DELFT UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF AEROSPACE ENGINEERING

Report LR-330

THE STRESS INTENSITY FACTOR OF SMALL CRACKS AT NOTCHES

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ABSTRACT

It was shown before that stress fields around notches are very much similar quantitatively, if the peak stress at the notch root (σ_{peak}) and the notch root radius (ρ) are the same. As a result, small cracks (length ℓ) in similar stress fields should have the same stress intensity factor. This implies that the geometry factor C in $K = C\,\sigma_{peak}\sqrt{\pi\ell}$ should primarily depend on ℓ/ρ only and not on other dimensions. Available data from the literature was analysed, which confirmed the similarity concept. From the data an equation for C (ℓ/ρ) was obtained, which turned out to be an accurate approximation for small cracks.

Key words: stress intensity factor, crack, notch

NOTATION

a	crack length measured from center of notch
C	geometry factor in $K = C\,\sigma_{peak}\sqrt{\pi\ell}$
F	geometry factor in $K = F\,S\sqrt{\pi a}$
F_C	approximation of F based on C (eq.6)
F_{BK}	approximation of F based on Benthem/Koiter equation
F_{SM}	approximation of F based on Smith/Miller equation
K	stress intensity factor
K_t	stress concentration factor based on net section stress
K_{tg}	stress concentration factor based on gross stress
ℓ	crack length measured from edge of notch root
S	gross stress
W	width
α/β	major and minor axes of ellipse
α'	stress gradient coefficient
ρ	notch root radius
σ_{peak}	local maximum at root of notch

INTRODUCTION

Geometrical notches in a structure are causing local stress concentrations with the maximum stress at the root of the notch. Fatigue cracks will be initiated at this location, and this can occur early in the fatigue life. In many cases the crack will be small for a large part of the life. For this reason the stress distribution at the root of the notch is significant. In a previous publication [1] it was shown that the stress distribution in the proximity of the notch is approximately the same for a variety of notches and stress concentration factors (K_t), if the peak stress at the notch (σ_{peak}) and the root radius (ρ) are the same. This is illustrated by Figure 1 drawn from reference [1]. It then should be expected that small cracks occurring in similar stress fields should have the same stress intensity factor (K). In other words, for small cracks K should predominantly depend on σ_{peak} and ρ only. This idea is elaborated in the present report, adopting data from the literature on stress intensity factors of cracks at notches. A comparison is made with two other approximations for K values of small cracks [2, 3, 4].

K AS A FUNCTION OF σ_{peak} AND ρ

The stress intensity factor will be written here as:

$$K = C\, \sigma_{peak} \sqrt{\pi \ell} \qquad (1)$$

where ℓ is the crack length as measured from the notch root (Figure 2), and C is the geometry factor. If we now consider small cracks at different notches, the basic arguments mentioned in the introduction are:

(a) Approximately similar stress distributions in the uncracked condition are obtained if the same values of σ_{peak} and ρ apply, and:

(b) If small cracks of the same length ℓ are present, while σ_{peak} and ρ are the same, it is a consequence of (a) that again approximately similar stress distributions should apply, and thus the same stress intensity factors should be obtained. It implies that the geometry factor C in equation (1) should be a function of the ratio ℓ/ρ only.

$$C = C\ (\ell/\rho) \tag{2}$$

This will be checked by adopting K values calculated by Newman [5] for cracks occurring at the ends of an axis of an elliptical hole in an infinite sheet loaded in tension by a stress S. Newman made calculations for 5 different ellipses with axes ratios α/β = 0.25, 0.5, 1, 2 and 4, corresponding to K_t-values of 1.5, 2, 3, 5 and 9 respectively. Newman defined the stress intensity factor by:

$$K = F\ S\ \sqrt{\pi a} \tag{3}$$

with the crack length "a" measured from the center of the ellipse (Figure 2). The geometry factor F was calculated by Newman by an improved boundary collocation method. His results are reproduced here in Table 1. The F data are easily converted into C data by combining equations (1) and (3):

$$C = F \frac{S}{\sigma_{peak}} \sqrt{\frac{a}{\ell}} = \frac{F}{K_t} \sqrt{\frac{a}{\ell}}$$

For an ellipse in an infinite sheet: $K_t = \dfrac{\sigma_{peak}}{S} = 1 + 2\ (\alpha/\beta)$

With $a = \alpha + \ell$ (Figure 2) and $\rho = \beta^2/\alpha$ the final results are:

$$C = \frac{F}{1+2\,\frac{\alpha}{\beta}} \sqrt{1 + \frac{(\alpha/\beta)^2}{\ell/\rho}} \tag{4}$$

$$\ell/\rho = \left(\frac{\alpha}{\beta}\right)^2 \left(\frac{a}{\alpha} - 1\right) \tag{5}$$

With these equations Newman's data in Table 1 were recalculated to C values as a function of ℓ/ρ. The results are presented in Table 2 and in Figure 2. The horizontal scale in Figure 2 is linear for $\sqrt{\ell/\rho}$ which was done to expand the ℓ/ρ axis for small values of ℓ/ρ. This improves the comparison of the C-data of different ellipses for small cracks on which the major interest of the present paper is concentrated. It is encouraging to see both in Table 2 and in Figure 2 that similar C values are found indeed up to values of $\ell/\rho \sim 0.6$. This supports the basic idea that similar K values will apply to small cracks at notches if σ_{peak} and ρ are similar. (In Figure 2 some deviating results occur for $\alpha/\beta = 0.25$ and low ℓ/ρ values. In view of the unsystematic values these results are believed to be inaccurate.)

It is obvious that equation (1) with C depending on ℓ/ρ only can not apply to large cracks because other dimensions, apart from the root radius ρ, should become significant. Deviations can be observed in Table 2 and Figure 2 for $\ell/\rho \gtrsim 0.8$. Moreover, the data for $\alpha/\beta = 0.25$ ($K_t = 1.5$) and $\alpha/\beta = 0.5$ ($K_t = 2$) do not cover ℓ/ρ values larger than 0.0875 and 0.35 respectively. Another aspect is that Newman's data apply to elliptical holes of different shapes in an infinite sheet. For finite dimensions deviations may occur at lower ℓ/ρ values. This will be explored later in this report.

The curve in Figure 2 was approximated by a polynomial equation:

$$C = 1.1215 - 3.21\ (\ell/\rho) + 5.16\ (\ell/\rho)^{1.5} - 3.73\ (\ell/\rho)^{2} + 1.14\ (\ell/\rho)^{2.5} \qquad (6)$$

It is in excellent agreement with Newman's data up to $\ell/\rho \sim 1$ which is more than sufficient for the present purpose. Equation (1) with C according to equation (6) will be applied to other notches later in this report. A somewhat simpler equation than (6) is:

$$C = 1.1215 - 3\ (\ell/\rho) + 4\ (\ell/\rho)^{1.5} - 1.7\ (\ell/\rho)^{2} \qquad (7)$$

It hardly deviates from equation (6) up to $\ell/\rho \sim 0.6$ (differences $\lesssim 0.2$ percent).

