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## ORIGINAL ARTICLE OPEN ACCESS

# Risk-Aware Updating of Reliability Standards for Flood Defences

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## ABSTRACT

Objective of this paper is to study how reliability standards, expressed as probabilities of dike segment failure, can be practically updated to improve opportunities for risk-based dike design and planning. The approach to assess the economic optimal flood probability, used by the Dutch Delta Committee (1958, in this paper referred to as Van Dantzig), is adapted to reflect time-dependent effects of a.o. climate change and subsidence. Furthermore, the approach is adapted to reflect overtopping instead of overflow and it is extended to include reinforcements over time. A comparison of the results of the Adapted Van Dantzig approach with the economic optimal probabilities used as input for the recently formalised Dutch standards (2017) is performed for 73 dike segments in the Netherlands, showing good agreement. Following the Adapted Van Dantzig approach, an analytical relation is developed for economic optimal design horizons, dependent on the dike design, and characteristics of load, investment, climate effect, and economic growth. Finally, a dynamic and simple-to-use approach is developed to enable updating of the economic optimal reliability based on a proposed design and investment planning. This can serve to consider whether an existing reliability standard still fits adequately or needs updating.

## 1 | Introduction

The European Union established the Floods Directive 2007/60/EC (EU 2007), meant to guide the member states in their flood risk management, and to stimulate them to manage their flood risks based on the same rationale: to map risks, plan and take measures, and monitor. Nevertheless, despite the Floods Directive stimulated the application of quantitative approaches, still differences in risk approaches are present (Sayers 2017; Vonk et al. 2020). Managing their flood defences, different countries use different approaches for standardisation and performance assessment (CIRIA 2013; Klerk 2022; Vonk et al. 2020).

Risk management is a part of mature asset management. The three dimensions of risk management capabilities as presented in (Poljansek et al. 2019) are technical, financial and administrative.

A part of the administrative capability is the formulation of policies and strategies. A practical utilisation of a flood risk management strategy is the definition of a reliability standard, which can be used for performance analyses to decide on interventions.

For interventions on flood defences several organisations cooperate to initiate, budget, design and prepare and to maintain. Standardisation delivers the reason for the involved organisations to invest when and where, in a complex portfolio of flood risk reducing assets.

## 2 | Reliability Standards in the Netherlands

In the Netherlands the flood risk standards are introduced in 1956 by van Dantzig (1956), after the disaster in the southern

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part of the country in 1953. It is the first known quantitative risk-based derivation of an economic optimal safety level for flood defences. Herein, the probability of an undesired flood event is based on water level exceedance frequencies, and the consequences of flooding are based on complete economic loss in the polder. Being quantitatively derived for the western part of the Netherlands which is prone to sea floods, the standards were qualitatively extended to other parts of the country, depending on the character of the threat (rivers, estuaries, sea), and the consequences at risk. These water level based standards, are established by law in 1996 (Ministerie van Verkeer en Waterstaat 1996).

Due to sea level rise and economic growth, flood risks are time dependent. In 2017 the standards are updated, expressed as acceptability-limits for the probability of failure of a dike segment per year. To derive economic optimal safety levels, the risks are based on the failure mechanism wave overtopping, and on calculations for the extension of floods and their consequences (Eijgenraam et al. 2017; Kind 2014). The costs for reinforcement take into account a standard dike shape, which is assumed to be sufficient to withstand geotechnical failure mechanisms.

Next to the economic optimal safety level, in the Netherlands the acceptable individual and group risks on victims are used to choose the standards (Kok et al. 2017). Despite the economic optimal reliability is time-dependent (Kind 2014), the standards are not dynamic in the Dutch law (Ministerie van Infrastructuur en Milieu 2016).

### 3 | The Need

Given flood defence management use standardisation expressed as probabilities of failure, the need for updating of reliability standards is threefold. Firstly, the type of dike construction affects the risk (den Heijer and Kok 2022), and therewith it affects the risk-optimal design. A dike with a clay core causes delay in the breach process and breach dimensions will be reduced with respect to a dike with a sand core. This reduces the flood volume and consequently the consequences of flooding. The present application of the National database of Flood Simulations (Helpdesk Water 2020) can serve as an example in the Dutch context. The breach-widths most likely correspond with breaches in sand dikes (den Heijer 2025), named as brittle in (den Heijer and Kok 2022). Therewith, the consequences in the database are overestimated in case of assessment of ductile dikes with a behaviour like clay dikes, affecting the optimal design probability.

Secondly, the economic optimal design reliability is ageing, as demonstrated in (Eijgenraam et al. 2014; Kind 2014). Therewith, in case the planning of an reinforcement shifts considerably with respect to the year for which the standard is derived, for example, due to availability of resources, the design reliability needs to be updated.

Thirdly, new or updated or rectified information may come available, which affect the optimal design probabilities of failure.

## 4 | Knowledge Gap

Given a reliability standard is in place, expressed as the probability of flooding, despite these are risk-based as is the case in the Netherlands, it is a challenge to keep focus on risk-aware decisions for reinforcements of individual dikes and systems. Especially the measures which focus on reducing consequences are prone to be dropped or even to be not considered, because these measures as such do not satisfy the standard. Nevertheless, benefits are in place, because the economic optimal safety level depends on the construction (den Heijer and Kok 2022) and order and planning of reinforcements in system (den Heijer and Kok 2024). However, there is no existing method to update or adjust a reliability standard dependent on a reinforcement proposal, taking into account the effect on consequences.

The gap addressed in this paper is to develop a risk-based method to derive or update flood defence reliability standards, for a variety of measures which both increase reliability of the flood defence and reduce consequences in case of flooding. The challenge is to use the reliability standards in a way to benefit from consequence reducing measures, while keeping the operational context, to avoid formal or juridical discussions about protection levels, additional extensive calculations, and other practical problems which would be cumbersome to overcome.

## 5 | Objective and Approach

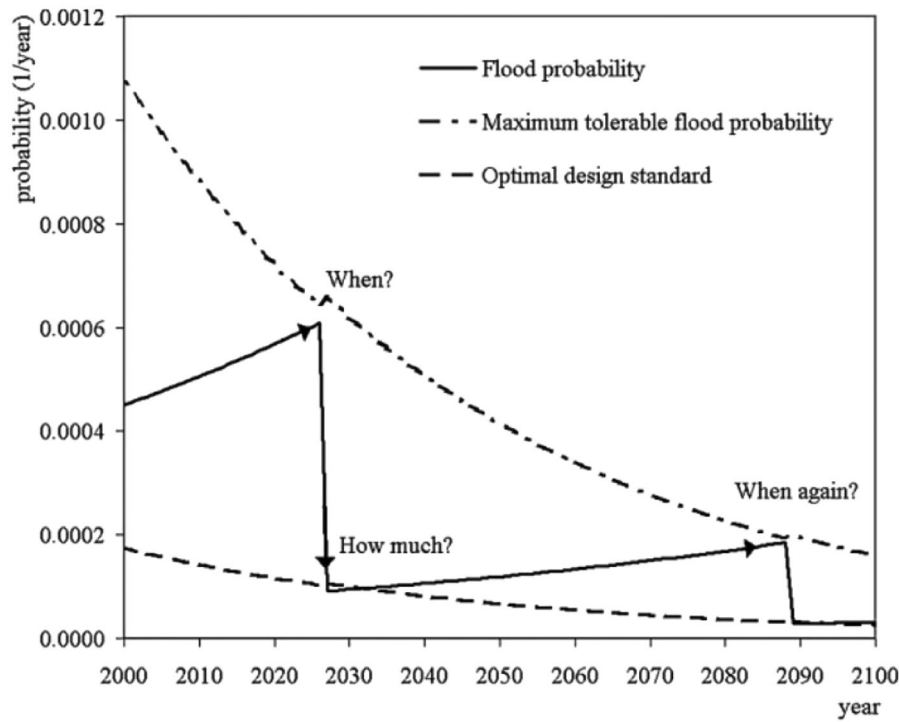
The objective of this paper is to study how reliability standards can be risk-aware updated to obtain better opportunities for risk-based dike designs. The hypothesis is that the simple formula of Van Dantzig, adopted by the first Dutch Delta Committee (Deltacommissie 1960), which is easy to use in an operational context, can be adapted to meet this objective.

The approach consists of several steps. Firstly, a time-dynamic component is added to Van Dantzig's formula, and it is adapted to consider the failure mechanism wave overtopping instead of overflow. Secondly, the results of application for the flood defences in the Netherlands are compared with the time-dynamic approach, derived based on the advice of the second Delta Committee (Delta Commision 2008), which has been used for the present Dutch standards established in 2017. An analytical relation is developed for economic optimal design horizons, dependent on the dike design. Finally, a dynamic and simple to use approach is developed to enable updating of the economic optimal design reliability based on a proposed design and planning.

## 6 | Time Dependent Optimal Reliability—Adapting Van Dantzig's Formula

The Delta Committee (Deltacommissie 1960; van Dantzig 1956) developed an approach to determine the economically optimal safety standard as the sum of present value of risks and investments:

$$C_{tot} = I + R^{PV} \quad (1)$$



**FIGURE 1** | Saw-tooth pattern for the probabilities of flooding including upper and lower limits (Source: Kind (2014)).

with  $C_{tot}$  the total societal costs,  $I$  the investment costs and  $R^{PV}$  the present value of risks. In (Jonkman et al. 2016) is illustrated that, considering only the failure mechanism overflow and assuming an exponential water level distribution, this leads to a relatively simple equation for the optimal safety standard by minimising the total societal costs  $C_{tot}$ :

$$P_{f_{opt}} = \frac{I' B r}{D} \quad (2)$$

with  $P_{f_{opt}}$  the economic optimal probability of flooding,  $I'$  the marginal reinforcement costs,  $B$  the scale parameter of the exponential water level distribution,  $r$  the discount rate and  $D$  the economic damage in the polder of interest, given a flooding due to dike failure. However, herein the time dependence due to economic growth, climate change and deterioration is not expressed. Therefore, in Kind (2014), the optimal safety standard for the dike stretches in a series of dike segments is modeled as an optimization approach of total costs, summing the time series of intervention costs and the economic risks of flooding. This resulted in a saw-tooth pattern for the probabilities of flooding over time, presented in Figure 1. The solid curve reflects the actual flood probability of an deteriorating dike segment. Furthermore, it shows an upper limit curve (dotted), which determines the economically optimal time of intervention at the point in time when the solid line intersects it, and a lower limit curve (dashed), determining the optimal design probabilities.

