

A Study of the Application of Safety Formats in Non-linear Finite Element Analysis of Reinforced Concrete Tunnel Segment

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1. Introduction

Recently, finite element method (FEM) is playing an important role in structural analysis, including the description of real structure, the modelling of structural behavior and the insight of structural mechanical properties. FEM is an effective tool to help people get an approximate solution, which can guide the process of calculation and design. Non-linear finite element analysis (NLFEA) is contained in the FEM and it can deal with complex tasks by introducing non-linearity such as geometry, material and contact. During the NLFEA, iteration and convergence are applied to reach reasonable equilibrium.

A well-designed reinforced concrete (RC) structure will fail in flexure rather than shear when it is under extreme loading^[1]. This type of failure has obvious warning phenomena of approaching failure. However, unlike the flexure failure, shear failure in RC structure is brittle and can occur without warning, especially when there is no stirrups. Thus the shear capacity of reinforced concrete (RC) structure is important to study. When calculating the shear capacity, fracture energy is an important fracture parameter that cannot be avoided. However, concrete as a material with uncertainties, whose fracture properties are difficult to predict. The new research method, such as NLFEA simulation, is expected to perform better prediction of the fracture properties and shear failure of RC structure^[1].

In this thesis, a case study stated by Michael P. Collins and Daniel Kuchma^[2] is performed, which is an experiment of shear capacity of subway tunnel segment of Toronto. The work from Larsen^[3], the NLFEA of the tunnel segment experiment, is also studied. A model and NLFEA are reproduced referring to Larsen's work and the results are compared. Then safety formats interpreted in the Guideline^[4] are applied, while the fracture energy calculation follows the way Larsen did, which is stated in Model Code 1990^[5]. And a comparative analysis of those results is performed. During the study, it is found that the fracture energy calculation method in Larsen's work is different from the latest Model Code 2010^[6], which will produce different values of fracture energy. Thus a sort of NLFEA, applying the fracture energy resulted from the Model Code 2010, is performed and the results from two sorts of analysis are compared and analyzed.

The goals of this thesis are firstly to study the performance of NLFEA on simulation of shear capacity of the real tunnel segment experiment. What's more, the application of safety formats in NLFEA is studied. Last but not the least, the influence of fracture energy towards shear capacity of RC structure is also studied.

2. Non-linear finite element approach

2.1 Introduction of non-linear finite element approach

There are three main nonlinearities in structural mechanics: material, geometry and contact. And the stiffness matrix varies due to the nonlinearities. Applying the fixed stiffness, the program will lead to the **non-equilibrium** between internal and external forces. In NLFEA, incremental-iterative procedure is applied to search for the equilibrium. Figure 2-1 describes the process of NLFEA. Once the difference between internal and external forces is less than the desired tolerance, the program will result in the equilibrium, then iteration will stop. Otherwise the iteration will be repeated until it reach the maximum steps and non-convergence will be concluded. The specific NLFEA method applied in this thesis will be introduced in later chapter.

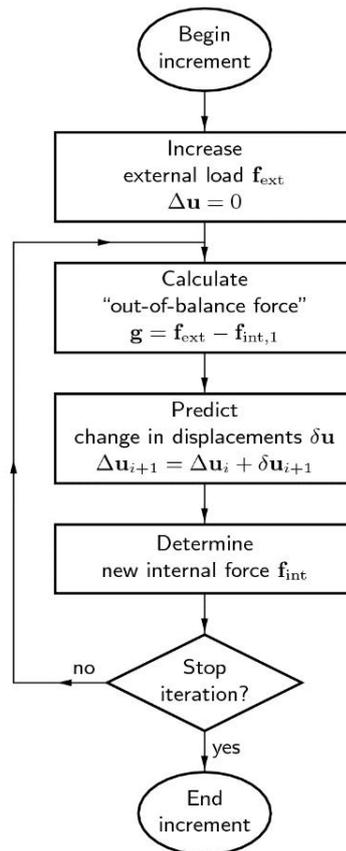


Figure 2-1 Calculation process of non-linear finite element analysis

2.2 Inputs as mean values without safety format

According to the Model Code 2010, the design principles based on global resistance design method is applied in NLFEA, which integrate the effect of various uncertainties, such as material properties and geometrical dimensions, in a global design resistance. In order to study how safety formats perform in the NLFEA, different analysis results by inputting only mean material properties and three safety formats are compared in this thesis.

In terms of the NLFEA without safety format, inputs are derived from the mean value of material properties. The result from the finite element analysis is just the design resistance. The calculation of inputs will be shown in Table 2-1 below.

Table 2-1 Concrete properties of mean value without safety format

$f_c (MPa)$	$f_{ct} (MPa)$	$E_c (MPa)$	$G_F (N / mm)$	$G_c (N / mm)$
$f_{cm} = f_{ck} + \Delta f$	$f_{ctm} = 0.3 \cdot (f_{ck})^{2/3}$	$E_{ci} =$ $0.85 \cdot 22000 \cdot \left(\frac{f_{cm}}{10}\right)^{1/3}$	$G_F =$ $0.073 \cdot f_{cm}^{0.18}$	$G_c = 250G_F$

Table 2-2 Steel properties of mean value without safety format

$f_y (MPa)$
$f_{ym} \geq f_{yk} + 10$

In Larsen's work, the fracture energy is calculated as follow:

$$G_F = G_{F0} \left(\frac{f_{cm}}{f_{cm0}} \right)^{0,7}$$

$D_{max} (mm)$	$G_{F0} (Nmm/mm^2)$
8	0,025
16	0,030
32	0,058

Where D_{max} is the maximal size of aggregate.

The ultimate load of the failure stage of NLFEA is considered the design resistance.

2.3 Inputs as mean values with safety formats

In order to perform the Ultimate Limit State (ULS) verification, the design resistance should be obtained and compared with the design loads. According to the fib Model Code 2010, three methods for the ULS verification are demonstrated in the Guideline: the Global Resistance Factor method (GRF), the Partial Factor method (PF) and the Estimate of Coefficient of Variation Of resistance method (ECOV). These methods contain the safety factors for the ultimate limit state, thus they are also known as safety formats.

2.3.1 The Global Resistance Factor method (GRF)

According to this method, the global resistance of the structure is a random variable. The effects of various uncertainties are integrated in a global design resistance and can be expressed by a global safety factor. In other words, a global safety coefficient is applied to reduce the uncertainties and make the results safe enough.

