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EFFICIENT APPROACHES FOR FLUID STRUCTURE INTERACTION WITH FULLY ENCLOSED INCOMPRESSIBLE FLOW DOMAINS

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Abstract. *Many popular partitioned approaches to fluid-structure interaction (FSI) problems fail to work for an interesting subset of problems if highly deformable structures are interacting with incompressible flows. This is particularly true for coupling approaches based on Dirichlet-Neumann substructuring, both for weak and strong coupling schemes. The subset is characterized by the absence of any unconstrained outflow boundary at the fluid field, that is the fluid domain is entirely enclosed by Dirichlet boundary conditions. The inflating of a balloon with prescribed inflow rate constitutes a simple problem of that kind. The commonly used coupling algorithms will not satisfy the fluid's incompressibility during the FSI iterations in such cases. That is because the structure part determines the interface displacements and the structural solver does not know about the constraint on the fluid field. To overcome this deficiency of partitioned algorithms a small augmentation is proposed that consists in introducing the fluid volume constraint on the structural system of equations. This allows to circumvent the dilemma of the fluid's incompressibility. At the same time the use of a Lagrangian multiplier to introduce the volume constraint allows to obtain the pressure level of the fluid domain. However, the customary applied relaxation of the interface displacements has to be abandoned in favor of the relaxation of coupling forces. These modifications applied to a particular strong coupled Dirichlet-Neumann partitioning scheme result in an efficient and robust approach that exhibits only little additional numerical effort. Numerical examples with largely changing volumes of the enclosed fluid show the capabilities of the proposed scheme.*

1 INTRODUCTION

Partitioned coupling approaches are appreciated for solving fluid-structure interaction (FSI) problems. Many particularly efficient and robust algorithms are based on the Dirichlet-Neumann approach where the fluid serves as Dirichlet partition inheriting velocity boundary conditions from the structure while the structure, that is the Neumann partition, is loaded by the fluid forces[15,8,12,6,5,4,13].

The added Dirichlet boundaries to the fluid domain present a difficulty that leads to failure of the algorithm if there is no unconstrained outflow boundary. This situation occurs e.g. in balloon type of problems where an enclosing structure is filled by a prescribed flow rate. Another example are flexible tube systems where velocity profiles are prescribed at certain cross sections.

A close look reveals that there are two parts to this difficulty. First of all, the incompressible flow in a deforming domain requires a domain size change equivalent to the fluid volume difference due to in- and outflow. This constitutes a condition on the structure solution. And secondly the absolute pressure level inside the fluid needs to be determined. Oftentimes the fluid pressure is adjusted to the external pressure at a Neumann boundary Γ_N of the fluid field. Inside an enclosed fluid, however, the forces from the deforming structural parts determine the fluid pressure.

For such cases we propose to enrich the structural equations by the fluid volume condition. This augmentation of the classical Dirichlet-Neumann algorithm allows to obtain the fluid pressure level from the corresponding Lagrange multiplier, thereby providing the unknown pressure level information.

Partitioned block Gauß-Seidel FSI algorithms require relaxation to ensure and accelerate convergence[15,12,11,1]. But since the structural displacements have to obey a side condition relaxation of displacements at the coupling interface Γ cannot be employed in the modified algorithm. Thus the relaxation has to be done on the coupling forces.

2 GOVERNING EQUATIONS

2.1 Fluid domain

The fluid is described by the incompressible Navier-Stokes equations with Newtonian stresses. Thus the flow velocities $\mathbf{u}$ and pressure p are described by

$$\left.\frac{\partial \mathbf{u}}{\partial t}\right|_{\boldsymbol{\chi}} + \left(\mathbf{u} - \mathbf{u}^G\right) \cdot \boldsymbol{\nabla} \mathbf{u} - 2\nu \boldsymbol{\nabla} \cdot \boldsymbol{\varepsilon}(\mathbf{u}) + \nabla p = \mathbf{f}^F \quad \text{in } \Omega^F \times (0,T), \tag{1}$$

$$\boldsymbol{\nabla} \cdot \mathbf{u} = 0 \quad \text{in } \Omega^F \times (0,T). \tag{2}$$

with the grid velocity $\mathbf{u}^G$. Here the kinematic viscosity is $\nu = \mu/\rho^F$ where μ represents the viscosity and ρ^F the fluid's density. The vector field $\mathbf{f}^F$ denotes the specific body force on the fluid. The kinematic pressure is represented by p, and accordingly $\bar{p} = p\,\rho^F$ yields the physical pressure.

The stress tensor of a Newtonian fluid is given by

$$\boldsymbol{\sigma}^F = -\bar{p}\,\mathbf{I} + 2\mu\boldsymbol{\varepsilon}(\mathbf{u}) \tag{3}$$

with the strain rate

$$\boldsymbol{\varepsilon}(\mathbf{u}) = \frac{1}{2}\left(\boldsymbol{\nabla}\mathbf{u} + \boldsymbol{\nabla}\mathbf{u}^T\right). \tag{4}$$

The partial differential equations (1) and (2) are subject to initial and boundary conditions

$$\begin{gathered}\mathbf{u} = \mathbf{u}_0 \quad \text{in } \Omega^F \text{ at } t = 0 \\ \mathbf{u} = \hat{\mathbf{u}} \quad \text{on } \Gamma_D^F, \qquad \boldsymbol{\sigma}\cdot\mathbf{n} = \hat{\mathbf{h}}^F \quad \text{on } \Gamma_N^F,\end{gathered} \tag{5}$$

where the present study is concerned with cases where no Neumann portion of the fluid boundary is present i.e. $\partial\Omega^F = \Gamma_D^F$.

The matrix representation of a finite element discretization of equations (1) and (2) is

$$\mathbf{M}^F\dot{\mathbf{u}} + \mathbf{N}^F(\mathbf{u})\mathbf{u} + \mathbf{K}^F\mathbf{u} + \mathbf{G}^F\mathbf{p} = \mathbf{f}^F, \qquad \left(\mathbf{G}^F\right)^T\mathbf{u} = \mathbf{0}, \tag{6}$$

with the fluid mass matrix $\mathbf{M}^F$, the convective matrix $\mathbf{N}^F(\mathbf{u})$ and viscous matrix $\mathbf{K}^F$ and the vector of nodal body forces $\mathbf{f}^F$. The matrix $\mathbf{G}^F$ represents the discrete gradient operator.

