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Power-Imbalance Allocation Control of Power Systems-Secondary Frequency Control[☆]

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ABSTRACT

The traditional secondary frequency control of power systems restores nominal frequency by steering *Area Control Errors* (ACEs) to zero. Existing methods are a form of integral control with the characteristic that large control gain coefficients introduce an overshoot and small ones result in a slow convergence to a steady state. In order to deal with the large frequency deviation problem, which is the main concern of the power system integrated with a large number of renewable energy, a faster convergence is critical. In this paper, we propose a secondary frequency control method named *Power-Imbalance Allocation Control* (PIAC) to restore the nominal frequency with a minimized control cost, in which a coordinator estimates the power imbalance and dispatches the control inputs to the controllers after solving an economic power dispatch problem. The power imbalance estimation converges exponentially in PIAC, both overshoots and large frequency deviations are avoided. In addition, when PIAC is implemented in a multi-area controlled network, the controllers of an area are independent of the disturbance of the neighbor areas, which allows an asynchronous control in the multi-area network. A Lyapunov stability analysis shows that PIAC is locally asymptotically stable and simulation results illustrate that it effectively eliminates the drawback of the traditional integral control based methods.

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1. Introduction

Rapid expansion of the contribution of distributed renewable energy sources has accelerated research efforts in controlling the power grid. In general, frequency control is implemented at three different levels distinguished from fast to slow timescales (Ilić & Zaborszky, 2000; Schavemaker & van der Sluis, 2008). In a short time scale, the power grid is stabilized by decentralized droop control, which is called *primary control*. While successfully balancing the power supply and demand, and synchronizing the power frequency, the primary control induces frequency deviations from the nominal frequency, e.g., 50 or 60 Hz. The *secondary frequency control* regulates the frequency back to its nominal frequency in a slower time scale than the primary control. On top of the primary and secondary control, the *tertiary control* is concerned with global economic power dispatch over the networks in a large time scale. Consequently it depends on the energy prices and markets.

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The secondary frequency control is the focus of this paper. An interconnected electric power system can be described as a collection of subsystems, each of which is called a control area. The secondary control in a single area is regulated by *Automatic Generation Control* (AGC), which is driven by *Area Control Error* (ACE). The ACE of an area is calculated from the local frequency deviations within the area and power transfers between the area and its neighbor areas. The AGC controls the power injections to force the ACE to zero, thus restores the nominal frequency. Due to the availability of a communication network, other secondary frequency control approaches have recently been developed which minimize the control cost on-line (Dörfler, Simpson-Porco, & Bullo, 2016), e.g., the Distributed Average Integral Method (DAI) (Zhao, Mallada, & Dörfler, 2015), the Gather-and-Broadcast (GB) method (Dörfler & Grammatico, 2017), economic AGC (EAGC) method (Li, Zhao, & Chen, 2016), and distributed real time power optimal power control method (Liu, Qu, Xin, & Gan, 2017). These methods suffer from a common drawback, namely that they exhibit overshoot for large gain coefficients and slow convergence for small gain coefficients (Berger & Schweppe, 1989; Elgerd & Fosha, 1970; Ibraheem, Kumar, & Kothari, 2005). This is due to the fact that they rely on integral control which is well-known to give rise to the two phenomena mentioned above. Note that the slow convergence speed results in a large frequency deviation which is the main concern of power systems integrated with a large amount of renewable energy.

The presence of fluctuations is expected to increase in the near future, due to weather dependent renewable energy, such as solar and wind energy. These renewable power sources often cause serious frequency fluctuations and deviation from the nominal frequency due to the uncertainty of the weather. This demonstrates the necessity of good secondary frequency control methods whose transient performance is enhanced with respect to the traditional methods. We have recently derived such a method called *Power Imbalance Allocation Method* (PIAC) in [Xi, Dubbeldam, Lin, and van Schuppen \(2017b\)](#), which can eliminate the drawback of the integral control based approach. This paper is the extended version of the conference paper ([Xi et al., 2017b](#)) with additional stability analysis and the extension of PIAC to multi-area control.

We consider power systems with lossless transmission lines, which comprise traditional synchronous machines, frequency dependent devices (e.g., power inverters of renewable energy or frequency dependent loads) and passive loads. We assume the system to be equipped with the primary controllers and propose the PIAC method in the framework of Proportional–Integral (PI) control, which first estimates the power imbalance of the system via the measured frequency deviations of the nodes of the synchronous machines and frequency dependent power sources, next dispatches the control inputs of the distributed controllers after solving the economic power dispatch problem. Since the estimated power imbalance converges exponentially at a rate that can be accelerated by increasing the control gain coefficient, the overshoot problem and the large frequency deviation problem are avoided. Hence the drawback of the traditional ACE method is eliminated. Furthermore, the control gain coefficient is independent of the parameters of the power system but only relies on the response time of the control devices. Consequently the transient performance is greatly enhanced by improving the performance of the control devices in PIAC. When implemented in a multi-area power network, PIAC makes the control actions of the areas independent, while the controllers of each area handle the power imbalance of the local area only.

The paper is organized as follows. We introduce the mathematical model of the power system in Section 2. We formulate the problem and discuss the existing approaches in Section 3, then propose the secondary frequency control approach, *Power Imbalance Allocation Control* (PIAC), based on estimated power imbalance in Section 4 and analyze its the stability invoking the Lyapunov/LaSalle stability criterion in Section 5. Finally, we evaluate the performance of PIAC by simulations on the IEEE-39 New England test power system in Section 6. Section 7 concludes with remarks.

2. The model

A power system is described by a graph $G = (\mathcal{V}, \mathcal{E})$ with nodes $\mathcal{V}$ and edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, where a node represents a bus and edge (i, j) represents the direct transmission line connection between nodes i and j . We consider a power system as a lossless electric network with constant voltage (e.g., transmission grids where the line resistances are neglected) and an adjacency matrix $(\hat{B}_{ij})$ where $\hat{B}_{ij}$ denotes the susceptance between node i and node j . The system consists of three types of nodes, synchronous machines, frequency dependent devices and passive loads, the sets of which are denoted by $\mathcal{V}_M$, $\mathcal{V}_F$ and $\mathcal{V}_P$ respectively. Thus $\mathcal{V} = \mathcal{V}_M \cup \mathcal{V}_P \cup \mathcal{V}_F$. The frequency dependent devices are for example frequency dependent loads, inverters of renewable energy, buses equipped with droop controllers. Denote the number of the nodes in $\mathcal{V}$, $\mathcal{V}_M$, $\mathcal{V}_F$, $\mathcal{V}_P$ by n , n_M , n_F , and n_P respectively, hence $n = n_M + n_F + n_P$. The model is described by the following *Differential Algebraic Equations* (DAEs),

see e.g., [Dörfler, and Grammatico \(2017\)](#),

$$\dot{\theta}_i = \omega_i, \quad i \in \mathcal{V}_M \cup \mathcal{V}_F, \quad (1a)$$

$$M_i \dot{\omega}_i + D_i \omega_i = P_i - \sum_{j \in \mathcal{V}} B_{ij} \sin(\theta_i - \theta_j) + u_i, \quad i \in \mathcal{V}_M, \quad (1b)$$

$$D_i \omega_i = P_i - \sum_{j \in \mathcal{V}} B_{ij} \sin(\theta_i - \theta_j) + u_i, \quad i \in \mathcal{V}_F, \quad (1c)$$

$$0 = P_i - \sum_{j \in \mathcal{V}} B_{ij} \sin(\theta_i - \theta_j), \quad i \in \mathcal{V}_P, \quad (1d)$$

where θ_i is the phase angle at node i , ω_i is the frequency deviation from the nominal frequency, i.e., $\omega_i = \bar{\omega}_i - f^*$ where $\bar{\omega}_i$ is the frequency and $f^* = 50$ Hz or 60 Hz is the nominal frequency, $M_i > 0$ denotes the moment of inertia of a synchronous machine, $D_i > 0$ is the droop control coefficient, P_i is the power injection or demand, $B_{ij} = \hat{B}_{ij} V_i V_j$ is the effective susceptance of line (i, j) , V_i is the voltage at node i , $u_i \in [\underline{u}_i, \bar{u}_i]$ is a secondary frequency control input. Note that u_i is a constrained input of the secondary frequency control, $\underline{u}_i$ and $\bar{u}_i$ are its lower and upper bounds, respectively. Furthermore, the set of nodes equipped with the secondary controllers is denoted by $\mathcal{V}_K \subseteq \mathcal{V}_M \cup \mathcal{V}_F$ and $u_i = 0$ for $i \notin \mathcal{V}_K$. Here, we have assumed that the nodes that participate in secondary control are equipped with primary controllers. Note that the loads can also be equipped with primary controllers ([Zhao, Topcu, Li, & Low, 2014](#)). The dynamics of the voltage and reactive power is not modeled, since they are irrelevant for control of the frequency. More details on decoupling the voltage and frequency control in the power system can be found in [Kundur \(1994\)](#), [Simpson-Porco, Dörfler, and Bullo \(2016\)](#), and [Trip, Bürger, and De Persis \(2016\)](#). The model with linearized sine functions in (1) is also widely studied to design primary and secondary frequency control laws, e.g., [Andreasson, Dimarogonas, Sandberg, and Johansson \(2014\)](#), [Li et al. \(2016\)](#) and [Zhao, Mallada, Low, and Bialek \(2016\)](#). For the validity of the linearized model with lossless network, we refer to [Ilić and Zaborszky \(2000\)](#), and [Van Hertem \(2006\)](#).

3. Secondary frequency control of power systems

3.1. Problem formulation

In practice, the frequency deviation should be in a prescribed range in order to avoid damage to the devices in the power system. We assume droop controllers to be installed at some nodes such that $\sum_{i \in \mathcal{V}_M \cup \mathcal{V}_F} D_i > 0$. When the power supply and demand are time-invariant, the frequencies of all the nodes in $\mathcal{V}_M \cup \mathcal{V}_F$ synchronize at a state, called *synchronous state* defined as follows,

$$\theta_i = \omega_{syn} t + \theta_i^*, \quad i \in \mathcal{V}, \quad (2a)$$

$$\omega_i = \omega_{syn}, \quad i \in \mathcal{V}_M \cup \mathcal{V}_F, \quad (2b)$$

$$\dot{\theta}_i = \omega_{syn}, \quad i \in \mathcal{V}, \quad (2c)$$

$$\dot{\omega}_i = 0, \quad i \in \mathcal{V}_M \cup \mathcal{V}_F, \quad (2d)$$

where ω_{syn} is the synchronized frequency deviation, and the phase angle differences at the steady state, $\{\theta_i^* - \theta_j^*, (i, j) \in \mathcal{E}\}$, determine the power flows in the transmission lines. The explicit synchronized frequency deviation ω_{syn} of the system is obtained by substituting (2) into (1) as

$$\omega_{syn} = \frac{\sum_{i \in \mathcal{V}} P_i + \sum_{i \in \mathcal{V}_K} u_i}{\sum_{i \in \mathcal{V}_M \cup \mathcal{V}_F} D_i}. \quad (3)$$

If and only if $\sum_{i \in \mathcal{V}} P_i + \sum_{i \in \mathcal{V}_K} u_i = 0$, the frequency deviation of the steady state is zero, i.e., $\omega_{syn} = 0$. This implies that a system with only droop control, i.e., $u_i = 0$, for $i \in \mathcal{V}_K$, can never converge to a steady state with $\omega_{syn} = 0$ if the power demand and supply are unbalanced such that $\sum_{i \in \mathcal{V}} P_i \neq 0$. This shows the need

for the secondary control. As more renewable power sources are integrated into the power system, the power imbalance $\sum_{i \in \mathcal{V}} P_i$ may fluctuate severely leading to frequency oscillations. We aim to design an effective method to control ω_{syn} to zero by solving the following problem, e.g., Dörfler and Grammatico (2017) and Dörfler et al. (2016).

