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Shear tension resistance of prestressed girders with a low stirrup ratio

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ABSTRACT: In the Netherlands, existing bridges are being assessed to investigate whether they are still capable to resist current and future traffic loads. A part of these bridges consists of prestressed I- and T-shaped girders with a low shear reinforcement ratio. This ratio is low because the principle stress criterion was used to verify sufficient shear resistance at the time of engineering. These bridges are now assessed with the present Eurocode. It appears that it is difficult to demonstrate sufficient shear tension resistance. In this paper it is investigated whether existing models can accurately predict shear tension resistance for girders with a low shear reinforcement ratio. The model used in the CSA was found to predict this resistance conservatively and consistent. However, for single span girders with low shear reinforcement ratios, it was also found that it is difficult to demonstrate additional capacity compared to the resistance to diagonal tension cracking.

1 INTRODUCTION

In the Netherlands, existing bridges are being assessed to investigate whether they are still capable to resist current and future traffic loads. A part of these bridges consists of prestressed I- and T-shaped girders that are applied in combination with in-situ slabs. Both single span and continuous bridges are constructed in this way. In these girders both pre- and posttensioned tendons are applied. These tendons are always bonded. A part of the bridges were engineered using the principle stress criterion to verify sufficient shear resistance (bridges that were built with predecessors of the Dutch concrete codes of 1974). As a consequence, low shear reinforcement ratio ρ_w are present. The shear reinforcement ratio is typically less than 0.30%. These existing bridges are now assessed with the present Eurocode (CEN, 2005). It appears that it is difficult to demonstrate sufficient structural safety using for this kind of bridges. Especially in the area that is free of flexural cracks in the ultimate limit state, the shear capacity appears to be insufficient. In other words, sufficient resistance to shear tension failure ($V_{R,STF}$) cannot be demonstrated for these bridges.

Girders with insufficient shear reinforcement could abruptly fail after a diagonal tension cracking occurs. The resistance to shear tension failure of these girders is equal to the shear resistance to diagonal tension cracking ($V_{R,STF} = V_{R,DTC}$). However, if sufficient stirrups are applied, additional load can be resisted after diagonal tension cracking ($V_{R,STF} > V_{R,DTC}$). According to the Eurocode, sufficient stirrups are present if the shear reinforcement ratio ρ_w is higher than a minimum value $\rho_{w,min}$. According to the Eurocode, this minimum value $\rho_{w,min}$ is equal to 0.08 $\sqrt{f_{ck}}/f_{yk}$. In this expression f_{ck} is the characteristic value of the concrete cylinder compressive strength and f_{yk} is the characteristic yield strength of the reinforcement. The research in this paper focusses on girders with a shear reinforcement ratio between this minimum and the maximum applied in older existing bridges ($\rho_{w,min} \leq \rho_w \leq 0.30$). Possibly additional capacity can be demonstrated compared to the resistance to diagonal tension cracking.

The resistance for $\rho_w > \rho_{w,min}$ can be determined using the variable angle truss model according to the Eurocode. However, according to the Eurocode the angle of the compressive strut θ is conservatively limited to 21.8°. This could possibly lead to an underestimation of the resistance for girders with a low shear reinforcement ratio.

In this paper the accuracy of the Eurocode model regarding shear tension resistance for girders with a low shear reinforcement ratio is considered. Moreover, research on the accuracy of some other existing models is reported that could possibly predict shear tension resistance more accurately. For this aim, models used in the American Code (ACI, 2018) and the Canadian Code (CSA, 2004) are investigated.

2 SHEAR TENSION FAILURE DATABASE

A shear tension failure database is composed to make it possible to study shear tension failure for girders with stirrups. Data is gathered from experiments

Table 1. Characteristics of collected specimens in the shear tension failure database.