The sudden reductions of the probabilities in the solid curve reflect the interventions (when, how much, when again) for a dike segment. In fact, two risk-based assessments are performed. First, to decide whether a reinforcement is economically optimal, if in Figure 1 the solid line intersects the dotted line. Second, in case the decision is made to strengthen, to decide

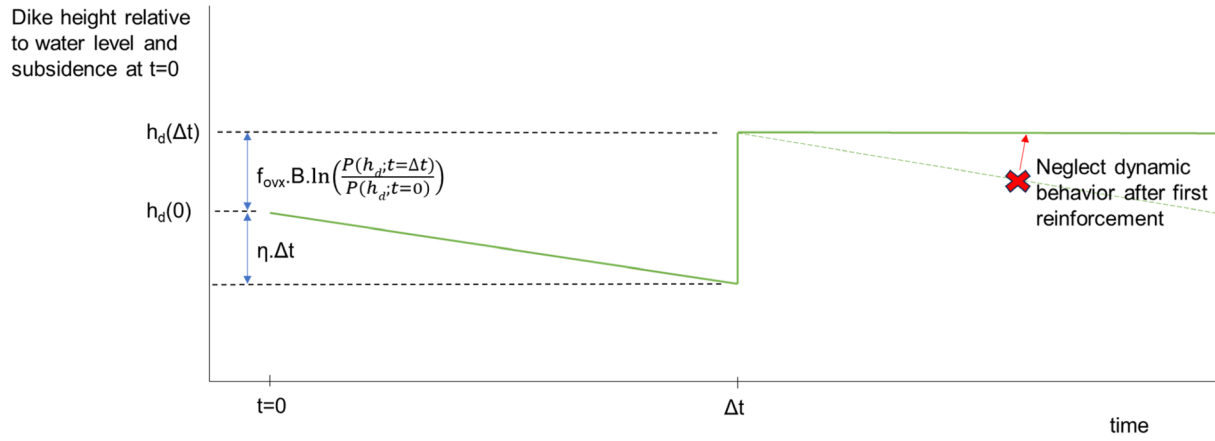
about the optimal safety requirements for strengthening, in Figure 1 represented by the dashed line.

In the Netherlands the middle probability, the average between upper and lower curve, is proposed as a representative value for standardisation. In (Kind 2014) a linear relationship is observed between the middle probabilities and the ratio of damage and cost to decrease the probability tenfold. Appendix 1 underpins this observation is in line with the formula of Van Dantzig in Equation (2). This raises the question whether Van Dantzig can be adapted to reflect the time dynamic effects. This would provide practical benefits for standardisation, because an analytical relation provides the results without the need of extensive numerical calculations which are needed to solve the equations in Eijgenraam et al. (2014) and Kind (2014). In the next subsections first the lower limit and upper limit are derived. A comparison is made with available data in Kind (2014). Then, the design horizon is derived, with the economic optimal horizon as a special case.

### 6.1 | Lower Limit of Economic Optimal Failure Probabilities

The approach of van Dantzig (1956) is adapted to enable a comparison with the lower limit in Kind (2014). Firstly, we introduce a shift over time in the exponential distribution of exceedance of water levels due to relative deterioration  $\eta$  per unity of time representing subsidence and climate change effect:

$$P_h(t) = \exp\left(-\frac{h - (A + \eta t)}{B}\right) \quad (3)$$



**FIGURE 2** | Schematic representation of the relative dike height  $h_d$  in time.

with  $h$  the water level,  $A$  and  $B$  the parameters of its exponential distribution and  $t$  the time. Note, this distribution is valid for  $A < h - \eta t$  to make sure that  $P_h(t) < 1$ .

Secondly, we consider the failure mechanism wave overtopping instead of overflow. The dike height is denoted by  $h_d$  corresponding to a probability  $P_f$  to be overtopped by a discharge exceeding a critical volume  $ovx$  per metre per second. The probability distribution of dike heights is assumed to follow an exponential distribution, shifting over time just as the water levels. In deviation from Equation (2) the factor  $B$  now refers to the required dike height. Therefore, the parameter  $B$  based on water levels is increased with a factor  $f_{ovx}$ . The dike height increase over time is assumed to increase proportionate with the water level increase  $\eta$ . Therewith the exponential distribution of exceedance of dike heights is:

$$P_f(t) = \exp\left(-\frac{h_d - (A + \eta t)}{f_{ovx} B}\right) \quad (4)$$

Since the theoretical lowest dike height design  $h_d$  would be based on the failure mechanism overflow, the lower limit of  $f_{ovx} = 1$  which means the dike heights are based on the water level distribution as used in Equation (2). In fact, there is no theoretical upper limit for  $f_{ovx}$ . Some practical considerations and results based on elaborative calculations are provided in Appendix 2, leading to a provisional upper limit for  $f_{ovx}$  of about 4 for exposed locations.

Thirdly, some practical starting points are used for the comparison. We optimize per dike segment as a whole, thus neglecting the differences of loads, strength and consequences within a dike segment. The assumption is that this will only affect the result marginally, in case the dike and the hydraulic loads do not change that much along the dike segment. Furthermore, we determine the timing of the first reinforcement to come at  $t = \Delta t$ , and neglect changes after the first reinforcement. This means that the dynamic effects on risks are neglected after  $t = \Delta t$ , see Figure 2.

Therewith, in Appendix 3 the probability of flooding is derived corresponding with the time dependent economic

optimal design, taking the economic damage  $D_\delta(\Delta t)$  equal to  $D(0) \cdot (1 + \delta)^{\Delta t}$ :

$$P_{f_{opt}}(\Delta t) = \frac{I' B r}{D(0)} \cdot \frac{f_I \cdot f_{ovx}}{(1 + \delta)^{\Delta t}} \quad (5)$$

with  $D(0)$  the damage of a flooding at the start year of the analysis,  $f_I$  the parameter indicating the reinforcement cost increase more than proportionate with dike height increase, and  $\delta$  the economic growth. This equation looks quite alike the original Van Dantzig formula in Equation (2). For  $\Delta t = 0$  and  $f_I = 1$  the second part of the formula after the equal sign is  $f_{ovx}$ , which means that the transformation to a dike height scale parameter is the only difference with the original formula of Van Dantzig.

## 6.2 | Upper Limit of Economic Optimal Failure Probabilities

To determine the upper limit of the probability of failure in time, the utility criterion in Deltacommissie (1960) and Jonkman et al. (2016) is used: an intervention at time  $\Delta t$  is economic beneficial if the economic risk reduction transcends the investments:

$$\Delta R^{PV}(\Delta t) - I^{PV}(\Delta t) > 0 \quad (6)$$

with  $\Delta R^{PV}(\Delta t)$  the present value of the risk difference before and after a reinforcement at time  $\Delta t$  with a present value of the cost  $I^{PV}(\Delta t)$ . The risk reduction is mainly determined by the probability reduction. This is the difference between the actual flood probability before a reinforcement and the reliability target for a reinforcement  $P_{f_{opt}}(\Delta t)$ , here generally denoted by  $P_{standard}$ . The assumption that the dike height after reinforcement meets the reliability target  $P_{standard}$  provides the reinforcement height  $\Delta h_d$ , including the relative decrease of dike height due to subsidence and climate change (see Figure 2):

$$\Delta h_d = \eta \Delta t + f_{ovx} B \cdot \ln\left(\frac{P_f(t=0)}{P_{standard}}\right) \quad (7)$$



Therewith, the probability  $P_f(\Delta t^-)$  just before intervention can be derived (see Appendix 3):

$$P_f(\Delta t^-) = f_{\Delta h_d} \cdot \left( P_{standard} + \frac{f_I \cdot r}{D_\delta(\Delta t)} \cdot \left( I_0 + I' \left( \eta \Delta t + f_{ovx} B \cdot \ln \left( \frac{P_f(0)}{P_{standard}} \right) \right) \right) \right) \quad (8)$$

With  $f_{\Delta h_d}$  a factor larger than 1 indicating the damage after an intervention is larger than before, given a flooding. This equation clearly shows the probability just before intervention is per definition larger than the target design reliability  $P_{standard}$  just after a reinforcement, which corresponds to Figure 1. In fact, Equation (5) provides the target design reliability and Equation (8) provides the target reliability for safety assessment.

### 6.3 | Optimal Horizons

The upper limit in Equation (8) can be used to derive the first beneficial time to intervene, given by:

$$\Delta t = \frac{f_{ovx} B}{\eta} \cdot \ln \left( f_{\Delta h_d} \cdot \frac{P_{f_{opt}}(\Delta t)}{P_f(0)} \cdot \left( \frac{I_0}{I' \cdot f_{ovx} B} + \frac{\eta \Delta t}{f_{ovx} B} + \ln \left( \frac{P_f(0)}{P_{standard}} \right) + \frac{P_{standard}}{P_{f_{opt}}(\Delta t)} \right) \right) \quad (9)$$

Despite the formula is still not completely explicit for  $\Delta t$  it is quickly converging since the  $\Delta t$  at the right side of the

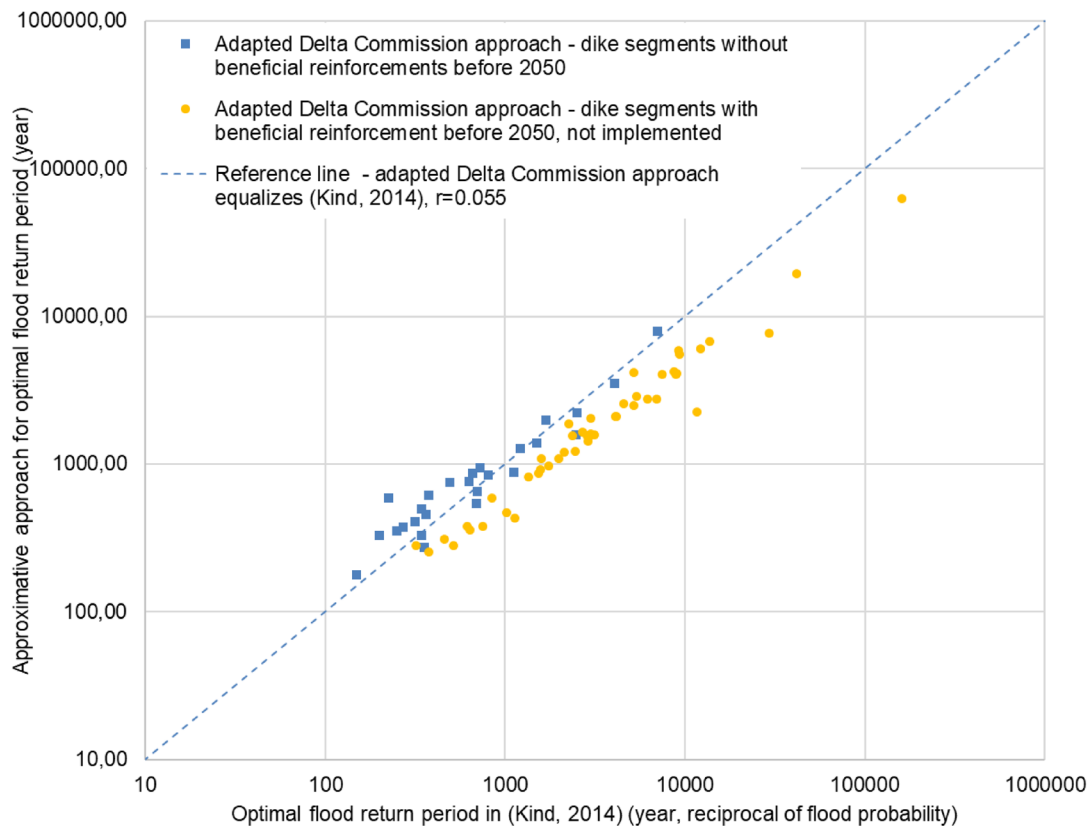
equality sign is under the logarithm sign. In case the first moment  $\Delta t$  that investment is beneficial has already been passed at  $t=0$ , Equation (8) leads to negative values of  $\Delta t$ . Then  $\Delta R^{PV}(0) - I^{PV}(0) > 0$ . This indicates the solid line in Figure 1 is above the dotted one, and it is beneficial to reinforce as soon as possible. Some special cases enable to further simplify Equation (9). A first special case is if the dike is compliant to  $P_{standard}$  at  $t=0$ , because the logarithm of  $P_f(0)/P_{standard}$  becomes zero and will disappear. A second special case emerge as the design probability  $P_{standard}$  is defined as the economic optimal probability, because the last term will become equal to 1. A third special case is as the dike at  $t=0$  is compliant to the economic optimal probability,  $P_{f_{opt}}(0)$ , and the design probability  $P_{standard}$  at time  $\Delta t$  is defined as the economic optimal probability,  $P_{f_{opt}}(\Delta t)$ . This special case is figured out resulting in a remarkable handy equation, which could be simply applied to indicate the economically optimal design horizons:

$$\Delta t = \frac{f_{ovx} B}{\eta + \delta f_{ovx} B} \cdot \ln \left( f_{\Delta h_d} \cdot \left( \frac{I_0}{I' \cdot f_{ovx} B} + \frac{\eta \Delta t}{f_{ovx} B} + \delta \Delta t + 1 \right) \right) \quad (10)$$