In this method, the mean mechanical properties of GRF are used as inputs. In GRF, the safety approach is achieved by applying a sort of safety factors, then the new mean values are derived from characteristic value. This method accounts for more random uncertainties of concrete and reinforcement, thus it also contains greater safety. The concrete mechanical property is calculated as:

$$f_{cm,GRF} = 0.85 \cdot f_{ck}$$

The reinforcement mechanical properties are as follow:

$$f_{ym,GRF} = 1.1 \cdot f_{yk}$$

$$f_{yk} = f_{ym} e^{(-1.65 \times 0.06)}$$

Then other concrete parameters can then be derived from the $f_{cm,GRF}$ via standard relation mentioned in 2.2. Then the new global safety coefficient for both concrete and reinforcement is derived to express the relation between new mean values and design values^[4].

$$\gamma_{GL} = 1.27$$

The design resistance R_d is taken as the design value of the ultimate load calculated as:

$$R_d = \frac{P_u}{\gamma_{GL}}$$

where P_u is the ultimate load obtained from the NLFEA.

2.3.2 The Partial Factor method (PF)

This method calculates the design resistance using the design values of different material properties. The design values are calculated applying the partial factors to the characteristic values, which is shown below:

$$f_{cd} = \frac{f_{ck}}{\gamma_c}$$

$$f_{ctd} = \frac{f_{ctk}}{\gamma_c}$$

$$f_{yd} = \frac{f_{yk}}{\gamma_s}$$

$$f_{yk} = f_{ym} e^{(-1.65 \times 0.06)}$$

where the concrete partial safety coefficient equal to 1.5, the steel partial safety coefficient equal to 1.15. The other concrete parameters can then be derived from the f_{cd} via standard relations. The ultimate load P_u obtained from the analysis by inputting the design mechanical properties is already the design resistance R_d .

In partial factor method, the model uncertainties are taken into account by the partial factors that derive the design values. This leads to lower inputs in all locations and is not correspond to the probabilistic concept. Thus the design resistance is deviated compared to the reality. However, partial factor method is proved to be applicable in practice and can perform a safe estimation.

2.3.3 The Estimate of Coefficient of Variation Of resistance method (ECOV)

Based on the probabilistic studies, the random distribution of resistance of RC structure can be described as lognormal distribution. This distribution can be described by mean resistance R_m and coefficient of variation of resistance V_R . Thus by applying safety factors and dealing with lognormal distribution relation, the uncertainties are taken into account. According to this method an estimate of mean and characteristic values of resistance shall be calculated using corresponding values of material parameters.

Calculation of mean and characteristic resistance should be performed by inputting the mean and characteristic mechanical properties in the NLFEA respectively. Then the coefficient of variation V_R is calculated as:

$$V_R = \frac{1}{1.65} \ln\left(\frac{R_{u,m}}{R_{u,k}}\right)$$

The global resistance factor γ_R is calculated as:

$$\gamma_R = e^{\alpha_R \beta V_R}$$

Where α_R is sensibility factor for the reliability of resistance and β is a reliability index.

Then the design resistance is:

$$R_d = \frac{P_{u,m}}{\gamma_{RD} \cdot \gamma_r}$$

where R_d is the ultimate load obtained from the analysis by inputting mean measured mechanical properties, γ_{RD} is the model uncertainty coefficient equal to 1.06.

3. Methods and results

In this chapter, the physical tunnel segment behavior, experiment and Larsen's work will be introduced first. Then it will mainly focus on the modelling method demonstration and showing the valuable and critical results from the NLFEA.

3.1 Introduction of case study

3.1.1 Realistic tunnel segment

The experiment stated in the work by Michael P. Collins and Daniel Kuchma^[1] is a simulation of Toronto subway tunnel segment. In reality, the RC structure is a typical single-cell box. The top one-way slab is a typical example of the very thick but lightly reinforced structure, which is 1.4 m thick and has 0.5 percent of flexural reinforcement. What's more, such members are constructed with no stirrups. The tunnel segment of Toronto subway is shown below:

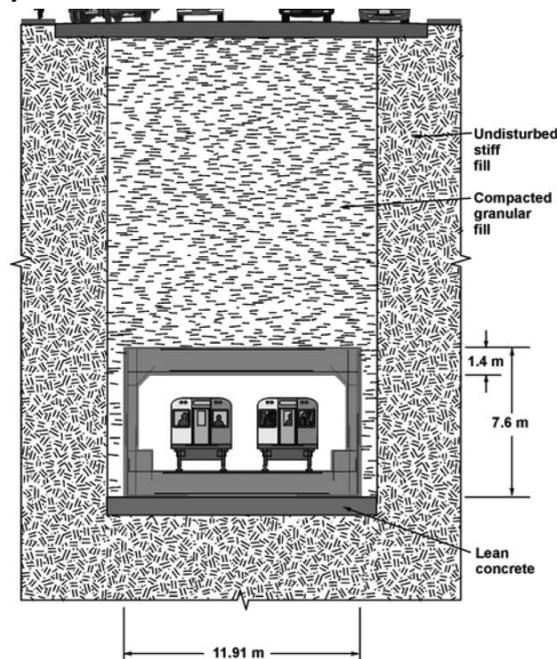


Figure 3-1 Tunnel segment of Toronto subway

3.1.2 Experiment from Michael P. Collins and Daniel Kuchma

According to the work by Michael P. Collins and Daniel Kuchma^[1], a 46% scale model of the tunnel segment is built. The experimental model can be seen in Figure 3-2.

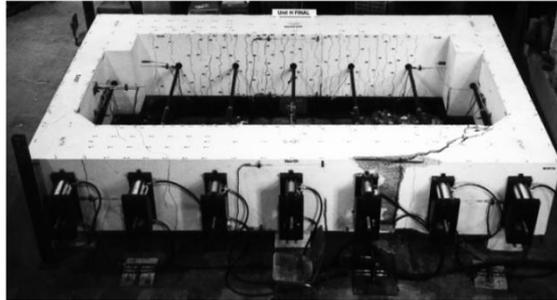


Figure 3-2 Experiment

The model was built with the depth of 960 mm. Seven sets of jacks are set to simulate a distributed load on top and bottom respectively. The jacks can simulate loading condition underground. The concrete has the compression strength of 45 MPa. The reinforcement of No. 15, No. 20 and US #3 has the yielding strength 460, 490 and 508 MPa. The concrete and reinforcement are set as is shown below.

