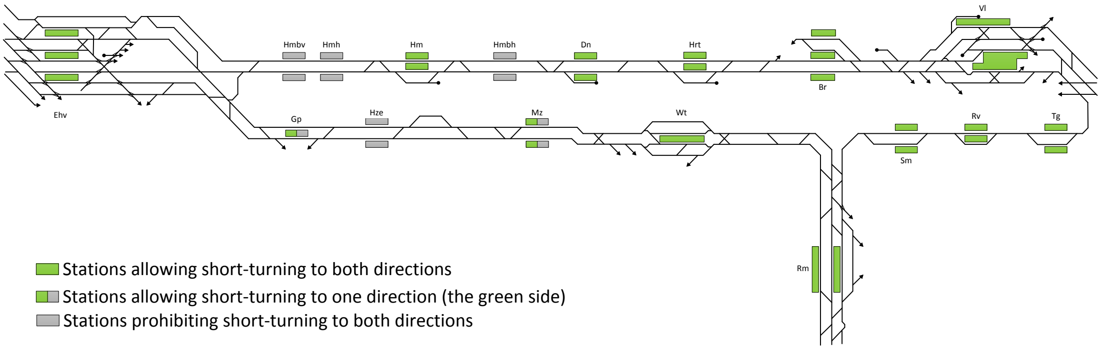
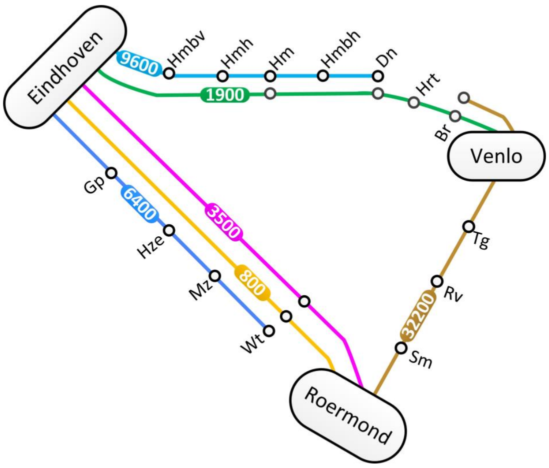


Figure 6.14: The schematic track layout in the considered network



Train line	Terminal
IC800	—
IC1900	Venlo (Vl)
IC3500	—
SPR6400	Wt and Eindhoven (Ehv)
SPR9600	Dn
SPR32200	Roermond (Rm)

Figure 6.15: The train lines operating in the considered network

Table 6.7: Parameter settings

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
$\ell_{e,e'}^{\text{trans}}$	5 min	β_{trans}	10 min	D	25 min	t^{stop}	30 s
$\ell_{\text{trans}}^{\max}$	30 min	β_{vehicle}	1	R	25 min	T^{stop}	300 s
$\ell_{\text{wait}}^{\max}$	30 min	β_{wait}	2.5	μ	2		

We consider four cases with increasing disruption length. We consider the passengers whose arrival times at the origin stations are during the period of $[t_{\text{start}}, t_{\text{end}} + R]$. We form the passengers who share the same expected journey in terms of the planned timetable into the same group $g \in G$. The number of passenger groups $|G|$ varies with the disruption starting/ending time and the required recovery time length. Table 6.8 indicates the total numbers of passenger groups and the total numbers of passengers considering different disruption durations but the same required recovery time length 25 min. The numbers of passengers in different groups can be different. In each case of Table 6.8, the largest group contains 126 passengers, while the smallest group contains 1 passenger only. Figure 6.16 shows the numbers of passenger groups considering different group sizes. Recall that n_g refers to the number of passengers in a group g.

In each case, most groups contain less than 10 passengers, and few groups contain 30 passengers or more.

Table 6.8: Disruption and passenger demand cases

Case	Disruption start	Disruption end	Travel starting period	Total number of passenger groups $\|G\|$	Total number of passengers $\sum_{g\in G} n_g$
I	8:00	8:30	[8:00,8:30+00:25]	334	2,557
II	8:00	9:00	[8:00,9:00+00:25]	477	3,320
III	8:00	9:30	[8:00,9:30+00:25]	618	3,897
IV	8:00	10:00	[8:00,10:00+00:25]	728	4,357

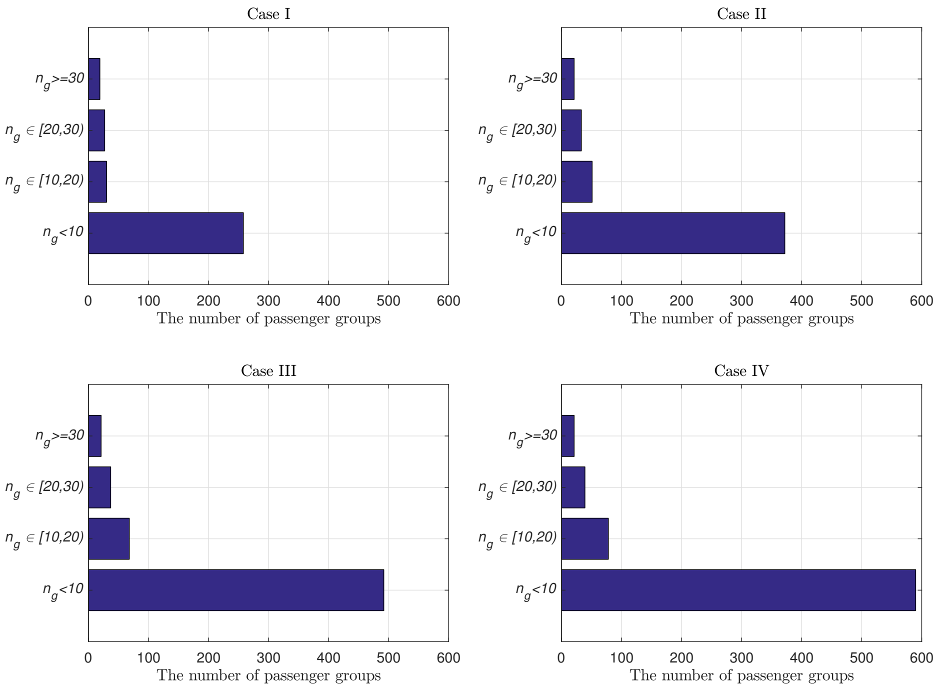


Figure 6.16: Passenger group sizes

Table 6.9: Notation relevant to a passenger-oriented (operator-oriented) solution

Notation	Description
z_o	The objective of the OOTR model: minimizing train cancellations and arrival delays, $z_o = \sum_{e\in E_{\text{ar}}^{\text{plan}}} w_c c_e + d_e$
w_c	The penalty of cancelling a train service between neighbouring stations, $w_c = 100$
z_p	The objective of the POTR model: minimizing the generalized travel times over all passengers, $z_p = \sum_{g\in G}\sum_{a\in A_g^*} n_g w_a^g$
z_o^*	The objective value of the *optimal* rescheduled timetable obtained by the OOTR model
$\tilde{z}_o$	The resulting train cancellations and arrival delays of a rescheduled timetable obtained by the POTR model, $\tilde{z}_o = \sum_{e\in E_{\text{ar}}^{\text{plan}}} w_c c_e + d_e$,
$\tilde{z}_p$	The resulting generalized travel times over all passengers of a rescheduled timetable obtained by the OOTR model, $\tilde{z}_p = \sum_{g\in G}\sum_{a\in A_g^*} n_g w_a^g$
Δ	The maximum allowed deviation of $\tilde{z}_o$ from z_o^*

We consider cases in which a section is completely blocked during four disruption periods (as Table 6.8 shown), respectively. The operator-oriented timetable rescheduling model (OOTR) uses the objective z_o of minimizing train cancellations and arrival delays, which is used to solve each disruption case to obtain optimal z_o^*. The passenger-oriented timetable rescheduling model (POTR), which uses the objective z_p of minimizing generalized travel times, is used to solve each disruption case by requiring that the resulting train cancellations and arrival delays cannot exceed $z_o^* + \Delta$, where $\Delta \geq 0$. The resulting generalized travel times of the rescheduled timetables obtained by the OOTR model are evaluated and denoted by $\tilde{z}_p$. The resulting train cancellations and arrival delays of the rescheduled timetables obtained by the POTR model are evaluated and denoted by $\tilde{z}_o$. Table 6.9 gives the notation relevant to a passenger-oriented (operator-oriented) solution. Note that the penalty w_c of cancelling one service is set to 100.

6.7.2 The performance of the passenger-oriented timetable rescheduling model

We consider section Mz-Hze (between Eindhoven and Roermond) to be completely blocked during the considered four disruption periods (see Table 6.8), respectively. Table 6.10 shows the optimal solutions obtained from both the operator-oriented and the passenger-oriented timetable rescheduling models by using Gurobi directly (i.e. not using the iterative approach). Due to different objectives, the optimality gap of a solution obtained by the operator-oriented model is different from the optimality gap of a solution obtained by the passenger-oriented model, and thereby we use 'O-gap (z_o)' and 'O-gap (z_p)' to distinguish them. In each case, the passenger-oriented solution reduced generalized travel times ($z_p - \tilde{z}_p$) by over 5000 min with at most 10 min additional train delay ($\tilde{z}_o - z_o$) than the optimal operator-oriented solution when setting Δ to 10 in the passenger-oriented model.