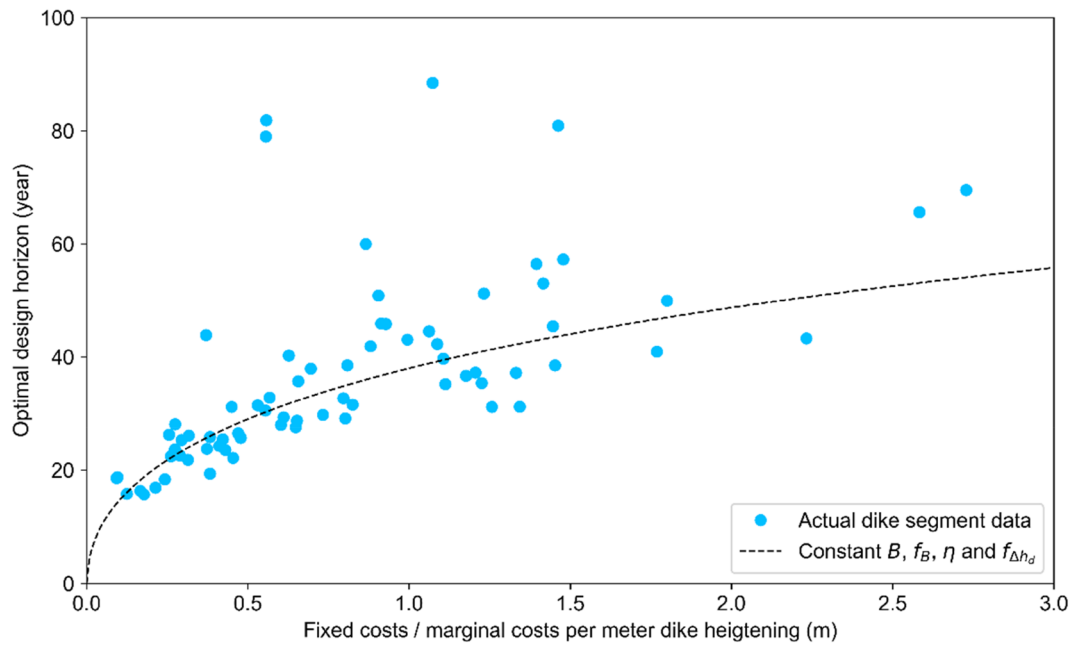
Therewith, the optimal standard for the reinforcement to come can be derived with Equation (5).

## 7 | Application

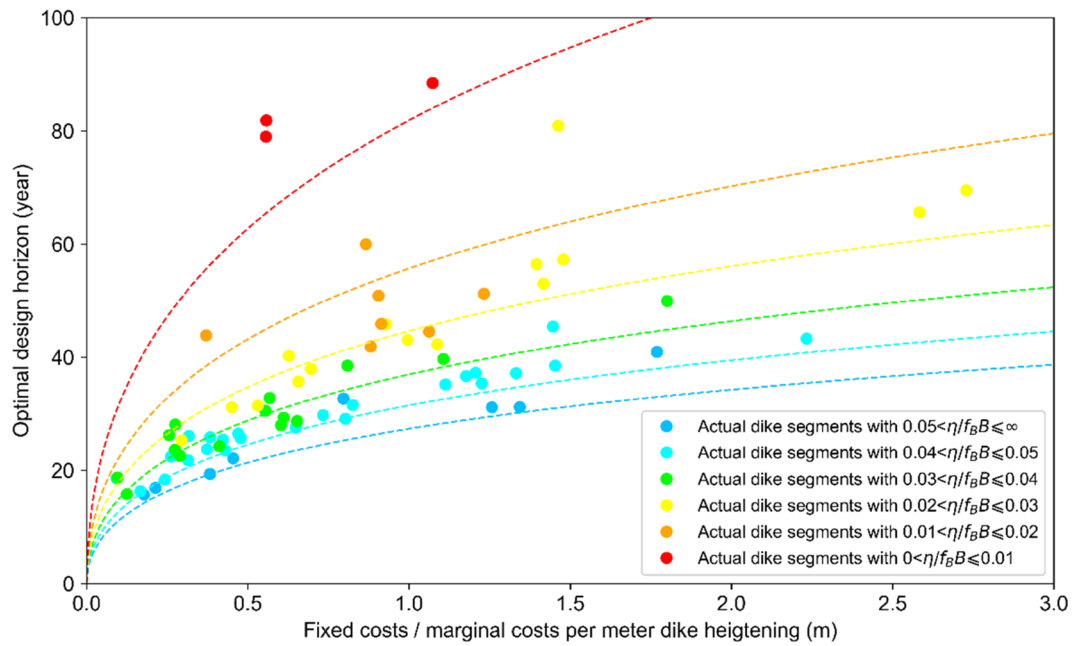
For determination of the lower limit with Equation (5), the upper limit with Equation (8), and the optimal horizon with



**FIGURE 3** | Comparison of the adapted delta commission approach with the results of Kind (2014). The results took no reinforcements into account, despite there being dike segments for which they would be beneficial (orange dots).



**FIGURE 4** | Optimal intervention interval for the dike segments in Kind (2014) (blue dots). The curve is obtained with constant values for  $B$  (0.3 m),  $f_{ovx}$  (1.5),  $\eta$  (0.01 m/year) and  $\zeta$  (0.2).



**FIGURE 5** | Optimal intervention interval for the dike segments in (Kind 2014) divided in classes for  $\eta/(f_{ovx}B)$  (coloured dots). The coloured curves corresponding to these classes are obtained with constant values per class for  $B$ ,  $f_{ovx}$ ,  $\eta$  and  $\zeta$ . In blue, light-blue, green, yellow, orange and red respectively the following values are used for  $B$  (0.38, 0.23, 0.19, 0.17, 0.15, 0.14),  $f_{ovx}$  (1.05, 1.15, 1.25, 1.35, 1.45, 1.55),  $\eta$  (0.0004, 0.004, 0.006, 0.008, 0.01, 0.012) and  $\zeta$  (0.2 for all).

Equation (9) or Equation (10) no additional data is required. In this paper, the dynamic optimization approach with this set of three formulas is referred to as “Adapted Van Dantzig.” The set of equations contains parameters which are all needed for traditional dike design. The comparison between Adapted Van Dantzig and the approach in Kind (2014) is performed for the

results as presented in Kind (2014), with the following starting points:

- For the practical reason of availability of data in Kind (2014), the comparison is carried out for the year 2050, per dike segment, with a yearly economic growth of 1.9% and a discount

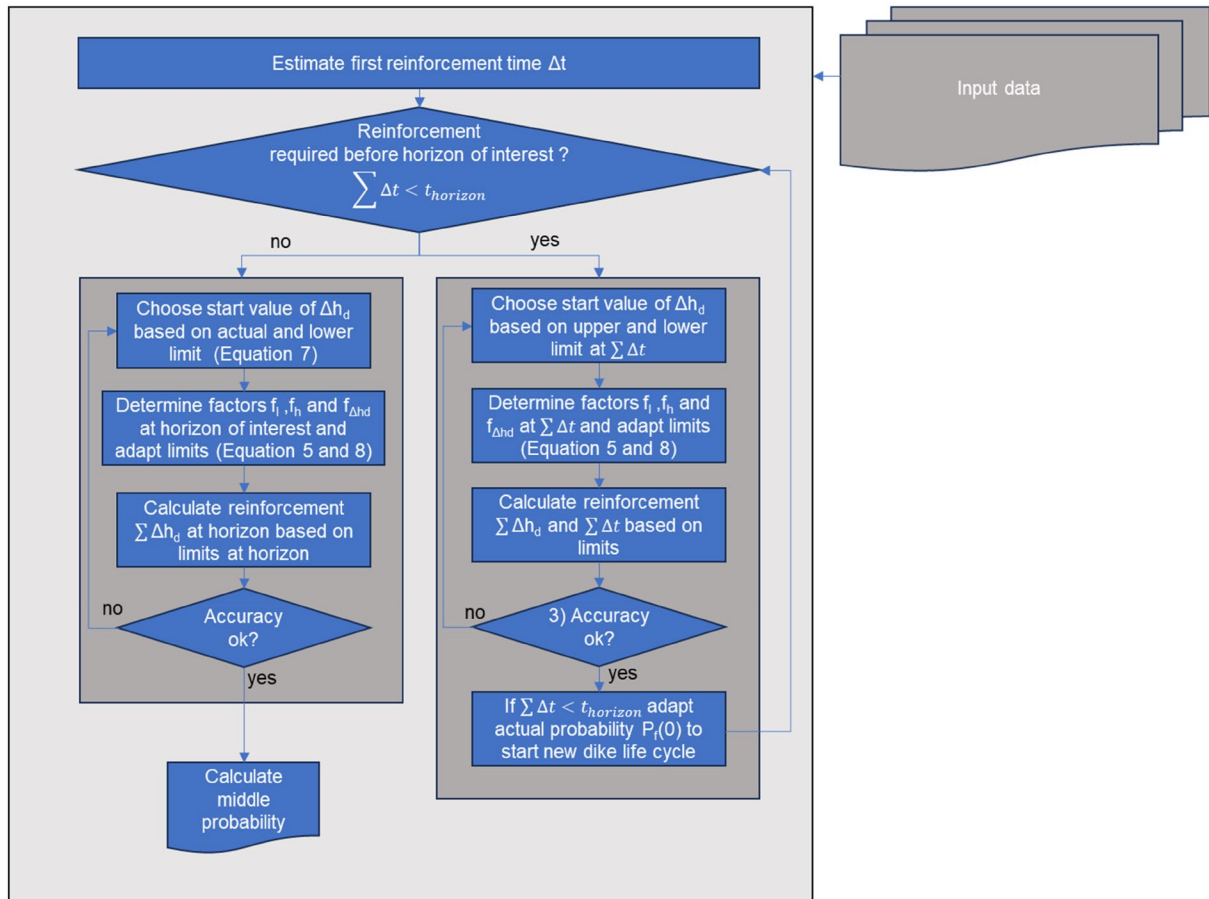
rate of 5.5%. The data consists of 73 dike segments. The listed values for  $I$ ,  $D$  and  $P_{f_{opt}}$  were used. Start year of the analysis is 2011.