	Shear tension failure database	
	minimum	maximum
ρ_w	0.09%	0.79%
σ_{cp}	2.3 N/mm^2	11.3 N/mm^2
f_{cm}	24 N/mm^2	96 N/mm^2
h	457 mm	970 mm
a_g	9 mm	15 mm

described in literature. Specimens are selected if shear tension failure is observed. Shear tension failure is defined as failure that can be related to diagonal tension cracking. Diagonal tension cracks are diagonal cracks that develop independent of the formation of flexural cracks. In prestressed girders diagonal tension cracks occur in the web. A typical shear tension failure mode for girders with a low shear reinforcement ratio is the opening of the diagonal tension crack and the yielding and eventually rupture of the stirrups, without crushing of the concrete. When more stirrups are applied, crushing of the concrete between the diagonal tensions cracks is a typical shear tension failure mode, that occurs together with yielding of the stirrups. If the shear reinforcement ratio is high, crushing of the concrete can occur without yielding of the stirrups. In codes this is referred as 'the upper limit for shear resistance'.

Specimens that have a shear reinforcement ratio lower than $\rho_{w,min}$ are excluded in the database, as for those specimens the resistance to diagonal tension cracking is assumed to be governing. For these specimens the presence of stirrups has no effect on the resistance. Also specimens with a high shear reinforcement ratio are not included in the database as this paper focusses on low shear reinforcement ratios. However also shear reinforcement ratio higher than 0.30% are included to be able to gain insight in the trend.

Furthermore specimens with a shear span (a) to effective depth (d) ratio of less than 2.4 are not included. This is because the models are intended to describe sectional behavior. For specimens with a ratio of a/d smaller than 2.4, direct load transfer mechanism are considered to increase the resistance. This is a conservative approach and is a common selection criterion in databases (Bentz, 2000, Reineck et al., 2012). Finally, failures within the transmission length of the pre-tensioned tendons are not included, as this failure mechanism is outside the scope of this paper.

The database consists of 48 prestressed I shaped specimens as described by Hanson (Hanson, 1964), Elzanaty (Elzanaty et al., 1986), Choulli (Choulli, 2005), Xie (Xie, 2009), Leonhardt (Leonhardt et al., 1973), Rupf (Rupf et al., 2013) and Mattock (Mattock and Kaar, 1961). Both single span and continuous girders are included. Selected specimens contain both post-tensioned and pretension tendons. The applied

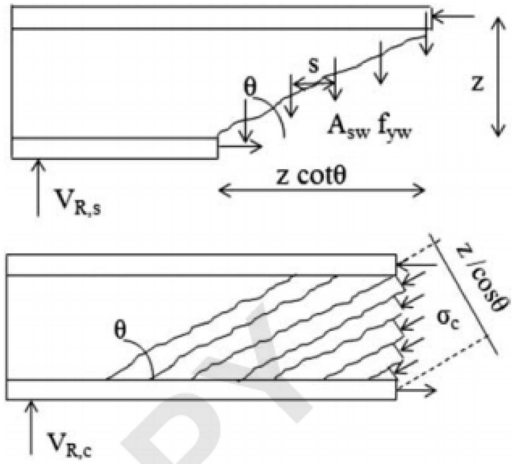

Figure 1. Variable angle truss model.

geometries of the tendons of the specimens are straight, draped and curved. In the experiment of Mattock a combination of a prestressed girder with an in-situ slab is applied. In Table 1 some characteristics of the specimens are given. In this table ρ_w is the shear reinforcement ratio, σ_{cp} is the compressive stress in concrete from prestressing, f_{cm} is the mean value of the concrete cylinder compressive strength, h is the overall depth of the cross section and a_g is the largest nominal maximum aggregate size.

3 TENSION RESISTANCE MODELS

3.1 *Variable angle truss model*

The variable angle truss model is a lower bound approach of the theory of plasticity (Walraven, 2002). The variable angle truss model is simply based on equilibrium, assuming the presence of a truss in the girder in which the reinforcement and the concrete represent the different components.