OTHER APPROXIMATIONS

Smith and Miller [2] have proposed a very simple formula for the stress intensity factor of small cracks at the root of a notch. The small crack (length ℓ) is compared to a larger one (length L) in the unnotched configuration. Both cracks have the same stress intensity factor. As a result of evaluating the same Newman data the relation between L and ℓ was supposed to be represented on the average by:

$$L = \ell + e \quad \text{with } e = 7.69\ \ell \sqrt{D/\rho} \tag{8}$$

where D is the depth of the notch (or the semi axis of an ellips, $D = \alpha$).

Equation (8) was proposed to be applicable up to a crack length

$$\ell = 0.13 \sqrt{D\rho} \tag{9a}$$

Substitution in equation (8) gives

$$L = \ell + D \tag{9b}$$

which implies that the depth of the notch is to be added to the physical crack length (ℓ) to obtain the apparent crack length (L) for calculating K. For a crack length exceeding the value in equation (9a) it was proposed to maintain equation (9b) as an approximation.

For an infinite sheet with an elliptical hole ($\rho = \beta^2/\alpha$) equation (8) implies:

$$K = S\sqrt{\pi L} = S\sqrt{\pi\ell}\ \sqrt{1 + 7.69\ (\alpha/\beta)} \tag{10}$$

A comparison with equation (1) indicates:

$$C = \frac{\sqrt{1 + 7.69\ (\alpha/\beta)}}{1 + 2\ (\alpha/\beta)} \tag{11}$$

In other words: For a given α/β the geometry factor C should be a constant, which is not dependent on ℓ/ρ. For the α/β values involved in Figure 2 (α/β from 0.25 to 4) the value of C according to equation (11) varies from 1.140 to 0.626. In view of the results plotted in Figure 2 it appears that the averaging idea behind equation (8) is a rather drastic one. More results will be presented later.

Another method to estimate the K value for a small crack was adopted by Karlsson and Bäcklund [4]. The method is based on the work of Benthem and Koiter [3] who gave a K solution for an edge crack in a semi-infinite sheet with a linear distribution of the tensile stress in the sheet. The solution is the same as for an edge crack with a linear load distribution on the crack edges (see Figure 3), for which Benthem and Koiter derived:

$$K = (1.122\ p + 0.439\ q)\ \sqrt{\pi\ell} \qquad (12)$$

(The numerical factor 1.122 is the same as 1.1215 used in equation (6) and (7). It is the wellknown geometry factor for an edge crack in a semi-infinite sheet). Karlsson and Bäcklund assumed that this equation could be adopted as a first approximation for a small edge crack at the root of the notch, where q/ℓ was supposed to be equal to the stress gradient at the notch root ($x = 0$) and $p + q$ is equal to the peak stress. Equation (12) was thus rewritten as:

$$K = \left\{1.122\ \sigma_{peak} + 0.683 \left[\partial\sigma_y/\partial x\right]_{x=0} \ell\right\} \sqrt{\pi\ell} \qquad (13)$$

In reference [1] the stress gradient was written as:

$$\left(\frac{\partial\sigma_y}{\partial x}\right)_{x=0} = -\ \alpha'\ \frac{\sigma_{peak}}{\rho} \qquad (14)$$

with the stress gradient coefficient α' still depending on the shape of the geometry involved. Substitition of equation (14) into equation (15) gives:

$$K = \left(1.122 - 0.683\ \alpha'\ \frac{\ell}{\rho}\right)\ \sigma_{peak}\ \sqrt{\pi\ell} \tag{15}$$

For an elliptical hole there is a simple relation for the stress gradient coefficient:

$$\alpha' = 2 + \frac{1}{K_t} = \frac{3 + 4\ \alpha/\beta}{1 + 2\ \alpha/\beta} \tag{16}$$

Comparing equation (15) to equation (1) and substitution of equation (16) gives:

$$C = 1.122 - 0.683\ \frac{\ell}{\rho}\ \left(\frac{3 + 4\ \alpha/\beta}{1 + 2\ \alpha/\beta}\right) \tag{17}$$

In this case C for small cracks is depending on ℓ/ρ, but also on the shape of the ellipse (α/β). However, the latter dependence is small as shown by the two curves drawn in Figure 2 for the minimum and maximum α/β values adopted by Newman: $\alpha/\beta = 0.25$ ($K_t = 1.5$) and $\alpha/\beta = 4$ ($K_t = 9$). The curves also show that there is a systematic difference between equation (17) and the results of Newman. This should not be too surprising because there are some differences between the two cases of Figure 3. In Figure 3b the edge is not a straight line and the stress distribution $\sigma_y(x)$ is not a linearly decreasing function. Nevertheless equation (17) gives a reasonable approximation up to $\ell/\rho \sim 0.2$, the differences with Newman's results being in the order of a few percent only. More results for other notches will be given later.

APPLICATIONS

1. Elliptical edge notch

Nisitani [6] has calculated stress concentration factors for a semi-elliptical edge notch in a semi-infinite sheet, loaded in tension by a stress S. For three notches of this type, v.i.z. for $\alpha/\beta = 0.5$, 1 and 2 respectively, he also calculated the stress intensity factor for cracks at the tip of the ellipse (see table 3). The configurations are shown in Figures 4a, b and c. Nisitani presents his results as values of F defined in the same way as by Newman

$$K = FS\sqrt{\pi a} = FS\sqrt{\pi(\alpha+\ell)} \qquad (3)$$

A conversion of C in equation (1) to F of equation (3) gives:

$$F_C = \frac{K_t\, C}{\sqrt{1+\alpha/\ell}} = \frac{K_t\, C}{\sqrt{1+\left(\frac{\alpha}{\beta}\right)^2\Big/\left(\frac{\ell}{\rho}\right)}} \qquad (18)$$

The subscript C is used to indicate that F_C has been obtained from C as a function of ℓ/ρ (eq. 6). F_C as a function of ℓ/ρ is also shown in Figures 4a, b and c. The agreement with Nisitani's data is very good for small cracks. For large ℓ/ρ values (> 0.4) deviations are found.

In a similar way the approximations for small cracks by the Smith/Miller formula (equation 10) and the Benthem/Koiter equation as adopted by Karlsson and Bäcklund (equation 13) can also be compared with Nisitani's data. Equation (10) according to the Smith/Miller concept has to be rewritten for an edge crack:

$$K = 1.1215\, S\sqrt{\pi L} = 1.1215\, S\sqrt{\pi\ell}\ \sqrt{1+7.69\,(\alpha/\beta)} \qquad (10a)$$

Combining this equation with equation (3) gives:

$$F_{SM} = \frac{1.1215 \quad \sqrt{1+7.69\ (\alpha/\beta)}}{\sqrt{1 + \left(\frac{\alpha}{\beta}\right)^{2} \Big/ \left(\frac{\ell}{\rho}\right)}} \tag{19}$$

The subcript SM refers to Smith and Miller.

