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Least Squares Calibration of MIMO Radars with Collocated Arrays

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***Abstract*— The calibration of a collocated MIMO radar is addressed by means of independent calibration of transmit and receive arrays. The solutions for both arrays' elements gain and phase terms only and the full coupling matrix estimation are presented. The proposed solution significantly improves over the conventional calibration of the virtual array in terms of calibration accuracy and reduced measurements requirement, as demonstrated by numerical simulation and validated by calibration of a commercial automotive radar.**

***Keywords*— MIMO radar, calibration, least squares**

I. Introduction

Multiple-input multiple-output (MIMO) radar systems have received significant attention in the last decade due to their instantaneous large-angle coverage, ability to provide high angular resolution and their low cost, essential for commercial radars. These promising benefits of MIMO radars can be archived only if the impact of prominent hardware errors is minimal, thus system calibration is crucial for radar production and maintenance. Array calibration is a procedure that corrects hardware imperfections such as gain, phase, array element locations, mutual coupling between elements and I/Q imbalance [1]. Improper array calibration leads to severe degradation of radar performance, in particularly the accuracy and target response of (high-resolution) direction-of-arrival (DOA) estimation techniques [2], interference cancellation [3] and target detection.

Most of the conventional methods for MIMO radar calibration use the virtual array representation and benefit from a wide variety of calibration techniques for phased array radars, e.g. [4], [5]. These methods provide reliable estimation of the calibration coefficients [5] or the coupling matrix [4] with sufficient amount of calibration data, but they do not benefit the particular structure of the MIMO beamforming.

In this paper paper we propose a simple yet efficient calibration technique for independent calibration of transmit and receive arrays of a coherent MIMO radar. We demonstrate that the proposed approach improves over the conventional calibration of the virtual array in terms of calibration accuracy and lowers the minimal required number of independent measurements.

Notations: Hereinafter we use lowercase boldface letters for vectors and uppercase boldface letters for matrices. The superscripts $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^*$ indicate matrix/vector transpose, Hermitian transpose and complex conjugate, respectively.

II. Coupling-free model - gain and phase calibration

A. Virtual array calibration

Consider a virtual uniform linear array (ULA) of M elements observing a point-like target at angle ϕ_i from the array pointing direction. The target is characterised by its complex back-scattering coefficient $\alpha_i = |\alpha_i|e^{j\varphi_i}$ with $\varphi_i \sim \mathcal{U}(0, 2\pi)$, which comprises signal attenuation due to two-way propagation and processing gain with no loss of generality. The amplitude and phase distortion in the m-th element of the array is characterised by a complex-valued coefficient γ_m, $m = 0, \dots, M-1$. Then, the response of the i-th target in the m-th antenna element $\kappa_{m,i}$ is given by:

$$\kappa_{m,i} = \alpha_i \gamma_m \exp\left(-j2\pi \frac{dm}{\lambda} \sin(\phi_i)\right) + n_{m,i}, \qquad (1)$$

where λ stands for the carrier wavelength, d is the inter-element spacing of the ULA and $n_{m,i} \sim \left(0, \sigma^2\right)$ represents the receiver noise.

To remedy the dependence on the target back-scattering coefficient α_i, we consider channel $m = 0$ (another m can be used with no loss of generality) as the reference point [1], [4]. This implies that the calibration coefficient at $\gamma_0 = 1$ is fixed. The new data set $\mathbf{P} \in \mathbb{C}^{(M-1)\times N_i}$ is obtained by normalising the target response in every channel via:

$$p_{m,i} = \frac{\kappa_{m,i}}{\kappa_{0,i}} = \gamma_m h_{m,i} + n'_{m,i}. \qquad (2)$$

Note that normalisation by a constant does not affect target SNR nor its DOA estimation. We assume that the angles of all observed targets $\phi_i, i = 1, \dots, I$ are known, and we use them to form the steering vectors for the corresponding targets in matrix $\mathbf{H} \in \mathbb{C}^{(M-1)\times I}$ with elements:

$$h_{m,i} = \exp\left(-j2\pi \frac{dm}{\lambda} \sin(\phi_i)\right). \qquad (3)$$

