Dancing by numbers

In search of the perfect rotation

BY MAARTEN VAN DER SANDEN

Try as we may, art and science appear to be mutually exclusive. Exact knowledge and ostensibly emotionless arguments don't seem to mix with fiction and feelings. Even so, there are moments when the two come very close, and even cooperate. For some years, Dr Johan Mebius, assistant professor in mathematics and information technology at TU Delft has been researching mathematical formulae to describe the perfect rotation for choreographic systems. Existing computer programs for dance and ballet lack smooth rotation sequences. Transitions from the one rotation to the other tend to look angular in these systems. Gripped by the beauty of his favourite pastime, dancing, Dr Mebius decided to find out in what way the art of dancing could benefit from mathematics. A smart combination of mathematical theories enabled him to develop ARTIBODIES, a computer program that choreographers can use to compose dances on an ordinary personal computer.

The kinematics of solid bodies is not really a terribly complicated subject. Whatever the movement of a solid body in space, at any instant it is a translation in some direction, or a rotation about some axis, or a helical motion along some axis with some pitch. When looking at a time interval rather than at a single instant, one observes that an arbitrary displacement of a solid body from a to b can still always be performed by a single translation or a single rotation or a single helical motion. That is the simplest but not the only way to get from a to b. One has innumerable possibilities to get from a to b: that is the main reason why motion can be made into the art of dance. Mebius: 'If you master each of these steps separately, as well as any combinations of them, you know everything there is to know about 3D kinematics. That includes not only the net displacement from a to b, but also the actual motion.' Computers as design aids have not yet taken hold in the world of choreography. But one of the few systems available - the Canadian program Life Forms – happens to be used by the world's veteran choreographers Merce Cunningham (b. 1919 Centralia, Wa., usa). According to Mebius though, the problem with Life Forms is that it doesn't include smooth rotations. Life Forms seems to be limited to fixed-speed rotations around fixed axes. Mebius: 'I was convinced I could do better than that. At the same time, this was the first part of the rotation problem that I wanted to solve, i.e. the angularity of the movements. Other systems exist that use goniometric equations, which do manage to properly display rotation, but those programs would be too slow on an average PC. That was the second part of the problem. The system I've developed, ARTIBODIES, will work on an ordinary off-the-shelf computer, which makes it affordable to many more dance groups. In addition, it features lifelike rotations.'

Petrushka

The love of dancing came early to the Delft scientist. Mebius: 'I can't remember exactly when it was, but my first experience may well have been a performance of the Dutch National Ballet dancing Petrushka, which I attended with my school. I loved it, I was really moved.' Almost at the same time, the young Mebius was introduced to the world of numbers, pure mathematics, algebra, and geometry. 'My old schoolmaster unveiled the secrets of algebra to me in the sixth form of primary school,' Mebius recalls, `when he took out a three-dimensional model and showed us what a plus b to the power of three looked like. That was another thing that made a deep impression on me as a child.' At secondary school his interest in mathematics was stimulated to the point where he decided to read mathematics at Utrecht University. 'I completely immersed myself in pure mathematics,' Mebius recalls. 'My fascination with the purely theoretical side of it was such that I almost ignored the need for practical applications.' This changed when Mebius saw John Travolta dance in the 1983 film, «Staying Alive». 'I saw how those people in the film were having the time of their lives. I thought it was brilliant, a modern kind of Petrushka. Shortly afterwards, I decided to take dancing lessons at the International Dance Centre in The Hague, which was run by Marjolein Briër en Marlène Maharay, both well-know in dance circles. Funny, the way things work out. First, you are gripped by the beauty of a dance performance, then you start to like mathematics, and before you know it, you're completely engrossed in pure theoretical mathematics. A little later, you come into contact with dance again, your old love flares up again, and so you combine the two. I managed to combine my love for movement with the mathematical description of those movements, and of ballet movements in particular.'