The application of equation (15) still offers a problem, because the stress gradient at the root of the notch is unkwown. However, the difference between the K_t-values of elliptical holes (K_t = 1+2 α/β) and semi-elliptical edge notches are small for α/β = 0.5, 1 and 2 respectively, as shown by the work of Nisitani. It then may be expected that the exact equation for the stress gradient of the elliptical hole (eq. 16) will still be a good approximation for the semi-elliptical edge notches. A comparison of equations (15) and (3) then leads to:

$$F_{BK} = \frac{K_t \left\{1.122 - 0.683\ \frac{\ell}{\rho}\ \left(\frac{3+4\ \frac{\alpha}{\beta}}{1+2\ \frac{\alpha}{\beta}}\right)\right\}}{\sqrt{1 + \left(\frac{\alpha}{\beta}\right)^{2} \Big/ \left(\frac{\ell}{\rho}\right)}} \tag{20}$$

The subscript BK refers to the Benthem/Koiter equation. Equations (19) and (20) were used to show the results of the corresponding approximations in Figures 4a, b and c. Differences between F_{BK} and F_c are rather small up to $\ell/\rho \sim 0.2$. F_{SM} takes different positions as compared to the other approximations, depending on K_t.

2. Circular hole in finite width strip

Newman [5] has calculated stress intensity factors for two hole diameter to width ratios, viz. $2\rho/W = 0.25$ and 0.50 respectively, see table 4. In a similar way as for the previous application we now obtain:

$$F_c = \frac{K_{tg} \quad C}{\sqrt{1 + (\ell/\rho)^{-1}}} \tag{21}$$

K_{tg} is the stress concentration factor based on the gross stress in the finite width strip. The values given in Figure 5 were obtained from a graph given by Peterson [7] representing calculated data of Howland. The strip in Newman's calculations also had a finite length (H), viz. H = 2W. According to data of Schulz (also quoted by Peterson [7]) for a finite width strip of infinite length with an infinite row of holes, it should be expected that H = 2W is a sufficient length for the applicability of the Howland K_{tg} results.
F_c calculated with equation (21) is shown in Figures 5a and 5b. Again the agreement with Newman's data is very good for small cracks.

With respect to the Smith/Miller concept equation 10 has now to be rewritten for a central crack in a finite width specimen, and with $\alpha/\beta = 1$ it becomes

$$K = \sqrt{\sec(\pi a/W)} \;\; S \sqrt{\pi L} = \sqrt{8.69 \sec\;(\pi a/W)} \;\; S \sqrt{\pi \ell} \tag{10b}$$

Combining this with equation (3) and $a = \rho+\ell$ gives:

$$F_{SM} = \left[\frac{8.69\,\frac{\ell}{\rho}}{\left(1 + \frac{\ell}{\rho}\right) \cos\left\{\frac{\pi\rho}{W}\left(1 + \frac{\ell}{\rho}\right)\right\}}\right]^{1/2} \tag{22}$$

For application of equation (15) the difficulty is again that the stress gradient is not exactly known. Estimates were made in [1] (extended version) based on the stress distributions as calculated by Howland [8] . From these data the stress gradient coefficient was found to be $\alpha' = 2.40$ for $\rho/W = 0.125$ and $\alpha' = 2.75$ for $\rho/W = 0.250$. Combining equations (15) and (3), and $a = \rho+\ell$ gives:

$$F_{BK} = \frac{K_{tg}\left[1.122 - 0.683\,\alpha'\,(\ell/\rho)\right]}{\sqrt{1 + (\ell/\rho)^{-1}}} \qquad (23)$$

Calculated results for F_{SM} and F_{BK} according to equations (22) and (23) are also shown in Figures 5a and 5b. Initially F_{BK} is fairly close to F_c again. Differences with F_{SM} are small in Figure 5a and large in Figure 5b.

3. Loaded hole in lug specimen

In a recent study [9] crack propagation in a lug type specimen with a pin loaded hole was studied. The dimensions are shown in Figure 6. K-values as derived from the crack growth rate were compared with calculated K-values in the literature [10, 11] . The agreement was good. A close approximation of the results was:

$$K = S\sqrt{\pi\ell}\left[5.38 - 12.3\left(\frac{\ell}{\rho}\right) + 19.1\left(\frac{\ell}{\rho}\right)^2 - 14.6\left(\frac{\ell}{\rho}\right)^3 + 4.55\left(\frac{\ell}{\rho}\right)^4\right] \qquad (24)$$

The lug specimen is still symmetric with respect to the loading direction (Y-axis), but not with respect to the transverse direction (X-axis). Moreover, the loading on the hole is applied rather close to the critical section, where σ_{peak} occurs. It is fairly optimistic to expect still a similar stress distribution as for fully symmetric configurations with remote loading. Nevertheless, a comparison of K according to equation (24) with the K-value based on C (equations 1 and 6) will be made. In conformity with the previous applications the comparison will be made for the geometry factor as defined in equation (3). From this equation and the empirical result in equation (24) we obtain (with $a = \rho+\ell$)

$$F_{emp} = \frac{\left[5.38 - 12.3\left(\frac{\ell}{\rho}\right) + 19.1\left(\frac{\ell}{\rho}\right)^2 - 14.6\left(\frac{\ell}{\rho}\right)^3 + 4.55\left(\frac{\ell}{\rho}\right)^4\right]}{\sqrt{1 + (\ell/\rho)^{-1}}} \tag{25}$$

The predicted F_c value based on C follows from a comparison between equations (1) and (3)

$$F_c = \frac{K_{tg} \quad C}{\sqrt{1 + (\ell/\rho)^{-1}}} \tag{26}$$

A comparison between F_c and F_{emp} is made in Figure 6. For $\ell/\rho < 1$ the difference is smaller than 3 percent, whereas it has increased to 9 percent at $\ell/\rho = 0.2$. At larger ℓ/ρ values the comparison is meaningless.

DISCUSSION

The basic idea of the present paper is that the stress intensity factor of small cracks at the root of notch is predominantly depending on the local peak stress (σ_{peak}) and the root radius (ρ) only. The main argument for this assumption is that the stress distribution around notches are quantitatively very much similar if σ_{peak} and ρ are the same while other dimensions and K_t may be different. As a result small cracks in similar stress fields should imply similar stress intensity factors. Calculated K values of Newman [5] and Nisitani [6] for a variety of notches have amply confirmed the above suppositions. Calculated data as presented in [5] and [6] are assumed to have a high accuracy (with a few exceptions in [5] as referred to before). Differences between comparative data in [5] and [6] are in the order of 0.1 percent at most.

Part of the proof of the above similarity concept could already be derived from two graphs presented by Nisitani [6] . For small cracks he plotted

$$F' = \frac{K}{1.1215\ K_t\ S\ \sqrt{\pi \ell}} \tag{27}$$

as a function of ℓ/ρ. For elliptical holes in an infinite sheet and for elliptical edge notches his graphs show almost similar curves for α/β = 0.5, 1 and 2 respectively. Nisitani did not present any discussion on this result. However, if equation (27) is compared to the present definition of C:

$$K = C\ \sigma_{peak}\ \sqrt{\pi \ell} \tag{1}$$

it follows that:

$$C = 1.1215\ F' \tag{28}$$

In other words, the fact that C for small cracks depends on ℓ/ρ only could have been deduced from Nisitani's graphs.