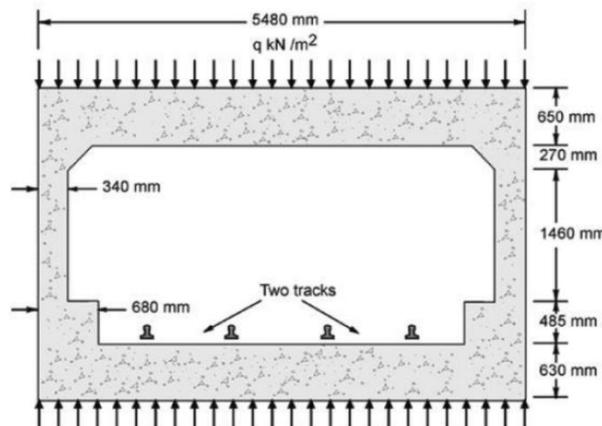


Figure 3-3 Concrete dimension

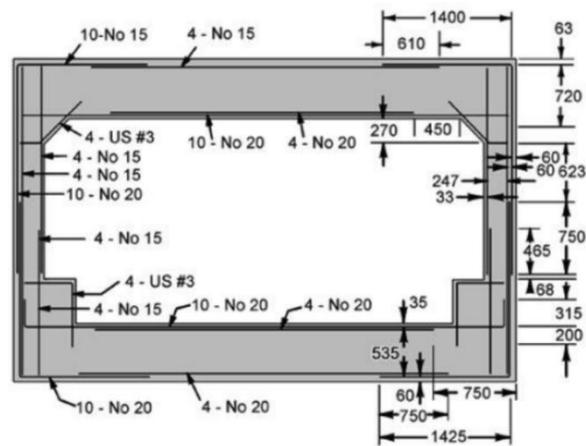


Figure 3-4 Details of reinforcement

In the figure 3-5, a typical shear crack appears in the top slab. Thus the maximum load in the experiment can be regarded as the shear capacity. From the experiment, the maximum load is 347 kN/mm², which can be seen in the figure below.

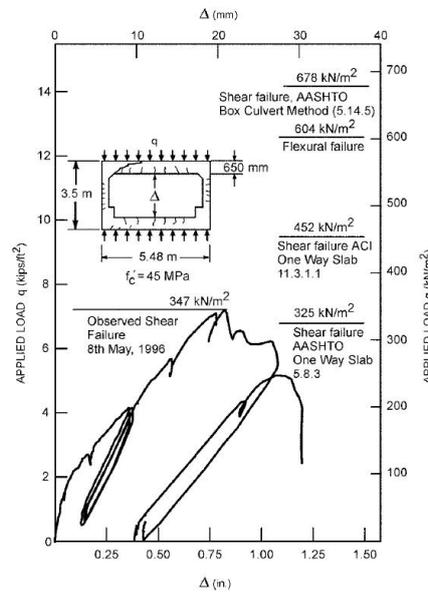


Figure 3-5 Load-deformation response of the experiment

3.1.3 NLFEA by Larsen

Larsen made four different finite element analysis to simulate the experiment. They are linear and non-linear finite element analysis with beam element and plane stress element respectively. In this thesis, only non-linear finite element analysis with plane stress element is concerned. In Larsen's work, the program inputs are the mean value of material properties mentioned in experiment. The reinforcement is embedded into the concrete and fully bonded. Other unknown parameters are derived following the principle in chapter 2.2. The inputs are shown in the table 3-1.

Larsen only built a quarter of the structure to reach the convergence in NLFEA. This model is shown below.

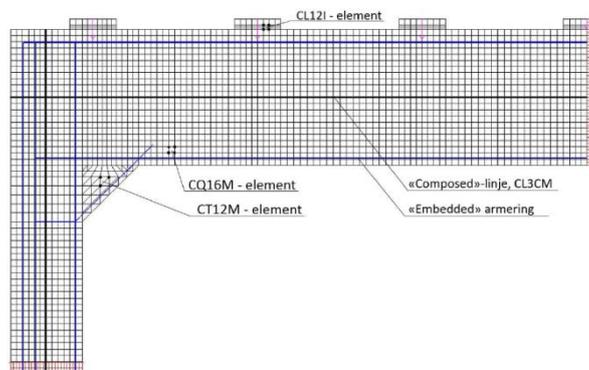


Figure 3-6 Larsen's NLFEA model

According to Larsen's work, the maximum load of the structure is 289 kN/mm². The load-deformation curve of experiment, hand calculation and NLFEA are shown in the figure 3-7

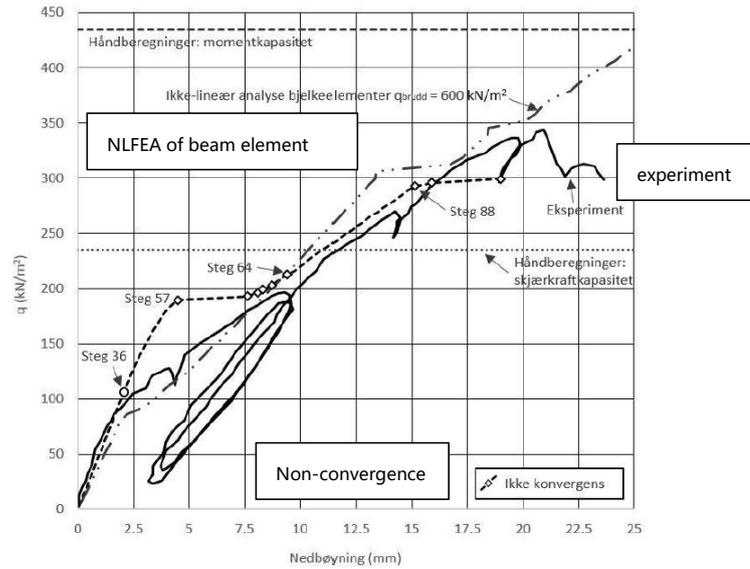


Figure 3-7 Load-deformation response of Larsen's work

It is important to emphasize that the fracture energy calculation in Larsen's work is stated in Model Code 1990 as is mentioned in chapter 2.2. The latest Model Code 2010 indicates a different fracture energy calculation that leads to a higher value compared with the old one. Because of the fracture energy has influence on the shear capacity, the comparative analysis of NLFEA with and without safety formats is performed twice corresponding to different fracture energy calculation.

In the following chapter, a reproduction of Larsen's work is done to check the modelling. Then three safety formats are applied. These four NLFEA are again repeated with a different fracture energy calculation.