The noise vector after normalization is $n'_{m,i} \sim \left(0, \sigma_i^2\right)$, where

$$\sigma_i^2 = \left(\frac{|\alpha_i|^2}{\sigma^2} + 1\right)^{-1} = (\mathrm{SNR}_i + 1)^{-1}. \qquad (4)$$

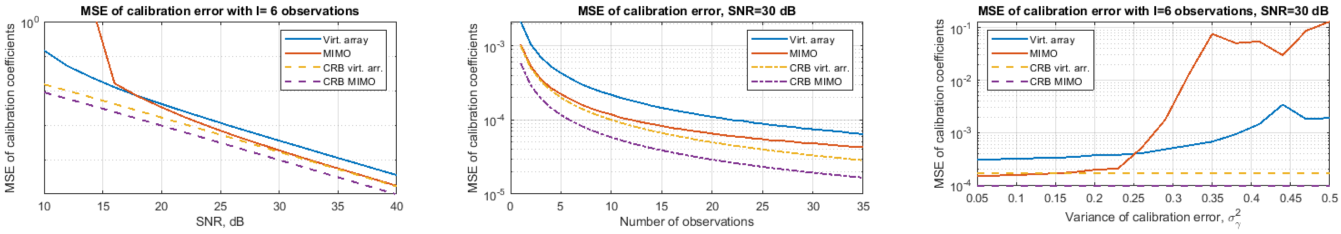


Fig. 1. Gain and phase calibration comparison: MSE of calibration coefficients (*a*) vs SNR, (*b*) vs number of targets, (*c*) vs calibration error.

The equality follows directly from the normalization (2).

To perform calibration in a stationary scenario we collect the normalised responses of all the observed targets $\mathbf{P}$ and compare them to the expected array response in the same channel. The calibration coefficients $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_{M-1}]^T$ are then estimated by the least squares method:

$$\hat{\gamma}_m = \frac{\sum_i^I h_{m,i} p_{m,i}^*}{\sum_i^I |p_{m,i}|^2}, \quad m = 1, \ldots, M-1. \tag{5}$$

The accuracy of the calibration depends on the SNR and the number of available observations.

For I independent measurements, the observation model (2) can be written in the form:

$$\mathbf{p}_m = \gamma_m \mathbf{h}_m + \mathbf{n}'_m. \tag{6}$$

with $\mathbf{p}_m = [p_{m,1}, \ldots, p_{m,I}]^T$, $\mathbf{n}'_m \sim \mathcal{CN}(\mathbf{0}_I, \boldsymbol{\Sigma})$ and $\boldsymbol{\Sigma} = \operatorname{diag}\left(\sigma_1^2, \ldots, \sigma_I^2\right)$. The representation (6) is a linear Gaussian estimation problem, for which the Cramer-Rao bound (CRB) of the estimated calibration coefficients can be given by:

$$\mathrm{CRB}_{\gamma_m} = \left(\mathbf{h}_m^H \boldsymbol{\Sigma}^{-1} \mathbf{h}_m\right)^{-1} \Big|_{|h_{m,i}|=1} = \left(I + \sum_{i=1}^{I} \frac{|\alpha_i|^2}{\sigma^2}\right)^{-1}. \tag{7}$$

The coupling-free model is widely used because it gives a reasonable estimation even with one measurement [6].

B. Tx and Rx array calibration

Consider a coherent MIMO system with K transmit and L receive channels and let $k = 0, \ldots, K-1$ and $l = 0, \ldots, L-1$. The measured signal for the i-th target in the k-th Tx and l-th Rx channel is:

$$\kappa_{k,l,i} = \alpha_i \tilde{\gamma}_k \breve{\gamma}_l \exp\left(-j2\pi \frac{d_t k + d_r l}{\lambda} \sin(\phi_i)\right) + \tilde{n}_{k,l,i}, \tag{8}$$

where d_t and d_r are the inter-element spacing of the uniform linear arrays used for transmit and receive respectively and $\tilde{n}_{k,l,i} \sim \left(0, \sigma^2\right)$.