Table 6.10: General results by using a solver directly: disrupted section Mz-Hze

Case	Operator-oriented (solver)				Passenger-oriented (solver): Δ=10					
	z_o [min]	$\tilde{z}_p$ [min]	Time [sec]	O-gap (z_o) [%]	$\tilde{z}_o$ [min]	z_p [min]	Time [sec]	O-gap (z_p) [%]	$z_o - \tilde{z}_o$ [min]	$z_p - \tilde{z}_p$ [min]
I	848	116,211	5	0.00	857	110,757	28	0.00	9	- 5,454
II	2,742	161,057	7	0.00	2,752	154,568	150	0.00	10	- 6,489
III	4,639	195,944	10	0.00	4,649	189,322	370	0.00	10	- 6,622
IV	6,536	221,773	12	0.00	6,546	216,481	250	0.00	10	- 5,292

Table 6.11: Train-related results by using a solver directly: disrupted section Mz-Hze

Case	Operator-oriented (solver)				Passenger-oriented (solver): Δ=10			
	# Cancelled services	Train arrival delays [min]	# Extra stops	# Skipped stops	# Cancelled services	Train arrival delays [min]	# Extra stops	# Skipped stops
I	6	248	3	2	6	257	6	4
II	24	342	3	2	24	352	6	6
III	42	439	3	2	42	449	6	7
IV	60	536	4	2	60	546	5	8

Table 6.12: Passenger-related results by using a solver directly: disrupted section Mz-Hze

Case	Operator-oriented (solver)					Passenger-oriented (solver): Δ=10				
	# Dropped groups	# Dropped passengers	Total in-vehicle time [min]	Total wait wait [min]	Total # transfer	# Dropped groups	# Dropped passengers	Total in-vehicle time [min]	Total wait time [min]	Total # transfer
I	21	228	45,205	21,975	85	18↓	200↓	45,824↑	20,004↓	83↓
II	54	593	49,527	27,654	92	45↓	514↓	51,714↑	25,683↓	91↓
III	88	820	53,612	31,544	165	81↓	800↓	54,126↑	29,145↓	164↓
IV	126	1,028	56,710	33,754	176	123↓	1,025↓	56,685↓	31,723↓	176

Table 6.11 gives train-related results in more detail. In each case, the numbers of cancelled services were the same in the operator-oriented and the passenger-oriented solutions. This is because the deviation of a passenger-oriented solution from the optimal operator-oriented solution cannot exceed 10 ($\Delta = 10$) while the penalty of cancelling one train service is set to 100 (w_c=100). The resulting total train arrival delays were different in the operator-oriented and the passenger-oriented solutions, as well as the numbers of extra stops and skipped stops. The number of both extra stops and skipped stops in a passenger-oriented solution was more than in the operator-oriented solution of the same case, because larger operation deviation was allowed in the passenger-oriented model which thereby made more changes on train stopping patterns to reflect on passenger needs. We want to emphasize that in the passenger-oriented model the decisions of adding or skipping stops were made with the aim of reducing generalized travel times, whereas in the operator-oriented model these decisions were made with the aim of reducing train cancellations and arrival delays. For example in the operator-oriented model an extra stop will be added to a train at the station where this train was originally planned to pass through but now has to dwell at this station for waiting on platform capacity to be released in a downstream station where this train will be short-turned.

Table 6.12 gives passenger-related results in more detail, where the symbol $\downarrow$ ($\uparrow$) is used to denote the decrease (increase) in a passenger-oriented solution compared to the operator-oriented solution of the same case. We can see that compared to the operator-oriented solutions, the passenger-oriented solutions resulted in less passenger groups leaving the railways, and the total number of passengers in these groups was also smaller. The passenger-oriented solutions also helped to shorten passenger waiting times at stations in all cases and reduce the number of transfers in most cases. In cases I-III, the passenger-oriented solutions resulted in longer passenger in-vehicle times, because in the passenger-oriented objective waiting times at stations were penalized 2.5 times of in-vehicle times considering passenger preferences (β_{wait}=2.5 and β_{vehicle}=1). Under this circumstance, the passenger-oriented model tends to delay the departures of specific trains at specific stations, which is beneficial to passengers who could now catch the train. The waiting times of these passengers were reduced by earlier boarding because of the delayed train departures, whereas other passengers who were on-board the delayed trains experienced longer in-vehicle times.

To investigate the impact of maximum allowed operation deviation Δ on the solutions obtained by the passenger-oriented model, we performed 10 more experiments on case IV using different values of Δ. The results are shown in Figure 6.17. Each green triangle indicates the performance of a solution obtained by the passenger-oriented model using a specific Δ, and each blue circle indicates the performance of the optimal solution obtained by the operator-oriented model. On the one hand, with the increase of Δ the passenger-oriented model resulted in larger weighted train cancellations and train arrival delays ($\tilde{z}_o$). The number of cancelled services always remained the same while train arrival delays increased gradually with the growth of Δ. More extra stops and skipped stops were created under larger Δ. On the other hand, with larger operation deviation allowed the passenger-oriented model resulted in shorter generalized travel times (z_p). The generalized travel time of a passenger is the sum of the weighted waiting time, in-vehicle time and the number of transfers. It can be seen that larger Δ led to shorter waiting times but longer in-vehicle times, whereas the number of transfers almost remained the same. This is because waiting time is perceived 2.5 times of the same length of in-vehicle time by passengers so that the model tends to reduce the waiting times of some passengers at the expense of longer in-vehicle times of other passengers. Under whichever Δ, the number of passengers who chose to leave the railways was always smaller in a passenger-oriented solution compared to the operator-oriented solution.

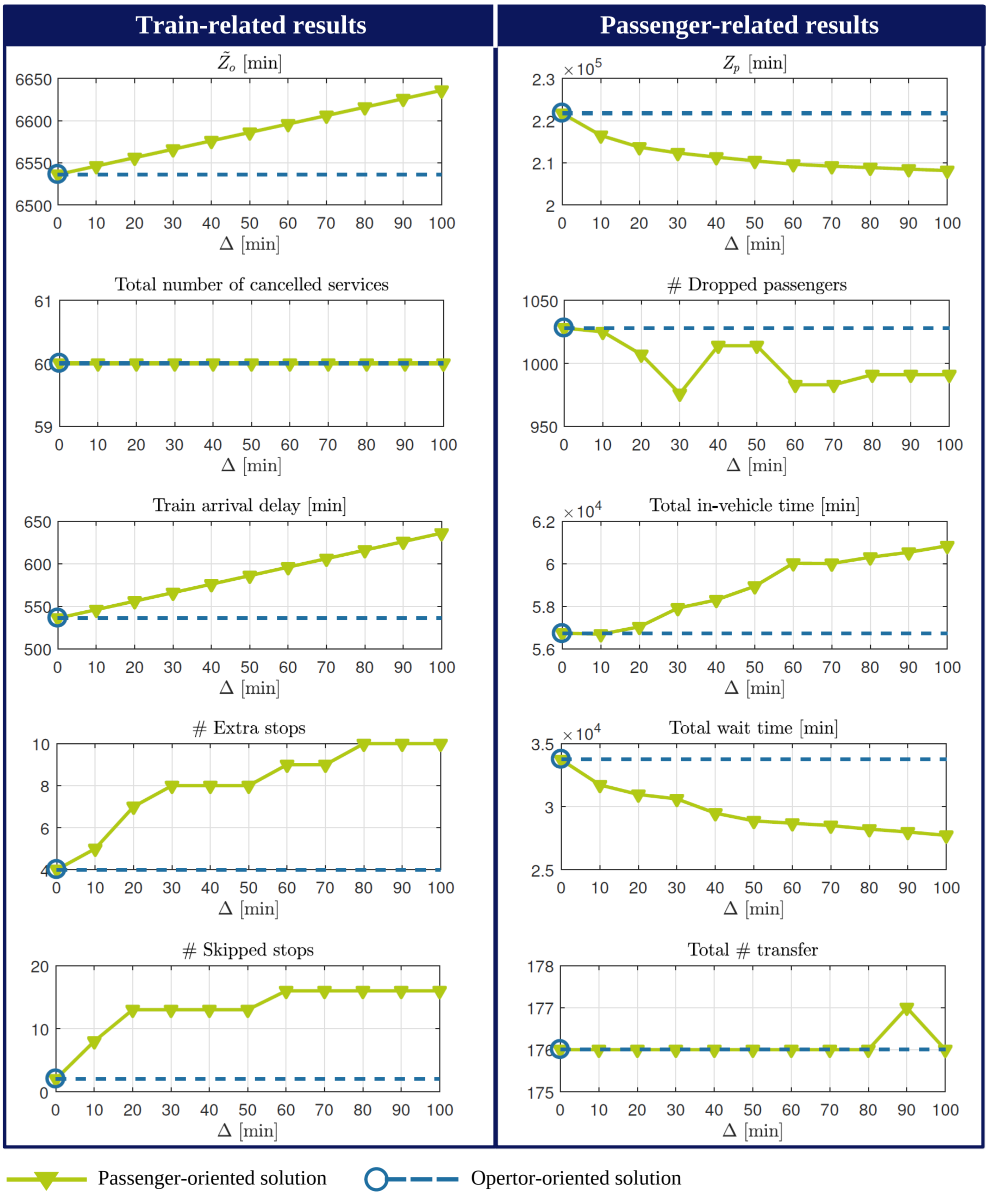


Figure 6.17: The optimal passenger-oriented solutions for case IV under different settings of Δ: disrupted section Mz-Hze

We take case IV as an example to show the operator-oriented solution in Figure 6.18, and the passenger-oriented solutions of $\Delta = 10$ and $\Delta = 30$ in Figure 6.19 and Figure 6.20, respectively. The grey rectangle indicates the time-distance disruption window, the dashed (dotted) lines represent the original scheduled services that were cancelled (delayed), the solid lines represent the services scheduled in the rescheduling solution, and the red triangles (circles) represent the extra (skipped) stops. The differences of train stopping patterns in each of the passenger-oriented solutions (Figure 6.19 or Figure 6.20) compared to the operator-oriented solution (Figure 6.18) are highlighted by dashed black rectangles. Because station Hze lacks turning facilities, downstream trains from line SPR6400 (in dark blue) and line IC800 (in yellow) were both short-turned at an earlier station Gp. Because station Mz lacks turning facilities for short-turning upstream trains, an upstream train from line IC3500 (in pink) had to be delayed until the disruption ended, and a train from line SPR6400 (in dark blue), which reached its destination (station Wt) around 8:00, had to wait until the disruption ended to operate in opposite direction. These happened in both the operator-oriented solution (Figure 6.18) and the passenger-oriented solutions (Figures 6.19 and 6.20).