Problem 1. Compute the inputs $\{u_i, i \in \mathcal{V}_K\}$ of the power system so as to achieve the control objective of a balance of power supply and demand in terms of $\omega_{syn} = 0$, or equivalently, $\sum_{i \in \mathcal{V}} P_i + \sum_{i \in \mathcal{V}_K} u_i = 0$.

The following assumption states the basic condition for which a feasible solution of Problem 1 exists.

Assumption 1. During a small time interval the values of power supply and demand are constant. In addition, for these values there exist control inputs $\{u_i \in [\underline{u}_i, \bar{u}_i], i \in \mathcal{V}_K\}$, such that $\sum_{i \in \mathcal{V}} P_i + \sum_{i \in \mathcal{V}_K} u_i = 0$.

In a small time interval, the tertiary control, which calculates the operating point stabilized by the secondary control, guarantees the existence of a steady state and its local stability (Ilić & Zaborszky, 2000; Wood & Wollenberg, 1996). In addition, the main task of the generators is to provide electricity to the loads and maintain the nominal frequency, so Assumption 1 is realistic.

For different controllers in the system, the control cost might be different for various reasons such as different device maintenance prices. From the global perspective of the entire network, we aim to minimize the secondary frequency control cost which leads to Problem 2.

Problem 2. Compute the inputs $\{u_i, i \in \mathcal{V}\}$ of the power system so as to achieve the control objective of minimal control cost, in addition to the control objective of a balance of power supply and demand with $\omega_{syn} = 0$.

Corresponding to Problem 2, the following economic power dispatch problem needs to be solved, e.g., Dörfler and Grammatico (2017) and Trip and De Persis (2016).

$$\begin{aligned} \min_{\{u_i, i \in \mathcal{V}_K\}} & \sum_{i \in \mathcal{V}_K} J_i(u_i) \\ \text{s.t.} & \sum_{i \in \mathcal{V}} P_i + \sum_{i \in \mathcal{V}_K} u_i = 0 \end{aligned} \quad (4)$$

where $J_i(u_i)$ is the control cost of node i , which incorporates the cost money and the constraints $u_i \in [\underline{u}_i, \bar{u}_i]$. Note that to solve (4), the power imbalance $\sum_{i \in \mathcal{V}} P_i$ should be known and the solution is for the small time interval mentioned in Assumption 1. Here, with respect to the existence of the solution of the economic power dispatch problem, we make the second assumption.

Assumption 2. The cost functions $J_i : \mathbb{R} \rightarrow \mathbb{R}$, $i \in \mathcal{V}_K$ are twice continuously differentiable and strictly convex such that $J''(u_i) > 0$ where $J''(u_i)$ is the second order derivative of $J(u_i)$ with respect to u_i .

Assumption 2 is also realistic because the constraint $u_i \in [\underline{u}_i, \bar{u}_i]$ can be incorporated in the objective function $J_i(u_i)$ for $i \in \mathcal{V}_K$ in a smooth way.

A necessary condition for solving the economic dispatch problem is (Ilić & Zaborszky, 2000),

$$J'_i(u_i) = J'_j(u_j) = \lambda, \quad i, j \in \mathcal{V}_K \quad (5)$$

where $J'_i(u_i)$ is the derivative of $J_i(u_i)$, which is the marginal cost of node i , $i \in \mathcal{V}_K$, and $\lambda \in \mathbb{R}$ is the nodal price. At the optimum of

(4) all the marginal costs of the controllers are equal to the nodal price. The market clearing price λ^* is obtained as the solution of the equation

$$0 = \sum_{i \in \mathcal{V}} P_i + \sum_{i \in \mathcal{V}_K} J_i^{-1}(\lambda^*) \quad (6)$$

where $J_i^{-1}(\cdot)$ is the inverse function of $J_i(\cdot)$ which exists by Assumption 2. Since in practice the power imbalance is uncertain with respect to the fluctuating power loads, the economic power dispatch problem (4) cannot be solved directly.

3.2. A brief review of secondary frequency control

Before embarking on solving Problems 1 and 2, we briefly outline existing secondary frequency control methods and discuss their relevance for finding a solution to Problems 1 and 2.

ACE based AGC (Ilić & Zaborszky, 2000): The Area Control Error (ACE) of an area is defined as

$$ACE = B\omega + P_{ex} - P_{ex}^*, \quad (7)$$

where B is a positive constant, ω is the frequency deviation of the area, P_{ex} is the net power export, and P_{ex}^* is the nominal value of P_{ex} . The adjustment of the power injection of the area is given as follows

$$\dot{u} = -k \cdot ACE$$

where $k \in (0, \infty)$ is a control gain coefficient. In the traditional Automatic Generation Control (AGC) method, the frequency deviation is measured at a local node and communicated by a coordinator as the ACE to the controllers in the system, which calculate their control inputs according to their participation factors. When the interconnected system is considered as a single area, the AGC has the form (Dörfler & Grammatico, 2017)

$$\dot{\lambda} = -k\omega_{i^*}, \quad i \in \mathcal{V}, \quad u_i = J_i^{-1}(\lambda), \quad (8)$$

where $k \in (0, +\infty)$ is a control gain coefficient, ω_{i^*} is the measured frequency deviation at a selected node i^* . λ can be seen as the nodal price which converges to the market clearing price λ^* as the power supply and demand are balanced. Note that the participation factor is involved in the derivative of the cost function, $J'_i(u_i)$. The frequency deviation of the area is not well reflected in (8) since it is measured at only one node. Furthermore, the communication network is not used so efficiently because it only communicates the nodal price λ from the coordinator to the controllers.

Gather-and-Broadcast (GB) Control (Dörfler & Grammatico, 2017): In order to well reflect the frequency deviation of the area and use the communication network efficiently, the GB method measures the frequency deviations at all the nodes connected by the communication network. It has the form

$$\dot{\lambda} = -k \sum_{i \in \mathcal{V}} C_i \omega_i, \quad i \in \mathcal{V}, \quad u_i = J_i^{-1}(\lambda), \quad (9)$$

where $k \in (0, +\infty)$ is a control gain coefficient and $C_i \in [0, 1]$ is a set of convex weighting coefficients with $\sum_{i \in \mathcal{V}} C_i = 1$. The ACE in the GB method is actually the weighted average of the frequency deviations. As in the ACE based AGC method (8), a coordinator in the network also broadcasts the nodal price to the controllers and the controllers compute the control inputs according to their own cost functions.

Distributed Averaging Integral control (DAI): Unlike the ACE based AGC method and GB method implemented in a centralized way, DAI is implemented in a distributed way based on the consensus control principle (Andreasson et al., 2014). In the DAI method, there are no coordinators and each controller computes its own

nodal price and communicates to its neighbors. A local node in the system calculates its control input according to the local frequency deviation and the nodal prices received from its neighbors. As the state of the interconnected system reaches a new steady state, the nodal prices of all the nodes achieve a consensus at the market clearing price, thus Problem 2 is solved. It has the form (Zhao et al., 2015)

$$\dot{\lambda}_i = k_i(-\omega_i + \sum_{j \in \mathcal{V}} w_{ij}(\lambda_j - \lambda_i)), \quad u_i = J_i'^{-1}(\lambda_i), \quad (10)$$

where $k_i \in (0, +\infty)$ is a gain coefficient for the controller i and (w_{ij}) denotes the undirected weighted communication network. When $w_{ij} = 0$ for all the lines of the communication network, the DAI control law reduces to a *Decentralized Integral* (Decl) control law. The DAI control has been widely studied on both the traditional power grids and Micro-Grids, e.g., Schiffer, Seel, Raisch, and Sezi (2016) and Simpson-Porco, Shafiee, Dörfler, Vasquez, Guerrero, and Bullo (2015). Wu, Shen, and Iravani (2017) proposed a distributed secondary control method where it is not necessary to know the nominal frequency for all the nodes in $\mathcal{V}_K$.

When a steady state exists for the nonlinear system (1), all the approaches above can restore the nominal frequency with an optimized control cost. However, it can be easily observed that the control approaches, e.g., (8) and (9), are in the form of integral control where the control inputs are actually the integral of the frequency deviation. This will be further explained in Section 4. A common drawback of integral control is that *the control input suffers from an overshoot problem with a large gain coefficient or slow convergence speed with a small one, which causes extra oscillation or slow convergence of the frequency* (Berger & Schweppe, 1989; Elgerd & Fosha, 1970; Ibraheem et al., 2005).

The methods that we discussed above, concern controlling the nonlinear system (1) based on passivity methods. However, the linearized version of the evolution equations (1) was also addressed based on primal–dual method in the literature. For example, Li et al. proposed an Economic AGC (EAGC) approach for the multi-area frequency control (Li et al., 2016). In this method each controller exchanges control signals that are used to successfully steer the state of the system to a steady state with optimized dispatch, by a partial primal–dual gradient algorithm (Boyd, Parikh, Chu, Peleato, & Eckstein, 2010). Unfortunately, transient performance was not considered in the method. A potentially very promising method to the control of the linear system was recently proposed by Zhao et al. (2016) a novel framework for primary and secondary control, which is called Unified Control (UC), was developed. The advantage of UC is that it automatically takes care of the congestions that may occur in the transmission lines. Numerical simulations results show that the UC can effectively reduce the harmful large transient oscillations in the frequency. However, so far a theoretical analysis on how the UC improves the transient performance is lacking. Other recently reported studies are by Liu et al. (2017) and by Wang, Liu, Low, Zhao, and Mei (2017). These methods optimize both control costs and manage power flow congestion using the principle of consensus control, but cannot prevent large frequency deviations at short times. For more details of distributed frequency control, we refer to the survey paper (Molzahn, Dörfler, Sandberg, Low, Chakrabarti, Baldick, & Lavaei, 2017).

Finally, we mention here control methods whose underlying principle is neither based on integral control nor on primal–dual method. The optimal load-frequency control framework by Liu et al., described in Liu, Song, Ma, Mei, and Lu (2003), is an example of such a method. The goal is still to optimize the control costs and frequency deviation, but rephrasing it as a finite horizon optimization problem. This can only be solved when the loads are precisely known within the selected time horizon. Obviously, this will require very precise forecasting of the loads. A more robust

approach based on the concept of the Active Disturbance Rejection Control (ADRC) (Han, 2009), was pursued by Dong, Zhang, and Gao (2012). The method is robust against model uncertainties, parameter variations and large perturbations which was employed to construct a decentralized load frequency approach for interconnected systems. However, the decentralized control employed in this method prevents a solution to Problem 2.