It still has to be recognized that for larger cracks the small-crack approximation can no longer be valid, and C will depend on other dimensions as well. An impression of the validity can be obtained from Figures 4 and 5 by comparing F as based on C (F_c) with the calculated results. Differences smaller than 2 percent are found up to maximum ℓ/ρ values compiled in the table below.

		K_t	Maximum ℓ/ρ
Semi-infinite sheet, elliptical edge notch	α/β = 0.5	2.016	0.25
	1	3.065	0.4
	2	5.221	0.8
Finite width strip, central hole	$2\rho/W$ = 0.25	2.43	0.35
	0.50	2.16	0.15

Table 5: Maximum ℓ/ρ values for F_c as a good approximation of F.

The lower maximum ℓ/ρ values are found for the shallow elliptical edge notch (α/β = 0.5) and the relatively large central hole ($2\rho/W$ = 0.5). In the former case the free edge is relatively close to the tip of the root of the notch, and in the latter case the same applies to the finite width edges. In such cases the similarity of the stress field around the notch will be more affected than in the other cases. Note that the specimens in Figure 5 have been drawn to scale. Apparently free edges close to the notch root will disturb the similarity approach adopted in the present paper. For the lug specimen (Figure 6) the 2 percent criterion gives a maximum $\ell/\rho \sim 0.08$, which is still lower than for the other notches. However, in this case the asymmetric load transmission, partly close to the notch root, will more easily violate the similarity of root notch stress fields.

For large cracks the K factor can be approximated by assuming that the notch is part of the crack. For elliptical edge notches the result is $F = 1.1215$ and for central holes in finite width strips $F = \sqrt{\sec(\pi a/W)}$. Both results have been indicated in Figures 4 and 5. For the elliptical side notches (Figure 4) Nisitani's results are going asymptotically to the "large-crack" approximation. For the central hole in the strip the same trend is observed in Figure 5a, but it is not so obvious in Figure 5b. In the latter case the large-crack approximation remains fairly poor for all values of ℓ/ρ. This is not unexpected because the hole diameter is equal to 50 percent of the width of the strip.

The present results indicate a good approximation of the K value of small cracks up to certain limits of the crack size. It still is desirable to obtain accurate K values for other types of notches in view of the applicability of equation (1), especially with respect to the maximum ℓ/ρ values, where deviations become significant.
Another line to follow as suggested by the present stress field similarity approach, is the application to small cracks with curved crack fronts, such as semi-elliptical surface cracks and quarter elliptical corner cracks.

CONCLUSION

1. For notched elements with various dimensions and K_t factors the stress fields close to the root of the notch are quantitatively very much similar, if the same peak stress (σ_{peak}) and the same root radius (ρ) are present. As a result of the similarity the stress intensity factor of a small crack (length ℓ) should primarily depend on σ_{peak} and ρ, and not on other dimensions. In other words in $K = C\,\sigma_{peak}\sqrt{\pi\ell}$ the geometry factor C should depend on ℓ/ρ only. This was confirmed by an analysis of calculated data available in the literature.

2. For the geometry factor C a polynomial expression was obtained, which gives accurate K predictions for small cracks up to ℓ/ρ values depending on the geometry of the specimen. The ℓ/ρ range in which C is applicable is larger if material edges, apart from the notch profile, are more remote.

3. Two other methods for obtaining K values of small cracks at notches were found to be less accurate and less systematic.

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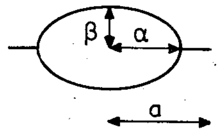


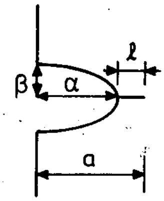
a/α	Values of F in $K = F S \sqrt{\pi a}$				
	$\alpha/\beta=4$	$\alpha/\beta=2$	$\alpha/\beta=1$	$\alpha/\beta=0.5$	$\alpha/\beta=0.25$
1.01	-	-	0.3256	-	-
1.02	0.9050	0.6757	0.4514	0.3068	0.2114
1.03	0.9597	0.7742	-	-	-
1.04	0.9865	0.8398	0.6082	0.4297	-
1.05	1.0013	0.8861	-	-	0.3137
1.06	1.0098	0.9206	0.7104	0.5164	-
1.08	1.0179	0.9664	0.7843	0.5843	0.4463
1.10	1.0206	0.9925	0.8400	0.6401	0.5027
1.15	1.0202	1.0258	0.9322	0.7475	0.5901
1.20	1.0176	1.0357	0.9851	0.8241	-
1.25	-	-	1.0168	-	0.7248
1.30	-	1.0366	1.0358	0.9255	-
1.40	-	1.0317	1.0536	0.9866	0.8494
1.50	-	-	1.0582	1.0246	-
1.55	-	-	-	-	0.9279
1.60	-	-	1.0571	1.0483	-
1.80	-	-	1.0495	1.0714	1.0063
2.00	-	-	1.0409	1.0777	-
2.10	-	-	-	-	1.0551
2.20	-	-	1.0336	1.0766	-
2.40	-	-	1.0252	1.0722	1.0788
3.00	-	-	1.0161	-	-
4.00	-	-	1.0077	-	-

Table 1: Numerical data calculated by Newman [5]

Table 2: Values of C in eq. (2) derived from values of F (eq.1) as calculated by Newman [1]

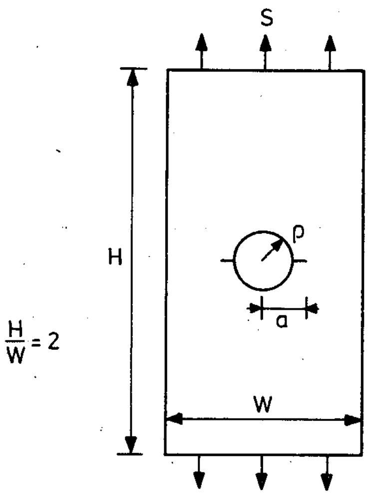
ℓ/ρ	C			
	α/β=2	α/β=1	α/β=0.5	α/β=0.25
0.00125				1.0065
0.003125				0.9584
0.005			1.0955	1.0932
0.00625				1.1115
0.009375				1.0893
0.01		1.0907	1.0955	
0.015			1.0853	
0.015625				1.0805
0.02		1.0745	1.0734	
0.025			1.0615	1.0594
0.034375				1.0385
0.0375			1.0349	
0.04		1.0337		
0.05			1.0093	1.0063
0.06		0.9953		
0.06875				0.9719
0.075			0.9633	
0.08	0.9651	0.9606		
0.0875				0.9417
0.1		0.9287	0.9229	
0.12	0.9073			
0.125			0.8873	
0.15		0.8604	0.8559	

ℓ/ρ	C			
	α/β=4	α/β=2	α/β=1	α/β=0.5
0.16		0.8564		
0.2		0.8121	0.8043	0.8036
0.24		0.7739		
0.25			0.7579	0.7620
0.3			0.7187	0.7289
0.32	0.7181	0.7102		
0.35				0.7019
0.4		0.6584	0.6570	
0.48	0.6248			
0.5			0.6110	
0.6		0.5681	0.5754	
0.64	0.5589			
0.8	0.5098	0.5074	0.5247	
0.96	0.4716			
1			0.4907	
1.2		0.4316	0.4665	
1.28	0.4156			
1.4				
1.6	0.3761	0.3860		
2			0.4148	
2.4	0.3139			
3			0.3879	
3.2	0.2770			



