TRANSIENT ENERGY GROWTH MODULATION BY TEMPERATURE DEPENDENT TRANSPORT PROPERTIES IN A STRATIFIED PLANE POISEUILLE FLOW

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Abstract We investigate the effect of temperature dependent thermal conductivity λ and isobaric specific heat c_P on the transient amplification of perturbations in a thermally stratified laminar plane Poiseuille flow. It is shown that for decreasing thermal conductivity the maximum transient energy growth is amplified with respect to the $\lambda = 1$ case, while the opposite occurs for increasing λ. A reversed mechanism is induced by a variable c_p. Substantial maximum growth enhancement/suppression is found in the range of Prandtl numbers Pr which encompasses most fluids of practical interest. The relative growth modulation shows an optimum Pr under spanwise perturbations. For energy amplifying property distributions a speed-up of the transient to reach the maximum energy growth is observed at low Pr, while a slow-down is found at large Pr. The opposite is true when the property variations suppress the growth of perturbations.

INTRODUCTION

The study of non-modal stability is of paramount importance to understand the path that leads a laminar flow to a turbulent state. The transient amplification of flow perturbations can be large enough to by-pass the natural mode of transition (Tollmien–Schlichting wave), such that a turbulent state can occur at lower Reynolds numbers than predicted by the eigenvalue analysis [1, 2]. This mechanism is often dominant in flows described by non-normal operators, such as Poiseuille and Couette flows. The effect of a temperature dependent viscosity $\mu(T)$ on the modal and non-modal stability of a heated plane Poiseuille flow was studied by several authors, e.g., [3, 4, 5]. Viscosity stratification can either stabilize or destabilize the flow, and it affects the perturbations transient energy growth. This work aims at the study of more realistic fluid flows considering the effect of variable thermal conductivity $\lambda(T)$ and isobaric specific heat $c_P(T)$. A plane Poiseuille flow subject to a cross-stream linear temperature profile is considered. Results are shown for a fixed temperature dependent viscosity profile and for streamwise and spanwise perturbations at several Prandtl numbers Pr.

EQUATIONS AND NUMERICAL MODEL

The non-dimensional form of the linearized Navier–Stokes equations is written in the v-η formulation. For a fluid with temperature dependent μ, λ and c_P the system reads

$$\begin{aligned} \mathrm{i}\alpha[(v''-k^2v)(U-c)-U''v] = \frac{1}{Re}\Big\{&\mu[v''''-2k^2v''+k^4v]+\frac{d\mu}{dT}T'2[v'''-k^2v']+\frac{d\mu}{dT}T''[v''+k^2v] \\ &+\frac{d^2\mu}{dT^2}(T')^2[v''+k^2v]-\frac{d\mu}{dT}\mathrm{i}\alpha[U'\theta''+2U''\theta'+(k^2U'+U''')\theta]-2\mathrm{i}\alpha\frac{d^2\mu}{dT^2}U'T'\theta' \\ &-\mathrm{i}\alpha\frac{d^2\mu}{dT^2}T''U'\theta-2\mathrm{i}\alpha\frac{d^2\mu}{dT^2}U''T'\theta-\mathrm{i}\alpha\frac{d^3\mu}{dT^3}U'(T')^2\theta\Big\}-Ri\,k^2\theta, \end{aligned} \tag{1}$$

$$\mathrm{i}\alpha(U-c)\eta+\mathrm{i}\beta U'v = \frac{1}{Re}\left\{\mu[\eta''-k^2\eta]+\frac{d\mu}{dT}T'\eta'+\mathrm{i}\beta\frac{d\mu}{dT}(U''\theta+U'\theta')+\mathrm{i}\beta\frac{d^2\mu}{dT^2}T'U'\theta\right\}, \tag{2}$$

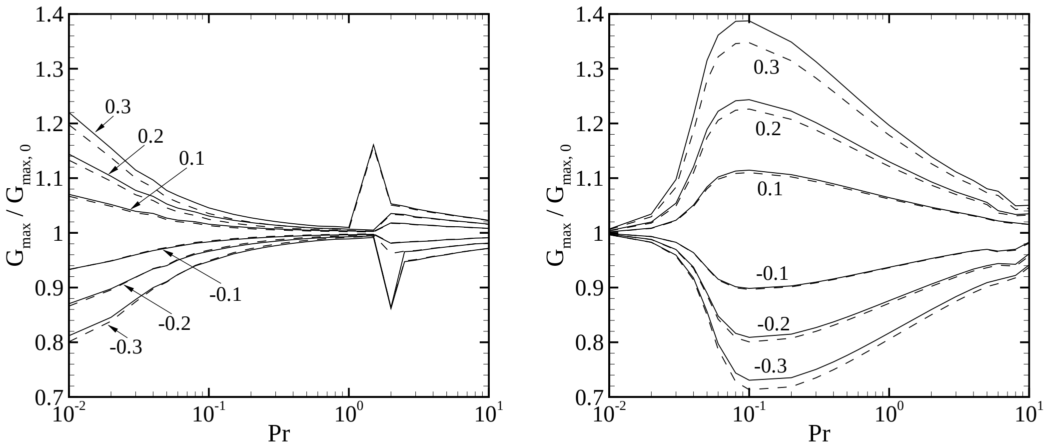
$$\mathrm{i}\alpha(U-c)\theta+T'v = \frac{1}{RePr}\frac{\lambda}{c_p}[\theta''-k^2\theta]+\frac{1}{RePr}\frac{2}{c_p}\frac{d\lambda}{dT}T'\theta'. \tag{3}$$