The verticals of the truss are represented by the stirrups. The resistance of the shear reinforcement $V_{R,s}$ is given by expression (1). In this expression A_{sw} is the area of shear reinforcement, s is spacing of bars, z is the internal lever arm and θ is the angle of the inclined struts in the web. These parameters are illustrated in Figure 1.

$$V_{R,s} = \frac{A_{sw}}{s}\, z\, f_{yw}\, \cot\theta \quad (1)$$

$$V_{R,c} = \frac{\alpha_c b_w\, z\, \nu\, f_{cm}}{\sin\theta \cdot \cos\theta} \quad (2)$$

The diagonal struts of the truss are represented by the concrete, see also Figure 1. The area perpendicular to the compressive struts is equal to $b_w \ z/\cos\theta$, in which b_w is the width of the web. The force in the

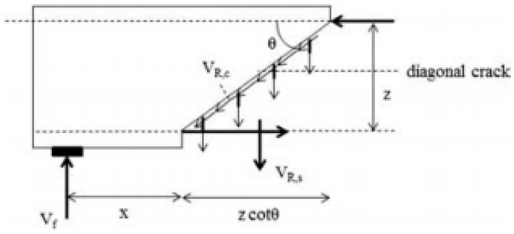

Figure 2. Basis model for shear resistance according to the CSA.

compressive struts is equal to $V_{R,c}/\sin\theta$. Assuming the concrete to crush at a stress of $\alpha_c\, \nu\, f_{cm}$, shear resistance of the concrete struts ($V_{R,c}$) is given by expression (2). The factor ν addresses the effect that the concrete compression strength in a web is less than f_{cm} and the factor α_c is intended to address the effect of prestressing.

According to the theory of plasticity, the largest resistance is found when the shear reinforcement yields and the concrete struts crush simultaneous. That is, if expression (1) is equal to expression (2). However, because the redistribution capacities are limited, strut rotation is allowed between 45° and 21.8°. The calculated inclinations of the struts are compared with measured values, which show reasonable similarity (Walraven and Stroband, 1999).

The resistance model does not distinguish between flexural shear resistance and shear tension resistance.

3.2 *CSA aggregate interlock model*

The aggregate interlock model according to the CSA (Bentz and Collins, 2006) is based on the Modified Compression Field Theory (MCFT). The MCFT is a model intended to describe the load-deflection response of membrane elements (Vecchio F.J., 1986). The MCFT is made applicable for girders in the CSA model .(Bentz and Collins, 2006). According to the CSA model, the shear in the crack is resisted by two components, see Figure 2 and expression (3).

The first component of the shear resistance is the shear reinforcement ($V_{R,s}$). $V_{R,s}$ is the sum of the vertical forces in the shear reinforcement that cross the diagonal crack, assuming the shear reinforcement yields (see expression (4)). As a simplification of the MCFT the angle of inclination of the principle stresses θ in concrete only depends on the longitudinal strain ε_x at the mid-depth of a cross section. The relation is given by the expression $\theta = 29 + 7000\,\varepsilon_x$. The strain is derived assuming a tie representing the steel and prestressing. The force in the tie is calculated from the sectional forces. The strain can be calculated from this tensile force and the stiffness of the tie.

$$V_R = V_{R,c} + V_{R,s} \tag{3}$$

$$V_{R,s} = \frac{A_{sw}\, f_{yw}\, z}{s} \cot\theta \tag{4}$$

$$V_{R,c} = \frac{0.4}{(1 + 1500\varepsilon_x)} \sqrt{f_c}\; b_w\, z \tag{5}$$

The second component of the shear resistance in the crack is the sum of shear stresses in the diagonal crack due to aggregate interlock ($V_{R,c}$). Also the contribution of the aggregate interlock is assumed to be only depending on the longitudinal strain ε_x which is also a simplification of the MCFT. It is assumed that the aggregate interlock resistance of the complex crack geometry may be estimated from ε_x at the mid depth of the cross section and that this can represent the entire crack surface. The shear resistance of the flexural compression region, which is not explicitly considered, is assumed to be larger than that of the cracked region (Bentz and Collins, 2006). The vertical component of any prestress force ($V_{R,p}$) is, in contrast to the CSA, not inserted in expression (3). This is because it is common in Europe to consider $V_{R,p}$ as reduction of the load instead of a component of the shear resistance.

