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Ship motions

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SHIP MOTIONS

Guest Lectures

by

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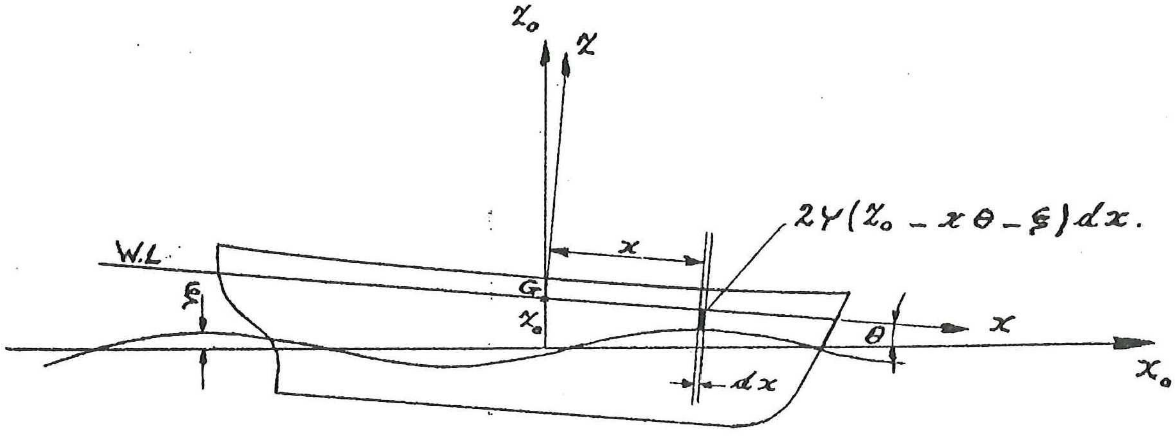
1. Introduction.

In this lecture I shall treat for you some aspects of the problem of shipmotions in still water. The solution of this problem is important for the calculation and analysis of shipmotions and related phenomena such as bending moments, slamming, the shipping of green water and so on, in regular and irregular seas.

To show you the importance of the oscillations in still water we shall study first of all, in a very simplified way, the motions of a ship in long regular and longitudinal head waves.

The ship has no forward speed and the cross sections are wall sided in the vicinity of the load waterline. Consider a right hand coordinate system, fixed in space, $(x_o y_o z_o)$. A second system is attached to the ship: $(x\,y\,z)$. The origin of this system lies in the centre of gravity of the ship (G). For the ship at rest the two coordinate systems coincide.



The surging motion is neglected and the heave and pitch of the ship are described by $z_o(t)$ and $\theta(t)$.

A long regular wave ζ, with small amplitude r runs in the direction of the negative x_o axis. The surface elevation of this wave is given by:

$$\zeta = r \cos (k x_o + \omega t),$$

where $k = \frac{2\pi}{\lambda}$ is the wave number and $\omega = \frac{2\pi}{T}$ is the circular frequency of the wave.

Because we neglect the surging motion and by assuming that the motion amplitudes are very small, we may write:

$$x \cong x_o .$$

The vertical displacement of a strip of the ship at a distance x from the centre of gravity is given by:

$$z_o - x \theta .$$

The displacement relative to the water surface is:

$$z_o - x \theta - \zeta .$$

The relative vertical velocity is given by:

$$\dot{z}_o - x \dot{\theta} - \dot{\zeta} .$$

The relative vertical acceleration will be:

$$\ddot{z}_o - x \ddot{\theta} - \ddot{\zeta} .$$

The vertical forces acting on a strip of length dx of the ship are related by the following equation: (Newton)

$$\left(\frac{w'}{g} dx\right)(\ddot{z}_o - x\ddot{\theta}) = \rho g A_x dx - 2\rho g Y(z_o - x\theta - \zeta) dx - w' dx$$
$$- N'(\dot{z}_o - x\dot{\theta} - \dot{\zeta}) dx +$$
$$- m'(\ddot{z}_o - x\ddot{\theta} - \ddot{\zeta}) dx,$$

where: w' is the weight of the ship per unit length;
A_x is the cross section under the load waterline;
Y is the half width of the waterline at x;
N' is a coefficient (the sectional damping coefficient);
m' is a coefficient (the sectional added mass).

Here we assume that the pressure in the wave is not disturbed by the presence of the ship.

Because the wave length is very large in comparison with the ship length, the pressure in the wave is taken as the hydrostatic pressure.

The total vertical force on the ship is found by integration. We have: $\rho g \int_L A_x \, dx = \int_L w' \, dx$, because at rest the weight of the ship equals the weight of the displacement. Also we have:

$\int_L \frac{w'}{g} dx = \rho \nabla$ the mass of the displacement,

$2 \rho g \int_L Y \, dx = \rho g A_{wL} = c$ the waterline area times the specific gravity of the water.

This coefficient is called: the restoring force coefficient.

$2 \rho g \int_L Y x \, dx = + \rho g S_{wL} = g$ the first moment of the waterplane area with respect to a transverse axis through G, assuming that G lies in the waterplane;

$\int_L N' \, dx = b$ the damping coefficient for heave;

$\int_L N' \, x \, dx = e$ a dynamic cross coupling coefficient;

$\int_L m' \, dx = a$ the added mass;

$\int_L m' \, x \, dx = d$ a dynamic cross coupling coefficient.

Re-arranging after integration we find the heave equation:

$$(a + \rho V)\ddot{z}_o + b \dot{z}_o + c z_o - d \ddot{\theta} - e \dot{\theta} - g \theta = 2 g \int_L Y \zeta \, dx +$$

$$+ \int_L N' \dot{\zeta} \, dx + \int_L m' \ddot{\zeta} \, dx = F_a \, e^{i(\omega t + \varepsilon_z)}.$$

By taking the moment of the forces on the strip with respect to G, we find in a similar way the pitch equation:

$$(A + k_{yy}^2 \rho \nabla)\ddot{\Theta} + B\dot{\Theta} + C\Theta - D\ddot{z}_o - E\dot{z}_o - Gz_o = M_a e^{i(\omega t + \varepsilon_\Theta)}.$$

These equations for heave and pitch are secondorder linear differential equations, which are coupled. In the heave equation we have terms which contain the pitch angle "Θ" and in the pitch moment equation the heave "z_o" appears to be present. The coupling arises from the fact that the distribution of the forces along the hull is not symmetric with respect to G. For instance the term $g\Theta$ exists for a for and aft non symmetric waterline, because $g = \rho g S_{wL}$, where S_{wL} is the statical moment of the waterplane with respect to a transverse axis through G. For a symmetrical waterline $g = 0$. For a symmetrical shipform at zero speed the other coupling coefficients are also equal to zero. In that case:

$$d = e = g = D = E = G = 0.$$

It has to be emphasized that up till now our problem is very simplified: the wave length is very large in comparison with the ship length and consequently the boyancy in the wave is taken as in the case of a hydrostatic calculation; also the effect of forward speed is not considered.

The left hand sides of the equations contain only quantities which describe the motion in still water. The wave elevation (ζ) terms are situated in the right hand sides. These right hand sides give the forces and moments on a non heaving and non pitching ship in waves. We have the same situation when a more refined analysis is applied to the problem, at least as a first approximation.

The calculation of shipmotions in regular head waves by using a strip theory, has been discussed in a number of papers. Recent contributions were given by Korvin-Kroukovsky and Jacobs [1], Fay [2], Watanabe [3] and Fukuda [4].

In these papers the influence of forward speed on the hydrodynamic forces is considered and dynamic cross-coupling terms are included in the equations of motion, which are assumed to describe the heaving and pitching motions.

In earlier work [5] it was shown that a relatively small influence of speed exists on the damping coefficients, the added mass and the exciting forces, at least for the case of head waves and for speeds which are of practical interest. On the other hand, forward speed has an important effect on some of the dynamic cross-coupling coefficients. Although, at a first glance these terms could be regarded as second order quantities, it was pointed out by Korvin-Kroukovsky [1] and also by Fay [2] that they can be very important for the amplitudes and phases of the motions. This has been confirmed in [5] where the coupling terms are neglected in a calculation of the heaving and pitching motions. In this calculation we used coefficients of the motion equations, which were determined by forced oscillation tests. In comparison with the calculation where the cross-coupling terms are included and also in comparison with the measured motions, an important influence is observed, as shown in Figure 1, which is taken from reference [5]. Further analysis showed that the discrepencies between the coupled and uncoupled motions were mainly due to the damping cross-coupling terms.

The influence of forward speed has been discussed to some extent in Vossers's thesis [6]. From a first order slender body theory it was found that the distribution of the hydrodynamic forces along an oscillating slender body is not influenced by forward speed. Vossers concluded that the inclusion of speed dependent damping cross-coupling terms is not in agreement with the use of a strip theory. In view of the above mentioned results such a simplification does not hold for actual shipforms.

For symmetrical shipforms at forward speed, it was shown by Timman and Newman [7] that the damping cross-coupling coefficients for heave and pitch are equal in magnitude, but opposite in sign. Their conclusion is valid for thin or slender submerged of surface ships and also for non slender bodies.

Golovato's work [8] and some of our experiments [5] on oscillating shipmodels confirmed this fact for actual surface ships to a certain extent.

The effects of forward speed are indeed very important for the calculation of shipmotions in waves. The two-dimensional solutions for damping and added mass of oscillating cylinders on a free surface, as given by Grim [9] and Tasai [10] show a very satisfactory agreement with experimental results. When the effects of forward speed can be estimated with sufficient accuracy, such two-dimensional values may be used to calculate the total hydrodynamic forces and moments on a ship, provided that integration over the shiplength is permissible.

In order to study the speed effect on an oscillating shipform in more detail, a series of forced oscillating experiments was designed. The main object of these experiments was to find the distribution of the hydrodynamic forces along the length of the ship as a function of forward speed and frequency of oscillation.

2. The experiments.

The oscillation tests were carried out with a 2.3 meter model of the Sixty Series, having a blockcoefficient C_B = .70. The main dimensions are given in Table 1. The model is made of polyester, reinforced with fibreglass, and consists of seven separate sections of equal length. Each of the sections has two end-bulkheads. The width of the gap between two sections is one millimeter. The sections are not connected to each other, but they are kept in their position by means of stiff strain-gauge dynamometers, which are connected to a longitudinal steel box girder above the model.

Table 1.

Main particulars of the shipmodel.

Length between perpendiculars	2.258 m
Length on the waterline	2.296 m
Breadth	0.322 m
Draught	0.129 m
Volume of displacement	0.0657 m^3
Blockcoefficient	0.700
Coefficient of mid length section	0.986
Prismatic coefficient	0.710
Waterplane area	0.572 m^2
Waterplane coefficient	0.785
Longitudinal moment of inertia of waterplane	0.1685 m^4
L.C.B. forward of $L_{pp}/2$	0.011 m
Centre of effort of waterplane after $L_{pp}/2$	0.038 m
Froude number of service speed	0.20

The dynamometers are sensitive only for forces perpendicular to the baseline of the model.

By means of a Scotch-Yoke mechanism a harmonic heaving or pitching motion can be given to the combination of the seven sections, which form the shipmodel. The total forces on each section could be measured as a function of frequency and speed.

A non segmented model of the same form was also tested in the same conditions of frequency and speed to compare the forces on the whole model with the sums of the section results. A possible effect of the gaps between the sections could be detected in this way. The arrangement of the tests with the segmented model and with the whole model is given in Figure 2.

The mechanical oscillator and the measuring system is shown in Figure 3. In principle the measuring system is similar to the one described by Goodman [11]: the measured force signal is multiplied by $\cos\omega t$ and $\sin\omega t$ and after integration the first harmonics of the in-phase and quadrature components can be found without distortion due to vibration noise. In some details the electronic circuit differs somewhat from the description in [11]. In particular synchro resolvers are used instead of sine-cosine potentiometers, because they allow higher rotational speeds.

The accuracy of the instrumentation proved to be satisfactory which is important for the determination of the quadrature components, which are small in comparison with the in-phase components of the measured forces.

