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**DESIGN METHODOLOGY FOR A METAMATERIAL-BASED
CUSHION FOR REDUCING UNDERWATER NOISE FROM
IMPACT-DRIVEN OFFSHORE MONOPILES**

**DESIGN METHODOLOGY FOR A METAMATERIAL-BASED
CUSHION FOR REDUCING UNDERWATER NOISE FROM
IMPACT-DRIVEN OFFSHORE MONOPILES**

Dissertation

for the purpose of obtaining the degree of doctor
at Delft University of Technology
by the authority of the Rector Magnificus, prof. dr. ir. T.H.J.J. van der Hagen,
chair of the Board for Doctorates
to be defended publicly on
Wednesday 9 April 2025 at 12:30 o'clock

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Keywords: Mechanical metamaterials, local resonance, pile hammering, wave filtering, noise attenuation, design framework, numerical modeling, experimental evaluation

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We will make it through somehow.

Trent Reznor

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SUMMARY

The global energy sector is increasingly shifting towards sustainable solutions, driving the expansion of offshore wind farm installation to meet rising energy demands. The upcoming wind farms will be constituted of larger wind turbines, requiring larger monopile foundations. The monopiles are typically installed using impact hammers that blow several strikes until the monopile reaches a specific penetration depth. The installation process generates high underwater noise levels, posing severe risks to marine life, including hearing loss, behavioral changes, and even death.

Existing noise mitigation systems struggle to attenuate low-frequency noise below 1000 Hz due to the mass law limitations of natural materials. This thesis proposes a design methodology for a novel metamaterial-based noise mitigation system—the meta-cushion—designed to brake this limitation. Placed between the monopile and the hammer, the meta-cushion filters out mechanical waves generated by the hammer’s blow that cause high underwater noise levels. The design methodology ensures adaptability to various monopile installation specifications, facilitating the implementation of the meta-cushion in real-world pile driving cases.

The thesis begins with an analysis of underwater noise during pile driving and the limitations of existing mitigation solutions, emphasizing the need for novel approaches to reduce low-frequency noise. The analysis shows a strong relation between high underwater noise levels and the low-frequency vibrations of the monopile caused by the hammer’s blow. Then, the concept of metamaterials is introduced, highlighting how their unique properties contribute to reducing low-frequency vibrations. Based on the understanding of the underwater noise-vibration relationship, an initial meta-cushion design exhibiting such filtering features is defined.

The numerical modeling of the meta-cushion is then detailed, with finite element analyses being conducted to investigate the behavior of the meta-cushion when interacting with mechanical propagating waves. The attenuation capabilities are evaluated via analysis of dispersion-curve and transmission loss diagrams. To ensure both mechanical integrity and noise reduction under impact loads, a design optimization strategy is developed, from which genetic algorithm is used to investigate the trade-off behavior between mechanical and attenuation performances. Once verified, the meta-cushion’s functionality is experimentally validated through modal impact analysis and small-scale pile hammering tests, demonstrating effective noise reduction at frequencies below 1000 Hz as also indicated by the numerical results.

Finally, this thesis presents a systematic design methodology—encompassing meta-cushion design selection based on pile specifications, numerical verification, and experimental validation—that can be adapted to full-scale monopile installations. By integrating the meta-cushion into offshore pile driving operations, this work contributes to mitigating underwater noise and its environmental impact.

SAMENVATTING

De wereldwijde energiesector verschuift steeds meer naar duurzame oplossingen, wat een uitbreiding van offshore windparken stimuleert om te voldoen aan de groeiende energievraag. Toekomstige windparken zullen bestaan uit grotere windturbines, waarvoor grotere monopile fundaties nodig zijn. De monopiles worden doorgaans geïnstalleerd met slaghamers die meerdere slagen toedienen totdat de monopile de gewenste penetratiediepte bereikt. Dit installatieproces genereert hoge geluidsniveaus onder water, wat ernstige risico's met zich meebrengt voor het onder water leven, zoals gehoorverlies, gedragsveranderingen en zelfs sterfte van zoogdieren en vissen.

Bestaande geluidreductiesystemen hebben moeite met het dempen van laagfrequent geluid onder 1000 Hz vanwege de massawet beperkingen van natuurlijke materialen. Dit proefschrift presenteert een nieuw ontwerp van een op metamateriaal gebaseerd geluidreductiesysteem—de zogenaamde “meta-cushion”—dat deze beperking doorbreekt. Het meta-cushion wordt geplaatst tussen de bovenzijde van de monopile en de hamer, en dempt de mechanische golven uit die door de slag van de hamer worden gegenereerd en onderwater hoge geluidsniveaus veroorzaken. De gepresenteerde ontwerpmethodologie maakt het mogelijk het meta-cushion aan te passen aan verschillende installatiespecificaties van monopiles. Dit vergemakkelijkt de implementatie van het meta-cushion in de praktijk.

Het proefschrift begint met een analyse van onderwatergeluids tijdens de installatie van monopiles op zee en brengt de beperkingen van bestaande geluidsreductie oplossingen in kaart. De analyse toont een sterke relatie aan tussen hoge onderwater geluidsniveaus en de laagfrequente trillingen van de monopile veroorzaakt door de slag van de hamer. Vervolgens wordt het concept van metamaterialen geïntroduceerd en wordt besproken hoe hun unieke eigenschappen bij kunnen dragen aan de reductie van laagfrequente trillingen. Op basis van de relatie tussen onderwater geluid en trillingen is een eerste meta-cushion ontwerp gedefinieerd dat de filtereigenschappen vertoont.

Vervolgens wordt de numerieke modellering van het meta-cushion gedetailleerd beschreven, waarbij eindige-elementenanalyses worden uitgevoerd om het gedrag van het meta-cushion te onderzoeken bij interactie met mechanische voortplantingsgolven. De dempende capaciteit wordt geëvalueerd aan de hand van dispersiecurves en transmissieverliesdiagrammen. Om zowel mechanische integriteit als geluidsreductie bij impactbelastingen te waarborgen, is een ontwerpstrategie ontwikkeld, waarbij een genetisch algoritme toegepast wordt om de afweging tussen mechanische prestaties en demping inzichtelijk te maken. De functionaliteit van het meta-cushion is experimenteel gevalideerd via modale impactanalyse en kleinschalige tests. De numerieke en experimentele resultaten zijn in overeenstemming en tonen een effectieve geluidsreductie bij frequenties onder 1000 Hz aan.

Tot slot presenteert dit proefschrift een systematische ontwerpmethodologie—die de keuze

van het meta-cushion ontwerp op basis van pile specificaties, numerieke verificatie en experimentele validatie omvat—en welke kan worden aangepast aan grootschalige monopile installaties. Door de integratie van het meta-cushion in offshore paalfundering operaties levert dit werk een bijdrage aan de reductie van geluid en de bijbehorende effecten onder water leven.

PREFACE

Almost five years ago, I embarked on a new adventure in my life: starting my doctoral research in a completely new country, immersed in a different culture, and far from my family. I thought this experience would be similar to when I moved 2500 km from my hometown to pursue my master's studies in southeastern Brazil. However, after a few months, I realized I was wrong. Life has a way of teaching us—not only to hone our professional skills but also to strengthen our emotional resilience in challenging situations. Just as I evolved over the past years, this thesis reflects that growth. It traces a journey from exploring a slice of the metamaterial world to developing a novel noise mitigation system grounded in metamaterial concepts. Achieving this milestone would not have been possible without the direct and indirect support of many people, to whom I would like to express my deepest gratitude in the following paragraphs.

First, I would like to thank my supervisors, Jovana Jovanova and Dingena Schott, for guiding me to become more critical and independent, both professionally and personally. Your insights and support were crucial to the completion of this PhD project. Jovana, your encouragement and positivity were invaluable, especially during my first year at TU Delft. I am truly grateful for your supervision and for helping me navigate through challenges along the way. Dingena, I really admire your passion for research and attention to even the smallest details, which make a significant difference. Your speech during your inaugural address was quite inspiring, which I will never forget. I also appreciate the fun activities organized outside of work—these moments of relaxation were important for us PhDs, especially when we were dealing with challenging situations. I feel incredibly fortunate to have had both of you as my supervisors throughout this journey.

I would also like to extend my gratitude to my former supervisors in Brazil, Simone Hoefel and Josue Labaki, for believing in my potential, even when I doubted it myself. Your guidance and encouragement were instrumental in preparing me for this new chapter of my life. I am truly grateful for your teaching and support—without them, I would not be who I am today. I hope this work and my journey inspire other students to pursue their dreams, even when the path seems uncertain.

I am also grateful to the TU Delft staff, professors, and technicians who contributed directly and indirectly to the completion of this thesis. Your valuable suggestions, support, and guidance, particularly during my experiments, were greatly appreciated.

A special thanks to my colleagues at TU Delft, especially my officemates Raisa, Adrien, Ethan, Raisa, Sankarshan, and Yupeng for creating such a friendly environment filled with wonderful conversations. I am also grateful to my colleagues from JAM and MMI—Aaron, Ahmed, Chamod, Christof, Hao, Hesam, Javad, Marc, Marcel, Vera, and Zhaorui—and all my colleagues from MTT for the engaging talks, brainstorming sessions, coffee breaks, piano sessions, trips (especially the one to Macedonia), lunches, dinners, and, above all, for supporting me through tough times. I feel truly fortunate to be surrounded

by such amazing people, and I look forward to sharing more memorable moments with all of you in the future.

I would like to dedicate this paragraph to my dear Brazilian friends—David, Rafael, and Amanda—who have been incredibly important to me. Thank you for bringing joy to my life, listening to my struggles, celebrating my milestones, bringing the Brazilian spirit, and always making me feel positive. I will never forget your support, and I look forward to creating more wonderful memories together.

Finally, none of this would have been possible without the unwavering support and love of my family—my parents, Dauva and Nonato, my brother Ricardo, my aunt Maria (aunt Baby), and my cousin Gaby. Though not physically present, you were with me every single day in my heart. Your care and encouragement were essential in keeping me positive and motivated, especially during the challenging moments of my PhD journey. I would also like to express my gratitude to the van Loon family for welcoming me to the Netherlands and introducing me to Dutch culture. Thank you, Dora, for always keeping my spirits high, believing in me, and bringing the Brazilian warmth when I needed it most. I truly hope we can continue to share wonderful moments here in the Netherlands.

*Ana Carolina Azevedo Vasconcelos
Delft, April 2025*

1

INTRODUCTION

To achieve the EU's target of 42.5% renewable energy by 2030, annual wind energy installations must average 33 GW between 2024 and 2030. The growing demand for renewable energy has driven an increase in offshore wind farms, particularly in Europe, where 18.3 GW of new wind capacity was added in 2023 [1]. Offshore wind accounted for 21% of all European installations, with 3.8 GW connected to the grid. In the Netherlands, offshore installations comprised half of Europe's offshore wind power in 2023. Monopile foundations, used in 80.5 % of offshore wind installations in 2020 [2], are preferred for their simplicity in design and ease of manufacturing [3]. They consist of circular steel tubes welded into a single structure, reaching up to 65 m in length and 6-7 m in diameter. To meet the 2030 wind capacity goals, larger offshore wind turbines (OWT) are being developed, requiring larger monopile foundations, known as XXL foundations [4]. Furthermore, a decrease in the Levelised Cost of Energy (LCoE) from offshore wind is expected soon [5, 6], encouraging the installation of OWTs in deeper seas, thereby requiring monopiles of greater dimensions suitable for such depths [7].

The installation of monopiles involves the use of either hydraulic impact hammers or vibratory hammers positioned at the pile head, which drive the pile into seabed through repeated striking or vibrating. The impact hammer is generally favored for maintaining the pile's upright position on the seabed [8]. Each hammer strike induces stress waves that travel through the pile, causing longitudinal deformation and subsequent radial expansion/contraction of the pile wall. This pile wall motion interacts with surrounding water, leading to loud underwater noise waves that can travel for kilometers [9] (see Fig. 1.1). This noise has detrimental effects on marine environment, including seabed morphology and biodiversity, as well as economic activities such as fishery. In particular to marine life, studies demonstrated that underwater noise can cause both physical and behavioral changes in fish and marine mammals, including damage to internal organs and blood vessels [10–14], flight behavior, fright response, and potential for habituation [15, 16]. These adverse effects are exacerbated with the use of larger monopiles, which generate higher noise levels [17] (see Fig. 1.2).

To protect marine life, various countries have proposed guidelines and threshold values for regulating underwater noise [18]. In the Netherlands, frameworks have

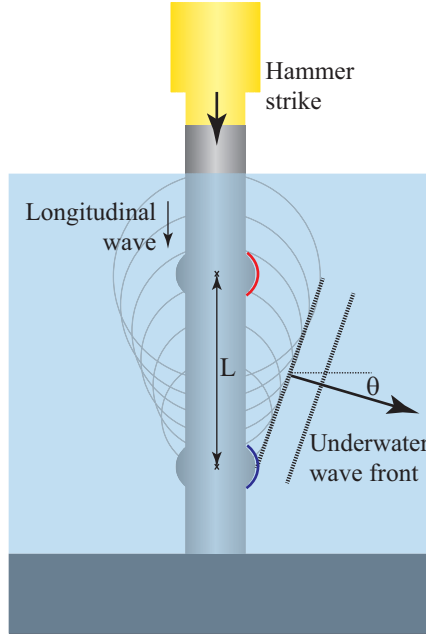


Figure 1.1.: Radial deformation of the monopile wall due to hammer strike. The pile deformation induces compressional sound waves into water in a cone shape. (adapted from [12]).

been developed to determine the cumulative effects of impulsive noise on marine mammals in the North Sea [19]. Table 1.1 outlines the threshold levels for causing temporary (TTS) and permanent (PTS) hearing injuries in harbour porpoises and seals, which are the predominant species of marine mammals found in the North Sea.

These limits are expressed in terms of $SPL_{(peak)}$, $SEL_{(ss)}$, and $SEL_{(cum)}$, representing the maximum sound peak pressure level and sound exposure level over single and multiple pulses, respectively. These levels are calculated by using the following expressions:

$$\begin{aligned}
 SPL_{(peak)} &= 20 \log \left(\frac{|p_{peak}|}{p_0} \right) [\text{dB re } 1\mu\text{Pa}], \\
 SEL_{(ss)} &= 10 \log \left(\frac{1}{T_0} \int_{T_1}^{T_2} \frac{p(t)^2}{p_0^2} dt \right) [\text{dB re } 1\mu\text{Pa}^2\text{s}], \\
 SEL_{(cum)} &= SEL_{(ss)} + 10 \log(\text{number of impulses}),
 \end{aligned} \tag{1.1}$$

where p_{peak} is the maximum – positive or negative – sound pressure, p_0 is the reference sound pressure, T_0 is 1 second, and T_1 and T_2 are, respectively, the starting and ending time that the noise event occurs.

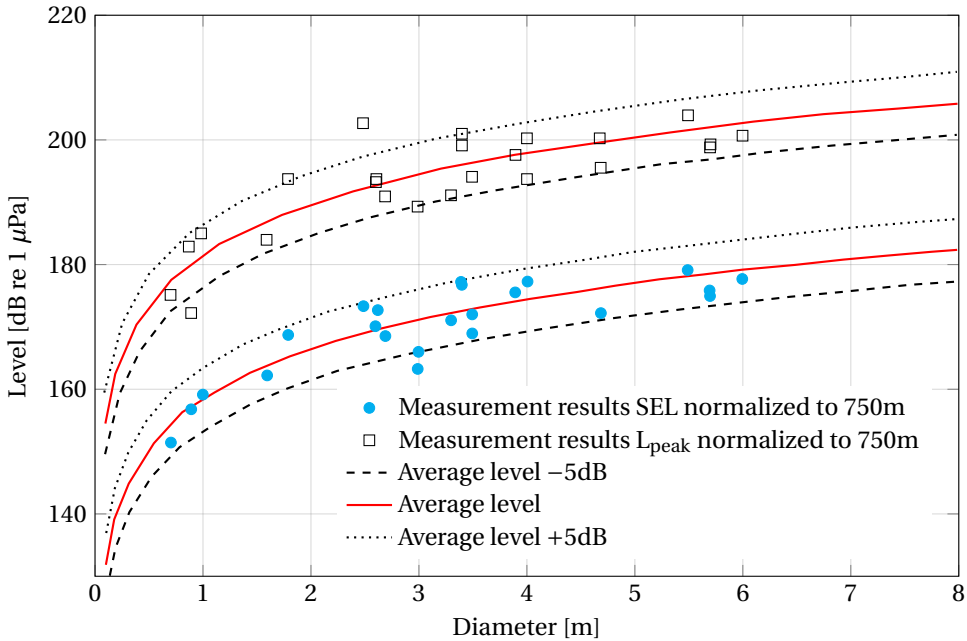


Figure 1.2.: Variation of sound exposure level (SEL) and sound peak level (L_{peak}) with respect to pile diameter [17].

Species	Effect	Value (dB re 1 $\mu\text{Pa}^2\text{s}$)	Source
Harbour porpoise	Flight	$\text{SEL}_{(\text{ss})}$ 140 dB	Intermezzo threshold values
	TTS onset	$\text{SEL}_{(\text{cum})}$ 164 dB	Lucke et al. (2009) [20]
	TTS-1 hour	$\text{SEL}_{(\text{cum})}$ 169 dB	TTS onset + 5 dB
	PTS onset	$\text{SEL}_{(\text{cum})}$ 179 dB	TTS onset + 15 dB
Seals	Flight	$\text{SEL}_{(\text{ss})}$ 145 dB	Kastelein et al. (2010) [21]
	TTS onset	$\text{SEL}_{(\text{cum})}$ 171 dB	PTS onset - 15 dB
	TTS-1 hour	$\text{SEL}_{(\text{cum})}$ 176 dB	TTS onset + 5 dB
	PTS onset	$\text{SEL}_{(\text{cum})}$ 186 dB	Southall et al. (2008) [22]

Table 1.1.: Noise levels (in decibels) that can cause temporary (TTS) or permanent (PTS) hearing injury in harbour porpoises and seals. The levels are indicated as $\text{SPL}_{(\text{peak})}$ (maximum sound peak pressure level), $\text{SEL}_{(\text{ss})}$ (sound exposure level over a single pulse), and $\text{SEL}_{(\text{cum})}$ (sound exposure level accumulated after a number of pulses in a certain period of time, ≥ 1 h). Adapted from [19].

These sound pressure quantifiers are described in units of decibels (dB) with reference to a standard pressure of 1 μPa for underwater sound – for instance, typical range of SPL varies from 50 to 250 dB re 1 μPa . Realistic sound levels of pile driving are approximately 170-180 dB for SEL_{ss} [18], which are higher than the

threshold values indicated in Table 1.1. To reduce such noise levels, engineers are then tasked with developing noise mitigation systems (NMS) that protect marine life from harsh environmental conditions caused by monopile installations.

1.1. STATE-OF-THE-ART OF NOISE MITIGATION SYSTEMS

Traditional NMSs consist of large structures enveloping the monopile and spanning the full water column height [17, 23]. Such NMSs can be categorized in three main groups according to the noise attenuation principle: air bubble curtains, steel casings and resonator-based NMSs (Fig. 1.3).

Bubble curtains (BCs) are commonly used to mitigate underwater noise, due to their efficacy and simplicity in their attenuation principle. The curtains are created when compressed air supplied by an external vessel is pushed through perforated tubes at the seabed, releasing bubbles [24]. The principle of noise attenuation in such curtains is based on two mechanisms [25]. First, the high contrast in density and compressibility between water and the released bubbles results in impedance mismatch, reflecting part of the incoming sound wave, thereby reducing the intensity of the transmitted wave. Second, the air bubbles resonate when waves propagate through the curtain, partially absorbing the propagating wave energy. The resonance of these air bubbles is tuned by changing their size so they resonate at specific wavelengths of the sound wave. Although these curtains demonstrate these two mechanisms, experimental studies have shown that their primary noise reduction mechanism is the impedance mismatch [26], which can be optimized for different operational conditions by varying bubble sizes and adjusting airflow.

The most typical curtains are small bubble curtains (SBCs) and big bubble curtains (BBCs), where the first only encloses the monopile, while the second covers the complete operation—i.e. the noise generated by pile installation and the vessels are damped. Double bubble curtains (DBBCs) are sometimes used to enhance noise attenuation (Fig. 1.3a). When it comes to low-frequency noise reduction, bubbles of large diameters are needed, posing two significant challenges. First, creating large bubbles requires more supplied energy, increasing operational costs. Second, such bubbles are unstable and can easily split into smaller parts, particularly when exposed to high currents. In the experiment carried out on the Baltic II offshore wind farm, a SEL reduction between 11 and 15 dB was obtained at a distance of 750 m from the pile, being the highest attenuation level occurring at frequencies between 1 and 10 kHz [27], however, these levels tended to decrease for frequencies lower than 1 kHz.

The second category of NMS consists of involving the monopiles with an isolating casing, which is basically a double-wall steel tube with an air-filled inter-space. One example of such system is the Noise Mitigation Screen developed by IHC IQIP (see Fig. 1.3b). During pilot tests performed in the North Sea as part of the FLOW-project, this system exhibited noise reduction values of 9 dB and 11 dB for monopiles with a diameter of 3.35 m, while a reduction of 20 dB in third octave bands between 150 Hz and 8 kHz was obtained, although for piles with diameters smaller than 0.9 m [28]. This system also relies on the impedance mismatch principle; the emitted

sound waves are reflected by the casing and are trapped in the region between the pile and the casing. Another part of the energy induces motion of the inner wall, radiating waves into the inter-space between the two walls. Because of the low density of the air, the radiated wave cannot induce motion of the outer wall, reducing the sound transmission from the outer wall to the water. To enhance its attenuation performance, the noise mitigation screen is combined with the BBCs, installed at the space between the monopile and the inner wall, reaching reduction levels in the order of 17 dB.

The third category is resonator-based systems, which utilize resonant elements strategically positioned around the pile to absorb the sound energy through their internal vibrations. One such system is the Hydro Sound Dampers (HSDs), which consist of multiple net planes containing elements made of PE foam and thin walled-balloons filled with gas that are adaptable to different water depths (Fig. 1.3c) [29, 30]. Noise reduction in HSD systems is not solely dependent on balloon resonance, but also involves the damping effects of the PE foams and injected air, as well as wave reflection due to the impedance mismatch between the water and the net. As a first offshore test within the ESRa project, the HSD system was able to reduce the SEL up to 23 dB within a frequency range between 100 Hz and 600 Hz during installation of a pile with diameter of 2.35 m [29]. Similar to BCs, the effectiveness of HSDs depends on the balloon size. Although lightweight and easily adaptable, HSDs are limited by their susceptibility to currents above 0.75 m/s and limited life-time of their elements.

Another resonator-based system, proposed by AdBm, features panels containing open-ended containers that trap air when submerged (Fig. 1.3d). The system mitigates noise through a mechanism similar to Helmholtz resonance, where incoming sound wave induces resonance of the trapped air, which re-radiate the sound from the air-water interface [31]. Similarly to free and encapsulated air bubbles presented in the previous systems, the noise reduction efficacy of resonator-based systems depends on the volume and frequency of the containers.

Figure 1.4 provides an overview of the noise attenuation levels achieved by traditional NMSs from each category in commercial projects at frequencies lower than 10 kHz. Notably, all NMSs exhibit significant noise reduction at frequencies above 1 kHz. However, in the lower frequency range (below 1 kHz), mainly the DBBC and the IHC-NMS combined with BBC show substantial attenuation. While these systems could be used for XXL monopiles, their implementation would considerably increase operational costs and time – once it relies on the manipulation of huge structures that require the combination of different NMS – potentially decreasing the speed of the achievement of the desired wind energy targets by 2030. Therefore, developing new NMSs that effectively reduce low-frequency noise and are easier to handle is crucial and forms the core focus of this work.

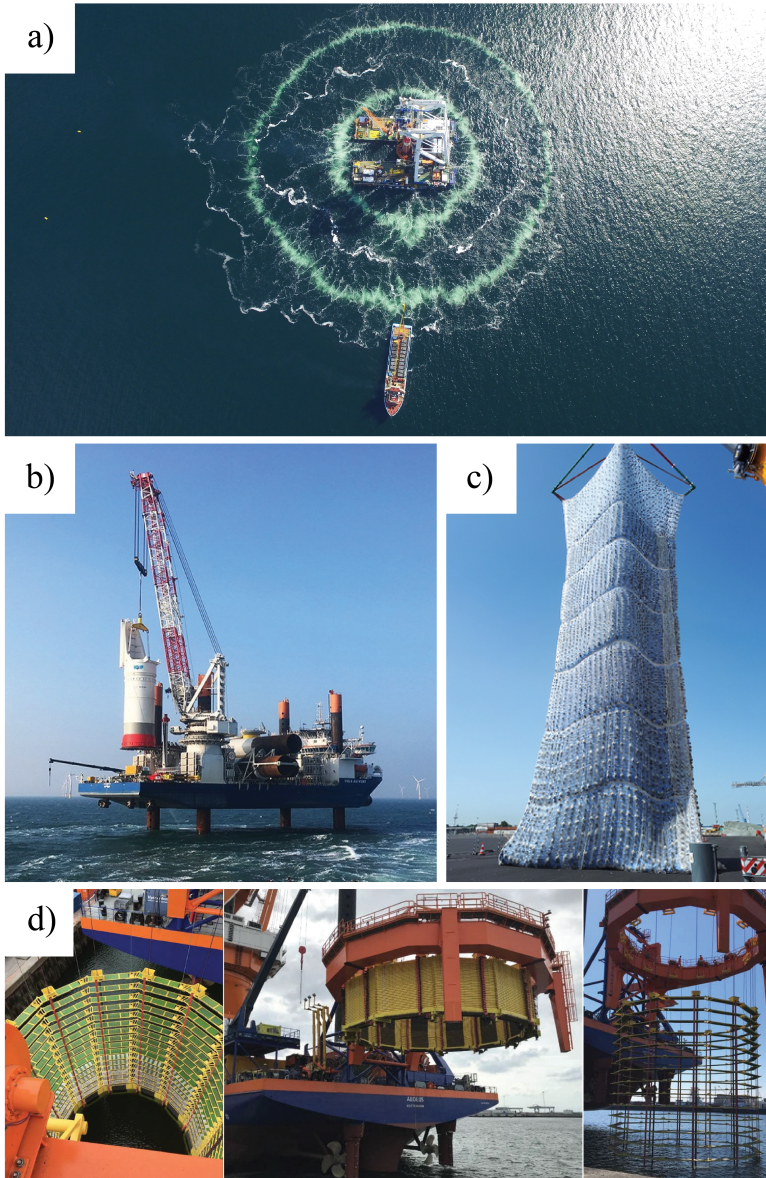


Figure 1.3.: Traditional noise mitigation systems: a) Bubble curtains used at the Arkona offshore wind project in the Netherlands. b) Noise mitigation screen (integrated monopile system) developed by IHC IQIP. c) The hydro sound damper system lifted by a crane (OffNoise Solution GmbH). d) Helmholtz resonator by AdBm.