Implicit one-step-θ and BDF2 schemes are used to discretize (6) in time. The occurring nonlinearities are dealt with using Newton or fixed-point like iteration schemes[14].

2.2 Structural domain

Geometrical nonlinearities due to large structural deformations have to be taken in account. Linear material response is assumed for simplicity while the formulation can easily be extended to more general material laws as can be seen in the examples in section 6. The structural displacements $\mathbf{d}$ are governed by the geometrically nonlinear elastodynamics equations

$$\rho^S\frac{\mathrm{D}^2\mathbf{d}}{\mathrm{D}t^2} = \boldsymbol{\nabla}\cdot\mathbf{S} + \rho^S\mathbf{f}^S \qquad \text{in } \Omega^S \times (0,T), \tag{7}$$

with the structural density ρ^S and the specific body force $\mathbf{f}^S$. The differential D denotes the material time derivative. The second Piola-Kirchhoff stress tensor $\mathbf{S}$ is related to the Green-Lagrangian strains via

$$\mathbf{S} = \mathbf{C} : \mathbf{E} \quad \text{with} \quad \mathbf{E} = \frac{1}{2}\left(\mathbf{F}^T\cdot\mathbf{F} - \mathbf{I}\right), \tag{8}$$

where $\mathbf{C}$ denotes the material tensor and $\mathbf{F} = \boldsymbol{\nabla}\mathbf{d}$ represents the deformation gradient. The time dependent problem (7) is subject to the initial and boundary conditions

$$\mathbf{d} = \mathbf{d}_0 \quad \text{and} \quad \dot{\mathbf{d}} = \dot{\mathbf{d}}_0 \quad \text{in } \Omega^S \text{ at } t = 0 \tag{9}$$

$$\mathbf{d} = \hat{\mathbf{d}} \quad \text{on } \Gamma_D^S, \qquad \mathbf{S} \cdot \mathbf{n} = \hat{\mathbf{h}}^S \quad \text{on } \Gamma_N^S, \tag{10}$$

where Γ_D^S and Γ_N^S denote the Dirichlet and Neumann partition of the structural boundary, respectively, and $\hat{\mathbf{d}}$ and $\hat{\mathbf{h}}^S$ denote the prescribed Dirichlet and Neumann values.

Using finite elements the semi-discrete structural equations read

$$\mathsf{M}^S \ddot{\mathsf{d}} + \mathsf{N}^S(\mathsf{d}) = \mathsf{f}^S, \tag{11}$$

with the structural mass matrix M^S, the internal forces N^S and the external forces f^S. The nodal displacement vector is given by d while the overset dot represents the velocities and accelerations. In the present approach this system is solved using the nonlinear version of the 'generalized-α method' of Chung and Hulbert[2] along with consistent linearization and a Newton-Raphson iterative scheme.

3 PARTITIONED SOLUTION OF THE FSI PROBLEM

Equations (6) and (11) together with an appropriate mesh moving algorithm form a highly nonlinear coupled system of equations. In the present contribution iteratively coupled solution schemes are considered. There is a vast amount of literature that presents and proposes these schemes[15,8,12,7,10,5] for their robustness and ease of implementation. In particular these schemes converge to the exact solution and yet allow to solve the different fields independent of each other.

The algorithmic framework of the partitioned FSI analysis is discussed in detail in Mok[11] and Deparis[3].

3.1 Coupling conditions

In the iterative coupling algorithm the wet structural surface acts as coupling interface Γ, for brevity being denoted as Γ in the following.

At each time level the coupling iteration aims to fulfill the discrete version of the kinematic and the dynamic continuity across the interface.

$$\mathbf{d}_\Gamma(t) \cdot \mathbf{n} = \mathbf{r}_\Gamma(t) \cdot \mathbf{n} \qquad \text{and} \qquad \mathbf{u}_\Gamma(t) \cdot \mathbf{n} = \mathbf{u}_\Gamma^G(t) \cdot \mathbf{n} = \left. \frac{\partial \mathbf{r}_\Gamma(t)}{\partial t} \right|_{\boldsymbol{\chi}} \cdot \mathbf{n} \tag{12}$$

$$\boldsymbol{\sigma}_\Gamma^S(t) \cdot \mathbf{n} = \boldsymbol{\sigma}_\Gamma^F(t) \cdot \mathbf{n} \tag{13}$$

with $\mathbf{r}$ denoting the fluid mesh displacements and $\mathbf{n}$ the unit normal vector on the interface.

3.2 Non-overlapping Dirichlet-Neumann partitioning

The Dirichlet-Neumann partitioning is particularly suited for partitioned FSI solutions. In that case the fluid domain acts as Dirichlet partition with prescribed velocities $\mathbf{u}_\Gamma$ and the structure domain acts as Neumann partition loaded with interface forces $\mathbf{f}_\Gamma$.

We consider the Dirichlet-Neumann coupling algorithm with synchronous time discretization and block Gauß-Seidel iteration that was proposed by Mok and Wall[15,12,11].

This formulation provides the framework that will subsequently be enhanced to work for temporally changing fully Dirichlet constraint fluid domains. The key idea of the enhancement is not limited to coupling schemes based on Gauß-Seidel iterations, the same reasoning can be applied to Newton-Raphson based field iteration schemes.

To highlight the coupling behavior the following outline abbreviates the nonlinear field equation (11) and (6) with the symbolic systems

$$\mathbf{A}^S \mathbf{d}^S = \mathbf{f}^S \qquad \text{and} \qquad \mathbf{A}^F \mathbf{u}^F = \mathbf{f}^F \tag{14}$$

for the structure field and the fluid field, respectively.

In the following $(\cdot)_I$ and $(\cdot)_\Gamma$ denote variables or coefficients in the interior of a subdomain Ω^j and on the coupling interface, respectively, while the absence of any subscript comprises degrees of freedom on the entire subdomain including interior and interface.

In every time step the following calculations have to be performed several times. The variable i denotes the loop counter.

1. Transfer the latest structure displacements $\mathbf{d}^S_{\Gamma,i+1}$ to the fluid field, calculate the fluid domain deformation and determine the appropriate fluid velocities at the interface $\mathbf{u}^S_{\Gamma,i+1}$.