Throughout the experiments only first harmonics were determined. It should be noted that non-linear effects may be important for the sections at the bow and the stern where the ship is not wall-sided. The forced oscillation tests were carried out for frequencies up to ω = 14 rad/sec. and four speeds of advance were considered, namely: Fn = .15, .20, .25 and .30. Below a frequency of ω = 3 to 4 rad/sec. the experimental results are influenced by wall effect due to reflected waves generated by the oscillating model.

The motion amplitudes of the shipmodel covered a sufficiently large range to study the linearity of the measured values (heave $\sim$ 4 cm, pitch $\sim$ 4.6 degrees). An example of the measured forces on section 2, when the combination of the seven sections performs a pitching motion, is given in Figure 4.

3. Presentation of the results.

3.1. Whole model.

It is assumed that the force F and the moment M acting on a forced heaving or pitching shipmodel can be described by the following equations:

$$\left.\begin{aligned} \text{Heave: } (a+\rho\nabla)\ddot{z}_o + b\dot{z}_o + cz_o &= F_z \sin(\omega t + \alpha) \\ D\ddot{z}_o + E\dot{z}_o + Gz_o &= -M_z \sin(\omega t + \beta) \end{aligned}\right\} \quad (1)$$

$$\left.\begin{aligned} \text{Pitch: } (A + k_{yy}^2\rho\nabla)\ddot{\theta} + B\dot{\theta} + C\theta &= M_\theta \sin(\omega t + \gamma) \\ d\ddot{\theta} + e\dot{\theta} + g\theta &= -F_\theta \sin(\omega t + \delta) \end{aligned}\right\} \quad (2)$$

For a given heaving motion: $z_o = z_a \sin\omega t$, it follows that:

$$\left.\begin{aligned} b &= \frac{F_z \sin\alpha}{z_a\,\omega} \\ a &= \frac{cz_a - F_z\cos\alpha}{z_a\,\omega^2} - \rho\nabla \\ E &= \frac{-M_z \sin\beta}{z_a\,\omega} \\ D &= \frac{Gz_a + M_z\cos\beta}{z_a\,\omega^2} \end{aligned}\right\} \quad (3)$$

Similar expressions are valid for the pitching motion. The determination of the damping coefficients b and B and the damping cross-coupling coefficients e and E is straightforward: for a given frequency these coefficients are proportional to the quadrature components of the forces or moments for unit amplitude of motion. For the determination of the added mass, the added mass moment of inertia, a and A, and the added mass cross-coupling coefficients d and D it is necessary to know the restoring force and moment coefficients c and C, and the statical cross-coupling coefficients g and G.

The statical coefficients can be determined by experiments as a function of speed at zero frequency. For heave the experimental values show very little variation with speed; they were used in the analysis of the test results.

In the case of pitching there is a considerable speed effect on the restoring moment coefficient C. C decreases approximately 12% when the speed increases from Fn = .15 to .30. This reduction is due to a hydrodynamic lift on the hull when the shipmodel is towed with a constant pitch angle. Obviously this lift effect also depends on the frequency of the motion. Consequently, the coefficient of the restoring moment, as determined by an experiment at zero frequency, may differ from the value at a given frequency.

As it is not possible to measure the restoring moment and the statical cross-coupling as a function of frequency, it was decided to use the calculated values at zero speed. This is an arbitrary choise, which affects the coefficients of the acceleration terms: for harmonic motions a decrease of C by ΔC results in an increase of A by $\frac{\Delta C}{\omega^2}$ when C is used in the calculation.

The results for the whole model are given in the Figures 5 and 6. The results for the heaving motion were already published in [13]; they are presented here for completeness.

3.2. Results for the sections.

The components of the forces on each of the seven sections were determined in the same way as for the whole model. As only the forces and no moments on the sections were measured two equations remain for each section:

$$\text{Heave:}\quad (a^* + \rho\nabla^*)\ddot{z}_o + b^*\dot{z}_o + c^* z_o = F_z^* \sin(\omega t + \alpha^*)$$
$$\text{Pitch:}\quad (d^* + \rho\nabla^* x_i)\ddot{\theta} + e\dot{\theta} + g\theta = -F_\theta^* \sin(\omega t + \delta^*) \qquad (4)$$

where $\rho\nabla^* x_i$ is the mass-moment of the section i with respect to the pitching axis. The star (*) indicates the coefficients of the sections. The section coefficients divided by the length of the sections give the mean cross-section coefficients, thus:

$$\frac{a^*}{L_{pp}/7} = \bar{a}' ,$$

and so on. Assuming that the distributions of the cross-sectional values of the coefficients: a', b' etcetera, are continuous curves, these distributions can be determined from the seven mean cross-section values. In the Figures 7, 8, 9 and 10 the distributions of the added mass a, the damping coefficient b and the cross-coupling coefficients d and e are given as a function of speed and frequency. Numerical values of the section results, a*, b* etcetera, are summarized in the Tables 2, 3, 4 and 5.

In Figure 8 it is shown that the distribution of the damping coefficient b depends on forward speed and frequency of oscillation. The damping coefficient of the forward part of the shipmodel increases when the speed is increasing. At the same time a decrease of the damping coefficient of the afterbody is noticed. For high frequencies negative values for the cross-sectional damping coefficients are found.

The added mass distribution, as shown in Figure 7, changes very little with forward speed but there is a shift forward of the distribution curve for increasing frequencies.

Negative values for the cross-sectional added mass are found for the bow-sections at low frequencies. For higher frequencies the influence of frequency becomes very small.

The distribution of the damping cross-coupling coefficient e varies with speed and frequency as shown in Figure 10. From Figure 9 it can be seen that the added mass cross-coupling coefficient depends very little on speed. For higher frequencies the influence of frequency is small.

As a check on the accuracy of the measurements the sum of the results for the sections were compared with the results for the whole model. The following relations were analysed:

$$\sum a^* = a \qquad \int_L d' \, x \, dx = A$$

$$\sum b^* = b \qquad \int_L e' \, x \, dx = B$$

$$\sum d^* = d \qquad \int_L a' \, x \, dx = D$$

$$\sum e^* = e \qquad \int_L b' \, x \, dx = E$$

The results are shown in Figure 11 for a Froude number Fn = .20. For the other speeds a similar result was found. A numerical comparison is given in the Tables 2, 3, 4 and 5. It may be concluded that the section results are in agreement with the values for the whole model. No influence of the gaps between the sections could be found.

4. Analysis of the results.

The experimental values for the hydrodynamic forces and moments on the oscillating shipmodel will now be analysed by using the strip theory, taking into account the effect of forward speed. For a detailed description of the strip theory the reader is referred to [1], [2] and [3]. For convenience a short description of the strip theory is given here. The theoretical estimation of the hydrodynamic forces on a cross-section of unit length is of particular interest with regard to the measured distributions of the various coefficients along the length of the shipmodel.

4.1. Strip theory.

A right hand coordinate system $x_o y_o z_o$ is fixed in space. The z_o-axis is vertically upwards, the x_o-axis is in the direction of the forward speed of the vessel and the origin lies in the undisturbed water surface. A second right hand system of axis x y z is fixed to the ship. The origin is in the centre of gravity. In the mean position of the ship the body axis have the same directions as the fixed axis.

Consider first a ship performing a pure harmonic heaving motion of small amplitude in still water. The ship is piercing a thin sheet of water, normal to the forward speed of the ship, at a fixed distance x_o from the origin.

At the time t a strip of the ship at a distance x from the centre of gravity is situated in the sheet of water. From $x_o = Vt + x$ it follows that $\dot{x} = -V$, where: V is the speed of the ship.

The vertical velocity of the strip with regard to the water is $\dot{z}_o$, the heaving velocity. The oscillatory part of the hydromechanical force on the strip of unit length will be:

$$F'_H = -\frac{d}{dt}(m'\dot{z}_o) - N'\dot{z}_o - 2\rho g y z_o ,$$

where: m' is the added mass and N' is the damping coefficient for a strip of unit length and y is the half width of the strip at the waterline. Because:

$$\frac{dm'}{dt} = \frac{dm'}{dx} \cdot \dot{x} ,$$

it follows that:

$$F'_H = -m'\ddot{z}_o - (N' - V\frac{dm'}{dx})\dot{z}_o - 2\rho g y z_o \qquad (5)$$

For the whole ship we find, because: $\int_L \frac{dm'}{dx} dx = 0$:

$$F_H = -(\int_L m' dx)\ddot{z}_o - (\int_L N' dx)\dot{z}_o - \rho g A_w z_o \qquad (6)$$

where A_w is the waterplane area.

The moment produced by the force on the strip is given by:

$$M'_H = -x F'_H = (x m')\ddot{z}_o + (N' x - V x \frac{dm'}{dx})\dot{z}_o + 2\rho g x y z_o \qquad (7)$$

Because $\int_L x \frac{dm'}{dx} dx = -m$, we find for the whole ship:

$$M_H = (\int_L x m' dx)\ddot{z}_o + (\int_L N' x dx + V m)\dot{z}_o + \rho g S_w z_o \qquad (8)$$

where S_w is the statical moment of the waterplane area.

For a pitching ship the vertical speed of the strip at x with regard to the water will be: $-x\dot{\theta} + V\theta$, and the acceleration is: $-x\ddot{\theta} + 2V\dot{\theta}$. The vertical force on the strip will be:

$$F'_p = -\frac{d}{dt} m'(-x\dot{\theta} + V\theta) - N'(-x\dot{\theta} + V\theta) + 2\rho g y x \theta,$$

or:

$$F'_p = m' x\ddot{\theta} + (N' x - 2 V m' - x V \frac{dm'}{dx})\dot{\theta} + (2\rho g y x + V^2 \frac{dm'}{dx} - N'V)\theta \qquad (9)$$

The total hydromechanical force on the pitching ship will be:

$$F_p = (\int_L m' x dx)\ddot{\theta} + (\int_L N' x dx - V m)\dot{\theta} + (\rho g S_w - V \int_L N' dx)\theta \qquad (10)$$

The moment produced by the force on the strip is given by:

$$M'_p = -x F'_p = -m' x^2 \ddot{\theta} - (N' x^2 - 2 V m' x - x^2 V \frac{dm'}{dx})\dot{\theta} -$$

$$- (2\rho g y x^2 + V^2 x \frac{dm'}{dx} - N' V x)\theta. \qquad (11)$$

The total moment on the pitching ship will be:

$$M_p = -(\int_L m' x^2 dx)\ddot{\theta} - (\int_L N' x^2 dx)\dot{\theta} - (\rho g I_w - V^2 m - V \int_L N' x dx)\theta, \qquad (12)$$

because:

$$\int_L x^2 V \frac{dm'}{dx} dx = -2 V \int_L m' x dx.$$

A summary of the expressions for the various coefficients for the whole ship according to the notation in equations (1) and (2) is given in Table 6.

Table 6.

Coefficients for the whole ship according to the strip theory.

$$a = \int_L m' \, dx \qquad d = \int_L m' x \, dx + \frac{Vb}{\omega^2}$$

$$b = \int_L N' \, dx \qquad e = \int_L N' x \, dx - V m$$

$$c = \rho g A_w \qquad g = \rho g S_w \qquad (13)$$

$$A = \int_L m' x^2 \, dx + \frac{VE}{\omega^2} \qquad D = \int_L m' x \, dx$$

$$B = \int_L N' x^2 \, dx \qquad E = \int_L N' x \, dx + V m$$

$$C = \rho g I_w \qquad G = \rho g S_w$$

For the cross-sectional values of the coefficients similar expressions can be derived from the equations (5) to (12). For the comparison with the experimental results two of these expressions are given here, namely:

$$b' = N' - V \frac{dm'}{dx}$$

$$e' = N' x - 2 V m' - x V \frac{dm'}{dx} \qquad (14)$$

Also it follows that:

$$A = \int d' x \, dx$$

and: (15)

$$B = \int e' x \, dx$$

4.2. Comparison of theory and experiment.

For a number of cases the experimental results are compared with theory. First of all the damping cross-coupling coefficients are considered. From equations (13) it follows that:

$$E = \int_L N' \, x \, dx + V m$$
$$e = \int_L N' \, x \, dx - V m \qquad (16)$$

The first term in both expressions is the cross-coupling coefficient for zero forward speed. For a for and aft symmetrical ship this term is equal to zero. For such a ship the resulting expressions are equal in magnitude but have opposite sign, which is in agreement with the result found by Timman and Newman [7]. The experiments confirm this fact as shown in Figure 13 where e and E are plotted on a base of forward speed as a function of the frequency of oscillation. The magnitude of the speed dependent parts of the coefficients is equal within very close limits. Extrapolation to zero speeds shows that the e and E lines intersect in one point which should represent the zero speed cross-coupling coefficient.