ℓ/α	values of F in K = F S $\sqrt{\pi a}$ α/β=0.5 K_t =1.503	α/β=1 K_t =2.016	α/β=2 K_t =3.065
0.1	0.654	0.862	1.053
0.2	0.831	1.017	1.109
0.4	1.000	1.102	1.122
0.6	1.068	1.118	1.122
0.8	1.098	1.121	1.121
1.0	1.111	1.122	1.121

Table 3: Numerical data calculated by Nisitani [6]



values of F in K = F S $\sqrt{\pi a}$			
2a/W	2ρ/W=0.25	2a/W	2ρ/W=0.50
0.25	0	0.50	0
0.26	0.6593	0.51	0.6527
0.27	0.8510	0.52	0.8817
0.28	0.9605	0.525	0.9630
0.29	1.0304	0.53	1.0315
0.30	1.0776	0.54	1.1426
0.35	1.1783	0.55	1.2301
0.40	1.2156	0.60	1.5026
0.50	1.2853	0.70	1.8247
0.60	1.3965	0.78	2.1070
0.70	1.5797	0.85	2.4775
0.80	1.9044	0.90	2.9077
0.85	2.1806		
0.90	2.6248		

Table 4: Numerical data calculated by Newman [5]

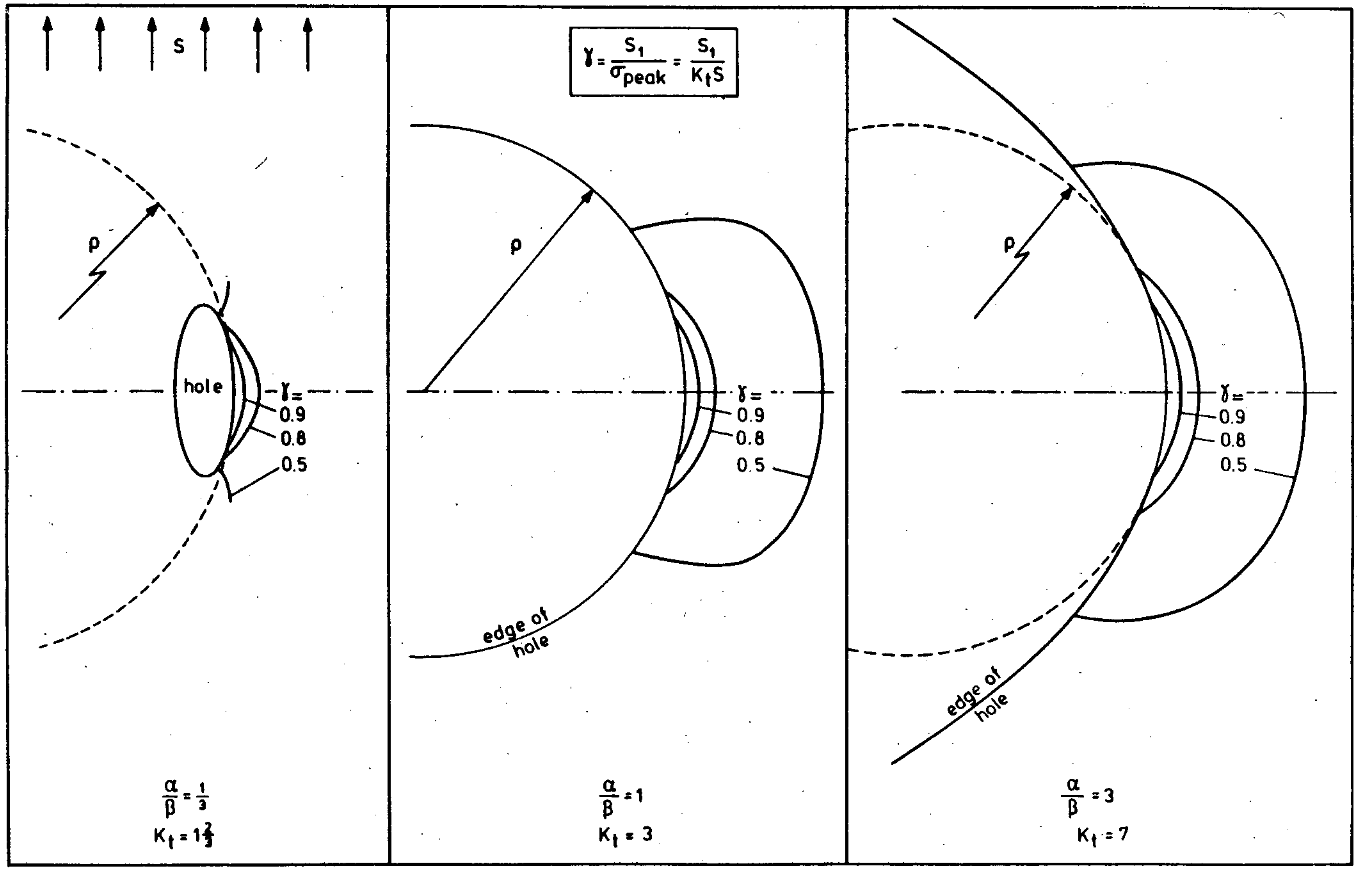


Figure 1 : Lines of constant maximum principal stress (S_1) around three elliptical holes in an infinite sheet loaded in tension (holes scaled to obtain same ρ). Figure from reference [1]