Also this resistance model does not distinguish between flexural shear resistance and shear tension resistance.

3.3 *Empirical model according to the ACI*

The empirical model for shear tension resistance, as used in the ACI code provisions, is based on experiments done by MacGregor on prestressed single span girders with shear reinforcement (Mac Gregor J.C., 1960). The model is empirically derived from specimens that failed in shear. Although the model covers both flexural shear and shear tension failure, only the shear tension resistance model will be explained in this paper. McGregor assumes the shear tension resistance to be equal to diagonal cracking resistance ($V_{R,DCT}$) plus a contribution of shear reinforcement ($V_{R,s}$). This is shown in expression (6).

$$V_{R,STF} = V_{R,DTC} + V_{R,s} \tag{6}$$

$$V_{R,DTC} = \left(0.291\sqrt{f_{cm}} + 0.3\,\sigma_{pc}\right) b_w\, d \tag{7}$$

$$V_{R,s} = \frac{A_{sw}\, f_{yw}\, z}{s} \tag{8}$$

The shear stress that can be resisted by the concrete tensile strength at an arbitrary point can be approached with the simplified expression $0.291\ \sqrt{f_{cm}}+0.3\sigma_{cp}$. The shear tension resistance is approached by multiplying this stress by $b_w\, d$, what results in expression (7). It appears that resistance to diagonal tension cracking can be predicted accurately for single span girders using this expression (Elzanaty et al., 1986). The contribution of the shear reinforcement was derived empirically, fixing the ratio of the experimentally to predicted resistance to one. This results in expression (8). Originally this value was multiplied with a factor 1.1 which was adapted later to unity as a conservative approach.

Table 2. Mean values and coefficients of variations for $V_{R,exp}/V_{R,model}$ for 48 specimens of the database.

	All		Single Span		Continuous	
	Mean	Cov	Mean	Cov	Mean	Cov
VAT	2.02	35%	2.17	36%	1.73	22%
CSA	1.23	16%	1.33	12%	1.06	11%
ACI	1.14	17%	1.22	11%	0.98	18%

4 PREDICTIONS OF SHEAR TENSION RESISTANCES

For the 48 specimens of the shear tension database $V_{R,exp}/V_{R,model}$ is determined. Mean values and coefficients of variation (CoV) are listed in Table 2. Beside the results for all specimens also results for only the single span and only continuous specimens are listed.

For the variable angle truss model (in the table abbreviated as VAT) first ν and α_c are determined. Subsequently θ could be determined by equating expression (1) and (2). For almost all specimens the minimum angle of 21.8° was governing. Only for one specimen with a ρ_w of 0.70% a steeper angle was found.

As the specimens failed in shear tension the predictions for the CSA model are made assuming a strain ε_x at the ultimate fiber of zero. This results in a resistance model specifically intended for shear tension resistance, see expressions (9), (10) & (11). Finally it was verified if the 'the upper limit for shear resistance' according to the CSA was governing. This appeared not be the case for any of the specimens.

$$V_{R,SFT} = V_{R,c} + V_{R,s} \tag{9}$$

$$V_{R,s} = \frac{A_{sw}\, f_{yw}\, z}{s} \cot 29° \tag{10}$$

$$V_{R,c} = 0.4\sqrt{f_c}\, b_w\, z \tag{11}$$

As the specimens failed in shear tension, the predictions for the ACI model are made using expressions (6), (7) & (8). This model is especially intended for shear tension resistance. As the specimens failed as result of shear tension it was not further verified whether flexural shear failure was predicted as governing failure mechanism. Also for the ACI model it was verified whether the 'upper limit for shear resistance' according to the ACI was governing. Also for this model, this appeared not the case for any of the specimens.