Using Grim's two-dimensional solution for damping and added mass at zero speed [9] the coefficients e and E were also calculated according to the equations (16). The distribution of added mass and damping coefficient for zero speed is given in Figure 12 and the calculated damping cross-coupling coefficients are shown in Figure 13.

The calculated values are in line with the experimental results. The natural frequencies for pitch and heave are respectively ω = 7.0/6.9 rad/sec and in this important region the calculation of the damping cross-coupling coefficients is quite satisfactory. The zero speed case will be studied in the near future by oscillating experiments in a wide basin to avoid wall influence.

Another comparison of theory and experiment concerns the distribution along the length of the shipmodel of the damping coefficient and of the damping cross-coupling coefficient e.

From equation (14):

$$b' = N' - V \frac{dm'}{dx}$$

$$e' = N' x - 2 V m' - x V \frac{dm'}{dx}$$

Again using Grim's two-dimensional values for N' and m', these distributions could be calculated. An example is given in Figure 14. Also in this case the agreement between the calculation and the experiment is good. For high speeds negative values of the cross-sectional damping in the afterbody can be explained on the basis of the expression for b', because in that region $\frac{dm'}{dx}$ is a positive quantity.

Finally the values for the coefficients A, B, a and b for the whole model, as given by the equations (13) were calculated and compared with the experimental results. Figure 15 shows that the damping in pitch is over estimated for low frequencies. The other coefficients agree quite well with the experimental results.

TABLE 2.

Added mass for the sections and the whole model.

kg sec^2/m.

Fn = .15.

ω rad/sec	a*							a	
	1.	2	3	4	5	6	7	sum of sections	whole model
4	-1,21	0,59	-	0,54	0,87	0,41	-0,17	-	1,84
6	0,31	0,66	1,08	1,38	1,26	0,65	0,02	5,36	5,37
8	0,24	0,60	1,09	1,37	1,28	0,76	0,10	5,44	5,26
10	0,20	0,69	1,29	1,48	1,34	0,85	0,14	5,99	5,91
12	0,18	0,78	1,40	1,60	1,45	0,90	0,17	6,48	6,39

Fn = .20.

4	0,59	0,83	1,29	1,59	1,15	0,22	-0,27	5,40	5,63
6	0,32	0,65	1,00	1,40	1,23	0,64	0	5,24	5,19
8	0,21	0,55	1,08	1,38	1,21	0,75	0,12	5,30	5,18
10	0,19	0,65	1,23	1,49	1,33	0,83	0,14	5,86	5,78
12	0,20	0,77	1,37	1,60	1,45	0,88	0,17	6,44	6,32

Fn = .25.

4	0,86	1,09	1,26	1,66	1,20	0,16	-0,32	5,91	4,99
6	0,33	0,65	1,01	1,38	1,19	0,55	-0,02	5,09	4,89
8	0,20	0,54	1,03	1,39	1,26	0,68	0,08	5,18	5,13
10	0,18	0,62	1,19	1,48	1,34	0,77	0,12	5,70	5,65
12	0,20	0,76	1,37	1,60	1,45	0,83	0,16	6,37	6,21

Fn = .30.

4	0,70	0,91	1,49	1,58	1,07	-0,10	-0,22	5,43	5,59
6	0,25	0,44	1,15	1,39	1,07	0,45	0,07	4,82	4,51
8	0,16	0,42	1,14	1,45	1,08	0,58	0,13	4,96	4,93
10	0,15	0,55	1,26	1,47	1,22	0,68	0,17	5,50	5,48
12	0,17	0,69	1,41	1,57	1,35	0,81	0,19	6,19	6,18

TABLE 3.

Damping coefficients for the sections and the whole model.

kg sec/m.

Fn = .15.

ω rad/sec	b^*							b	
	1	2	3	4	5	6	7	sum of sections	whole model
4	2,03	9,78	-	5,78	3,80	4,80	2,00	-	35,63
6	1,82	4,42	4,55	4,58	4,52	4,78	1,67	26,34	26,53
8	1,61	2,31	2,26	2,75	3,35	3,94	1,53	17,75	17,49
10	1,36	1,08	0,76	1,39	2,36	3,43	1,49	11,87	11,63
12	0,95	0,47	0,44	0,87	1,89	3,09	1,50	9,21	8,54

Fn = .20.

4	1,53	4,53	5,08	5,05	5,73	6,63	2,50	31,05	31,33
6	1,95	3,95	4,32	4,45	4,52	5,07	2,07	26,33	26,15
8	1,50	1,91	2,25	2,81	3,49	4,38	1,94	18,28	17,78
10	1,10	0,37	0,62	1,54	2,70	4,01	1,90	12,24	12,14
12	0,74	-0,15	0,21	1,01	2,18	3,84	1,93	9,76	9,03

Fn = .25.

4	2,13	4,80	5,38	5,20	5,98	7,63	2,85	33,97	35,88
6	1,97	3,43	4,17	4,23	4,62	5,68	2,35	26,45	27,63
8	1,48	1,58	2,28	2,83	3,68	5,21	2,19	19,25	18,75
10	0,95	-0,06	0,60	1,68	3,00	4,96	2,20	13,33	12,69
12	0,52	-0,56	-0,03	1,03	2,63	4,74	2,29	10,62	9,78

Fn = . 30.

4	1,78	4,40	4,40	5,15	6,78	7,60	2,98	33,09	38,10
6	1,75	2,77	3,50	4,10	5,18	6,32	2,55	26,17	28,45
8	1,21	0,99	1,70	2,81	4,50	5,73	2,51	19,45	20,40
10	0,64	-0,87	0,17	1,88	4,07	5,42	2,59	13,90	13,95
12	0,42	-0,56	-0,63	1,37	3,72	5,28	2,66	11,26	10,4[illegible]

TABLE 4.

Added mass cross-coupling coefficients for the sections and the whole model.

kg sec^2.

ω rad/sec	d*							d	
	1	2	3	4	5	6	7	sum of sections	whole model
Fn = .15.									
4	-	-	-	-	+0,59	+0,28	-	-	-
6	-0,42	-0,47	-0,33	+0,02	+0,46	+0,57	+0,13	-0,04	+0,09
8	-0,27	-0,44	-0,40	-0,01	+0,38	+0,50	+0,13	-0,11	-0,16
10	-0,19	-0,43	-0,40	-0,01	+0,37	+0,49	+0,15	-0,02	-0,10
12	-0,19	-0,45	-0,40	-0,01	+0,40	+0,51	+0,15	+0,01	-0,04
Fn = .20.									
4	-0,57	-0,67	-	-	-	+0,78	+0,32	-	-
6	-0,39	-0,52	-0,34	+0,01	+0,46	+0,59	+0,13	-0,06	-0,06
8	-0,24	-0,45	-0,40	-0,01	+0,39	+0,51	+0,11	-0,09	-0,14
10	-0,20	-0,45	-0,40	-0,01	+0,38	+0,51	+0,13	-0,04	-0,08
12	-0,20	-0,47	-0,41	-0,01	+0,40	+0,53	+0,14	-0,02	-0,03
Fn = .25.									
4	-0,62	-0,59	-0,01	+0,12	+0,72	+0,86	+0,21	+0,69	+0,15
6	-0,39	-0,50	-0,32	+0,02	+0,46	+0,59	+0,13	-0,01	0,00
8	-0,23	-0,48	-0,40	-0,01	+0,39	+0,52	+0,14	-0,07	-0,13
10	-0,18	-0,46	-0,42	-0,01	+0,38	+0,51	+0,13	-0,05	-0,08
12	-0,20	-0,46	-0,42	-0,01	+0,40	+0,51	+0,15	-0,03	-0,05
Fn = .30.									
4	-0,62	-0,61	+0,13	+0,08	+0,64	+0,93	+0,20	+0,75	+1,09
6	-0,29	-0,47	-0,36	+0,01	+0,43	+0,59	+0,21	+0,12	+0,01
8	-0,21	-0,47	-0,44	-0,01	+0,38	+0,53	+0,16	-0,06	-0,11
10	-0,19	-0,46	-0,44	-0,02	+0,38	+0,51	+0,15	-0,07	-0,10
12	-0,20	-0,46	-0,44	-0,02	+0,39	+0,52	+0,16	-0,05	-0,06

TABLE 5.

Damping cross-coupling coefficients for the sections and the whole model.

kg sec.

Fn = .15.

ω rad/sec	e*							e	
	1	2	3	4	5	6	7	sum of sections	whole model
4	-	-	-	-	+1,63	+1,34	-	-	- 2,43
6	-1,65	-2,58	-2,12	-1,19	-0,09	+1,70	+1,21	- 4,72	- 5,32
8	-1,71	-2,49	-2,45	-1,81	-0,68	+1,20	+1,09	- 6,84	- 6,75
10	-1,40	-2,01	-2,43	-2,10	-1,21	+0,88	+1,05	- 7,22	- 7,04
12	-1,07	-1,55	-2,28	-2,39	-1,52	+0,63	+1,05	- 7,13	- 6,88

Fn = .20.

4	-1,22	-3,07	-	-	-	+2,39	+1,77	-	- 6,63
6	-1,68	-2,43	-2,40	-2,06	-0,68	+1,52	+1,42	- 6,31	- 6,65
8	-1,59	-2,36	-2,83	-2,50	-1,25	+1,11	+1,32	- 8,10	- 8,23
10	-1,29	-2,04	-3,02	-2,87	-1,75	+0,82	+1,29	- 8,86	- 8,86
12	-0,98	-1,65	-2,99	-2,97	-2,06	+0,61	+1,30	- 8,74	- 8,75

Fn = .25.

4	-1,52	-3,04	-3,47	-3,03	-0,96	+2,16	+1,91	- 7,95	- 6,70
6	-1,50	-2,21	-2,85	-2,66	-1,36	+1,47	+1,61	- 7,50	- 7,38
8	-1,50	-2,26	-3,21	-2,97	-1,79	+1,11	+1,51	- 9,11	- 9,30
10	-1,22	-2,14	-3,56	-3,39	-2,27	+0,86	+1,49	-10,23	-10,18
12	-0,85	-1,81	-3,66	-3,58	-2,53	+0,66	+1,47	-10,30	-10,31

Fn = .30.

4	-1,37	-2,82	-3,61	-3,06	-1,22	+2,19	+1,98	- 7,91	- 7,55
6	-1,23	-1,93	-3,16	-3,06	-1,84	+1,43	+1,72	- 8,07	- 7,95
8	-1,30	-1,96	-3,55	-3,42	-2,32	+1,03	+1,67	- 9,85	- 9,81
10	-1,19	-2,06	-3,94	-3,90	-2,70	+0,76	+1,67	-11,36	-11,25
12	-0,91	-1,97	-4,08	-4,19	-2,97	+0,56	+1,69	-11,87	-11,84

List of symbols.