Proceeding in a similar manner as in (2), it is possible to calibrate Tx and Rx arrays separately. Thus, for the Tx array, consider the element $k = 0$ as the reference one $\tilde{\gamma}_k = 1$ and obtain the normalized data set via:

$$\tilde{p}_{k,l,i} = \frac{\kappa_{k,l,i}}{\kappa_{0,l,i}} = \tilde{\gamma}_k \exp\left(-j2\pi \frac{d_t k}{\lambda} \sin(\phi_i)\right) + \tilde{n}'_{k,l,i}, \tag{9}$$

$k = 1, \ldots, K$. Denote $\tilde{h}_{k,i} = \exp\left(-j2\pi \frac{d_t k}{\lambda} \sin(\phi_i)\right)$, then the calibration of Tx array can be performed via:

$$\hat{\tilde{\gamma}}_k = \frac{\sum_{l=0}^{L-1} \sum_i^I \tilde{h}_{k,i} \tilde{p}_{k,l,i}^*}{\sum_{l=0}^{L-1} \sum_i^I |\tilde{p}_{k,l,i}|^2}, \quad k = 1, \ldots, K-1. \tag{10}$$

The measurement model (9) can be given in a from (6) by defining new variables: $\tilde{\mathbf{h}}_k = [\tilde{h}_{k,0,1}, \ldots, \tilde{h}_{k,L-1,1}, \ldots, \tilde{h}_{k,0,I}, \ldots, \tilde{h}_{k,L-1,I}]^T$, and similarly for $\tilde{\mathbf{p}}_k$. Assuming that $E\{|\gamma_l|\} = 1, l = 0, \ldots, L-1$, the noise vector becomes $\tilde{\mathbf{n}}'_k \sim \mathcal{CN}(\mathbf{0}_{IL}, \mathbf{I}_L \otimes \boldsymbol{\Sigma})$ respectively. Then the CRB for Tx calibration coefficients is:

$$\mathrm{CRB}_{\tilde{\gamma}_k} = \left(\tilde{\mathbf{h}}_k^H \left(\mathbf{I}_L \otimes \boldsymbol{\Sigma}\right)^{-1} \tilde{\mathbf{h}}_k\right)^{-1} = \frac{1}{L}\left(I + \sum_{i=1}^{I} \frac{|\alpha_i|^2}{\sigma^2}\right)^{-1}. \tag{11}$$

where the second equality is based on $|\tilde{h}_{k,i}| = 1$.

Proceeding in a similar manner for the Rx array calibration and setting the reference channel $l = 0, \breve{p}_l = 1$, we obtain:

$$\breve{p}_{k,l,i} = \frac{\kappa_{k,l,i}}{\kappa_{k,0,i}} = \breve{\gamma}_l \exp\left(-j2\pi \frac{d_r l}{\lambda} \sin(\phi_i)\right) + \breve{n}'_{l,i}, \tag{12}$$

with $l = 0, \ldots L-1$. The calibration is performed similarly to the examples above:

$$\hat{\breve{\gamma}}_l = \frac{\sum_{k=0}^{K-1} \sum_i^I \breve{h}_{l,i} \breve{p}_{k,l,i}^*}{\sum_{k=0}^{K-1} \sum_i^I |\breve{p}_{k,l,i}|^2}, \quad l = 1, \ldots, L-1, \tag{13}$$

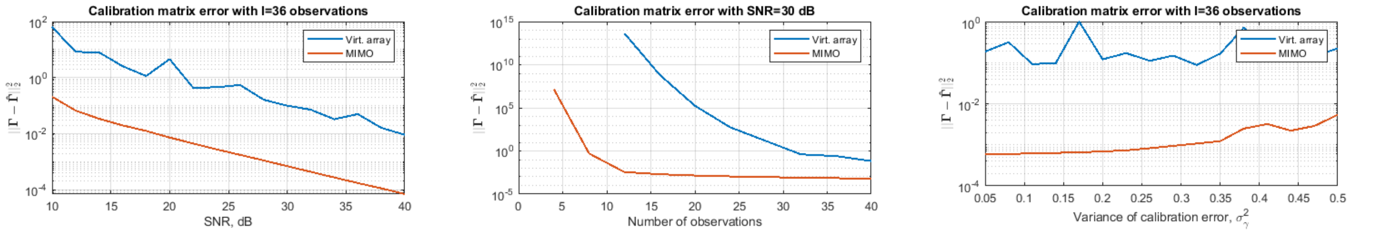
where $\breve{h}_{l,i} = \exp\left(-j2\pi \frac{d_r l}{\lambda} \sin(\phi_i)\right)$. Stacking the responses of I targets in K Tx channels in an array $\breve{\mathbf{h}}_l$, the CRB of the estimation of Rx array calibration is given by:

$$\mathrm{CRB}_{\breve{\gamma}_l} = \left(\breve{\mathbf{h}}_l^H \left(\mathbf{I}_K \otimes \boldsymbol{\Sigma}\right)^{-1} \breve{\mathbf{h}}_l\right)^{-1} = \frac{1}{K}\left(I + \sum_{i=1}^{I} \frac{|\alpha_i|^2}{\sigma^2}\right)^{-1}. \tag{14}$$

Here we assume that $E\{|\gamma_k|\} = 1, k = 0, \ldots, K-1$.

To compare the performance of Tx and Rx arrays calibration to that of virtual array, we can notice from (2) and (8) the relation $\gamma_m = \mathbf{g}\left(\tilde{\gamma}_k, \breve{\gamma}_l\right) = \tilde{\gamma}_k \breve{\gamma}_l$ with $m = Lk + l$ for dense receive and sparse transmit arrays. Define Jacobian $J^T(\mathbf{g}) = \left[\frac{\partial g(\tilde{\gamma}_k, \breve{\gamma}_l)}{\partial \tilde{\gamma}_k}, \frac{\partial g(\tilde{\gamma}_k, \breve{\gamma}_l)}{\partial \breve{\gamma}_l}\right]$, then the *asymptotic* CRB for the product of two parameters is [7]:

$$\begin{aligned} \mathrm{CRB}_{\tilde{\gamma}_k \breve{\gamma}_l} &= J(\mathbf{g}) \operatorname{diag}\left[\mathrm{CRB}_{\tilde{\gamma}_k}, \mathrm{CRB}_{\breve{\gamma}_l}\right] J^T(\mathbf{g}) \\ &= \frac{K+L}{KL}\left(I + \sum_{i=1}^{I} \frac{|\alpha_i|^2}{\sigma^2}\right)^{-1} = \frac{K+L}{KL} \mathrm{CRB}_{\gamma_m}, \end{aligned} \tag{15}$$



a *b* *c*

Fig. 2. Error in the estimated full coupling matrix: (*a*) vs SNR, (*b*) vs number of targets, (*c*) vs calibration error.

where CRB_{γ_m} is defined in (7). It follows that $\mathrm{CRB}_{\tilde{\gamma}_k\breve{\gamma}_l} \leq \mathrm{CRB}_{\gamma_m}$ for MIMO configuration: $(K \geq 2) \wedge (L \geq 2)$. This can be explained by the smaller number of unknown parameters $(K + L - 2)$ in the proposed approach compared to the virtual array calibration $(M - 1 = KL - 1)$.

The comparison of virtual array and MIMO calibration is demonstrated in Fig. 1 as a function of SNR for $I = 6$ (Fig. 1, *a*) and as a as the function of number of observation for $SNR = 30$ dB (Fig. 1, *b*). In these simulations the calibration error in each Tx and each Rx element was modelled as: $\tilde{\gamma}_k, \breve{\gamma}_l \sim \mathcal{CN}(1, \sigma_\gamma^2)$ with $\sigma_\gamma = 0.2$. It can be seen that MIMO calibration outperforms virtual array approach for SNR ≥ 18 dB case, while it has larger errors for small SNR. On the other hand, for large calibration errors $\sigma_\gamma \geq 0.25$, virtual array calibration provides more accurate estimation (Fig. 1, *c*).