3.2 Modelling method

DIANA is used to run the NLFEA. The element type of concrete is CQ16M, which is an eight-node quadrilateral isoparametric plane stress element. The thickness of the model is applied as 960 mm. In terms of the cutting edge of the tunnel, the CT12M, a six-node triangular isoparametric plane stress element, is generated automatically^[7].

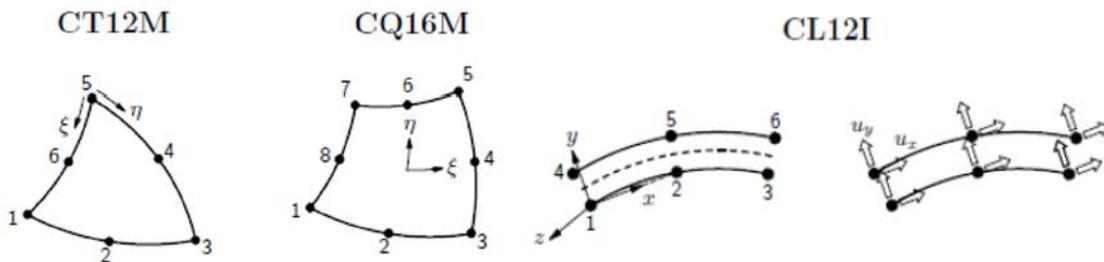


Figure 3-8 Element type

According to the experiment, 14 jacks are connected to the structure to apply load. In this model, load from jacks is simulated by point load. However, this will cause unreliable results in NLFEA because of the enormous displacement caused by point load. To avoid this, seven small plates are connected to the structure to distribute point load. The plate has much higher Young's modulus than concrete and is connected to concrete by a layer of interface. The interface is assigned CL12I element, which is an interface element between two lines in a two-dimensional configuration. The interface is stiffer in vertical direction than horizontal, which means that the plate can not deform but can slip along the edge. Same as Larsen's work, reinforcement is embedded into concrete and fully bonded.

The concrete and reinforcement modelling are shown below. The model refers to Larsen's work. Only a quarter of the structure is applied in order to reach the convergence. This model is using the smeared cracking strategy, based on total strain crack. The crack orientation is fixed. The Poisson ratio reduction is considered as the type of damage based. The details of modelling are shown in the appendix.

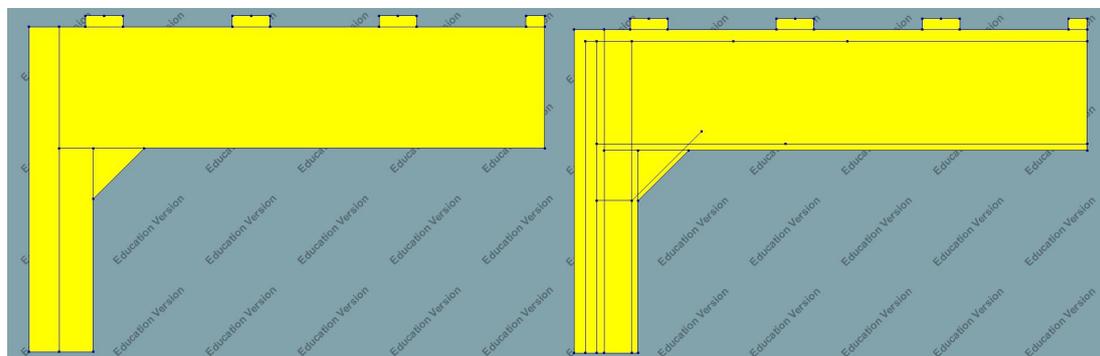


Figure 3-9 Concrete and reinforcement

In this thesis, arc-length control is used as increment method referring to Larsen's work. In addition, according to the load-displacement curve in Larsen's work, displacement control can also work. The iteration method is based on regular Newton-Raphson with maximum 100 iterations per increment. The convergence norm is based on energy. The convergence tolerance is 0.001.

When the model is proved valid, parameters calculated according to different design methods are inputted. The inputs for all analysis are shown below.

Table 3-1 Inputs for all analysis

Inputs	unit	Larsen's work mean value	fib Model Code 2010 (fracture energy calculated as Larsen's work)					fib Model Code 2010						
			Reproduction of Larsen's work		GRF	PF	ECOV		Reproduction of Larsen's work		GRF	PF	ECOV	
			mean value	mean value	mean value from GRF	design value	Mean	Characteristic	mean value	mean value from GRF	design value	Mean	Characteristic	
f_c	N/mm ²	45	45	31.45	24.7	45	37	45	31.45	24.7	45	37		
f_{ct}	N/mm ²	3.35	3.35	3	1.55	3.33	2.33	3.35	3	1.55	3.33	2.33		
f_y of No.15	N/mm ²	460	460	458	400	460	414	460	458	400	460	414		
f_y of No.20	N/mm ²	490	490	485	426	490	444	490	485	426	490	444		
f_y of #3	N/mm ²	508	508	503	442	508	460	508	503	442	508	460		
G_F	N/mm	0.075	0.075	0.055	0.047	0.072	0.062	0.145	0.136	0.13	0.149	0.14		
G_c	N/mm	18.84	18.84	14	11.76	17.9	15.62	36.21	34.00	32.50	37.25	35.00		
E_c	N/mm ²	29363	29363	26371	24517	29363	27689	29363	26371	24517	29363	27689		
E_s	N/mm ²						210000							
$V_{concrete}$							0.15							
V_{steel}							0.3							
b	mm						30							
β_σ^{\min}							0.4							

3.3 Comparative results based on Larsen's work

To compare with Larsen's work, a reproduction of Larsen's work and safety formats including GRF, PF and ECOV are performed. The analysis results can be seen in figure 3-10, 3-11 and table 3-2.