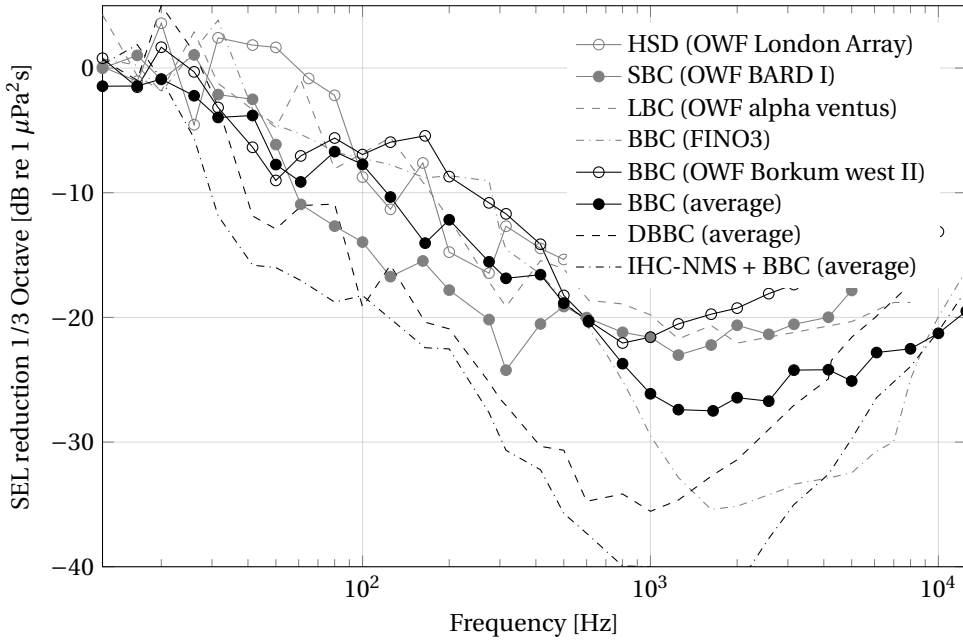


Figure 1.4.: Sound exposure level reduction in terms of frequency for several noise mitigation systems (adapted from [17]). The majority noise attenuation performed by all NMS occurs in frequencies around 1000 Hz, while only DBBC and IHC-NMS BBC show significant attenuation for lower frequencies.

1.2. A NEW DIRECTION OF UNDERWATER NOISE CONTROL: A METAMATERIAL-BASED NMS

A major challenge in installing XXL offshore monopiles lies in the limitations of current NMS to effectively attenuate low-frequency underwater noise, especially for frequencies below 1000 Hz. This has motivated researchers to seek innovative solutions to minimize the environmental impact of the pile driving process. One promising approach involves developing novel NMS designs that take advantage of the concept of metamaterials to specifically target these critical frequencies. Before diving into the design of a metamaterial-based NMS, it is crucial to understand the definition of metamaterials and their attenuation mechanisms.

Metamaterials are engineered structures that possess unique effective properties not typically found in nature materials (the Greek prefix "meta-" means "beyond"). They consist of building blocks or unit cells, the smallest periodic unit within the structure. These unit cells exhibit specific properties that create frequency zones where wave propagation is prohibited, known as band gaps. The concept was first introduced by Veselago in 1967 [32], who theoretically demonstrated a material with simultaneous negative values for both the dielectric constant and the magnetic permeability. Almost thirty years later, Pendry [33] proposed a periodic structure

of thin copper wires that exhibited negative dielectric constant values. In 1999, he also discovered that periodic split ring resonators (SRR) produce a frequency bandwidth with negative magnetic permeability [34]. By combining these findings, Shelby et al. [35] experimentally verified Veselago's theory and introduced the first electromagnetic metamaterial.

The substantial advances in electromagnetic metamaterials paved the way for developing metamaterials to controlling other wave types, such as acoustic and elastic waves. The first acoustic/elastic metamaterial, introduced in 2000 by Liu et al. [36], utilized resonant unit cell to achieve negative elastic parameters at specific frequency ranges. In Liu's work, the unit cell was composed of a stiff core covered by a soft material exhibiting out-of-phase motion, named as negative mass, in relation to the external excitation within those frequency ranges. Inspired by this discovery, researchers have developed new acoustic/elastic metamaterial designs, expanding their applications to areas such as acoustic cloaking [37, 38], energy harvesting [39–42], seismic wave mitigation [43–46], vibration control, among others. These innovative discoveries hold significant potential for the development of NMS capable of filtering low-frequency noise.

1.3. RESEARCH QUESTIONS AND SCOPE

The combination of multiple NMSs to achieve the required noise reduction levels at low-frequency ranges (lower than 1000 Hz) appears to be a viable solution for installing large offshore monopiles. Nevertheless, this approach is not optimal due to increased operational costs and handling time. For example, bubble curtain systems necessitate separate vessels and compressors to supply air, adding complexity to offshore logistics with vessels [47] and energy costs – typical bubble curtain projects may require 20 - 26 compressors [48]. Although HSD nets offer advantages in handling and cost-efficiency compared to other NMS, they must be robust enough to withstand strong tidal currents and flexible enough to enable flow-through [28]. Furthermore, these NMSs consist of large structures covering the entire water column, as noise control occurs in the seawater environment. A more efficient solution would be a systematically designed NMS that controls low-frequency underwater noise directly at its source, i.e. at the point of impact when the hammer strikes the monopile. This would reduce handling and operational costs, eliminating the need for additional vessels or energy suppliers.

The objective of this thesis is to propose a methodology for designing a novel, metamaterial-based NMS, positioned between the hammer and monopile, that reduces low-frequency noise during offshore monopile hammering. Using numerical models, the new NMS is designed to partially filter the hammer's impact load while withstanding high induced stresses. The methodology targets underwater noise generated from a single hammer blow, specifically focusing on noise transmission from the pile directly to the water. In the final stage, the NMS concept is validated experimentally with a scaled-down prototype. This objective raises key research questions that guide the structure of this thesis:

What design methodology is required to effectively apply the meta-cushion

principle in mitigating noise during monopile installation via hammering?

- *What is the state-of-art of metamaterials designed for reducing vibrations caused by impact loads?*
- *How to numerically evaluate the ability of the meta-cushion to mitigate underwater noise?*
- *How can mechanical integrity and attenuation performance be ensured simultaneously during the meta-cushion design?*
- *How to experimentally demonstrate the meta-cushion principle?*
- *How can the design steps be structured to ensure that the meta-cushion principle effectively mitigates noise across various offshore monopile installation scenarios?*

1.4. THESIS OUTLINE

This thesis is structured into six chapters, as described in Fig. 1.5.

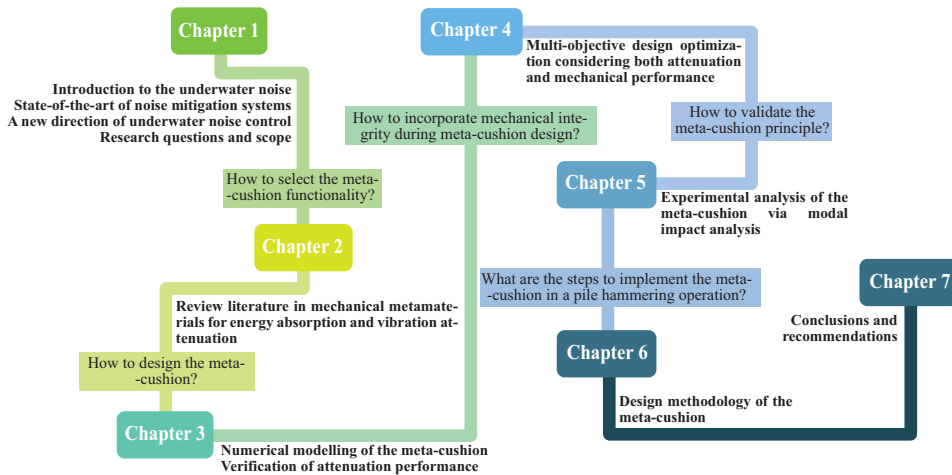


Figure 1.5.: Schematic representation of thesis outline.

Chapter 2 introduces the concept of mechanical metamaterials where a review of their mechanisms for mitigating vibrations and absorbing energy is presented. This analysis will guide the selection of the mechanism for addressing the underwater noise problem.

Based on the selected mechanism, a suitable metamaterial-based NMS design is chosen, which is termed as meta-cushion. The numerical model of the meta-cushion is explained in Chapter 3, including a finite element analysis to evaluate its

functionality. Additionally, a second finite element model simulates the full-scale pile-soil-water interaction, with the pile and soil modeled as continuum media and the water domain as an acoustic medium. An impact load replaces the impact hammer at the top of the monopile, and sound levels are then measured at distances specified by Dutch underwater noise regulations. A case study is then proposed to quantify the attenuation performance of the meta-cushion. The effectiveness of the meta-cushion is then assessed by comparing the noise levels of a pile system with and without the meta-cushion.

Since the hammer strikes deliver high stress levels to the pile system, assessing the mechanical integrity of the meta-cushion is crucial. In addition to that, the meta-cushion should keep its vibration attenuation performance when such high impact loads are applied. In Chapter 4, a multi-objective design optimization is presented. In this optimization approach, the mechanical and vibration attenuation aspects of the meta-cushion's resonant elements are simultaneously evaluated. A set of feasible solutions is obtained from which unit cell designs can be selected for a specific vibration control problem.

Chapter 5 validates the meta-cushion's performance in filtering mechanical waves across various frequencies. Selectively designed meta-cushions are fabricated and tested using modal impact analysis, where a modal impact hammer induces a load with a wide frequency spectrum and accelerometers measure the mechanical responses of the cushion. The experimental results are finally compared with the predictions of the numerical model.

Building on the meta-cushion design stages outlined in earlier chapters, Chapter 6 proposes a methodology using the input pile-system parameters highly related to the critical noise levels parameters to develop customizable meta-cushions with optimized noise attenuation and structural performance. Meta-cushion prototypes are fabricated via a 3D printing technique and tested in a small-scale experimental setup. The designs showing significant noise attenuation performance are selected, from which nondimensional dispersion diagram analyses are conducted to verify the scalability of such performance.

Chapter 7 summarizes the major findings of this thesis and provides recommendations for further research.

2

AN INTRODUCTION TO MECHANICAL METAMATERIALS

Mechanical metamaterials are architected structures with unique functionalities, such as negative Poisson's ratio and negative stiffness, which are widely employed for absorbing energy of quasistatic and impact loads, giving improved mechanical response. Acoustic/elastic metamaterials, their dynamic counterparts, rely on frequency-dependent properties of their microstructure elements, including mass density and bulk modulus, to control the propagation of waves. To design a meta-cushion, it is important to initially understand what mechanisms are suitable for mitigating low-frequency mechanical waves. From this knowledge, the function that most suits the underwater noise problem is selected.

In this chapter, a state-of-the-art in the design processes of mechanical metamaterials is presented in terms of their functionality in attenuating vibrations and absorbing energy. This chapter is a transcription of the journal paper published in *Heliyon*¹.

¹Vasconcelos, A.C.A., Schott, D., Jovanova, J., Hybrid mechanical metamaterials: advances of multi-functional mechanical metamaterial with simultaneous static and dynamic properties, *Heliyon*, 2025.

LIST OF ABBREVIATIONS

AEMM acoustic/elastic metamaterials.

CNN convolutional neural network.

EBM effective bulk modulus.

EMD effective mass density.

ESM effective shear modulus.

EYM effective Young's modulus.

GAN generative adversarial network.

HMMM hybrid mechanical metamaterials.

IA inertial amplifications.

MM metamaterials.

MMM mechanical metamaterials.

NS negative stiffness.

PnC phononic crystals.

PPN property prediction network.

QZS quasi-zero stiffness.

SGN structure generation network.

SMP shape memory polymer.

2.1. INTRODUCTION

Dynamic loads significantly contribute to the behavior and performance of engineered structures, often leading to undesirable structural responses characterized by nonlinearities, resonances, and structural failure. Understanding and mitigating these responses are crucial for ensuring the safety and reliability of engineered structures across diverse fields, including civil infrastructure, aerospace, mechanical systems and offshore installations.

Shielding structures relying on large plastic deformation or significant cracking are conventionally utilized to protect other engineered structures from diverse loading conditions. Cellular structures, for instance, have been widely used in aerospace and automotive industries due to their improved strength-to-relative density ratio and high energy absorption capability in comparison to their constituent material [49, 50], although suffering from crushing [51]. Relying on irreversible mechanisms may hinder material reuse under repeated loads, significantly escalating the costs of manufacturing new shielding materials and generating material waste. This has instigated the search for shielding structures that are capable of suppressing the transmission of mechanical waves caused by such loads, while retaining their structural integrity.

Decades ago, researchers unveiled [metamaterials \(MM\)](#)s, which are materials exhibiting unique properties capable of manipulating electromagnetic and optical waves [32, 33]. This breakthrough boosted the development of a novel subset of MMs for absorbing mechanical energy, which has been named as [mechanical metamaterials \(MMM\)](#)s. MMMs show improved mechanical properties compared to nature materials, such as negative Poisson's ratio and negative stiffness, stemming from the strategic design of their building blocks or unit cells. Such unique features have enabled the development of light-weight MMM structures with high-strength capacity [52], which have been widely applied in aircraft structures.

Another application of [MMMs](#) includes the control of mechanical vibrations, such as airborne vibrations and structural vibrations. These [MMMs](#) are usually referred to as [acoustic/elastic metamaterials \(AEMM\)](#)s and their controlling capacity stems from two mechanisms: Bragg scattering and local resonance [53]. [AEMMs](#) exhibiting the Bragg scattering mechanism are usually called [phononic crystals \(PnC\)](#)s, which consist of periodically arranged microstructures—or unit cells—serving as scatterers. Propagating acoustic/elastic waves interact with these scatterers and produce dispersion relations showing band gaps—or attenuation zones—in specific frequency ranges. In these band gaps, waves with wavelength in the order of unit cell size are attenuated, while outside the range, waves propagate without loss. Locally-resonant [AEMMs](#) exhibit frequency-dependent mechanical properties due to the local resonators arranged in their micro-structure. This enables the mechanical wave control in specific frequency ranges called “resonant band gap”. The induced resonant band gaps originate from the interference between incoming waves and the waves re-radiated from the local resonators, also named as Fano-type interference [54]. The resonance can be adjusted by modifying the resonator's mass or stiffness. Unlike Bragg scattering, this approach enables the formation of subwavelength band gaps, facilitating low-frequency wave mitigation for diverse

applications, such as vibration control [55, 56], blast loading resistance [57], sound isolation [58], and seismic isolation [43].

Although MMMs have shown important contributions to energy absorption in static problems and vibration control in dynamic problems, complex applications involving static and dynamic load conditions demand MMMs integrating multiple functionalities. For example, MMMs subjected to dynamic loads may require vibration attenuation performance to prevent resonance in protected structure, while concurrently exhibiting energy absorption capabilities for load-bearing purposes. The recent developments in computational design tools and additive manufacturing (AM) processes have facilitated the investigation and fabrication of complex designs with enhanced functionalities [59]. Among these designs, **hybrid mechanical metamaterials (HMMM)**s are emerging as a promising solution for such complex problems, as they integrate multiple mechanisms to achieve diverse functionalities (see Fig. 2.1).

One of the biggest challenges of the current MMM designs are their implementation in real-world engineering problems faced in automotive and aerospace, infrastructure design, underwater acoustics, among others. However, this trend may change with the development of **HMMMs**, once they are moving towards single-integrated systems that target different properties. Thus, understanding the individual mechanisms governing energy absorption and vibration control is essential to tailor **HMMM**s for specific applications. While numerous review papers have documented the mechanisms of **MMMs** [60] and **AEMMs** [61], an overview of the existent **HMMMs** and emerging design trends for engineering applications remains absent.

This review paper elucidates traditional **MMM** mechanisms for energy absorption and vibration isolation, how these mechanisms are integrated into **HMMM** designs to overcome the challenges faced by traditional **MMMs**, and explores future directions for their implementation as engineering solutions. Section 2 introduces **MMM** mechanisms for energy absorption capability, while Section 3 presents **AEMMs** containing unusual effective values of mass and stiffness. Section 4 highlights the design process of **HMMMs** and their potential multifunctional properties. The section ends with a discussion of the next generation of **HMMMs** and how they can be implemented in engineering applications.

2.2. MECHANICAL METAMATERIALS FOR ENERGY ABSORPTION

The mechanical properties of energy-absorbing materials, such as cellular materials (for instance, foams), polymers and gels, are crucial for engineering applications where structures face various loading conditions. However, cellular materials have practical limitations as they often rely on non-recoverable absorption mechanisms, leading to plastic deformation. This feature may prove unsuitable for applications involving repetitive loads and can result in increased material wastage.

In response to the demand for materials that can endure challenging loading conditions, researchers have developed mechanical metamaterials characterized by exceptional energy absorption and high specific strength, which stem from

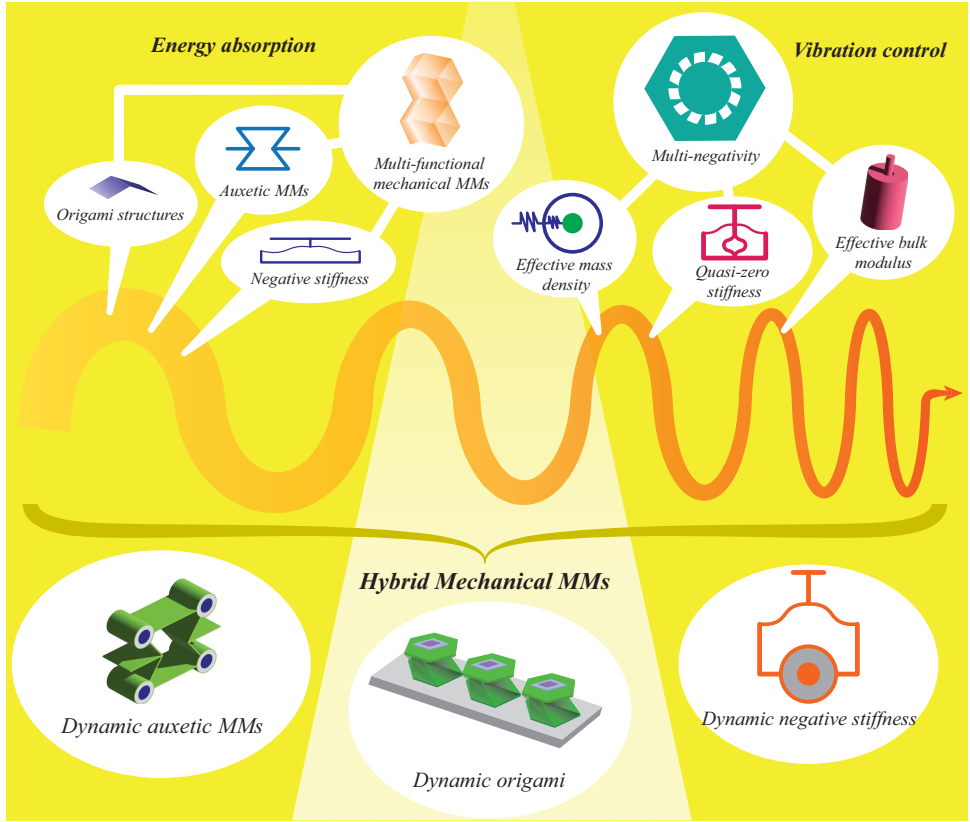


Figure 2.1.: Hybrid mechanical metamaterials origin: the combination of energy-absorbing mechanisms from mechanical metamaterials and vibration-controlling mechanisms of elastic/acoustic metamaterials.

the microstructure geometry—also referred as unit cell. The energy-absorbing performance is attributed to the deformation modes and loading conditions—notice that material properties also play an important role, however, only the geometric aspects will be discussed here. In this context, we dedicate this section to summarize and discuss the energy absorption mechanisms of mechanical metamaterials. Then, the response of such mechanisms under diverse load conditions is investigated. Finally, the unique properties of mechanical metamaterials are presented according to the designed mechanisms and loading conditions.

2.2.1. ENERGY ABSORPTION MECHANISMS

The unique macroscopic properties behind mechanical metamaterials emerge from the absorption mechanisms of their unit cell. Such unit cells can be formed by several structural elements, such as rods, beams and shell structures, and they can give distinct deformation modes when interacting with an external stimuli.

Figure 2.2 summarizes the main deformation modes related to the energy absorption ability of mechanical metamaterials. Those can be either stretching-dominated, bending-dominated, buckling deformation or a combination of such mechanisms, particularly when designing hierarchical materials. Stretching-dominated unit cells deform through uniaxial tension or compression of their inner elements, while bending-dominated ones deform by bending of their elements. Because of that, stretching-dominated unit cells can exhibit higher stiffness-to-weight ratio in comparison to their bending-dominated counterparts, being the former practical for dealing with loads in structural applications and the latter suitable for developing compliant structures [52, 62, 63]. For instance, microlattice materials formed by octet-truss unit cells exhibited a linear scaling relationship of $E/E_s \sim \rho/\rho_s$, unlike those made of bending-dominated unit cells, which relationship is $E/E_s \sim (\rho/\rho_s)^2$. Here, E/E_s is the ratio of the Young's modulus of the macroscopic structure and the Young's modulus of the material, while ρ/ρ_s is the relative density defined by the ratio of the macroscopic density and the density of the basis material. This means that stretching-dominated unit cells can exhibit high strength with a lower relative density in comparison to the bending-dominated counterparts. Even though stretching-dominated unit cells exhibit improved strength density ratio, local buckling may occur under large compressive strains as well as high stress concentration since the unit cell elements cannot rotate or bend due to the high nodal connectivity. In contrary, bending-dominated ones are compliant and lightweight, and their internal elements can rotate, fold or bend reducing the risks of buckling. To combine the benefits of both stretching- and bending-dominated unit cells, hybrid lattice structures [64, 65] and multiscale hierarchical structures [66] have been developed. The combined elements in hybrid structures have similar scale, while in hierarchical structures, the scale varies. Although such structures demonstrated enhanced properties, local buckling can still occur, preventing their reuse. Composite bending-dominated lattices were later developed to suppress buckling while maintaining high strength weight ratio and good cyclic performance [67]. To reduce the stress concentration, smoothness was introduced in the connections of shell lattices, which resulted in improved strength and energy absorption capacity [68, 69].

Other deformation mechanisms are reached through motion structures or deployable structures, from which their shape can be reconfigured and changed through folding and unfolding to the cost of zero (or near zero) deformation energy [70]. Such folding motion has been achieved by rigid deployable structures that consist of one or multiple degrees of freedom kinematic chains made of rigid structural elements connected by joints. Some examples are truss structures containing scissor joints, which expand and collapse their shapes, and plate structures connected by hinges at their edges. In addition to that, flexibility can be embedded to deployable structures, which gives flexible elements that under external constraints are folded in a small volume, storing potential energy, as commonly observed in automotive airbags and shield for space telescopes. Deployable structures formed by rigid and flexible elements have been also proposed to provide high stiffness, high flexibility, and lightness. Examples of such structures

are compliant and pantographic mechanisms—rigid elements connected by flexible joints— and tensegrity structures—rigid elements connected by flexible cables.

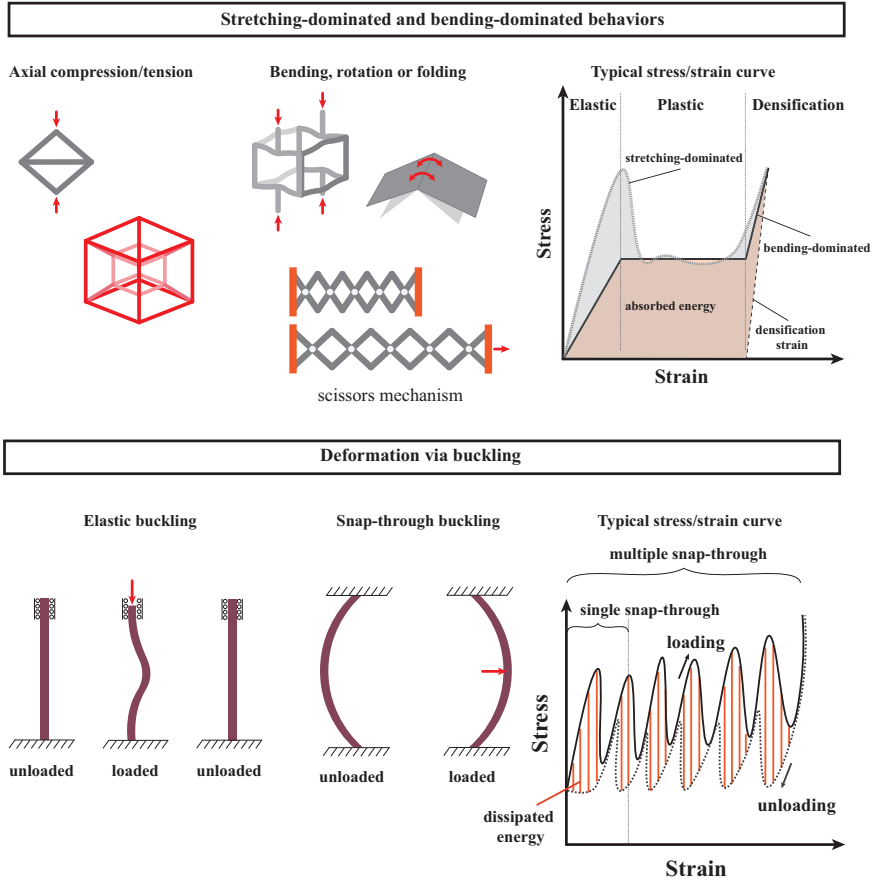


Figure 2.2.: Main deformation mechanisms observed in mechanical metamaterials for absorbing energy.

Although elastic and plastic buckling are often considered as an undesirable mechanical instability, many mechanical metamaterials are designed to possess such mechanisms tailored in such a way that energy absorption is enhanced and the metamaterial can be recoverable and reusable. In fact, incorporating buckling can result in geometrical nonlinearities—while the constituent materials could still be in the linear regime—, converted into mechanical bistability [71, 72], multistability [73, 74], or programmable behavior [75, 76]. Snap-through behavior is a common case of bistability, where a state shifts quickly to the second state when the load applied to a structures exceeds the critical load, as illustrated in Fig. 2.2. During the transition stage, the initial positive stiffness converts to negative stiffness during deformation and returns to positive stiffness as second state is reached. When

looking to the energy involved in such process, the structure absorbs energy until an unstable point and it releases energy when changing states. This gives the ability of snap-through structures in absorbing-energy and reusable properties. Finally, bistable structures can be combined into multistable structures that can provide different stable configurations with a vast variety of deformation modes, enhancing their energy absorption properties.

2.2.2. ENERGY ABSORPTION PERFORMANCE UNDER DIVERSE LOADING CONDITIONS

When defining the material and the mechanism for absorbing energy in a mechanical metamaterial, it is important first to investigate the nature of the load at which the metamaterial undergoes. Loading conditions can be described by their velocity, which can be classified as quasi-static, low-velocity impacts, and high-velocity impacts. This results in structural responses with distinct strain rates, implying the relevance of considering inertial effects and wave propagation [77].