As more renewable power sources are integrated into the power system, the fluctuations in the power supply become faster and larger. There is a need to design a control law that has a good transient performance without the overshoot problem and with a fast convergence speed. The traditional method to eliminate the overshoot is to calculate the control gain coefficients by analyzing the eigenvalue of the linearized systems (Wu, Dörfler, & Jovanovic, 2016; Wu & Shen, 2017). However, the improvement of the transient performance obtained by the eigenvalue analysis is still poor because of the dependence of the eigenvalues on the control law structure, and the large scale, complex topology and heterogeneous power generation and loads of the power system.

Based on the framework of PI control this paper aims to design a secondary frequency control method that remedies the drawbacks mentioned above of the existing methods. To this end, we consider the following problem concerning the transient performance of the power system (1) after a disturbance.

Problem 3. For the power system (1) with primary and secondary controllers, design a secondary frequency control law for $\{u_i, i \in \mathcal{V}_K\}$ so as to eliminate the extra oscillation of the frequency caused by the overshoots of the control inputs, thus improve the transient performance of the system after a disturbance through an accelerated convergence of the control inputs.

To address Problem 3, various control laws have been proposed for the frequency control of power systems, e.g., the sliding mode based control laws (Mi, Fu, Wang, & Wang, 2013; Vrdoljak, Perić, & Petrović, 2010) and $\mathcal{H}_2/\mathcal{H}_\infty$ control based control laws (Bevrani, 2014, chap. 5) and (Rerkpreedapong, Hasanovic, & Feliachi, 2003), which are able to shorten the transient phase while avoiding the overshoots and large frequency deviations. However, they focus on the linearized system and did not consider the economic power dispatch problem (4) at the steady state.

4. Power imbalance allocation control

In this section, we introduce the *Power-Imbalance Allocation Control* (PIAC) method to solve Problems 1–3.

The communication network is necessary for solving Problem 2, for which we make the following assumption.

Assumption 3. All the buses in $\mathcal{V}_M \cup \mathcal{V}_F$ can communicate with a coordinator at a central location via a communication network. The frequency deviations ω_i , $i \in \mathcal{V}_M \cup \mathcal{V}_F$, can be measured and subsequently communicated to the coordinator. For $i \in \mathcal{V}_K$, the control input u_i can be computed by the coordinator and dispatched to the controller at node i via the communication network.

In Assumption 3, the local nodes need to provide the coordinator the local frequency deviation which are the differences between the measured frequencies by meters and the nominal frequency. We remark that there usually are time-delays in the measurement and communications which are neglected in this paper.

In the following, we first define an abstract frequency deviation for the power system, for which we design a control law to eliminate the overshoot of the control input. Then we introduce the PIAC method for the power system controlled as a single area in subsection 4.1 followed by a multi-area control approach in subsection 4.2.

Definition 1. For the power system (1), define an abstract frequency deviation ω_s such that

$$M_s \dot{\omega}_s = P_s - D_s \omega_s + u_s, \quad (11)$$

where $M_s = \sum_{i \in \mathcal{V}_M} M_i$, $P_s = \sum_{i \in \mathcal{V}} P_i$, $D_s = \sum_{i \in \mathcal{V}} D_i$, and $u_s = \sum_{i \in \mathcal{V}_K} u_i$.

Note that $\omega_s(t)$ is a virtual global frequency deviation, which neither equals $\omega_i(t)$ nor $\omega_{syn}(t)$ in general. However, if the power loads (or generation) P_i change slowly and the frequencies $\{\omega_i, i \in \mathcal{V}_M \cup \mathcal{V}_F\}$ synchronize quickly, the differences between $\omega_i(t)$ and $\omega_s(t)$ are negligible, i.e., $\omega_i(t) = \omega_s(t)$, by summing all the equations of (1), (11) is derived.

At the steady state of (11), we have $\omega_s^* = (P_s + u_s)/D_s$ which leads to $\omega_s^* = \omega_{syn}^*$ by (3). Then the objective of the secondary frequency control that restores the nominal frequency of the system (1) is equivalent to controlling ω_s of (11) to zero, i.e., $\lim_{t \rightarrow \infty} \omega_s(t) = 0$.

Since the frequencies $\{\omega_i, i \in \mathcal{V}_M \cup \mathcal{V}_F\}$ of system (1) with primary controllers synchronize quickly, the extra oscillations of ω_i are actually introduced by the overshoot of the total amount of control inputs u_s in the traditional control approach (8). This is because it is in the form of integral control. It will be explained in an extreme case where system (1) is well controlled by primary controllers such that $\omega_i = \omega_s$ for all $i \in \mathcal{V}$. It can be obtained easily for (8) by substituting ω_i by ω_s that the total amount of control inputs, u_s , is calculated as follows,

$$\dot{\lambda}_s = \omega_s, \quad (12a)$$

$$u_s = -k\lambda_s, \quad (12b)$$

which is in the form of integral control. Note that (12) can also be obtained similarly for the GB method (9) and for the DAI method (10) with a special setting of control gain coefficients k_i and communication weight w_{ij} e.g., $\{k_i, i \in \mathcal{V}_K\}$ are all identical and w_{ij} forms a Laplacian matrix, $L = (w_{ij})$, such that $\sum_{i=1}^{n_K} w_{ij} = 0$ for all $j = 1, \dots, n_K$ (n_K is the number of nodes in $\mathcal{V}_K$).

In order to accelerate the convergence of the frequency to its nominal value without introducing extra oscillations, the overshoot of u_s should be avoided. Similar to the PI cruise control of a car (Aström & Murray, 2008), for ω_s in (11), we introduce the following control law,

$$\dot{\eta}_s = D_s \omega_s, \quad (13a)$$

$$u_s = -kM_s \omega_s - k\eta_s, \quad (13b)$$

where $k \in (0, +\infty)$ is a parameter to be chosen by engineers. Details on the setting of this parameter will be further discussed after introducing the PIAC method in this section. From (11) and (13), we obtain

$$\dot{u}_s(t) = -k(P_s + u_s(t)), \quad (14)$$

which indicates that u_s is an estimate of $-P_s$ and converges to the power imbalance $-P_s$ exponentially. Hence, the overshoot of u_s is eliminated. With the value of u_s obtained from (13), the control input u_i for node $i \in \mathcal{V}_K$ is computed by solving the following economic power dispatch problem,

$$\begin{aligned} \min_{\{u_i \in \mathbb{R}, i \in \mathcal{V}_K\}} \quad & \sum_{i \in \mathcal{V}_K} J_i(u_i), \\ \text{s.t.} \quad & -u_s(t) + \sum_{i \in \mathcal{V}_K} u_i(t) = 0. \end{aligned} \quad (15)$$

However, u_s cannot be calculated as in (13) since ω_s is a virtual frequency deviation which cannot be measured in practice. In the following, we introduce the PIAC method where u_s also converges to $-P_s$ exponentially as in (14).

4.1. Single-area implementation of piac

We consider the power system controlled as a single area without any power export (or import). The PIAC method is defined as follows.

Definition 2 (PIAC). Consider the power system described by (1) with Assumptions 1–3, the PIAC control law is defined as the dynamic controller,

$$\dot{\eta}(t) = \sum_{i \in \mathcal{V}_M \cup \mathcal{V}_F} D_i \omega_i(t), \quad (16a)$$

$$u_s(t) = -k(\eta(t) + \sum_{j \in \mathcal{V}_M} M_j \omega_j(t)), \quad (16b)$$

$$0 = -u_s(t) + \sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda(t)), \quad (16c)$$

$$u_i(t) = J_i'^{-1}(\lambda(t)), \quad i \in \mathcal{V}_K, \quad (16d)$$

where $k \in (0, \infty)$ is a parameter of the control law, η is a state variable introduced for the integral term as η_s in (13a), and λ is an algebraic variable for solving the optimization problem (15), which is a function of time.

For the special case with quadratic cost function $J_i(u_i) = \frac{1}{2} \alpha_i u_i^2$ for node i , the control law becomes

$$\dot{\eta} = \sum_{i \in \mathcal{V}_M \cup \mathcal{V}_F} D_i \omega_i,$$

$$u_s = -k(\sum_{i \in \mathcal{V}_M} M_i \omega_i + \eta_s),$$

$$u_i = \frac{\alpha_s}{\alpha_i} u_s, \quad i \in \mathcal{V}_K,$$

$$\frac{1}{\alpha_s} = \sum_{i \in \mathcal{V}_K} \frac{1}{\alpha_i},$$

where $\alpha_i \in \mathbb{R}$ is the control price of node i .

PIAC is based on the design principle of coordination. The local nodes in $\mathcal{V}_M \cup \mathcal{V}_F$ send the measured frequency deviations to the coordinator. The coordinator computes the control inputs $\{u_i, i \in \mathcal{V}_K\}$ or marginal cost λ using the local measurements and sends them to all local controllers of the nodes indexed by $\mathcal{V}_K$. The procedure is similar to the GB method which gathers the locally measured frequency deviation from the nodes and broadcasts the nodal price to the controllers. The control procedure and properties of PIAC are summarized in the following procedure and theorem respectively.

Procedure 1. If Assumptions 1–3 hold, then the secondary frequency control approach of the power system (1), PIAC, is implemented as follows.

(i) Collect the measured frequency deviations

$$\{\omega_i, i \in \mathcal{V}_M \cup \mathcal{V}_F\},$$

(ii) Calculate the total amount $u_s(t)$ of control inputs by (16b),

(iii) Solve the optimization problem (15) by (16c), (16d),

(iv) Allocate the power compensation $\{u_i, i \in \mathcal{V}_K\}$ to the controllers.

Theorem 1. Consider the power system (1) and the PIAC control law, the controller has the following properties,

(a) at any time, $t \in T$, $u_s(t)$ satisfies (14). Thus it is an estimate of the power imbalance $-P_s$;

(b) at any time, $t \in T$, the input values $\{u_i(t), i \in \mathcal{V}_K\}$ are computed by solving the optimization problem (15). So the total amount of control inputs, $u_s(t)$, are dispatched to the local nodes of $\mathcal{V}_K$ economically;

- (c) at the steady state, $\omega_i^* = 0$ for all $i \in \mathcal{V}_M \cup \mathcal{V}_F$ and the power-imbalance P_s is optimally compensated by the local controllers. Hence both [Problems 1](#) and [2](#) are solved;
- (d) because

$$u_s(t) = -k \sum_{i \in \mathcal{V}_M} M_i \omega_i(t) - k \int_0^t \sum_{j \in \mathcal{V}_M \cup \mathcal{V}_F} D_j \omega_j(\tau) d\tau,$$

the PIAC control law is of Proportional-Integral type.

Proof. (a). From [\(1\)](#) it follows,

$$\sum_{i \in \mathcal{V}_M} M_i \dot{\omega}_i = \sum_{i \in \mathcal{V}} P_i - \sum_{i \in \mathcal{V}_M \cup \mathcal{V}_F} D_i \omega_i + \sum_{i \in \mathcal{V}} u_i. \quad (18)$$

It follows from [\(16a\)](#), [\(16b\)](#) that,

$$\dot{u}_s(t) = -k \left(\sum_{i \in \mathcal{V}_M \cup \mathcal{V}_F} D_i \omega_i(t) + \sum_{j \in \mathcal{V}_M} M_j \dot{\omega}_j(t) \right)$$

by [\(18\)](#),

$$= -k \left(\sum_{i \in \mathcal{V}_K} u_i(t) + \sum_{j \in \mathcal{V}} P_j \right)$$

by [\(16d\)](#),

$$= -k \left(\sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda) + \sum_{j \in \mathcal{V}} P_j \right)$$

by [\(16c\)](#),

$$= -k(u_s(t) + P_s),$$

Thus [\(14\)](#) is obtained which indicates that $u_s(t)$ is an estimate of $-P_s$ with $k \in (0, \infty)$, i.e., $u_s(t)$ converges to $-P_s$ exponentially with a speed determined by the control gain coefficient k .