5 CONSIDERATION OF THE RESULTS

5.1 *Variable angle truss model*

According the theory of plasticity the largest resistance is found when the shear reinforcement yields

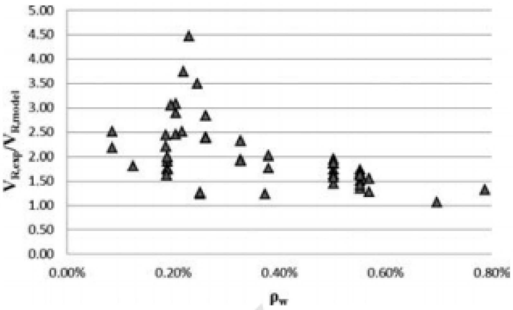

Figure 3. $V_{R,exp}/V_{R,model}$ for the variable angle truss model compared to ρ_w.

and simultaneously the concrete struts crush. For low reinforcement ratio's this condition would be met for inclinations smaller than 21.8°. However an angle of 21.8° is chosen as limit regarding the ability of the stresses to redistribute. For low shear reinforcement ratios angles of the truss model are found to be fixed values of 21.8°. This explains the significant underestimation of the resistance using the variable angle truss model for the considered specimens. Moreover the found CoV of 36% shows that the predictions are significantly inconsistent (see Table 2).

From Figure 3 it is found that for decreasing values of ρ_w, the underestimation of the resistance becomes more significant. The contributions of the uncracked concrete and aggregate interlock to the shear resistance can only be taken into account by allowing a smaller angle of the compressive struts. This value is found to be fixed for low values of ρ_w. Moreover the contribution of the stirrups is already small if the amount of shear reinforcement is limited. The lower bound approach only results in predictions closer to unity for higher values of the shear reinforcement ratios.

5.2 *Empirical model according to the ACI*

The model according to the ACI is an empirical model based on contributions of both shear reinforcement and concrete. The most accurate predictions for single span girders are found using the ACI model. This can be explained because the model is derived empirically using results for single span girders. As a consequence all resistance mechanisms are implicitly included. For continuous girders the predictions become less consistent. This is shown in Figure 4 where the results are differentiated between single span and continuous girders. Frequently the resistance is found to be underestimated for the considered continuous girders.

5.3 *CSA aggregate interlock model*

The shear resistance according to the CSA is based on contributions of aggregate interlock and shear reinforcement. A contribution of uncracked concrete is not explicitly described. In case of shear tension failure, both the bottom and top flange remain uncracked.

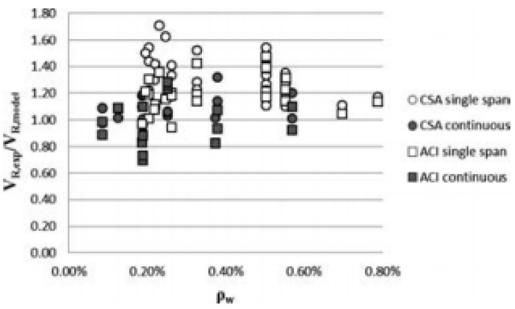

Figure 4. $V_{R,exp}/V_{R,model}$ of the ACI and CSA model compared to ρ_w.

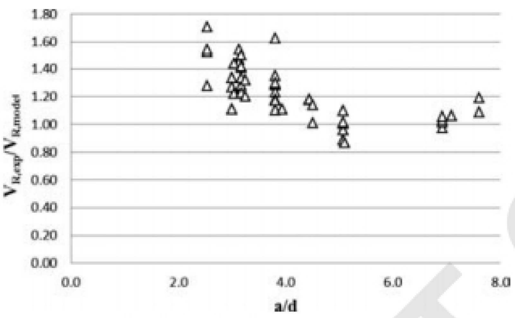

Figure 5. $V_{R,exp}/V_{R,model}$ of the CSA model compared to a/d.