- In Kind (2014) the damage is increased over time by two additional factors with respect to Equation (5):  $D_{\delta}(\Delta t) = D(0) \cdot (1 + \delta)^{\Delta t} \cdot f_h(\Delta t) \cdot f_{\Delta h_d}$ . Herein,  $f_h(\Delta t) = \exp(\Psi \eta \Delta t)$  and  $f_{\Delta h_d} = \exp(\zeta \cdot \Delta h_d(\Delta t))$  indicate the extra flood damage due to respectively water level increase and dike height increase over time.
- Additional data used in Kind (2014) is needed for load increase rate  $\eta$ , actual probability of failure  $P_f(0)$ , decimation heights  $h^{10}$  and  $h_d^{10}$ , and damage increase factors due to dike height and water level increase, determined by  $\zeta$  and  $\Psi$  respectively, and data for increase of investments for additional reinforcement height, determined by  $\lambda$ . This data is provided by its author on request, together with additional results for a discount rate of 3.0%. It is available for dike subsegments.
- The dike subsegments are merged to segment level. A dike segment level consists of about 2 or 3 subsegments on an average. The data on dike subsegment level is translated to segment level by a length based average (applied for  $\eta$ , the decimation heights  $h^{10}$  and  $h_d^{10}$ , and the parameters used to determine  $f_I$ ,  $f_h$ , and  $f_{\Delta h_d}$ , see Appendix 3). The actual probability  $P_f(0)$  for a dike segment is based on dependent failure of subsegments.

- The middle probability, presented in Kind (2014) as the end result, is calculated as the mean of upper limit in Equation (8) and the lower limit in Equation (5).

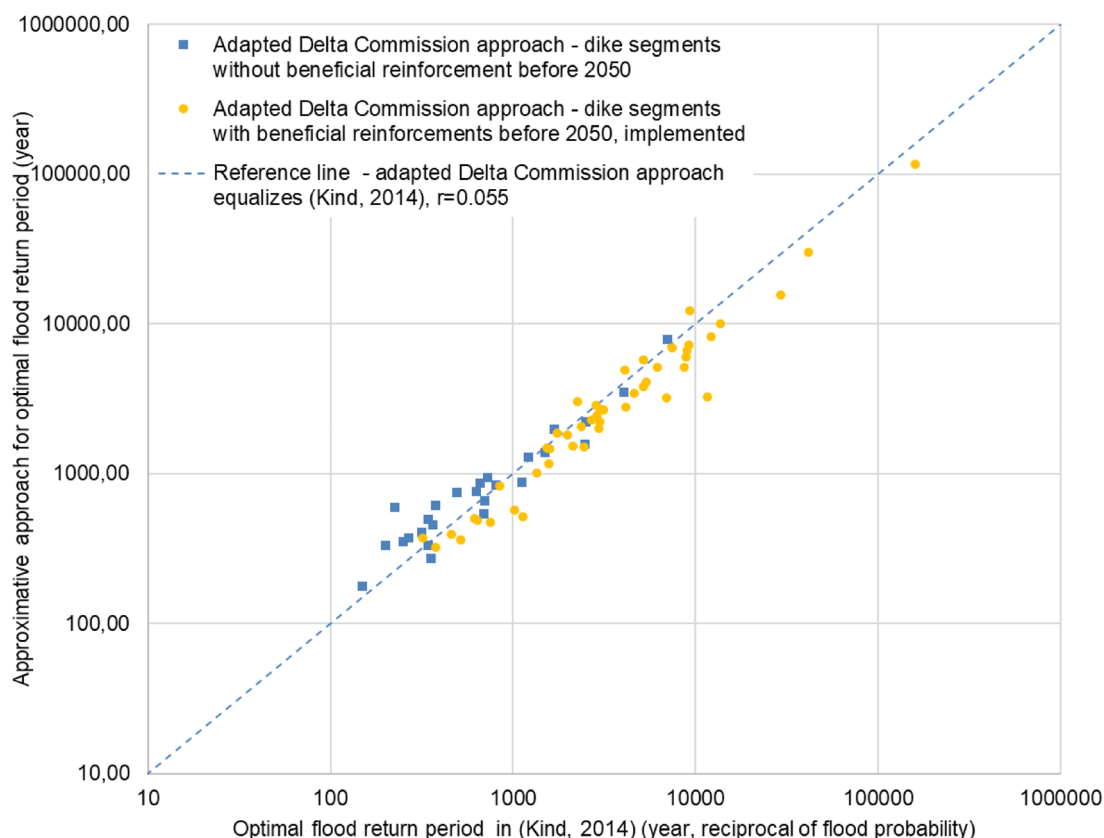
Due to the weak dependence of the factors  $f_I$  and  $f_{\Delta h_d}$  on the dike height increase, the calculation of the upper and lower limits is slightly implicit, requiring some iterations to obtain a stable solution. Figure 3. shows the comparison for the middle probabilities, for the case no reinforcements would have been executed until 2050. Based on the optimal design horizons, estimated with Equation (9), the data is distinguished in a part for which the first beneficial intervention timing is before 2050 (orange dots) and a part for which this is after 2050 (blue dots). The comparison for the dike segments for which the first beneficial intervention timing is beyond 2050, is rather good. For the other dike segments, despite not recorded in Kind (2014), reinforcement would be beneficial before 2050, due to exceedance of the upper limit before 2050. This may cause the orange dots in Figure 3. show a less good agreement. This indicates the relevance to include the effect of reinforcements in the present comparison.

Figure 4, representing the optimal life cycle obtained with Equation (10) and using the same data as used to obtain Figure 3, indicates the life time of reinforcements. A higher ratio between fixed and marginal costs will lead to a longer design horizon. Some data points seem to be outliers, but these are situated in controlled water systems in small lakes, with very small yearly

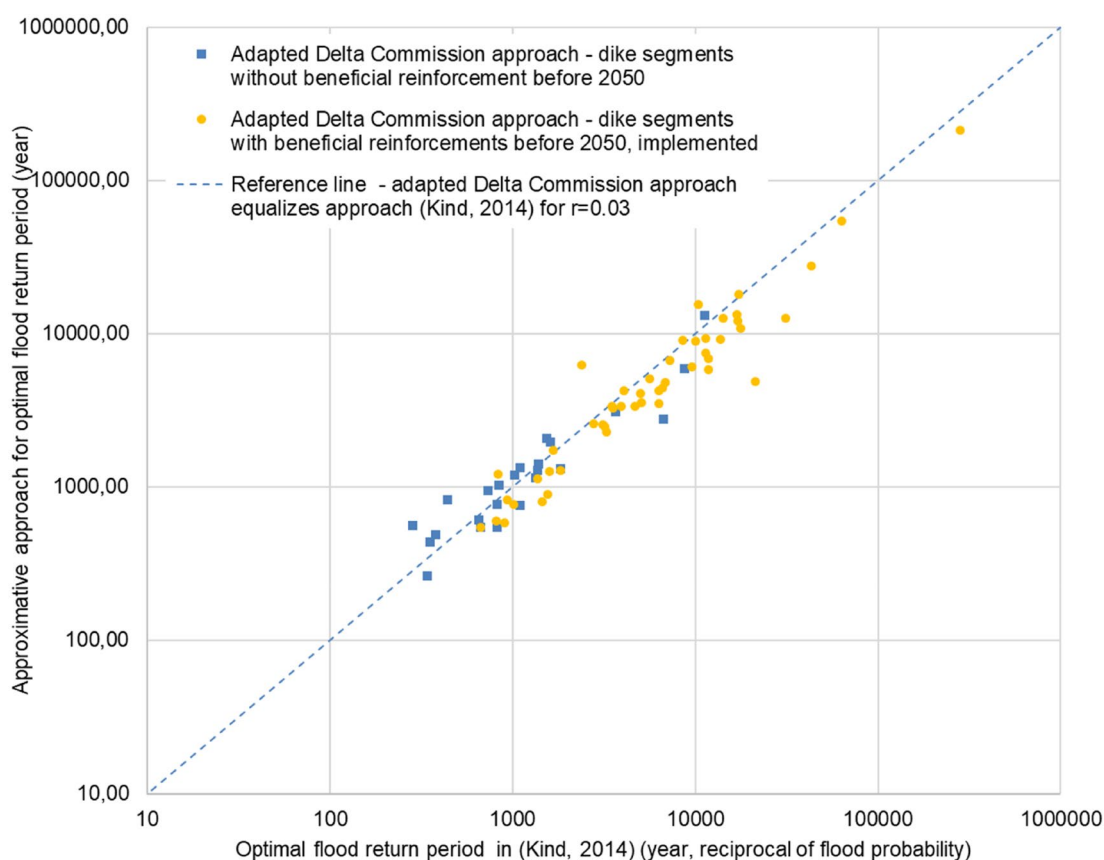


**FIGURE 6** | Flowchart determination of middle probabilities for the Adapted Van Dantzig approach.





**FIGURE 7** | Comparison of the adapted Delta Commission approach with the results of Kind (2014). The results took the reinforcements into account for dike segments for which that would be beneficial (orange dots).



**FIGURE 8** | Comparison of the adapted Delta Commission approach with the results of Kind (2014) derived for  $r = 3\%$ . The results took the reinforcements into account for dike segments for which it would be beneficial (orange dots).

water level increase  $\eta$ . This is demonstrated in Figure 5 in which the dots are divided in classes of  $\eta/(f_{ovx}B)$ , which is the reciprocal of the first term in Equation (9), determining the intervention time. Despite the average equals about 38 years, there are several dike segments with lifetimes of less than 20 years, because of relatively low fixed costs, indicating their reinforcement is beneficial before 2050. This confirms that inclusion of reinforcements

**TABLE 1** | Data for dike segment IJsseldelta for the case ‘safety over time’.

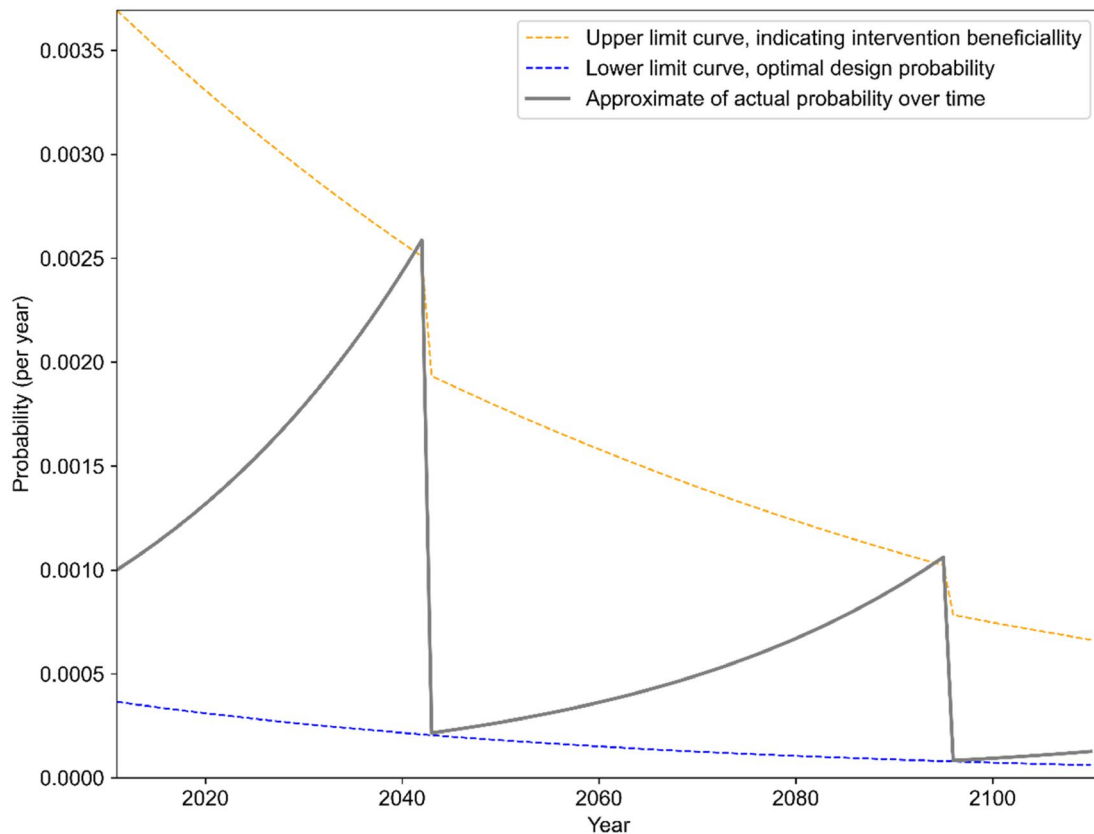
Parameter	Value	Unity
$P_{f,2011}(t=0)$	1/1000	per year
$I$	$71 \times 10^6$	€/m
$I_0$	$128 \times 10^6$	€
$B$	0.12	m
$r$	5.5	%
$\eta$	0.007	m/year
$D(t=0)$	$2477 \times 10^6$	€
$\delta$	0.019	per year
$\lambda$	0.16	per m
$\zeta$	0.088	per m
$\psi$	0.0	per m

is important to improve the comparability of both approaches for the year 2050.