Quasi-static loads cause relatively low strain rates ($10^{-5} \sim 10 \text{ s}^{-1}$) and inertia effects are negligible, which results in homogeneous deformation mode. In this mode, the deformation and collapse occurs at the weakest point of the structure [78]. In cellular structures formed by struts, the nodes are the weakest point where high stress concentration occurs. For instance, when chiral auxetic structures buckle, the nodes rotate, resulting in large local strains that brings to a structural failure [79]. The energy absorption performance of structures under quasi-static loads are attributed to such localized events.

In case of dynamic loads, cellular structures can exhibit different deformation modes due to the strain-rate and inertia effects [80]. In high-velocity loading conditions (strain rates in the range $10^3 \sim 10^5 \text{ s}^{-1}$), shock modes are identified, where transverse crushing of the structure is observed at the impact front, independently of the weakest structural point [81]. Structures under low-velocity loading conditions, where strain rates with range $10 \sim 10^3 \text{ s}^{-1}$ are observed, experience transition modes, where homogeneous and shock modes can be present. For instance, regular honeycombs under low-impact velocity show double V-shaped bands near the crushing edge and at the support edge. Auxetic sandwich panels under blast loading have also indicated improved energy absorption in comparison to traditional sandwich panels due to the material concentration at the impact zone attributed to the auxetic effect [82]. Fancher et al. (2023) [83] investigated the kinetic energy transmission performance in response to impact of an impactor with different mass and velocity conditions. They concluded that damping can affect significantly the performance, which can be underperform a more traditional materials if the material and impact conditions are not properly selected. For high impact velocity, inertia effect become significant and the V-shaped bands disappear and I-shaped bands are identified at the crushing edge [84]. In case of auxetic structures under quasi-static and low-velocity loads, the crushing stress was related to the angle related to the auxetic effect. However, under high-velocity loads (strain rates in the range $10^3 \sim 10^5 \text{ s}^{-1}$) the crushing stress was not affected by changing the angle [85]. Depending on the impact velocity, the energy absorption ability of mechanical

metamaterials can be attributed to localized events, such as buckling and shear bands, and to global events, such as densification at the impact zone.

2.2.3. UNIQUE PROPERTIES OF MECHANICAL METAMATERIALS

Based on the loading conditions and designed deformation mechanisms described earlier, mechanical metamaterials can exhibit their unique properties, which will be presented in this section.

AUXETIC METAMATERIALS

Metamaterials possessing a negative value of Poisson's ratio, which are also named as auxetic metamaterials, have the interesting phenomenon of contracting (expanding) transversely under longitudinal compression (tension) load [86, 87]. From the classical elasticity theory, the Poisson's ratio is related to other mechanical properties – Young's modulus and shear modulus – by the expression $G = \frac{E}{2(1 + \nu)}$. Therefore, when the Poisson's ratio approaches -1 , an infinite value of shear modulus is obtained. This results in many intrinsic properties, such as indentation resistance [88], thermal shock resistance, and fracture toughness [89]. Other works have also reported high shear resistance [90] and energy absorption [59].

Such auxetic behavior originates from the deformation mechanisms exhibited by the microstructure under uniaxial loading. The most common microstructure geometries are the re-entrant structures and chiral structures, as shown in Fig. 2.3. The deformation mechanism of re-entrant structures [91, 92] consists of mainly folding of their structural elements at the joints. The auxetic effect is tuned by adjusting the geometric parameters of the structure, such as the strut thickness, the length ratio, and the re-entrant angle. In chiral structures, the deformation mechanism is governed by the nodes rotation and cell elements flexure when the structures are under tensile/compressive loadings. For example, chiral honeycomb structures [93–95] achieve this effect through (un)wrapping of the ligaments around the circular nodes under external loads, while rib flexure and/or hinging is observed in star honeycomb structures [96]. Subsequent research has explored new structures to enhance the mechanical properties and energy absorption of auxetic metamaterials. This involves investigating different ligament geometries [97–99] and incorporating additional structural elements into existing re-entrant structures [100], leading to improved auxetic properties.

When analyzing auxetic structures under various loading speeds, it has been observed improvements of energy absorption in re-entrant honeycomb structures in comparison to regular hexagonal honeycomb due to the early densification. However, under high peak stress, the re-entrant structure may generate higher stress to dissipate amount of impact energy [101]. Although many studies presented the effect of energy absorption capacity of auxetic structures, a comprehensive understanding of the relation between the deformation mechanism and macrostructure properties under dynamic loads—especially high-strain rates—is still required.

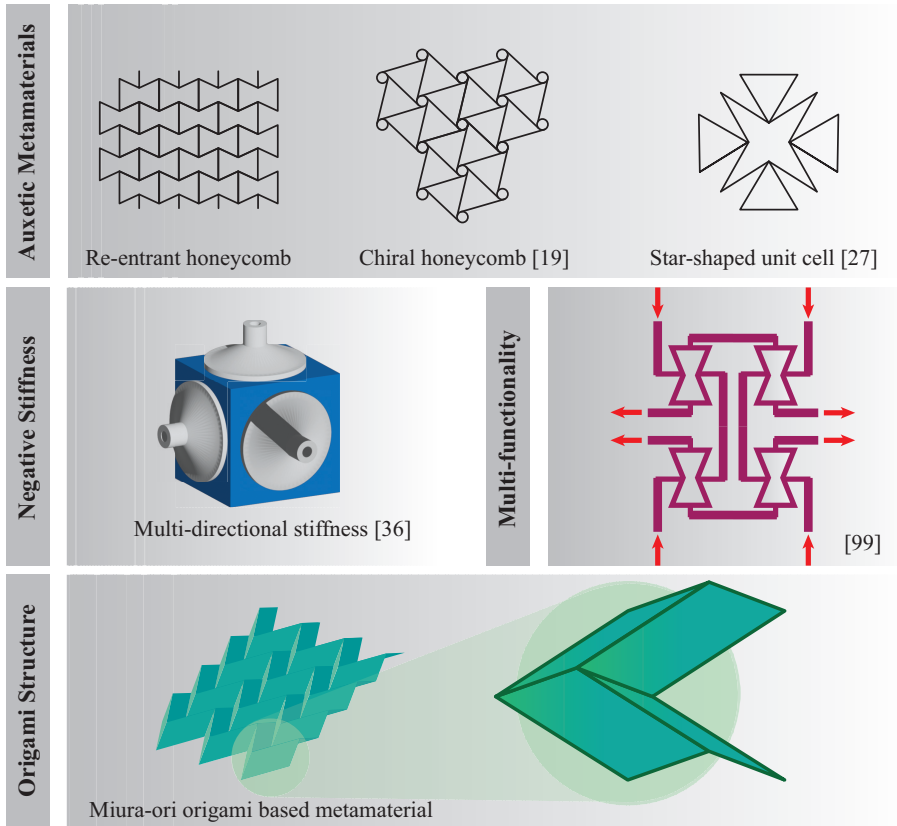


Figure 2.3.: Examples of mechanical metamaterials for energy absorption application.

NEGATIVE STIFFNESS

Metamaterials can also exhibit unique properties through structural instabilities. One such example is **negative stiffness (NS)** metamaterials, characterized as recoverable structures due to their energy absorption relying on elastic deformation of internal structural elements. Typically, negative stiffness metamaterials consist of axially constrained curved beams with snap-through behavior [102], where displacement increases as the force increases until the beams shift from one stable position to another followed by a force level reduction, from which the stored strain energy is released. These beams can be arranged in various configurations, including honeycomb [103, 104], cubic [105], and hexagonal structures [102]. Other structures capable of achieving negative stiffness behavior include disk structures [106, 107], sleeve-type structures [108, 109], among others.

The buckling effect observed in **NS** metamaterials has been explored in impact absorbers. The early models of **NS** metamaterials primarily absorbed energy in one direction and could withstand tensile or compressive loads in a single principal direction. Recent research has concentrated on enhancing **NS** metamaterial features

by expanding their energy-absorbing capacity to multiple directions [105, 110] as shown in Fig. 2.3, their ability of bidirectional buffering and absorbing energy [111], and their adaptability for specific engineering applications [112, 113].

Although improvements on energy absorption capacity have been extensively explored, mostly under controlled loading conditions, an in-depth investigation of the mechanical response of **NS** metamaterials under various loading conditions is necessary. A computational study on the dependence of impact mass and velocity on energy absorption capacity has recently shown that the performance of bistable metamaterials can range from worse to superior, especially when damping is introduced in the model [83]. When dealing with high-strain rate impact problems, material behavior can be crucial since the constituent material of the bistable elements may undergo plasticity and fracture, which could rather decrease the efficiency of the bistable mechanism. To solve such impact problems, real-time tunability of mechanical responses of **NS** metamaterials could be potentially used, since it allows better control of the deformation stages [114].

EXOTIC PROPERTIES OF FOLDABLE STRUCTURES: ORIGAMI, TENSEGRITY, AND PANTOGRAPHIC METAMATERIALS

Origami is a Japanese art of creating 3D objects from folding a paper sheet. This process creates patterns of flexible (crease) and stiff areas (thin panel), which usually results in two stable stages, named as folding and expanding stages. By controlling the number, order and orientation of folds, origami has served as inspiration for designing novel mechanical metamaterials possessing tunable stiffness, multi-stability and auxetic properties [115]. The initial studies on origami-based mechanical metamaterials were based on rigid origami, where the panels remain rigid during folding/unfolding and the crease deforms. Therefore, the strain energy is only related to the crease deformation. Miura-ori and Ron Resch patterns, for instance, have demonstrated negative Poisson's ratio and unusual load bearing capacity, respectively [116–118]. By changing the geometry of the crease and/or panel, the mechanical properties of such rigid origami can be tuned. Although rigid origami has a simple pattern of deformation, its application can be limited. To extend the applicability of origami-based mechanical metamaterials, works have included the flexibility of origami panels, which add the contribution of strain energy of the panel deformation. Zhai et al. (2020) [119] proposed curved origami panels to achieve in situ stiffness manipulation, which could be positive, zero or negative. A detailed experimental validation of the intrinsic properties of an origami-based cellular structure was presented by Yamaguchi et al. (2023) [120]. In-depth analysis is still required to understand the mechanisms behind the exotic features of origami metamaterials [121].

Unlike origami structures, where deformation occurs mainly at the crease, pantographic metamaterials exhibit unique mechanical properties due to the pantographic motif, which is a mechanism characterized by a zero-energy extension/compression deformation mode. This kind of mode is commonly seen, for example, in scissor mechanisms, where two structural elements are connected by rotating or flexible joints [122]. Such metamaterials can undergo large deformations

while maintaining their overall structural integrity, since their mechanism can enable effective load redistribution making them ideal for absorbing mechanical energy and conforming to complex surfaces, such as in smart textiles and wearable electronics [123]. 3D micro-metric pantographic structures have exhibited resilience against damage and different deformation patterns, due to their unique topology, where the recoverable deformation energy was mostly stored in the bending of the beams or in the torsion of the connecting elements [124]. Although the mechanical properties of pantographic metamaterials have been explored for cycling loads, the same is missing for impact waves with varied strain rates.

Tensegrity metamaterials differ from the previous foldable structures due to their unique internal load-transfer mechanism and deformation, which results from selfstressed framework formed by isolated compressive elements (bars) and tensile elements (cables). Under uniaxial compression tests, tensegrity metamaterials exhibited stiffening or softening response [125] and delocalized deformation mechanisms and a stable stress-strain response in comparison to octet architecture and Kelvin foam [126], which gives improved failure resistance and energy absorption in comparison to conventional lattice structures. However, the experimental realization of metamaterials of tensegrity is still in its early stages, particularly due to the complexity of assembling the network of bars and cables. To address this issue, have recently reported tensegrity-like structures, where the bars-cables framework is replaced by only bars framework connected to compliant hinges, giving a monolithic structure that can be fabricated by additive manufacturing techniques [127]. The only-bars framework has been also applied to the fabrication of micro-scale tensegrity structures through the AM technique of multiphoton lithography [128].

2.2.4. MULTI-FUNCTIONAL MECHANICAL METAMATERIAL

Recent advancements in mechanical metamaterials for energy absorption have emphasized the design of structures with multi-functionalities, rendering them suitable for addressing more intricate engineering challenges. This section presents examples of current multi-functional mechanical metamaterials.

Hewage et al. (2016) [129] proposed a double-negative mechanical metamaterial, comprising an auxetic host embedded with negative stiffness elements. This design offers both negative stiffness and negative Poisson's ratio, potentially resulting in improved vibration damping. Inspired by the double negative design reported by Hewage et al. (2016) [129], other works have explored different 2D metamaterial designs displaying multiple negative properties [74, 130]. Some designs have demonstrated a strong coupling between two negative properties, which may affect the energy dissipation efficiency [74]. To overcome that, a realization of double negative in 3D metamaterials was presented by Pan et al. (2024) [131], where a less strong coupling between the negative mechanical properties was induced.

In their study, Zhang et al. (2021) [132] integrated re-entrant and chiral structures to investigate two deformation mechanisms: bending at the re-entrant scale and chirality. This integration led to enhancements in both in-plane effective stiffness and Poisson's ratio. Additionally, Yasuda et al. (2015) [118] proposed combining re-entrant structures with origami, demonstrating analytically and experimentally

both negative Poisson's ratio and structural bistability. In terms of energy absorption for different load conditions, re-entrant circular honeycombs exploring structural hierarchy on the meso-scale and functional gradient on the macro-scale have shown improved performance in comparison to traditional re-entrant structures [133].

2.2.5. APPLICATION OF MMMs IN ENGINEERING PROBLEMS

In this section, we outline the potential applications of MMM mechanisms for absorbing energy. Table 2.1 provides a summary of MMM designs featured in this review, categorized by property, their deformation mechanisms at unit cell level, potential applications, challenges and limitations related to the functionality, and scalability issues.

While several studies have validated the MMM's mechanisms experimentally, most focus on prototypes developed for laboratory testing, often manufactured via 3D printing technology. When adapting such MMM designs to scales differing from the numerical and experimental studies, it is important to consider possible size effects. Such effects could be attributed to geometrical scaling, microstructural effect, anisotropy and non-linearity, boundary interface effects, and manufacturing and defect sensitivity. By investigating these effects, MMMs can be applied to their corresponding potential applications.

2.3. VIBRATION CONTROL THROUGH ACOUSTIC/ELASTIC METAMATERIALS

Acoustic/elastic metamaterials can control vibrations by means of their frequency-dependent effective mechanical properties. At specific frequency ranges, the propagating wave interacts with the AEMM resonant inclusions, causing them to resonate in a certain vibration mode associated to the frequency-dependent mechanical property. The first work demonstrating the vibration control through a resonant mechanism was proposed by Liu et al. [36]. In their work, a composite containing locally resonant structural units showed negative **effective mass density (EMD)** at frequency ranges near the microstructure resonance. The negative EMD is associated to the out-of-phase translation motion between the main structure and its resonant element, which is called dipolar resonance [135]. Similarly to the EMD, negative values of **effective bulk modulus (EBM)** [136] and **effective shear modulus (ESM)** [137] have been also obtained due to monopolar and quadrupolar resonant behaviors of the MM microstructure, respectively. Some works have also reported interesting mechanical properties by specific spring arrangements, as observed in structures possessing **effective Young's modulus (EYM)** [138]. To create ultra-low band gap frequencies by using resonator arrangements, a system formed by heavy masses and low stiffness spring would be required, which could result in unpractical designs. To overcome such limitation, **quasi-zero stiffness (QZS)** and **inertial amplifications (IA)** designs have been proposed to control the effective stiffness and effective mass, respectively. In this section, a brief description of each mechanical property is presented. Spring-mass systems are used to illustrate the mechanism

Table 2.1.: Summary of MMM in terms of deformation mechanism at unit cell level. Their potential engineering applications are highlighted along with the challenges, limitations and scalability issues.

Feature	Deformation mechanism	Potential application	Challenges	Scalability issues
Auxetic behavior	Bending-dominated, stretching-dominated	1) Multi-axial energy absorption [96]	1) Energy absorption investigation restricted to compressive loads	1) Anisotropic behavior for printed structures, due to interlayer defects and pores [100]
	Chirality	2) Improved stiffness and auxetic behavior for larger deformations [100]	2) Energy absorption process relies on plastic deformation, reducing the use to a single application	
Negative stiffness	Elastic buckling due to geometry nonlinearity,	1) Multi-axial energy absorption with self-recovery snap-through response [105, 111]	1) Further investigation of energy absorption under distinct impact conditions	1) Study required for NS lattice structure with sufficient number of cells per side [105] 2) Numerical models cannot predict well asymmetric modes caused by manufacturing imperfections [112] 3) Boundary constraint alters stiffness of outer layers
	Snap-through behavior	2) Energy absorbing feature by using cylindrical structures [112, 113] 3) Development of stretchable devices [113]	2) Damping effects of functionality [112]	
Foldable structures	Crease folding and panel bending; scissor mechanism	1) In-situ stiffness manipulation and curved origami-based grippers [119] 2) Automotive airbags, implants [120], smart textiles, and wearable electronics	1) Manually tunability of stiffness [119] 2) Property evaluation in linear-elastic regime [120]	1) Connection of the panels [119] 2) Planar configurations not suitable for three-dimensional construction
Multi-functional	Snap-through behavior	1) Shape reconfiguration and shock isolation [74] 2) Impact protectors in aerospace, soft robotics [129–131, 134]	1) Require investigation under different loading conditions 2) Large friction coefficient [131]	1) Amount of unit cells not enough for demonstrating scalability

behind the negative phenomenon. Then, special metamaterial designs possessing multiple negative properties are presented.

2.3.1. EFFECTIVE NEGATIVE PROPERTIES

Inspired by the initial evidence of exotic mechanical properties [36, 139], research on negative EMD was pioneered by [140–142]. This property was first theoretically evaluated using a mass-in-mass system connected by springs [143], as illustrated in Fig. 2.4.

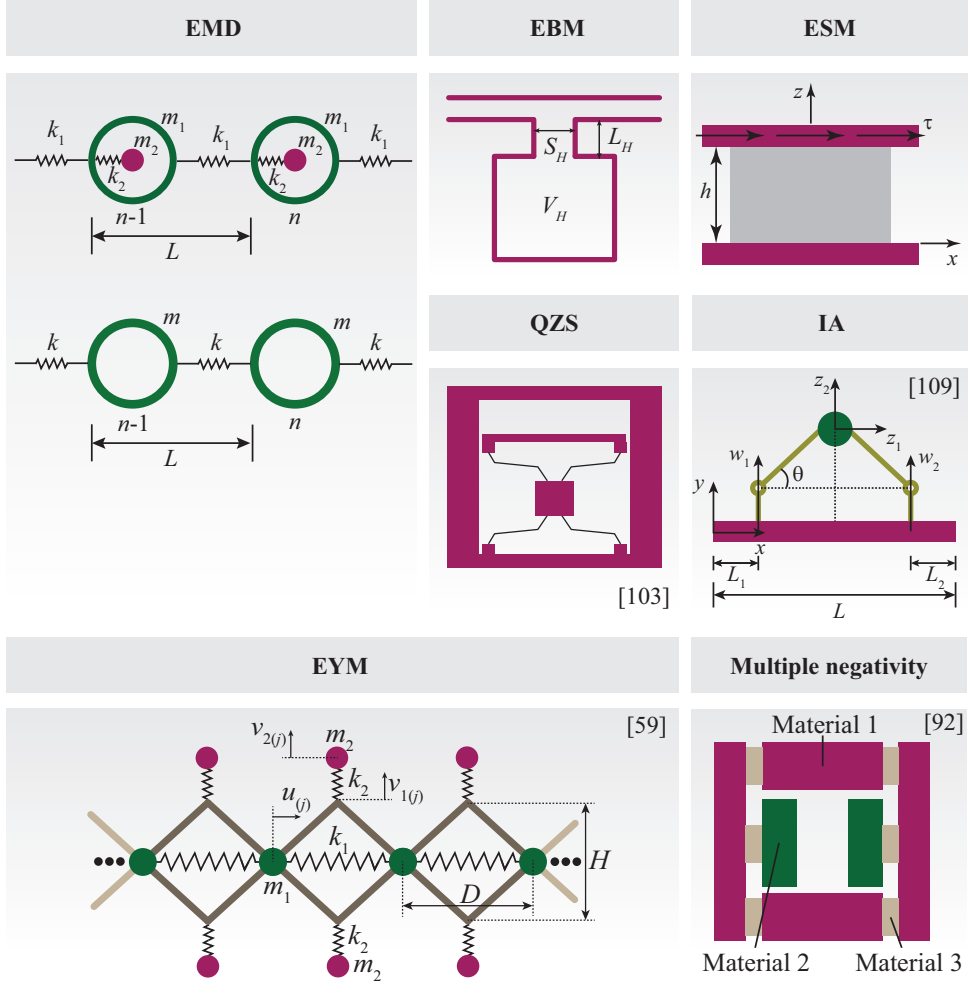


Figure 2.4.: Examples of mechanical metamaterials possessing effective properties for controlling vibrations.

The dynamic behavior of such mass-in-mass systems was further investigated through effective systems whose effective mass m_{eff} is given as

$$m_{\text{eff}} = m_1 + \frac{m_2 \omega_0^2}{\omega_0^2 - \omega^2}, \quad (2.1)$$

where $\omega_0 = \sqrt{k_2/m_2}$ is the local resonance frequency related to the mass m_2 . Figure 2.5a shows the variation of the effective mass over frequency for a mass-in-mass system with properties $m_1 = 1$, $m_2 = 9$, $k_1 = 1$, $k_2 = 0.1$, and $L = 1$. It is worth noting that when the frequency approaches the resonance of the inclusion ($\omega/\omega_0 = 1$), the effective mass becomes negative. Physically, this means that the acceleration of the effective mass is opposite to the direction of the applied excitation, hence the response amplitude is reduced.

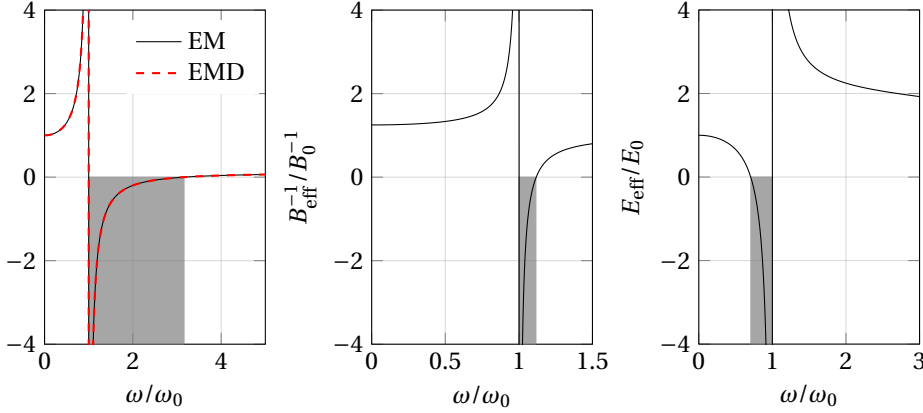


Figure 2.5.: Nondimensional effective mass $m_{\text{eff}}/m_{\text{st}}$ and mass density $\rho_{\text{eff}}/\rho_{\text{st}}$ (left); nondimensional effective bulk modulus $B_{\text{eff}}^{-1}/B_0^{-1}$ (center); and, nondimensional effective Young's modulus E_{eff}/E_0 (right). The gray rectangle indicates the region with negative values of the respective effective properties.

When modeling the equivalent monoatomic system as a elastic solid, the effective mass density is expressed as

$$\frac{\rho_{\text{eff}}}{\rho_s} = \frac{\theta}{\delta(1+\theta)(\omega/\omega_0)^2} \left\{ \cos^{-1} \left\{ 1 - \frac{\delta}{2\theta} \frac{(\omega/\omega_0)^2 [(\omega/\omega_0)^2 - (1+\theta)]}{(\omega/\omega_0)^2 - 1} \right\} \right\}^2, \quad (2.2)$$

where $\theta = m_2/m_1$, $\delta = k_2/k_1$, $\omega_0^2 = k_2/m_2$, and the static mass density $\rho_s = (m_1 + m_2)/L$. Figure 2.5a shows the variation of effective mass density as a function of frequency for the same mass-in-mass system's features. It is also noticed the presence of a negative region at a frequency range nearby the resonance of the mass-in-mass system, which is characterized by the out-of-phase motion of the system with respect to the excitation. Since the longitudinal wave speed is given by $c_L = \sqrt{E/\rho}$, a negative value of ρ results in a complex wave speed, which physically means that the wave attenuates.

Building upon findings from simple discrete models, several continuum designs have been developed to attain the EMD. These designs employ various approaches, including impedance mismatch by pairing a stiff core with a soft material [144] and spring-mass systems [145]. Alternatively, single-phase unit cells are explored to achieve EMD through the microstructure topology of the unit cell [8, 146].

External resonators, such as metamaterial plates formed by resonant pillars [147, 148], have also been utilized to create EMD, particularly for waveguiding and confinement applications. Anisotropic EMD MMs, which contain independent EMD values for each propagation direction, have been investigated for converting wave types [149] and mitigating waves with different orientations [150]. While metamaterials possessing negative EMD have made significant contributions to wave attenuation, the limited range of negative EMD in specific designs, as depicted in Fig. 2.5), may constrain their applicability to problems involving waves with multiple frequencies, for instance, impact loads. To address this constraint, strategies such as grading resonant structures and exploring parallel arrangements of resonators have been proposed to broaden the attenuation bandwidth [8, 144, 146, 151].

Elastic materials exhibit a positive bulk modulus, representing their resistance to compression/expansion. In case of local resonant metamaterials, the interaction between propagating wave and resonant subwavelength elements can yield a frequency-dependent effective bulk modulus. Fang et al. (2006) [136] initially observed negative EBM in an acoustic metamaterial composed of Helmholtz resonators, leading to out-of-phase bulk mode propagation. Considering a single Helmholtz resonator of volume V_H , neck size S_H and effective length L_H , as illustrated in Fig. 2.4b, the EBM B_{eff} is defined as [152]

$$B_{\text{eff}}^{-1} = B_0^{-1} \left(1 + \frac{\omega_1^2 - \omega_0^2}{\omega_0^2 - \omega^2} \right), \quad (2.3)$$

where $\omega_1 = \omega_0 \sqrt{(1 + V_H/V)}$ and ω_0 is the resonant frequency of the Helmholtz cavity defined as

$$\omega_0 = \sqrt{\frac{B_0 S_H}{V_H \rho_0 L_H}}, \quad (2.4)$$

being B_0 and ρ_0 the bulk modulus and density of the air in the tube. Similarly to the negative EMD, the EBM can be also explained by a spring-mass system, where the fluid in the neck works as an effective oscillating mass and the fluid in the chamber body acts as an effective spring. As illustrated in Fig. 2.5b, the EBM becomes negative close to the resonance.

While EBM has predominantly been studied in acoustic contexts, some studies reported elastic metamaterials engineered to achieve negative bulk modulus through rotational resonance of the unit cell. EBM in elastic metamaterials has contributed significantly to wave filtering applications [153–155].