Compared to the operator-oriented solution (Figure 6.18), the passenger-oriented solution of $\Delta = 10$ (Figure 6.19) added 1 more extra stop to an upstream train from line IC3500 (in pink) at station Gp around 10:15, skipped 6 more scheduled stops of two downstream trains from line SPR9600 (in light blue) at stations Hm, Hmh, and Hmbv, and delayed more train services, e.g., the departures of two downstream trains from line SPR9600 (in light blue) at station Hmbh. These departure delays reduced the waiting times of the passengers who arrived at station Hmbh just after the original departure times of these two trains and originally had to board other trains departing later. Due to the delayed departures, the passengers who were on-board these two trains at station Hmbh would experience arrival delays at their destinations so that the model skipped the following stops at stations Hm, Hmh and Hmbv to avoid the destination arrival delays of these passengers. Compared to these on-board passengers, there were much fewer passengers who would board/leave these two trains at station Hm, Hmh or Hmbv, which was another reason why the model decided to skip these stops. Due to the additional stop at station Gp around 10:15 in the passenger-oriented solution of $\Delta = 10$ (Figure 6.19), one passenger group (including six passengers) that arrived at station Gp at 10:00 and expected to station Ehv benefitted from earlier boarding by shorter generalized travel times, and another three passenger groups chose not to leave the railways. These three groups include (1) one passenger who arrived at station Rm at 9:00, (2) one passenger who arrived at station Wt at 9:15, and (3) one passenger who arrived at station Rm at 9:45, whose destinations were all station Gp. Given the passenger-oriented solution (Figure 6.19), these passengers all took the same train from line IC3500 (in pink), which additionally stopped at station Gp around 10:15. Without this stop (as in the operator-oriented solution shown by (Figure 6.18), these passengers would have to take an upstream train to station Ehv first and then transfer to another downstream local train from line SPR6400 (in dark blue) to reach the destination Gp. This would cost much longer than what these passengers would tolerate, which is why they were observed to leave the railways under the operator-oriented solution. Recall that we assume the maximum generalized travel time a passenger is willing to accept under a rescheduled timetable is twice of his/her expected generalized travel time in terms of the planned timetable.

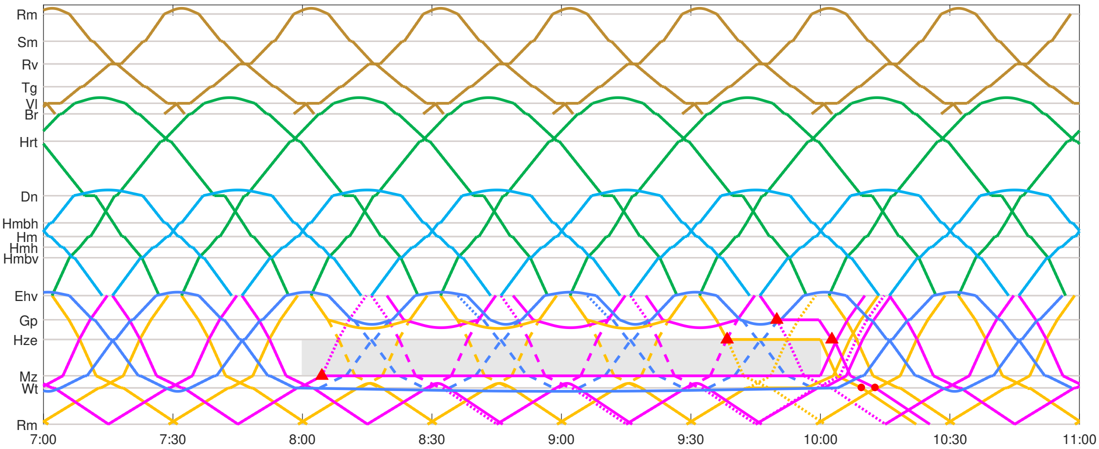


Figure 6.18: The optimal operator-oriented rescheduling solution for case IV: disrupted section Mz-Hze

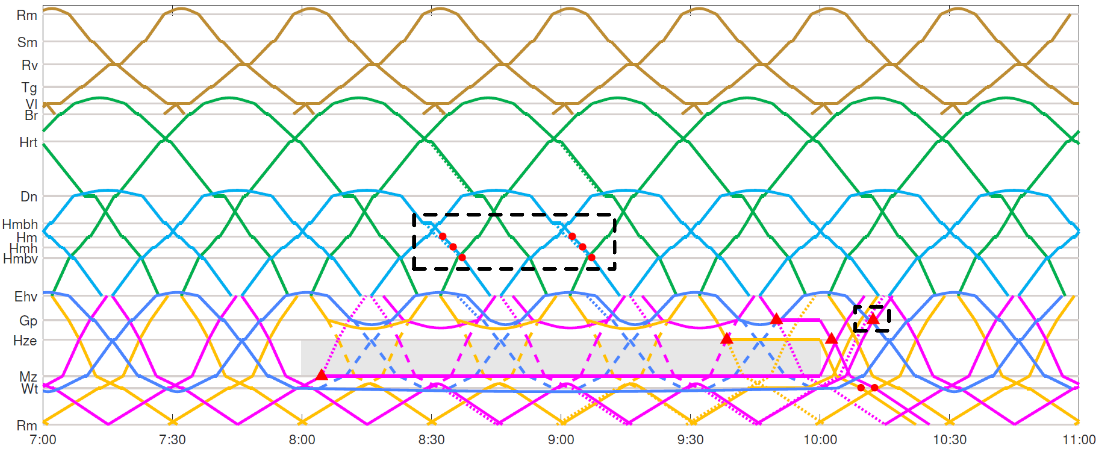


Figure 6.19: The optimal passenger-oriented rescheduling solution for case IV: disrupted section Mz-Hze and $\Delta = 10$

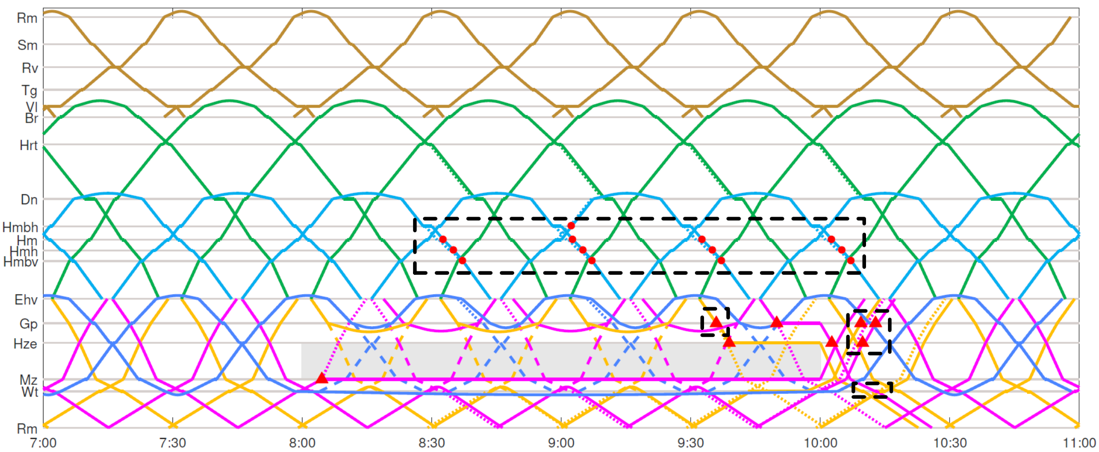


Figure 6.20: The optimal passenger-oriented rescheduling solution for case IV: disrupted section Mz-Hze and $\Delta = 30$

More differences on train stopping patterns from the operator-oriented solution (Figure 6.18) were observed in the passenger-oriented solution of $\Delta = 30$ (Figure 6.20) due to the increase of Δ, which helped to shorten generalized travel times further. For example, the four more extra stops (highlighted by dashed black rectangles in Figure 6.20) helped to shorten the generalized travel times of 31 passengers. Eight of these passengers were observed to leave the railways under the operator-oriented solution (Figure 6.18) or the passenger-oriented solution of $\Delta = 10$ (Figure 6.19), who however chose to travel by train under the passenger-oriented solution of $\Delta = 30$ (Figure 6.20). When $\Delta = 10$ the passenger-oriented solution (Figure 6.19) skipped two scheduled stops at station Wt as in the operator-oriented solution (Figure 6.18) in order to enable the additional train delay below the current Δ. When increasing Δ to 30 the passenger-oriented solution (Figure 6.20) kept these two scheduled stops although leading to more train delay that was acceptable under the current Δ. By keeping these two scheduled stops at station Wt, two more passenger groups chose to not leave the railways compared to either the operator-oriented solution (Figure 6.18) or the passenger-oriented solution of $\Delta = 10$ (Figure 6.19). These two passenger groups include (1) 15 passengers who arrived at station Ehv at 9:30, and (2) 14 passengers who arrived at station Ehv at 9:45, whose destinations were all station Wt. These results indicate that the proposed passenger-oriented timetable rescheduling model is able to provide better alternative train services during disruptions with shorter generalized travel times and also helps railway operators to keep more passengers within the railways. By allowing only 10 min additional train delay than the optimal operator-oriented solution, the passenger-oriented model reduced generalized travel times by thousands of minutes, which is a significant improvement to passengers. By allowing more operation deviations from the optimal operator-oriented solution, the passenger-oriented model can reduce generalized travel times further.

Compared to the operator-oriented model, the passenger-oriented model is able to find better rescheduling solutions to passengers while the needed computation times are longer. From Table 6.10 we know that by using a solver directly an optimal solution can be obtained from the operator-oriented model in seconds, while obtaining an optimal solution from the passenger-oriented model took 370 s in the worst case. With the increase of disruption duration, more passenger groups need to be taken into account, which is why in case I (0.5 h disruption) the passenger-oriented model only took 28 s to get an optimal solution, but in case II (1 h disruption), case III (1.5 h disruption), or case IV (2 h disruption) it consumed longer time to generate an optimal solution (see Table 6.10). Although case III considered a half-hour shorter disruption than case IV, it took a longer computation time than case IV. This is because the disruption durations in both cases are not significantly different and the computation time can be affected by the starting/ending time of a disruption.