III. Full Coupling Matrix Calibration

A. Virtual array calibration

If the mutual coupling is present, the received signal of the i-th target $\boldsymbol{\kappa}_i$ of the virtual array can be given by:

$$\boldsymbol{\kappa}_i = \alpha_i \boldsymbol{\Gamma} \mathbf{h}_i + \mathbf{w}_i, \tag{16}$$

where $\boldsymbol{\Gamma}$ is the full $M \times M$ mutual coupling matrix, $\mathbf{h}_i = [h_{0,i}, \ldots, h_{M-1,i}]^T$ with $h_{m,i}$ defined in (3) and $\mathbf{w}_i$ is $M \times 1$ vector of noise. Set the channel $m = 0$ as the reference one and normalize the array data by its measurements:

$$\mathbf{p}_i = \frac{\boldsymbol{\kappa}_i}{\kappa_{0,i}} = \boldsymbol{\Gamma} \mathbf{h}_i + \mathbf{w}'_i, \tag{17}$$

with $\boldsymbol{\kappa}_i = [\kappa_{0,i}, \ldots, \kappa_{N-1,i}]^T$ and $\mathbf{p}_{0,i} = 1$. In [4] the equivalent procedure is implemented by multiplication of the steering vector by the received signal in the reference channel. Next, we stack I measurements column-wise in $M \times I$ matrix $\mathbf{P} = [\mathbf{p}_1, \ldots, \mathbf{p}_I]$ and do the same procedure with the steering vectors: $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_I]$ to obtain the least-squares estimation of the virtual array coupling matrix via:

$$\hat{\boldsymbol{\Gamma}} = \mathbf{P}\mathbf{H}^H \left(\mathbf{H}\mathbf{H}^H\right)^{-1}. \tag{18}$$

The inverse of $\hat{\boldsymbol{\Gamma}}$ is used for the virtual array calibration.

B. Tx and Rx array calibration

For a general MIMO radar, there exist two types of antenna mutual coupling: within each array and from one array to another (Tx-Rx). For collocated MIMO transmitting FMCW chirps, Tx-Rx coupling is concentrated in the vicinity of zero range. It implies that the distortion of the virtual array beam-pattern for targets of interest is due to the mutual coupling of adjacent elements in Tx and Rx subarrays respectively.

For the separate calibration of Tx and Rx arrays, received data from the i-th target is given by the $K \times L$ matrix:

$$\boldsymbol{\mathcal{K}}_i = \alpha_i \tilde{\boldsymbol{\Gamma}} \boldsymbol{\mathcal{H}}_i \breve{\boldsymbol{\Gamma}} + \mathbf{W}_i, \tag{19}$$

where $\mathrm{vec}(\boldsymbol{\mathcal{H}}_i) = \mathbf{h}_i$ is the counterpart of the steering vector for the matrix data model and $\mathbf{W}_i$ is the matrix of noise. For the Tx array calibration, we reshape the data matrix as $\tilde{\boldsymbol{\mathcal{K}}} = [\boldsymbol{\mathcal{K}}_1, \ldots, \boldsymbol{\mathcal{K}}_I] = [\tilde{\boldsymbol{\kappa}}_{1,1}, \ldots, \tilde{\boldsymbol{\kappa}}_{1,L}, \ldots, \tilde{\boldsymbol{\kappa}}_{I,1}, \ldots, \tilde{\boldsymbol{\kappa}}_{I,L}]$ such that every column represents an independent measurement corresponding to the Tx array. Normalize the data as before:

$$\tilde{\mathbf{p}}_{i,l} = \frac{\tilde{\boldsymbol{\kappa}}_{i,l}}{\tilde{\kappa}_{0,i,l}} = \tilde{\boldsymbol{\Gamma}} \tilde{\mathbf{h}}_{i,l} + \mathbf{w}'_{i,l}, \tag{20}$$

where $\tilde{\boldsymbol{\kappa}}_{i,l} = [\tilde{\kappa}_{0,i,l}, \ldots, \tilde{\kappa}_{K-1,i,l}]^T$. Construct the matrix $\tilde{\mathbf{P}} = [\tilde{\mathbf{p}}_{1,1}, \ldots, \tilde{\mathbf{p}}_{1,L}, \ldots, \tilde{\mathbf{p}}_{I,1}, \ldots, \tilde{\mathbf{p}}_{I,L}]$ and do the same rearrangement to $\boldsymbol{\mathcal{H}}_i$ to get $\tilde{\mathbf{H}}$. Then Tx array calibration is done via:

$$\hat{\tilde{\boldsymbol{\Gamma}}} = \tilde{\mathbf{P}}\tilde{\mathbf{H}}^H \left(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H\right)^{-1}. \tag{21}$$