$a \ldots g$ / $A \ldots G$	Coefficient of the motion equations (hydromechanical part).
$a^* \ldots g^*$ / $A^* \ldots G^*$	The same for a section of the ship.
$a' \ldots g'$ / $A' \ldots G'$	The same for a cross-section of the ship.
C_B	Blockcoefficient.
Fn	Froude number.
F_z, F_Θ	Amplitude of vertical force on a heaving or pitching ship.
F_H, F_P	Oscillatory part of the hydromechanical force on a heaving or pitching ship.
g	Acceleration of gravity.
k_{yy}	Longitudinal radius of inertia of the ship.
L_{pp}	Length between perpendiculars.
M_z, M_Θ	Amplitude of moment on a heaving or pitching ship.
M_H, M_p	Oscillatory part of the hydromechanical moment on a heaving or pitching ship.
m'	Added mass of a cross-section (zero speed).
N'	Damping coefficient of a cross-section (zero speed).
t	Time..
V	Forward speed of ship.
$x\,y\,z$	Right hand coördinate system, fixed to the ship.
x_o, y_o, z_o	Right hand coordinate system, fixed in space.
z_o	Vertical displacement of ship.
x_i	Distance of centre of gravity of a section to the pitching axis.
$\alpha, \beta, \gamma, \delta$	Phase angles.
Θ	Pitch angle.

ρ Density of water.

ω Circular frequency.

∇ Volume of displacement of ship.

∇^* Volume of displacement of section.

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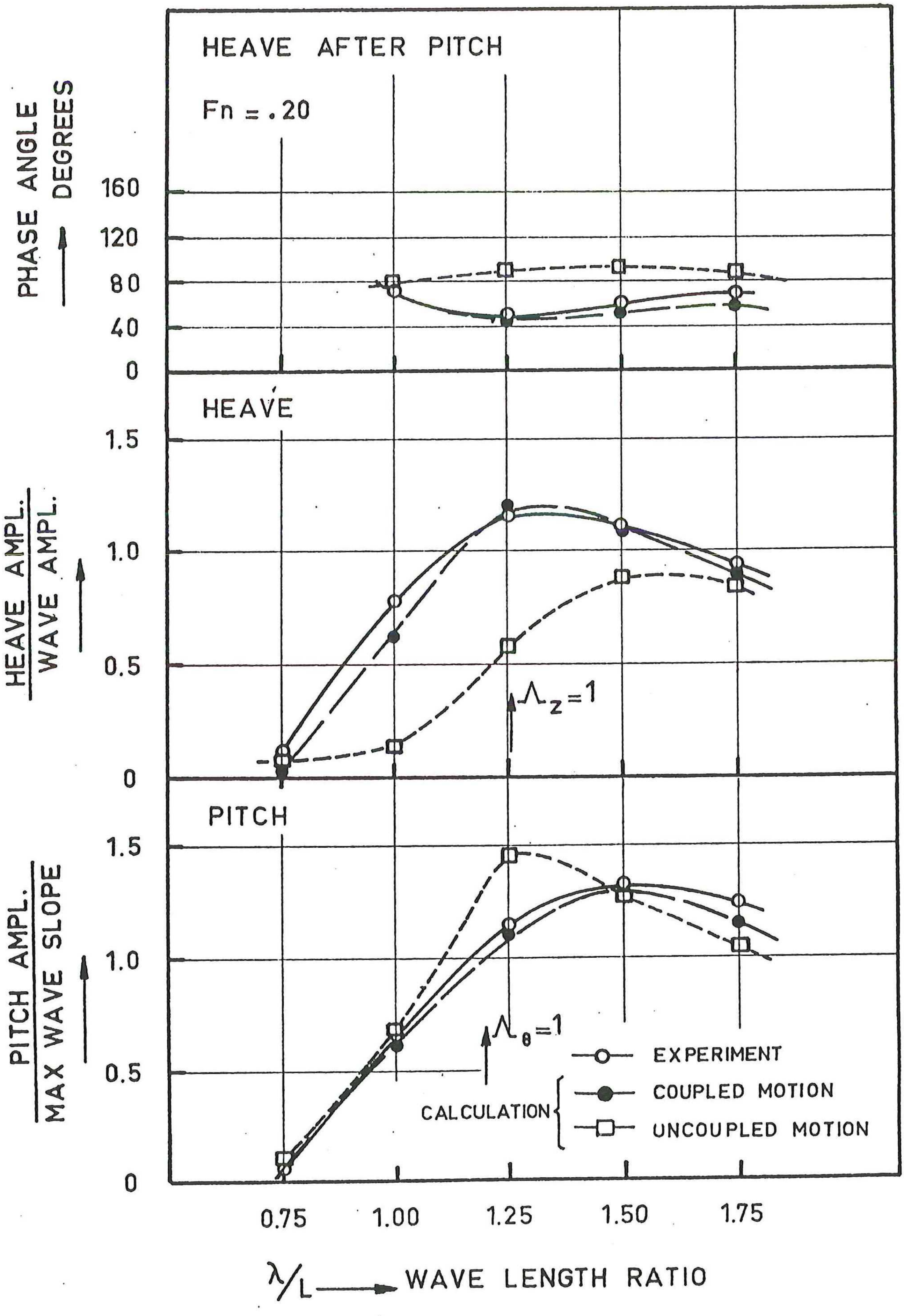
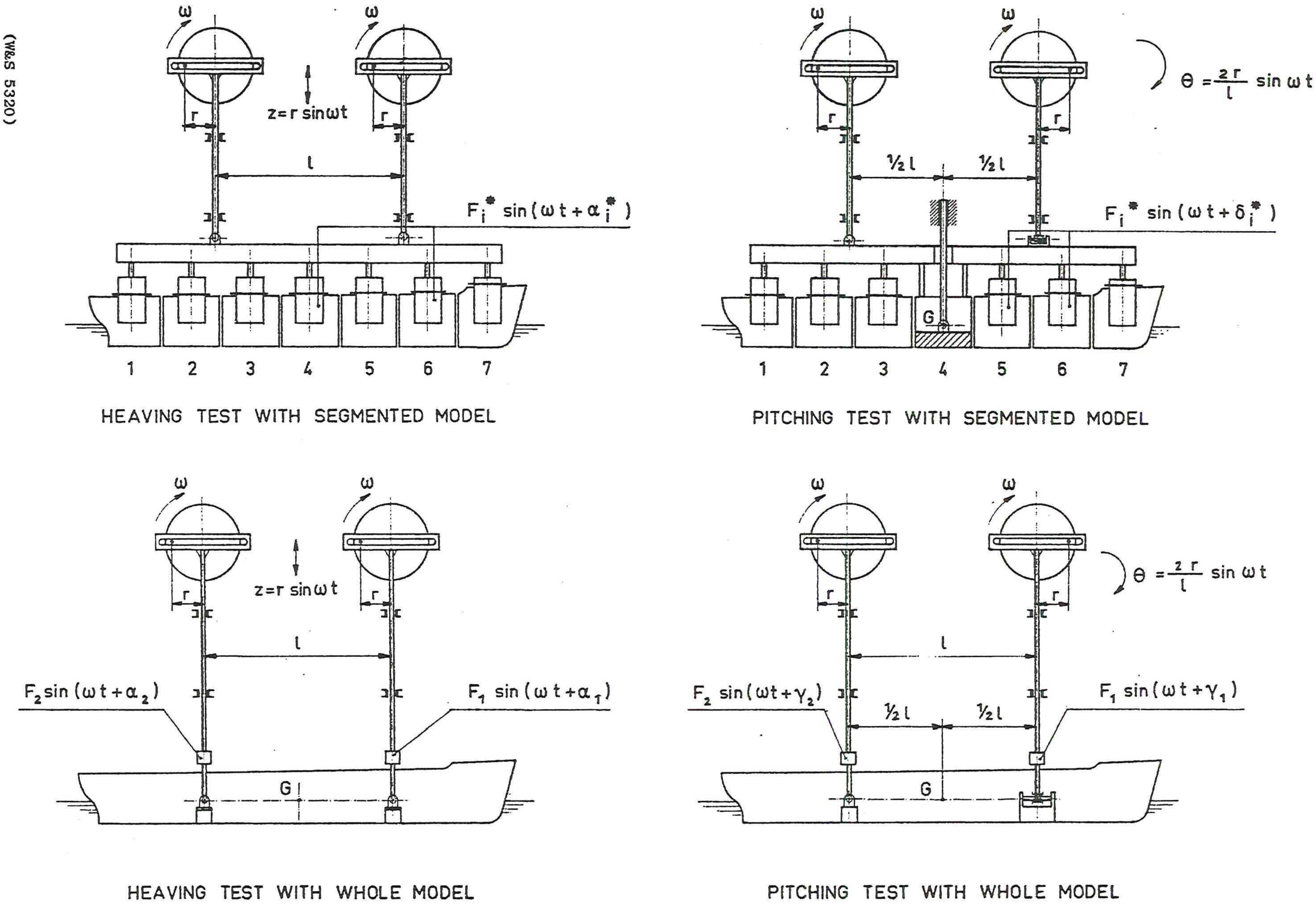


INFLUENCE OF CROSS_COUPLING

(W&S 5320)

FIGURE 1

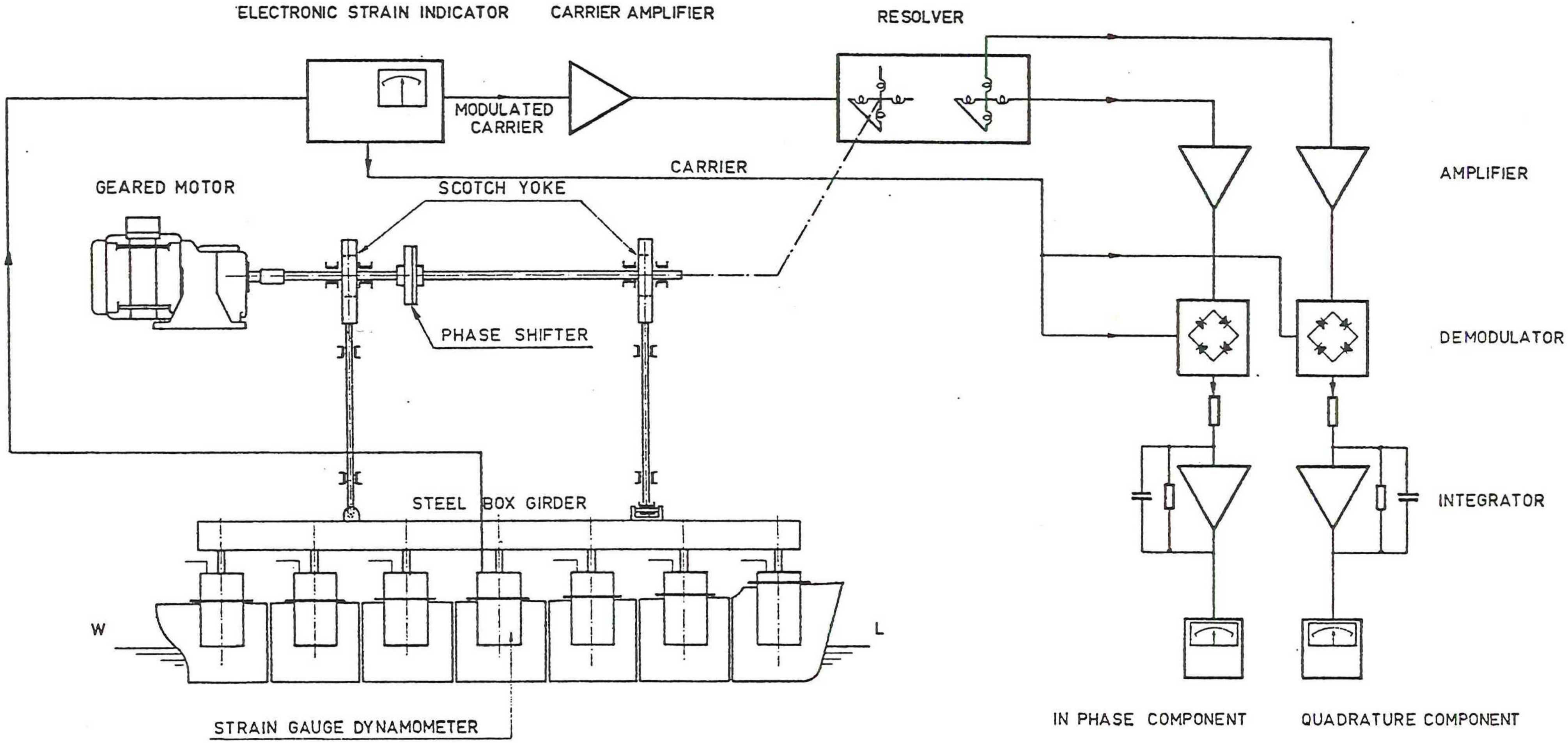
(W&S 5320)