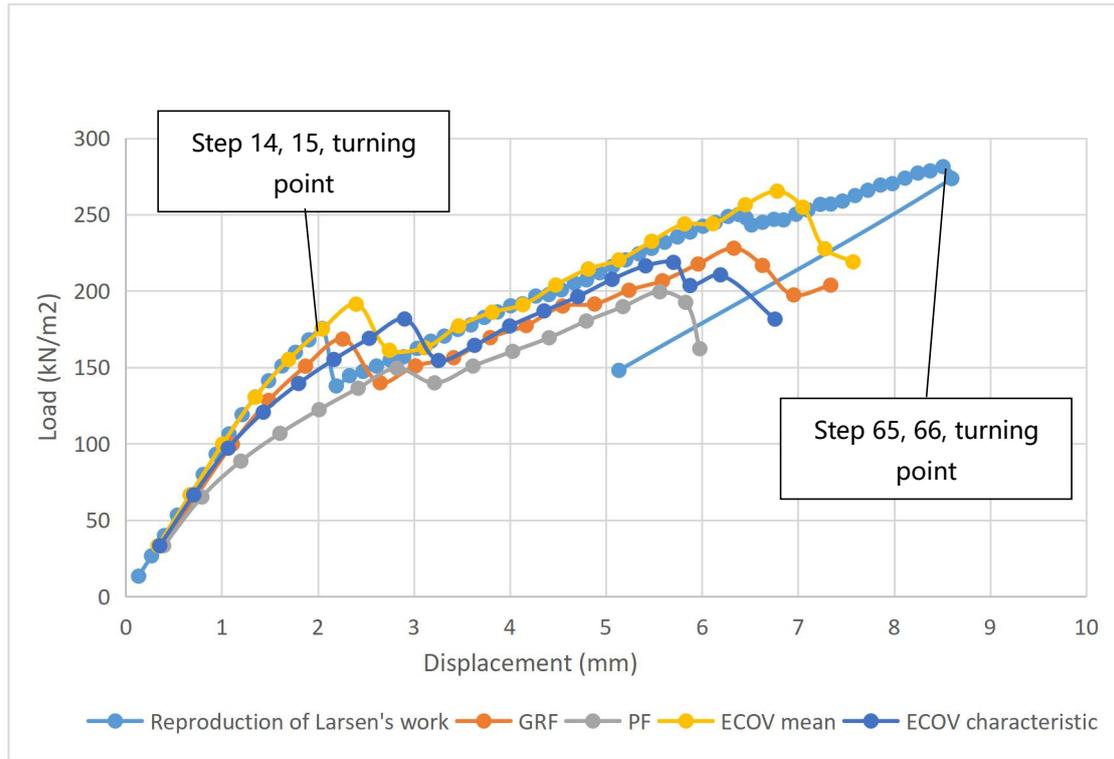


Figure 3-10 Load-displacement of all analysis

Table 3-2 NLFEA results

Method	Reproduction of Larsen's work	GRF	PF	ECOV	Larsen's work	Experiment
Design resistance (kN/m ²)	281.3	179.5	199.4	179.0	289	347
Percentage (%)	81.08%	51.73%	57.47%	51.59%	83.29%	100.00%

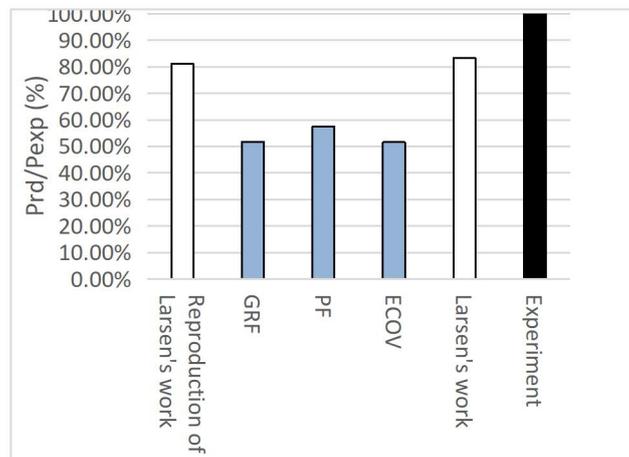


Figure 3-11 Comparison of NLFEA

Within each NLFEA, structural development with load increment of four analysis is similar. Thus, to emphasize the critical points on the load-displacement curve, the curve of reproduction of Larsen's work is taken as example. The curve is plotted referring to the value of node which located at the mid-bottom of top slab.

From figure 3-10 the critical steps are highlighted in the curve of reproduction of Larsen's work. Below are the Diana results of these steps, including the horizontal stress in concrete, horizontal stress in reinforcement, crack pattern and crack width, from which the important development of the structural behavior can be observed.