(b). According to the definition of the PIAC control law that at any time, $t \in T$,

$$J_i'(u_i(t)) = J_j'(u_j(t)) = \lambda(t), \quad \forall i, j \in \mathcal{V}_K.$$

Thus the necessary condition [\(5\)](#) for economic dispatch of $u_s(t)$ to all the local nodes is satisfied and at any time $t \in T$, the control inputs $\{u_i, i \in \mathcal{V}_K\}$ solve the optimization problem [\(15\)](#). Hence, $\{u_i(t), i \in \mathcal{V}\}$ achieve the minimal cost.

(c). It follows from [\(14\)](#) that $u_s^* = -\sum_{i \in \mathcal{V}} P_i$ at the steady state, which yields $\omega_{syn} = 0$ by [\(3\)](#). Thus $\omega_i^* = 0$ for all $i \in \mathcal{V}_M \cup \mathcal{V}_F$ by [\(2\)](#), [Problem 1](#) is solved. It follows from (b) that the control inputs $\{u_i, i \in \mathcal{V}\}$ achieve minimal cost at the steady state. So the optimization problem [\(4\)](#) is solved at the steady state. Hence, [Problem 2](#) is solved.

(d). This follows directly from the definition of the PIAC control law. $\square$

Note that in [Theorem 1\(c\)](#), we have assumed that the steady state exists which will be further described in [Assumption 4](#) in [Section 5](#).

In order to clearly illustrate how PIAC improves the transient performance of the system, with the abstract frequency deviation ω_s and control inputs u_s , we decompose the dynamic process into three sub-processes,

- (i) the convergence process of u_s to $-P_s$ as in [\(14\)](#) with a speed determined by k .
- (ii) the convergence process of ω_s to zero as in [\(11\)](#) with a speed determined by u_s and D_s .
- (iii) and the synchronization process of $\{\omega_i, i \in \mathcal{V}_M \cup \mathcal{V}_F\}$ to ω_s described by [\(1\)](#) and with the synchronization speed mainly determined by $\{D_i, i \in \mathcal{V}_M \cup \mathcal{V}_F\}$.

The transient performance of the power system can be improved by tuning the corresponding control gain coefficients for the sub-processes. PIAC focuses on the first two sub-processes only in which the transient behaviors of u_s and ω_s can be improved by increasing the parameter k , and the primary control focuses on the synchronization of $\{\omega_i, i \in \mathcal{V}_M \cup \mathcal{V}_F\}$ which can be improved by tuning the coefficient $\{D_i, i \in \mathcal{V}_M \cup \mathcal{V}_F\}$ as in [Dörfler et al. \(2016\)](#), and [Motter, Myers, Anghel, and Nishikawa \(2013\)](#). As u_s converges to $-P_s$ exponentially, the control inputs $\{u_i, i \in \mathcal{V}_K\}$ in [\(16d\)](#) also converge to the desired optimal values in an exponential way without any overshoots and their convergence speed increase with a large k . Thus the extra oscillation caused by the overshoot of u_s is avoided and [Problem 3](#) is solved by PIAC.

With the process decomposition, the improvement in the transient performance by the UC control law ([Zhao et al., 2016](#)) can be explained, and the control gain coefficients of the primal-dual based control laws, e.g., [Wang et al. \(2017\)](#) without triggered constraints of line flows, which also estimate the power disturbance, can be tuned to improve the transient performance.

In the following, we introduce the properties of PIAC in the implementation for the power system controlled as a single area.

On the communications in PIAC, note that the communicated data include dynamic data and static data. The dynamic data are the frequency deviations $\{\omega_i, i \in \mathcal{V}_M \cup \mathcal{V}_F\}$ and the control inputs $\{u_i, i \in \mathcal{V}_K\}$, both should be communicated frequently. The static data are the moment inertia $\{M_i, i \in \mathcal{V}_i\}$, the droop control coefficients $\{D_i, i \in \mathcal{V}_M \cup \mathcal{V}_F\}$ and the cost functions $\{J_i(u_i), i \in \mathcal{V}_K\}$, which are constant in a short time as in [Assumption 1](#) and are not necessary to communicate as frequently as the dynamic data.

In the computing of the control inputs $\{u_i, i \in \mathcal{V}_K\}$, solving the optimization problem [\(15\)](#) is equivalent to solving Eqs. [\(16c\)](#) and [\(16d\)](#). So the computation includes calculating the integral of frequency deviation in [\(16a\)](#) and solving the algebraic equations [\(16c\)](#), [\(16d\)](#). For quadratic cost functions $J_i(u_i) = \frac{1}{2} \alpha_i u_i^2$, the computation requires approximately $4n_M + n_F$ arithmetic operations that are $\times, +, -$ etc. For nonlinear cost functions, an iteration method is used to solve the one dimension nonlinear algebraic equation [\(16c\)](#) for λ , which needs more computing than for the quadratic cost functions.

Remark 1. PIAC is a centralized control where the communications and computations increase as the scale of the power systems increases. In a large-scale power system, a large number of devices communicate to the coordinator simultaneously. In that case, the time-delay due to the communications and computing is not negligible, in such a situation further investigation for the transient performance and stability of the nonlinear system is needed.

On the dynamics of u_s , it can be observed from [\(14\)](#) that u_s converges exponentially with a speed that can be accelerated by increasing the control gain coefficient k which does not depend on any parameters of the power system and the cost functions. Hence, when the dynamics of the voltages are considered which does not influence the dynamics of u_s in [\(14\)](#), the power supply and demand can also be balanced.

On the control gain coefficient k , we remark that it does neither depend on the parameters of the system nor on the economic power dispatch problem, and can be set very large from the perspective of theoretical analysis. However, it relies on how sensitive the control devices are to the fluctuation of power imbalance. In this case, it can be tuned according to the response time of the control devices and the desired range of the frequency deviation. The control actuators of traditional hydraulic and steam turbines are their governor systems. For details of the model of the governor system and its response time, we refer to [Kundur \(1994, chap. 9, 11\)](#).

By [Assumption 3](#), the PIAC method requires that all the nodes in $\mathcal{V}_M \cup \mathcal{V}_F$ can communicate with the coordinator. However, our initial numerical experiments show that PIAC can still restore the nominal frequency even without all the frequency deviation collected, where the estimated power imbalance converges to the actual power imbalance although not exponentially. This is because PIAC includes the integral control which drives the synchronized frequency deviation to zero.

Remark 2. In practice, the state of the power system is never at a true equilibrium state, because of the fluctuations of the power imbalance caused by the power loads. Furthermore, the fluctuations become even more serious when a large amount of renewable power sources are integrated into the power system. In this case, it is more practical to model the power imbalance as a time-varying function. For the power system with time-varying power imbalance, analysis and numerical simulations show that PIAC is also able to effectively control the synchronized frequency to any desired range by increasing the control gain coefficient k ([Xi, Lin, & van Schuppen, 2017c](#)).

4.2. Multi-area implementation of PIAC

When a power system is partitioned into several control areas, each area has either an export or an import of power. After a disturbance occurred, the power export or the power import of an area must be restored to the nominal value calculated by tertiary control. In this subsection, we introduce the implementation of PIAC for multi-area control where each area has power export (or import).

Denote the set of the nodes in the area A_r by $\mathcal{V}_{A_r}$, the set of the boundary lines between area A_r and all the other areas by $\mathcal{E}_{A_r}$. Denote $\mathcal{V}_{M_r} = \mathcal{V}_M \cap \mathcal{V}_{A_r}$, $\mathcal{V}_{F_r} = \mathcal{V}_F \cap \mathcal{V}_{A_r}$ and $\mathcal{V}_{K_r} = \mathcal{V}_K \cap \mathcal{V}_{A_r}$. The multi-area control of PIAC in the area A_r is defined as follows.

Definition 3. Consider the power system (1) controlled as several areas with [Assumptions 1–3](#), the multi-area implementation of PIAC for area A_r is defined as the dynamic controllers,

$$\dot{\eta}_r(t) = \sum_{i \in \mathcal{V}_{M_r} \cup \mathcal{V}_{F_r}} D_i \omega_i(t) + P_{ex}(t) - P_{ex}^*, \quad (20a)$$

$$u_r(t) = -k_r \left(\sum_{i \in \mathcal{V}_{M_r}} M_i \omega_i(t) + \eta_r(t) \right), \quad (20b)$$

$$0 = -u_r + \sum_{i \in \mathcal{V}_{K_r}} J_i'^{-1}(\lambda_r), \quad (20c)$$

$$u_i = J_i'^{-1}(\lambda_r), \quad i \in \mathcal{V}_{M_r} \cup \mathcal{V}_{F_r} \quad (20d)$$

where $P_{ex} = \sum_{i \in \mathcal{V}_{A_r}, (i,j) \in \mathcal{E}_{A_r}} B_{ij} \sin(\theta_i - \theta_j)$ is the export power of area A_r , P_{ex}^* is the nominal value of P_{ex} , $k_r \in (0, \infty)$ is a control gain coefficient, η_r is a state variable for the integral term as η_s in (13a) and λ_r is an algebraic variable for the controller.

It can be observed from (20) that the control procedure for the coordinator in area A_r is similar to [Procedure 1](#) but the power export deviation, $P_{ex} - P_{ex}^*$, should be measured. The sum of the three terms on the right hand side of (20a) is actually the ACE of the area. PIAC has the proportional control input included in secondary frequency control, which is consistent with the PI based secondary frequency control ([Machowski, Bialek, & Bumby, 2008](#), chap. 9) and ([Bevrani, 2014](#), chap. 4). In PIAC, the weights of the frequency deviation of node i are specified as the inertia M_i and the droop coefficient D_i for the proportional input and integral input respectively. The proportional input is used to estimate the power stored in the inertia at the transient phase, which is usually neglected in the traditional ACE method. The control gain coefficient

k_r can be different for each area, which can be tuned according to the sensitivity of the control devices in the area. By (20a), as the synchronous frequency deviation is steered to zero, P_{ex} also converges to the nominal value P_{ex}^* . Similar as the derivation of (14) for PIAC (16), we derive for (20) that

$$\dot{u}_r(t) = -k_r \left(\sum_{i \in \mathcal{V}_{A_r}} P_i + P_{ex}^* + u_r(t) \right)$$

which indicates that the controllers in area A_r only respond to the power imbalance $\sum_{i \in \mathcal{V}_{A_r}} P_i$. Hence, in the network, the control actions of all the areas can be done in an asynchronous way where each area can balance the local power supply–demand at any time according to the availability of the devices.