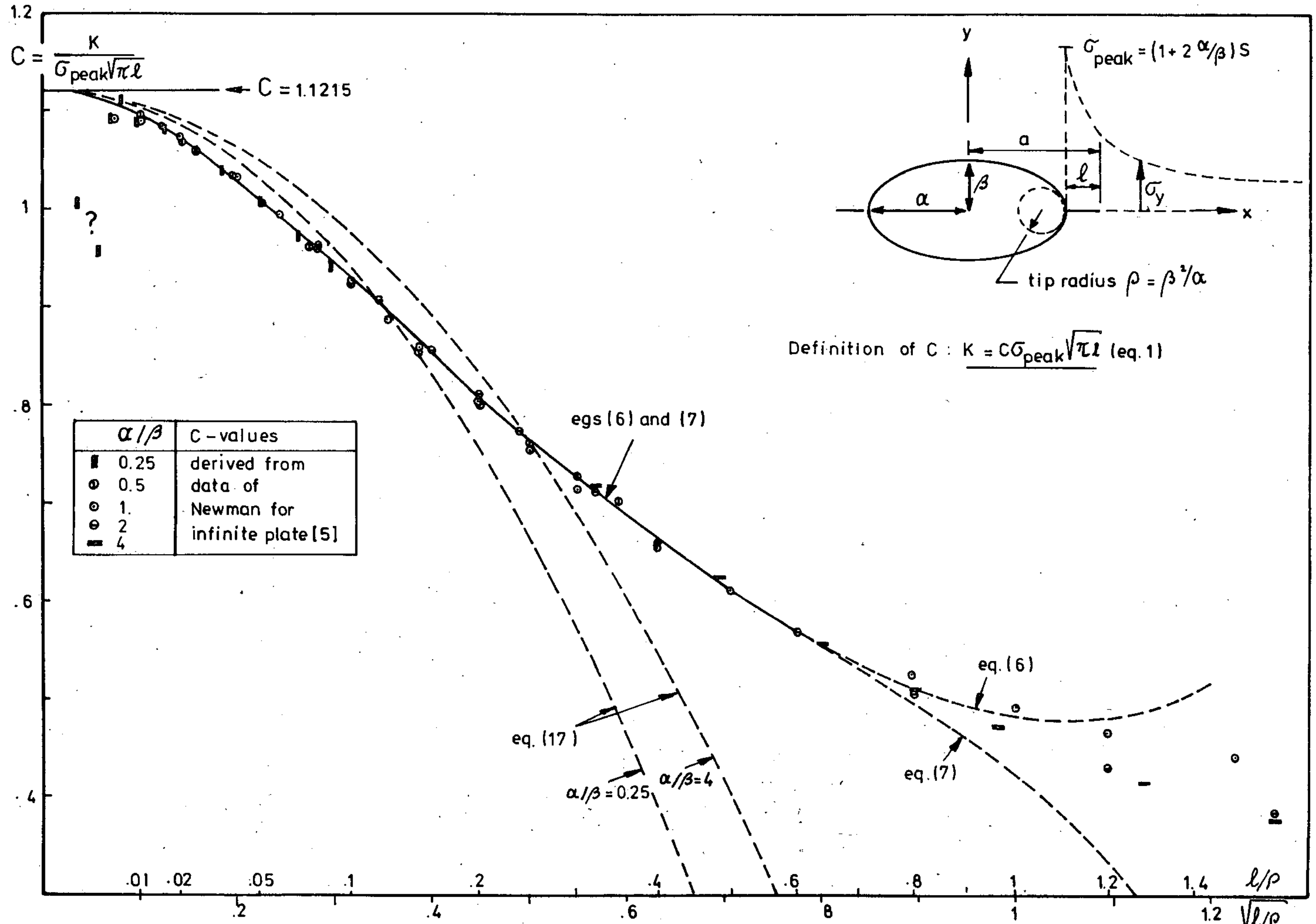
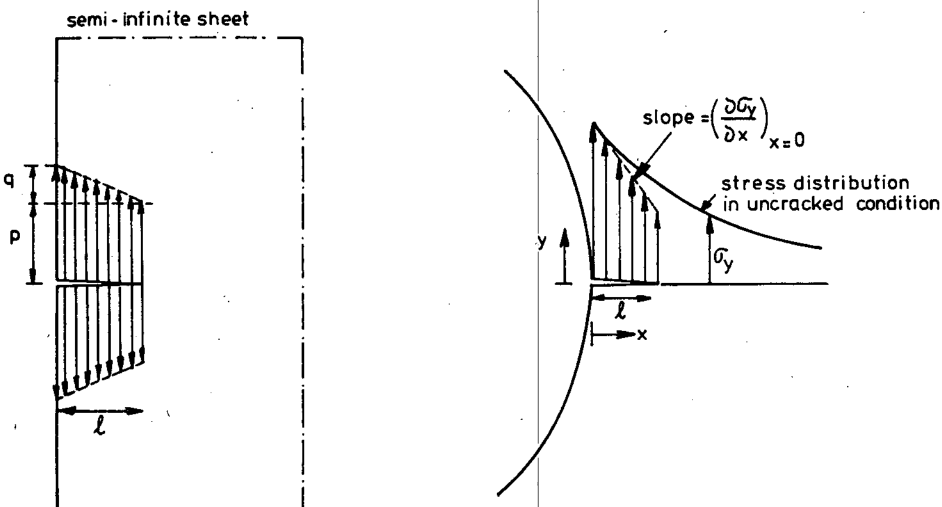


Figure 2: C-values for different ellipses in an infinite sheet loaded in tension.



a. Problem solved by Benthem and Koiter [3].

b. Analogous situation for a small edge crack at a notch according to Karlsson and Bäcklund [4].

Figure 3: Two approximately similar cases of edge cracks.

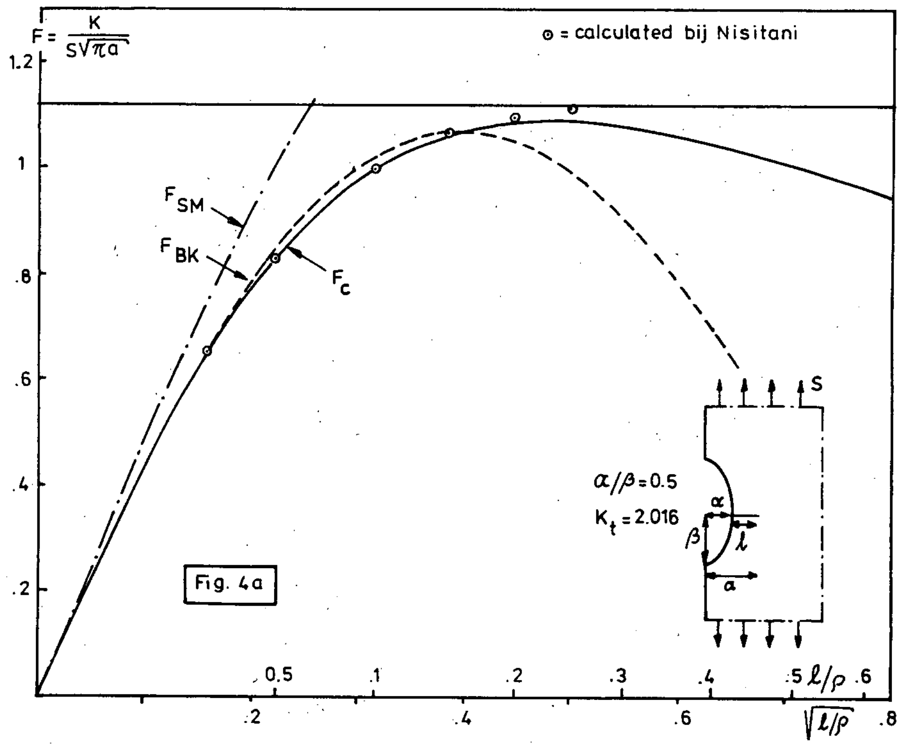


Figure 4: Elliptical edge notch in semi-infinite sheet. Comparison between data of Nisitani 6 and approximations.