Note that rearranging the matrix and normalization (20) decouples the Tx and Rx array calibration, thus they can be estimated independently.

For the received array, a similar procedure is applied over the other dimension, namely: $\breve{\boldsymbol{\mathcal{K}}} = \left[\boldsymbol{\mathcal{K}}_1^T, \ldots, \boldsymbol{\mathcal{K}}_I^T\right]^T = \left[\breve{\boldsymbol{\kappa}}_{1,1}, \ldots, \breve{\boldsymbol{\kappa}}_{1,K}, \ldots, \breve{\boldsymbol{\kappa}}_{I,1}, \ldots, \breve{\boldsymbol{\kappa}}_{I,K}\right]^T$ where $\boldsymbol{\mathcal{K}}_i^T = \left[\breve{\boldsymbol{\kappa}}_{i,0}, \ldots, \breve{\boldsymbol{\kappa}}_{i,K-1}\right]$ and $\breve{\boldsymbol{\kappa}}_{i,k} = \left[\breve{\kappa}_{0,i,k}, \ldots, \breve{\kappa}_{L-1,i,k}\right]^T$. Every column in $\breve{\boldsymbol{\mathcal{K}}}$ is an independent measurement of the Rx array for the i-th target and the k-th Tx channel respectively. Applying normalization similarly to the above, we obtain:

$$\breve{\mathbf{p}}_{i,k} = \frac{\breve{\boldsymbol{\kappa}}_{i,k}}{\breve{\kappa}_{0,i,k}} = \breve{\mathbf{h}}_{i,k} \breve{\boldsymbol{\Gamma}} + \mathbf{w}''_{i,k}. \tag{22}$$

Here $\breve{\mathbf{h}}_{i,k}$ are $\breve{\mathbf{p}}_{i,k}$ the Rx array steering vector and the normalized received data for the i-th target and the k-th transmit channel respectively. Merge all the normalized data into matrix $\breve{\mathbf{P}} = [\breve{\mathbf{p}}_{1,1}, \ldots, \breve{\mathbf{p}}_{1,K}, \ldots, \breve{\mathbf{p}}_{I,1}, \ldots, \breve{\mathbf{p}}_{I,K}]$ and all the steering vectors into matrix $\breve{\mathbf{H}}$ with the same rearrangement of elements. Then Rx array calibration is done via:

$$\hat{\breve{\boldsymbol{\Gamma}}} = \left(\breve{\mathbf{H}}^H \breve{\mathbf{H}}\right)^{-1} \breve{\mathbf{H}}^H \breve{\mathbf{P}}. \tag{23}$$