The negative EYM was theoretically proposed by Huang and Sun. (2011) [138]. Their model consists of a resonator chain as presented in Fig. 2.4, where a spring with stiffness k_1 and length D is connected to four rigid rods whose mass were concentrated at the mass m_1 . The perpendicular responses are given by two sets of mass-spring systems with elastic constant k_2 and mass m_2 connected to the rods. The effective Young's modulus of this system is given by

$$\frac{E_{\text{eff}}}{E_0} = \frac{\delta}{\theta} \eta^2 \left\{ \cos^{-1} \left[1 - \frac{(\eta^2 - 1)\eta^2}{2[(\theta/\delta + \theta\mu/2)\eta^2 - \theta/\delta]} \right] \right\}^{-2}. \quad (2.5)$$

where $\theta = m_2/m_1$, $\delta = k_2/k_1$, $\mu = (D/H)^2$, and $\eta = \omega/\omega_0$. Figure 2.5c illustrates the relation between the nondimensional frequency and the EYM for a lattice structure with properties $\theta = 2$, $\delta = 0.5$, and $\mu = 4$, where a negative EYM is also observed close to the resonance.

Inspired by the effective Young's modulus model proposed by Huang and Sun (2011) [138], new MMs have been developed to suppress the propagation of longitudinal [156] and flexural waves [157] in continuum structures by transforming the direction of wave propagation, which have been fundamental for controlling vibration in common engineering structures, such as beams and plates.

Different from acoustic metamaterials, from which the acoustic medium is defined by the effective mass density and bulk modulus, the elastic counterparts have an additional parameter, the effective shear modulus (ESM), since the solid medium supports both longitudinal and transversal waves. As the negativity of the effective mass density and bulk modulus was result of an anti-parallel motion between the inclusions and the matrix, the similar behavior occurs for negative shear modulus MMs. Fig. 2.4 shows an elastic material under a shear test where the bottom surface is fixed and a shear stress τ acts on the top surface, considering an infinite long elastic material, the shear modulus μ_{eff} can be expressed as

$$\mu_{\text{eff}} = \tau \frac{z}{u_x(z)} = \tau \frac{h}{U_x}, \quad (2.6)$$

where $u_x(z)$ is the displacement of a material layer at z along the x direction, and $U_x = u_x(h)$ is the top surface displacement. As observed in Eq. 2.6, a negative shear modulus is obtained when the direction of the displacement field of the material layer at z is anti-parallel to the direction of the applied shear stress. When it comes to dynamics, some works have derived the effective shear modulus based on effective medium and analytical models [137, 158, 159]. The property of negative shear modulus can be widely explored as selective wave filtering. For instance, Wang et al. (2020) [160] has demonstrated the control of shear waves by exploring the torsional resonance of an elastic metamaterial plate with a stub on one side.

2.3.2. AEMMs WITH MULTIPLE NEGATIVITY

Researchers have delved into understanding various resonant mechanisms to combine effective properties within the same frequency range, leading to a negative refractive index. This results in an anti-parallel phase and group velocities. The initial designs of MMs showcasing double negativity involved a combination of solid and fluid media [135, 161, 162]. However, these designs were complex to manufacture, prompting the development of double-negative metamaterials using only solid materials (Fig. 2.4). This was initially performed by developing theoretical models [163], followed by proposals for more compact designs aiming to achieve simultaneous negative effective mass density and negative bulk modulus through chiral structures [153, 164, 165]. Such structures induce rotational and translational resonance, thereby producing negative effective bulk modulus and negative effective mass density at specific frequencies. Different unit cells have been suggested to independently regulate the negative mass density and the negative modulus [151,

154, 155, 166]. Due to the strong coupling among different deformation modes in elastic medium, the independent tuning of density and stiffness modulus becomes more complex. To address such issue, Oh et al. (2016) [167] introduced an elastic metamaterial made of two independent resonators each of which realizing negative density and stiffness.

In elastic MMs, achieving double negativity also occurs when both the mass density and shear modulus are negative within specific frequency ranges [160, 168, 169], leading to dipolar and quadrupolar resonances. Thus, a negative transverse wave velocity is obtained, i.e. $c_t = \sqrt{\mu}(\sqrt{1/\rho}) < 0$, implying a negative refraction. This property could be used for mode conversion from transversal waves to longitudinal waves. Further studies also reported triple negativity – negative EMD, EBM, and ESM – at the same frequency range [170], which provides various dynamic features [171], such as fluid-like behavior and negative phase velocity for longitudinal and transversal waves. Therefore, having control of the effective parameters of elastic metamaterials enables the control of hybrid waves by means of polarized band gaps [172].

2.3.3. QUASI-ZERO STIFFNESS

Previously discussed negative effective properties arose from simple mass-spring systems, resulting in low-frequency band gaps. However, achieving ultra-low band gaps in such systems required a heavy mass connected to a low-stiffness spring, which poses impractical design challenges. To address this, researchers have explored resonator designs with extremely low stiffness while maintaining structural strength. Some studies have introduced high-static-low-dynamic-stiffness resonators, generating band gaps at very low frequencies [173, 174]. The high-static-low-dynamic-stiffness property involves a negative stiffness mechanism counteracted by a positive stiffness system. While previous designs employed spring-mass systems to achieve high-static-low-dynamic-stiffness [173–175], recent research by Cai et al. (2020) [176] investigated negative stiffness systems using compact bistable elastic structures. Additionally, compact QZS metamaterials have been developed using resonators comprising programmable curved beams [134, 177], magnet rings and coil springs [178, 179], and parallel multi-segment beam systems [180–182] (Fig. 2.4).

2.3.4. INERTIAL AMPLIFIERS

Another approach to reduce the band gap involves amplifying the effective mass of a component through displacement amplification mechanisms, also known as IA (Fig. 2.4g). The amplified motion generates enhanced inertial forces, thereby elevating the effective mass of the system and consequently lowering the resonance frequency. Discrete models have been utilized to elucidate this mechanism in both infinite [183] and finite media [184]. Building upon this concept, an extension to continuum models was proposed by [185–187]. Another mechanism of inertia amplification involves employing chiral links to couple the axial and rotational motion of metastructure elements, resulting in an additional inertia contribution without altering the total mass [188, 189]. Further chiral inertial amplification

structures with distinct unit cell arrangements were investigated, from which syndiotactic arrangement exhibit ultra-broad low-frequency band gap, although still limited by the stiffness and mass of the structure [190]. Novel chiral structures are being investigated to break the mass and low-frequency band gap relation [191].

2.3.5. APPLICATION OF AEMMs IN ENGINEERING PROBLEMS

In this section, we explore the practical applications of the numerical and experimental analysis of wave control mechanisms in AEMMs. Table 2.2 categorizes AEMM models and prototypes, similar to the previous section. Notably, experimental validation is limited, primarily focused on small-scale unit cell designs, while numerical investigations have tackled larger engineering problems. The main challenges include managing geometry complexity, tuning dynamic properties, and addressing nonlinearities, which are crucial considerations for future mechanical metamaterial designs.

2.4. FUTURE DIRECTION OF MECHANICAL METAMATERIALS: HYBRID METAMATERIALS FOR SIMULTANEOUS VIBRATION CONTROL AND ENERGY ABSORPTION

The demand for single integrated metamaterials that exhibit multiple functionalities allowing improved mechanical performance and vibration control is increasing nowadays to solve complex engineering problems. With the advances in computational science (optimization schemes [194] and inverse design by artificial intelligence) and additive manufacturing technology, multi-functional metamaterials, termed as hybrid mechanical metamaterials, are being developed to attain the static and dynamic properties discussed in the previous sections. To understand the development of HMMMs, this section summarizes the most recent design processes employed for obtaining the multi-functionality of HMMMs. We highlight the limitations of such design processes and potential solutions to overcome them. Finally, the potential application of HMMMs in real-world problems is discussed.

2.4.1. DESIGN PROCESSES OF HYBRID MECHANICAL METAMATERIALS

The multi-functionality of HMMMs can be attained by two main design processes: the forward and the inverse design. The steps of each process are indicated in Fig. 2.6.

FORWARD DESIGN PROCESS

The forward design of HMMMs has been performed by two approaches: step-based and optimization-based approach. In the first, a structural design and its design variables are initially defined. The structural design is commonly chosen from traditional, multi-scale and bio-inspired designs, which will be described later. Numerical or experimental simulations are performed to quantify each functionality. In case one of the desired functionalities is not achieved, the structural design is

Table 2.2.: Summary of **AEMMs** in terms of desired dynamic property. Their typical band gap principles are highlighted along with the potential engineering applications, challenges, limitations and scalability issues.

Feature	Band gap principle	Potential application	Challenges	Scalability issues
EMD	nonlinearity due to geometric change [145], flexural mode excitation [8, 144, 146], axial and shear rod deformation [8]	vibration attenuation at quasi-static frequency [145], non-destructive evaluation and structural health monitoring [148], wave redirection mechanisms [150], seismic shields [192]	Material damping effect on attenuation values [146],	Friction effect for small-scale nonlinear metamaterial [145], perfect interface coupling assumed in numerical models
EYM	Inner resonance due to stiffness contrast [156], interaction between four-link mechanism and lateral resonators [157]	Device for controlling the direction of transmitted elastic waves [156]; wave conversion [157]	Damping effect reflecting on resonance performance [157]	Principle of wave control based on homogenization method, therefore wave length should be larger than unit cell size [156]
QZS	Energy transfer from metamaterial to energy source [179], combination of negative- and positive-stiffness structure [176], large deformation of compliant structure [180]	Ultralow vibration isolation (lower than 20 Hz) [179, 182]	Damping effect reduces wave attenuation [182]; enhanced stiffness nonlinearity can degrade wave attenuation [176, 180]; manufacturing imperfections shift band gap [182]	Band gap is only broadened when scaling up the mass and stiffness of the resonator [181]
IA	Increase of effective inertia by generating rotational motion from axial motion [188, 189]	Isolate seismic surface waves [187]	Band gap induced by IA is quite narrow [187]; geometry relies on thin structures, requiring stress evaluation [189]	Geometry of the ligaments should be changed to shift the stop band, which may induce different behaviors for distinct scales [189]
Double	translational motion of the resonator [151, 166], combination of torsional, bending and compressional resonance [160]	Polarization filtering, abnormal refraction, and mode conversion (elastic switches and seismic waves control) [160, 164, 165, 167]; structural protection [193]	Broad band gap induced by inclusion of more resonators, increasing geometry complexity; complex coupling of longitudinal and shear waves in 2D and 3D problems	Sensitivity of negative refraction to imperfections can be significant when cell size approaches the wavelength of propagating wave [164]; boundary and material inhomogeneity effects on band gap prediction

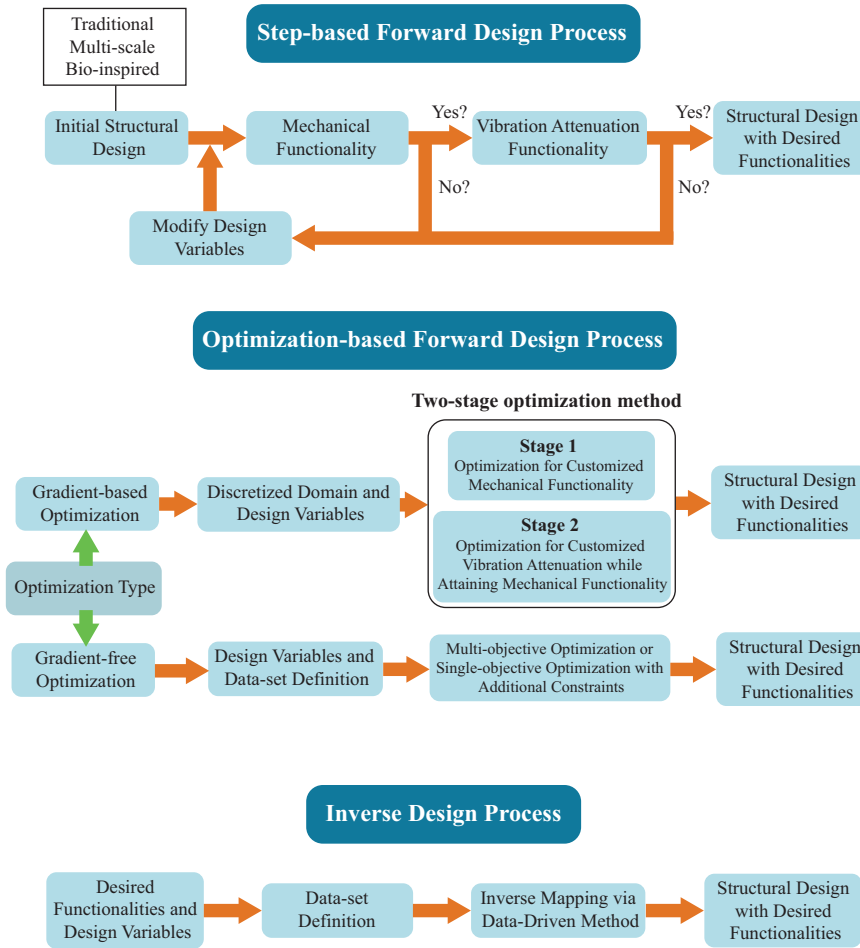


Figure 2.6.: Design processes of hybrid mechanical metamaterials.

changed by modifying its design variables from which new studies are performed. This process repeats until a structural design with the desired functionalities is obtained.

In step-based forward design, the initial structural design often uses traditional geometry, focusing on evaluating the dynamic or static properties of metamaterials tailored for their respective purposes. The performance of traditional mechanical metamaterials were initially focused on their ability to support quasi-static, low-velocity, and high-velocity impact loads, as we highlighted to be dependent on their deformation modes. However, it was just recently that studies were performed to relate the deformation process with the vibration characteristics of the metamaterial. Some works have shown that band gap tuning can be achieved by the different deformation states of NS metamaterials [79, 195–197]. Chirality has been

used to induce tunable phonon band gaps in 3D micropolar metamaterials [198]. The development of dynamic mechanical properties based on auxetic metamaterials has been possible by investigating the deformation modes of their unique topology – the local rotation of nodes and bending of ligaments can produce wave mitigation features. For instance, Chen et al. (2017) [199] showed that deformation modes of lattice metamaterials formed by sinusoidal-shaped beams with negative Poisson's effect result in tuned band gaps. Both properties were attributed to the sinusoidal shape of the lattice beams. Tao et al. (2022) [200] explores the tunability of acoustic resonant band gaps via the auxetic behavior of a negative Poisson's ratio structure. It was shown that by changing the compressive strain the width and position of the band gap is shifted. In other applications, band gaps within specific frequency ranges must be preserved under deformation. Such response has been achieved in micro-scale mechanical metamaterials exhibiting two distinct configurations and positive and negative Poisson's ratio [201]. Band gap tunability has been also observed in foldable metamaterials. Xu et al. (2021) [202] presented an origami-based solution for controlling torsional vibration in beams, employing a bistable resonator made of Kresling-pattern origami. The compression-torsional coupling behavior of Kresling origami was leveraged to create torsional band gaps. The same coupling behavior has been used in a graded origami-inspired structure made of beams and plates, whose dynamic features involve programmable, directional wave control [203]. Furthermore, a metamaterial consisting of various Kresling origami modules, leading to different deformation stages, has been designed [204]. The several deformation stages were able to open band gap in different frequency ranges, which exemplify the tunability of such structures. Static compression of tensegrity-inspired lattices structures also induce band gap tunability [205, 206], while the same ability has been explored by varying geometric parameters of flexible hinges in pantographic metamaterials [207]. In these studies, the metamaterial is subjected to compressive static loads, from which the deformation stages are obtained for different strain values. The dispersion characteristics of the metamaterial at each deformation stage is obtained, which usually give band gaps at distinct frequency ranges. This gives the metamaterial the ability of tuning band gaps when under static loads, while the same ability for larger strain rates still needs to be explored.

Multi-scale (hierarchical) approaches have been also used as initial structural designs, from which each functionality is attributed to structures of different scales. Zhu et al. (2014) [208] reported a chiral-lattice-based mechanical metamaterial beam embedded with multiple local resonators to achieve broadband vibration attenuation while keeping its load-bearing capacity. Qi et al. (2019) [209] proposed a hybrid latticed formed by a re-entrant and an anti-chiral structure. The vibration attenuation was realized by adding rubber coated mass inclusions into the local nodes of the hybrid lattice. Other nodal shapes resulted in ultralight lattice structures possessing negative Poisson's ratio and vibration control performance [210]. Guo et al. (2022) [211] proposed a lattice truss core sandwich metastructure exhibiting coupled Bragg scattering and local resonance mechanism. Since the resonator periodicity is different from that of the host lattice structure, the metastructure can be modeled as a multiscale structure. The analysis of such structure is performed via a multiscale

modeling method based on the combination of the dynamic homogenization method and the spectral element method. The metastructure achieved a wide low-frequency band gap using less local resonators compared with traditional metastructures, which gives a high load-bearing capacity. As an alternative for the high additional mass ratio requirement for low-frequency attenuation, the mass component in the macroscopic metamaterial is replaced by microscopic particle materials. The multi-scale design showed broader attenuation while retaining the lightweight feature [212]. In terms of enhancing energy absorption, Zhou et al. (2022) [213] performed impact testings with distinct load conditions on re-entrant structures embedded with local resonators. The experimental results showed that the impact mitigation capacity of the local resonant re-entrant structure was approximately 22.6 % better than the conventional re-entrant structure. Dual-functional hierarchical mechanical metamaterials with vibration insulation and energy absorption features for quasi-static compression was proposed by Zhang et al. (2022) [214]. Recently, Zheng et al (2024) [215] investigated the energy absorption performance of a multi-resonator honeycomb metamaterial, where a reduction of 80 % of reaction force due to a blast load was observed, although crushing stress levels have not been reported. Hierarchical metamaterials have also been developed to attain band gap tunability. For instance, small re-entrant unit cells are added to the nodes of re-entrant structures to obtain a tunable band gap. The band gaps were attributed to the deformation mode of the small re-entrant designs, while tunability was achieved by applying compressive (tensile) strain to the overall structure [216]. In the same direction, Ma et al. (2023) [217] added square unit cells with mass inclusion to the nodes of the re-entrant structure. The band gap tunability was explored by changing the sizes of the resonators. Experimental studies were conducted in hexachiral local resonant metamaterial to demonstrate improved mechanical and vibration attenuation performance by exploring the deformation modes of the chiral frame [218]. In foldable metamaterials, Jiang et al. (2023) [219] proposed a tunable hybrid origami metamaterial by integrating local resonators into traditional Miura origami panels. The proposed metamaterial demonstrated improved wave mitigation capabilities, with a 25.7 % reduction in reaction peaks compared to traditional Miura origami structures.

The unique characteristics exhibited by biological structures provide insights for designing novel mechanical metamaterials [220]. Several works have proposed energy-absorbing materials and structures inspired by animal bones, shells, and wings or leaves of plants and mushroom gills [221, 222]. To account for both impact energy absorption and impact-induced stress wave attenuation, composites inspired by the beak of woodpeckers were designed, where distributed soft inclusions were embedded in hard matrices [223]. Nacre-like materials have shown enhanced mechanical properties due to an alternating pattern of soft protein layers and stiff aragonite tablets [224]. An extended analysis of their vibrational response was performed, revealing that the incorporating hollow platelets together with the application of specific prestress states result in improved wave absorption capability. Tensegrity metamaterials resembling biological tissues have demonstrated tailored static and dynamic properties by tuning the topology, geometry, and induced prestress [225].

While the step-based forward design is straightforward, it often requires multiple iterations to achieve the desired functionalities. This process becomes increasingly time-consuming when additional functionalities are introduced or when handling complex 3D geometries with numerous design variables. To address these challenges, topology optimization has been integrated into the forward design stage, enabling systematic identification of the optimal material distribution for maximum performance within design constraints [226].

The optimization process has been performed via gradient-informed and gradient-free methods, as indicated in Fig. 2.6. Gradient-informed methods use derivatives of the objective function with respect to the design variables to evaluate how small changes in the design variables affect the objective function and constraints. Solid isotropic material with penalization [227], level set [228], and evolutionary structural optimization [229] are some examples of gradient-based methods that have been applied for optimizing metamaterial designs to provide improved static or dynamic properties. Simultaneously optimizing metamaterials with static and dynamic properties increases the complexity of the problem, necessitating the exploration of new optimization methodologies. Recently, a two-stage topology optimization methodology was proposed for designing multifunctional metamaterials with simultaneous negative thermal expansion and phononic band gap [230], and for designing hybrid metamaterials with simultaneous negative Poisson's ratio and low-frequency band gap [231]. For the latter, unit cells with negative Poisson's ratio are initially determined by optimizing the topological distribution of the matrix material region, while the band gap issue is disregarded. In the second stage, the Poisson's ratio and band gap – induced by adding stiff and soft elements – are considered simultaneously. Here, the band gap objective function was converted into constraints on geometric parameters, reducing the problem to single-objective optimization. Strict constraints and objectives are then applied to refine the Poisson's ratio. This two-stage optimization offers a promising framework that could be extended to other mechanical functionalities.

In contrast, gradient-free optimization bypasses gradient information, relying instead on heuristic methods, random sampling, or direct search strategies to explore the design space. The optimization starts by defining a set of designs represented as discretized domains that respect imposed geometric constraints. Objective functions are evaluated for each design, from which gradient-free methods are employed to update the design variables iteratively. Examples of gradient-free methods used for optimizing mechanical metamaterial designs are genetic algorithms [232], particle swarm optimization [233], simulated annealing [234], and Nelder-Mead method [235]. For simultaneous optimization of mechanical and vibration attenuation properties, single-objective optimization with additional constraints can be applied, whose approach is similar to the second stage of the two-stage optimization method. Another approach is using multi-objective topology optimization, where objective functions related to each desired functionality are derived. As vibration attenuation and mechanical improvement could be two contradictory objective functions, the optimization provides a set of Pareto-front solutions that identifies such tradeoff behavior. The optimization iteratively uses the operations of the gradient-free

methods until an optimal Pareto-front solution is obtained, which gives the set of optimized designs with desired functionalities. Such approach has been used to customize in-plane mechanical properties and broadband band gaps via genetic algorithms [232, 236]. Since massive amount of designs is evaluated per objective functions, efforts have been made to mitigate the computational time issue, mainly through parallel computing and model simplifications, such as plain stress and plain strain assumptions, reduced order modeling [237], surrogate models [238], and homogenization techniques [239].

INVERSE DESIGN PROCESS

The step-based and optimization-based forward design processes have in common a creation of a mapping between structural design and desired functionalities. As an alternative for modeling mechanical metamaterials, inverse design approaches have been used to map directly the desired functionalities to the mechanical metamaterial geometry, as indicated in Fig. 2.6 [240–244]. This is performed via two neural networks; the first maps geometries into mechanical properties and the second maps mechanical properties into geometries [245]. Data-driven models, such as machine and deep learning, have been applied to link the relation between dispersion curves and structure topology of elastic metamaterials [242], where the forward prediction is performed via [convolutional neural network \(CNN\)](#)s and inverse prediction is performed via conditional [generative adversarial network \(GAN\)](#) (cGANs). When designing hybrid mechanical metamaterials, the mapping dimensionality increases, since a link among static and dynamic properties and structural topology is required. Therefore, more complex inverse design approaches, such as multi-network systems built based on the topology optimization data set [246] could be employed, although with a cost of computational time [247]. In such systems, structural database with distinct material properties is defined by using multi-objective topology optimization algorithm. The generated data set is used to build the multi-network system formed by three networks—GAN, [property prediction network \(PPN\)](#), and [structure generation network \(SGN\)](#)—that will realize the performance evaluation and the structural customization. With respect to [HMMMs](#), the structural database with distinct functionalities could be defined through the multi-objective optimization and further used on the multi-network system.

2.4.2. FURTHER ADVANCES IN DESIGNING HMMMS

The [HMMM](#) designs presented previously possess multi-functionalities that cannot be modulated after manufacturing, which could limit their application in case they are exposed to varied external factors, such as different loading conditions or change on vibration bands. Incorporating elements that respond to external stimuli into the metamaterial design allow for active control of static and dynamic properties after fabrication, which could be programmable for specific operational conditions. For instance, metamaterials fabricated with smart materials, such as [shape memory polymer \(SMP\)](#) offer excellent environmental adaptability and high deformation capacity [248–250]. Topological transformation through heat stimulus

in active lattice structures was observed, enabling switching between stretching- and bending-dominated topology [251]. This topological transformation combined with metamaterials with programmable coefficient of thermal expansion [252, 253] could result in unique mechanical performance. Some studies have also demonstrated the tunability of dynamic features through external stimuli, including magnetic field [198, 254, 255], variation of thermal conditions [249, 256], pre-stress induced by shunted piezoelectric patches [257–262]. For instance, programmable deformation configurations are enabled when combining soft materials with hard-magnetic particles [263]. The different deformation modes resulted from changing of magnetic field and mechanical loading causes the shifting of band gaps [255]. By incorporating smart materials like shape memory polymers or using external factors such as magnetic fields, heat, or pre-stress, these metamaterials can achieve programmable deformation and tunable performance for specific operational conditions. This adaptability enhances their functionality, making them suitable for a wide range of applications where environmental conditions or operational requirements may vary.

Rapid advancements in manufacturing processes enable the integration of such active elements, leading to more intelligent and unified designs. The advent of 3D printing technology significantly broadened mechanical metamaterial design possibilities [264], allowing the exploration of new mechanical properties in designs ranging from micro- to macro-scale [99, 128, 265, 266]. Emerging 4D printing technology [264, 267] integrates multi-physics properties into manufacturing, enabling passive metamaterial features to become active and programmable [268]. These advancements in additive manufacturing will facilitate the rapid implementation of hybrid mechanical metamaterials to address real-world engineering challenges.

2.4.3. HYBRID MECHANICAL METAMATERIALS AS SOLUTION FOR ENGINEERING PROBLEMS

The development of hybrid mechanical metamaterials has been motivated by the increase of complexity of engineering problems combined with the advances in computational design methods and manufacturing process. From the unique properties developed by HMMM designs, their potential applications in engineering problems can be drawn and categorized as shown in Table 2.3.

Table 2.3.: Potential application of HMMs in engineering problems in terms of their unique features.