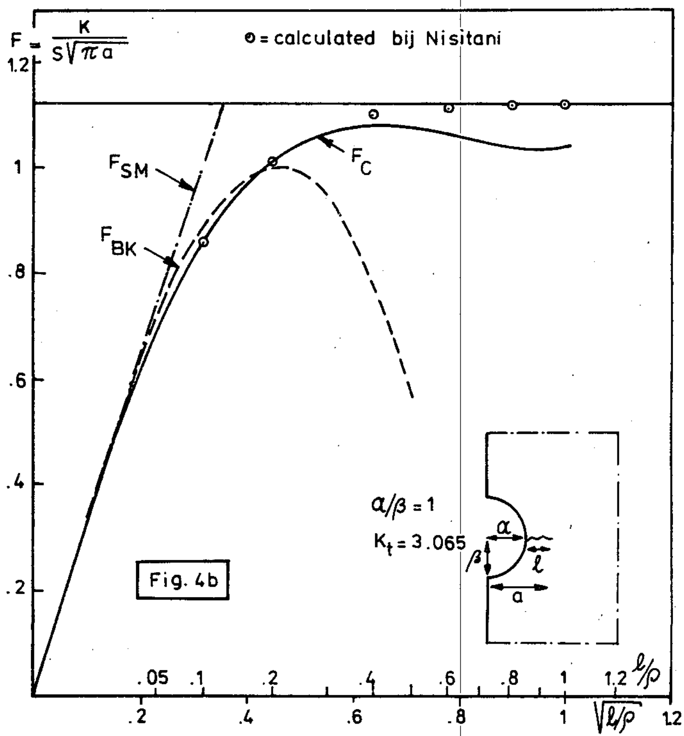


Figure 4 (continued)

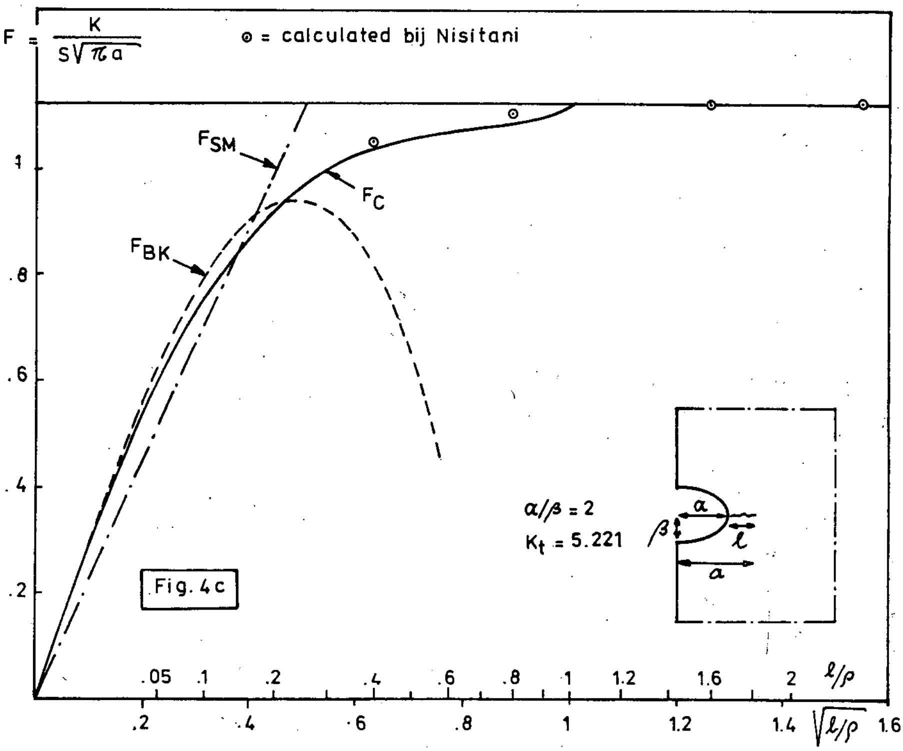


Figure 4 (continued)