2. Solve the fluid equation for the inner fluid velocities and pressures $\mathbf{u}^F_{I,i+1}$.

$$\mathbf{A}^F_{II} \mathbf{u}^F_{I,i+1} = \mathbf{f}^F_{I\,ext} - \mathbf{A}^F_{I\Gamma} \mathbf{u}^S_{\Gamma,i+1} \tag{15}$$

3. Find the fluid forces $\mathbf{f}^F_{\Gamma,i+1}$ at the interface Γ.

$$\mathbf{f}^F_{\Gamma,i+1} = \mathbf{A}^F_{\Gamma I} \mathbf{u}^F_{I,i+1} + \mathbf{A}^F_{\Gamma\Gamma} \mathbf{u}^S_{\Gamma,i+1} \tag{16}$$

4. Apply the fluid forces $\mathbf{f}^F_{\Gamma,i+1}$ to the structure. Solve the structure equations for the structural displacements.

$$\begin{bmatrix} \mathbf{A}^S_{\Gamma\Gamma} & \mathbf{A}^S_{\Gamma I} \\ \mathbf{A}^S_{I\Gamma} & \mathbf{A}^S_{II} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{d}}^S_{\Gamma,i+1} \\ \mathbf{d}^S_{I,i+1} \end{bmatrix} = \begin{bmatrix} \mathbf{f}^S_{\Gamma\,ext} - \mathbf{f}^F_{\Gamma,i} \\ \mathbf{f}^S_{I\,ext} \end{bmatrix} \tag{17}$$

5. The calculation is finished when the difference between $\tilde{\mathbf{d}}^F_{\Gamma,i+1}$ and $\mathbf{d}^F_{\Gamma,i}$ is sufficiently small.

6. Relax the interface displacement using a suitable ω_i.

$$\mathbf{d}^S_{\Gamma,i+1} = \omega_i \tilde{\mathbf{d}}^S_{\Gamma,i+1} + (1 - \omega_i) \mathbf{d}^S_{\Gamma,i} \tag{18}$$

7. Update i and return to step 1.

4 DILEMMA WITH FULLY ENCLOSED FLUID DOMAINS

The Dirichlet-Neumann algorithm described above fails if there are prescribed velocities on all boundaries of the fluid domain. A fluid domain fully enclosed by Dirichlet boundaries can only be solved if the domain size change of the incompressible fluid matches the prescribed velocities. Additionally a constraint is needed that fixes the pressure level. The Dirichlet-Neumann algorithm fails on both conditions.

4.1 The fluid mass balance constraint

The constraint on the fluid domain size, that is the constraint on the fluid mass balance, amounts to a constraint on the interface velocities $\mathbf{u}_\Gamma$. This follows from the mass volume balance of the incompressible fluid

$$\Delta V^{n+1} = V^{n+1} - V^n = \int_{\Delta t} \int_{\Gamma^F} \mathbf{u} \cdot \mathbf{n} \, d\Gamma \, dt \tag{19}$$

where Γ^F represents the boundary of the fluid domain and V^{n+1} and V^n denote the volume of the enclosed fluid domain at the discrete time steps t^{n+1} and t^n, respectively. Because inflow and outflow velocities in condition (19) are fixed the volume change ΔV^{n+1} depends on the interface velocity $\mathbf{u}_\Gamma$ at the coupling interface Γ.

The Dirichlet-Neumann coupling algorithm determines the interface displacements $\mathbf{d}_\Gamma$, and thereby the interface velocities $\mathbf{u}_\Gamma$, along with the structural solution in the structural solver. Physically the fluid's incompressibility constrains the admissible structure solutions while within standard partitioned algorithms this constraint is not known by the structure.

Consequently within the Dirichlet-Neumann algorithm the first attempt of a fluid solution on a not yet converged domain will create an ill posed problem yielding to the failure of the overall formulation.

4.2 Pressure level

The absolute pressure value of an incompressible fluid problem can be arbitrary and is determined by the boundary conditions only. If there are prescribed velocities on all boundaries of the fluid domain, however, the absolute pressure remains undetermined. Thus an additional condition is needed to fix the pressure values.

FSI problems, on the other hand, do specify the pressure level of the fluid domain even if both inflow and outflow velocities are prescribed. The structure's coupling forces and the fluid pressure at the interface are closely related according to the equilibrium coupling condition (13). That is in order to find the solution to the overall problem the correct pressure has to be calculated by the fluid solver. Otherwise the wrong fluid forces applied to the structure will causes the FSI algorithm to diverge. Thus the structure does require a specific pressure level from the fluid, the fluid solver, however, lacks any means to know it. Without modification the Dirichlet-Neumann algorithm does not provide the required exchange of pressure information.

4.3 Remedies to overcome the dilemma

The incompressible dilemma consists in the case that both sides need information from the other one to find their solution and the required information is not available. This picture together with the Dirichlet-Neumann algorithm sketched above provides the means to attack the dilemma. From the many strategies that could be pursued to obtain a working coupling algorithm for fully Dirichlet bounded fluid domain at least three seem to be viable.

- The incompressibility constraint of the fluid has to be satisfied by the interface displacements, that is by the structure solution. Thus the introduction of the constraint to the structure equations allows to fulfill the constraint. The fluid pressure level will need to be calculated from the structure solution.
- The start from the pressure level coupling between fluid and structure suggests to transfer interface forces from the structural domain to the fluid. This solves the pressure level determination issue, however the fluid has to calculate its own displacements in turn. Consequently the Dirichlet-Neumann coupling is reversed to a Neumann-Dirichlet approach and a free surface like fluid solver is needed.
- The whole incompressibility issue is avoided, of course, if the incompressibility constraint can be circumvented in the first place. The artificial compressibility approach aims at that and can be applied to the dilemma.

In the following the algorithm for the first approach is given. A detailed discussion of these approaches can be found in Küttler et al[9].

5 AUGMENTED DIRICHLET-NEUMANN APPROACH — MODIFIED DIRICHLET-NEUMANN COUPLING WITH VOLUME CONSTRAINT

The modification to the iterative Dirichlet-Neumann coupling algorithm from section 3.2 consists in the introduction of the fluid volume constraint to the structural solver. Additionally the relaxation of the interface displacements has to be replaced by the relaxation of the coupling forces at the interface. Consequently the order of the field solvers in the FSI iteration is reversed.