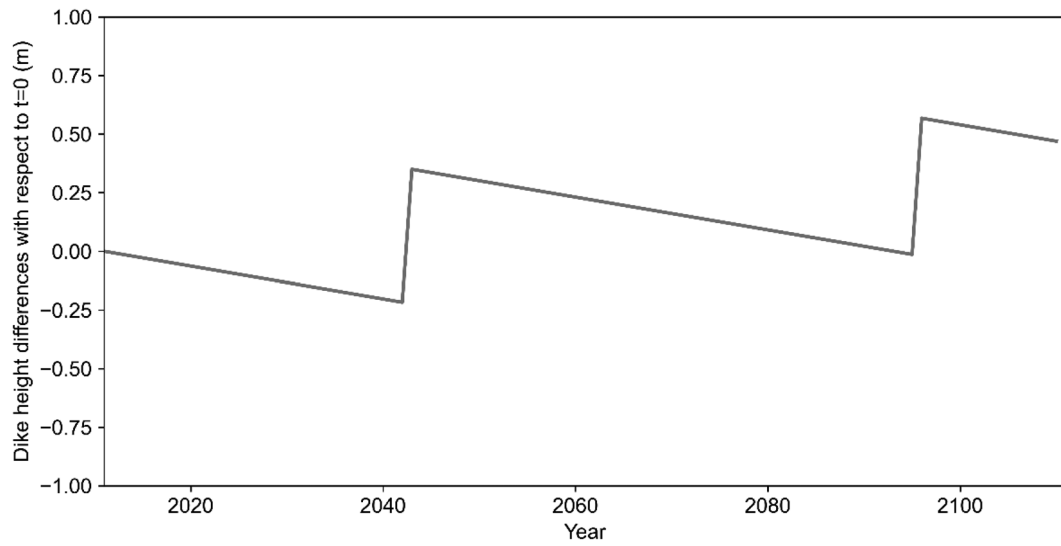
To include the effect of reinforcements the calculation schedule is used as presented in Figure 6. Therein, the “single”- reinforcement concept of this paper is extended to a “multi”- reinforcement concept. Based on a first estimate to find out whether the first reinforcement would be beneficial before the horizon of interest, it is decided to virtually reinforce before the horizon. The grey loop on the right performs a reinforcement. The first beneficial time to intervene at  $t=\Delta t'$  is determined with Equation (9), assuming the reinforced dike is designed at the safety level of the lower limit  $P_{f_{opt}}(\Delta t')$ . To start a second loop in the ‘single’- reinforcement concept of this paper, Equation (9) is used again with the probability  $P_{f_{opt}}(\Delta t')$  substituted as the new  $P_f(0)$  just after the reinforcement, and the term  $\eta \cdot \Delta t$  changed in  $\eta \cdot (t - \Delta t')$ . In some of the 73 dike segments the second reinforcement loop ended before the year 2050, urging to proceed with the loop again, changing  $\Delta t'$  in  $\Sigma \Delta t'$  in the procedure. In case the next reinforcement interval would exceed the horizon of interest, the left grey block in the schedule is entered to estimate the upper and lower limits at the horizon of interest.

The additional starting points for the calculations including the effect of interventions are:

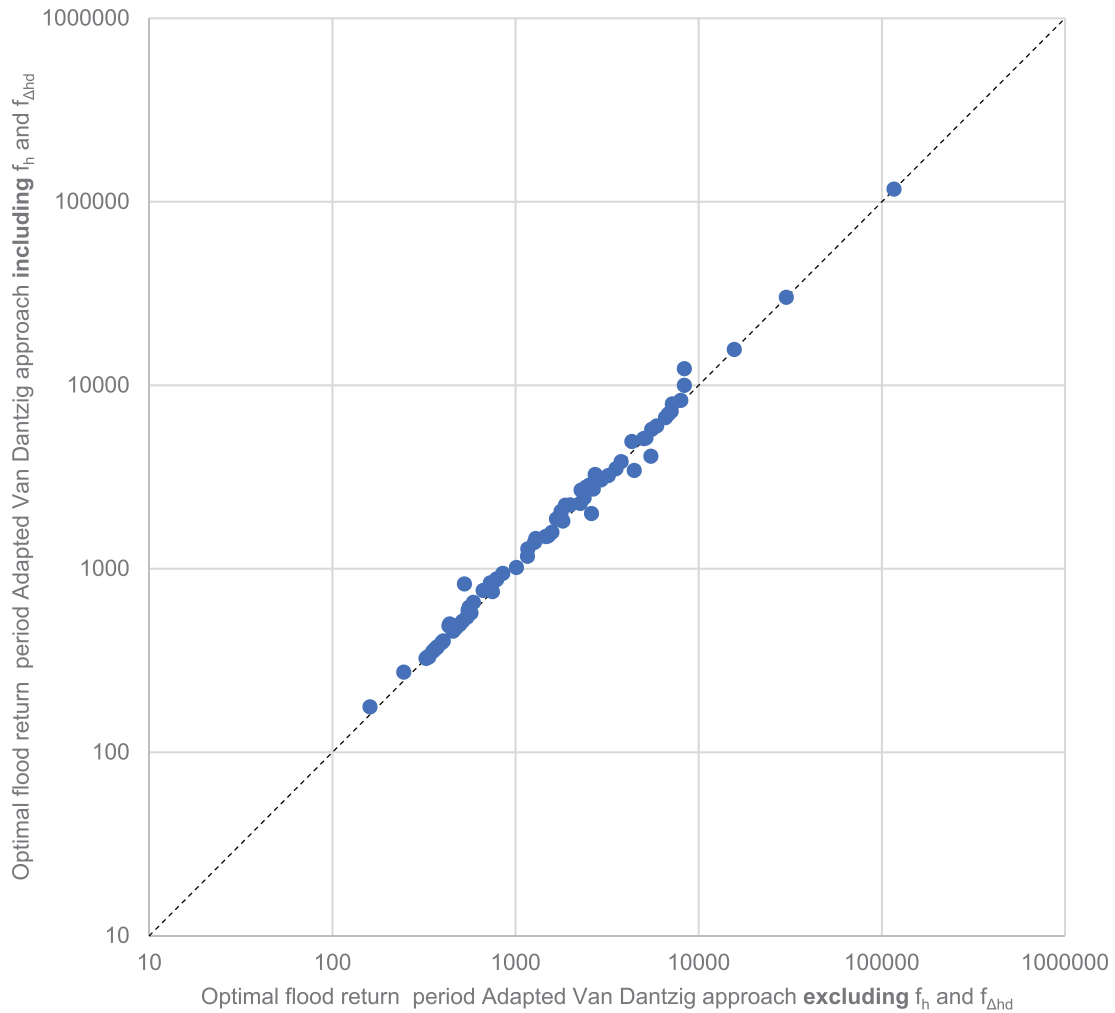
- The upper limit is taken absolute. Sufficient budget and sufficient execution capacity is assumed to be available. Therefore, if the probability exceeds the upper limit given by Equation (9) before 2050 ( $\Delta t = 39$ ), then a reinforcement is executed.



**FIGURE 9** | Probabilities over time, together with its upper and lower limits, for dike segment IJsseldelta, in Kind (2014) denoted by segment 11–1.



**FIGURE 10** | Dike height over time, relative to  $t=0$ , for dike segment IJsseldelta, in Kind (2014) denoted by segment 11–1.



**FIGURE 11** | Comparison of the Adapted Van Dantzig approach with and without the damage factors  $f_h$  and  $f_{\Delta hd}$  ( $r=5.5\%$ ).

- In case reinforcement appeared to be beneficial before the start year of analysis, 2011 is taken as the first reinforcement time.
- Calculating the upper limit with Equation (8),  $P_{standard}$  is taken equal to the economic optimal probability for a design,  $P_{f_{opt}}(\Delta t)$ .

- The analysis contains a minor step for convergence of the factors  $f_I$  and  $f_{\Delta h_d}$  which are based on the magnitude of the reinforcements itself. For completed reinforcements before the horizon of interest, these factors are based on a reinforcement height equal to  $\Delta h_d(\Delta t') = f_{ovx} B \ln \left( \frac{P_f(\Delta t')}{P_{f_{opt}}(\Delta t')} \right)$  with  $\Delta t'$  the time of the reinforcement. In case the last reinforcement loop is not ending with a reinforcement before the time horizon of interest, these factors are based on a reinforcement height based on a reinforcement at the horizon:  $\Delta h_d(horizon) = f_{ovx} B \ln \left( \frac{P_f(horizon)}{P_{f_{opt}}(horizon)} \right)$ . Iterations have been performed to find stable values for  $f_I$  and  $f_{\Delta h_d}$  for the reinforcement of a dike segment.

Figure 7 shows the comparison for the middle probabilities in year 2050, for the case the beneficial reinforcements are executed. The comparison is much better than in Figure 3. The orange dots, now in contrast to Figure 3 containing the effect of reinforcements in the period until the year 2050, are about centered around the reference line. The average difference between the return periods based on the Adapted Van Dantzig approach and the results in Kind (2014) is only about 5% on an average. The comparison is carried out again for results for a discount rate of 3%, see Figure 8, for which the average differences with the approach of Kind (2014) is about 10%. Based on these results the mainly analytical Adapted Van Dantzig approach, based on the lower and upper limits as well as the estimates of the intervention timing, is considered to be in the same order of accuracy as the calculations in Kind (2014).

To extent the case so far, which concerned only the year 2050 for the reason of availability of results in Kind (2014), the course of the flood probabilities over time is figured out. This is performed to serve as qualitative verification on the adapted Van Dantzig approach. The situation without reinforcement steps is straight forward. Equation (5) is the lower limit curve and Equation (8) is the upper limit curve in Kind (2014), both with  $\Delta t$  substituted by time  $t$ . The time beyond the first reinforcement step is assessed like the flow chart presented in Figure 6. Therewith, a time dependent safety level and dike height pattern can be developed. Dike height over time is found rewriting Equation (4) and substituting the time with respect to the last executed reinforcement  $t - \sum \Delta t$ :

$$h_d(t) = A + f_{ovx} B \ln \left( P_{f_{opt}}(\Delta t) \right) - \eta \left( t - \sum \Delta t \right) \quad (11)$$

This case 'safety over time' is figured out for dike segment IJsseldelta, which in Kind (2014) is denoted with segment 11–1. The data used is given in Table 1. Figures 9 and 10 provide respectively the probability development over time, and the dike height differences over time relative to its value at the start of the analysis,  $t = 0$ . The latter show clearly the deterioration over time, the reinforcements (the dike height jumps), and the effect of climate change since the dike height after the second reinforcement is higher than the dike height after the first reinforcement.

