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The influence of anomalous Doppler waves on the Hyperloop stability

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Summary. The Hyperloop, a developing transportation system, reduces air resistance by housing the vehicle within a depressurized tube and eliminates contact friction through an electro-magnetic suspension/levitation system. Maintaining system stability poses a challenge due to the exceptionally high target velocities. The interplay between electro-magnetic and wave-induced instability has been previously studied by the authors, showing that stability domains drastically change above a certain vehicle velocity. The current study demonstrates that the anomalous Doppler waves (i.e., wave-induced instability) are causing this drastic change. This investigation offers physical insight into the mechanisms that can cause instability in the Hyperloop system.

Introduction

The Hyperloop is an innovative transportation system that is currently under development. It minimizes air resistance by enclosing the vehicle in a de-pressurized tube and eliminates wheel-rail contact friction through the use of an electromagnetic suspension/levitation, similar to Maglev trains. This design can potentially achieve much higher velocities compared to traditional railways, positioning the Hyperloop as an environmentally friendly alternative to air transportation.
A potential challenge for Hyperloop is ensuring the dynamic stability at large velocities, where multiple instability sources can be present. An apparent source is the electro-magnetic suspension (adopted by some designs) making a control strategy mandatory to ensure stability even at quasi-static velocities. A less obvious instability mechanism is that the vibration of a vehicle on an elastic guideway can become unstable when surpassing a critical velocity [1].
The authors have previously investigated the interplay between the electro-magnetic and wave-induced instability mechanisms [3], and showed that the stability space changes significantly above a certain velocity. In other words, the control strategy can ensure the overall system stability only for a very limited range of its gains. The cause for this drastic change was attributed to the wave-induced instability mechanism [3]. Metrikin [2] demonstrated that this instability arises with the radiation of anomalous Doppler waves, which introduce more energy to the vehicle's vibration than normal Doppler waves radiate away from the vehicle. The current study demonstrates that the change of stability domain is indeed caused by the anomalous Doppler waves. While identifying unstable velocity regimes is practical for Hyperloop design, gaining insight into the contribution of individual instability mechanisms can be crucial for efficient mitigation.

Model, linearization, and stability investigation

The model considers a moving mass and an infinite Euler-Bernoulli beam on a visco-elastic foundation interacting through a nonlinear electro-magnetic force F. Since we focus on the guideway response under the moving vehicle, we express its governing equation as a convolution between the guideway Green's function G_0 (or equivalent compliance) under the vehicle (i.e., $x = vt$) and F. Fig. 1 depicts the system, while its governing equations are presented in Eqs. (1)–(5) [3], where the overdots denote partial derivatives in time t, g is the gravitational acceleration, $w_0 = w(x = vt)$ is the beam displacement under the moving mass, I is the current intensity, $\Delta = w_0 - u$ is the air-gap, and C is a constant [3]. Eq. (4) is a nonlinear differential equation governing the current intensity where U is the voltage and R is the circuit resistance. Finally, w_0^{ic} represents the guideway free vibrations due to initial conditions corresponding to the system's equilibrium position. This term is necessary because the convolution integral does not account for non-trivial initial conditions.

$$w_0(t) = -\int_0^t G_0(t-\tau)F(\tau)\mathrm{d}\tau + w_0^{\mathrm{ic}}(t), \quad (1)$$

$$M\ddot{u} = F(t) - Mg, \quad (2)$$

$$F(t) = C\frac{I^2}{\left(w_0-u\right)^2}, \quad (3)$$

$$\dot{I} = \frac{w_0-u}{2C}\Big(U - IR + 2C\frac{I}{\left(w_0-u\right)^2}\left(\dot{w}_0 - \dot{u}\right)\Big), \quad (4)$$

$$U(t) = K_\mathrm{p}\left(w_0 - u - \Delta^{\mathrm{ss}}\right) + K_\mathrm{d}\left(\dot{w}_0 - \dot{u}\right) + U^{\mathrm{ss}}. \quad (5)$$

$$w_0^{\mathrm{tr}}(t) = -\int_0^t G_0(t-\tau)F^{\mathrm{tr}}(\tau)\mathrm{d}\tau, \quad (6)$$

$$M\ddot{u}^{\mathrm{tr}} = F^{\mathrm{tr}}, \quad (7)$$

$$F^{\mathrm{tr}}(t) = \frac{2CI^{\mathrm{ss}2}}{\Delta^{\mathrm{ss}3}}\Big(\frac{\Delta^{\mathrm{ss}}}{I^{\mathrm{ss}}}I^{\mathrm{tr}} + u^{\mathrm{tr}} - w_0^{\mathrm{tr}}\Big), \quad (8)$$

$$\dot{I}^{\mathrm{tr}} = \frac{\Delta^{\mathrm{ss}}}{2C}\Big[-I^{\mathrm{tr}}R + K_\mathrm{p}\left(w_0^{\mathrm{tr}} - u^{\mathrm{tr}}\right) + \left(K_\mathrm{d} + \frac{2CI^{\mathrm{ss}}}{\Delta^{\mathrm{ss}2}}\right)\left(\dot{w}_0^{\mathrm{tr}} - \dot{u}^{\mathrm{tr}}\right)\Big]. \quad (9)$$

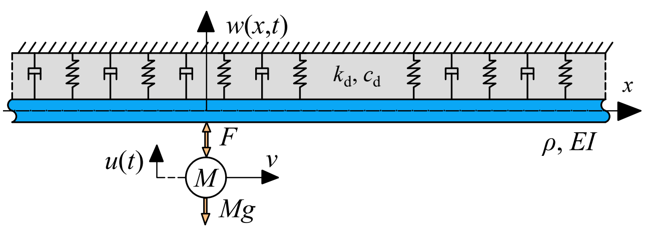

Figure 1: Schematic of the system: a visco-elastic foundation continuously supports an infinite Euler-Bernoulli beam travelled by a moving mass. The interaction between the vehicle and the structure is dictated by the nonlinear electro-magnetic suspension.