Using the abbreviated symbolic structural and fluid system (14) the following calculations have to be performed in every time step.

1. Solve for the structure displacements loaded with the fluid forces $\mathbf{f}^F_{\Gamma,i}$, but respect the volume constraint as required by the fluid.

$$\begin{bmatrix} \mathbf{A}^S_{\Gamma\Gamma} & \mathbf{A}^S_{\Gamma I} & -V_{,\mathbf{d}^S_\Gamma} \\ \mathbf{A}^S_{I\Gamma} & \mathbf{A}^S_{II} & 0 \\ -V_{,\mathbf{d}^S_\Gamma} & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{d}^S_{\Gamma,i+1} \\ \mathbf{d}^S_{I,i+1} \\ \lambda_{i+1} \end{bmatrix} = \begin{bmatrix} \mathbf{f}^S_{\Gamma\, ext} - \mathbf{f}^F_{\Gamma,i} - V_{,\mathbf{d}^S_\Gamma}\lambda_i \\ \mathbf{f}^S_{I\, ext} \\ V_c - V_{,\mathbf{d}^S_\Gamma}\mathbf{d}^S_{\Gamma,i} \end{bmatrix} \tag{20}$$

2. Transfer the interface displacements $\mathbf{d}^S_{\Gamma,i+1}$ to the fluid and determine the interface velocities $\mathbf{u}^S_{\Gamma,i+1}$.

3. Solve for the inner fluid velocities and pressures $\mathbf{u}^F_{I,i+1}$

$$\mathbf{A}^F_{II}\mathbf{u}^F_{I,i+1} = \mathbf{f}^F_{I\, ext} - \mathbf{A}^F_{I\Gamma}\mathbf{u}^S_{\Gamma,i+1} \tag{21}$$

4. Find the fluid forces at the FSI interface Γ

$$\tilde{\mathbf{f}}^F_{\Gamma,i+1} = \mathbf{A}^F_{\Gamma I}\mathbf{u}^F_{I,i+1} + \mathbf{A}^F_{\Gamma\Gamma}\mathbf{u}^S_{\Gamma,i+1} \tag{22}$$

5. End the iteration is the interface forces $\tilde{\mathbf{f}}^F_{\Gamma,i+1}$ are suitably close to the preceding ones $\mathbf{f}^F_{\Gamma,i}$.

6. Relax the fluid forces

$$\mathbf{f}^F_{\Gamma,i+1} = \omega_i\tilde{\mathbf{f}}^F_{\Gamma,i+1} + (1-\omega_i)\mathbf{f}^F_{\Gamma,i} \tag{23}$$

   The relaxation parameter ω_i can be calculated by any of the methods suggested by Mok[14,15].

6 NUMERICAL EXAMPLES

The examples have been calculated by the modified Dirichlet-Neumann algorithm with volume constraint in the structural equation.

6.1 An academic balloon-like problem

The system of the first example is depicted in figure 1. It consists of a structure with neo-Hookean material that surrounds the fluid area fully but for the inflow boundary. The fluid velocities $\mathbf{u}_{in}$ are prescribed. The time step for the simulation is $\Delta t = 0.05s$ and the stepping curve $\sin(t/2s \cdot \pi)$ is applied for $t < 1s$.

During the coupling iteration the fluid pressure is set to 0 at the indicated point. The fluid's pressure level is therefore defined. The Lagrangian multiplier λ does not vanish and has to be added to the pressure calculated by the fluid to obtain the pressure that is in equilibrium with the structure. Figure 2 shows the increasing Lagrangian multiplier λ

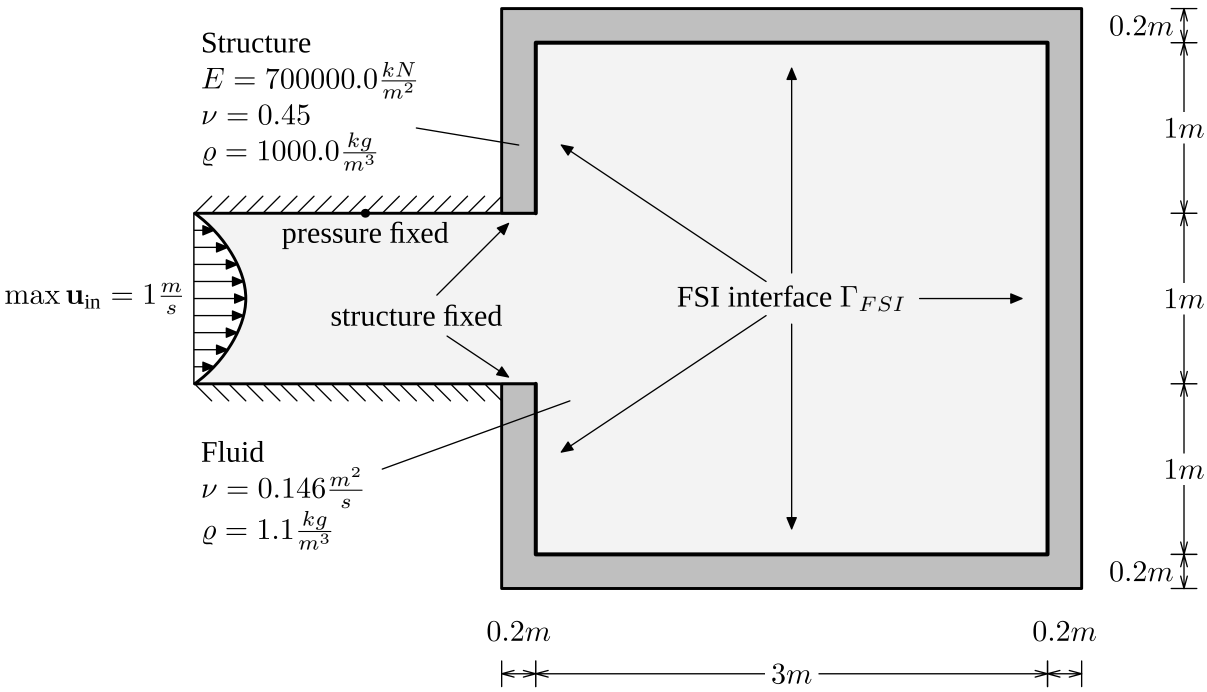

Figure 1: An academic "balloon problem".