Feature	Static Property	Dynamic Property	Application
Vibration damping and isolation	Load-bearing capacity	Tunable band gap	1) Infrastructure design: mitigate vibrations caused by seismic activity or machinery [269] 2) Automotive and aerospace: isolate sensitive components from engine vibrations or mechanical shocks [270] 3) Machinery and equipment: in industrial settings for reducing vibrational impact on sensitive tools, enhancing precision [271]
Sound insulation and control	Structural integrity	Attenuation or redirection of sound waves	1) Noise barriers: In urban environments and around highways to reduce noise pollution [272] 2) Underwater acoustics: reduction of noise emission during offshore foundation installation [273], while keeping resistance to high hydrostatic pressure
Wave manipulation and energy harvesting	High durability	Wave propagation control for guiding, trapping or focusing energy	1) Energy harvesting from vibrations: In bridges, railways, buildings, or vehicles where the metamaterial converts mechanical vibrations into electrical energy [274]
Impact and shock absorption	High strength, flexibility or lightweight structure to absorb static loads	High energy absorption capacity under dynamic impacts or shocks	1) Personal protective equipment (PPE): helmets, body armor, and protective clothing that absorb impacts while remaining lightweight and flexible [275] 2) Crash-resistant structures: in automotive and aerospace industries to absorb energy during collisions, protecting passengers and sensitive components [276] 3) Sports equipment: helmets, pads, and shoes to enhance performance and safety by absorbing impact forces [201]
Biomedicine	Structural support and stiffness for implants or prosthetics	Tunable mechanical responses and absorb dynamic loads	1) Bone implants: hybrid metamaterials designed to mimic the mechanical properties of natural bone, combining strength with dynamic load absorption [277] 2) Wearable medical devices: flexible, adaptive materials for devices that monitor and respond to the body's mechanical movements [278]

2.5. OUTLOOK

Beginning with the first local resonant MM presented by Liu et al. (2000) [36] to contemporary hybrid metamaterial designs, there is a discernible trajectory towards increasingly single integrated systems. This trend is poised to persist in the next generation of hybrid metamaterials, albeit with the introduction of new

elements aimed at achieving not only multifunctionality but also multi-physics and adaptability capabilities. In essence, the evolution leads towards the development of intelligent **HMMM**s. Expanding functionalities and incorporating additional physics to the **MM** models will increase the complexity of theoretical models, numerical simulations, and manufacturing processes. Therefore, future advancements are anticipated to focus on enhancing algorithms and machine learning tools to manage this complexity effectively.

One prospective avenue for intelligent **HMMM**s involves their integration into real-world applications. Current research has predominantly focused on verifying the functionality of metamaterial designs through numerical simulations, such as a metamaterial-based cushion for noise reduction in offshore installations [273] or laboratory experimental validation [279–281]. Broader implementation of the next **HMMM**s in real-world applications hinges on simplifying manufacturing operations. Future investigations could explore various manufacturing techniques, such as 3D printing, laser cutting, metal stamping, or metal forming, to incorporate their mechanical properties, while active and programmable features could be achieved through 4D printing. Furthermore, attention should be directed towards the life cycle of intelligent **HMMM**s, aiming to meet engineering demand with minimal impact. Streamlining manufacturing processes will accelerate the integration of intelligent **HMMM**s into real-world applications, such as engineering problems faced on railways, buildings, and underwater and aerospace devices.

2.6. CONCLUSIONS

Metamaterials (**MM**s) have made significant contributions to vibration control through the manipulation of stiffness-mass ratios in their microstructures – referred to as acoustic/elastic metamaterials (**AEMM**s) – and to energy absorption, by taking advantage of deformation modes at certain load velocity conditions, mainly quasi-static and low-velocity impacts, – named as mechanical metamaterials (**MMM**s). However, designing **MM**s with complex geometries for single tasks may not be conducive to real-world applications, as unforeseen events could necessitate redesigning such **MM**s with new properties. Additionally, **MM**s are often designed under controlled environment conditions, which may not accurately reflect the harsh conditions they could encounter in real-world applications. These limitations have prompted researchers to explore novel designs with adaptable multi-functionalities, leading to the emergence of hybrid mechanical metamaterials (**HMMM**s).

This review sheds light on the development of **HMMM** designs, which combine the dynamic features of **AEMM**s with the unconventional static properties of **MMM**s into a single integrated system. This integration allows for structures that can perform their functions under varying external conditions, although analysis on the effect of high-strain rates on **HMMM**s is still unexplored. With advancements in computational methods and manufacturing techniques, we anticipate the integration of additional functions from diverse physical principles into future **HMMM**s. These advancements aim to deliver multidisciplinary, adaptable and fast performance, leading to the emergence of what we term as intelligent **HMMM**s. Such

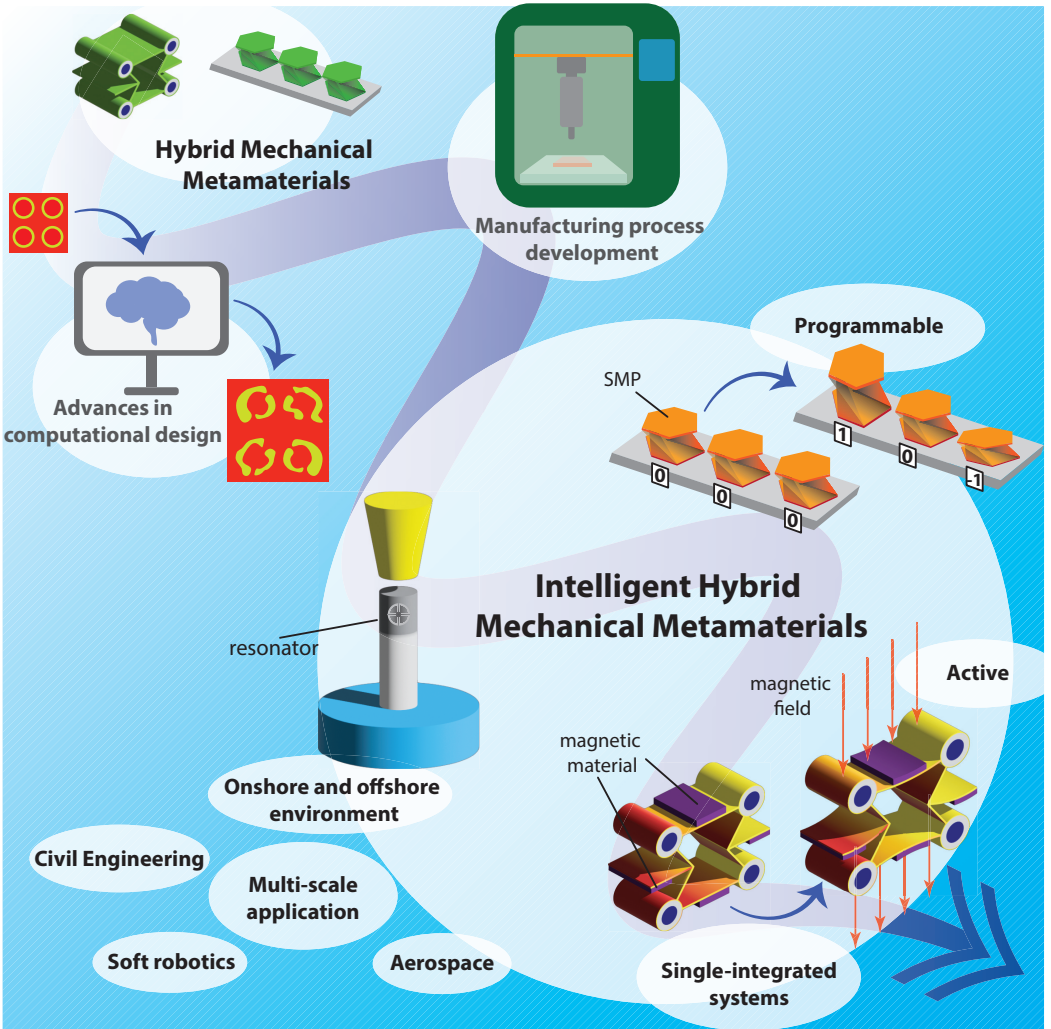


Figure 2.7.: The next generation of hybrid mechanical metamaterials road map.

enhancements in [HMMM](#) designs will have the potential to overcome resistance to their implementation in real-world applications.

3

DESIGN OF THE META-CUSHION

In the previous chapter, the main metamaterials mechanisms for attenuating vibrations and absorbing energy were discussed followed by the explanation of their design processes. This gives a full glance of how metamaterials can be designed to address a specific application.

In this chapter, the underwater noise problem is initially investigated, from which a numerical model for predicting the noise levels is developed. The frequencies where high noise occurs are identified and used for designing the meta-cushion. The attenuation mechanism is explored via analyses of dispersion curve and transmission loss diagrams. The meta-cushion functionality is finally compared to a cushion without metamaterial elements. This chapter is a transcription of the journal paper published in *Engineering Structures*².

3.1. INTRODUCTION

Most offshore wind turbines are installed on top of monopiles, which are the foundation of choice (80.5% in 2020) due to their straightforward manufacturing [3]. However, monopiles are driven into the ground by hammering, which results in sound pressure waves that propagate for distances in order of kilometers in seawater that cause injuries to marine life [11–13]. To mitigate the high noise levels produced by monopile driving, several mechanisms have been proposed, such as enclosing the pile with casings [28], air-bubble curtains [282], and a combination of encapsulated bubbles with foam elements [283]. Such techniques have proven to be effective at reducing noise higher than 1 kHz. Bellmann [17] reported that the highest noise reduction, among all available noise mitigation systems, was achieved in the 1-2 kHz range. For the next generation of monopiles, high noise levels can be reached at lower frequencies as a consequence of the increasing average capacity of wind turbines—from 2018 to 2019, such capacity increased by 1 MW [3]. This is because larger monopiles are required to support the higher-capacity wind turbines, which incidentally shift the high-noise frequencies—associated to the

²Vasconcelos, A.C.A., Valappil, S.V., Schott, D., Jovanova, J., Aragon, A.M., A metamaterial-based interface for the structural resonance shielding of impact-driven offshore monopiles, *Engineering Structures* 300, 117261, 2024.

monopile's eigenmodes—below 1 kHz. Since traditional noise mitigation techniques are not suited for frequencies below 1 kHz, several mechanisms need to be combined to reach the necessary noise reduction, which make it time consuming and operationally expensive. The IHC Noise Mitigation System (IHC-NMS), for instance, combines a double-wall steel pipe filled with air and a bubble curtain between the pile and the pipe walls, resulting in a noise reduction of about 7.5 dB at 800 Hz [28].

Acoustic/elastic metamaterials (A/E MMs), architected structures with unusual properties not commonly found in nature, have shown to be viable for LFN mitigation [284, 285]. The attenuation mechanism of A/E MMs originates from locally resonant structures [36]; these resonators produce resonant band gaps (BGs), which are frequency ranges where waves are attenuated due to the resonator's motion. A/E MMs can therefore realize subwavelength frequency attenuation—*i.e.*, the dimensions of the periodic unit cells (PUCs) can be orders of magnitude smaller than the target attenuated wavelength. The resonant effect of the first A/E MMs were initially attributed to the combination of materials with high impedance mismatch [286–288] and recently extended to single-material configurations, due to the advances of additive manufacturing processes and laser-cutting techniques [8, 43, 44, 289–291]. Although showing such intrinsic contributions in wave control, the potential of single-phase A/E MMs has not been fully explored in real-world applications, such as noise/vibration control in offshore environments, specially because of the time demanded to manufacture complex A/E MM designs for large-scale applications.

In this work we propose a novel single-phase A/E MM-based structure—called henceforth a meta-interface—which reduces the low-frequency underwater noise caused by the installation of offshore monopiles. Here we explore a single-phase unit cell with a planar geometry, so that straightforward manufacturing processes can be used. During the installation, the impact load's energy is transferred to the monopile through longitudinal waves [11]. Due to Poisson's effect, those waves induce radial displacements of the monopile, which produce a pressure field in water [292] (See Fig. 3.1). The meta-interface is then properly designed to selectively filter high-noise levels at specific frequencies—particularly, those related to the monopile's eigenmodes—so that the energy transmitted to the pile is enough to drive it into the ground. Frequency ranges causing high noise levels are identified via the analysis of the sound pressure level (SPL), which is a metric used by international regulations to quantify the sound levels in water [18]. Finally, the noise attenuation is compared to that of a standard cushion also made of nylon commonly used for controlling the amplitude and duration of the impact load, removing the stress concentrations on the pile head, and reducing the noise level [293]. Noteworthy, the focus of this work is on the noise reduction properties of the meta-interface. However, we also note that the pulse used will undoubtedly result in high stresses within the meta-interface that can severely limit its functionality. Therefore, designing the meta-interface needs to account for mechanical performance and will be the subject of our future work. For sake of demonstration, here we design a meta-interface containing resonators with the same geometry, however, it is worth mentioning that multiple resonators with different ranges of attenuation could be used. This would allow us to control

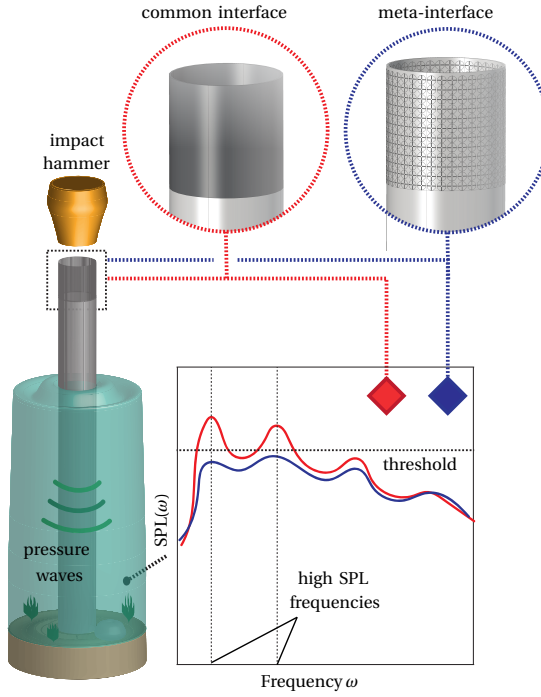


Figure 3.1.: Schematic representation of the meta-interface device for controlling wave propagation. The meta-interface, which is formed by periodic single-phase unit cells, transfers the impact's energy from the hammer to the monopile's head. The radial displacement of the monopile induces the propagation of pressure waves in water. The energy associated to high SPL (determined by the SPL threshold) is removed due to the functionality of the meta-interface.

the amount of energy removed from the pile driving process. The proposed solution shows a significant attenuation of SPL at low-frequency ranges, which suggests its potential application in reducing the harmfulness in the underwater environment.

3.2. METHODS

3.2.1. DYNAMIC CHARACTERIZATION OF META-INTERFACE

The proposed meta-interface is composed by unit cells properly designed to filter energy from the impact load. To analyze the unit cell's dynamic characteristics, the band structure diagram is determined for wave vectors $\mathbf{k}$ varying through the boundaries of the irreducible Brillouin zone Γ -X-M- Γ . For a periodic medium, the

displacement field $\mathbf{u}$ satisfies the Bloch-Floquet theorem [53]

$$\mathbf{u}(\mathbf{r}) = \bar{\mathbf{u}}(\mathbf{r}, \mathbf{k}) e^{i(\mathbf{k} \cdot \mathbf{r})}, \quad (3.1)$$

where $\mathbf{r}$ is the position vector, $\bar{\mathbf{u}}(\mathbf{r}, \mathbf{k})$ is a periodic displacement vector, and $i = \sqrt{-1}$. The finite element method (FEM) is used to define the eigenvalue problem, which is written as

$$[\mathbf{K}(\mathbf{k}) - \omega_n^2 \mathbf{M}] \Psi = \mathbf{0}, \quad (3.2)$$

where $\mathbf{K}$ and $\mathbf{M}$ are the stiffness and mass matrices, respectively; Ψ is the eigenmode related to the angular frequency ω_n . The band structure is then calculated by varying $\mathbf{k}$ along the IBZ's boundary. In this study, a wavenumber sweep of $\Delta k = 0.001$ and a total of 30 eigenfrequencies were used to determine the band structure. The obtained eigenfrequencies $\omega = \omega(\mathbf{k})$ can be normalized as $\Omega = \omega a / c$, where c is the longitudinal wave speed of the constituent material and a is the unit cell size.

The transmission loss diagram is obtained by exciting the finite meta-interface with a harmonic input signal $\mathbf{U} = \bar{\mathbf{U}} \cos \omega t$, where $\bar{\mathbf{U}}$ is the displacement vector. The structure analyzed consists of an array of 14 unit cells, being $\mathbf{U}_1$ and $\mathbf{U}_2$ the displacements at points $\mathbf{x}_1$ (before) and $\mathbf{x}_2$ (after) the array, respectively. The input signal, which is applied perpendicularly along the top boundary of such array, consists of a prescribed displacement with an amplitude of 0.1 mm. Symmetric boundary conditions are applied at the left and right boundaries. A PML is assumed at the bottom boundary to prevent the contribution of reflected waves on the displacement of the measure points. The input signal frequency is swept from 1 Hz to 1000 Hz with steps of 1 Hz. The frequency response is then evaluated by solving,

$$(\mathbf{K} - \omega^2 \mathbf{M}) \bar{\mathbf{U}} = \bar{\mathbf{F}}, \quad (3.3)$$

where $\bar{\mathbf{F}}$ denotes the amplitude vector of external harmonic forces ($\mathbf{F} = \bar{\mathbf{F}} \cos \omega t$). The transmission loss TL is defined as

$$\text{TL(dB)} = 20 \log_{10} \left(\frac{\|\mathbf{U}_2\|}{\|\mathbf{U}_1\|} \right). \quad (3.4)$$

3.2.2. DETERMINATION OF THE SOUND PRESSURE LEVEL (SPL)

The SPL resulted from the pile driving process is calculated through three steps. Initially, the contact problem for the hammer strike is replaced by a compressive impact load applied at the top of the cushions. This assumption neglects the rebound effect of the hammer, which could induce a second impact load that may excite different pile modes and would require different resonator designs. However, the main part of the energy is transmitted to the pile through the first impact [294]. This is an assumption usually used by underwater noise prediction models and can give a good indication of the underwater noise spectrum. In the second step, the interaction between the impact load and the cushions are analyzed separately from the pile-soil-water system. Through a frequency domain analysis, the reaction force is determined at the bottom surface of both cushions. Finally, the reaction force is

introduced as a boundary condition in the pile-water-soil model, from which the SPL is calculated in the frequency domain. More details about each step are given further.

The impact load applied is modeled as a symmetric vertical Gaussian pulse with amplitude 250 MN and a duration of 5 ms. It is worth emphasizing that these load's characteristics were selected so that a realistic representation of SPL is guaranteed. Its spectrum of excitation is determined by applying a fast Fourier transform (FFT), as shown in Fig. 3.2a. Notice that the load energy is concentrated at frequencies lower than 800 Hz, which restricts the sound measurements to this frequency range. The load spectrum is then uniformly applied to the top of both cushions, whereas the pile interaction at their bottom boundary is replaced by an axial spring with stiffness $k_p = E_p A_p / L_p$, where E_p , A_p , and L_p are the Young's modulus, the cross-sectional area, and the length of the pile, respectively. Damping is considered in the cushions by applying an isotropic loss factor for Nylon of $\eta_n = 0.024$ [295]. A frequency domain analysis provides the reaction spectrum at the bottom of the cushions.

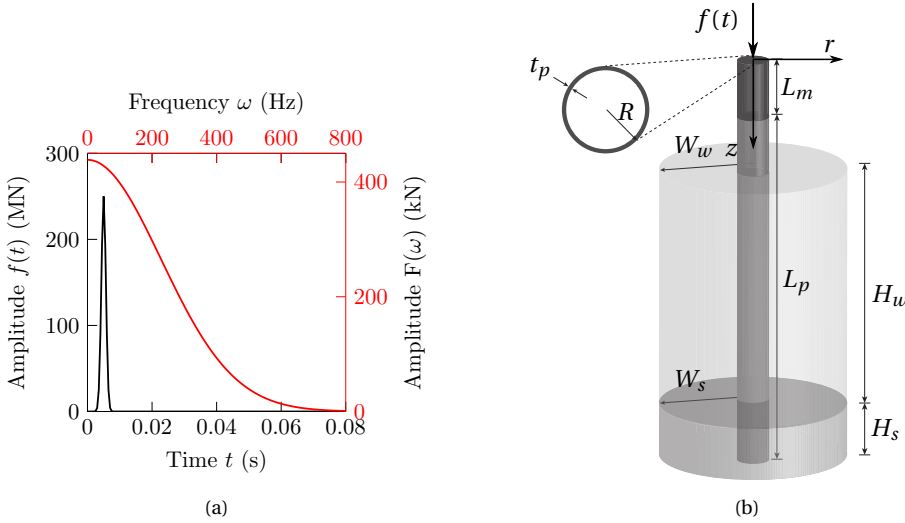


Figure 3.2.: (a) The impact load $f(t)$ applied on the top of the cushions is represented by a Gaussian pulse (in black) with corresponding frequency spectrum $F(\omega)$ in red; (b) Geometric characterization of underwater sound prediction model. The darker gray cylinder represents the common cushion or the meta-interface. The geometric variables are presented in Tab. 3.1.

The reaction spectrum is then transferred to the pile-driving system—which describes the pile-soil-water interaction—as the new forcing function responsible for driving the monopile (See more details about the reaction spectra in Section A:2). Since we assume that the impact load is symmetric, the pile-driving system is treated as a 2D axisymmetric model. For the monopile-water-soil system, a zero displacement is prescribed at the bottom of the monopile. The seasurface is

modeled as a boundary with total pressure $p_t = 0$, while the lateral sea ends have a cylindrical radiation condition to eliminate wave reflections. Low-reflecting boundary conditions are also assumed at the external boundaries of the soil domain. Once the maximum frequency of the excitation spectrum F_{max} is determined—the energy's load is concentrated at frequencies below 800 Hz—the minimum wavelength λ can be calculated through the expression $\lambda = c_{min}/F_{max}$, where c_{min} is the minimum wave speed of a specific domain. Therefore, the wavelengths in water and soil are $\lambda_w = 1.85\text{m}$ and $\lambda_s = 0.19\text{m}$, respectively. A minimum of 5 finite elements per wavelength is considered in the model, which provides a minimum element size of 0.37m for water and 0.04m for soil. A frequency domain linear analysis of a structural-acoustic finite element model is then performed to predict SPLs at the underwater environment [11, 296].

To quantify the noise propagation in the water, the SPL is measured at different locations. The SPL, in dB, is calculated as [297]

$$\text{SPL}(\omega) = 20 \log_{10} \left(\frac{\sqrt{2}}{2} \frac{|\bar{p}_t(r, z, \omega)|}{p_r} \right), \quad (3.5)$$

where r and z are the coordinates of the measurement points, $\bar{p}_t$ is the total pressure in the frequency domain and p_r is the reference pressure, which for water is $p_r = 1\mu\text{Pa}$. Since the modeled water domain has finite extension, an exterior field calculation by means of the Helmholtz-Kirchhoff integral [298] is set up to obtain the SPL at the distance defined by German regulations (190dB at 750m) based on the pressure and its normal derivative at the external underwater boundaries.

3.3. RESULTS AND DISCUSSION

3.3.1. CRITICAL SOUND PRESSURE LEVELS

Figure 3.2b describes the model used to predict the underwater noise that results from the pile driving process. The monopile is represented by a cylindrical shell made of steel, whereas the common interface, which represents a cushion commonly used, is placed on top. The diameter and thickness of the interface are equal to those of the monopile, and its length is $L_m = 5\text{m}$. The material used for the interface is Nylon, with Young's modulus $E = 2\text{GPa}$, mass density $\rho = 1150\text{kg/m}^3$, and Poisson's ratio $\nu = 0.4$. The water column is mathematically described by an acoustic semi-infinite medium, in which perfectly matched layers (PMLs) are used to avoid wave reflections and the total pressure at the sea surface is $p_t = 0$. A semi-infinite elastic continuum is used to represent the soil. PMLs are also assumed at the soil's lateral and bottom surfaces. The geometric and material properties of the pile-water-soil system are given in Table 3.1. Finally, the hammer's loading is represented by a Gaussian impulse (see Fig. 3.2a) and is prescribed as a natural boundary condition applied at the top of the common interface. The steps described in Section 3.2 are then followed to determine the SPLs.

Several international regulations have defined this threshold according to the species living in the environment, the distance to the noise source, the number

Pile	Value	Water	Value	Soil	Value
Length L_p (m)	55	Column Height H_w (m)	45	Depth of layer H_s (m)	5
External radius R (m)	2.5	Column Width W_w (m)	30	Soil Width W_s (m)	30
Thickness t_p (mm)	90	PML Width (m)	5	PML Width (m)	5
Modulus E_p (GPa)	210	Density ρ_w (kg/m ³)	998	Density ρ_s (kg/m ³)	1600
Density ρ_p (kg/m ³)	7850	Wave speed c_p^w (ms ⁻¹)	1480	Wave speed c_p^s (ms ⁻¹)	1592
Poisson's ration ν	0.28			Wave speed c_s^s (ms ⁻¹)	159

Table 3.1.: Geometric and material parameters of the pile-soil-water system. The subscripts p and s denote pressure (longitudinal) and shear wave speeds, respectively.

of hammer blows, among other factors. From those regulations, the German one defined a maximum SPL of 190dB at 750m from the pile [18]. Figure 3.3 summarizes the SPL response of the model at a point located 25m above the seabed and 750m away from the pile wall. To identify the frequency ranges that correspond to high sound pressure levels, the sound threshold is also drawn in the figure. Since the water domain considered here is finite, the external pressure field is calculated by means of the Helmholtz-Kirchhoff integral (see Section 3.2 in Methods). The inset in Fig. 3.3 highlights the peak at which the sound level is higher than the threshold. From an eigenfrequency analysis of a clamped monopile, the first five natural frequencies are calculated and marked in Fig. 3.3 with vertical lines. Their values are $f_1 = 22.955\text{Hz}$, $f_2 = 68.739\text{Hz}$, $f_3 = 114.1\text{Hz}$, $f_4 = 158.58\text{Hz}$, and $f_5 = 201.29\text{Hz}$. It is worth noting that these frequencies correspond to some SPL peaks. More specifically, the highest SPL occurs near the frequency associated to the fifth monopile eigenmode f_5 . For the remaining eigenfrequencies, we observe a relation between their values and some lower-amplitude peaks. Such relation occurs due to the coupling between the radial displacement of the monopile and the water particle motion [11, 12, 293]. During the hammer impact, a radial expansion/compression of the monopile undergoes as a result of the Poisson effect. The energy associated to this motion is then transferred to the surrounding water, which consequently causes the high SPL. Based on this analysis, the unit cell is designed to realize a BG at the frequency range corresponding to the highest SPL.

3.3.2. META-INTERFACE DESIGN

In order to design the meta-interface, we first look at the single-phase periodic unit cell, which consists of a cylindrical mass of radius R_2 connected to an external frame with squared-size a (and cavity's radius R_1) through four identical beams of length l_b and width w_b . The beam's paths have width w_g and depth l_g . Finally, the unit cell has a uniform thickness h , as shown in Fig. 3.4. These parameters were tuned to create a resonant BG that falls in the frequency range corresponding to the highest SPL. Now that the PUC is defined, the meta-interface is realized by replacing it concentrically (with respect to the monopile's center), and then each layer is replicated vertically 12 times.