6.7.3 The performance of the iterative solution method

To solve the passenger-oriented model in a more efficient way, we proposed an iterative solution method in Section 6.6, which is used here to solve the passenger-oriented model for the same cases considered in Table 6.10. The results from the iterative method are indicated in Table 6.13, in which n_{new} refers to the number of passenger groups newly considered in an iteration. I_{need} and I_{finish} refer to the number of required iterations and the number of completed iterations within T^{stop}, respectively. Recall that we set the computation time limit of each iteration to 30 s and the total computation limit to 300 s. Table 6.13 showed that when n_{new}=10 the iterative solution method terminated when reaching the required computation limit in case III or IV, whereas when n_{new}=50 or 100 the required iterations were all completed within the required computation limit in each case. It was observed that with more passenger groups newly considered in an iteration, the total computation time was shorter and the obtained solution was also better. This is because larger n_{new} requires less iterations, at each of which the quality of the solution obtained can also be improved. For each disrupted section the appropriate value of n_{new} to obtain a good solution in short time can be different. For disrupted section Mz-Hze, $n_{\text{new}} = 100$ is better than $n_{\text{new}} = 50$ while we found that for disrupted section Hze-Gp $n_{\text{new}} = 50$ is better than $n_{\text{new}} = 100$. This will be introduced later.

For disrupted section Mz-Hze, under whichever setting of n_{new} the optimality gaps of the obtained solutions were all small, among which the worst case was 1.23% and the best case was 0.00%. By comparing Table 6.13 with Table 6.10, it was found that the passenger-oriented solutions obtained by the iterative method were all better than the operator-oriented solutions of the same case regarding the generalized travel times of all passengers z_p. For the most difficult case considered: case IV that is associated with 2 h disruption and 728 passenger groups, the iterative method obtained a solution with optimality gap of 0.25% in 107 s if n_{new}=100.

In addition, we used case IV as an example to investigate the computational efficiency of the iterative method when allowing larger maximum operation deviation in the passenger-oriented model. Table 6.14 shows the results by using a solver directly and the iterative method to solve the passenger-oriented model under different values of Δ. The time needed to find an optimal solution by a solver directly became longer with the increase of Δ, because a larger solution space needed to be explored. On average, the solver took 1,156 s to find an optimal solution, which would not be acceptable for real-time application. In contrast, the iterative solution with $n_{\text{new}} = 100$ method took 125 s on average to find a near-optimal solution. It was observed that the computation time needed by the iterative method was much less sensitive to the increase of Δ compared to the solver. The passenger-oriented solutions of $\Delta = 10$ and $\Delta = 30$ by the iterative method are shown in Figure 6.21 and Figure 6.22, respectively. The dashed rectangles highlight the differences on train stopping patterns compared to the operator-oriented solution (Figure 6.18). Compared to the optimal passenger-oriented solution (Figure 6.19 or Figure 6.20), the passenger-oriented solution by the iterative

Table 6.13: Results of the iterative solution method to the passenger-oriented model: disrupted section Mz-Hze, and $\Delta = 10$

Case	n_{new}=10					n_{new}=50					n_{new}=100				
	z_p [min]	I_{need}	I_{finish}	Time [sec]	O-gap (z_p) [%]	z_p [min]	I_{need}	I_{finish}	Time [sec]	O-gap (z_p) [%]	z_p [min]	I_{need}	I_{finish}	Time [sec]	O-gap (z_p) [%]
I	112,140	34	34	107	1.23	110,770	7	7	33	0.01	110,758	4	4	29	0.00
II	156,311	48	48	225	1.12	155,408	10	10	70	0.54	154,668	5	5	58	0.06
III	191,078	62	50	300	0.92	189,493	13	13	120	0.09	189,500	7	7	93	0.09
IV	217,262	73	48	300	0.36	217,018	15	15	155	0.25	217,018	8	8	107	0.25

method (Figure 6.21 or Figure 6.22) was slightly different on the stopping patterns of trains that were originally planned to run through the disrupted section. This is because the passengers who planned to travel through the disrupted section were handled at later iterations due to their shorter expected generalized travel times in this case, while it was observed that the iterative method determined the solution mainly according to the needs of passengers handled at earlier solutions.

Table 6.14: The passenger-oriented solutions for case IV under different Δ: disrupted section Mz-Hze

	Solver			Iterative method (n_{new}=100)				
Δ	z_p [min]	Time [sec]	O-gap (z_p) [%]	z_p [min]	I_{need}	I_{finish}	Time [sec]	O-gap (z_p) [%]
0	221,704	213	0.00	221,704	8	8	104	0.00
10	216,481	250	0.00	217,018	8	8	107	0.25
20	213,693	246	0.00	214,010	8	8	110	0.15
30	212,321	472	0.00	212,365	8	8	124	0.02
40	211,331	640	0.00	211,656	8	8	132	0.15
50	210,439	527	0.00	210,986	8	8	131	0.26
60	209,649	834	0.00	210,038	8	8	131	0.19
70	209,175	1,291	0.00	209,369	8	8	128	0.09
80	208,835	1,935	0.00	209,236	8	8	136	0.19
90	208,477	2,212	0.00	208,842	8	8	134	0.17
100	208,129	4,101	0.00	208,312	8	8	135	0.09
	Average	1,156	0.00			Average	125	0.14

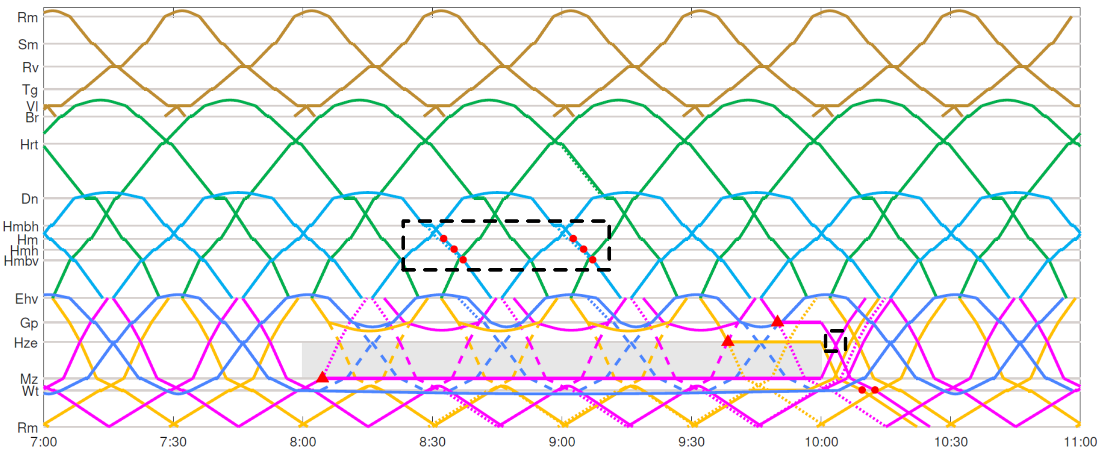


Figure 6.21: The sub-optimal passenger-oriented solution by the iteration method for case IV: disrupted section Mz-Hze and $\Delta = 10$

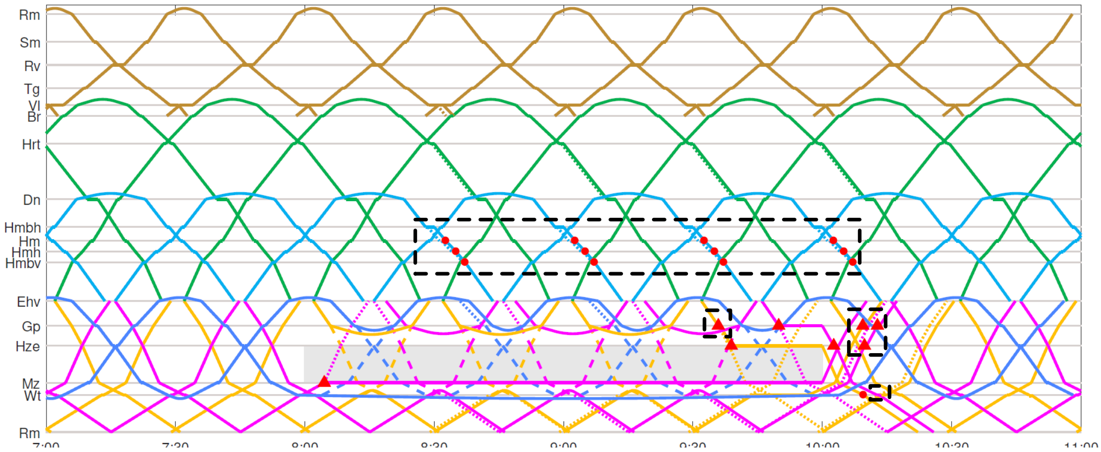


Figure 6.22: The sub-optimal passenger-oriented solution by the iteration method for case IV: disrupted section Mz-Hze and $\Delta = 30$

The previous experiments were carried out on the same disrupted section Mz-Hze. To investigate the performance of the passenger-oriented rescheduling model and the iterative solution method on other disrupted locations, we performed experiments to all sections shown in Figure 6.15. In each of these experiments, the maximum allowed deviation from the optimal operator-oriented solution Δ is set to 10 in the passenger-oriented model, and the number of passenger groups newly considered in each iteration n_{new} is set to 100 in the iterative method. Table 6.15 shows the resulting generalized travel times $\tilde{z}_p$ of the optimal operator-oriented solutions, and the resulting generalized travel time z_p of the optimal passenger-oriented solutions obtained by the solver directly and by the iterative solution method. It is observed that an optimal operator-oriented solution was obtained quickly for each disrupted section, but the resulting total generalized travel time is longer than either the one of the optimal passenger-oriented solution or the one of the passenger-oriented solution from the iterative method. We use $\downarrow$ to highlight the decrease in a passenger-oriented solution compared to the corresponding optimal operator-oriented solution. The computation time of generating an optimal passenger-oriented solution by the solver directly varied across disrupted sections. Disrupted section Hrt-Br took the shortest computation time of 41 s, while disrupted section Wt-Mz took the longest computation time of 5,666 s. The reason is relevant to the number of train lines that were originally scheduled to run through a disrupted section and the starting/ending time of the considered disruption, which both affect the solution space to be explored.