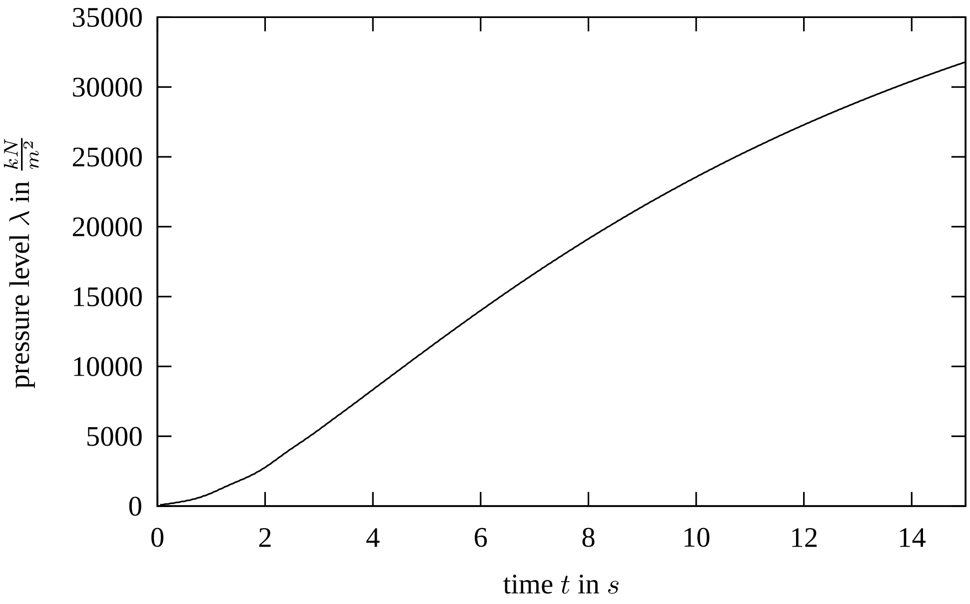

Figure 2: The pressure λ of the fluid domain at the indicated point calculated by the structural solver.

over time, that is the real fluid pressure at the point where for algorithmic reasons 0 is prescribed.

In this example the pressure variation in the fluid region, the pressure gradient, is very small compared to the pressure level that is needed to push the structure. It is therefore advisable to visualize the pressure gradient without the pressure level λ. As an example figure 3 shows the pressure gradient at time $t = 15s$ as calculated by the fluid solver. The corresponding velocity is shown in figure 4.

The effectivity of the proposed coupling algorithm is depicted by the number of field iterations needed per time step as shown in figure 5. The small number of iterations needed suggests that Aitken style relaxation of the interface forces is again a suitable approach.

6.2 Damped structural instability

A second example consists of a bended fluid domain that is surrounded by two thin structures with neo-Hookean material and different stiffness. The system is shown in figure 6. The structures are fixed at their short edges, the long edges are free respectively interacting with the fluid. At both inflow boundaries velocities are prescribed with the left one a little less than the right in order to avoid perfect symmetry. The fluid is loaded with the body force $\mathbf{f}_y = -1N/m^2$ in y direction. The simulation is carried out with a uniform time step size $\Delta t = 0.005s$.

The constant inflow increases the fluid pressure so that first mainly the soft flexible

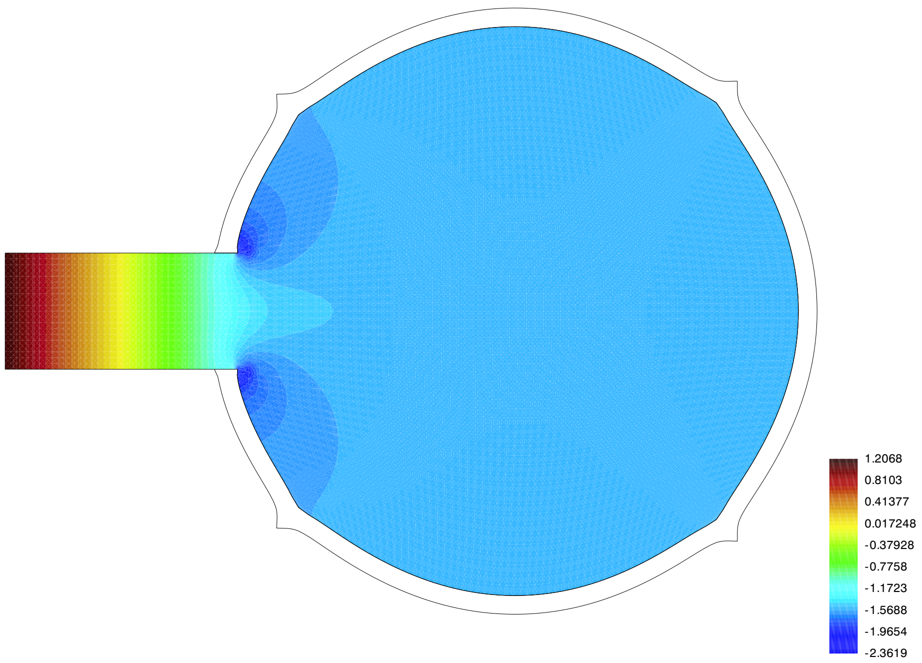

Figure 3: Pressure p in $\frac{kN}{m^2}$ which adds to the constant pressure level λ in the deformed balloon at time $t = 15s$.