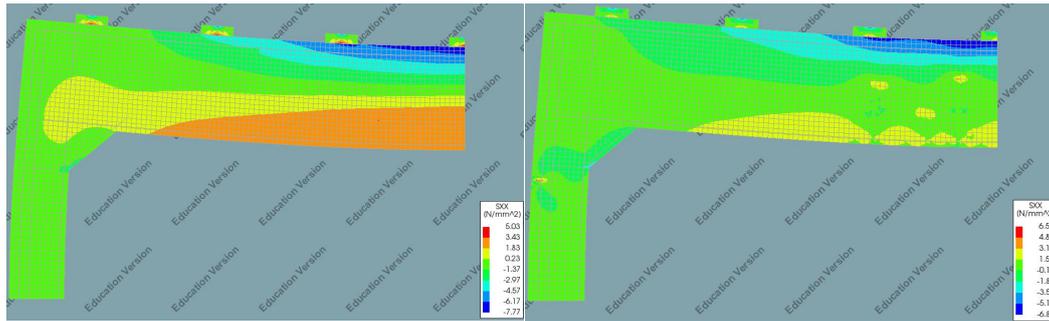


Figure 3-12 Element stress in x-direction of step 14, 15

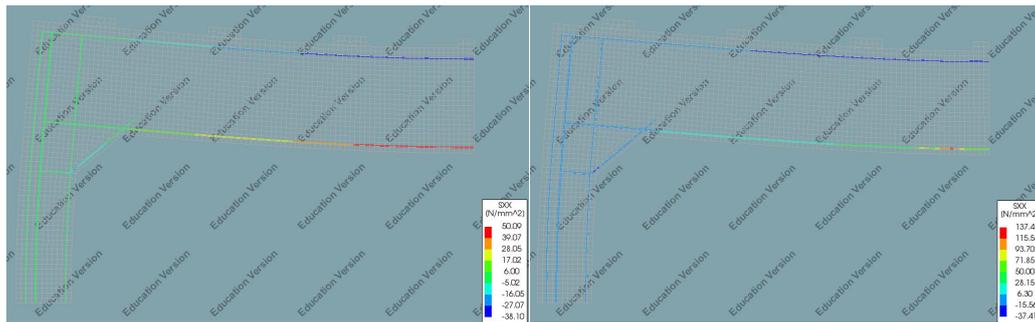


Figure 3-13 Reinforcement stress in x-direction of step 14, 15

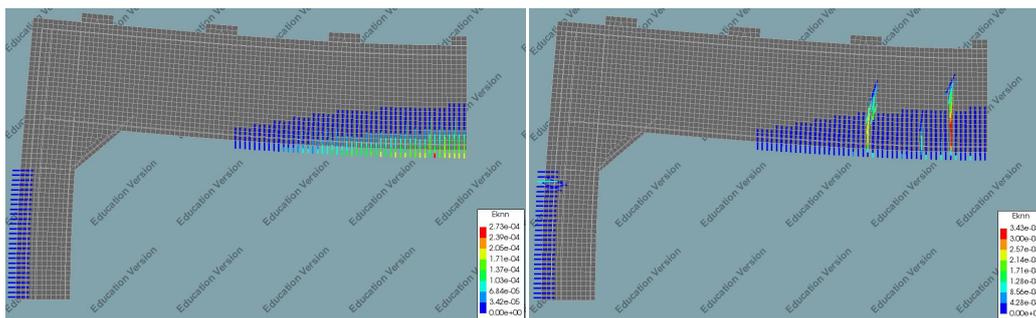


Figure 3-14 Crack pattern of step 14, 15

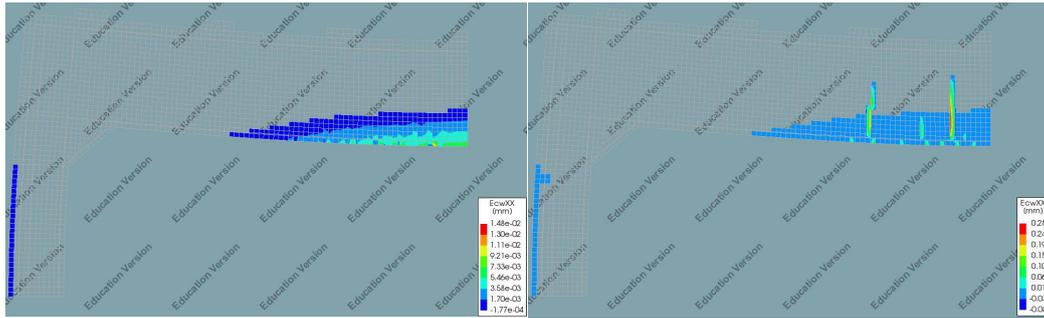


Figure 3-15 Crack width in x-direction of step 14, 15

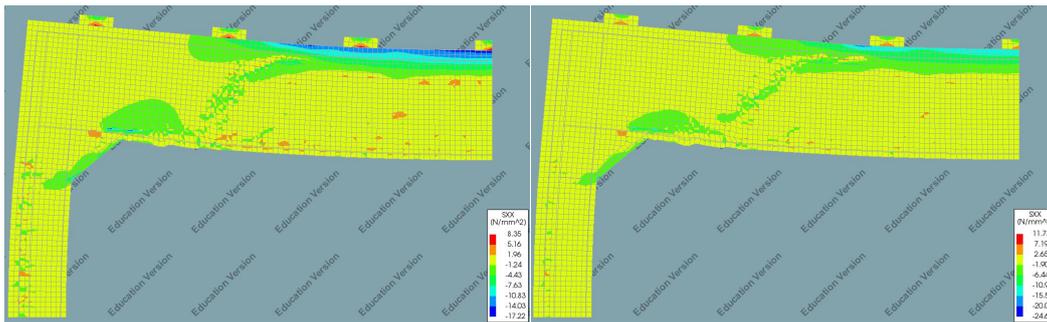


Figure 3-16 Element stress in x-direction of step 65, 66

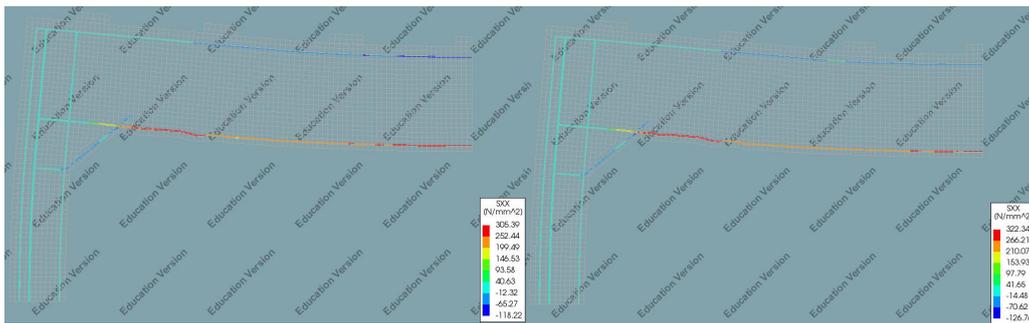


Figure 3-17 Reinforcement stress in x-direction of step 65, 66

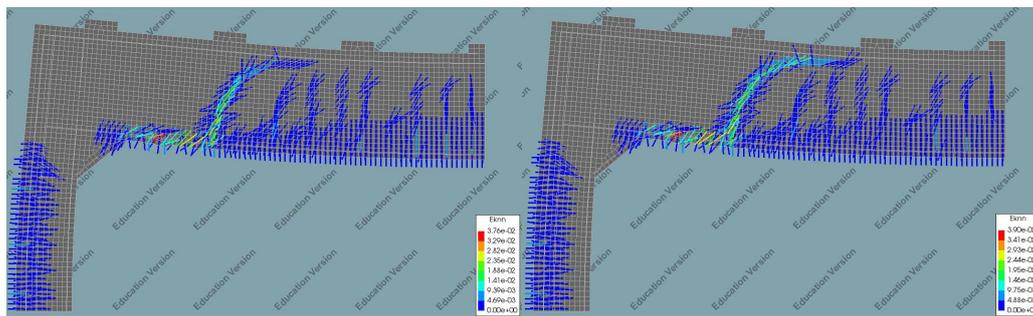


Figure 3-18 Crack pattern of step 65, 66

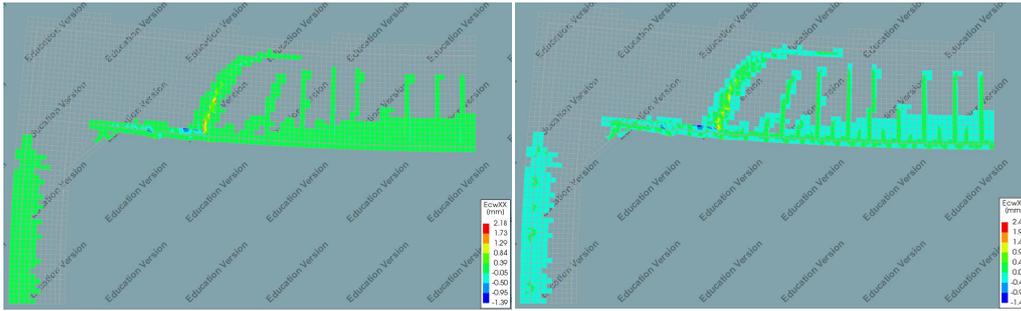


Figure 3-19 Crack width in x-direction of step 65, 66

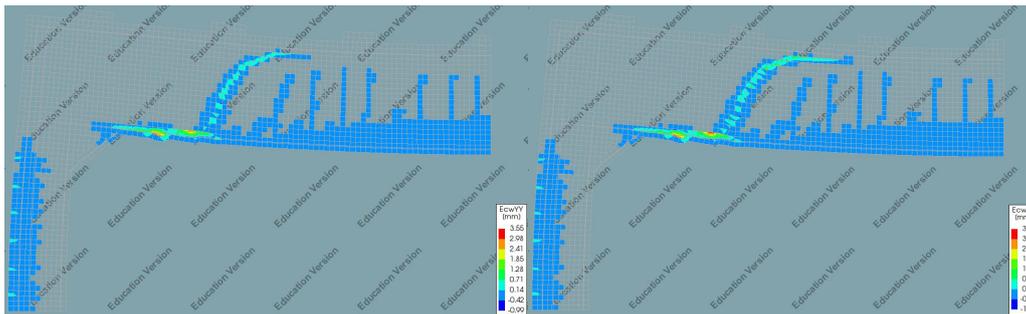


Figure 3-20 Crack width in y-direction of step 65, 66

3.4 Comparative results based on Model Code 2010

It is found that the fracture energy calculation method that Larsen use is different from the latest Model Code 2010. This leads to different analysis result of the design