ARRANGEMENT OF OSCILLATION TESTS

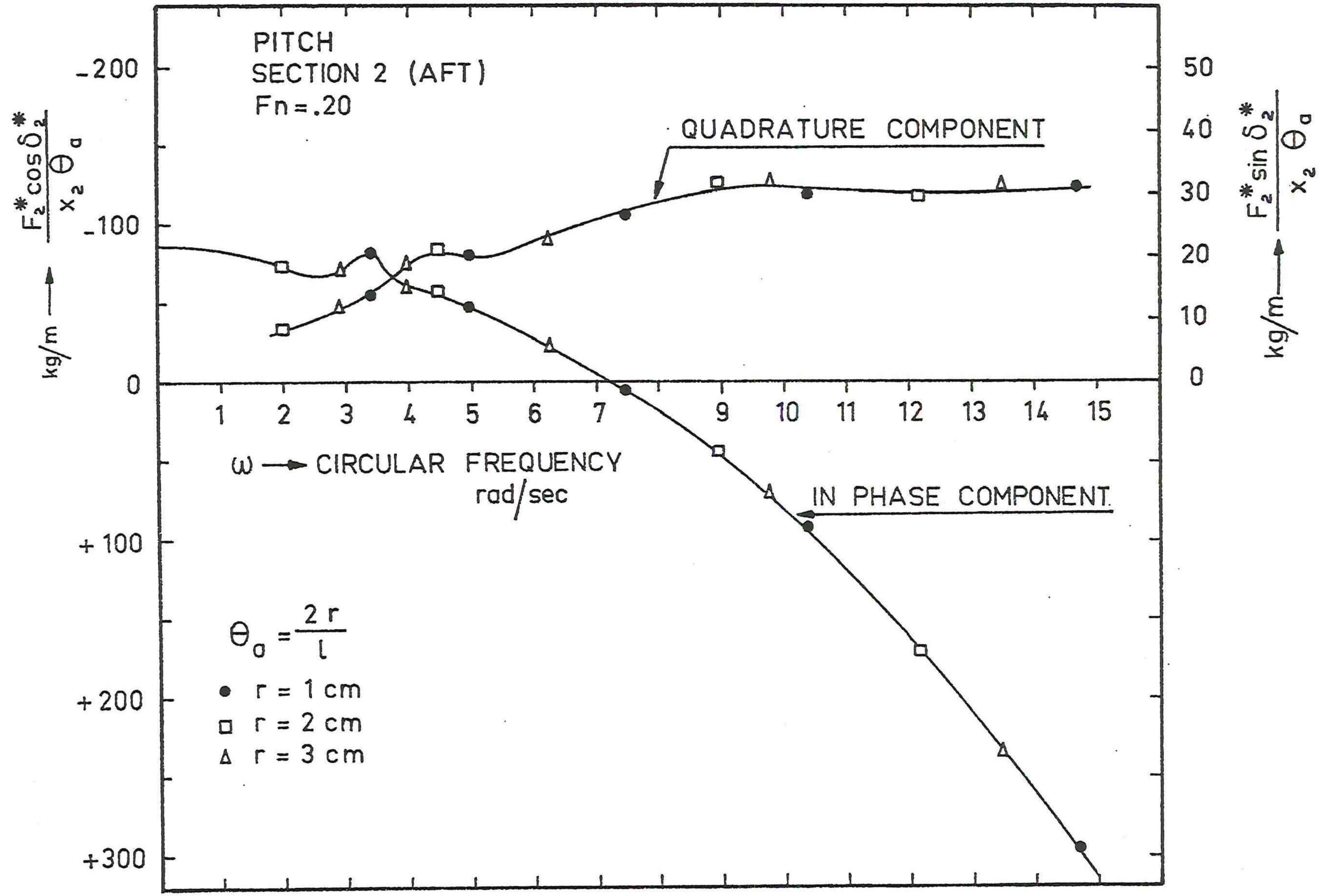
FIGURE 2

(W&S 5320)



PRINCIPLE OF MECHANICAL OSCILLATOR AND ELECTRONIC CIRCUIT

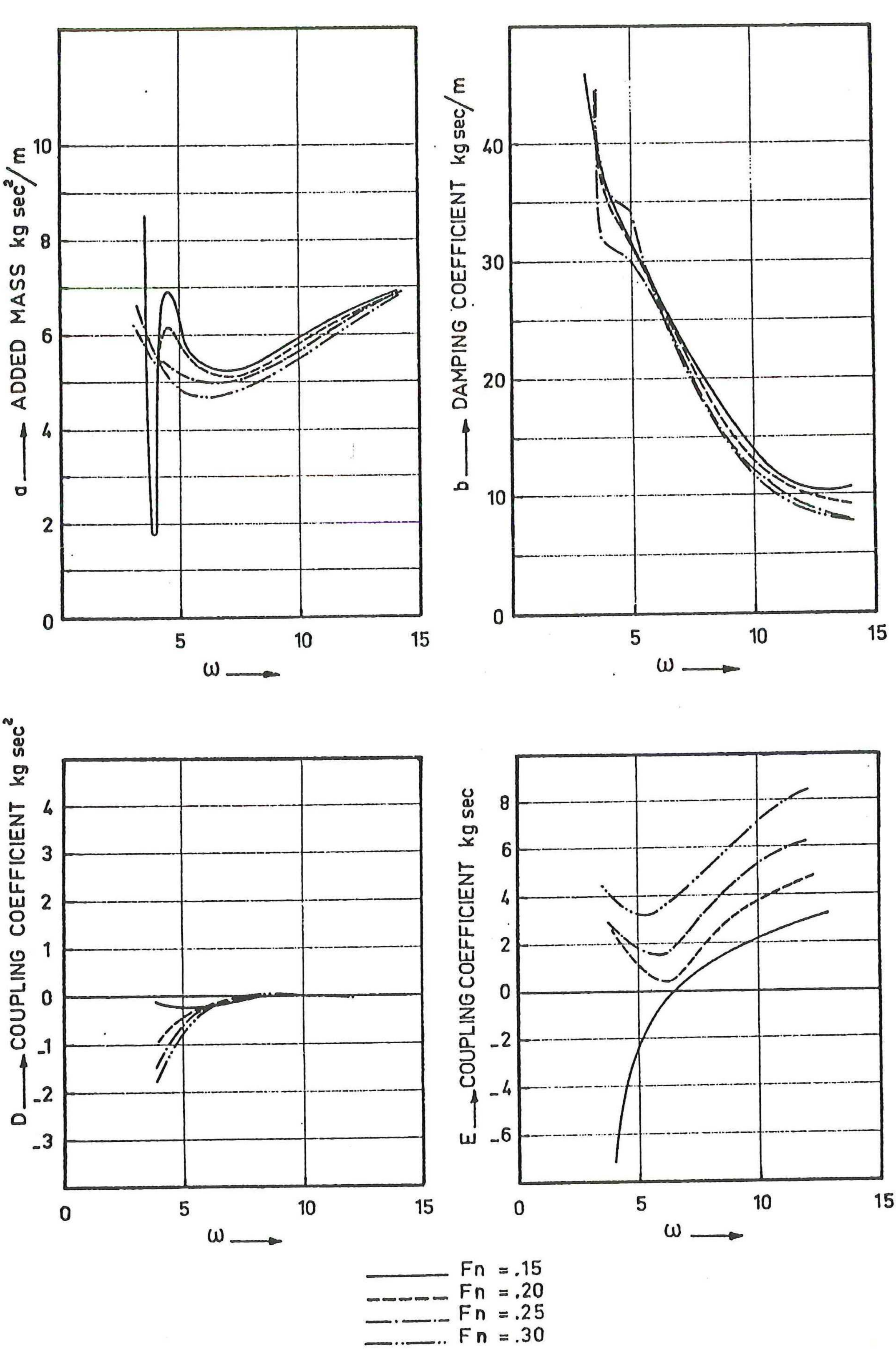
FIGURE 3



COMPONENTS OF FORCE ON SECTION 2. PITCHING MOTION

FIGURE 4

HEAVING MOTION

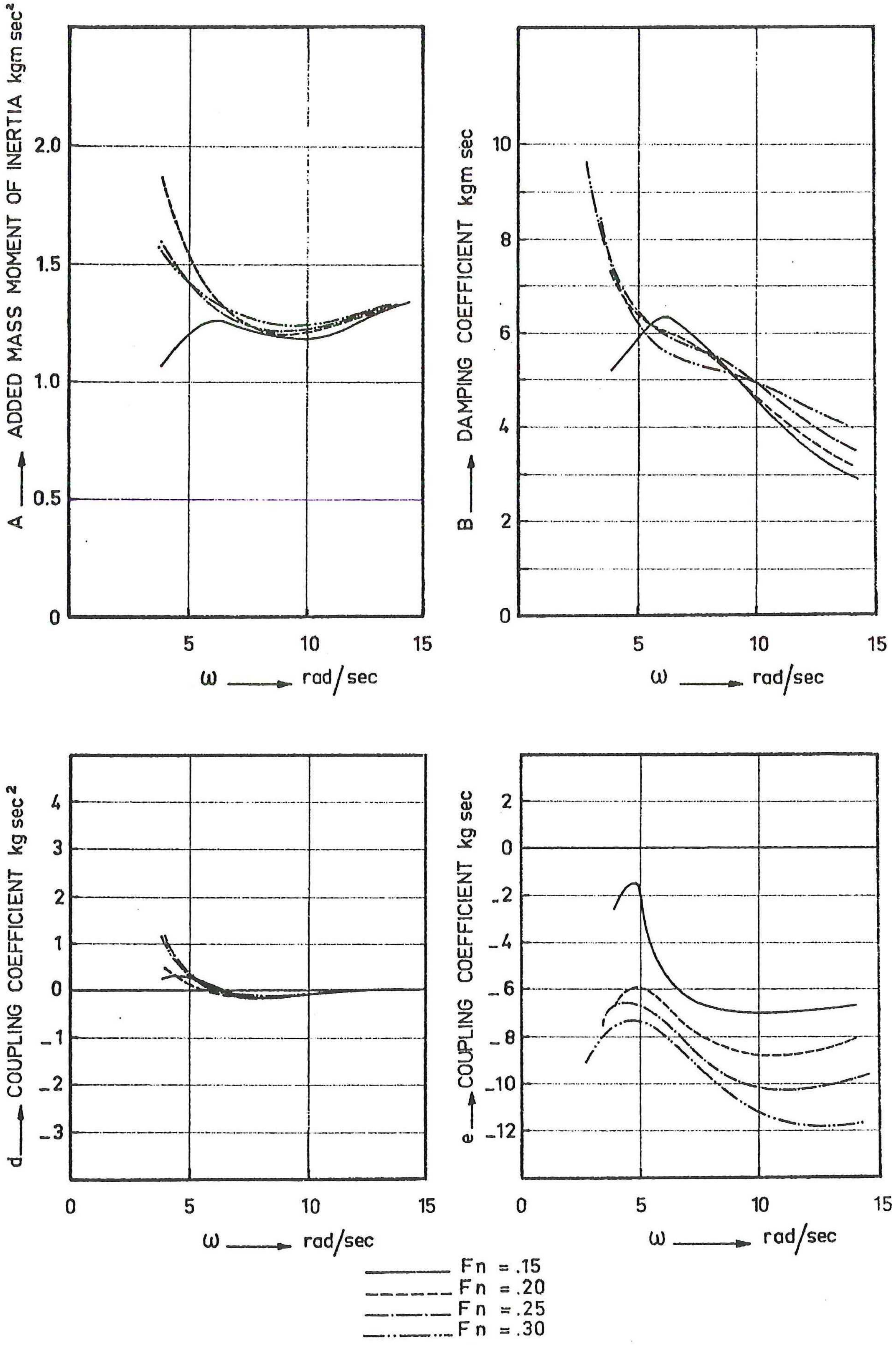


EXPERIMENTAL RESULTS FOR WHOLE MODEL

(W&S 5320)