The single-phase unit cell presented in Fig. 3.4 has dimensions $a = 0.4$ m,

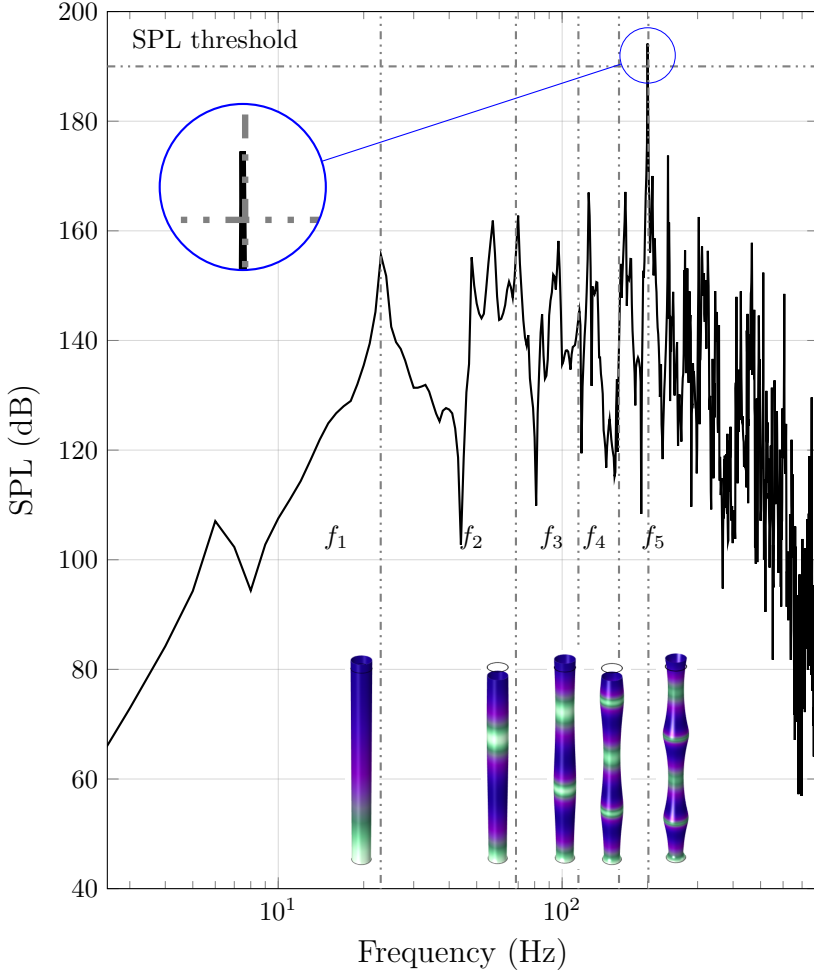


Figure 3.3.: Sound pressure level in the water domain that results from the hammer's impact. The measuring point is located at 25m above the seabed and 750m away from the monopile wall. Vertical lines indicate the monopile's first five eigenmode frequencies. The peak above 190dB (SPL threshold) is located near the fifth monopile eigenfrequency ($f_5 = 201.29\text{Hz}$).

$R_1 = 0.42a$, $R_2 = 0.365a$, $w_b = 0.02a$, $l_b = 0.99a$, $l_g = R_2 - 0.08a$, $w_g = 0.05a$, and $h = 0.225a$. Such dimensions were determined through a parametric study (See Section A.1 of Supplementary Information). The numerical characterization is conducted by obtaining the dispersion relation (band structure) and transmissibility analysis. While the former can be used to reveal band gaps by assuming an infinite metamaterial (which is done by enforcing Bloch-Floquet periodic conditions), the latter is used to obtain the actual attenuation of a finite-dimensional structure.

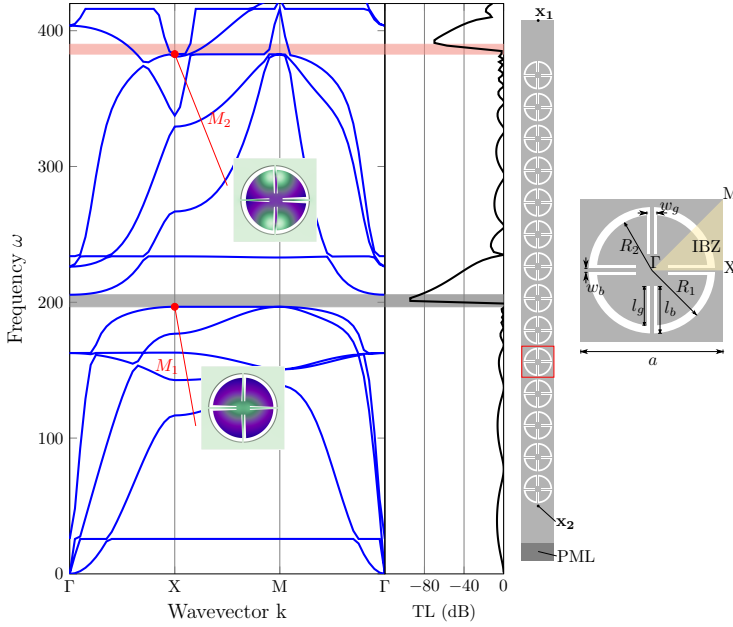


Figure 3.4.: Band structure of a periodic unit cell (PUC)—with an Irreducible Brillouin Zone (IBZ) defined in the yellow triangle, whose boundaries are given by the path Γ -X-M- Γ —and transmission loss diagrams for an array composed by 14 PUCs. The gray area identifies a complete BG with range $\omega = [196.69\text{ Hz}, 205.54\text{ Hz}]$. A partial BG is also identified at a frequency around 382.77 Hz (salmon area). The inset in the band structure diagram highlights the eigenmodes at the frequencies defining the band gaps in the Γ -X direction. In the vibration modes, the amplitude of displacement is indicated by the colormap from light green (minimum) to dark purple (maximum).

Notice that the 8-fold symmetry of the unit cell requires its dispersion evaluation along the boundaries of the Irreducible Brillouin Zone (IBZ). Fig. 3.4 reveals a complete resonant BG at a frequency range extending from 196.69 Hz to 205.54 Hz (gray area). This means that waves with wavelength associated to the complete BG are attenuated, regardless of their direction of propagation. In the case of the transmission loss at a frequency of 388.77 Hz, this is attributed to the resonant mode of the unit cell (indicated by M_2) and to the presence of a negative group velocity (negative slope of the dispersion curve), which indicates a negative refraction index [167] in the Γ -X direction (direction of the impact load). Another observation is that the wavelength associated to both frequency ranges is much longer than the dimensions of the unit cell, which characterizes subwavelength frequency attenuation. Finally, the asymmetric transmission loss observed along the BG frequency ranges also demonstrates resonant-type BGs.

3.3.3. UNDERWATER NOISE REDUCTION

To ensure that SPL analyses (Section 3.2 of Methods) in a pile system containing a common cushion and a meta-interface are comparable, their heights are kept the same, (thus, the meta-interface possesses 12 ring-shaped layers; each layer contains 36 unit cells), as well as, the impact load (see Fig. 3.2a). Figure 3.5 summarizes the results of this comparison. A reduction of approximately 98dB is observed at the critical noise region. Such reduction is more than enough to guarantee that SPL for this frequency range is kept below 190dB. As discussed in Section 3.3.2, the wave attenuation at such frequency range arises from the complete band gap. For more details about such discussion, please refer to Section A:3 of Supplementary Information. Other noise reduction behavior is observed at frequency around 382.77Hz. For such case, a noise gap of 34dB is obtained due to the presence of a locally resonant bandgap as shown in Fig 3.4. It is important to notice that SPL associated to bands below 20Hz are almost kept unchanged, which means that the energy associated to those bands is transmitted to the pile unaltered. At frequencies between 20Hz and 40Hz, we observe an increase in SPL due to the excitation of the meta-interface's natural frequencies (see Section A:2). Although only the measurement at the middle of the underwater column is shown here, we observed a similar behavior at different underwater heights and close to the monopile (see Section A:4).

Fig. 3.5 shows that the SPL drops drastically at the band gaps (i.e, at SPLs much lower than the threshold), which implies that less energy is transmitted to the pile. The advantage of our system is that the transmitted energy can also be controlled by adding or removing unit cell layers—the minimum number of layers is chosen based on the minimum attenuation required to reach the sound pressure level threshold.

3.4. CONCLUSIONS

This work demonstrated numerically a meta-interface—a metamaterial-based structure composed of single-phase unit cells—for shielding the structural resonance of an offshore monopile to reduce the high underwater low-frequency noise levels. The periodic unit cell's resonator was designed according to critical frequency ranges causing high noise levels, which were shown to be associated to the natural frequencies of the monopile. The dynamic characteristics of the single-phase unit cells were analyzed via the dispersion relation and the transmission loss, which have identified a resonant BG corresponding to the critical frequency range. Such intrinsic features lead to a list of advantages of the presented noise mitigation approach. Compared with the common cushion, the proposed meta-interface guarantees a noise attenuation at the critical frequency range. Even though the inclusion of more unit cells allows us to obtain higher attenuation, the aim of the meta-interface is to remove energy from the impact load to a point that the SPL keeps acceptable levels and enough energy is transmitted to drive the monopile. Regarding the manufacturing process, the single-phase unit cell offers advantages for straightforward fabrication, which can be performed not only via 3D-printing but also with cutting techniques, such as laser cutting machines. Finally, the

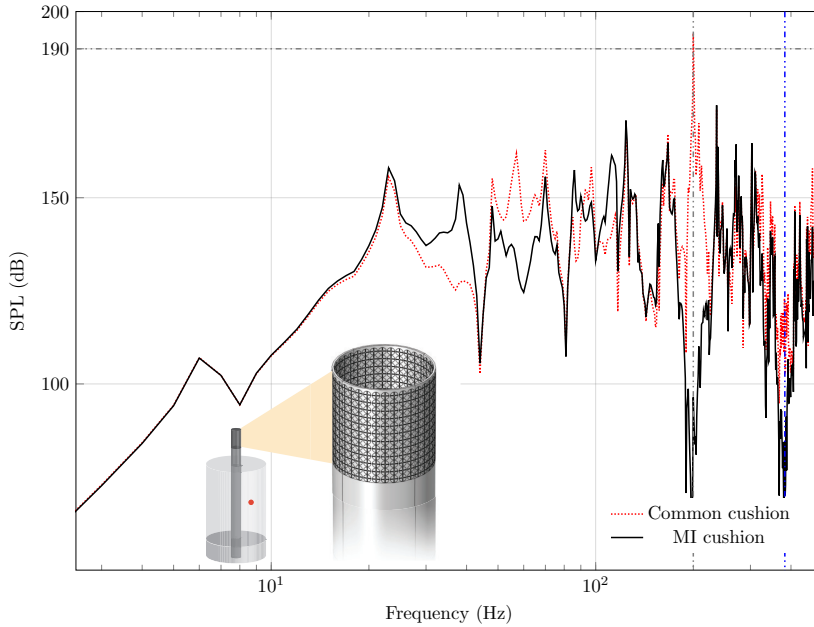


Figure 3.5.: SPL comparison between the common cushion (red dotted curve) and the meta-interface (solid black curve) at the measuring point shown in Fig. 3.3. The meta-interface contains 12 layers of periodically distributed unit cells; each layer has 36 unit cells at the radial direction. The SPL threshold is highlighted by the dashed horizontal line. The gray vertical lines indicate the frequencies where the noise reduction was obtained by the local resonance of the meta-interface. A reduction of 98dB is observed in the region with the highest SPL, which occurs around 200Hz (first vertical dashed line).

sound control is performed directly at the source, unlike traditional noise-reduction techniques that attempt at mitigating noise away from the impact loading. Mitigating sound waves at their origin then avoids the use of huge structures that are used to encase the monopile, which further reduces costs and installation complexity and time.

Although the meta-interface concept predicts numerically the noise reduction during the pile installation, further studies are required to mature the concept. It is worth noting that the meta-interface design shown was tailored for a monopile with specific dimensions; therefore, changing the monopiles dimensions—thickness, diameter, or length—would require a completely different PUC design for attenuating different high noise levels effectively. Trial-and-error approaches for designing unit cells with specific attenuation properties is not straightforward, leading to the development of new design strategies, such as gradient-based optimization procedures [299], evolutionary algorithms [300], and inverse design by means of machine learning techniques [301]. Due to its local resonant nature, the meta-interface's band gap covers a narrow bandwidth, which may result in efficiency

loss in the presence of manufacturing imperfections. To avoid this, a precise manufacturing of the resonator should be guaranteed. It is also important to highlight that other pile driving processes may generate noise with wider noise bands. Some works have proposed strategies to increase the attenuation band—for instance, by using MMs formed by multi-resonators [33-35], which could be well implemented in the presented unit cell design. Regarding the design, it is worth noting that computational techniques such as topology optimization can be used to create robust designs to manufacturing variability [302, 303]. In addition, the mechanical integrity of the meta-interface needs to be considered early in the design process, since it is also necessary to withstand repeated impact loads. In subsequent research, we will explore a multi-objective optimization approach aimed at deriving unit cell geometries that can withstand high impact loads while preserving their attenuation characteristics within specified frequency ranges. The investigation will focus on combining these optimal geometries to broaden the attenuation band. Finally, an experimental validation is essential to demonstrate the effectiveness and applicability of the meta-interface in reducing SPL. Addressing such tasks might enable the potential use of meta-interfaces as a new mitigation noise technique, which will greatly improve the installation process of offshore monopiles.

4

A MULTI-OBJECTIVE DESIGN OPTIMIZATION STRATEGY FOR ACOUSTIC/ELASTIC METAMATERIALS UNDER IMPACT LOADING

An important aspect when designing a metamaterial to attenuate structural vibrations is to evaluate its ability to support high impact loads, such as the load provided by the impact hammer. Therefore, it is crucial to design resonators that resist such loading conditions, while attaining their ability to attenuate vibrations.

This chapter introduces a multi-objective optimization strategy that can be utilized during the design stage of metamaterials undergoing high impact loads. The optimization provides in the end designs that are suitable for the real load conditions without losing the attenuation performance. This chapter is a transcription of the journal paper published in *Structures*³.

4.1. INTRODUCTION

Metamaterials (MMs) refer to architected structures exhibiting unique effective properties that are not found in their constituent materials. Due to such properties, MMs have been widely explored for solving wave propagation problems. The sub-category of MMs possessing properties for manipulating mechanical waves is named as acoustic/elastic metamaterials (A/E MMs). These MMs are comprised of locally resonant periodic (or non-periodic) unit cells, which retain subwavelength frequency attenuation ranges (also known as resonant band gaps). The dynamic interaction between the local resonators and the propagating mechanical wave

³Vasconcelos, A.C.A., Schott, D., Jovanova, J., A multi-objective design optimization strategy for acoustic/elastic metamaterials under impact loading, *Structures* 73, 108229, 2025.

results in frequency-dependent effective properties, such as negative mass density [143], negative bulk modulus [304], and double negativity [164] (both mass density and bulk modulus have negative values). As a result of that, A/E MMs have been extensively investigated within the field of noise/vibration control [36, 289, 305–307].

Regarding to the attenuation of mechanical waves induced by dynamic loads (such as impact loads), some works reported the use of the dynamic effective properties of A/E MMs. For instance, Tan et al. (2014) [308] demonstrated numerically the attenuation performance of single- and dual-resonator models during the transmission of blast waves. Both models consisted of spring-mass systems retaining the negative effective density at two distinct frequencies. An experimental validation of A/E MMs for impact wave attenuation was presented by Khan et al. (2018) [309]. They showed a transmission reduction at the frequency related to the resonance of an unit cell made of aluminium and steel. The effective mass density of meta-panels, a metamaterial-based sandwich structure, was also applied to attenuate blast waves [57]. Although these studies have developed A/E MM designs for impact load attenuation, a verification of their mechanical performance is still to be demonstrated. The lack of such analysis can result in two main drawbacks. First, the design may not resist the impact load due to the use of thin elements and significantly large masses to create the subwavelength resonant band gap. Second, the deformation of such MMs can modify their dynamic effective properties. In the works reported by Wang et al. (2014) and Bertoldi et al. (2017) [154, 310], the buckling effect was exploited to tune the band gaps of acoustic MMs. It was highlighted that the pre-deformation of MMs can induce the band gap formation at different frequency ranges. Including the mechanical performance of A/E MMs in their design stage is crucial to retain the MM effective properties for the desired application.

The design of A/E MMs with a desired functionality typically relies on experiments or trial-and-error approaches. Depending on the complexity of the geometry, the required manufacturing constraints, and the method used to determine the A/E MM's effective properties, the current iterative design process can be computationally expensive and time consuming. To avoid that, optimization techniques have been explored to systematically design A/E MMs, i.e., by implying the desired functionalities and respecting the imposed geometric constraints in the design stage. Gradient-based methods are commonly used to optimize A/E MMs. For instance, Oh et al. (2015) [303] conducted topology optimization using the homogenization method to maximize the operating frequency range of hyperbolic elastic MMs. Topology optimization has also been used to design acoustic MMs slabs exhibiting a prescribed negative refraction at a specific incidence angle and frequency [311]. Lu et al. (2013) [299] proposed level-set based topology optimization to design acoustic MMs with negative bulk modulus at specific frequencies. At the same direction, Noguchi et al. (2022) [312] designed acoustic labyrinthine MMs with a negative refractive index by means of level-set based topology optimization. Since gradient-based methods are not effective for non-convex problems, suboptimal configurations (local minima) may be found. In a worst scenario, the solution can diverge in case of the initial design is not well selected. Furthermore, gradient-based

methods require the sensitivity of the fitness function with respect to some design variable, which could be a challenge task for some optimization problems. To avoid that, efforts have been devoted to design MMs via non-gradient-based methods such as evolutionary methods; these algorithms borrow concepts from evolutionary biology—a population of individuals evolves over generations so as to optimize a fitness function (it is worth noticing that evolutionary algorithms may not guarantee a global solution, since there is no optimality criterion). To mention some works, Dong et al. (2017) [300] introduced a topology optimization approach to design two-dimensional single-phase elastic MMs with double-negative effective material properties by using the single-objective genetic algorithm (GA). Shape optimization approaches combining effective properties and GA have also been introduced to design single-phase chiral elastic MMs [313]. Broadband vibration attenuation of one-dimensional and two-dimensional meta-structures was achieved via optimization procedures based on genetic algorithms [266, 314–316]. In the field of active metamaterials, whose properties are actively adjusted by external stimuli, genetic algorithm procedures have been employed to achieve improved vibration suppression [262, 317, 318]. Besides that, evolutionary algorithms can take full advantage of parallel computing—reducing significantly the computational time—, since the fitness evaluation of all individuals is independent. Despite the significant contributions made by GA, investigations concerning the the study of vibration attenuation and mechanical performance relation in early design stage remains still unexplored.

In this paper, we investigate the influence of integrating the mechanical performance into the design process of A/E MMs through a multi-objective optimization using GA. The unit cell's geometry is initially parameterized, from which a vector of design variables is defined and employed to update the two fitness functions required in this optimization problem; the first related to the vibration attenuation analysis, while the second evaluates the maximum stress due to an impact load. Adequate geometric constraints are defined so that the manufacturing of the optimized MMs is feasible. The multi-objective optimization provides in the end a Pareto front, which shows a set of non-dominated optimal solutions. We analyze the trade-off behavior between mechanical and vibration attenuation performance and discuss the importance of incorporating these features into the design stage of A/E MMs. Through such multi-objective design optimization, the A/E MM can be used for attenuating vibrations at specific frequencies of structures undergoing high amplitude impact loads, such as foundations installed in onshore or offshore environment through an impact hammer and structures exposed to shock waves.

4.2. PERFORMANCE INDICATORS OF ACOUSTIC/ELASTIC METAMATERIALS

In this section, the evaluation of the vibration attenuation and mechanical performance of A/E MMs is introduced. The unit cell geometry used in this work is initially described and its dynamic features are investigated. Finally, a strategy to

determine the stresses caused by an impact load is presented.

4.2.1. UNIT CELL MODEL AND DISPERSION CURVES

The dynamic characteristics of A/E MMs will provide information about the vibration attenuation functionality. The characterization starts by initially defining the unit cell design. The design steps of the unit cell are presented in Figs. 4.1a and 4.1b. A square unit cell with side length a contains a resonator, with a mass represented by a rounded-vertex square structure of length $2R_2$ containing four rectangular-shape cavities of length l_g and width w_g , and four identical beams with length l_b and width w_b . The round shape of the square vertices is introduced by a fillet with radius R_2^f . The resonator is embedded in an external rounded-vertex square frame of size $2R_1$ containing a fillet of radius R_1^f . A fillet r_f was also included in the edges connecting the beams to the external frame. The unit cell has a thin out-of-plane thickness t_c , enabling the assumption of plane stress analysis. We consider an unit cell made of Nylon, with properties are Young's modulus $E_n = 2$ GPa, density $\rho_n = 1150 \text{ kg/m}^3$, and Poisson's ratio $\nu_n = 0.4$.

Dispersion curves are commonly used to evaluate the dynamic features of A/E MMs, since they show the relation between the wavelength and frequency of a wave propagating in the medium. Consequently, band gaps can be identified as the frequency zones with no dispersion curves, which means no wave propagation. For that purpose, here we consider an infinite 2D medium formed by periodically repeated unit cells presented in Fig. 4.1b. The periodicity is introduced along the in-plane boundaries of the unit cell and it is conducted by Bloch-Floquet theorem, which states that the displacement field $\mathbf{u}$ satisfies

$$\mathbf{u}(\mathbf{x} + \mathbf{a}, \mathbf{k}) = \mathbf{u}(\mathbf{x}, \mathbf{k}) e^{i(\mathbf{k} \cdot \mathbf{a})}, \quad (4.1)$$

where $\mathbf{x}$ is the position vector, $\mathbf{a}$ is the lattice vector of the periodic material, $\mathbf{k}$ is the wave vector of the propagating wave, and $i = \sqrt{-1}$. Due to the periodicity of the unit cell, the vector $\mathbf{k}$ is defined at the edges of the irreducible Brillouin zone (green triangle in Fig. 4.1c). The band structure is obtained by solving the following eigenvalue problem

$$(\mathbf{K}(\mathbf{k}) - \omega^2 \mathbf{M})\mathbf{u} = \mathbf{0}, \quad (4.2)$$

where $\mathbf{K}(\mathbf{k})$ and $\mathbf{M}$ are, respectively, the global stiffness and global mass matrices, and ω is the angular frequency of the propagating wave. To investigate the mechanism of local-resonant band gaps, the band structure is determined for an unit cell with the following geometric parameters: $a = 0.042 \text{ m}$, $R_1 = 0.47a$, $R_2 = 0.46a$, $w_b = 1 \text{ mm}$, $l_g = 0.07a$, $w_g = 0.1a$, $l_b = R_2 - l_g$, $R_1^f = 0.85R_1$, $R_2^f = 0.85R_2$, $r_f = 0.5 \text{ mm}$, and $t_c = 10 \text{ mm}$. The commercial software COMSOL Multiphysics is used to calculate the band structure. The first six band modes are depicted in the left of Fig. 4.1c, with abscissas $\Gamma - X - M - \Gamma$. A band gap ranging from 488 Hz to 556 Hz is identified (gray rectangle) and its flat edges—or, close-to-zero group velocity—indicate that it results from local resonance [36]. This can be observed from modes M_1 and M_2 , where the motion is mostly confined to the resonator.

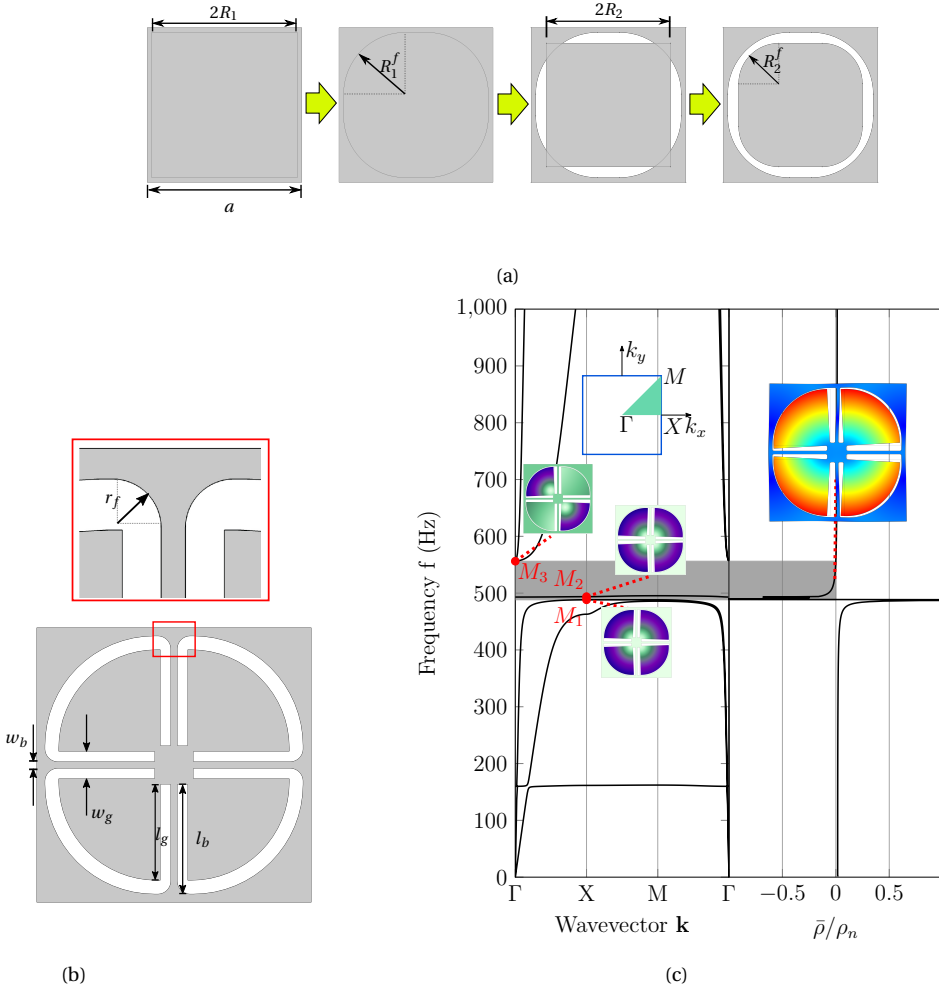


Figure 4.1.: (a) Steps for definition of the external frame of the unit cell and the resonator mass. Fillets R_1^f and R_2^f are used to create the round shape of the square's vertices. (b) Final design of unit cell with remaining geometric parameters. (b) Band structure and effective mass density of the initial unit cell design. The band structure shows the first six modes of the periodic unit cell for a finite set of wavevectors $\mathbf{k}$ along the IBZ (green triangle), where two band gaps are observed. The gray rectangle indicates the resonant band gap. The inset highlights the modes (M_1 , M_2 , and M_3) identified at the band gap edges and the displacement field obtained by exciting harmonically the unit cell at a frequency of 525 Hz.