resistance. Thus, in this chapter a new sort of NLFEA is performed totally follow the Model Code 2010 in order to make comparative analysis about how the two fracture energy calculation methods influence the design resistance.

The analysis results can be seen in figure 3-21, 3-22 and table 3-3.

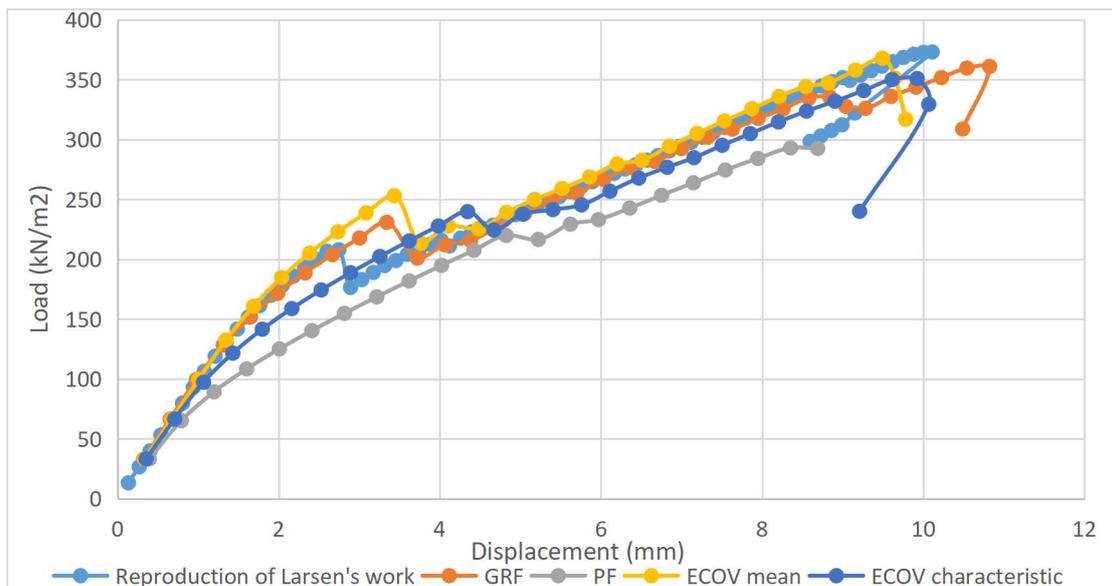


Figure 3-21 Load-displacement of all analysis

Table 3-3 Results comparison

Method	Reproduction of Larsen's work	GRF	PF	ECOV	Larsen's work	Experiment
Design resistance (kN/m ²)	373.1	284.3	293.0	315.6	289	347
Percentage (%)	107.51%	81.92%	84.43%	90.95%	83.29%	100.00%

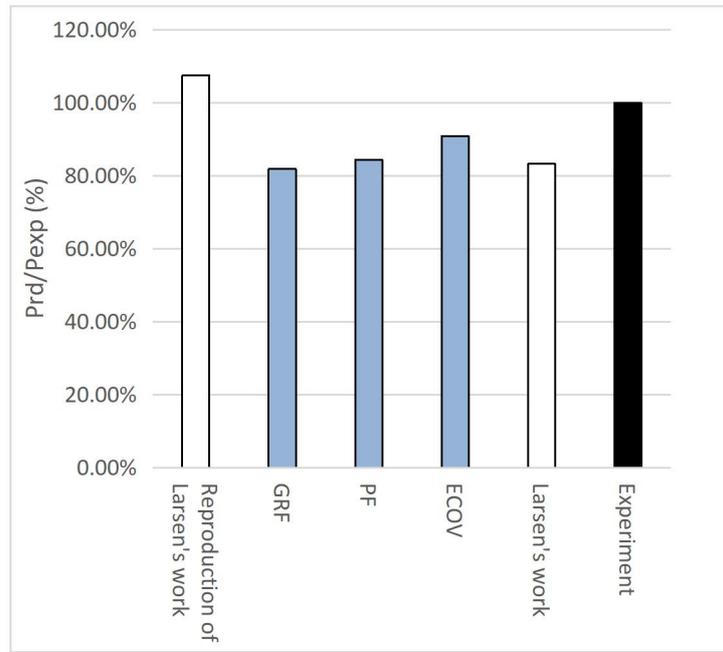


Figure 3-22 Comparison of NLFEA

The result of critical points in the load-displacement curve are similar to what is mentioned in chapter 3.3.

4. Discussion

In this chapter, the results and comparison are discussed, mainly focus on three topics: the development of structural behavior observed from NLFEA, the comparative analysis among design methods (with and without safety formats) and the comparative analysis of fracture energy effect.