From Table 6.15 we see that in all disrupted sections, the iterative method found better solutions in terms of generalized travel times than the corresponding operator-oriented solutions. It is observed that the passenger-oriented solution by the iterative method reduced generalized travel times by thousands of minutes in each disrupted section, which is indicated by $z_p - \tilde{z}_p$. The gap of a solution from the iterative method to the corresponding optimal passenger-oriented solution was 0.54% in average. The average computation time of obtaining a passenger-oriented solution from the iterative method was 99 s. In 16 out of 17 disrupted sections, the required iterations were completely

Table 6.15: Results of different disrupted sections considering case IV

Disrupted section	Operator-oriented (solver) $\tilde{z}_p$ [min]	Time [sec]	O-gap (z_o) [%]	Passenger-oriented: Δ=10 Solver z_p [min]	Time [sec]	O-gap (z_p) [%]	Iterative method (n_{new}=100) z_p [min]	I_{need}	I_{finish}	Time [sec]	O-gap (z_p) [%]	$z_p - \tilde{z}_p$ [min]
Rm-Wt	217,814	13	0.00	209,801 ↓	204	0.00	211,040 ↓	8	8	124	0.59	- 6,774
Wt-Mz	228,131	12	0.00	214,981 ↓	5,666	0.00	224,217 ↓	8	1	300	4.12	- 3,914
Mz-Hze	221,773	12	0.00	216,481 ↓	250	0.00	217,018 ↓	8	8	107	0.25	- 4,755
Hze-Gp	223,649	12	0.00	218,086 ↓	534	0.00	220,855 ↓	8	8	132	1.25	- 2,794
Gp-Ehv	230,820	12	0.00	225,795 ↓	302	0.00	225,858 ↓	8	8	102	0.03	- 4,962
Ehv-Hmbv	241,442	9	0.00	237,066 ↓	388	0.00	237,518 ↓	8	8	102	0.19	- 3,924
Hmbv-Hmh	241,508	9	0.00	236,809 ↓	343	0.00	237,316 ↓	8	8	100	0.21	- 4,192
Hmh-Hm	241,676	9	0.00	236,854 ↓	353	0.00	237,428 ↓	8	8	99	0.24	- 4,248
Hm-Hmbh	231,575	9	0.00	226,125 ↓	253	0.00	227,150 ↓	8	8	74	0.45	- 4,425
Hmbh-Dn	233,161	9	0.00	227,983 ↓	252	0.00	228,858 ↓	8	8	85	0.38	- 4,303
Dn-Hrt	209,022	9	0.00	201,304 ↓	49	0.00	201,439 ↓	8	8	76	0.07	- 7,583
Hrt-Br	205,075	9	0.00	197,694 ↓	41	0.00	198,523 ↓	8	8	69	0.42	- 6,552
Br-Vl	203,624	9	0.00	196,495 ↓	57	0.00	197,616 ↓	8	8	79	0.57	- 6,008
Vl-Tg	195,050	6	0.00	189,402 ↓	125	0.00	189,402 ↓	8	8	60	0.00	- 5,648
Tg-Rv	193,861	6	0.00	188,213 ↓	46	0.00	188,213 ↓	8	8	51	0.00	- 5,648
Rv-Sm	193,492	6	0.00	187,640 ↓	67	0.00	187,988 ↓	8	8	52	0.19	- 5,504
Sm-Rm	192,653	6	0.00	187,005 ↓	227	0.00	187,005 ↓	8	8	64	0.00	- 5,648
Average	217,902	9	0.00	211,631 ↓	539	0.00	212,827 ↓	–	–	99	0.54	- 5,075

finished with at most 132 s in total. An exception was disrupted section Wt-Mz, for which only one iteration was finished and the corresponding computation time reached the required computation time limit: 300 s. This is because disrupted section Wt-Mz was the most difficult section to be solved and thus including 100 new passenger groups in each iteration was still computation-consuming to the passenger-oriented model.

Therefore, we performed another experiment to disrupted section Wt-Mz by including 50 new passenger groups in each iteration. For each rescheduling solution computed in an iteration, we evaluated the resulting generalized travel times of all passengers. Figure 6.23 shows the iterative results of disrupted section Wt-Mz by setting n_{new} to 50 and 100, respectively. The operator-oriented solution was also indicated for comparison. The x-axis refers to the iteration number, and the y-axis refers to generalized travel times. When n_{new}=100, only one iteration was completed due to reaching the total computation time limit of 300 s, while the obtained passenger-oriented solution was still better than the operator-oriented solution. When n_{new}=50, the required 15 iterations were all completed. This indicates that for a disruption case that is difficult to be solved using a smaller value of n_{new} helps to find a better solution quickly. It was observed from Figure 6.23 that when $n_{\text{new}} = 50$ the passenger-oriented solution by the iterative method was the same as the operator-oriented solution at the 1st iteration, but was largely improved in the 2nd and 3rd iterations. From the 4th iteration until the final iteration, the passenger-oriented solution was barely improved. This indicates that the quality of the final solution obtained by the iterative method is mainly determined by earlier iterations. This is because the path choices of passenger groups who have already been considered at an earlier iteration were fixed in the iterative method at the following iterations where new passenger groups were included but reducing their generalized travel times may increase the ones of earlier considered passenger groups so that very few/none schedule adjustments were made to avoid affecting earlier considered passengers. Recall that in the case study the passenger groups with larger expected generalized travel times are considered at earlier iterations.

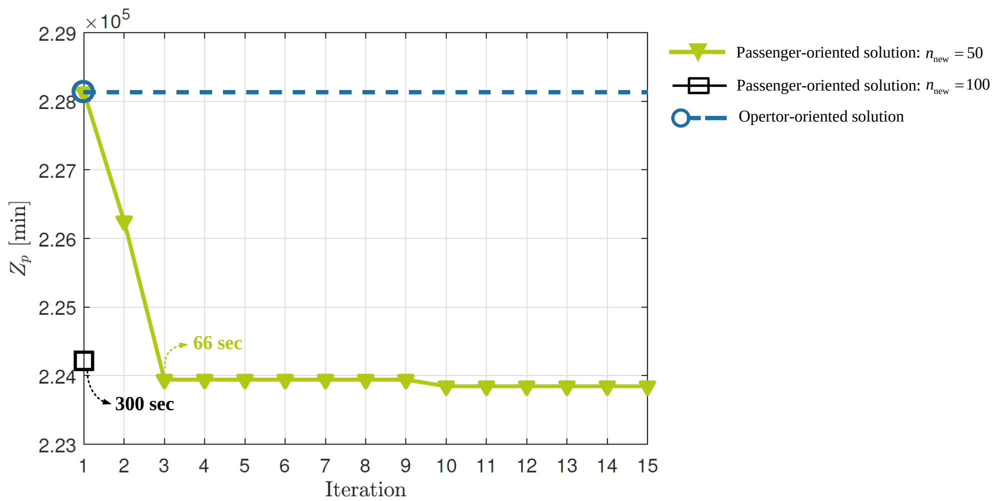


Figure 6.23: Results for disrupted section: Wt-Mz (Δ=10)

We also take disrupted section Mz-Hze and disrupted section Hze-Gp as two more examples to explore the performance of the iterative method. Figures 6.24 and 6.25 show the relevant results, respectively. It is observed that in both disrupted sections, the quality of the passenger-oriented solutions by the iterative method are mainly determined by earlier iterations when n_{new}=50 or 100, and are all better than the corresponding operator-oriented solutions. In disrupted section Mz-Hze (Figure 6.24), a *stable* passenger-oriented solution was obtained after 1 iteration when n_{new}=100, and after 2 iterations when n_{new}=50. Here, we describe a solution as stable when no/few improvements were made on this solution in all following iterations in the iterative method. The computation time for generating the stable solution was no longer than 30 s when n_{new}=50 or 100. In disrupted section Hze-Gp (Figure 6.25), a stable passenger-oriented solution was obtained after 4 iteration when n_{new}=100, and after 3 iterations when n_{new}=50. In these two situations, the computation times for generating the stable solutions were 66 s and 45 s, respectively. It is observed that in disrupted section Hze-Gp the quality of the solution when setting n_{new} to 100 is worse than the quality of the solution when setting n_{new} to 50. The reason is when n_{new}=100 the solution obtained at the 1st iteration was a suboptimal solution, which took 30 s reaching the required computation time limit of an iteration. The relatively poor quality of the 1st solution affects further improvements in the following iterations. Whereas when n_{new}=50, the solution obtained at the 1st iteration was an optimal solution, which helps for further improvements in later iterations. These results indicate that the performance of the iterative method is relevant to the number of passenger groups newly considered at an iteration, the computation time limit required at an iteration, the total computation limit, and the disrupted section. The passenger groups considered at earlier iterations play an important role in determining the quality of the solution finally obtained by the iterative method. In that sense, the computation time of obtaining a high-quality passenger-oriented solution by the iterative method can be improved further by only performing a few iterations.