Quaternion algebra

If a body turns to the right and then bends, none of the current software offerings manage to turn it into a smooth movement. Rotations in computer games appear smooth as a result of the speed at which they are displayed. This prevents the viewer from noticing any irregularities in the motion. A computer choreographer on the other hand, needs to look at movements in detail, and doesn't need high-speed images. At lower speeds however, angularity of movement tends to show up more clearly. According to Mebius, this is a problem that can be solved by the use of quaternions. The problem with using rotation matrices is that it involves lots of calculations. Multiplying a 3D matrix with another 3D matrix takes 27 multiplications and 18 additions. When hundreds of steps are involved, it all adds up to a lot of calculations. The number of motion calculations can be halved by representing the movements using what is known as quaternion algebra (see text in box).

Quaternion algebra can dispense with complicated cosine and sine calculations, and instead uses simple additions, subtractions, multiplications, and an occasional root extraction. Using this method, the number of calculations works out at 16 multiplications and 12 additions. Mebius: ‘Another drawback of the original matrix calculation method is that extended movements tend to cause the subject to be flattened or elongated. This doesn’t happen with the quaternion technique, which is guaranteed to produce on-screen movements that retain at least the original shape.’

Philippine Wine Dance

In 1985, Mebius got the use of a PC on which he could produce simple images. The first images of rotating bodies appeared on his display some two years later. ‘In 1991 I had the idea of using the quaternion representation technique for the Philippine Wine Dance’, Mebius says, ‘which is a dance in which the dancers rotate a hand holding a glass of wine under their own arm without spilling the wine. Initially, I wanted to write a program for just a single arm. I later added the rest of the dancer’s body, with the arm acting as one of its components. In the end, it took me exactly one month to finish a prototype.’ In the end, Mebius wasn’t entirely satisfied with the Philippine Wine Dance simulation. ‘Although it enabled me to create a reasonable rotation on a simple computer, the motion was still too angular. I had solved only one half of the problem.’ The other half of the quest for the perfect rotation has to do with the fact that the classic interpolation techniques don’t work in 3D rotation space. The classic techniques are all based on a two-dimensional space and linear algebra. Since the beginning of computer graphics, practical and theoretical work has been done on computer animations. Since 1985, a number of techniques have been available for interpolation in 3D rotation space, but these are all based on the logarithmic and exponential functions from Lie algebra and group theory. Mebius: ‘Unless you can use the fastest computers available, those interpolation techniques won’t get you very far, the load imposed by the calculations is simply too vast.’

Cartoons

So the task was to find a way to create smooth motion using quaternions, translations, and rotations. The angular motion problem is one that also crops up in the cartoon industry. Cartoon characters have a habit of moving in jerks. Mebius: ‘In cartoons, this problem is solved by using key-frame animations. This involves plotting the various critical points in the movement into separate images, so-called key frames, and then altering the positions frame by frame. In cartoons, this is the first rough draft of the storyboard, and in dance and ballet it is the first draft of the choreographer.’ Cartoon makers use so-called «in-betweenings» between the key frames. Replay the in-betweenings and the key frames at high speed, and you have your finished animation. Mebius: ‘If you were to use the key frames on a computer as they are, you’d end up with jerky movements. You would clearly see the points at which the movement enters the next stage in the key frame. The end of a rotation, followed by a bending over, for example, does not result in a nice, smooth movement. The result looks unnatural, so you need to add a final correction to the frames to eliminate the abruptness.'

Corner rounding
Mebius also uses key frames to produce a smooth rotation on the computer. In his ARTIBODIES software, he smoothens the transition from one frame to another by removing the corner from an abrupt transition. According to mathematicians, quaternions are indispensable for this technique, which is known as «corner rounding». 'Your average PC is quite capable of performing these calculations,' Mebius says, 'which makes the process suitable for use in ballet and dance, since powerful computers cost too much money. At last we have what we were looking for. The perfect rotation has come within reach of the ordinary PC.' Even so, the system can also be used for other purposes than to visualise ballet and dance: ergonomic simulations for instance. Mebius: 'I think I'll stick to the ballet side of things. What I miss in computerised ballet are things like facial expressions, the expression of the dance, fire and emotion. I don't think people will ever go into raptures over an image on a PC display, except perhaps for the odd computer nerd. A real ballet is something different altogether. Even so, if sums can become dancing and a smooth pirouette can be caught in an algorithm, we might be able to dance using numbers, and mathematics might become art.'

For more information, please contact
Dr. J. E. Mebius,
phone +31 15 278 3072,
e-mail j.e.mebius@its.tudelft.nl