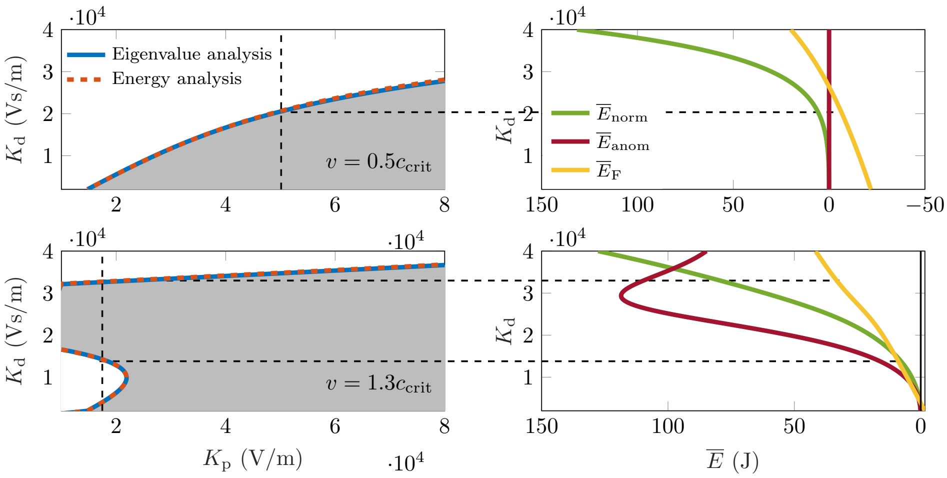

Figure 2: Left panels: stable (white) and unstable (grey) areas in the K_{p}–K_{d} space predicted by the eigenvalue (blue lines) and the energy (orange dashed lines) analyses for sub- (top panels) and super- (bottom panels) critical velocities. Right panels: the energy of normal (green line) and anomalous (red line) Doppler waves, and the energy dissipated by the electro-magnetic force (yellow line).

To investigate the system stability, Eqs. (1)–(5) are linearized around the physically meaningful equilibrium state. The resulting linear system describing the free-vibration response is presented in Eqs. (6)–(9) (tr and ss stand for transient and steady state, respectively). The stability of the linear system was studied in Ref. [3] by obtaining the system eigenvalues and investigating their real part for different combinations of K_{p}, K_{d}, and v. Some results are reproduced in Fig. 2.

Distinguishing between different instability sources

The eigenvalue analysis mentioned above and used in [3] is straightforward and offers insight into the system's stability, but it fails to differentiate between various sources of instability. Therefore, discerning the primary contributing mechanism for effective mitigation remains impossible. The approach presented hereafter offers a solution to this limitation.

At the stability boundary, the response of the linear system is harmonic in time. Consequently, a harmonic motion is imposed to the vehicle with frequencies obtained from the eigenvalue analysis. To isolate different instability mechanisms, we use the energy variation of the vehicle over time, as done by Metrikin [2]. The mass energy variation is governed by the energy E_{F} dissipated by the electro-magnetic force, and the energy radiated into the guideway, which can be divided into energy associated to anomalous E_{anom} and normal E_{norm} Doppler waves [2]. The mass energy variation ΔE reads

$$\Delta\overline{E} = \overline{E}_{\mathrm{anom}} - \overline{E}_{\mathrm{norm}} - \overline{E}_{\mathrm{F}}, \tag{10}$$

where the overbar indicates quantities averaged over one period of oscillation, and $\overline{E}_{\mathrm{anom}}$ and $\overline{E}_{\mathrm{norm}}$ are positive definite. The energy variation shows that while the normal Doppler waves are removing energy from the vehicle (i.e., stabilizing mechanism), the anomalous Doppler waves are feeding back energy (i.e., destabilizing mechanism). As for the electro-magnetic force, a positive E_{F} corresponds to a stabilizing mechanism while a negative one is destabilizing (i.e., negative dissipation). The balance of the stabilizing and destabilizing mechanisms ($\Delta\overline{E} = 0$) corresponds to a stability boundary.

Results and conclusions

The left panels in Fig. 2 show the stable/unstable areas in the K_{p}–K_{d} space predicted by the eigenvalue and by the energy analyses. The almost perfect match between the two demonstrates that the system stability is governed by the energy balance in Eq. (10). Fig. 2 also shows the stable space shrinking drastically at large velocities. The right panels in Fig. 2 show the contribution of each component to the energy balance Eq. (10). At relatively low velocities, the only contributors are the normal Doppler waves and the electro-magnetic force since anomalous Doppler waves are not excited. Also, the energy dissipated by the electro-magnetic force becomes negative at the right side of the K_{p}–K_{d} space, but the normal Doppler waves delay the onset of instability, enlarging the stability area. At relatively large velocities, the anomalous Doppler waves are governing the energy balance and, thus, are responsible for the drastic shrinkage of the stable area.

The interplay between electro-magnetic and wave-induced instability has been previously studied by the authors [3], showing a drastic change in the stability area above a certain vehicle velocity. The current study demonstrates that anomalous Doppler waves are causing this drastic change, and presents a methodology to isolate individual contributions to the system stability, thus offering physical insight into mechanisms that can cause instability in the Hyperloop system.

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