Furthermore, we studied the effect of the factors  $f_h$  and  $f_{\Delta h_d}$ , indicating the extra damage effects due to water level and dike height increase. Results with and without these factors are

presented in Figure 11. It appears the effect is minimal for the 73 dike segments in this study.

Therewith, the Adapted Van Dantzig approach can be used to derive economic optimal reliability standards. Assuming an intervention is performed with the objective to obtain an economic optimal design reliability, a good estimation of the first beneficial intervention timing can be performed with an explicit calculation scheme without iterations. Therefore, Equation (9) is slightly adapted: the factor  $f_{\Delta h_d}$  is excluded,  $\Delta t$  is substituted by  $\Delta t_{start}$ , and  $P_{standard}$  is substituted by  $P_{f_{opt}}(\Delta t_{start})$ :

$$\Delta t = \frac{f_{ovx} B}{\eta} \cdot \ln \left( \frac{P_{f_{opt}}(\Delta t_{start})}{P_f(0)} \cdot \left( \frac{I_0}{I' \cdot f_{ovx} B} + \frac{\eta \Delta t_{start}}{f_{ovx} B} + \ln \left( \frac{P_f(0)}{P_{f_{opt}}(\Delta t_{start})} \right) + 1 \right) \right) \quad (12)$$

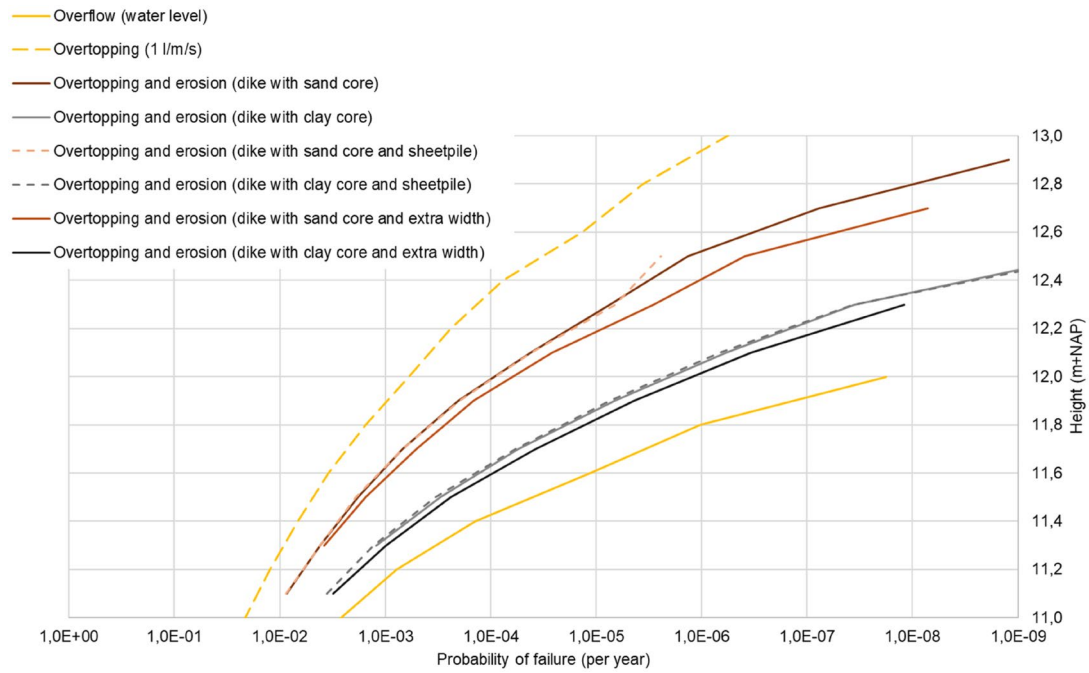
For  $\Delta t_{start}$  a value of, for example, 50 years can be taken in case of the dike is reinforced recently, or a value of 0 in other cases. With the  $\Delta t$  resulting from Equation (12) the design probability of failure can be derived with Equation (5). Note, Equation (12) can result in negative values in case the actual probability of failure  $P_f(0)$  is larger than the upper limit, meaning it is beneficial to reinforce as soon as possible.

## 8 | Risk Aware Updating of Standards

Updating reliability standards may come across contra-intuitive in case the standards are derived based on flood risk. However, in case the standards do not provide an optimal safety level related to a proposed construction type, a standardised updating of present reliability standards enables to tune the safety level to relate to the corresponding dike design.

In this section the data for case Grebbe (den Heijer and Kok 2022) is used for comparison to figure out whether the Adapted Van Dantzig approach agrees the economic optimal probabilities for different construction types. Firstly, the data for the different Construction Dimension Combinations in den Heijer and Kok (2022) are used to extract the values of marginal investments  $I'$  and damage  $D$ . Secondly, the probabilistic model in den Heijer and Kok (2022) is used to derive the values of  $f_{ovx} B$  corresponding to the construction. The probabilities of failure due to overtopping and erosion are calculated for a range of dike heights, see Figure 12. The probabilities of failure of the 6 different constructions are all in between those for the mechanisms overflow and overtopping of 1 L/m/s. The three types with sand cores have larger probabilities of failure than the types with clay cores. Since the tails are not that smooth due to the Monte Carlo sampling, the values for  $f_{ovx} B$  are derived fitting the tails. The curves do not follow an exponential distribution. Therefore, a few iterations are carried out to find the  $f_{ovx} B$  along the curves corresponding to the result of the Adapted Van Dantzig approach.

The calculations of the economic optimal probabilities of flooding are generated with the starting points of den Heijer and Kok (2022): a discount rate of 0.03 per year, and no economic



**FIGURE 12** | Probabilities of Failure for several dike construction types, for location Grebbe, Rhine river km 906.300.

**TABLE 2** | Results of Adapted Van Dantzig Approach compared with the numerical optimization in den Heijer and Kok (2022).

Dike construction type	$P_{f, \text{opt-numeric}}$ ( $10^{-5}/\text{year}$ )	$I'$ (M€/m)	$f_{\text{ovx}} B$ (m)	$D$ (B€)	$P_{f, \text{opt-AVD}}$ ( $10^{-5}/\text{year}$ )
Dike with sand core	0.11	7.6	0.10	25	0.09
Dike with clay core	0.16	9.7	0.08	20	0.12
Dike with sand core and sheetpile	0.42	6.3	0.11	11	0.19 <sup>a</sup>
Dike with clay core and sheetpile	0.03	8.7	0.08	8	0.26 <sup>b</sup>
Dike with sand core and extra width	0.19	9.9	0.09	23	0.12
Dike with clay core and extra width	0.18	11.6	0.08	20	0.14

<sup>a</sup>The numerical optimization provided a wide flat optimum, indicating the optimum is not unambiguous.

<sup>b</sup>The numerical optimization in den Heijer and Kok (2022) provided no global optimum because of the used grid. The edge optimum serves here as a lower bound of the global optimum found for the Adapted Van Dantzig approach.

growth. The results are presented in Table 2, for the numerical approach in the second column, and for the Adapted Van Dantzig method in column six. The results agree very good, except for the construction type Dike with clay core and sheetpile. The numerical optimization in den Heijer and Kok (2022) provided no global optimum for that construction type, since the grid is chosen based on reinforcement with respect to the existing situation. Therefore, the optimum found is an edge optimum: this means that changing the construction type, while keeping the actual dimensions, leads to the edge-optimal Construction Dimension Combination. The Adapted Van Dantzig approach provides a value irrespective of the existing situation. The resulting optimal probability is larger than in den Heijer and Kok (2022) which is not in contradiction with the edge optimum. In case the dike would be rebuilt the dike dimensions can be less than the existing ones.

Therewith, the Adapted Van Dantzig approach is considered to be sufficiently valid to update existing standards for different

designs. A translation factor  $f_d$  is introduced to translate existing flood probability standards to economic optimal probabilities belonging to a proposed design, using Equation (5):

$$P_{f, \text{risk-opt}}(\Delta t) = f_d \cdot P_{\text{standard}} \equiv f_d \cdot P_{f, \text{opt}}(\Delta t_{\text{standard}}) = f_d \cdot \frac{I' B r}{D_{\delta}(\Delta t_{\text{standard}})} \cdot f_I \cdot f_{\text{ovx, standard}} \quad (13)$$

with  $P_{f, \text{risk-opt}}(\Delta t)$  the economic optimal probability of failure adapted with respect to a specific dike design, and  $\Delta t_{\text{standard}}$  the period starting from present to the year for which the standards are derived. The translation factor  $f_d$  enables to 'replace' the variables which depend on a design in the Adapted Van Dantzig formulas, with the ones corresponding to a dike design or its reinforcement. For the lower limit it follows:

$$f_d = \frac{I'_{\text{design}} \cdot f_{\text{ovx, design}} \cdot D_{\delta}(\Delta t)_{\text{standard}}}{I'_{\text{standard}} \cdot f_{\text{ovx, standard}} \cdot D_{\delta}(\Delta t)_{\text{design}}} \quad (14)$$



in which the subscripts refer to the “standard” design used for derivation of the standards, and a specific “design” under consideration. In fact, this proposal requires insight in the investment and damage for the reference situation used for derivation of the standards as well as for the proposed situation. Furthermore, the same ‘replacement’ method can be used for assessment of the time to reinforce existing dikes with Equation (8), needed for determination of  $\Delta t_{design}$ , and for determination of optimal life cycles with Equation (10), needed for planning and program budgeting objectives.

## 9 | Discussion

If reliability standards are in place, derived regardless the dike construction, the presented Adapted Van Dantzig approach and the accompanying updating method can be used to homogenise the standards into values tailored to the actual construction of the dikes, or tailored to a proposed design.

Updating reliability standards provides opportunities for strategic planning to reduce system risks alternatively. An example is to plan structural robust dike constructions on high risk locations, decimating the risk on victims (de Bruijn and Klijn 2011).

In case of established reliability standards, such as the Dutch standards, the tactic and operational flood defence management turns into the management of failure probabilities. If the reliability standards are risk-based, the consequences are only explicitly involved during derivation of the standards at strategic decision level. The consequences are not involved in actual designs. On the one hand this supports to ease the flood defence manager. On the other hand, risk-awareness is no inherent part of the flood defence managers work. This could exclude alternative risk aware reinforcement options, or it leaves unnoticed a reinforcement can be postponed due to actual structural robustness. Flood defence managers strive for investment cost optimality given the standard, using cheapest materials and construction methods. Note, investment cost optimality differs from optimization of the Total Cost of Ownership, which includes the flood risk effect of investments as well.

The approach is applicable in a variety of countries which are exposed to flood risk. In the Netherlands, the actual management of the flood defences is rather straightforward, because the standards are laid down in Dutch Law. In some other countries, performance requirements are used, but not formalised in law. Some of them are based on quantitative risk analysis (Vonk et al. 2020). In all cases, if a performance measure is defined as a reliability for the flood defence, risk-aware updating is opportune. Also, in countries where standards are expressed as water level frequencies, the approach is applicable by using  $f_{ovx} = 1$  in Equations (5), (8), (9), and (14), which means the failure mechanism overflow is used instead of overtopping. Then, the effect of a construction type is included in damage  $D$  and marginal costs  $I'$ .