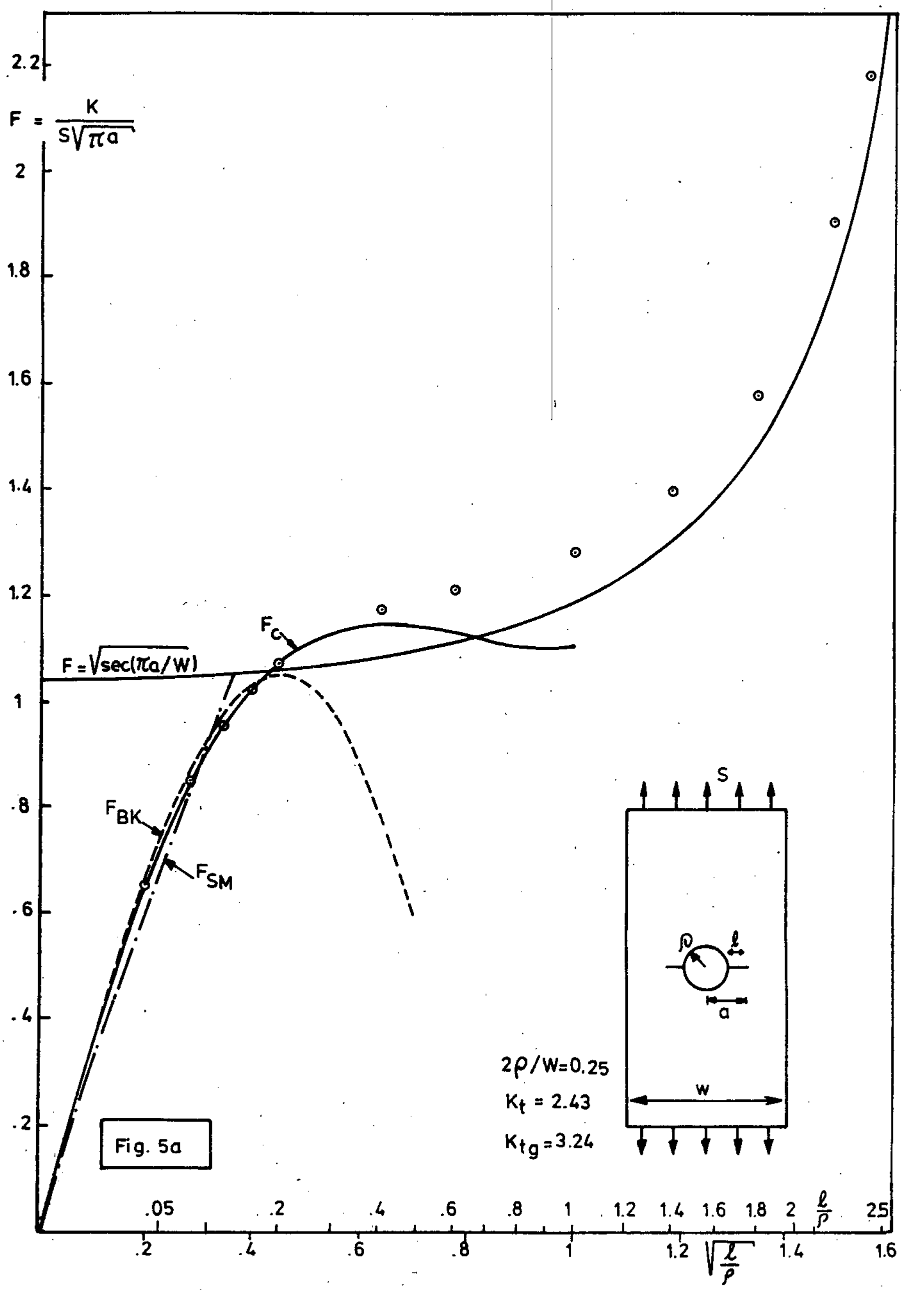
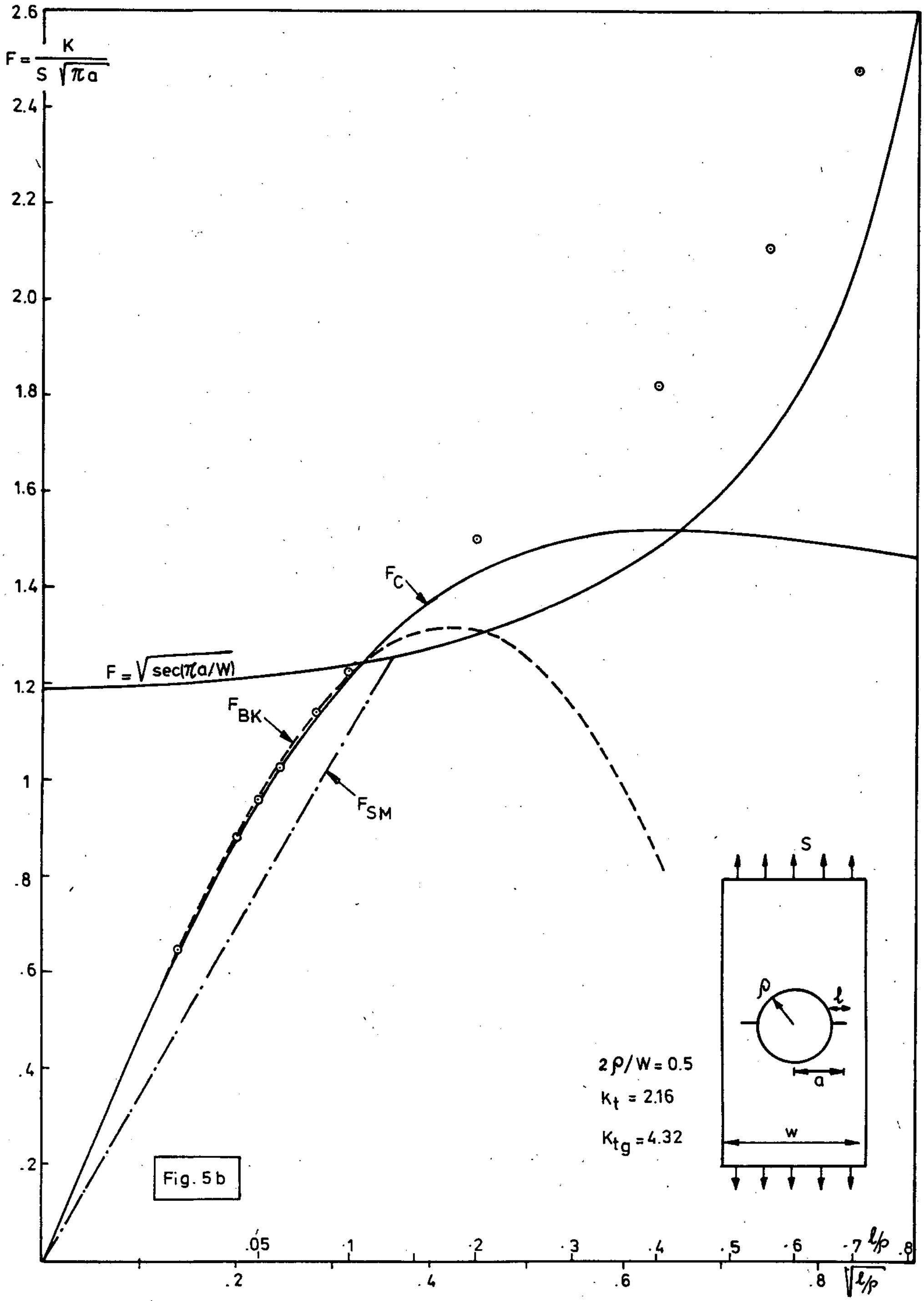


Figure 5: Central hole in finite width strip. Comparison between data of Newman [5] and approximations.



$F = \frac{K}{S\sqrt{\pi a}}$
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1
.8
.6
.4
.2
F_C
$F = \sqrt{\sec(\pi a/W)}$
F_{BK}
F_{SM}
S
ρ
ℓ
a
w
$2\rho/W = 0.5$
$K_t = 2.16$
$K_{tg} = 4.32$
.05
.1
.2
.3
.4
.5
.6
.7
.8
ℓ/ρ
.2
.4
.6
.8
$\sqrt{\ell/\rho}$

Fig. 5b

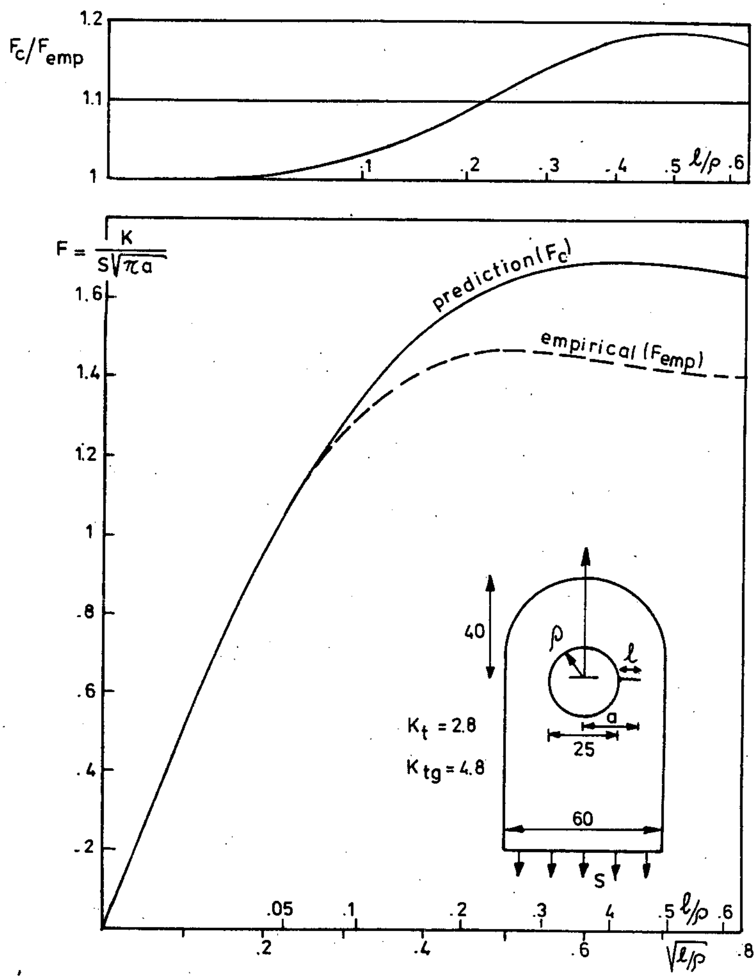


Figure 6: Lug specimen with loaded hole. Comparison between data of [9] and approximation.