4.2.2. EFFECTIVE MASS DENSITY

Under the long wavelength assumption, whereby the phase displacement difference among boundaries is ignored [153], the effective mass density of the unit cell can be determined. To that end, a harmonic prescribed displacement is applied to the four external boundary edges of the unit cell, and the corresponding reaction forces are evaluated by

$$(\mathbf{K} - \omega^2 \mathbf{M}) \mathbf{U} = \mathbf{F}, \quad (4.3)$$

where ω is the frequency of the driving excitation, and $\mathbf{U}$ and $\mathbf{F}$ are the global displacement and external force vectors, respectively. The effective mass density is then calculated by averaging the reaction force with the acceleration, in all edges, as [319]

$$\bar{\rho}(\omega) = -\frac{1}{V} \frac{F_b}{\omega^2 U_b}, \quad (4.4)$$

where F_b and U_b are the averaged reaction force and displacement in the edges, respectively, and V is the volume of the unit cell.

To further understand the physics behind the effective mass density, a harmonic prescribed displacement in the x direction of amplitude $u_x = 0.1$ mm is applied at the external boundaries of the unit cell described in Section 4.2.1. The reaction forces are obtained by Eq. (4.3) and replaced in Eq. (4.4) to determine the effective mass density. The right curve in Fig. 4.1c shows the frequency as a function of the normalized effective mass density (EMD) $\bar{\rho}/\rho_n$ of the proposed unit cell. We notice that the normalized EMD is slightly constant until 400Hz. Close to the resonant frequency of the unit cell, the EMD increases to an infinite positive value and suddenly drops to an infinite negative value. After that, the EMD increases and becomes constant again. The inset shows the displacement field of the unit cell at $f = 525$ Hz, where the EMD is negative. At such frequency, the positive external loads induce an out-of-phase motion of the resonator, which prevents the wave energy to propagate through the structure [142].

From such analysis, it is worth noticing that the negative EMD region starts nearby the frequency of the lower band gap's edge. Since the determination of the EMD is less computationally demanding than the band structure—here it requires solving the eigenvalue problem stated in Eq. (4.2) with Bloch-Floquet periodic BC for a finite set of vectors along the IBZ (Fig.4.1c)—the EMD was chosen as the method to optimize the unit cell's band gap.

4.2.3. TRANSMISSION LOSS DIAGRAM

We have shown that the determination of the band gap by means of dispersion curve analysis and the effective mass density requires some assumptions; in the first analysis, we assume that the unit cell is periodically repeated in a infinite domain, while in the second, the long wavelength assumption is used. To investigate the attenuation performance of the metamaterial in a finite domain, a transmission loss diagram is numerically computed. Here, we consider a plate containing a finite number of unit cells. The excitation is a sinusoidal horizontal prescribed displacement $u_I(\omega)$ of amplitude 0.1mm, which is applied at a point before the unit

cell arrangement. The output displacement $u_O(\omega)$ is then measured at a point after such arrangement. To reduce the effect of reflected waves, perfectly matched layers (PML) were included at the plate ends. The transmission loss (TL) is then computed by

$$TL(\omega) = 20 \log_{10} \frac{u_O}{u_I}, \quad (4.5)$$

4.2.4. DETERMINATION OF TRANSIENT STRESS

This section introduces the finite element model used for calculating stresses induced by an impact load. Figure 4.2 depicts the model, comprising an array of identical unit cells subjected to a transient load at the top edge, with zero displacement prescribed at the bottom edge. The load function is represented as $F(t) = F_0 e^{-t/0.0001}$, where $F_0 = 1$ kN is the maximum load amplitude and t is the time in seconds. As systems without damping may exhibit oscillatory behavior under impact loads, especially high-frequency content that leads to noisy responses, this would require more iterations and smaller time steps, slowing numerical convergence and increasing computational time. To enhance convergence, numerical damping is introduced as small Rayleigh damping coefficients, $\alpha = 0$ and $\beta = 1e-4$ (material damping has demonstrated influence on the vibration attenuation performance, therefore small values are also used in this work to restrict the vibration attenuation to the resonance effect of the unit cell [320]). The time step Δt and the maximum element mesh size e_{max} for the transient problem are determined based on the maximum frequency to be solved, f_{max} . Consequently, Δt is set as $1/(60f_{max})$, and e_{max} is defined as $c/(6f_{max})$, where c represents the wave speed in the medium and the number 6 indicates the minimum number of finite elements per wavelength. The end time for each transient analysis calculation is set at 0.0038s to also capture the response of reflected elastic waves.

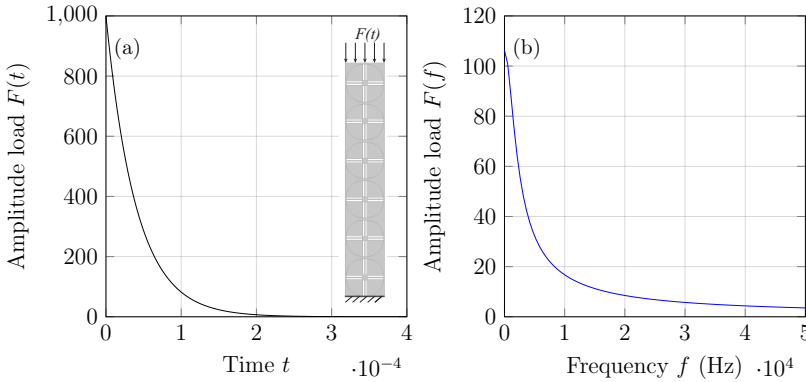


Figure 4.2.: (a) Impact load used in the transient analysis; and (b) frequency spectrum of the corresponding load.

From Figure 4.2b, it is noticed that the energy content of the impact is concentrated

within frequencies below 10 kHz. To accurately determine the maximum frequency, Figure 4.3 depicts the maximum transient von Mises stress response for five maximum frequency values ($f_i = [1, 2, 3, 4, 5]$ kHz, where $i = 1, \dots, 5$). Notably, selecting a maximum frequency $f_{max} = f_2 = 2$ kHz yields a favorable stress response, since frequencies exceeding 2 kHz provide comparable values of maximum von Mises stress, as indicated in Fig. 4.3b.

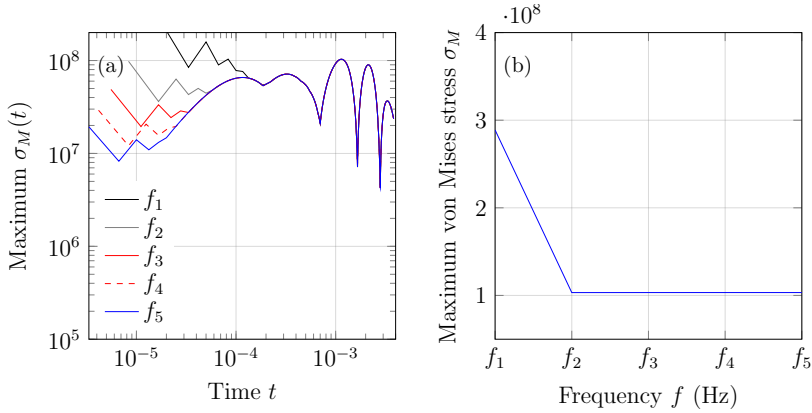


Figure 4.3.: (a) Maximum von Mises stress in function of time for different maximum frequencies f_i . (b) Maximum von Mises stress value obtained from transient simulations for each f_i .

4.3. DESIGN OPTIMIZATION STRATEGY

In this work, the performed optimization updates the unit cell design according to its geometric parameters. Due to the high complexity of the unit cell geometry (see Fig. 4.1a), the geometric parameters that contribute the most to the band gap performance are carefully selected to constitute the design variable (DV) vector. The selection is conducted through a parametric EMD analysis of the main resonant elements. A parametric sweep is defined according to the possible minimum and maximum dimensions for each geometric feature. Since only three parameters are used to define the external frame (R_1 , R_1^f and r^f) they are directly considered as DVs. It is worth to mention that the size of the unit cell a will remain constant, since our aim is to explore the possible resonator's shape within a fixed frame.

Figure 4.4 shows the relation between the band gap's width and the variation of the resonator's size R_2 , the beam's width w_b , the cavity's position l_g , and the cavity's width w_g . From those parameters, the band gap is less sensitive to the beam's width variation, which is due to the limited size range (the beams cannot be thinner than the manufacturing resolution nor too large to avoid contact with the resonator's mass). While the increase of R_2 and w_g results in a decrease of the central frequency of the band gap, the opposite effect is observed when the cavity's position

increases—i.e., when the beam's length is shorter. From these observations, R_2 (and its fillet R_2^f), l_g and w_g were chosen as design variables for further optimization.

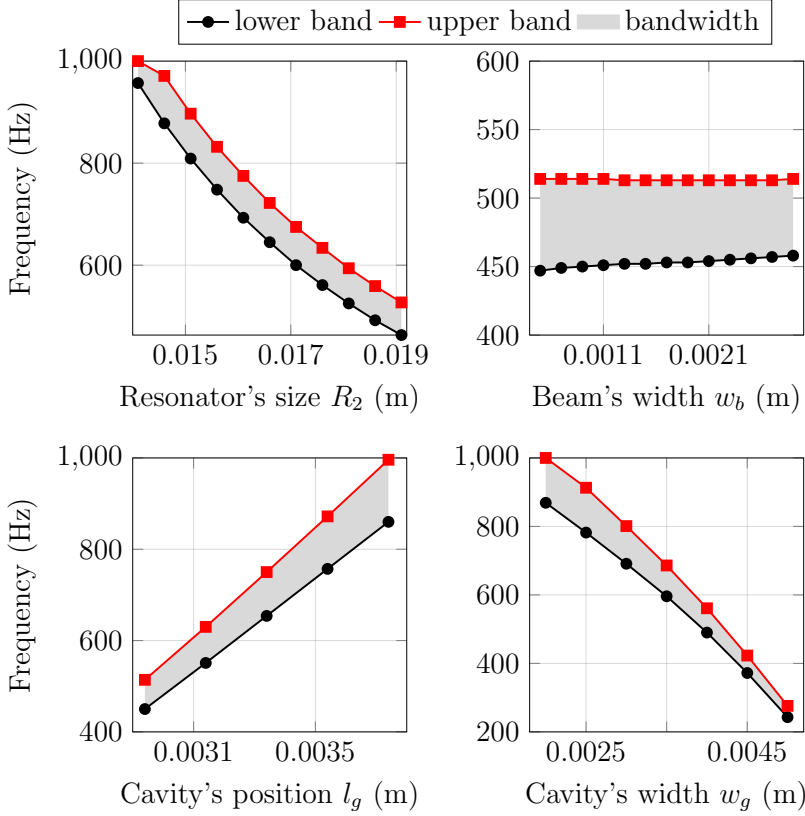


Figure 4.4.: Variation of the bandwidth and central frequency of the BG for different ranges of the resonator's size R_2 (top left), the beam's width w_b (top right), the cavity's position l_g (bottom left), and the cavity's width w_g (bottom right).

GAs have been widely employed to solve complex optimization problems, specially multi-modal ones containing several local minima. Unlike classical optimization methods, which rely on deterministic operators, GAs use stochastic operators that can explore better the search space to achieve the optimal solution. The algorithm works by initially defining a population P_0 containing l_0 individuals randomly generated [321]. Then, the algorithm starts a loop over a maximum number of generations G_m . In each iteration, the population is evaluated according to a defined objective function. The individuals containing lower fitness values are chosen by means of a selection function to constitute the parent population (P_p). The genetic operators crossover and mutation are applied to the individuals of P_p to create the individuals of the next iteration (offspring population P_o). The mutation operator makes random changes in a single individual from P_p , while the crossover operator

combines information of two individuals from P_p . The algorithm ends when one of the stopping criteria (for instance, maximum number of generations, maximum computational time or tolerance on the objective function) is reached.

Since in this work two independent parameters are investigated (the negative mass density and the maximum stress), a GA-based multi-objective optimization strategy is used. In this case, distinct solutions may generate a trade-off behavior between the objectives, which means that the improvement of one objective only occurs when compromising the other objective. This is shown in the end of the optimization by means of a Pareto front, which represents a set of optimal solutions in the space of objective functions. From such solutions, we select the one that is most suitable for a certain application.

The multi-objective optimization problem is then stated as,

$$\begin{aligned}
 \min \quad & \Phi = (\phi_1(\mathbf{d}), \phi_2(\mathbf{d})) \\
 \text{subjected to} \quad & 17.44 \text{ mm} \leq R_1 \leq 19.74 \text{ mm}, \\
 & 16.60 \text{ mm} \leq R_2 \leq 19.32 \text{ mm}, \\
 & 4.2 \text{ mm} \leq w_g \leq 4.5 \text{ mm}, \\
 & 2.94 \text{ mm} \leq l_g \leq 3.94 \text{ mm}, \\
 & 0.1 \text{ mm} \leq r_f \leq 1 \text{ mm}, \\
 & 98.7 \text{ mm} \leq R_1^f \leq 107.52 \text{ mm}, \\
 & 96.6 \text{ mm} \leq R_2^f \leq 100.8 \text{ mm}, \\
 & R_2 - R_1 \leq 0, \\
 & R_1^f - R_2^f \leq -1 \text{ mm},
 \end{aligned} \tag{4.6}$$

where $\mathbf{d}$ is the vector of design variables defined as $\mathbf{d} = [R_1, R_2, w_g, l_g, r_f, R_1^f, R_2^f]$.

The set of geometric constraints exhibits both the geometric bounds and the inequality constraints so that a gap of 1 mm is kept between two parameters. The first function ϕ_1 is related to the band gap optimization and it minimizes the ratio between the central frequency of the band gap and its bandwidth, i.e.,

$$\phi_1(\mathbf{d}) = \frac{0.5(f_2 + f_1)}{f_2 - f_1}, \tag{4.7}$$

f_1 and f_2 represent, respectively, the lower and upper frequencies defining the band gap (the negative EMD region). The second objective function is defined such as the ratio between the maximum transient von Mises stress and the yield stress is minimized. Therefore,

$$\phi_2(\mathbf{d}) = \frac{\sigma_{\max}}{\sigma_y}, \tag{4.8}$$

where σ_y is the yield stress of the constituent material (for Nylon $\sigma_y = 71.7 \text{ MPa}$).

A flow chart describing each step of the multi-objective optimization is presented in Fig. 4.5. The optimization problem is solved using Matlab's built-in function *gamultiobj*. It starts by calculating the EMD $\bar{\rho}$ and the maximum von Mises stress

$\sigma_{\max}$ for each individual of the initial population P_0 via COMSOL Multiphysics. By using the GA operations, the population evolves until it reaches the maximum number of generations or the function tolerance, from which a Pareto front is obtained (refer to schematic on the right from Fig. 4.5). We select then the set of DVs with $\phi_2 < 1$ that is most suitable for a certain application.

4.4. MULTI-OBJECTIVE DESIGN OPTIMIZATION RESULTS AND DISCUSSION

In this section, the trade-off behavior will be explored by performing the multi-objective optimization described in Section 4.3. Preliminary simulations revealed that a population size of 250 effectively provides a Pareto front with a sufficient amount of optimal solutions that will be further verified. The optimization stops when it reaches one of the stopping criteria; here it has been considered the maximum number of generations, which is equal to 20, and a function tolerance of $1e-3$, which indicates the minimum change in each objective function value that should occur between generations. The individuals constituting the parent population are chosen by the tournament function and the mutation children are produced by the mutation power function. The crossover fraction used to create the next generation is set to 0.8.

Figure 4.6a shows the cumulative optimal points in gray for generations $G < 20$ and the optimal Pareto front in black $G = 20$. Notably, the front depicts the trade-off between the objective functions, i.e., the improvement of ϕ_1 only occurs with a decrease in ϕ_2 and vice-versa. The gray area in the figure highlights the region of unfeasible designs, where the linear regime condition is violated, as proposed by Eq. (4.3). Three individuals on the Pareto front are identified, labeled P_A , P_B , and P_C . The individual P_A indicates the case with the lowest ratio between the central frequency of the band gap and its bandwidth; the second individual P_B represents an unit cell containing the lowest levels of von Mises stress; individual P_C highlights the limit case, i.e, the array formed by the optimized unit cell is near the limit of elastic regime.

The EMD curves of the three individuals are shown in Figure 4.6b with their respective band gaps highlighted in their corresponding colors. The individual P_C exhibits a band gap within ranges of [593 Hz, 693 Hz] (shaded green area), while individuals P_A and P_B demonstrate band gaps within the ranges of [564 Hz, 669 Hz] (shaded red area) and [657 Hz, 751 Hz] (shaded blue area), respectively. A comparison of the central frequency and bandwidth of the band gaps for these three individuals are indicated in Tab. 4.1. As expected by the obtained objective function ϕ_1 , individual P_A demonstrates the best optimized band gap, with the lowest central frequency and largest bandwidth, followed by individuals P_C and P_B . Such band gaps are also identified in the dispersion curves depicted in Fig. 4.7, where modes at the flat edges delineating the band gaps ($A_1, A_2, B_1, B_2, C_1, C_2$) indicate the resonant behavior of the internal structure.

To corroborate that the optimized unit cells effectively attenuate vibrations within the frequency ranges in the EMD curves, transmission loss diagrams are calculated.

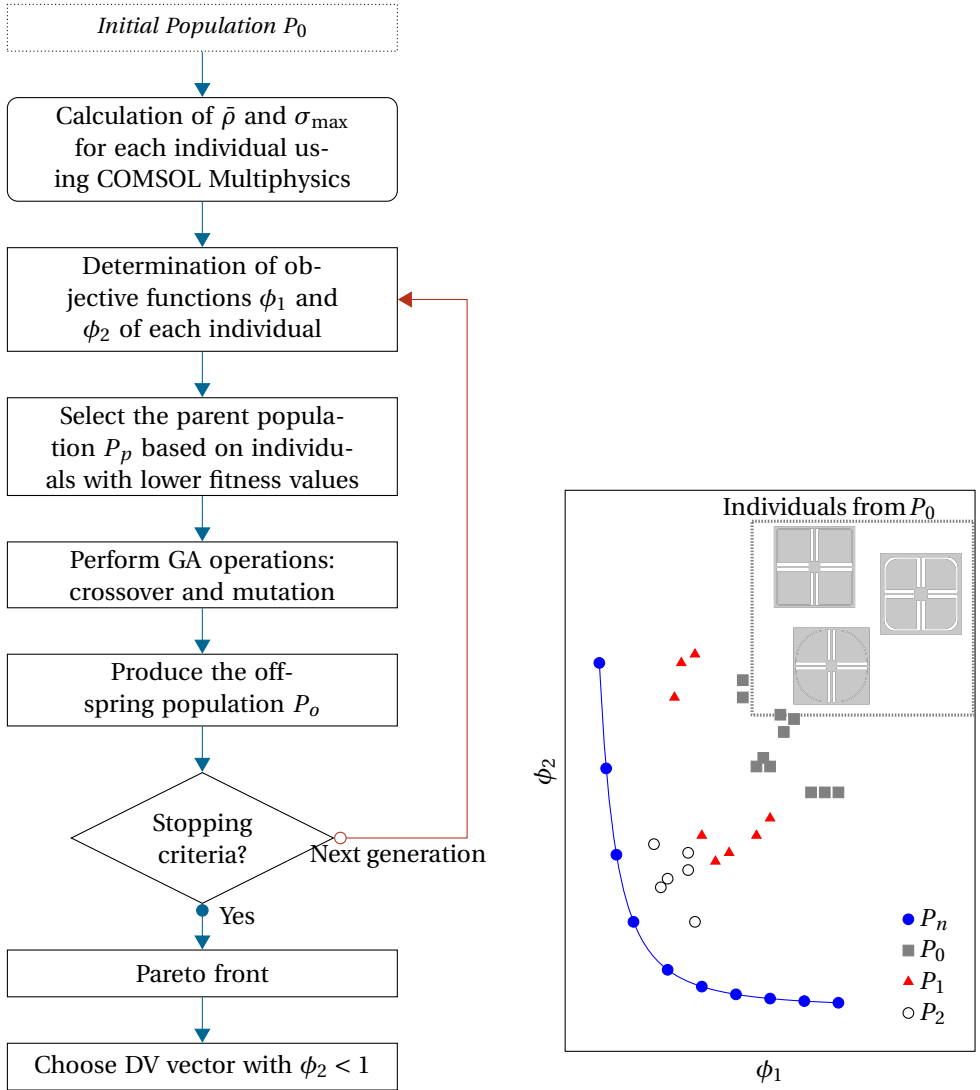


Figure 4.5.: Flowchart of the design optimization used to evaluate the trade-off behavior, which gives in the end a Pareto front with optimal solutions. On the right, it is shown a schematic of the Pareto front evolution through generations. The inset highlights three unit cells randomly generated in the first population P_0 .

As illustrated in Fig. 4.8, these diagrams correspond to three plates, each composed of the unit cells P_A , P_B , and P_C . Across all diagrams, a discernible reduction in transmission occurs with the band gaps identified in Fig. 4.6b and Fig. 4.7. Notably, the peaks situated at the lower edges of the band gaps emphasize the resonance as

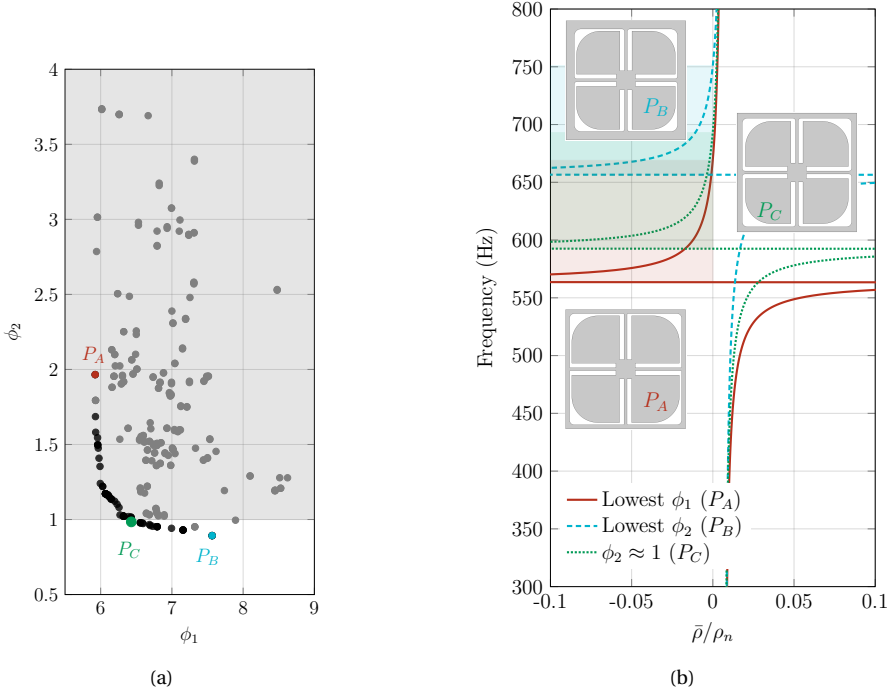


Figure 4.6.: (a) Optimal Pareto front obtained by solving the multi-objective optimization problem (black dots) and cumulative optimal solutions from previous generations (gray dots) with uniform beams. Individuals P_A , P_B , and P_C highlights the cases with lowest ϕ_1 , lowest ϕ_2 , and ϕ_2 close to 1, respectively. (b) Optimized EMD for individuals P_A , P_B , and P_C with their corresponding band gaps highlighted by the shaded areas and their unit cell designs.

Individual	Central frequency [Hz]	Bandwidth [Hz]	Objective function ϕ_1
P_A	616	105	5.867
P_B	704	94	7.489
P_C	643	100	6.430

Table 4.1.: Band gap features – central frequency and bandwidth – for individuals P_A , P_B , and P_C illustrated in Fig. 4.7.

the mechanism of attenuation, consistent with the discussion in [53]. This analysis thus demonstrates the feasibility of optimizing band gaps through EMD calculation.

We evaluate the significance of integrating stress analysis into the MM design process. Figure 4.9 illustrates the time step corresponding to the maximum transient von Mises stress within the arrays formed by unit cells P_A , P_B , and P_C . In Fig. 4.9a, the unit cell P_A comprises a large mass linked to a slender frame through thin beams, resulting in high stress concentration at the connection of these elements.

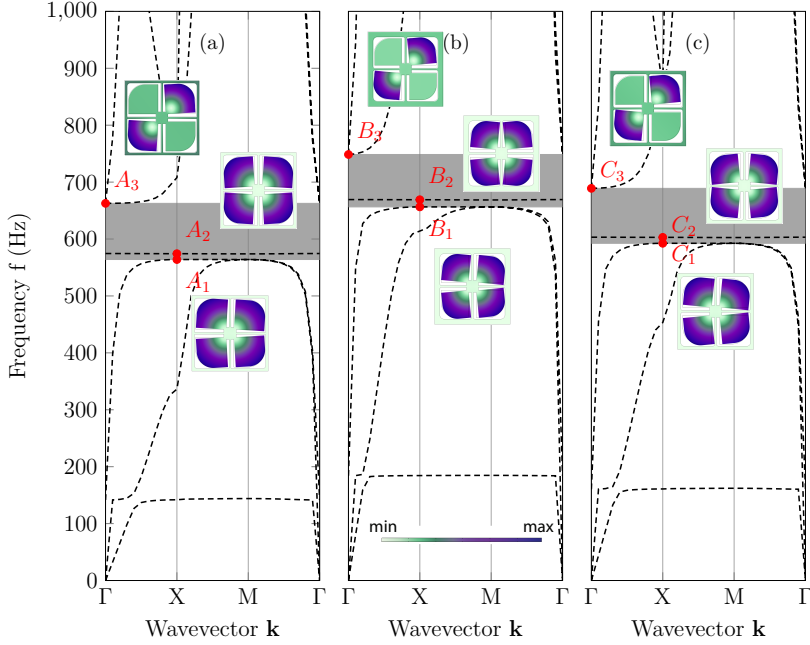


Figure 4.7.: Dispersion curves of the selected individuals (a) P_A , (b) P_B , and (c) P_C , respectively, from the Pareto front presented in Fig. 4.6. Complete band gaps are colored in gray. The inset shows the deformation motion of the unit cell at the flat edges defining the band gaps.

Conversely, Figs. 4.9b and 4.9c indicate maximum stress values below the yield stress of Nylon for arrays formed by P_B and P_C , respectively. While traditional optimization may provide the design of unit cell P_A due to its superior attenuation performance, it would not be indicated in scenarios involving high impact loads.