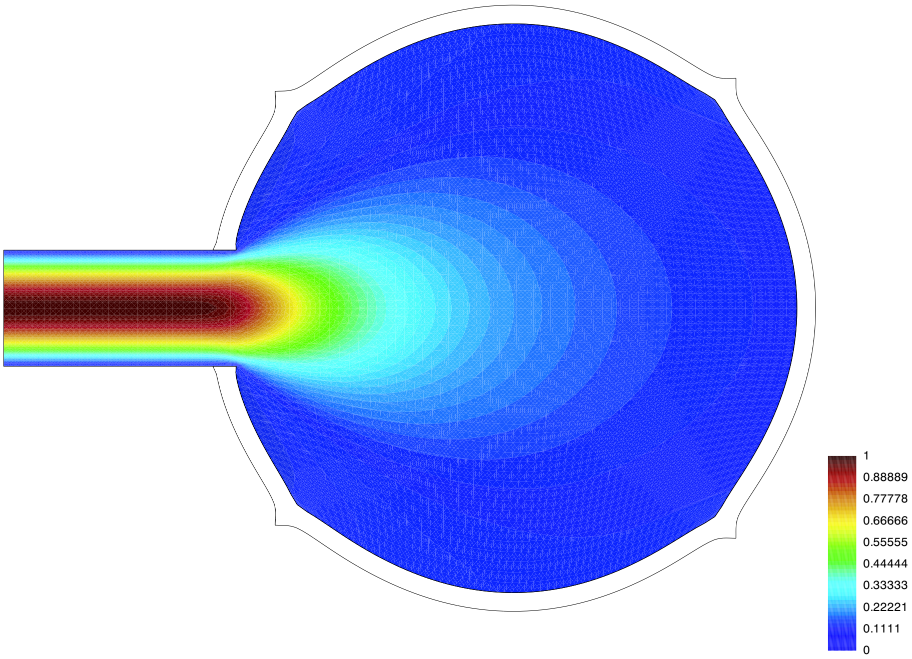

Figure 4: Velocity $|\mathbf{u}|$ in $\frac{m}{s}$ in the deformed balloon at time $t = 15s$.

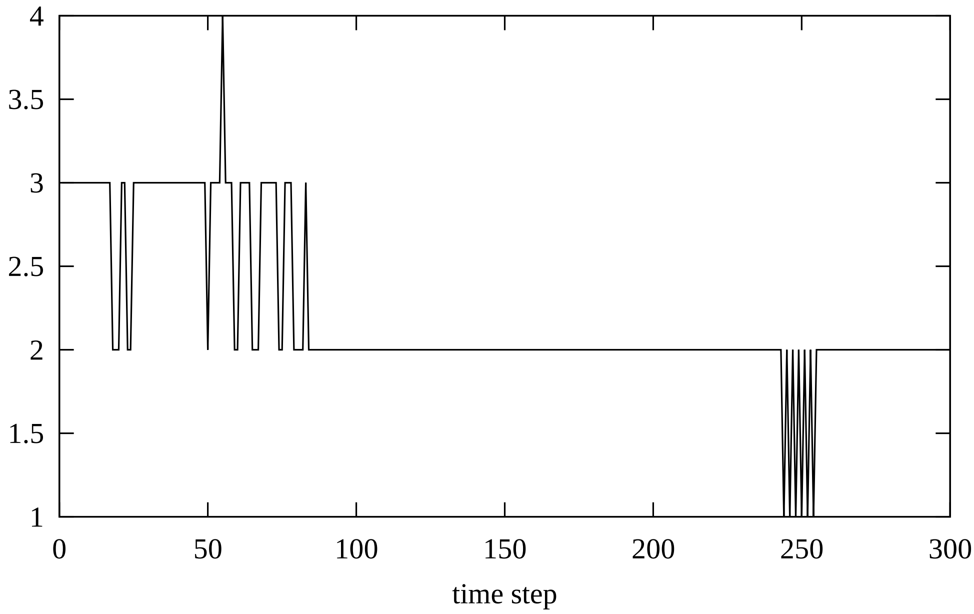

Figure 5: Number of field iterations of the academic balloon calculation.

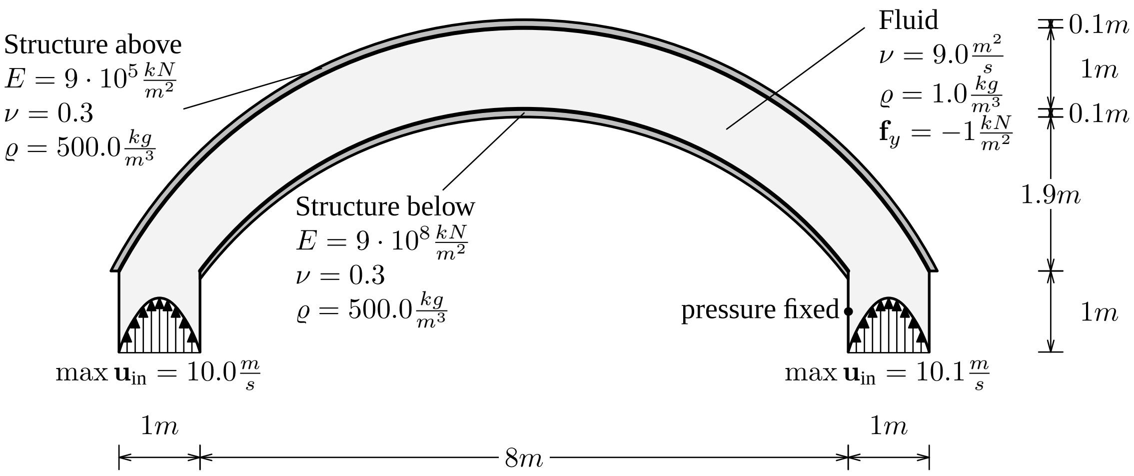

Figure 6: A bended fluid domain with two inflow boundaries constraint by structures of different stiffness.

structure above the fluid domain deforms to make room for the fluid. When a critical pressure value is reached the structure below the fluid collapses, however the instability is damped by the fluid volume constraint. That is why the deformation and the corresponding pressure decrease occur rather slowly. (Since this example is given just in order to demonstrate the augmented Dirichlet-Neumann approach, possible cavitation effects are not considered.) Afterward the system is in motion, the pressure varies rapidly in this phase. The pressure level development is depicted in figure 7.

Because the fluid pressure level is much larger than the pressure gradient variation there is again not much use in depicting the absolute pressure values. Instead figure 8 shows only the pressure gradient in the deformed configuration as calculated by the fluid. The pressure is fixed to 0 at the point indicated in figure 6. The negative pressures below that point are due to the body force $\mathbf{f}_y$. To obtain the real pressure the corresponding pressure level from figure 7 has to be added.