In particular, PIAC becomes a decentralized control method if each node is seen as a single area and $\mathcal{V}_P = \emptyset$, $\mathcal{V}_K = \mathcal{V}_M \cup \mathcal{V}_F$, i.e., for all $i \in \mathcal{V}_M \cup \mathcal{V}_F$,

$$\dot{\eta}_i = D_i \omega_i + \sum_{j \in \mathcal{V}} B_{ij} \sin(\theta_{ij}) - \sum_{j \in \mathcal{V}} B_{ij} \sin(\theta_{ij}^*),$$

$$u_i = -k_i(M_i \omega + \eta_i),$$

where $\{\theta_i^*, i \in \mathcal{V}\}$ are the steady state calculated by tertiary control. Since u_i is tracking $-P_i + \sum_{j \in \mathcal{V}} B_{ij} \sin(\theta_{ij}^*)$, each node compensates the power imbalance locally and the control actions of the nodes are irrelevant to each other. However, the control cost is not optimized by this decentralized control method.

5. Stability analysis of PIAC

In this section, we analyze the stability of PIAC with the Lyapunov/LaSalle approach as in [De Persis and Monshizadeh \(2018\)](#), and [Trip et al. \(2016\)](#). The stability proof makes use of [Theorem A.1](#) stated in the [Appendix](#). As indicated in Section 4.2, the control actions of the areas are decoupled in the multi-area control network. So we only need to prove the stability of PIAC implemented in a single-area network. Extension to multi-area control networks then follows immediately. With the control law (16), the closed-loop system of PIAC is

$$\dot{\theta}_i = \omega_i, \quad i \in \mathcal{V}_M \cup \mathcal{V}_F, \quad (22a)$$

$$M_i \dot{\omega}_i = P_i - D_i \omega_i - \sum_{j \in \mathcal{V}} B_{ij} \sin(\theta_{ij}) + J_i'^{-1}(\lambda), \quad i \in \mathcal{V}_M, \quad (22b)$$

$$0 = P_i - D_i \omega_i - \sum_{j \in \mathcal{V}} B_{ij} \sin(\theta_{ij}) + J_i'^{-1}(\lambda), \quad i \in \mathcal{V}_F, \quad (22c)$$

$$0 = P_i - \sum_{j \in \mathcal{V}} B_{ij} \sin(\theta_{ij}), \quad i \in \mathcal{V}_P, \quad (22d)$$

$$\dot{\eta} = \sum_{i \in \mathcal{V}_M \cup \mathcal{V}_F} D_i \omega_i, \quad (22e)$$

$$0 = \sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda) + k \left(\sum_{i \in \mathcal{V}_M} M_i \omega_i + \eta \right), \quad (22f)$$

$$0 = J_i'^{-1}(\lambda), \quad i \notin \mathcal{V}_K, \quad (22g)$$

where $\theta_{ij} = \theta_i - \theta_j$ is the phase angle differences between the nodes connected by a transmission line.

We denote the angles in the sets $\mathcal{V}_M$, $\mathcal{V}_F$, $\mathcal{V}_P$ by column vectors θ_M , θ_F , θ_P , the frequency deviations by column vectors ω_M , ω_F , ω_P , the angles in $\mathcal{V}$ by $\theta = (\theta_M^T, \theta_F^T, \theta_P^T)^T$, and the frequency deviations by $\omega = (\omega_M^T, \omega_F^T, \omega_P^T)^T$.

Note that the closed-loop system may not have a synchronous state if the power injections $\{P_i, i \in \mathcal{V}\}$ are much larger than the line capacity B_{ij} . For more details on the synchronous state of the power system, we refer to [Dörfler and Bullo \(2012\)](#) and [Xi, Dubbeldam, and Lin \(2017a\)](#). Therefore, we make [Assumption 4](#).

Assumption 4. For the closed-loop system (22), there exists a synchronous state $(\theta^*, \omega^*, \eta^*, \lambda^*) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R} \times \mathbb{R}$ with $\omega_i^* = \omega_{syn}$ and

$$\theta^*(t) \in \Theta = \{\theta \in \mathbb{R}^n \mid |\theta_i - \theta_j| < \frac{\pi}{2}, \forall (i, j) \in \mathcal{E}\}.$$

The condition $\theta^* \in \Theta$ is commonly referred to as a security constraint (De Persis & Monshizadeh, 2018) in power system analysis. It can be satisfied by reserving some margin of power flow when calculating the operating point in tertiary control (Ilic & Zaborsky, 2000).

Since the power flows $B_{ij} \sin(\theta_i - \theta_j)$, $(i, j) \in \mathcal{E}$ only depend on the angle differences and the angles can be expressed relative to a reference node, we choose a reference angle, i.e., θ_1 , in $\mathcal{V}_M$ and introduce the new variables

$$\varphi_i = \theta_i - \theta_1, i = 1, 2, \dots, n$$

which yields $\dot{\varphi}_i = \omega_i - \omega_1$. Note that $\varphi_1 = 0, \dot{\varphi}_1 = 0$ for all $t > 0$. With $\omega_i = \dot{\varphi}_i + \omega_1$ for $i \in \mathcal{V}_F$, the closed loop system (22) can be written in the DAE form as (A.1) in the Appendix,

$$\dot{\varphi}_i = \omega_i - \omega_1, i \in \mathcal{V}_M \cup \mathcal{V}_F, \quad (23a)$$

$$M_i \dot{\omega}_i = P_i - D_i \omega_i - \sum_{j \in \mathcal{V}} B_{ij} \sin(\varphi_{ij}) + J_i^{-1}(\lambda), i \in \mathcal{V}_M, \quad (23b)$$

$$D_i \dot{\varphi}_i = P_i - D_i \omega_1 - \sum_{j \in \mathcal{V}} B_{ij} \sin(\varphi_{ij}) + J_i^{-1}(\lambda), i \in \mathcal{V}_F, \quad (23c)$$

$$0 = P_i - \sum_{j \in \mathcal{V}} B_{ij} \sin(\varphi_{ij}), i \in \mathcal{V}_P, \quad (23d)$$

$$\dot{\eta} = \sum_{i \in \mathcal{V}_M \cup \mathcal{V}_F} D_i \omega_i, \quad (23e)$$

$$0 = \sum_{i \in \mathcal{V}_K} J_i^{-1}(\lambda) + k \left(\sum_{i \in \mathcal{V}_M} M_i \omega_i + \eta \right), \quad (23f)$$

where $\varphi_{ij} = \varphi_i - \varphi_j$, and Eqs. (23a)–(23d) are from the power system and (23e)–(23f) from the controllers. We next recast the system (23) into the form of the DAE system (A.1), the state variables are $x = (\varphi_M, \varphi_F, \omega_M, \eta) \in \mathbb{R}^{n_M-1} \times \mathbb{R}^{n_F} \times \mathbb{R}^{n_M} \times \mathbb{R}$, the algebraic variables are $y = (\varphi_P, \lambda) \in \mathbb{R}^{n_P} \times \mathbb{R}$, the differential equations are (23a)–(23c), (23e) and the algebraic equations are (23d), (23f). Here φ_M is with the components $\{\varphi_i, i \in \mathcal{V}_M\}$ besides φ_1 which is a constant, φ_F with components $\{\varphi_i, i \in \mathcal{V}_F\}$, and φ_P with components $\{\varphi_i, i \in \mathcal{V}_P\}$. Note that the variables $\{\omega_i, i \in \mathcal{V}_F\}$ are not included into the state variable or algebraic variables since the terms $\{D_i \omega_i, i \in \mathcal{V}_F\}$ in (23e) can be replaced by $\{P_i - \sum_{j \in \mathcal{V}} B_{ij} \sin(\varphi_{ij}) + J_i^{-1}(\lambda), i \in \mathcal{V}_F\}$. (22g) is neglected since it is irrelevant to the following stability analysis.

When mapping θ to the coordinate of φ , Assumption 4 yields

$$\varphi \in \Phi = \{\varphi \in \mathbb{R}^n \mid |\varphi_i - \varphi_j| < \frac{\pi}{2}, \forall (i, j) \in \mathcal{E}, \text{ and } \varphi_1 = 0\}.$$

We remark that each equilibrium state of (23) corresponds to a synchronous state of (22). In the new coordinate, we have the following theorem for the stability of the system (23).

Theorem 2. If the Assumptions 1–4 hold, for the system (23), then

- there exists a unique equilibrium state $z^* = (\varphi^*, \omega_M^*, \eta^*, \lambda^*) \in \Psi$ where $\Psi = \Phi \times \mathbb{R}^{n_M} \times \mathbb{R} \times \mathbb{R}$.
- there exists a domain $\Psi^d \subset \Psi$ such that for any initial state $z^0 \in \Psi^d$ that satisfies the algebraic equations (23d) and (23f), the state trajectory converges to the unique equilibrium state $z^* \in \Psi$.

Note that the cost functions are not required to be scaled for the local asymptotically stable of PIAC as assumed in Dörfler and Grammatico (2017), and the size of the attraction domain of the

equilibrium state z^* is not determined in Theorem 2 which states the stability of the PIAC method. The proof of Theorem 2 is based on the Lyapunov/LaSalle stability criterion as in Theorem A.1. In the following, we first present the verification of Assumptions 1 and 2 in the Appendix for the DAE system (23), then prove the stability of (23) by designing a Lyapunov function as $V(x, y)$ in Theorem A.1. Lemma 1 states that (23) possesses an equilibrium state, which verifies Assumption A.1, and Lemma 2 claims the regularity of the algebraic equations (23d), (23f), which verifies Assumption A.2.

Lemma 1. There exists at most one equilibrium state $z^* = (\varphi^*, \omega_M^*, \eta^*, \lambda^*)$ of the system (23) such that $z^* \in \Psi$ and

$$\omega_i^* = 0, i \in \mathcal{V}_M, \quad (24a)$$

$$\sum_{i \in \mathcal{V}} P_i + \sum_{i \in \mathcal{V}_K} J_i^{-1}(\lambda^*) = 0. \quad (24b)$$

$$\sum_{i \in \mathcal{V}} P_i - k \eta^* = 0. \quad (24c)$$

Proof. At the synchronous state, from (14) and (16b) it follows that

$$k \left(\sum_{i \in \mathcal{V}_M} M_i \omega_i^* + \eta^* \right) = \sum_{i \in \mathcal{V}} P_i. \quad (25)$$

Substitution of (25) into the algebraic equation (23f) and (24b). Substitution of (24b) into (3) with $u_i = J_i^{-1}(\lambda^*)$, we obtain $\omega_{syn} = 0$ which yields $\{\omega_i^* = 0, i \in \mathcal{V}_M\}$. Hence this and (25) yield

$$\sum_{i \in \mathcal{V}_K} J_i^{-1}(\lambda^*) + k \eta^* = 0.$$

which leads to (24c). It follows from Araposthatis, Sastry, and Varaiya (1981), and Skar (1980) that the system (22) has at most one power flow solution such that $\theta \in \Theta$. Hence there exists at most one equilibrium for the system (23) that satisfies $\varphi \in \Phi$. $\square$

With respect to the regularity of the algebraic equations (23d), (23f), we derive the following lemma.

Lemma 2. For any $\varphi \in \Phi$ and strictly convex functions of $\{J_i(u_i), i \in \mathcal{V}_K\}$ in the optimization problem (4), the algebraic equations (23d), (23f) are regular.

Proof. Since (23d) and (23f) are independent algebraic equations with respect to φ_P and λ respectively, the regularity of each of them is proven separately.