In case of the Netherlands, a risk-based framework of standards is in place, based on criteria on three different metrics: economic, individual risk and group risk. The presented risk-aware updating of reliability standards only yields for

locations where the economic criterion prevails the criteria for individual risk and group risk.

A limitation in the presented approach in this paper is to take into account only the failure mechanism overtopping, just as in the existing approaches. The effect of taking into account a second failure mechanism is studied in den Heijer (2025). The resulting economic optimal failure probabilities for seven locations in the Netherlands appear to be more or less the same.

Another limitation is that the Adapted Van Dantzig approach is derived using exponential distributions for the dike heights, with scale parameter  $f_{ovx}B$ . However, dike heights may be distributed differently. In those cases, the Adapted Van Dantzig approach can be used iteratively. Start with an exponential distribution fitted to the actual distribution for a frequency assumed not unrealistic. Use the resulting optimal reliability as a start for a second iteration, and so on.

## 10 | Conclusion

The Adapted Van Dantzig approach extends the existing analytic approach for the derivation of an optimal economic probability of failure, provided by van Dantzig (1956). It reflects dynamic effects such as climate change, subsidence and the effect of structural robustness. The shape of the lower limit in the Adapted Van Dantzig approach, reflecting the design safety level, looks quite alike the existing approach provided by Van Dantzig (see Equations 5 and 2). Furthermore, Equation (5) is simple to use, enabling application for all levels of flood defence asset management: in early decision stages for reinforcement (operational), what-if studies (tactical) or policy analysis over larger areas (strategic).

Comparison of the middle probabilities resulting from the Adapted Van Dantzig approach and the numeric approach as used in Kind (2014) shows the quality is satisfying for 73 dike segments in the Netherlands (see Figure 7). Furthermore, the economic optimal life cycle of flood defences can be estimated analytically (Equation 10). Next to the known dependence on the ratio of fixed and marginal investment costs, the optimal life cycle strongly depends on the ratio of water level increase rate and the dike height scale parameter  $\eta/(f_{ovx}B)$ . This means that in case the relative water level increase rate is low, it is beneficial to use long design horizons. The exercise for the 73 dike segments provides the insight that some of the flood defences have short optimal life cycles, even less than 20 years.

The Adapted Van Dantzig approach is applicable in a variety of countries for design of new flood defences, for reinforcement of existing flood defences, or for assessment of present flood defences. Additional to existing approaches, the approach can serve to consider whether existing standards still fit adequately. For the case if not, this paper proposes a translation factor to risk-aware update the existing standard. Risk-aware updating is opportune in case of new or adapted information with respect to the information used to choose or derive the standard. Examples of new information are a proposed design or reinforcement, the adapted consequences due to the structural robustness of a

design, new simulations of consequences, or decisions about the timing of reinforcements.

## Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

## References

- CIRIA. 2013. *International Levee Handbook*. CIRIA.
- de Bruijn, K., and F. Klijn. 2011. "Deltadijken: locaties waar deze het meest effectief slachtofferisico's reduceren."
- Delta Commission. 2008. "Samen werken met water - Een land dat leeft, bouwt aan zijn toekomst Bevindingen van de Deltacommissie 2008."
- Deltacommissie. 1960. *Rapport Deltacommissie - Eindverslag en interimadviezen*. Staatsdrukkerij- en uitgeversbedrijf.
- den Heijer, F. 2025. *Comprehensive Risk-Perspective for Flood Defence System Management*. Delft University of Technology.
- den Heijer, F., and M. Kok. 2022. "Assessment of Ductile Dike Behavior as a Novel Flood Risk Reduction Measure." *Risk Analysis* 43: 1–1794. <https://doi.org/10.1111/risa.14071>.
- den Heijer, F., and M. Kok. 2024. "Risk-Based Portfolio Planning of Dike Reinforcements." *Reliability Engineering & System Safety* 242: 109737. <https://doi.org/10.1016/J.RESS.2023.109737>.
- Duits, M. 2019. "HYDRA-NL gebruikershandleiding, versie 2.7."
- Eijgenraam, C., R. Brekelmans, D. Den Hertog, and K. Roos. 2017. "Optimal Strategies for Flood Prevention." *Management Science* 63, no. 5: 1644–1656. <https://doi.org/10.1287/mnsc.2015.2395>.
- Eijgenraam, C., J. Kind, C. Bak, et al. 2014. "Economically Efficient Standards to Protect The Netherlands Against Flooding." *Interfaces* 44, no. 1: 7–21. <https://doi.org/10.1287/inte2013.0721>.
- EU. 2007. "Directive 2007/60/EC of the European Parliament and of the Council of 23 October 2007 on the assessment and management of flood risks." <https://eur-lex.europa.eu/legal-content/EN/ALL/?uri=CELEX%3A32007L0060>.
- Helpdesk Water. 2020. "National Database Flood Simulations." <https://www.helpdeskwater.nl/onderwerpen/wetgeving-beleid/europese-richtlijn-overstromingsrisico/overstromingsgevaar-overstromingsrisicokaarten/>.
- Jonkman, S. N., R. D. J. M. Steenbergen, O. Morales-Nápoles, A. C. W. M. Vrouwenvelder, and J. K. Vrijling. 2016. "Probabilistic Design: Risk and Reliability Analysis in Civil Engineering." *Collegedictaat CIE4130*.
- Kind, J. M. 2014. "Economically Efficient Flood Protection Standards for The Netherlands." *Journal of Flood Risk Management* 7, no. 2: 103–117. <https://doi.org/10.1111/jfr3.12026>.
- Klerk, W. J. 2022. *Decisions on Life-Cycle Reliability of Flood Defence Systems*. Delft University of Technology.
- Kok, M., R. B. Jongejan, M. Nieuwjaar, and I. Tanczos. 2017. *Fundamentals of Flood Protection*. Ministry of Infrastructure and the Environment & Expertise Network for Flood Protection.
- Ministerie van Infrastructuur en Milieu. 2016. "Wet van 2 november 2016 tot wijziging van de Waterwet en enkele andere wetten (Waterwet)." <https://wetten.overheid.nl/BWBR0025458/2021-07-01>.
- Ministerie van Verkeer en Waterstaat. 1996. "Flood defence Act. Wet van 21 december 1995, houdende algemene regels." *Staatsblad* 1996, nr. 8.
- Poljansek, K., A. Casajus Valles, M. Marin Ferrer, et al. 2019. "Recommendations for National Risk Assessment for Disaster Risk Management in EU." <https://doi.org/10.2760/147842>.
- Sayers, P. B. 2017. *Evolution of Strategic Flood Risk Management in Support of Social Justice, Ecosystem Health, and Resilience*. Oxford Research Encyclopedia of Natural Hazard Science. <https://doi.org/10.1093/ACREFORE/9780199389407.013.85>.
- van Dantzig, D. 1956. "Economic Decision Problems for Flood Prevention." *Econometrica* 24, no. 3: 276. <https://doi.org/10.2307/1911632>.
- van der Meer, J. W. 2002. "Technisch Rapport Golfoploop en Golfoverslag bij Dijken."
- Vonk, B., W. J. Klerk, P. Fröhle, et al. 2020. "Adaptive Asset Management for Flood Protection: The FAIR Framework in Action." *Infrastructures* 5, no. 12: 109. <https://doi.org/10.3390/infrastructures5120109>.

## Appendix 1

### Optimal Probability Related to Marginal Costs and Damage

Based on a fit of results (Kind 2014) observed a linear relationship between the ratio of damage and the cost to decrease the probability tenfold on the one hand and the reciprocal optimal flood protection standard on the other hand, see Figure A1.

We consider the approach developed by the first Dutch Delta Committee, to explain this is understandable. Rewriting Equation (2) as its reciprocal, it follows:

$$\frac{1}{P_{f_{opt}}} = \frac{D}{I'Br} \quad (A1)$$

Assuming the water levels follow an exponential distribution, the cost to decrease the probability of flooding tenfold  $I^{10} = I' B \ln(10)$ . Therewith, it follows:

$$\frac{1}{P_{f_{opt}}} = \frac{D}{I^{10}} \frac{\ln(10)}{r} \quad (A2)$$

Therewith the equation based on the Delta Commissions approach is in the shape of the linear relation of Figure A1. This relationship is not dependent on the location and scale parameters in the exponential distribution, and therewith it is spatially independent, enabling to use it for multiple dike segments as shown in the Figure A1. The factor to the damage/cost ratio to obtain the reciprocal optimal flood protection standard is  $\ln(10)/r$ . With the discount equal to 5.5%, as taken in Kind (2014), this factor is calculated to be 42, which is quite comparable with the value of 38 found in Kind (2014).

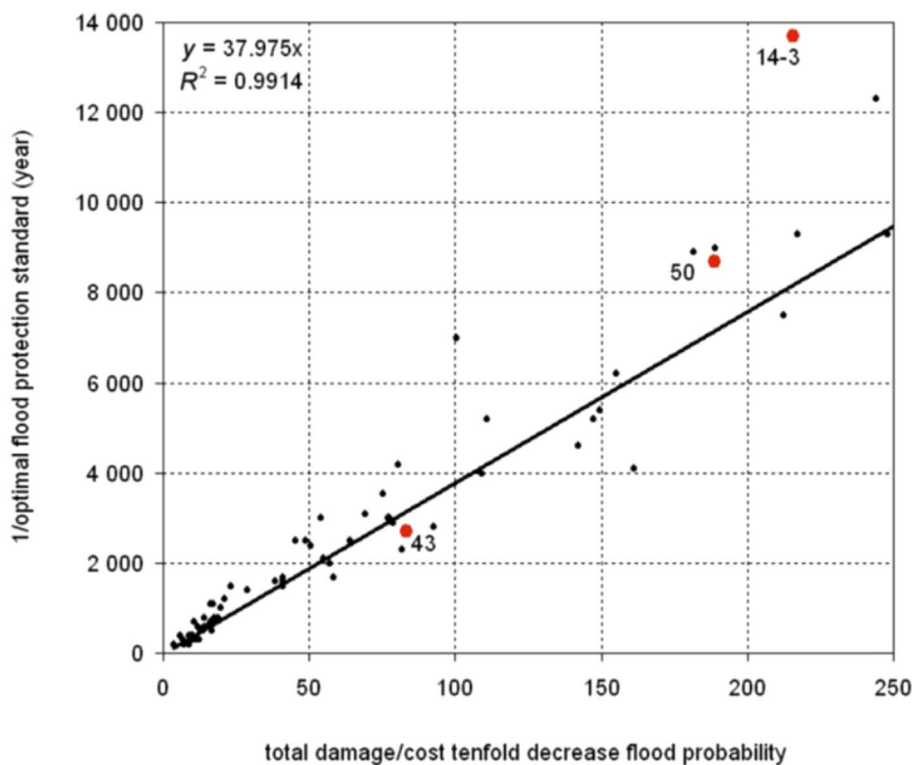


FIGURE A1 | Linear relation presented in Kind (2014).