Figure 9 shows the corresponding absolute velocities. And to demonstrate the algorithm's effectivity figure 10 shows the number of field iterations needed in each time step.

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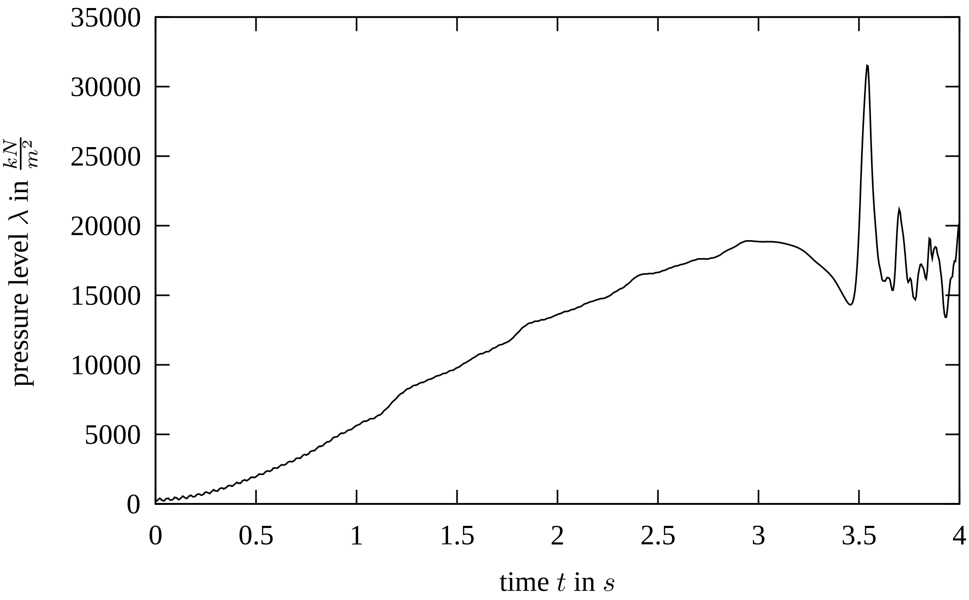

Figure 7: Pressure of the fluid domain of the bended example at the specified point.