4.1 Development of structural behavior observed from NLFEA

It is observed that critical points appear in load-displacement curve whose results are shown in chapter 3.3. The development of structural behavior can be seen from the results.

Firstly, from step 1 to step 14, small cracks appear in the mid-bottom of the top slab and they are developing from mid-span to sides along with the load increment. As is shown in the left of figure 3-14, the continuous small cracks are not consistent with reality. This is because of the assumption of embedded reinforcement that they are fully bonded, in other word, there is no slip between reinforcement and concrete. Then, in step 15, small cracks turns to big cracks as is shown in figure 3-14. In this step, the crack width of small cracks between big cracks decreases because the energy is released when big cracks appear. Figure 3-13 indicates that in step 15, reinforcement located in the big cracks takes more load while load is taken by a longer section of reinforcement in step 14. What's more, increasing load leads to the shear crack as is shown in figure 3-18. In step 65 and 66, the shear crack grows and penetrates the thickness of top slab. Then after step 66, the structure cannot take bigger load and fails. The crack width of shear crack should consider the results of both vertical and horizontal direction as is shown in figure 3-19 and 3-20. The reinforcement is not yielding even when structure fail.

4.2 Comparative analysis among design methods

The percentage of different results are shown in the figure 3-11 and table 3-2, comparing with the experimental results. The reproduction of Larsen's work obtains an approximate results the same as Larsen's work. This can prove that the modelling can work and simulate the experiment. It is obvious that the design resistance derived from safety formats are smaller than the reproduction of Larsen's work which is the method without safety formats. When comparing with the experimental results, safety formats will reduce the design resistance (shear capacity) about 45% while using mean values without safety formats can only reduce the maximum load about 20%. Further study is expected to discover how reliable these design methods are in practice.

4.3 comparative analysis of fracture energy effect

Applying the latest fracture energy calculation from the Model Code 2010, NLFEA results is shown in figure 3-22 and table 3-3. It is obvious that different fracture energy leads to big difference design resistance. The Percentage comparison of two sorts of NLFEA is shown again below.

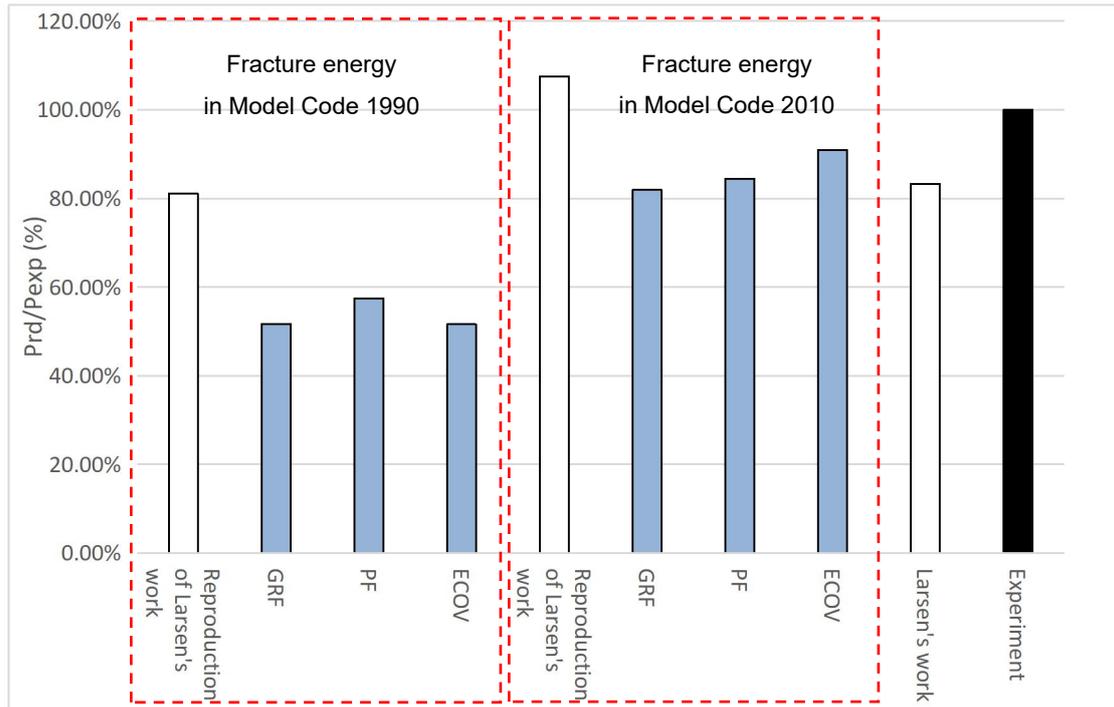


Figure 4-1 Comparison of results in terms of fracture energy

Results lead by fracture energy of Model Code 2010 increase about 30% compared to the former one. The later result without safety format exceed the experiment. Due to the others inputs are controlled the same, this change is mainly because of the different fracture energy.

It is observed that fracture energy has enormous influence on the design resistance, in this case it is also shear capacity. Fracture energy, as an important failure parameter, is difficult to predict because of the high uncertainties of concrete. The aggregate, additives, general composition and curing condition have influence on fracture energy according to the work by Alfred Strauss et al.^[6] The fib Model Code 2010 simplifies the problem of uncertainties by considering uncertain parameters to be deterministic, and addresses uncertainties through the use of empirical partial factors (Semi-probabilistic safety concept). To precisely simulate the shear capacity of RC structure in NLFEA, proper prediction of fracture failure parameters is needed.

According to the results above, design resistance (shear capacity) of RC structure is sensitive. The safety formats that dealing with the uncertainties of material is valid in the NLFEA. A reasonable range of conservation is important in practice. To have an insight of the modelling, the quarter model may lead to a larger result due to the ideal support condition. The unsymmetrical model about the horizontal axis may also cause the unexpected results.

5. Conclusion

This thesis mainly studies the non-linear finite element analysis with and without safety format according to the fib Model Code 2010, based on the tunnel segment experiment and the work from Leif Tore Larsen. The non-linear finite element analysis can simulate the real development of shear failure of RC structure. Then the application of safety format in NLFEA is effective and proper when compared with the expected results. Safety format in NLFEA can be applied as a guide in real design and research. Last but not the least, it is realized that fracture energy has big influence on the shear capacity. However fracture energy is hard to predict accurately. A more precise method to predict the fracture failure parameter is needed to launch better NLFEA in research and design, this may be reached by further probabilistic analysis.

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