First, we prove the regularity of (23d) by showing that its Jacobian is a principle minor of the Laplacian matrix of a weighted network. In the coordination of θ , we define function

$$\bar{U}(\theta) = \sum_{(i,j) \in \mathcal{E}} B_{ij} (1 - \cos(\theta_i - \theta_j)).$$

The Hessian matrix of $\bar{U}(\theta)$ is

$$\bar{L} = \begin{pmatrix} \bar{B}_{11} & -B_{12} \cos(\theta_{12}) & \dots & -B_{1n} \cos(\theta_{1n}) \\ -B_{21} \cos(\theta_{12}) & \bar{B}_{22} & \dots & -B_{2n} \cos(\theta_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ -B_{n1} \cos(\theta_{n1}) & -B_{n2} \cos(\theta_{n2}) & \dots & \bar{B}_{nn} \end{pmatrix},$$

where $\bar{B}_{ii} = \sum_{j \in \mathcal{V}} B_{ij} \cos(\theta_{ij})$ and $\theta_{ij} = \theta_i - \theta_j$. $\bar{L}$ is the Laplacian of the undirected graph G defined in Section 2 with positive line weights $B_{ij} \cos(\theta_i - \theta_j)$. Hence $\bar{L}$ is semi-positive definite and all its principle minors are nonsingular (Brualdi & Ryser, 1991, Theorem 9.6.1). In the coordination of φ , we define function

$$U(\varphi) = \sum_{(i,j) \in \mathcal{E}} B_{ij} (1 - \cos(\varphi_i - \varphi_j)), \quad (26)$$

with Hessian matrix

$$L = \begin{pmatrix} B_{22} & -B_{23} \cos(\varphi_{23}) & \dots & -B_{2n} \cos(\varphi_{2n}) \\ -B_{32} \cos(\varphi_{32}) & B_{33} & \dots & -B_{3n} \cos(\varphi_{3n}) \\ \vdots & \vdots & \ddots & \vdots \\ -B_{n2} \cos(\varphi_{n2}) & -B_{n3} \cos(\varphi_{n3}) & \dots & B_{nn} \end{pmatrix} \quad (27)$$

where $B_{ij} = \sum_{j \in \mathcal{V}} B_{ij} \cos(\varphi_{ij})$, $\varphi_{ij} = \varphi_i - \varphi_j$, and $\varphi_1 = 0$. Note that $B_{ij} \cos(\varphi_{ij}) = B_{ij} \cos(\theta_{ij})$, thus L is a principle minor of $\bar{L}$ and is nonsingular. Hence the Jacobian of (23d) with respect to φ_p , which is a principle minor of L , is nonsingular.

Second, because J_i is strictly convex by Assumption 2 such that $J_i'' > 0$, we obtain $(J_i'^{-1})' = \frac{1}{J_i''} > 0$ which yields $(\sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda))' > 0$. Hence (23f) is nonsingular. $\square$

What follows is the proof of Theorem 2 with the Lyapunov/LaSalle stability criterion.

Proof of Theorem 2. Lemma 1 and Assumption 4 state that the equilibrium state $z^* = (\varphi^*, \omega_M^*, \eta^*, \lambda^*) \in \Psi$ is unique. The proof of the statement (b) in Theorem 2 is based on Theorem A.1. Consider an incremental Lyapunov function candidate,

$$V(\varphi, \omega_M, \eta, \lambda) = V_1 + \alpha V_2 + V_3, \quad (28)$$

where V_1 is the classical energy-based function (De Persis & Monshizadeh, 2018),

$$V_1(\varphi, \omega_M) = U(\varphi) - U(\varphi^*) - \nabla_\varphi U(\varphi^*)(\varphi - \varphi^*) + \frac{1}{2} \omega_M^T M_M \omega_M,$$

and V_2, V_3 are positive definite functions

$$V_2(\lambda) = \frac{1}{2} \left(\sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda) - \sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda^*) \right)^2,$$

$$V_3(\omega_M, \eta) = \frac{k^2}{2} \left(\sum_{i \in \mathcal{V}_M} M_i \omega_i + \eta - \eta^* \right)^2.$$

Note that the definition of $U(\varphi)$ is in (26) and $V_2 = V_3$ by (23f). V_3 is introduced to involve the state variable η into the Lyapunov function.

First, we prove that $\dot{V} \leq 0$. From the dynamic system (23) and the definition of V_1 , it yields that

$$\dot{V}_1 = - \sum_{i \in \mathcal{V}_M \cup \mathcal{V}_F} D_i \omega_i^2 + \sum_{i \in \mathcal{V}_K} \omega_i (J_i'^{-1}(\lambda) - J_i'^{-1}(\lambda^*)), \quad (29)$$

by (23f), we derive

$$\begin{aligned} \dot{V}_2 &= -k \left[\sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda) - \sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda^*) \right] \left[\sum_{i \in \mathcal{V}_M} M_i \dot{\omega}_i + \dot{\eta} \right] \\ &\quad \text{by summing (23b–23e)} \\ &= -k \left[\sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda) - \sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda^*) \right] \left[\sum_{i \in \mathcal{V}} P_i + \sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda) \right] \\ &\quad \text{by (24b)} \\ &= -k \left[\sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda) - \sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda^*) \right]^2, \quad (30a) \\ &\quad \text{by expanding the quadratic} \\ &= -k \sum_{i \in \mathcal{V}_K} [J_i'^{-1}(\lambda) - J_i'^{-1}(\lambda^*)]^2 \\ &\quad - 2k \sum_{i \neq j, i, j \in \mathcal{V}_K} [J_i'^{-1}(\lambda) - J_i'^{-1}(\lambda^*)] [J_j'^{-1}(\lambda) - J_j'^{-1}(\lambda^*)] \\ &\quad \text{by Newton–Leibniz formula} \end{aligned}$$

$$\begin{aligned} &= -k \sum_{i \in \mathcal{V}_K} [J_i'^{-1}(\lambda) - J_i'^{-1}(\lambda^*)]^2 \\ &\quad - 2k \sum_{i \neq j, i, j \in \mathcal{V}_K} \left[\int_{\lambda^*}^{\lambda} (J_i'^{-1})' ds \right] \left[\int_{\lambda^*}^{\lambda} (J_j'^{-1})' ds \right]. \end{aligned} \quad (30b)$$

and by $V_3 = V_2$ and (30a), we obtain

$$\dot{V}_3 = -k \left[\sum_{i \in \mathcal{V}_K} (J_i'^{-1}(\lambda) - J_i'^{-1}(\lambda^*)) \right]^2. \quad (31)$$

Hence, with (29), (30b), (31), we derive

$$\begin{aligned} \dot{V} &= - \sum_{i \in \mathcal{V}_M \cup \mathcal{V}_F} D_i \omega_i^2 + \sum_{i \in \mathcal{V}_K} \omega_i [J_i'^{-1}(\lambda) - J_i'^{-1}(\lambda^*)] \\ &\quad - k \alpha \sum_{i \in \mathcal{V}_K} [J_i'^{-1}(\lambda) - J_i'^{-1}(\lambda^*)]^2 \\ &\quad - k \alpha \sum_{i \neq j, i, j \in \mathcal{V}_K} \left[\int_{\lambda^*}^{\lambda} (J_i'^{-1})' ds \right] \left[\int_{\lambda^*}^{\lambda} (J_j'^{-1})' ds \right] \\ &\quad - k \left[\sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda) - \sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda^*) \right]^2 \\ &= - \sum_{i \in \mathcal{V}_K} \frac{D_i}{2} \omega_i^2 - \sum_{i \in \mathcal{V}_K} \frac{D_i}{2} \left[\omega_i - \frac{J_i'^{-1}(\lambda) - J_i'^{-1}(\lambda^*)}{D_i} \right]^2 \\ &\quad - \sum_{i \in \mathcal{V}_K} \left[k \alpha - \frac{1}{2D_i} \right] [J_i'^{-1}(\lambda) - J_i'^{-1}(\lambda^*)]^2 \\ &\quad - 2k \alpha \sum_{i \neq j, i, j \in \mathcal{V}_K} \left[\int_{\lambda^*}^{\lambda} (J_i'^{-1})' ds \right] \left[\int_{\lambda^*}^{\lambda} (J_j'^{-1})' ds \right] \\ &\quad - k \left[\sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda) - \sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda^*) \right]^2 - \sum_{i \in \mathcal{V}_M \cup \mathcal{V}_F / \mathcal{V}_K} D_i \omega_i^2 \end{aligned}$$

where the equation

$$\sum_{i \in \mathcal{V}_M \cup \mathcal{V}_F} D_i \omega_i^2 = \sum_{i \in \mathcal{V}_K} D_i \omega_i^2 + \sum_{i \in \mathcal{V}_M \cup \mathcal{V}_F \setminus \mathcal{V}_K} D_i \omega_i^2$$

is used due to the fact that $\mathcal{V}_K \subseteq \mathcal{V}_M \cup \mathcal{V}_F$.

Since $J_i(u_i)$ is strictly convex and $(J_i'^{-1})' = \frac{1}{J_i''} > 0$, we derive

$$k \alpha \sum_{i \neq j, i, j \in \mathcal{V}_K} \left[\int_{\lambda^*}^{\lambda} (J_i'^{-1})' ds \right] \left[\int_{\lambda^*}^{\lambda} (J_j'^{-1})' ds \right] > 0,$$

Thus by setting $\alpha > \frac{1}{kD_i}$, we obtain $\dot{V} \leq 0$.

Second, we prove that $z^* = (\varphi^*, \omega_M^*, \eta^*, \lambda^*)$ is a strict minimum of $V(\varphi, \omega_M, \eta, \lambda)$ such that $\nabla V|_{z^*} = 0$ and $\nabla^2 V|_{z^*} > 0$. It can be easily verified that $V|_{z^*} = 0$ and

$$\nabla V|_{z^*} = \text{col}(\nabla_\varphi V, \nabla_{\omega_M} V, \nabla_\eta V, \nabla_\lambda V)|_{z^*} = 0 \in \mathbb{R}^{n+n_M+1}$$

where

$$\nabla_\varphi V = \nabla_\varphi U - \nabla_\varphi U^*,$$

$$\nabla_{\omega_M} V = M \omega_M + M \left(k^2 \left(\sum_{i \in \mathcal{V}_M} M_i \omega_i + \eta - \eta^* \right) 1_{n_M} \right),$$

$$\nabla_\eta V = k^2 \left(\sum_{i \in \mathcal{V}_M} M_i \omega_i + \eta - \eta^* \right),$$

$$\nabla_\lambda V = \alpha \left(\sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda) - \sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda^*) \right) \left(\sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda) \right)'.$$

Here $k^2(\sum_{i \in \mathcal{V}_M} M_i \omega_i + \eta - \eta^*) 1_{n_M}$ is a vector with all components equal to $k^2(\sum_{i \in \mathcal{V}_M} M_i \omega_i + \eta - \eta^*)$. The Hessian matrix of V is

$$\nabla^2 V|_{z^*} = \text{blkdiag}(L, H, \Lambda),$$

which is a block diagonal matrix with block matrices L , H , and Λ . L is positive definite by (27),

$$\Lambda = \alpha \left(\left(\sum_{i \in \mathcal{V}_K} J_i'^{-1}(\lambda^*) \right)' \right)^2 > 0$$

which is a scalar value, and H is the Hessian matrix of the function

$$\bar{V} = \frac{1}{2} \omega_M^T M_M \omega_M + \frac{k^2}{2} \left(\sum_{i \in \mathcal{V}_M} M_i \omega_i + \eta - \eta^* \right)^2$$

which is positive definite for any $(\omega_M, \eta - \eta^*)$, thus H is positive definite. Hence, we have proven that z^* is a strict minimum of V .