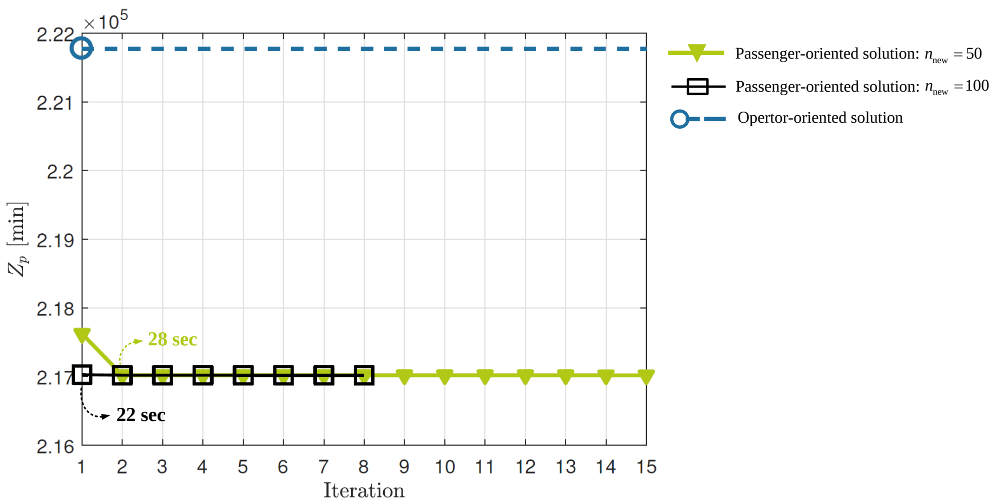


Figure 6.24: Results for disrupted section: Mz-Hze (Δ=10)

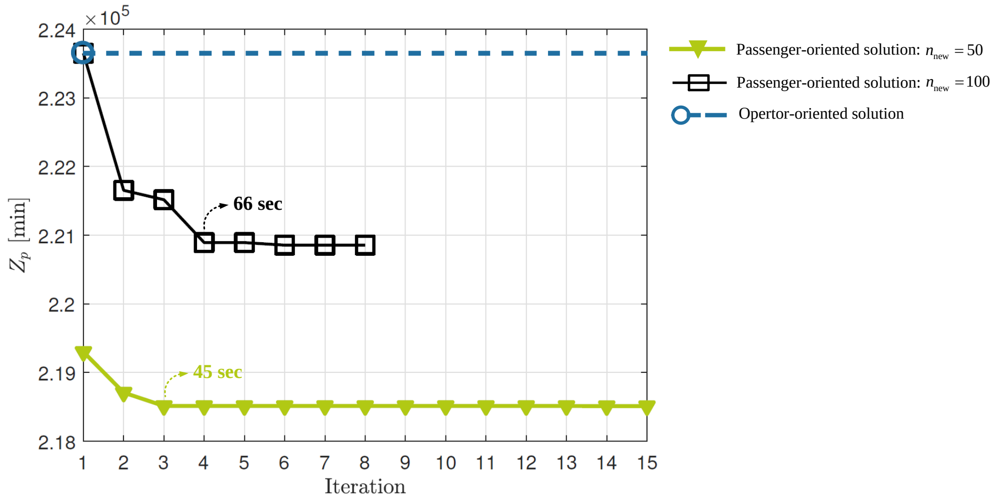


Figure 6.25: Results for disrupted section: Hze-Gp (Δ=10)

6.8 Conclusions and future directions

This chapter developed a novel MILP model that integrates timetable rescheduling with passenger reassignment to compute passenger-oriented rescheduled timetables in case of railway disruptions. The objective is minimizing generalized travel times of passengers, which consist of in-vehicle times, waiting times at origin/transfer stations and the number of transfers. Multiple dispatching measures were adopted to adjust the timetable with respect to passenger needs, including re-timing, re-ordering, cancelling, flexible stopping and flexible short-turning trains. An iterative solution method was proposed to solve the model efficiently, by considering restricted passenger groups at each iteration.

The passenger-oriented timetable rescheduling model was applied to a part of the Dutch railways, and compared to an operator-oriented timetable rescheduling model that does not formulate passenger reactions so that the objective is minimizing train cancellations and arrival delays. It was observed that the passenger-oriented model was able to shorten generalized travel times by thousands of minutes with only 10 min more train arrival delay than the optimal operator-oriented solution. With more operation deviations allowed, the passenger-oriented model is able to shorten generalized travel times further. When given a passenger-oriented rescheduling solution, more passengers chose to continue their train travels after the disruption started, compared to an operator-oriented solution for the same disruption case. By the proposed iterative solution method, high-quality rescheduling solutions were obtained by the passenger-oriented model in an acceptable time. It was found that the quality of the final solution obtained by the iterative method is mainly determined by the number of new passenger groups considered at earlier iterations.

In future, we will apply the passenger-oriented model to a larger railway network, by which the computation time will increase further as more train services and passengers will be considered. For this situation, the iterative solution method proposed in this work might also be able to obtain a good solution efficiently as long as representative passenger groups that determine the solution quality can be identified. Then, only these passenger groups need to be handled. During disruptions, trains could become crowded due to detouring passengers whose planned trains were cancelled, and then some passengers would be denied to board specific trains because of lacking capacities. Therefore, we also will take limited vehicle capacity into account to handle both timetable rescheduling and rolling stock rescheduling for providing passengers with more reliable alternative train services in case of railway disruptions. Besides disruption management, the proposed passenger-oriented timetable rescheduling model can be applied to disturbance management by few modifications, which is also promising to be used for improving an existing non-cyclic timetable in terms of generalized travel times. For example, because our model formulates flexible stopping it can be used to determine the planned train stopping patterns of a timetable according to passenger needs.

Appendix 6.A.

Table 6.16: Parameters and sets

Symbol	Description
$head(a)$	The event which activity a starts from
$tail(a)$	The event which activity a directs to
f_e	Binary parameter with value 1 indicating that arrival event e is a train destination arrival, and 0 otherwise.
r_e	Binary parameter with value 1 indicating that (duplicate) departure event e is a train origin departure, and 0 otherwise.
In_e	Set of activities going in event e
Out_e	Set of activities going out from event e
G	Set of passenger groups
G'	Set of passenger groups that are considered in an iteration in the iterative solution method: $G' \subseteq G$
G_{new}	Set of passenger groups that are newly considered in an iteration in the iterative solution method: $G_{\text{new}} \subseteq G'$
O_g	The origin of passenger group g
D_g	The destination of passenger group g
t_g^{ori}	The origin arrival time of passenger group g
n_g	The number of passengers in passenger group g

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Symbol	Description
T_g^{plan}	The expected generalized travel time of passenger group g in terms of the planned timetable
μT_g^{plan}	The maximum acceptable generalized travel time of passenger group g in terms of a rescheduled timetable: $\mu \geq 1$
$\ell_{\text{trans}}^{\text{min}}$	The minimum transfer time needed at a station
$\ell_{\text{trans}}^{\text{max}}$	The maximum transfer time which a passenger would like to spend at a station
$\ell_{\text{wait}}^{\text{max}}$	The maximum waiting time which a passenger would like to spend at a station
β_{wait}	The multiplier of waiting times perceived by passengers at stations
β_{vehicle}	The multiplier of in-vehicle times perceived by passengers
β_{trans}	The fixed time penalty perceived by passengers on one transfer
t^{stop}	The computation time limit of each iteration in the iterative solution method
T^{stop}	The total computation time limit of the iterative solution method
n_{new}	The number of passenger groups newly considered in each iteration of the iterative solution method
t_{start}	Start time of disruption
t_{end}	End time of disruption
R	The time length required for the planned timetable to be recovered after the disruption ends
D	Maximal allowed delay per event
M	A sufficiently larger number whose value is set to 2880
M^*	A sufficiently larger number whose value is set to $\beta_{\text{wait}}M$

Chapter 7

Conclusions

This thesis is dedicated to improving railway disruption management so that it becomes more efficient, operator-friendly, and passenger-friendly. Several research questions were posed to achieve the research objective, which are answered throughout Chapters 2 to 6. In this chapter, Section 7.1 presents the main findings. Sections 7.2 and 7.3 give future research directions and recommendations to practice, respectively.

7.1 Main findings

The main research question proposed in this thesis is: "*How to support railway disruption management by rescheduled timetables that are operator-friendly and passenger-friendly?* " . To answer this main question, 5 sub-questions were defined. The answers to these questions are summarized as follows.

- How to predict and affect passenger flows for a given rescheduled timetable? (Chapter 2)

  In Chapter 2, a schedule-based dynamic passenger assignment model has been developed to formulate passenger responses towards major service variations like short-turn/cancelled trains due to disruptions. The model considers denied boarding due to insufficient vehicle capacity. On-board passengers are given priority over waiting passengers, and waiting passengers are boarding under the first-come-first-serve rule. The model can cope with different information provisions to inform passengers with service variations and/or train congestion at different locations, to investigate how passengers will change their path choices depending on the information received. Experiments showed that when vehicle capacities were always sufficient (no denied boarding), informing passengers with service variations at both stations and trains helped to shorten their travel times while additionally publishing train congestion did not make any sense.

When vehicle capacities were in short supply (denied boarding exists), additionally publishing train congestion was able to shorten generalized travel times depending on how a train was defined as highly congested to proactively avoid some passengers boarding the next run of the train.

- How to obtain a rescheduled timetable that minimizes the impact on passengers' travel plans and has a high implementability in practice? (Chapter 3)

  In Chapter 3, an MILP model has been developed to deal with timetable rescheduling in case of disruptions. The dispatching measures of re-timing, reordering, cancelling, flexible stopping and flexible short-turning trains are all formulated in the model with the objective of minimizing the impact on passengers' expected travel paths that are estimated according to the planned timetable. The number of affected passengers and the resulting lateness/earliness to these passengers due to a decision of cancelling a service, delaying a train arrival, adding a stop or skipping a stop are both considered to estimate the passenger-dependent objective weight. In this way, the timetable rescheduling model rapidly computes a more passenger-friendly rescheduled timetable that can also be preferred by operators. Experiments showed that applying flexible stopping and flexible short-turning trains resulted in less passenger delays compared to applying either or neither of them.

  The proposed timetable rescheduling model distinguishes between platform tracks and pass-through tracks at a station level to assign each train with a platform at a station where passengers will board/leave the train. The rolling stock circulations at both short-turning and terminal stations of trains are considered, as well as whether a station has turning facilities for the rolling stock coming from different directions. All phases of a disruption are dealt with, which means that the model computes a rescheduling solution from the starting of a disruption until the planned timetable is recovered. Experiments showed that shortening the recovery duration mitigated the post-disruption consequence by less delay propagation but at the expense of more cancelled train services during the disruption.

  The timetable rescheduling model proposed in Chapter 3 assumes a fixed disruption duration, deals with the single-disruption case, and considers static passenger demand. This model is then extended in Chapter 4 to take uncertain disruption duration into account, in Chapter 5 to deal with multiple connected disruptions, and in Chapter 6 to consider dynamic passenger demand, respectively.