FIGURE 5

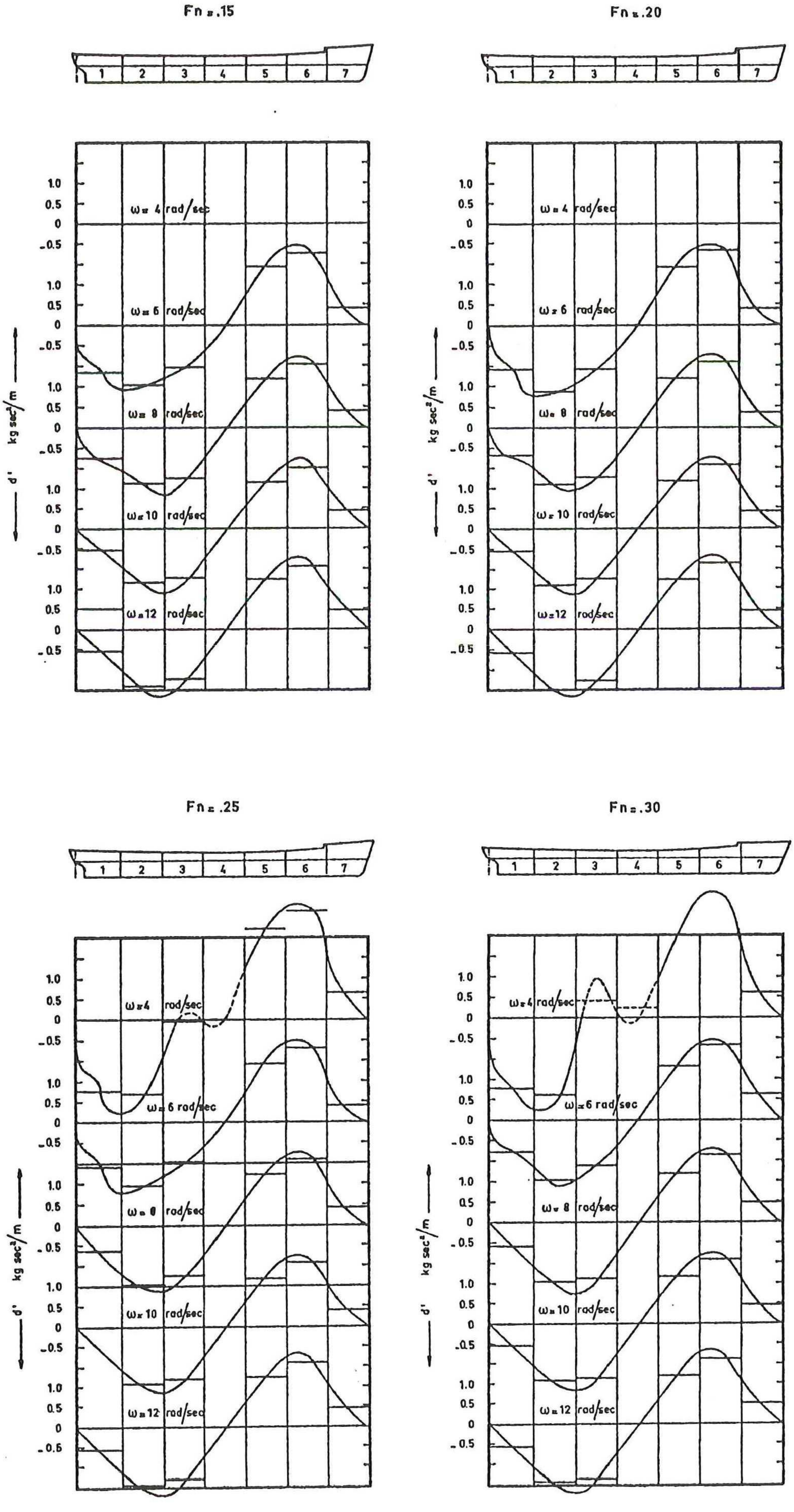
PITCHING MOTION



EXPERIMENTAL RESULTS FOR WHOLE MODEL

(W&S 5320)

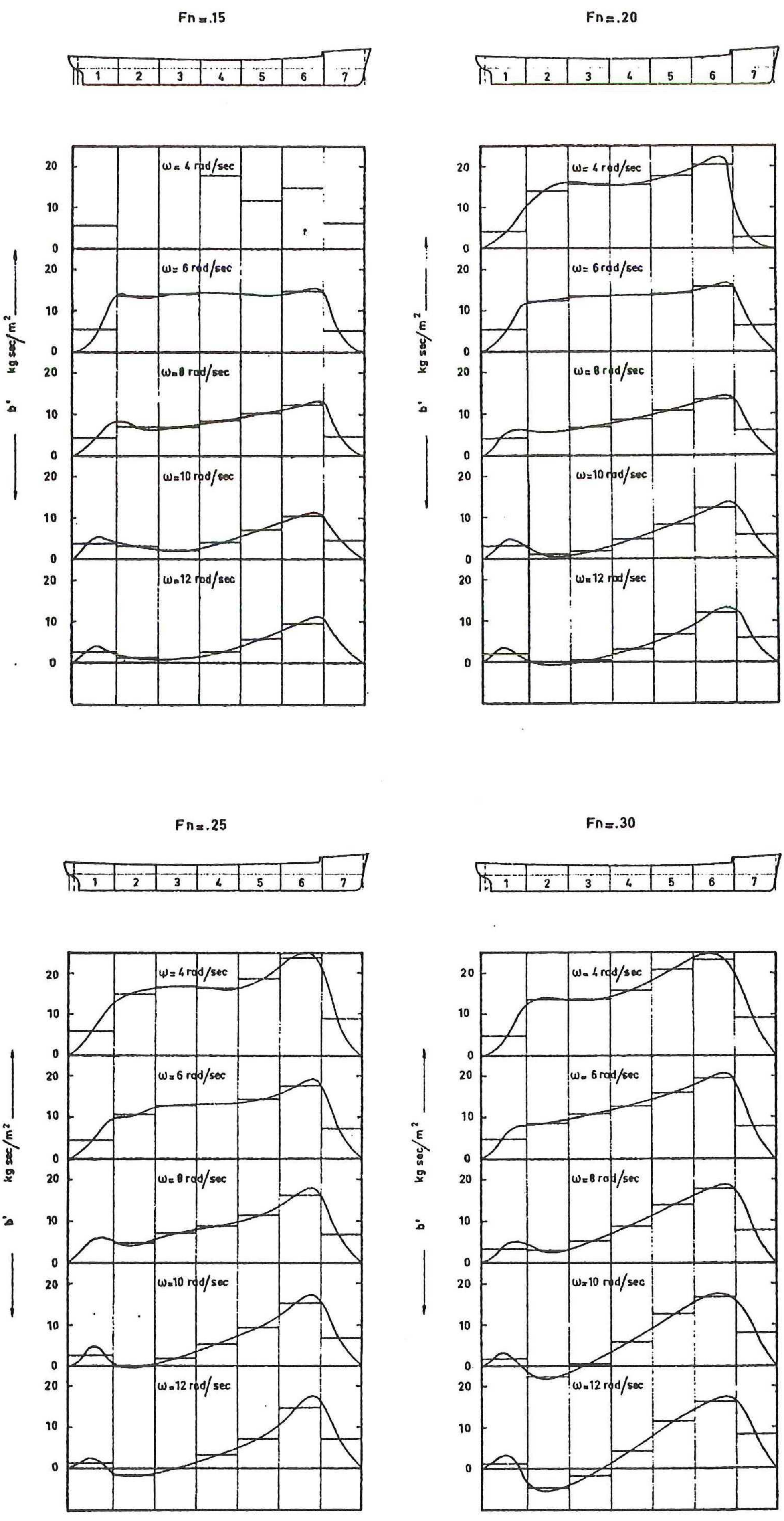
FIGURE 6



DISTRIBUTION OF d' OVER THE LENGTH OF THE SHIPMODEL

(W&S 5320)

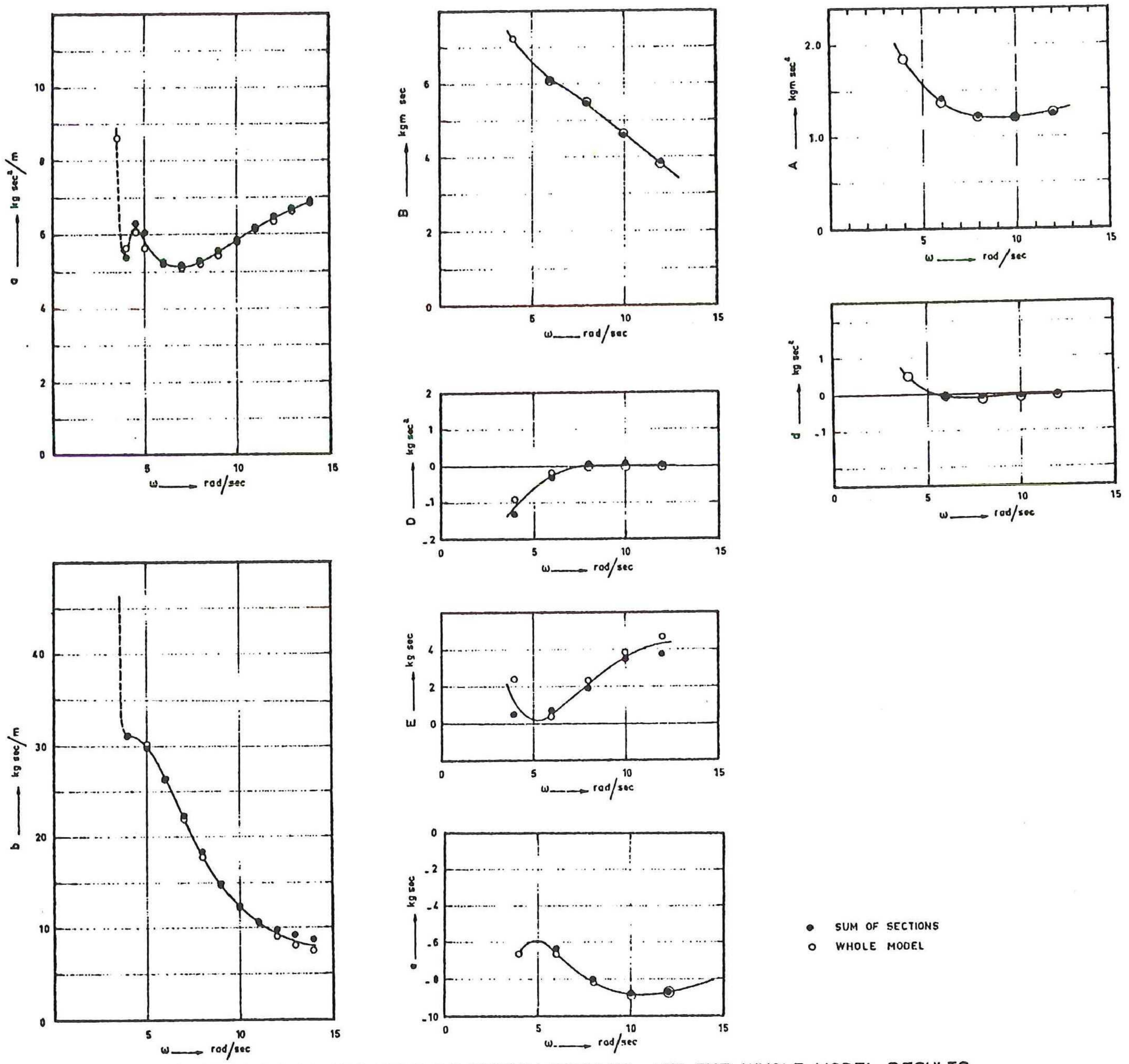
FIGURE 9



DISTRIBUTION OF b OVER THE LENGTH OF THE SHIPMODEL

(W&S 5320)