Finally, we prove that the invariant set

$$\{(\varphi, \omega_M, \eta, \lambda) | \dot{V}(\varphi, \omega_M, \eta, \lambda) = 0\}$$

contains only the equilibrium point. $\dot{V} = 0$ implies that $\{\omega_i = 0, i \in \mathcal{V}_M \cup \mathcal{V}_F\}$. Hence $\{\varphi_i, i \in \mathcal{V}\}$ are constants. By Lemma 1, there is at most one equilibrium with $\varphi \in \Phi$. In this case, z^* is the only one equilibrium in the neighborhood of z^* , i.e., $\Psi^d = \{(\varphi, \omega_M, \eta, \lambda) | V(\varphi, \omega_M, \eta, \lambda) \leq c, \varphi \in \Phi\}$ for some $c > 0$. Hence with any initial state z^0 that satisfies the algebraic equations (23d) and (23f), the trajectory converges to the equilibrium state z^* . $\square$

For the multi-area implementation of PIAC, we choose a Lyapunov candidate function as

$$V(\varphi, \omega_M, \eta, \lambda) = V_1 + \sum_{A_r} (\alpha V_{2r} + V_{3r}),$$

where $\lambda = \text{col}(\lambda_r)$ is a column vector consisting of the components λ_r , V_1 is defined as in (28) and V_{2r} and V_{3r} are defined for area A_r as

$$V_{2r} = \frac{1}{2} \left(\sum_{i \in \mathcal{V}_{K_r}} J_i'^{-1}(\lambda_r) - \sum_{i \in \mathcal{V}_{K_r}} J_i'^{-1}(\lambda_r^*) \right),$$

$$V_{3r} = \frac{k^2}{2} \left(\sum_{i \in \mathcal{V}_{M_r}} M_i \omega_i + \eta_r - \eta_r^* \right)^2.$$

Following the proof of Theorem 2, we can obtain the locally asymptotic stability of PIAC implemented in multi-area control.

Remark 3. The Assumptions 1–4 are realistic at the same time. Assumptions 1 and 3 are necessary for the implementation of PIAC to solve Problems 1 and 2. Assumptions 2 and 4 are general sufficient conditions for the stability of the nonlinear systems (1) controlled by PIAC. Assumptions 1 and 4 can be guaranteed by tertiary control and Assumption 3 by an effective communication infrastructure. Assumption 2 usually holds for frequently used convex cost functions, e.g., quadratic cost function, where the requirement of scale cost functions in Dörfler and Grammatico (2017, Assumption 1) is not needed.

6. Simulations of the closed-loop system

In this section, we evaluate the performance of the PIAC method and compare it with those of the GB, DAI and Decl control laws on the IEEE New England power grid shown in Fig. 1. The data are obtained from Athay, Podmore, and Virmani (1979). In the test system, there are 10 generators, and 39 buses and it serves a total load of about 6 GW. The voltage at each bus is a constant which is obtained by power flow calculation with the Power System Analysis Toolbox (PSAT) (Milano, 2008). In the network, there actually are 49 nodes, i.e., 10 nodes for the generators, 39 nodes for the buses. Each synchronous machine is connected to a bus and its phase angle is rigidly tied to the rotor angle of the bus if the voltages of the system are constants, e.g., the classical model of synchronous

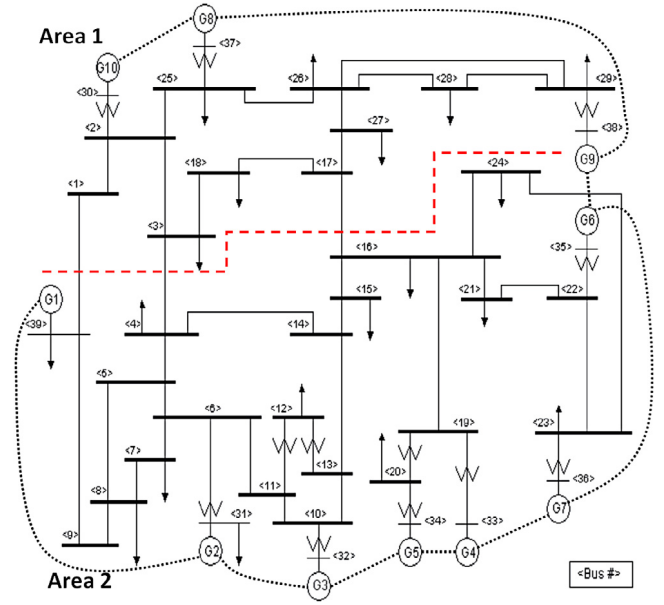


Fig. 1. IEEE New England test power system.

machines (Ilić & Zaborsky, 2000). We simplify the test system to a system of 39 nodes by considering the generator and the bus as one node. This is reasonable because the angles of the synchronous machine and the bus have the same dynamics. The 10 generators are in the set $\mathcal{V}_M = \{30, 31, 32, 33, 34, 35, 36, 37, 38, 39\}$ and the other buses are in the set $\mathcal{V}_F$ which are assumed to be frequency dependent loads. The nodes participating in secondary frequency control are the 10 generators, thus $\mathcal{V}_K = \mathcal{V}_M$. The inertias of the generators as stated in Athay et al. (1979) are all divided by 100 in order to obtain the desired frequency response as in Dörfler and Grammatico (2017), and Zhao et al. (2015). The buses in $\mathcal{V}_M \cup \mathcal{V}_F$ and controllers in $\mathcal{V}_K$ are connected by a communication network.

As in Dörfler and Grammatico (2017), and Zhao et al. (2015), we set the droop control coefficient $D_i = 1$ (p.u. power/p.u. frequency deviation) for $i \in \mathcal{V}_F \cup \mathcal{V}_M$ under the power base of 100 MVA and frequency base of 60 Hz with the nominal frequency $f^* = 60$ Hz, and we choose the quadratic cost function $J_i(u_i) = \frac{u_i^2}{2a_i}$, $i \in \mathcal{V}_K$. The economic dispatch coefficients a_i are generated randomly with a uniform distribution on $(0, 1)$. It can be easily verified that the quadratic cost functions are all strictly convex.

In the simulations, the system is initially at a supply–demand balanced state with the nominal frequency. At time $t = 0.5$ s, a step-wise increase of 33 MW of the loads at each of the buses 4, 12, and 20, amounting to a total power imbalance of 99 MW, causes the frequency to drop below the nominal frequency. The loads at other nodes do not change.

We conduct the simulations with the open source software PSAT and use the Euler-Forward method to discretize the ordinary differential equations and the Newton-Raphson method to solve the nonlinear system. We first evaluate the performance of PIAC on the network which is assumed as a single area. We compare the PIAC method with the GB, DAI and Decl control laws. For illustrations of the overshoot phenomena of other control laws, we refer to the simulation results of the literature published recently, e.g., Li et al. (2016) and Trip et al. (2016). Then we implement the PIAC method in the network separated into two areas and show that the control actions of the areas are totally decoupled by PIAC as described in the Section 4.2.

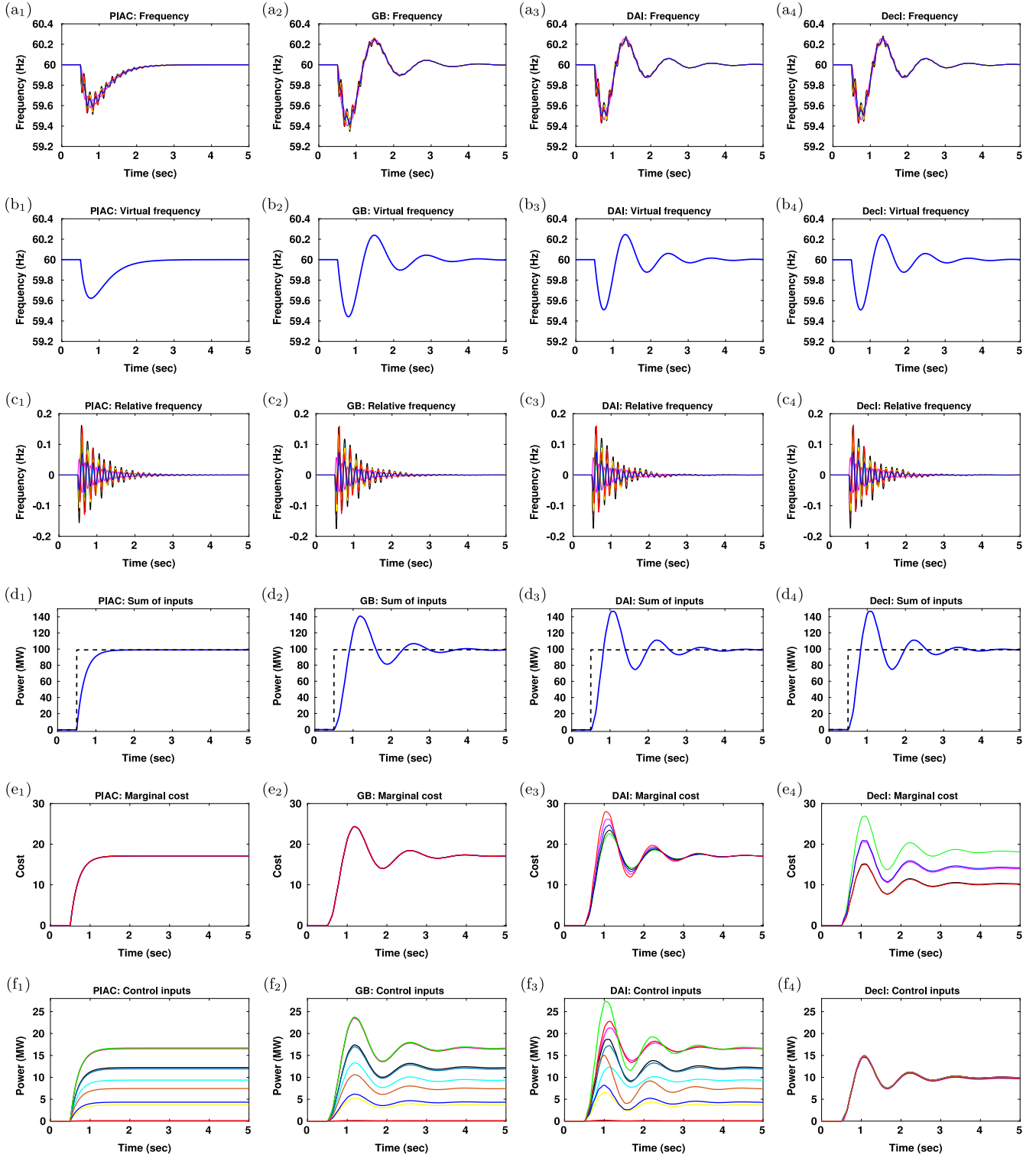


Fig. 2. Comparison of the PIAC, GB, DAI, Decl control laws (single area implementation). In (d₁–d₄), the black dashed lines denote the power imbalance of the network. Only the marginal costs of the generators {2, 3, 4, 5, 7} are shown in (e₃–e₄).