Therefore it is likely that the contribution of aggregate interlock is overestimated. On the other hand the contribution of shear transfer through uncracked concrete is missing. These assumptions could potentially affect the accuracy of the results. However, the predictions were found to be conservative and consistent for both single span and continuous prestressed girders, also for girders with a low shear reinforcement ratio (see Table 2 and Figure 4).

Furthermore it was found that for small ratios of a/d the predictions according to the CSA become more conservative (see Figure 5). This could indicate that the effect of direct load transfer is significant, even for specimens with a value a/d larger than 2.4. A contribution of direct load transfer is neglected in the CSA model as a simplification of the MCFT. However, even for high ratios of a/d. the predictions are found to be conservative.

6 DIAGONAL TENSION CRACKING

The resistance of the specimens with a low amount of shear reinforcement could also be determined by neglecting the stirrups. The resistance is then assumed to be equal to the resistance to diagonal tension cracking. According to the Eurocode this approach is only allowed for single span specimens. The resistance could be determined by calculating the maximum principle tensile stress $\sigma_{1,max}$ in the center of gravity and limiting this stress to 80% of the mean value of the

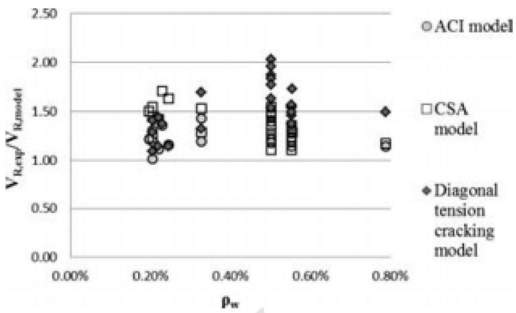

Figure 6. $V_{R,exp}/V_{R,model}$ for single span specimens, for the resistance model for diagonal tension cracking, the ACI model and the CSA model compared to ρ_w.

axial tensile strength of concrete, f_{ctm} (Roosen M.A., unpubl.). This is done for the single span specimens in the database with expressions (11) & (12). By dividing the prestress force by the area of the concrete cross section, $\sigma_{x,max,c.o.g.}$ was found.

$$V_{R,DTC} = \frac{I \cdot b_w}{S}\sqrt{f_{ct,web}^{\,2} - \sigma_x f_{ct,web}} \tag{11}$$

$$f_{ct,web} = 0.80 f_{ctm} \quad for\ \sigma_x = \sigma_{x,max,c.o.g.} \tag{12}$$

In expressions (11) I represents the second moment of area and S the first moment of area. Both are based on concrete cross section. The results of the predictions of the resistances to diagonal tension cracking are shown in Figure 6. Also the resistances according to the ACI and CSA are shown. For low shear reinforcement ratios the difference between the models is found to be less significant.

7 CONCLUSIONS

For small shear reinforcement ratios the variable angle truss model was found to be significantly underestimating the shear tension capacity.

The model according to the CSA is found to be significantly more accurate than the variable angle truss model. The model predicts shear tension capacity consistently, both for single span as continuous girders. This is also the case for low shear reinforcement ratios. However, the CSA model is somewhat conservative, especially for low values of shear span to effective depth ratios. For single span girders, also the model according to the ACI is found to be consistent. For single span girders, this model is found to be more accurate than the CSA model. However, the predictions for continuous girders are less consistent for the ACI model and frequently the resistance is found to be underestimated.

For single span girders with a low shear reinforcement ratio not much additional capacity can be demonstrated using the existing CSA or ACI models compared to the resistance to diagonal tension cracking (see Figure 6). This is because the CSA and ACI models are somewhat conservative and the resistances of the models with and without stirrups are relatively close. The difference only becomes significant for higher values of the shear reinforcement ratio.

For practical assessments it is recommended to base the shear tension resistance for single span girders with a low shear reinforcement ratio on the maximum of the resistance to diagonal tension cracking (ignoring the stirrups) and the ACI model (considering the stirrups). Using this approach could result in somewhat higher resistances. Moreover this approach can replace the use of $\rho_{w,min}$ as criterion to choose between a resistance model with or without stirrups.

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