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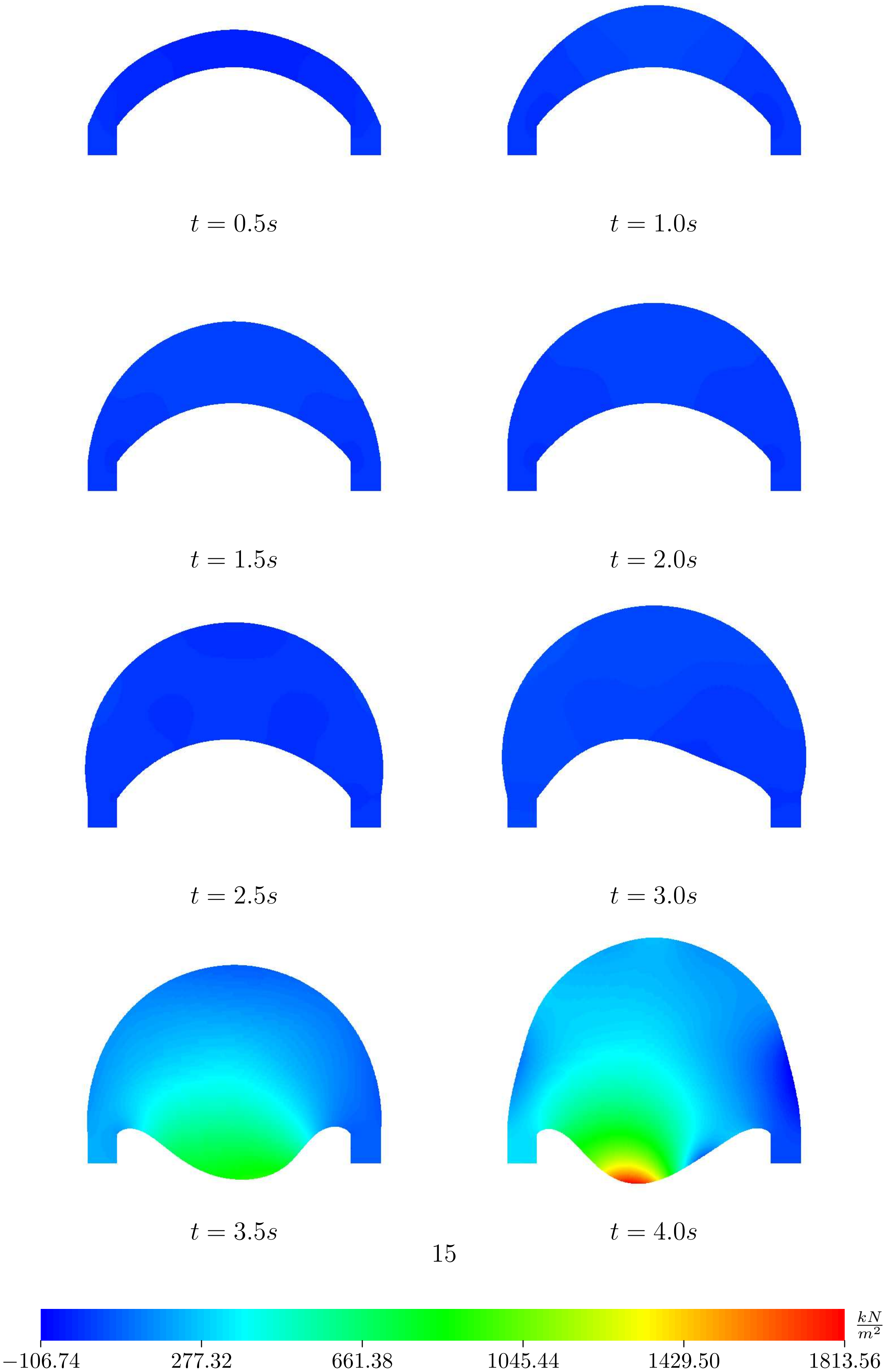
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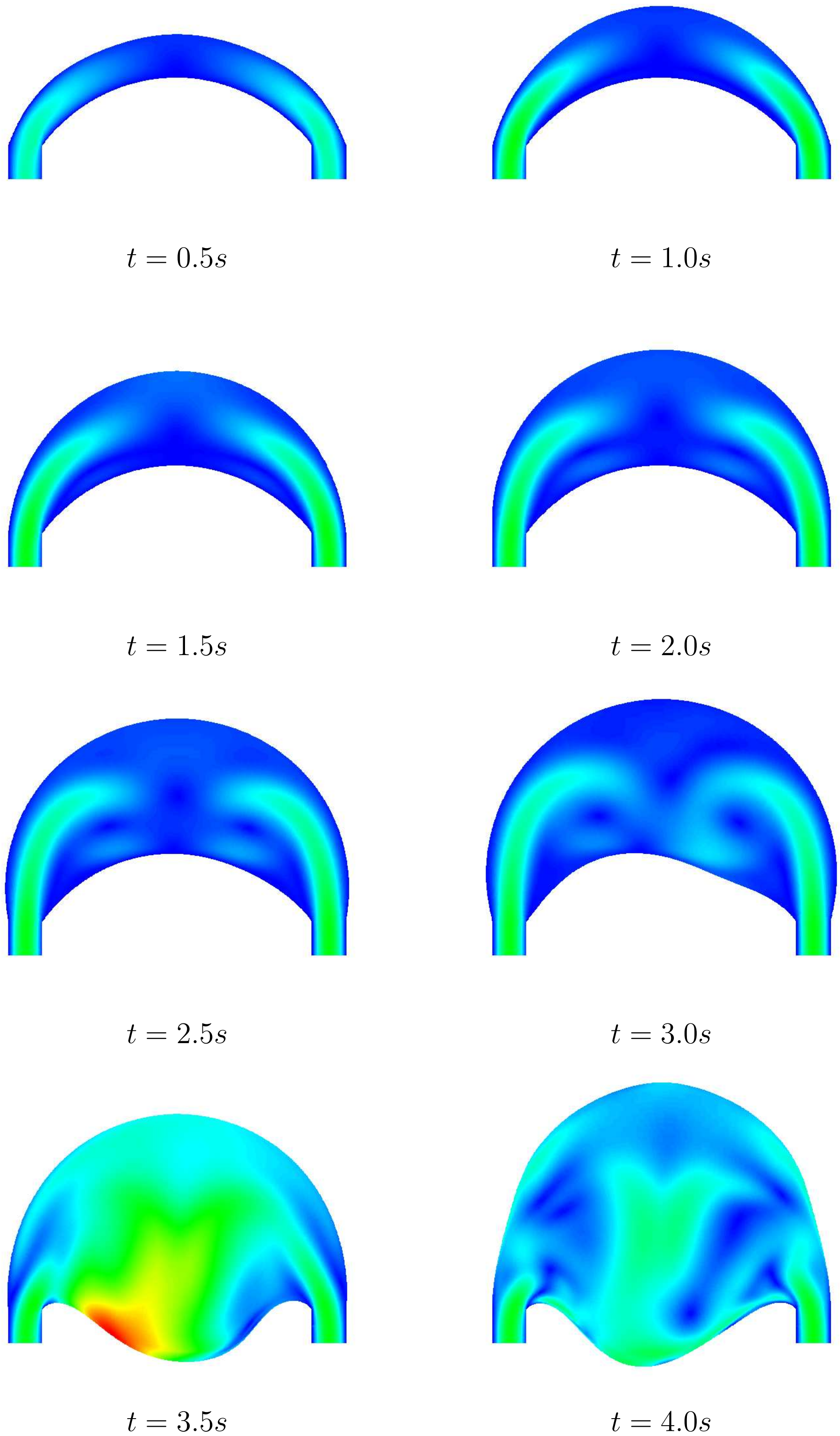
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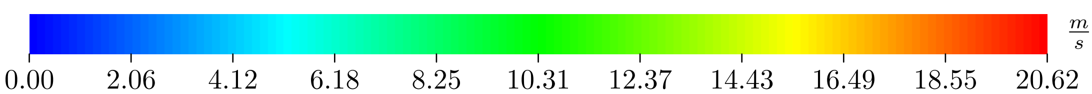
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20.62
$\frac{m}{s}$

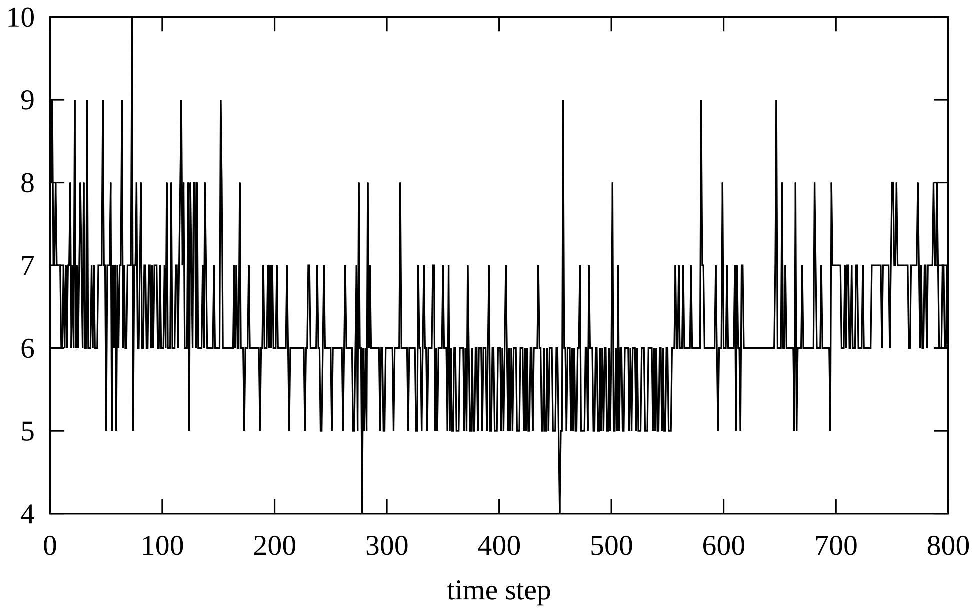

Figure 10: Number of field iterations of the bend fluid domain snap-through calculation with Aitken-style relaxation.

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