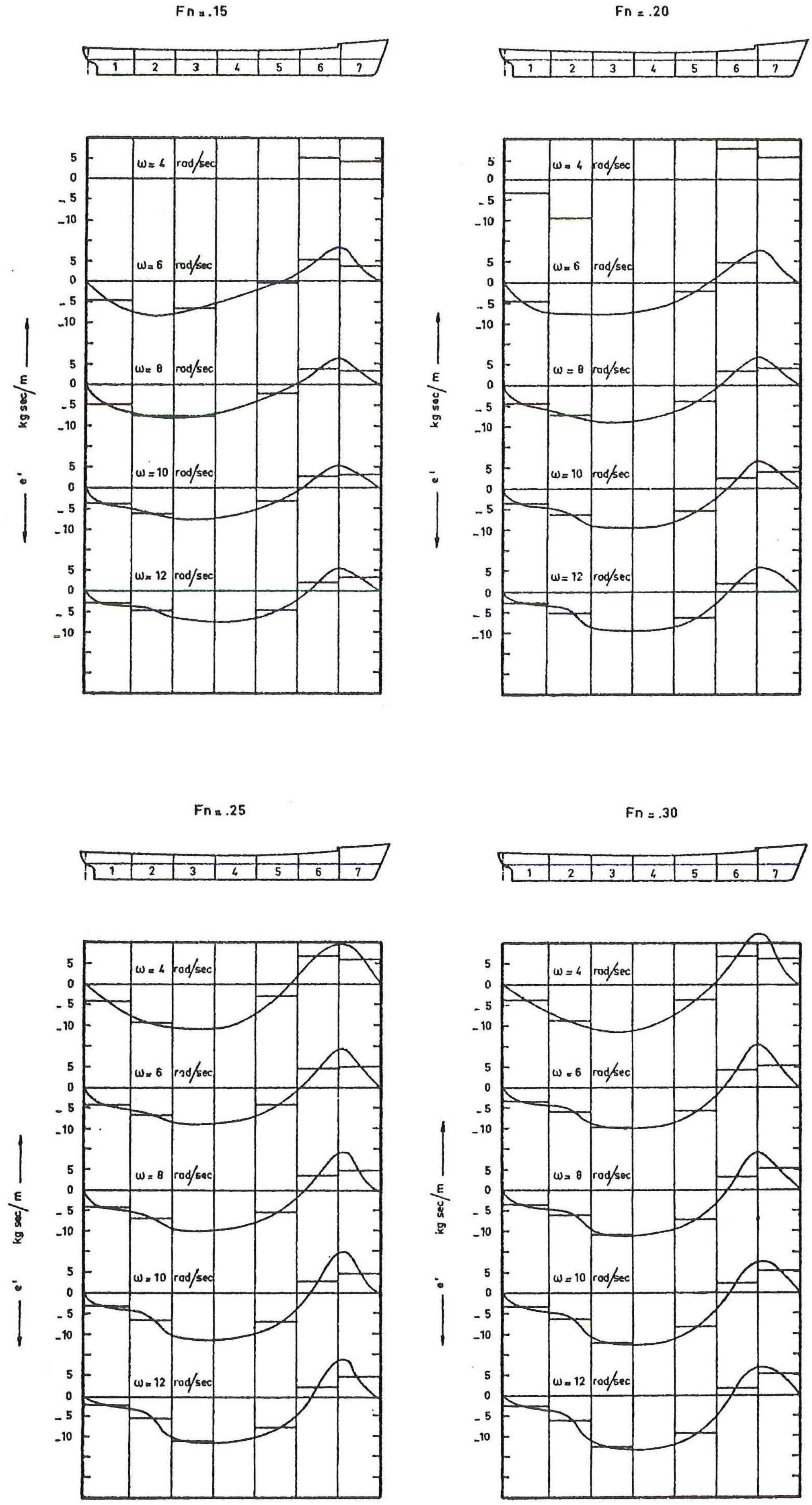
FIGURE 8



COMPARISON OF THE SUMS OF SECTION RESULTS AND THE WHOLE MODEL RESULTS
FOR FROUDE NUMBER Fn = .20

FIGURE 11

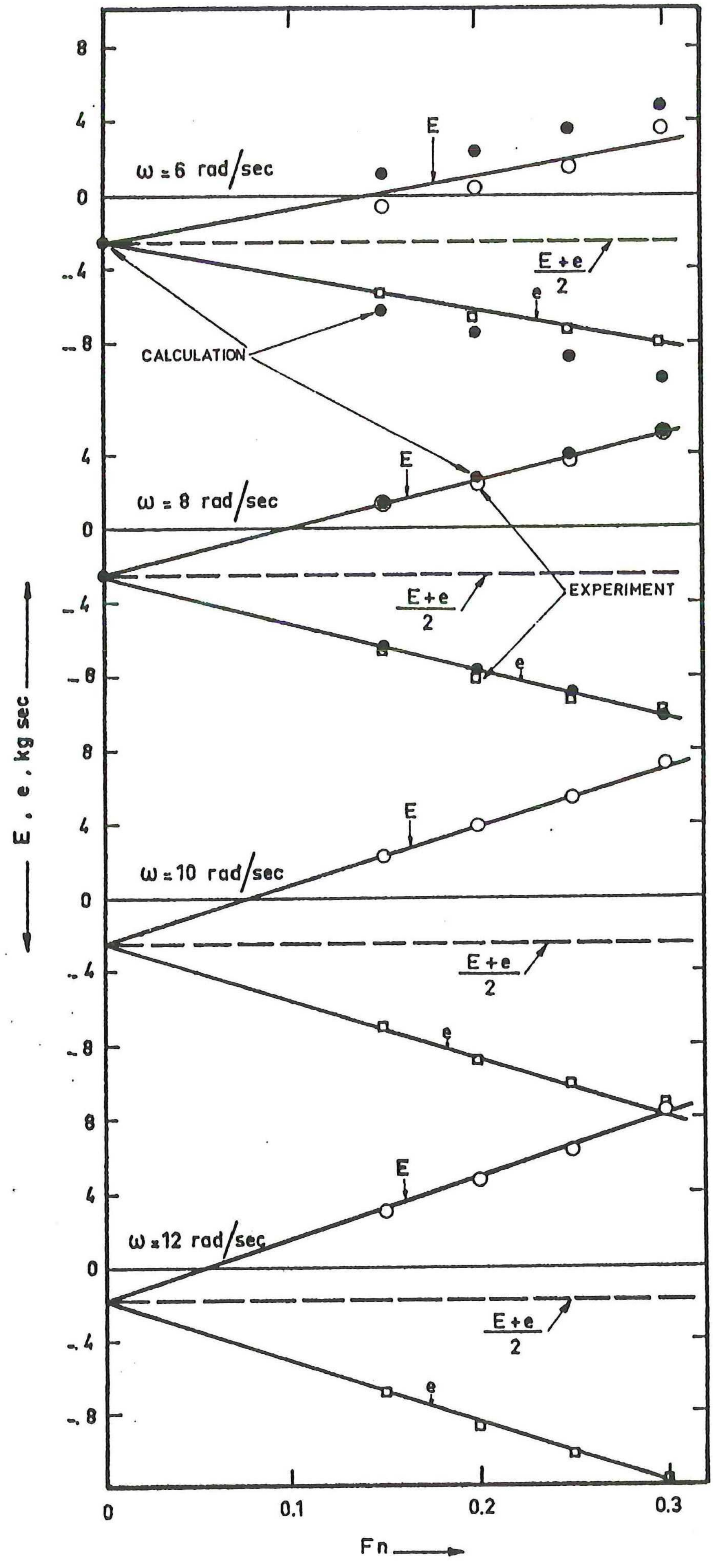
(W&S 5320)



DISTRIBUTION OF e OVER THE LENGTH OF THE SHIPMODEL

(W&S 5320)

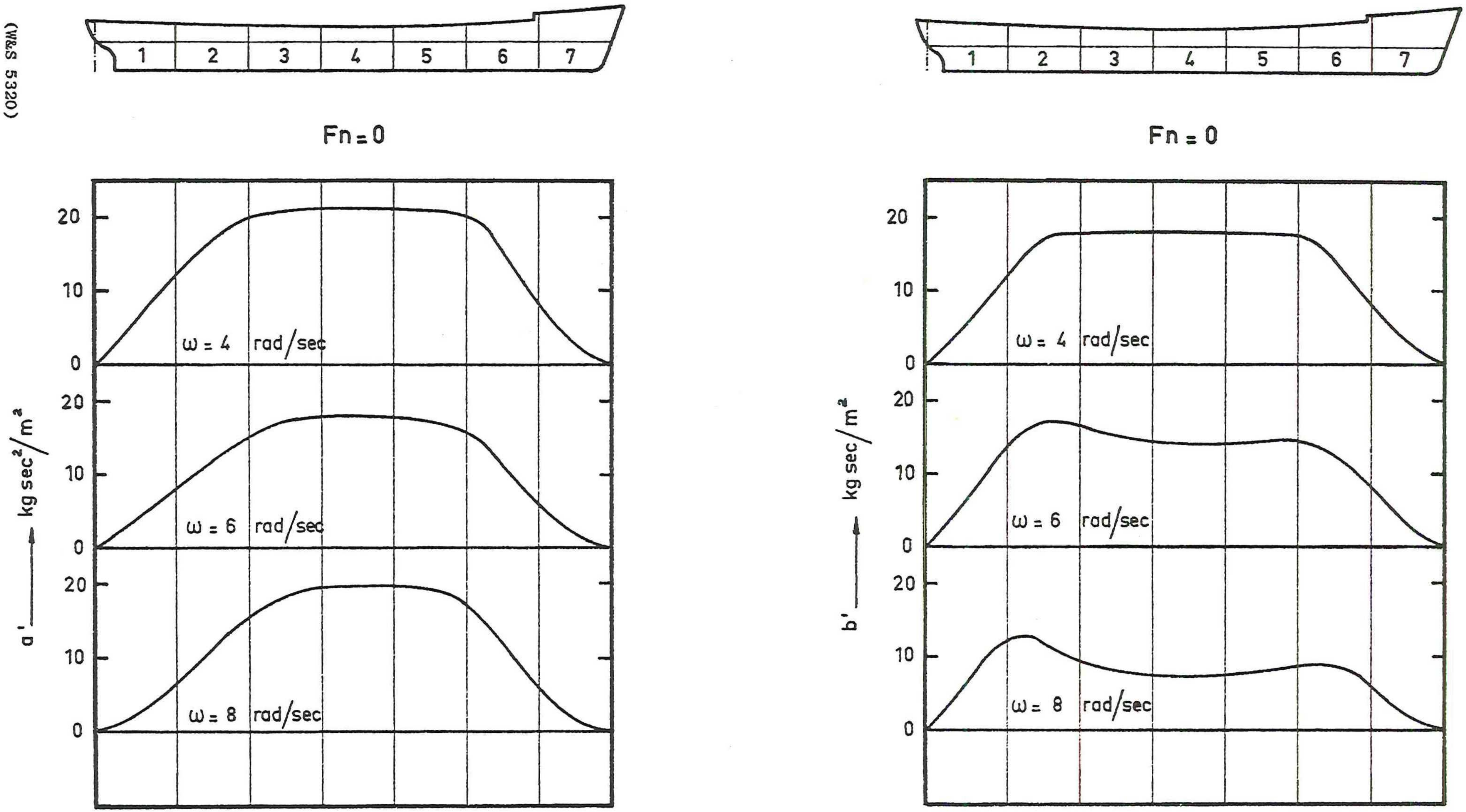
FIGURE 10



(W.S 5320)