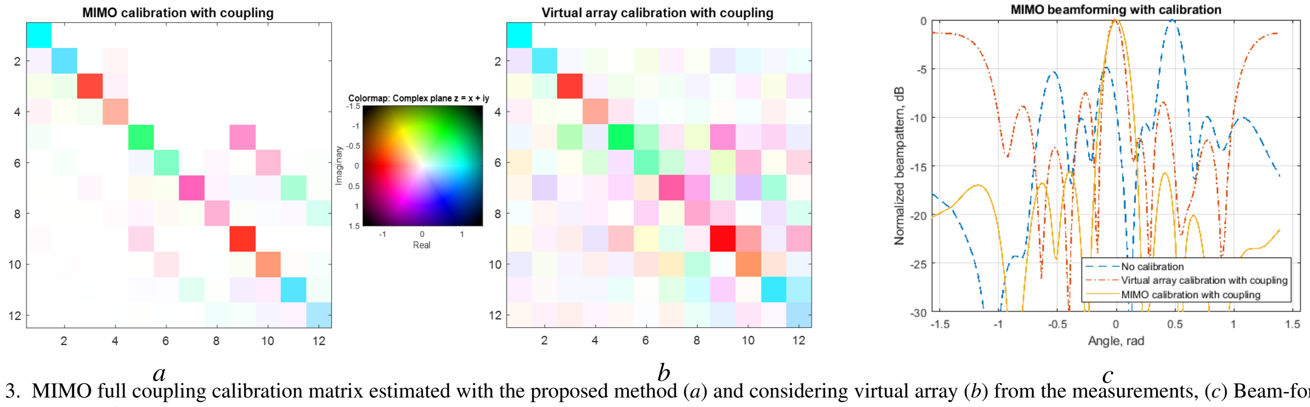


Fig. 3. MIMO full coupling calibration matrix estimated with the proposed method (*a*) and considering virtual array (*b*) from the measurements, (*c*) Beam-forming in the direction $\phi = 0$ with the calibration matrices depicted in (*a*) and (*b*).

The estimated calibration matrices (21) and (23) can be directly applied for the calibration of Tx and Rx arrays independently. To compare the results of independent Tx and Rx array calibration to that of the virtual array (18), the matrix $\mathbf{\Gamma} = \tilde{\mathbf{\Gamma}} \otimes \breve{\mathbf{\Gamma}}$ is analyzed (assuming sparse Tx array).

Simulation results are demonstrated in Fig. 2. There we considered 3 Tx $\times$ 4 Rx MIMO system. The coupling matrix in each array is modelled via lower triangular matrix ($K \times K$ for Tx and $L \times L$ for Rx) with elements: $\gamma_{r,s} = \exp(-(r-s))(1+\omega_{r,s})$, where the exponential part defines the average coupling and $\omega_{r,s} \sim \mathcal{N}(0, \sigma_\gamma^2)$ models the random part. Fig. 2, *a* shows the performance of calibration vs SNR and demonstrate significant improvement of separate calibration over virtual array calibration for limited number of observations, $I = 36$. The comparison of the calibration approaches as the function of measurements is presented in Fig. 2, *b*. It can be seen that the requirement of the full virtual array calibration $I \geq N$ is relaxed to $I \geq \max\{K, L\}$ for the proposed techniques. Finally, the sensitivity to the calibration error is demonstrated in Fig. 2, *c*.

IV. Experimental validation

We compare the performance of the full coupling matrix calibration methods described in Section III by applying them to a commercial mm-wave radar with a dense Rx array having $L = 4$ elements and sparse Tx array with $K = 3$ elements and $d_t = 2\lambda$. The radar transmits FMCW chirps of $B = 1.05$ GHz with time-division multiplexing (TDM) at the carrier frequency $f_c = 79.3$ GHz. The calibration setup was made in anechoic chamber with a dihedral corner reflector installed at the distance of 3 m in from of the radar. The measurements were taken by rotating the radar at angles from $\phi_{\min} = -80^o$ to $\phi_{\min} = 80^o$ with a step of 5^o . The measurement at $\phi_{\min} = 0^o$ was not used for calibration, but for validation. The estimated calibration matrix using (18) and its counterpart obtained with (21), (23) are depicted in Fig. 3, *a, b* [8]. It can be seen that enforced structure of MIMO calibration matrix (Fig. 3, *a*) makes it less fluctuating compared to the virtual array calibration (Fig. 3, *b*). This results in lower sidelobes of beam-forming after applying calibration, as shown in Fig. 3, *c* for the beam-forming in the direction $\phi = 0^o$. The improvement is significant for small to moderate number of measurements (here $I = 32$), which is typically of interest.

V. Conclusion

In this paper two novel methods of MIMO array calibration are proposed. The methods perform independent calibration of transmit and receive arrays by means of the complex gains of arrays elements or their coupling matrices. Both methods can be implemented for calibration with a limited number of measurements, e.g. in garage for an automotive radar. Simulation results and real data processing demonstrate that proposed approach significantly improves the calibration accuracy and decreases the required number of independent observation compared to conventional virtual array calibration.

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