6.1. Numerical results of the single-area implementation

In this subsection, the network is seen as a single area. The parameters for PIAC, GB, DAI and Decl are listed in Table 1. For the DAI method, the communication network is a weighted network as shown in Fig. 1 by the black dotted lines. The weight w_{ij} for line (i, j) is set as in Table 1 and $w_{ii} = -\sum_{(i,j)} w_{ij}$. The settings of these gain coefficients are chosen for a fair comparison in such a

way that the slopes of the total control inputs, which reflect the required response time of the actuators, show close similarity (see Fig. 2d₁–2d₄).

Fig. 2 shows the comparison of the performances between the four control laws. Fig. 2a–2d show the responses of frequencies $\bar{\omega}_i = \omega_i + f^*$ for all $i \in \mathcal{V}_M$, virtual abstract frequency $\bar{\omega}_s = \omega_s + f^*$, relative frequency $\{\omega_i - \omega_s, i \in \mathcal{V}_M\}$ and control input u_s respectively. The latter three illustrate the three-subprocesses

Table 1

The control parameters.

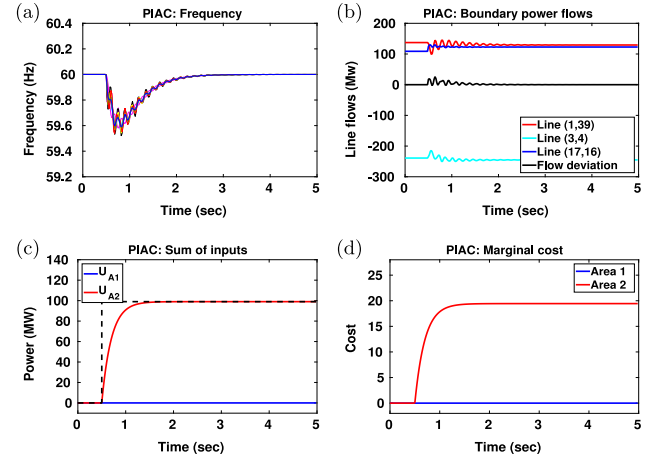
PIAC	GB		DAI		Decl
k	k_{GB}	C_i	k_i	w_{ij}	k_i
5	60	$\frac{1}{39}$	50	-20	50

decomposed from the dynamics of the system (1). Here, the responses of ω_s are obtained from (10) with u_s as the total amount of control inputs of the PIAC and GB methods respectively. It can be observed from Fig. 2a₁ and 2a₄ that all the control laws can restore the nominal frequency. However, the frequency deviation under the PIAC method is much smaller than under the other three control laws which introduce extra oscillations to the frequency. This is because the sum of control inputs of the GB, DAI, Decl methods overshoots the desired value as Fig. 2d₂–2d₄ show, while the one of the PIAC method converges exponentially as Fig. 2d₁ shows. Because of the overshoot, the GB, DAI and Decl control laws require a maximum mechanical power input of about 140 MW from the 10 generators after the disturbance while the PIAC method only requires 99 MW in this simulation. This scenario is also well reflected in the response of the virtual frequency $\bar{\omega}_s$ as shown in Fig. 2b₁–2b₄. Note that the convergences of the relative frequencies $\{\omega_i - \omega_s, i \in \mathcal{V}_M\}$ to zero as shown in Fig. 2c₁–2c₄ are the main concern of primary frequency control (Dörfler et al., 2016; Motter et al., 2013). Since the economic power dispatch is solved on-line, it can be observed in Fig. 2e₁–2e₂ that the marginal costs of all the controllers are the same during the transient phase under the control of PIAC and GB. In contrast with PIAC and GB, the marginal costs of DAI are not identical during the transient phase even though they achieve a consensus at the steady state. Because there are no control coordinations between the controllers in Decl, the marginal costs are not identical even at the steady state as shown in Fig. 2e₄. Since $u_i = \alpha_i \lambda$, the control inputs of the PIAC method and the GB method have similar dynamics as those of their marginal costs as shown in Fig. 2f₁–2f₂. Note that as shown in Fig. 2f₄, the control inputs of Decl are very close to each other because of the identical setting of k_i and the small differences between the frequency deviations. As shown in Fig. 2f₁–Fig. 2f₃, the control inputs of some generators in the PIAC, GB and DAI control laws are small due to the high control prices.

We remark that the larger the gain coefficient k_{GB} of the GB control law the larger are the oscillations of frequency deviations even though the frequencies converge to the nominal frequency faster. However, the control inputs of the PIAC method converge to the power imbalance faster under larger control gain k , which leads to smaller frequency deviations. As shown in Fig. 2a₁, the frequency drops about 0.4 Hz which can be even smaller when k is larger. However, k is related to the response time of the control devices and hence cannot be infinitely large. If the step-wise increase of the loads is too big and the gain coefficient k is not large enough, i.e., the controllers cannot respond quickly enough, the frequency might become so low that they damage the synchronous machines.

6.2. Numerical results of the multi-area implementation

In this subsection, the network is separated into two areas by the red dashed line. All the parameters of the controllers are the same to the ones in the single-area implementation. There are 3 generators in area A_1 and 7 generators in the area A_2 i.e., $\mathcal{V}_{M_1} = \{30, 37, 38\}$, $\mathcal{V}_{M_2} = \{31, 32, 33, 34, 35, 36, 39\}$. The boundary lines of area A_1 are in $\mathcal{E}_{A_1} = \{(1, 39), (3, 4), (17, 16)\}$. As in subsection 6.1, the secondary frequency controllers are installed at the nodes of the generators. After the step-wise increase of the loads at buses 4, 12 and 20 with the total amount of 99 MW in the area A_2 , the multi-area implementation of PIAC recovers the nominal

**Fig. 3.** Multi-area implementation of PIAC.

frequency as shown in Fig. 3a and the power export deviation of area A_1 converges to zero as shown by the black dashed lines in Fig. 3b. Fig. 3b also shows that as the system converges to a new state, the power flows in the three line in $\mathcal{E}_{A_1}$ are different from the ones before the step-wise increase of the loads in area A_2 . A characteristic of PIAC is that it decouples the control actions of the areas, which can be observed in Fig. 3c. Since the step-wise increase of the power loads at the buses 4, 12 and 20 happens in area A_2 , the control inputs of area A_1 are zero and the power is balanced by the controllers in the area A_2 . This shows that with the PIAC method, the power can be balanced locally in an area without influencing to its neighbors. *This characteristic of PIAC is attractive for such a non-cooperative multi-area control of a power system that different areas might have different amount of renewable energy. It is fair for the area with a large amount of renewable energy to respond to the disturbance in its own area. As mentioned in Section 4.2, this characteristic allows the controllers in different areas control the system in an asynchronous way at any time according to the power imbalance within the area.*

7. Conclusion

In this paper, we proposed a secondary frequency control approach, called PIAC, to restore the nominal frequency of power systems with a minimized control cost. A main feature of PIAC is that the estimated power imbalance converges exponentially with a speed that only depends on a gain coefficient which can be tuned according to the sensitivity of the control devices. Hence PIAC eliminates the drawback of the traditional integral control based secondary frequency control approach, in which large control gain coefficients lead to an overshoot problem of the control inputs and small ones result in a large frequency deviation of the power systems. When implemented in a network with multiple areas, PIAC decouples the control actions of the areas such that the power supply and demand of an area can be balanced locally in the area without any influences to the neighbors. For the power systems with a large amount of integrated renewable energy, the large transient frequency deviation can be reduced by PIAC with advanced control devices and communication networks.

However, in practice, there is usually some noise from the measurement of frequency and time delays and even information losses in the communication. In addition, the resistance of the transmission lines cannot be neglected in some power networks, e.g., distribution grids of power systems or Micro-Grids. Hence, further investigation on the performance of PIAC on such a

lossy power network with noisy measurements and time delays is needed. Further investigation is also required into the performance of PIAC in the power system with dynamic actuators.

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Appendix. Preliminaries on DAE systems

Consider the following *Differential Algebraic Equation* (DAE) systems

$$\dot{x} = f(x, y), \quad (\text{A.1a})$$

$$0 = g(x, y), \quad (\text{A.1b})$$

where $x \in \mathbb{R}^n$ are state variables, $y \in \mathbb{R}^m$ are algebraic variables and $f : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ and $g : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^m$ are twice continuously differentiable functions. (A.1a) and (A.1b) are differential and algebraic equations respectively. $(x(x_0, y_0, t), y(x_0, y_0, t))$ is the solution with the admissible initial conditions (x_0, y_0) satisfying the algebraic constraints

$$0 = g(x_0, y_0) \quad (\text{A.2})$$

and the maximal domain of a solution of (A.1) is denoted by $\mathcal{I} \subset \mathbb{R}_{\geq 0}$ where $\mathbb{R}_{\geq 0} = \{t \in \mathbb{R} | t \geq 0\}$.

Before presenting the Lyapunov/LaSalle stability criterion of the DAE system, we make the following two assumptions.

Assumption A.1. The DAE system possesses an equilibrium state (x^*, y^*) such that $f(x^*, y^*) = 0$, $g(x^*, y^*) = 0$.

Assumption A.2. Let $\Omega \subseteq \mathbb{R}^n \times \mathbb{R}^m$ be an open connected set containing (x^*, y^*) , assume (A.1b) is regular such that the Jacobian of g with respect to y is a full rank matrix for any $(x, y) \in \Omega$, i.e.,

$$\text{rank}(\nabla_y g(x, y)) = m, \quad \forall (x, y) \in \Omega.$$

Assumption A.2 ensures the existence and uniqueness of the solutions of (A.1) in Ω over the interval $\mathcal{I}$ with the initial condition (x_0, y_0) satisfying (A.2).

The following theorem provides a sufficient stability criterion of the equilibrium of DAE in (A.1).

Theorem A.1 (Lyapunov/LaSalle Stability Criterion (Hill & Mareels, 1990; Schiffer & Dörfler, 2016)). Consider the DAE system in (A.1) with Assumptions A.1 and A.2, and an equilibrium $(x^*, y^*) \in \Omega_H \subset \Omega$. If there exists a continuously differentiable function $H : \Omega_H \rightarrow \mathbb{R}$ such that (x^*, y^*) is a strict minimum of H i.e., $\nabla H|_{(x^*, y^*)} = 0$ and $\nabla^2 H|_{(x^*, y^*)} > 0$, and $\dot{H}(x, y) \leq 0$, $\forall (x, y) \in \Omega_H$, then the following statements hold:

(1) (x^*, y^*) is a stable equilibrium with a local Lyapunov function $V(x, y) = H(x, y) - H(x^*, y^*) \geq 0$ for (x, y) near (x^*, y^*) ,

(2) Let $\Omega_c = \{(x, y) \in \Omega_H | H(x, y) \leq c\}$ be a compact sub-level set for some $c > H(x^*, y^*)$. If no solution can stay in $\{(x, y) \in \Omega_c | \dot{H}(x, y) = 0\}$ other than (x^*, y^*) , then (x^*, y^*) is asymptotically stable.

We refer to Hill and Mareels (1990) and Schiffer and Dörfler (2016) for the proof of Theorem A.1.

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