- How to handle a disruption with uncertain duration by robust rescheduled timetables? (Chapter 4)

  In Chapter 4, a rolling-horizon two-stage stochastic programming model has been developed to generate robust rescheduled timetables for uncertain railway disruptions. The ending time of a disruption is assumed to be a random variable that has a finite number of realizations, called scenarios, with given probabilities. In each scenario, the considered time horizon is divided into a scenario-independent *control horizon* and a scenario-dependent *look-ahead horizon*. The dispatching decisions (e.g. cancelling a train service) relevant to the control horizons are forced to be the same in all scenarios, while the decisions relevant to the look-ahead horizons can be different among scenarios. In this way, the stochastic model computes the optimal rescheduling solution for the control horizon, which is robust to all scenarios. The stochastic model is embedded in a rolling-horizon approach so that every time a prediction about the range of the disruption end time is updated new scenarios are defined and the model recomputes a robust rescheduling solution accordingly. The stochastic rolling-horizon method is compared to a deterministic rolling-horizon method that uses the deterministic timetable rescheduling model from Chapter 3 to compute a rescheduled timetable with one possible disruption ending time considered. Experiments showed that the stochastic method was able to reduce train cancellations and/or delays compared to the deterministic method.

- How to deal with multiple connected disruptions in an efficient and operator-friendly way? (Chapter 5)

  In Chapter 5, two approaches have been developed to deal with multiple disruptions that occur at different geographic locations but have overlapping periods and are pairwise connected by common train lines. One is the sequential approach, in which a single-disruption rescheduling model is applied to handle each new disruption with the last solution as reference. Another one is the combined approach, in which a multiple-disruption rescheduling model is proposed to handle each extra disruption by considering the combined effects of all ongoing disruptions. The interactions among the dispatching decisions associated with different disruptions are explicitly formulated in the multiple-disruption rescheduling model. Experiments showed that the combined approach was able to find rescheduling solutions with less cancelled train services and/or delays than the sequential approach. For long disruptions, we propose a rolling-horizon solution method, which considers the periodic pattern of the rescheduled services in the stable phase of a disruption to speed up the computation. Experiments showed that the rolling-horizon approach is able to generate high-quality rescheduling solutions in an acceptable time.

- How to formulate a timetable rescheduling model considering dynamic passenger flows, and obtain a high-quality solution rapidly? (Chapter 6)

  In Chapter 6, timetable rescheduling and passenger reassignment have been formulated into one MILP model to compute passenger-friendly rescheduled timetables. The objective is minimizing generalized travel times, which include waiting times at origin/transfer stations, in-vehicle times, and the number of transfers. An improved event-activity network formulation is proposed to describe passenger path choices in terms of a timetable. Passengers are allowed to drop the railways if they cannot find preferred alternative travel paths from a timetable. We introduce a method of formulating a transition network that enables to include dynamic event-activity networks in the proposed passenger-oriented timetable rescheduling model so that the model is able to consider timetable-dependent passenger behaviours. An iterative solution method has been developed to solve the passenger-oriented rescheduling model in an efficient way. In each iteration, the timetable rescheduling problem is solved for all train services with restricted passenger groups taken into account. Experiments showed that the passenger-oriented timetable rescheduling model was able to reduce the number of dropped passengers and the generalized travel times of passengers significantly. The iterative solution method was able to generate high-quality solutions in an acceptable time.

7.2 Future research directions

This section points out several directions for future research, which are elaborated as follows.

The timetable rescheduling models developed to deal with uncertainty in disruption duration (Chapter 4) and multiple connected disruptions (Chapter 5) are both operator-oriented with the same objective of minimizing train cancellations and delays. Incorporating dynamic passenger behaviour and combining the models will be one of the future directions. Furthermore, the passenger-oriented timetable rescheduling model proposed in Chapter 6 is for disruption cases, but it can be adapted also to passenger-oriented timetabling and delay management with few modifications.

Improving passengers' travelling experiences can be in conflict with the benefits of operators. In Chapter 6 we found that shortening the generalized travel times of passengers came at the expense of introducing more deviations from the planned timetable. Hence, when rescheduling a timetable we can also explicitly consider a trade-off between passengers and operators so that both can be satisfied to a large extent.

It is found from Chapter 6 that a passenger-oriented solution obtained under the objective of minimizing generalized travel times is sensitive to passenger preferences on different journey components including waiting times at stations, in-vehicle times and the number of transfers. Currently, these preferences are extracted from the existing literature that focuses on passenger behaviour on normal days (no service variations), whereas during major service variations passengers attitudes towards the journey components could be different. More research needs to be done on analysing passenger behaviour during major service variations to provide accurate passenger preferences in the passenger-oriented rescheduling models so that train services can be adjusted to cater for passenger actual needs.

Besides providing passengers with better train services, it is also important to notify passengers with more reliable and timely information on service variations and station/train congestion. How to generate this information considering the uncertainties during railway operations (e.g. uncertain disruption duration) will be another future research direction.

Passengers' path choices vary with the information received. Chapter 2 investigated how passenger flows were affected given different information provisions under a given rescheduled timetable. In the future, it is recommended to include the impact of information provisions into passenger-oriented timetable rescheduling models, in order to formulate passenger behaviour in a more realistic way. When vehicle capacity is in short supply, the path choice of one passenger can affect the one of another passenger. Therefore, a further step will be taking limited vehicle capacity into account in passenger-oriented timetable and/or rolling stock rescheduling, in which the influence of train congestion on passenger path choices need to be considered as well.

More efficient solution approaches should be explored in the future to ensure that the passenger-oriented optimization models can be applied in the real world. It is found from Chapter 6 that the computation time of the passenger-oriented model increases with more passengers considered, while the solution quality is mainly determined by a few of the considered passengers. This indicates that a column generation method may be promising to solve the passenger-oriented models. The basic idea of column generation is to split the problem into a master problem that corresponds to the original problem but with only a subset of variables, and a sub-problem that aims to find the key variables that are able to improve the objective value of the master problem. In a passenger-oriented model, finding the key variables will be identifying the variables relevant to the passengers who play an important role in determining the solution quality. Besides focusing on integrated passenger-oriented models as proposed in Chapter 6, it is also recommended to design a closed-loop framework where a dynamic passenger assignment model and a timetable rescheduling model are performed iteratively with the output from the assignment model as a feedback to the rescheduling model. This helps to find a feasible solution rapidly. The challenge is how to design the feedback mechanism to improve the solution in each new iteration and ensure that the improvement is as large as possible so that a high-quality solution can be obtained after a few iterations.

7.3 Recommendations to practice

The timetable rescheduling models developed in Chapters 3 to 6 can be used either for off-line application of automatically generating improved contingency plans that deal with all phases of disruptions or for on-line application of real-time timetable rescheduling to a similar size of railway network as considered in this thesis. Adding additional stops and skipping scheduled stops are not commonly applied in the railways nowadays, but are recommended to be used in future. Using these two measures may provide more alternative train services that help to shorten passengers' travel times during disruptions. They can also be used for overcrowding situations. For example skipping a stop of a train if the train is already full or adding a train with an extra stop where many passengers are waiting at the station.

The passenger assignment model developed in Chapters 2 and 6 can help traffic controllers to foresee the impact of possible dispatching decisions on passengers so that the timetable can be adjusted in a more passenger-friendly way. The model of Chapter 2 can also be used to identify potential congested trains, based on which operators can take some mitigation strategies in advance such as allocating more vehicles to the potential congested trains and suggesting some passengers to take other uncrowded trains. This helps to prevent denied boarding on the one hand, and avoid prolonged train running times due to overcrowding (which may incur additional delays) on the other hand.

Improving information provision system is highly recommended. Passengers should be informed with any service adjustments determined by traffic controllers in a more timely manner so that they can re-plan their journeys in time, which gives more choices to passengers than what they have given delayed information. Besides information on service variations, information on train congestion also need to be provided to passengers to reduce the possibility of denied boarding, particularly during rush-hours.

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Summary

Disruptions are unavoidable in daily operations of railways due to a variety of unexpected events such as blocked tracks due to rolling stock breakdown or switch failures. In case of a disruption, the planned timetable will be inapplicable because of unavailable scheduled routes. In current practice, traffic controllers then have to manually adjust the timetable based on pre-designed contingency plans. This is a time-consuming procedure, in which the impact to passengers is rarely considered. As a result, the rescheduled timetable may be of worse quality to passengers than necessary.

This thesis aims to develop a methodological framework for improving railway disruption management so that it becomes more efficient, operator-friendly and passenger-friendly. To this end, a novel passenger assignment model has been developed to estimate passenger inconvenience under a given rescheduled timetable and to investigate the effectiveness of information on reducing the inconvenience. A timetable rescheduling model has been developed to deal with disruptions of given durations considering static passenger demand. Extensions to this model have been developed resulting in three more timetable rescheduling models respectively for handling disruptions with uncertain durations, multiple connected disruptions, and dynamic passenger demand during the rescheduling. A summary for each of the five developed models is given as follows.

First, we applied the technique of discrete-event simulation to formulate a schedule-based dynamic passenger assignment model. The model simulates passenger path choices during disruptions where cancelling/short-turning of trains are unavoidable. Train arrivals and departures at stations are events, which trigger changes of passenger behaviour that are also influenced by the information received. We designed different information provisions and simulated the impact on passengers. Experiments showed that informing passengers on service variations both at stations and in trains was helpful to shorten their travel times, while additionally publishing train congestion information was able to avoid denied boarding when vehicle capacity was in short supply.

Second, we developed a Mixed Integer Linear Programming (MILP) model to rapidly compute feasible rescheduled timetables from the beginning of disruptions until the normal schedule was recovered. The model applies re-timing, re-ordering, cancelling, flexible short-turning and flexible stopping trains. The impact of dispatching measures

on passengers' planned travels are quantified as weights to decision variables included in the objective of minimizing passenger delays. Platform track capacity and rolling stock circulations at terminal and short-turning stations are all considered to improve the solution implementability in practice. Experiments showed that the measures of flexible short-turning and flexible stopping trains were helpful to improve the performance of rescheduled timetables compared to using either or neither of them.