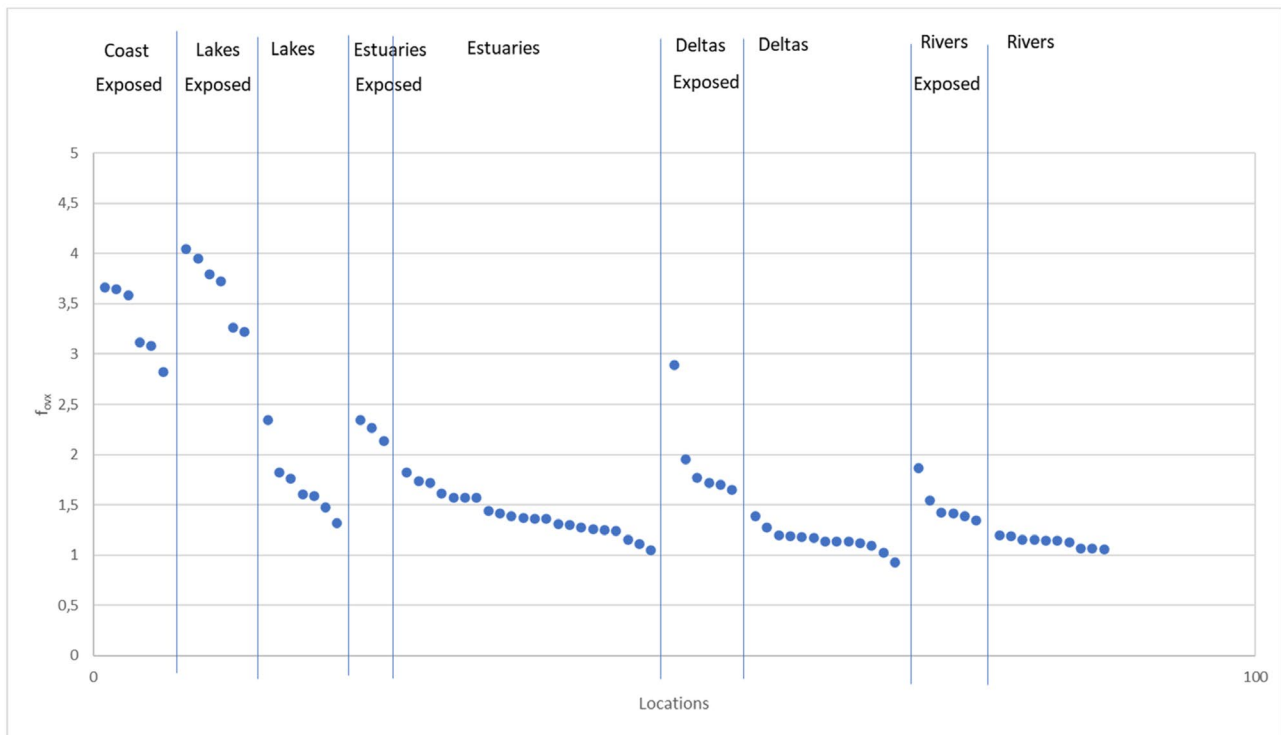
## Appendix 2

### Scale Parameter for Diike Height

The diike height is the sum of water level and freeboard. The scale parameter for diike height considering the failure mechanism wave overtopping is referred to in this paper with  $f_{ovx} B$ , with theoretically  $f_{ovx} \geq 1$  as explained in the main text.  $f_{ovx}$  is 1 in case only the failure mechanism overflow would be considered. The more wave attack, especially in case it is combined with water level set-up, the larger the scale parameter for diike height. Locations which are located exposed to wave attack have higher values of  $f_{ovx}$  than locations located lee. In case of exposed locations with respect to storm set-up, a rather high dependence between water levels and waves is expected. For depth limited wave heights applies  $H_s \approx 0.5$  to  $0.6$  times the water depth. The freeboard for mild sloped dikes (1:4) is approximately 2–2.5 times  $H_s$  (van der

Meer 2002). Together this leads to a freeboard of about 1 to 1.5 times the water depth. Together with water level, for which  $B = 1$  per definition, this leads to a scale parameter factor  $f_{ovx}$  of approximately 2 to 2.5.

For about 80 locations in the Netherlands  $f_{ovx}$  is calculated based on existing results of calculations with HYDRA-NL for diike heights based on an overtopping criterion of 1 L/m/s (Duits 2019). This resulted for different areas and coastal environments in different values of  $f_{ovx}$ , as shown in Figure A2. Figure A2 shows the range of values of  $f_{ovx}$  up to a value of 4. The more exposed the location, the larger  $f_{ovx}$ . All values exceeds 1, except one for which offshore wind directions are expected to cause this below-theoretical value. Note, the  $f_{ovx}$  decreases (theoretically) to the lower limit in case infinite large overtopping discharges would be acceptable. Thus, the larger the accepted overtopping discharge the lower  $f_{ovx}$ .



**FIGURE A2** | Determination of  $f_{ovx}$  for 78 locations in several water systems, with separate selections for exposed locations.

## Appendix 3

### Derivation Lower and Upper Limits for the Adapted Van Dantzig Approach

To derive the lower limit in Figure 1, the sum of risks and investments is minimised. The timing of the investment  $I$  is not necessarily at  $t=0$ , thus these cost has to be included as a present value:

$$C_{tot} = I^{PV} + R^{PV} = I^{PV}(\Delta t) + [P_f(t)D_\delta(t)]^{PV} \quad (A3)$$

With  $D_\delta(t)$  the economic damage caused by flooding at time  $t$ , and with  $P_f(t)$  dependent on time and whether or not a reinforcement has been implemented:

$$\begin{aligned} P_f(h_d(t)) &= \exp\left(-\frac{h_d(0)-(A+\eta t)}{f_{ovx}B}\right) & t < \Delta t \\ P_f(h_d(t)) &= \exp\left(-\frac{h_d(\Delta t)-(A+\eta t)}{f_{ovx}B}\right) & t \geq \Delta t \end{aligned} \quad (A4)$$

with  $h_d(t)$  the dike height at time  $t$ , and  $\Delta t$  the time of dike reinforcement. The dike reinforcement height is given by:

$$\Delta h_d(\Delta t) = h_d(\Delta t) - (h_d(0) - \eta \Delta t) = \eta \Delta t + f_{ovx}B \cdot \ln\left(\frac{P_f(0)}{P_{standard}}\right) \quad (A5)$$

With  $\Delta h_d(\Delta t)$  the dike reinforcement height at  $t=\Delta t$ . The investments at  $t=\Delta t$  are given by:

$$I(\Delta t) = f_I \cdot (I_0 + \Delta h_d(\Delta t) \cdot I') \quad (A6)$$

With  $f_I$  equal to  $\exp(\lambda \cdot \Delta h_d)$ , with  $\lambda$  small and positive, resulting in  $f_I$  slightly larger than 1, indicating the investments increase more than proportionate with the reinforcement height. Therewith, substituting Equations (A4–A6) in Equation (A3) the total societal costs can be given by:

$$\begin{aligned} C_{tot}^{PV} &= \frac{1}{(1+r)^{\Delta t}} \cdot f_I \cdot (I_0 + (h_d(\Delta t) - (h_d(0) - \eta \Delta t)) \cdot I') + \\ &\sum_{t=0}^{t=\Delta t} \frac{D_\delta(t)}{(1+r)^t} \cdot \exp\left(-\frac{h_d(0)-(A+\eta t)}{f_{ovx}B}\right) + \\ &\sum_{t=\Delta t}^{\infty} \frac{D_\delta(t)}{(1+r)^t} \cdot \exp\left(-\frac{h_d(\Delta t)-(A+\eta t)}{f_{ovx}B}\right) \end{aligned} \quad (A7)$$

The optimal societal costs can be derived by:

$$\frac{dC_{tot}^{PV}}{dh_d(\Delta t)} = 0 \quad (A8)$$

Therewith, the second term in Equation (A7) disappears. Given small values of  $r$ , and neglecting changes after the reinforcement, the third term in Equation (A7) can be simplified:

$$\begin{aligned} &\sum_{t=\Delta t}^{\infty} \frac{D_\delta(t)}{(1+r)^t} \cdot \exp\left(-\frac{h_d(\Delta t)-(A+\eta t)}{f_{ovx}B}\right) \\ &= \frac{D_\delta(\Delta t)}{(1+r)^{\Delta t}} \cdot \frac{1}{r} \cdot \exp\left(-\frac{h_d(\Delta t)-(A+\eta t)}{f_{ovx}B}\right) \end{aligned} \quad (A9)$$

Substituting in Equation (A8) an writing  $P_f(\Delta t)$  for  $\exp\left(-\frac{h_d(\Delta t)-(A+\eta t)}{f_{ovx}B}\right)$  this leads to:

$$\frac{dC_{tot}^{PV}}{dh_d(\Delta t)} = \frac{f_I I'}{(1+r)^{\Delta t}} - \frac{D_\delta(\Delta t)}{(1+r)^{\Delta t}} \cdot \frac{1}{f_{ovx}B} \cdot P_f(\Delta t) \quad (A10)$$

Rewriting, and writing  $P_{f_{opt}}$  for  $P_f$  for the case Equation (A8) is fulfilled, leads to the time dependent lower limit, see Equation (5) in the main text:

$$P_{f_{opt}}(\Delta t) = \frac{f_I I' f_{ovx} B r}{D_\delta(\Delta t)} = \frac{I' B r}{D_\delta(\Delta t)} \cdot f_I \cdot f_{ovx} \quad (A11)$$

To derive the upper limit in Figure 1 the utility criterion is:

$$\Delta R^{PV}(\Delta t) - I^{PV}(\Delta t) > 0 \quad (A12)$$

With  $\Delta R^{PV}(\Delta t)$  the risk difference before and after an intervention at  $t=\Delta t$ . A factor is introduced to indicate the relative difference between the damage before and after the intervention, given a flooding:

$$f_{\Delta h_d} = \frac{D_\delta(\Delta t)}{D_\delta(\Delta t^-)} \quad (A13)$$

Assuming the intervention leads to compliance to  $P_{standard}$ , the present value of the risk difference from  $\Delta t$  to  $\infty$  is:

$$\begin{aligned} \Delta R^{PV}(\Delta t) &= [P_f(\Delta t^-)D_\delta(\Delta t^-) - P_{standard}D_\delta(\Delta t)]^{PV} \\ &= \frac{1}{(1+r)^{\Delta t}} \cdot \frac{D_\delta(\Delta t)}{r} \cdot \left(\frac{P_f(\Delta t^-)}{f_{\Delta h_d}} - P_{standard}\right) \end{aligned} \quad (A14)$$

Substituting Equation (A5) in Equation (A6) the present value of the investment of an intervention at  $t=\Delta t$  is:

$$I^{PV}(\Delta t) = \frac{1}{(1+r)^{\Delta t}} \cdot f_I \cdot \left(I_0 + I' \left(\eta \Delta t + f_{ovx}B \cdot \ln\left(\frac{P_f(0)}{P_{standard}}\right)\right)\right) \quad (A15)$$

Since the probabilities of flooding are continuously increasing in time the first point in time for which it is beneficial to intervene can be found substituting Equations (A14) and (A15) in Equation (A12):

$$\begin{aligned} &\frac{D_\delta(\Delta t)}{r} \cdot \left(\frac{P_f(\Delta t^-)}{f_{\Delta h_d}} - P_{standard}\right) \\ &- f_I \cdot \left(I_0 + I' \left(\eta \Delta t + f_{ovx}B \cdot \ln\left(\frac{P_f(0)}{P_{standard}}\right)\right)\right) = 0 \end{aligned} \quad (A16)$$

This leads to Equation (8) in the main text:

$$P_f(\Delta t^-) = f_{\Delta h_d} \cdot \left(P_{standard} + \frac{f_I \cdot r}{D_\delta(\Delta t)} \cdot \left(I_0 + I' \left(\eta \Delta t + f_{ovx}B \cdot \ln\left(\frac{P_f(0)}{P_{standard}}\right)\right)\right)\right) \quad (A17)$$