Here, v, η and θ are the perturbations of wall-normal velocity, wall-normal vorticity and temperature, respectively, which are functions of the wall-normal coordinate only, $y = [-1, 1]$. The prime symbol denotes the derivative with respect to y. The system was transformed to the Fourier space as $(\tilde{v}, \tilde{\eta}, \tilde{\theta}) = \sum(v, \eta, \theta)\exp(-\mathrm{i}c\alpha t + \mathrm{i}\alpha x + \mathrm{i}\beta z)$. $T(y) = 1+y$ and $U(y)$ are the reference temperature and velocity profiles. Re indicates the Reynolds number, Pr the Prandtl number and Ri the Richardson number. Thermophysical properties are modeled using an exponential law, i.e., $\mu = e^{-K_\mu T}$, $\lambda = e^{-K_\lambda T}$ and $c_P = e^{-K_{c_P} T}$. The reference velocity profile is calculated numerically solving the streamwise momentum equation with a fixed non-dimensional pressure gradient chosen such that $U = 1 - y^2$ for $\mu = 1$. Equations (1)-(3) are discretized using collocated Chebyshev polynomials [1]. The software MATLAB is used to solve the linear system. The code was validated for constant and variable properties flow using as a reference [1, 3, 4, 6], however results are not included here for brevity.

RESULTS AND DISCUSSION

$Re = 1000$, $Ri = 0$, and $K_\mu = 0.2$ are considered hereafter. The effect of variable λ and c_P are assessed separately on a spanwise perturbation (SPW), $\alpha = 0$, $\beta = 0.5$, and on a streamwise perturbation (STW), $\alpha = 0.5$, $\beta = 0$.

The ratio between the maximum energy growth under properties variation, G_{max}, and the one obtained for the constant λ and c_P case, $G_{max,0}$, is shown in figure 1 for several values of K_λ and K_{c_P}, as a function of Pr. A relative amplification/suppression of the maximum perturbation energy is observed in both the SPW and STW case. However, the SPW perturbation shows a larger effect over a wider range of Prandtl numbers, which encompasses typical values for most fluids of practical interest. For any fixed coefficient K_{λ,c_P} there is an optimal Prandtl number for the SPW perturbation, while in the STW case the relative effect decreases as Pr increases. Figure 2 depicts the extent of the transient needed to reach the maximum growth in the SPW perturbation case. For energy amplifying λ distributions, the maximum growth is reached in a shorter time at low Prandtl numbers, if compared to the $\lambda = 1$ case. For higher Pr the process is slowed down. The opposite happens for a λ distribution which suppresses the maximum perturbations growth. The same behavior is observed for variable c_P. Further investigations will consider different viscosity profiles (stabilizing and destabilizing), the coupled effect of variable thermal conductivity and specific heat, and a different temperature profile (symmetric).



(a) STW, $\alpha = 0.5$, $\beta = 0$

(b) SPW, $\alpha = 0$, $\beta = 0.5$

Figure 1. $Re = 1000$ and $K_\mu = 0.2$. Numbers denote values of K_λ (dashed lines) and $-K_{c_p}$ (solid lines).

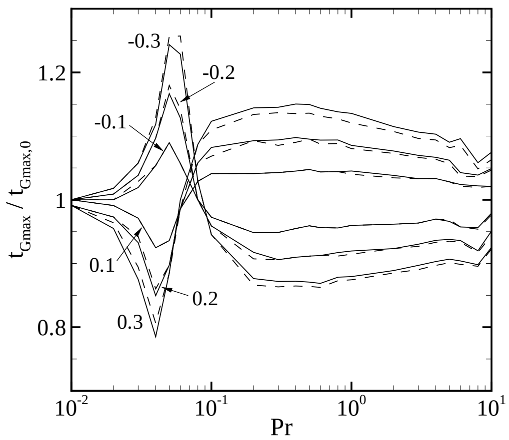


Figure 2. Same as in figure 1.

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