Third, we developed a rolling-horizon two-stage *stochastic* programming model to compute rescheduled timetables for disruptions with uncertain durations that were updated over time. The stochastic model considers multiple possible disruption durations (called scenarios with given probabilities) and computes the rescheduling solution that is optimal considering all scenarios. The optimal rescheduling solution results in the smallest expected consequence measured in train delays and cancellations over all scenarios. We also proposed a rolling-horizon *deterministic* timetable rescheduling model, which considers one possible disruption duration that is estimated using an optimistic, a pessimistic or an expected-value strategy. Experiments showed that the average performance of the stochastic method was better than the deterministic method using any strategy in terms of train cancellations and delays. The improvement percentages with respect to the value of the stochastic solution (*VSS*) were between 6.1% and 10.2% in our cases, demonstrating the benefit of the stochastic method.

Fourth, we proposed two approaches for rescheduling timetables in case of multiple disruptions that occur at different locations but are connected by common train lines. One is a sequential approach that uses a single-disruption timetable rescheduling model to handle each extra disruption with the last rescheduling solution as a reference. The second one is a combined approach, in which a multiple-disruption model was developed to handle each extra disruption while considering the combined effect on all ongoing disruptions. Experiments showed that compared to the sequential approach, the combined approach resulted in less cancelled trains and train delays but required longer times to compute optimal solutions. Therefore, a rolling horizon method was developed to solve the multiple-disruption model in the combined approach to rapidly generate sufficiently good solutions.

Fifth, we integrated the timetable rescheduling and passenger reassignment problems in a single MILP model to compute rescheduled timetables with the objective of minimizing the weighted travel times over all passengers. The model takes into account passengers' preferences on in-vehicle times, waiting times at stations and the number of transfers. An iterative solution method was developed to reschedule all train services with restricted passengers considered at each iteration. Experiments showed that the iterative method was able to find high-quality rescheduling solutions in an acceptable time, and that the solution quality is mainly determined by passengers that were considered at earlier iterations.

In summary, this thesis developed a methodological framework for passenger-oriented railway disruption management and demonstrated its effectiveness in improving passengers' travelling experiences during disruptions.

Samenvatting

In het dagelijkse treinverkeer zijn verstoringen onvermijdelijk door onverwachte gebeurtenissen als versperringen door materieeldefecten of wisselstoringen. Bij een storing is de geplande dienstregeling niet meer mogelijk omdat de geplande rijwegen niet meer beschikbaar zijn. In de huidige praktijk moeten verkeersleiders dan de dienstregeling handmatig aanpassen op basis van vooraf bepaalde versperringsmaatregelen. Dit is een tijdrovend proces waarin de invloed op de passagiers nauwelijks meegewogen kan worden. Als gevolg hiervan kan de aangepaste dienstregeling onvriendelijker zijn richting de reizigers dan nodig is.

Dit proefschrift heeft als doel een methodologische aanpak te ontwikkelen voor het verbeteren van maatregelen bij verstoringen van het treinverkeer, zodat die efficiënter worden en beter aansluiten bij de wensen van vervoerder en reiziger. Hiertoe is een nieuw reizigerstoedelingsmodel ontwikkeld dat het ongemak voor reizigers schat voor een gegeven aangepaste dienstregeling en waarmee de effectiviteit van informatie kan worden onderzocht op het verminderen van dit ongemak. Daarnaast is een optimaliseringsmodel ontwikkeld voor het aanpassen van de dienstregeling bij verstoringen met gegeven storingsduur gebaseerd op statische vervoervraag. Dit model is verder uitgebreid voor het afhandelen van storingen met onzekere tijdsduur, meerdere elkaar rakende verstoringen en dynamische vervoervraag tijdens de herplanning. Hieronder volgt een samenvatting van elk van deze vijf ontwikkelde modellen.

Discrete-event simulatie is gebruikt voor de formulering van een dienstregeling-gebaseerd dynamisch reizigerstoedelingsmodel. Het model simuleert de routekeuzes van reizigers tijdens verstoringen waarbij het onvermijdelijk is dat treinen uitvallen of vroeg keren op stations voor de versperringen (kortkeren). Aankomsten en vertrekken van treinen op stations zijn gebeurtenissen die het gedrag van passagiers beïnvloeden afhankelijk van de ontvangen informatie. We hebben de impact van verschillende informatievoorzieningen op het reizigersgedrag gesimuleerd. De experimenten toonden aan dat het informeren van passagiers over dienstregelingsaanpassingen op zowel stations als in treinen nuttig was om hun reistijden te verkorten, en daarnaast kon met het publiceren van informatie over drukte in de trein voorkomen worden dat reizigers niet konden instappen als de voertuigcapaciteit schaars was.

Ten tweede hebben we een gemengd-geheeltallig lineair programmeringsprobleem (MILP) ontwikkeld om snel toelaatbare aangepaste dienstregelingen te berekenen vanaf het begin van de storingen totdat de oorspronkelijke dienstregeling weer is hersteld. Het model kan tijden en volgordes aanpassen, treinen laten uitvallen of eerder laten keren, en flexibel laten stoppen. Het effect van de versperringsmaatregelen op de geplande reizen van passagiers wordt gekwantificeerd door gewichten toe te voegen aan de beslisvariabelen in de doelfunctie om reizigersvertragingen te minimaliseren. Perroncapaciteit en materieelomlopen op eindstations en kortkeerstations zijn meegenomen om de implementeerbaarheid van de oplossing in de praktijk te verbeteren. Experimenten toonden aan dat flexibel kortkeren en/of flexibel halteren nuttige maatregelen zijn om de prestaties van aangepaste dienstregelingen te verbeteren.

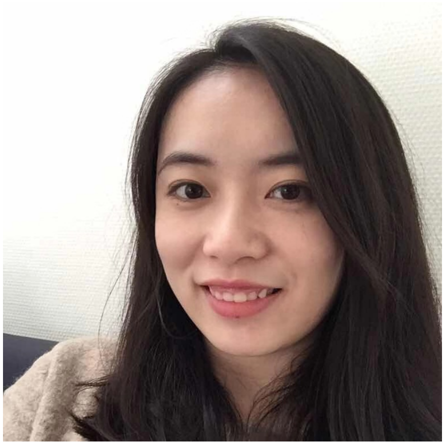
Ten derde hebben we een twee-staps stochastisch programmeringsmodel met rollende horizon ontwikkeld om aangepaste dienstregelingen te berekenen voor storingen met onzekere tijdsduur die in de loop van de tijd worden geüpdatet. Het stochastische model houdt rekening met meerdere mogelijke storingsduren in scenario's met gegeven kansen en berekent de aangepaste dienstregeling die optimaal is rekening houdend met alle scenario's. We hebben ook een deterministisch herplanningsmodel met rollende horizon ontwikkeld, waarbij de storingsduur wordt geschat met een optimistische, pessimistische of verwachte-waarde strategie. Experimenten toonden aan dat de gemiddelde prestaties van de stochastische methode beter waren dan de deterministische methode (ongeacht de strategie) in termen van vertragingen en het aantal vervallen treinen.

Ten vierde hebben we twee benaderingen voorgesteld voor het herplannen van dienstregelingen bij meerdere verstoringen die op verschillende locaties optreden, maar verbonden zijn door gemeenschappelijke treinseries. De eerste is een sequentiële aanpak die het optimaliseringsmodel voor individuele storingen gebruikt voor iedere extra storing met de laatste oplossing als referentiedienstregeling. De andere is een gecombineerde aanpak, waarbij een optimaliseringsmodel is ontwikkeld voor meerdere storingen tegelijk, waarbij de oplossing voor elke extra storing rekening houdt met de gecombineerde effecten van alle lopende verstoringen. Experimenten toonden aan dat in vergelijking met de sequentiële aanpak, de gecombineerde aanpak resulteerde in minder geannuleerde treinen en treinvertragingen, maar dat het langer duurde om optimale oplossingen te berekenen. Daarom is een rollende horizon methode ontwikkeld om voor het meervoudige storingsmodel met de gecombineerde aanpak snel goede oplossingen te vinden.

Ten vijfde hebben we een MILP model ontwikkeld waarin het herplanningsprobleem en de reizigerstoedeling zijn geïntegreerd met als doel de gewogen reistijden voor alle passagiers te minimaliseren. Het model houdt rekening met de voorkeuren van passagiers voor de reistijd in de trein, wachttijden op stations en het aantal overstappen. Een iteratieve oplossingsmethode is ontwikkeld om alle treindiensten te herplannen waarbij een toenemend aantal reizigers werd meegenomen in elke iteratie. Experimenten toonden aan dat de iteratieve methode in staat was om hoogwaardige aangepaste dienstregelingen te vinden in een acceptabele tijd, en dat de kwaliteit van de oplossing voornamelijk werd bepaald door de reizigers die bij de eerste iteraties werden meegenomen.

Samenvattend is in dit proefschrift een methodologische aanpak ontwikkeld voor reizigersgericht herplannen van treindienstregelingen bij versperringen en is de effectiviteit hiervan aangetoond op het verbeteren van de reiservaring van reizigers tijdens verstoringen.

About the author



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Her research interests include real-time traffic management, disruption management, passenger assignment, integrating passenger behaviour into decision making models, and optimization in transportation.

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4. **Zhu, Y.**, Goverde, R.M.P., (2019). Dynamic passenger assignment for major railway disruptions considering information interventions. *Networks and Spatial Economics*, in press.

3. **Zhu, Y.**, Goverde, R.M.P., (2019). Dynamic railway timetable rescheduling for multiple connected disruptions. *Transportation Research Part C: Emerging*

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2. **Zhu, Y.**, Goverde, R.M.P., (2019). Dynamic and robust timetable rescheduling for uncertain railway disruptions. *Best Papers Special Issue of RailNorrköping 2019 in Journal of Rail Transportation Planning & Management*, under review.

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