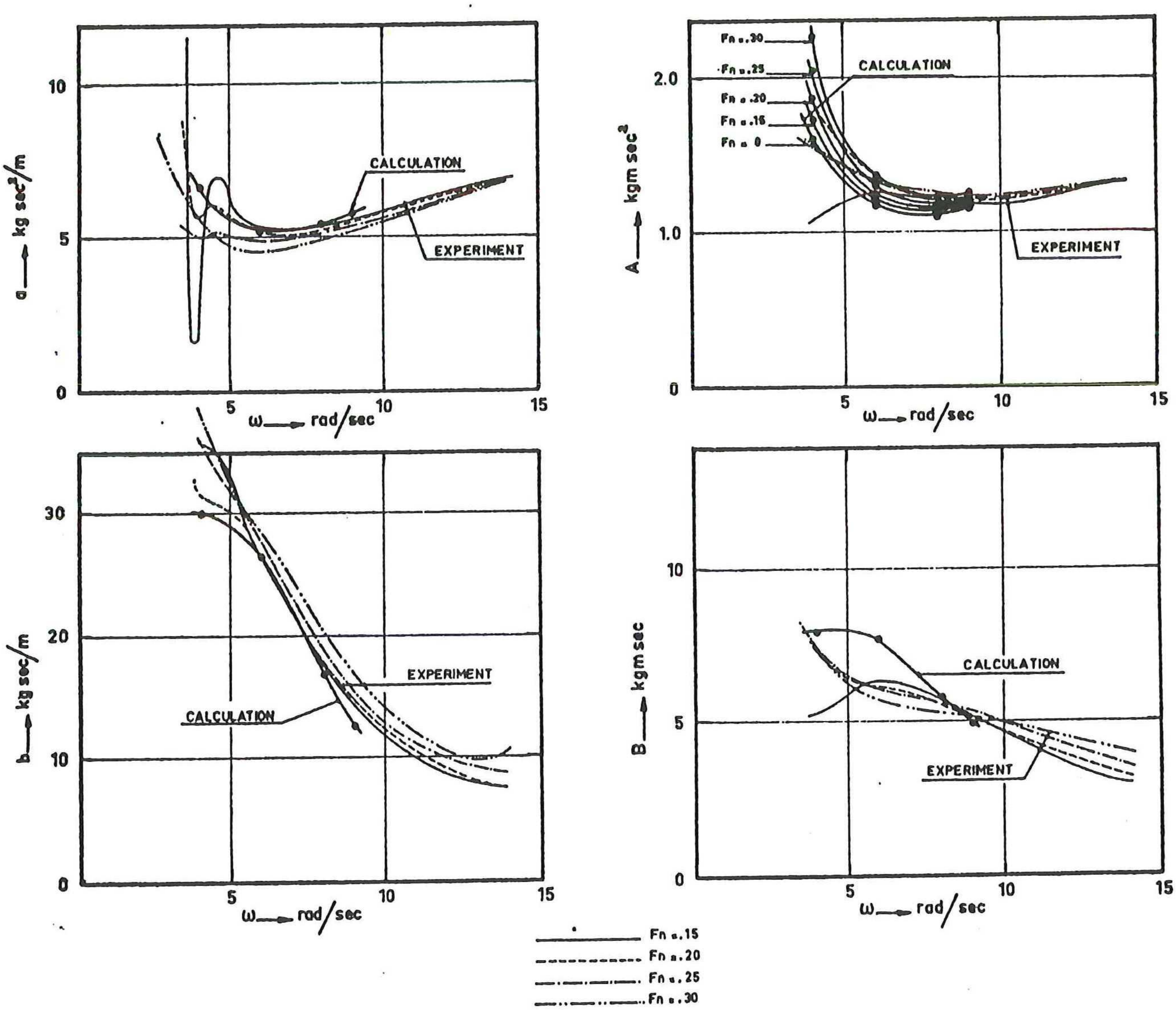
COMPARISON OF CALCULATED AND MEASURED VALUES FOR e AND E

FIGURE 13



CALCULATED DISTRIBUTION OF a AND b FOR ZERO SPEED

FIGURE 12

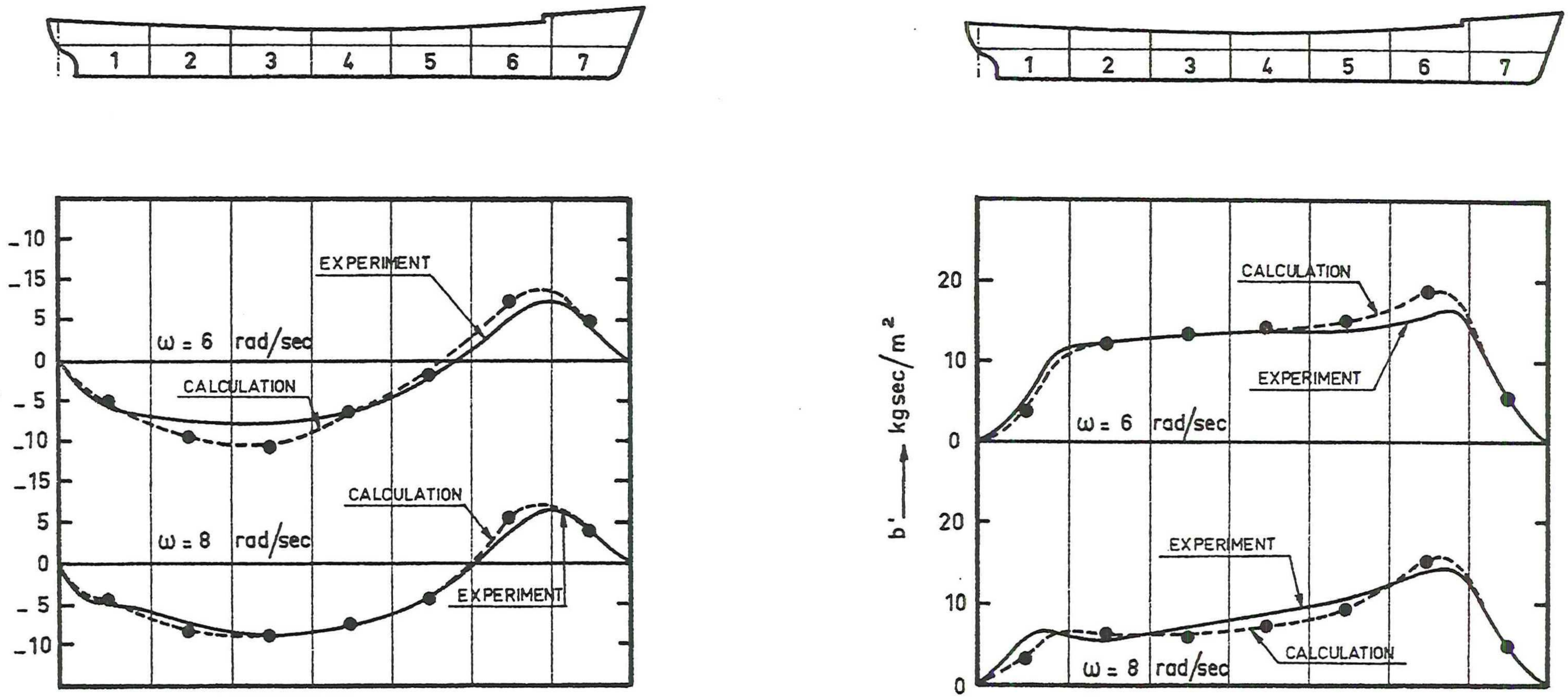


COMPARISON OF CALCULATED AND MEASURED VALUES FOR a, b, A AND B (WHOLE MODEL)

FIGURE 15

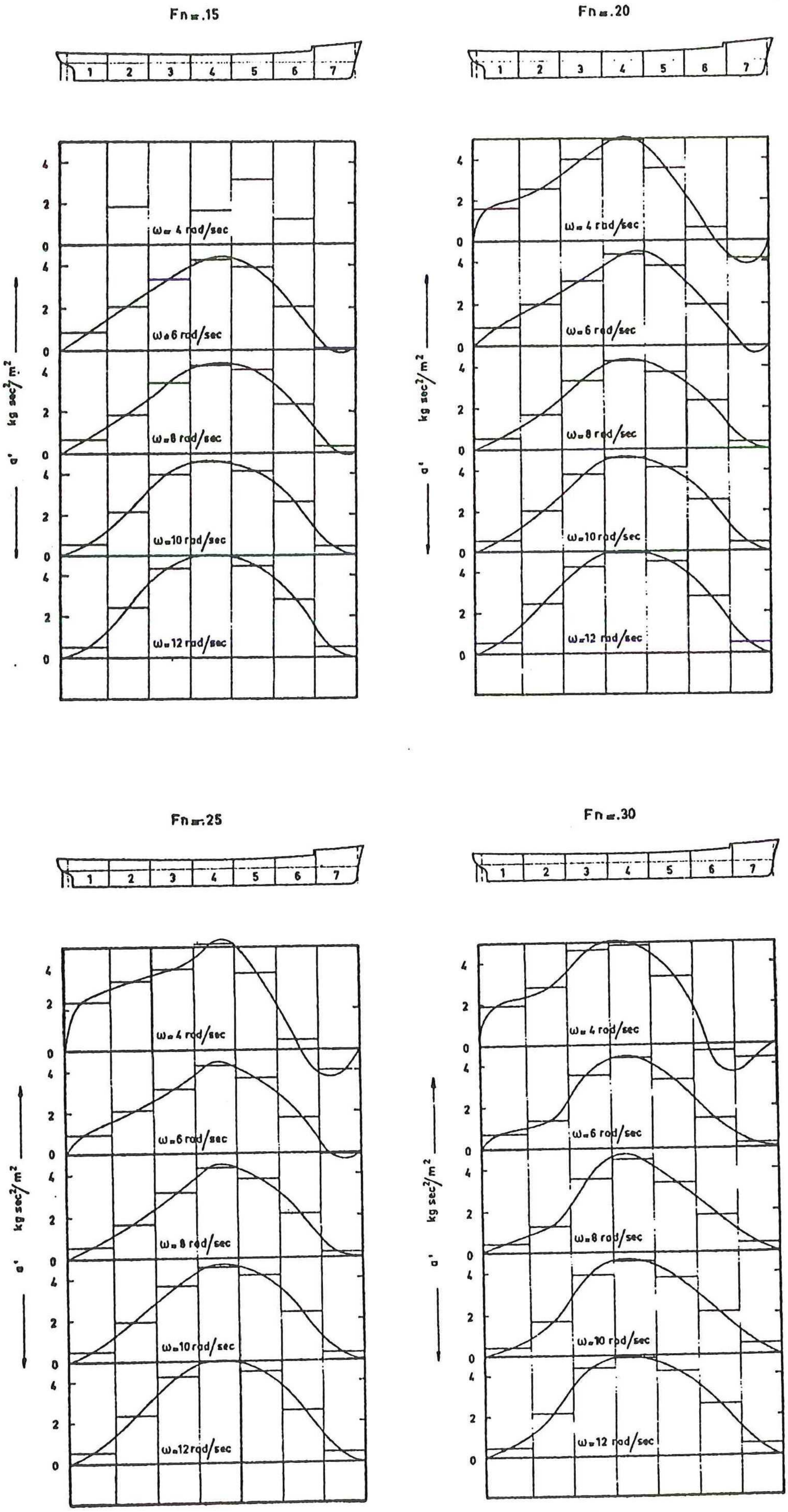
(W&S 5320)

(W&S 5320)



COMPARISON OF THE CALCULATED DISTRIBUTION OF e AND b WITH EXPERIMENTAL VALUES FOR FROUDE NUMBER .20

FIGURE 14



DISTRIBUTION OF a OVER THE LENGTH OF THE SHIPMODEL

(W&S 5320) FIGURE 7