In Figure 4.4, we have shown that the band gap formation is less sensitive to the variation of the beam's width. However, the investigation of beams with variable cross-section was not explored. To that end, we study the influence of non-uniform beams on the vibration attenuation and mechanical performance of the MM. Two geometric parameters will then define the variation of the beam's width: w_b and w_b^* ; the first parameter defines the width of the beam's end connected to the mass while the second defines the width of the beam's end connected to the external frame. The new set of design variables is then defined as $DV_3 = [R_1, R_2, w_b, r_f, R_1^f, R_2^f, w_b^*]$. Geometric constraints for w_b^* and w_b are defined as $0.3\text{mm} \leq w_b^* \leq 2\text{mm}$ and $0.5\text{mm} \leq w_b \leq 3\text{mm}$, respectively. Figure 4.10 highlights the Pareto front for the entire population after 20 generations, and the EMD for the three selected Pareto points, where the unit cells are illustrated in Fig. 4.10b. Here we can realize a clear distinction among the beam shape of the three points; while point P_B explores the non-uniformity of the beams to reduce the stress concentration, point P_A still shows the use of beams with constant width to obtain the best band gap. By observing

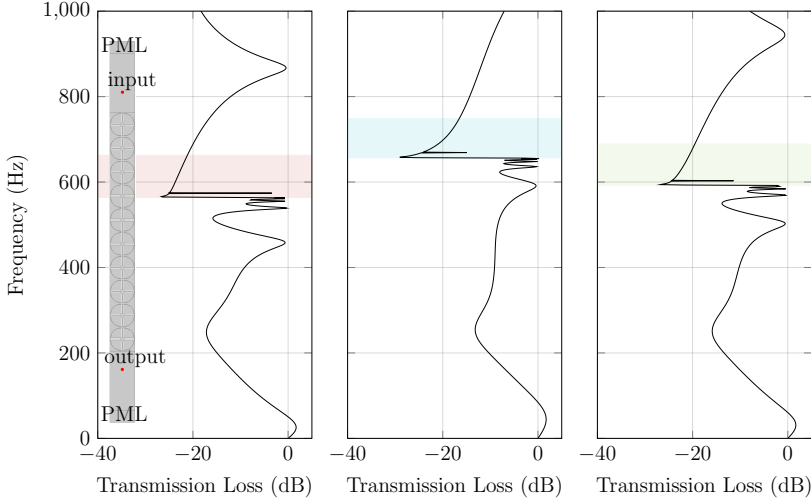


Figure 4.8.: Transmission loss diagrams for metamaterial plates containing unit cells P_A (left), P_B (middle), and P_C (right) presented in Fig. 4.6. The shaded areas indicate the band gap identified by the dispersion curves in Fig. 4.7 and the asymmetric peaks indicate the wave attenuation due to the unit cells' resonance. The inset shows a metamaterial plate for determination of the transmission loss diagram. A horizontal prescribed displacement is applied at the input and the resulting horizontal displacement is measured at the output. Perfectly matched layers are applied in the plate's ends to reduce the wave reflection.

the unit cell of point P_C , we realize a combination of the two previous points—the non-uniformity is less apparent in comparison to point P_B , however, still notorious to reduce the stress (refer to Fig. 4.12).

A comparison of the central frequency and bandwidth of the band gaps for these three individuals are indicated in Tab. 4.2. As expected, individual P_A demonstrates the best optimized band gap, with the lowest central frequency and largest bandwidth. Although, individuals P_B and P_C have similar bandwidth, the central frequency of individual P_C is lower than P_B . The band gaps delineated in Figure 4.10b are also evident in the dispersion curves depicted in Fig. 4.11. Interestingly, the dispersion curve of unit cell P_A depicts a negative slope, although attributed to a shear mode of the unit cell (as indicated by A_2). Additionally, the mode shapes at the lower edges of each band gap (A_1 , A_3 , B_1 , B_2 , C_1 , and C_2) indicate the resonant behavior inherent in the internal structure of the unit cell.

Figure 4.12 illustrates the time step corresponding to the maximum transient von Mises stress within the arrays formed by non-uniform unit cells P_A , P_B , and P_C . Despite unit cell P_A displaying a band gap with the lowest central frequency (as indicated in Figs. 4.10b and 4.11), the transient analysis reveals a maximum von Mises stress of 157MPa (Fig. 4.12a). This value exceeds twice the yield stress of Nylon, while acceptable stress levels are observed within the arrays illustrated in Figs. 4.12b

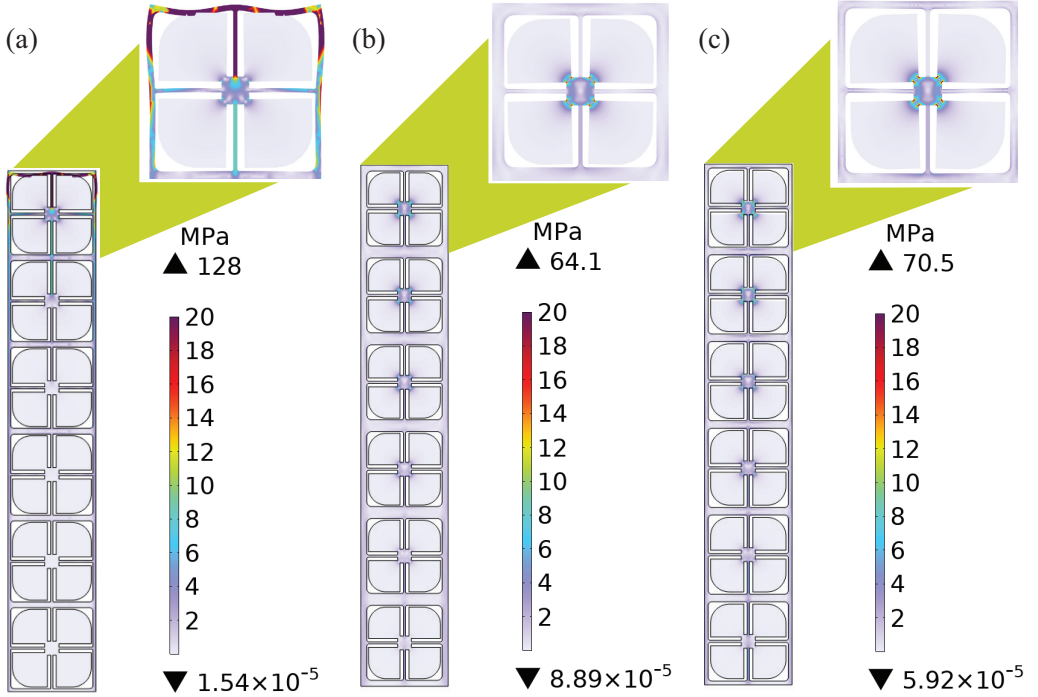


Figure 4.9.: von Mises stress field obtained in transient analysis of arrays formed by unit cells (a) P_A (b) P_B , and (c) P_C presented in Fig. 4.6. The stress fields were selected at time steps occurring the maximum von Mises stress for each array. The colormap indicates the variation of Von Mises stress with minimum and maximum values indicated by the triangles.

Individual	Central frequency [Hz]	Bandwidth [Hz]	Objective function ϕ_1
P_A	623.5	119	5.239
P_B	713.5	87	8.201
P_C	643	88	7.307

Table 4.2.: Band gap features – central frequency and bandwidth – for individuals P_A , P_B , and P_C illustrated in Fig. 4.11.

and 4.12c. Consequently, MMs formed by unit cells of configuration P_A may be unfeasible when subjected to high impact loads. Therefore, unit cell configurations P_B and P_C are recommended, given their maximum von Mises stresses of 57.8MPa and 63.3MPa, respectively.

As in the previous analysis, we also verify the vibration attenuation functionality of plates formed by the unit cells containing non-uniform beams. The transmission loss diagrams of such plates are shown in Fig. 4.13. In all diagrams, it is again

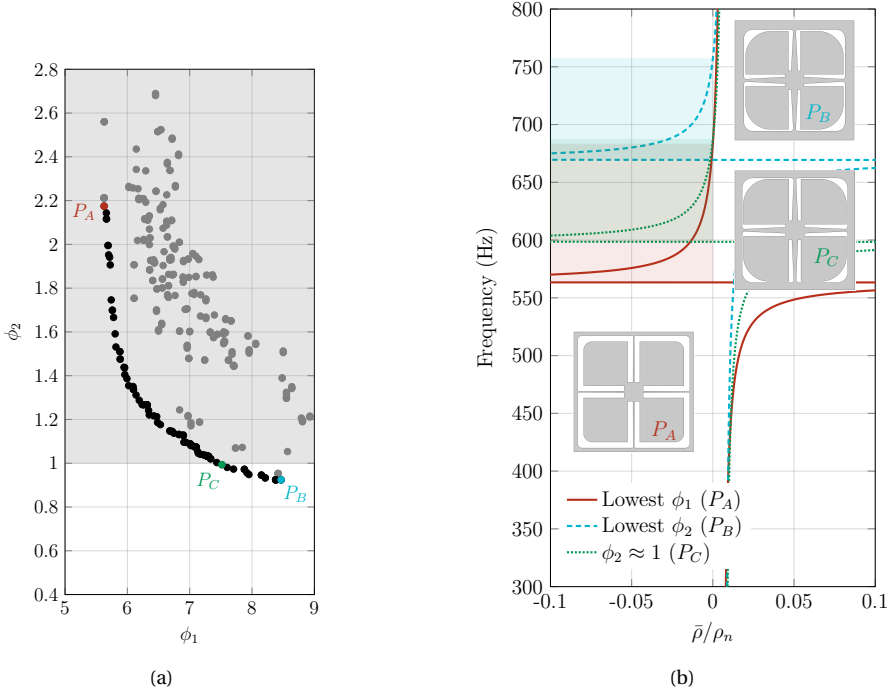


Figure 4.10.: (a) Optimal Pareto front obtained by solving the multi-objective optimization problem for an unit cell containing non-uniform beams. Individuals P_A , P_B , and P_C highlights the cases with lowest ϕ_1 , lowest ϕ_2 , and ϕ_2 close to 1, respectively. (b) Optimized EMD for individuals P_A , P_B , and P_C with their corresponding band gaps highlighted by the shaded areas and their unit cell designs.

realized a transmission attenuation at the same frequency ranges of the band gaps identified in Figs. 4.10 and 4.11.

From the analyses of the individuals highlighted in the Pareto fronts from Fig. 4.6 and Fig. 4.10, we can observe a certain trend in how the unit cells are designed. When considering the solutions where the first fitness functions ϕ_1 are dominant (points P_A), the unit cell is usually formed by thin uniform beams, the frame is narrow, and the fillet at the connection between the frame and the beams has a small radius. We also notice that the resonator's mass tends to fit the frame's cavity, i.e., the mass is maximized. For the unit cells related to the points P_B (dominant solution is associated to the second fitness function ϕ_2), the frame and the beams are thicker and non-uniform, the fillets have larger radius, and the resonator's mass is smaller. This explains the generation of higher-frequency band gaps. Lastly, we notice that the unit cells associated to the points P_C are a combination of the two previous points. The beams' width of such unit cells can slightly vary to create the low-frequency band gap and to have a better stress distribution. Moreover, the

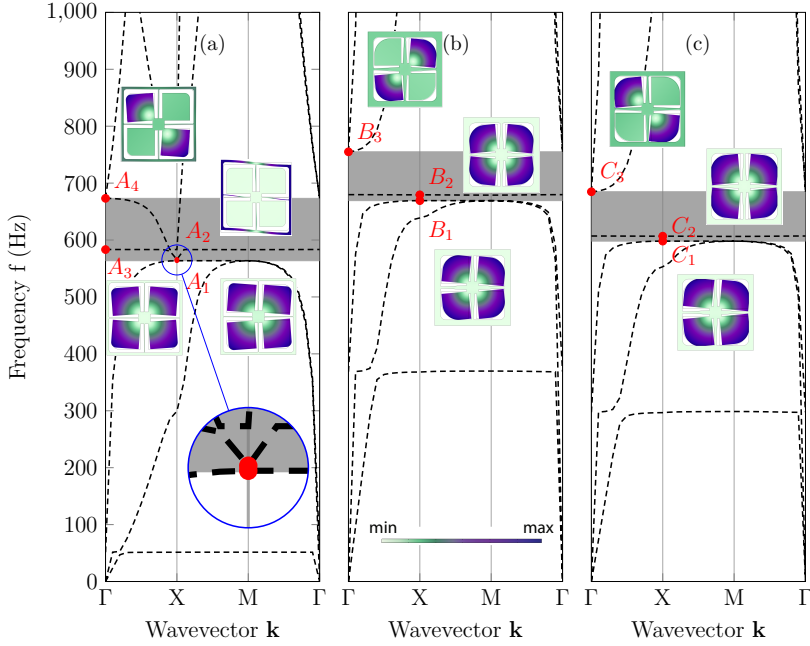


Figure 4.11.: Dispersion curves of the selected individuals (a) P_A , (b) P_B , and (c) P_C , respectively, from the Pareto front presented in Fig. 4.10. Complete band gaps are colored in gray. The inset shows the deformation motion of the unit cell at the flat edges defining the band gaps.

external frame is wide enough to ensure less deformation. Such design aspects will guarantee the overall performance of the A/E MM structure.

4.5. CONCLUSIONS

This chapter investigates the critical role of stress analysis in the design of impact-loaded A/E MMs, emphasizing its importance in ensuring optimal vibration attenuation and mechanical performance at the early design phase. Failure to evaluate structural integrity during the design process of A/E MMs may lead to fragile designs prone to breaking before achieving desired vibration attenuation. To mitigate this risk, we proposed a design optimization strategy that integrates stress analysis into A/E MM design to attenuate vibration induced by impact loads. Our research demonstrates that overlooking stress analysis can yield to structurally unfeasible configurations under such loading conditions. We concurrently obtained unit cell designs attaining comparable vibration attenuation performance and enhanced load resistance, suitable for practical applications.

By using a multi-objective optimization approach, we have generated a set of viable solutions, enabling the selection of the most appropriate design for applications with distinct impact load amplitudes. To confirm the attenuation

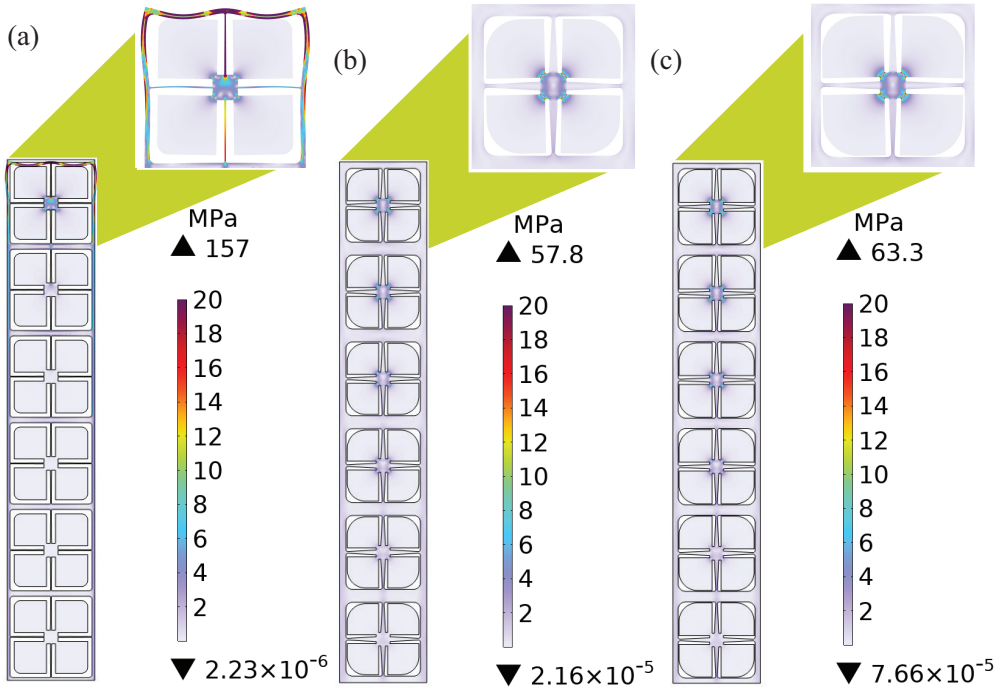


Figure 4.12.: von Mises stress field obtained in transient analysis of arrays formed by unit cells (a) P_A (b) P_B , and (c) P_C presented in Fig. 4.10. The stress fields were selected at time steps occurring the maximum von Mises stress for each array. The colormap indicates the variation of Von Mises stress with minimum and maximum values indicated by the triangles.

and mechanical performance of the metamaterial solutions, future work will focus on the experimental validation through modal impact analysis—to extract the vibration attenuation—and dropped-weight impact testing—to quantify the stress levels. Moreover, the diverse solutions provided by our strategy facilitate the design of graded A/E MM-based structures, significantly expanding the range of wave attenuation. This feature holds immense value, since impact loads are characterized by wide frequency spectrum. Thus, mitigating the adverse effects of impact loads across wide frequency bands is extremely necessary. The exploration of graded A/E MMs for absorbing impact loads will be numerically and experimentally considered in a future work. One of the challenges found in our approach is the computational time demanded by the evaluation of the design variables vector. As a future work, we will combine our optimization strategy with reduced order models (ROMs) that allow a fast calculation of the dynamic properties of the metamaterial. Although numerical damping was introduced to provide better convergent solutions, it is important to investigate the effect of material damping into the vibration and mechanical performances, which will be addressed in further studies. In essence, our proposed

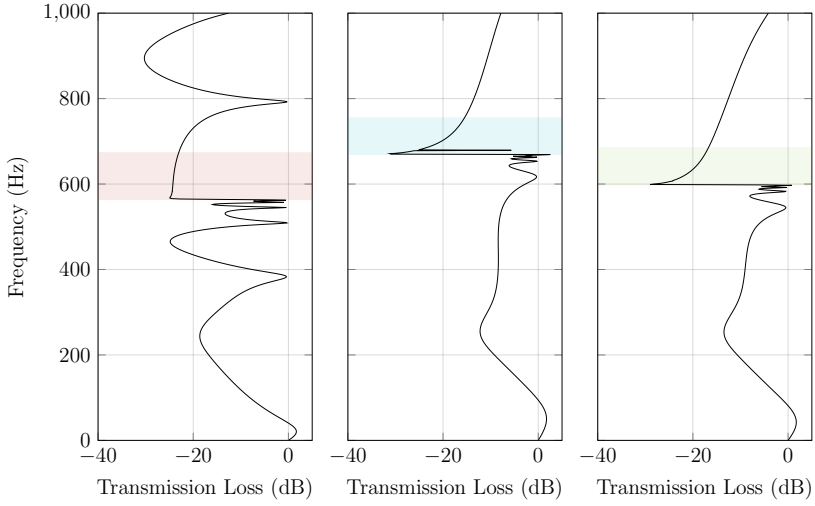


Figure 4.13.: Transmission loss diagrams for metamaterial plates containing unit cells P_A (left), P_B (middle), and P_C (right) with non-uniform beams (as presented in Fig. 4.10). The shaded areas indicate the band gap found by solving the EMD and the asymmetric peaks indicate the wave attenuation due to the unit cells' resonance.

multi-objective design optimization opens new avenues for the development of A/E MMs, enhancing their ability to filter vibration of certain frequencies from structures under challenging impact loading conditions, such as foundations that are installed in onshore and offshore environment through an impact hammer.

5

EXPERIMENTAL EVALUATION OF META-CUSHION PRINCIPLE IN FILTERING MECHANICAL VIBRATION

With the meta-cushion's functionality verified and optimized, it is essential to experimentally validate its attenuation performance before its implementation in offshore pile installations. Therefore, this chapter examines the meta-cushion's mechanical filtering ability by fabricating diverse prototypes with the ability to attenuate single and multiple frequency ranges. Such prototypes are subjected to modal analysis testing, where a modal impact hammer induces propagating mechanical waves with distinct wavelengths. The attenuation functionality is validated by comparing the blocked wavelengths exhibited in the experimental and numerical transmission loss diagrams. This chapter is a transcription of the journal paper under review in *Ocean Engineering*⁴.

5.1. INTRODUCTION

The increasing installation of offshore wind farms across Europe is boosted to meeting the 42.5% renewable energy target by 2030 [322]. To support such wind farms, monopiles are typically used as foundations due to their cost-effectiveness and ease of installation. Current monopile sizes range from 4 to 8 meters and 25 to 40 meters in diameter and length, respectively [179, 217, 323], however, future wind farms are expected to require even larger monopiles [1]. The installation process involves driving the monopile into the seabed through impact hammer blows, which can provide energy ranging from 150 kJ to 6600 kJ [324]. The high impact hammer blow induces longitudinal wave propagation, leading to pile resonance and consequently causing radial excitation of the pile wall. This radial motion interacts with the surrounding water, emitting noise waves that can travel long distances in

⁴Vasconcelos, A.C.A., Schott, D., Jovanova, J., Mechanical wave filtering through meta-cushions: an experimental study on vibration attenuation performance, *Ocean Engineering* (*under review*).

marine environment, causing potential risks to marine life, such as hearing loss and even death [11–13].

A major challenge in the installation of large monopiles is the generation of noise at frequencies below 1000 Hz, where current noise mitigation systems are less effective. For instance, bubble curtains, the most commonly used noise reduction system, can reduce noise by approximately 25 dB at 1000 Hz, however, their effectiveness decreases at lower frequencies [17]. This limitation leads to the use of multiple noise mitigation systems, increasing both operational costs and time. To address the low-frequency underwater noise problem, Vasconcelos et al.(2024) [273] proposed the meta-cushion, a metamaterial-based structure designed to filter impact load energy, particularly in the lower frequency range (see Fig. 5.1a). The meta-cushion is composed of periodic unit cells with resonant elements that behave as a spring-mass system. When longitudinal waves from hammer blows pass through the meta-cushion at frequencies associated with high underwater noise, the resonant elements move in the opposite direction, blocking wave transmission. This phenomenon, known as negative effective mass density, results from the metamaterial's unique geometry. [143, 286, 308]. Although Vasconcelos et al.(2024) [273] demonstrated the meta-cushion's functionality numerically, experimental validation remains pending. Further research is also needed to develop a meta-cushion that mitigates vibrations across multiple frequency ranges while simultaneously addressing potential monopile resonances that could cause high noise at other frequency ranges.

This chapter aims to experimentally demonstrate the vibration attenuation capabilities of the meta-cushion using a 3D-printed prototype (see Fig. 5.1b). The prototype features spiral-shaped resonators with unique geometric properties, chosen over those proposed by Vasconcelos et al. (2024) [273] for their ability to generate band gaps in similar frequency range while using smaller unit cells, potentially lowering manufacturing costs when scaled up. Numerical and parametric analyses are conducted to select spiral resonators' geometric features attenuating waves at distinct frequency ranges. By combining different spiral resonators, the meta-cushion generates multiple band gaps, enabling vibration mitigation across frequency ranges. Experimental validation is conducted using modal impact testing to measure the meta-cushion's response to an excitation load, identifying the frequency ranges where vibration is reduced and demonstrating the graded meta-cushion's filtering capability. These results highlight the potential of the meta-cushion as a noise mitigation system during monopile hammering.

5.2. DESIGN OF THE META-CUSHION

As depicted in Figure 5.1a, the meta-cushion is positioned between the hammer and the monopile to filter unwanted vibrations related to high noise levels. To assess this filtering ability, a prototype of the meta-cushion is developed (Fig. 5.1b) consisting of three components: the arrays of five spiral-shaped unit cells and the top and bottom plates where the arrays are inserted. This design facilitates the combination of arrays with distinct unit cell designs, enabling filtering across multiple frequency ranges. Each unit cell features four identical spiral paths, spaced 90° apart, which

define the inner mass and the curved beams connecting the mass to the external frame. By tuning the geometric parameters of the spiral paths, the unit cell can attenuate vibrations at specific frequency zones.

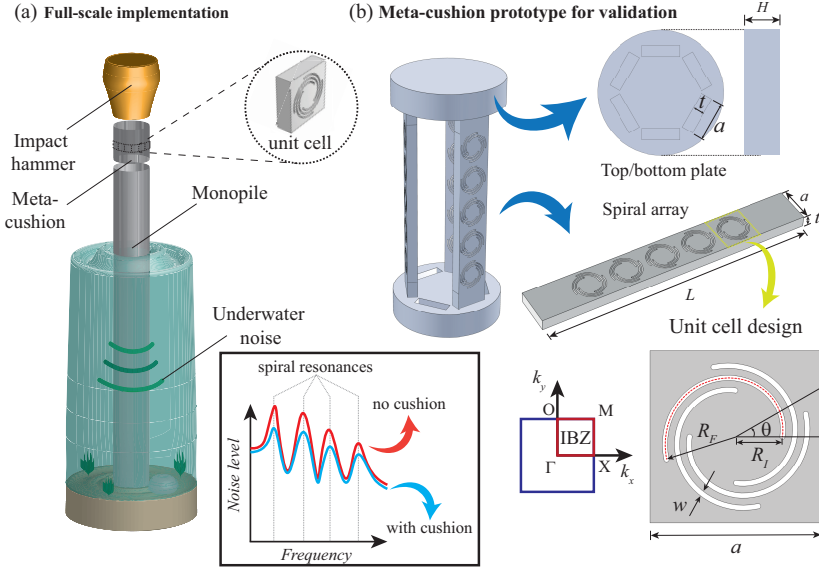


Figure 5.1.: Design of meta-cushion for attenuating multiple frequency ranges. The meta-cushion is formed by top and bottom plates containing six cavities where metamaterial arrays are inserted. The arrays are composed by unit cells consisting of spiral-shaped resonators where geometric parameters are tuned to reach several frequency ranges.

The spiral path is inspired by the Archimedean spiral, which maintains a constant distance between successive turns. The Archimedean spiral can be described by the following equation in polar coordinates:

$$R(\theta) = R_I + b\theta, \quad (5.1)$$

where R_I and b are parameters defining the initial radius of the spiral and the spiral growth rate, respectively, and θ is the angle in rad, being θ_I and θ_F the angles where the spiral curve starts and ends, respectively. The parameter b can be expressed as:

$$b = \frac{R_F - R_I}{2\pi n_t}, \quad (5.2)$$

where R_F is the final radius of the spiral and n_t is the desired number of turns.

Since the meta-cushion analysis is performed through COMSOL Multiphysics, it is convenient to define the spiral shapes by means of parametric curves. The spiral equations are then converted from polar to Cartesian coordinate system, where their

corresponding parametric forms are given by:

$$\begin{aligned} x &= r \cos \theta, \\ y &= r \sin \theta. \end{aligned} \quad (5.3)$$

Finally, the starting angle and the end angle of the spiral are defined as θ_I and θ_F . The distance d between the turns and the gap g are defined as:

$$\begin{aligned} d &= \frac{R_F - R_I}{n}, \\ g &= d - w, \end{aligned} \quad (5.4)$$

where w is the width of the curved beams. These geometric parameters will be used to tune the attenuation ability of the spiral resonator, which will be described later.

5.3. NUMERICAL ANALYSIS

In this section, the vibration attenuation capability of the meta-cushion is numerically evaluated. First, the resonant modes of a periodic unit cell are analyzed using dispersion curves. Then, the unit cells are assembled into arrays, from which their attenuation capability is further assessed through frequency response functions.

5.3.1. DISPERSION CURVES OF SPIRAL RESONATORS

The dynamic behavior of a metamaterial is characterized by infinite periodic structures composed of unit cells, the smallest repeating elements of the structure. Wave propagation in these structures is analyzed using Bloch's theorem, which describes it as the response of a wave in a reference unit cell combined with an exponential term that defines the amplitude and phase changes as the wave travels through adjacent unit cells. Thus, the displacement field is expressed as:

$$\mathbf{u}(\mathbf{r}) = \bar{\mathbf{u}}(\mathbf{r}, \mathbf{k}) e^{i(\mathbf{k} \cdot \mathbf{r})}, \quad (5.5)$$

where $\mathbf{r}$ is the position vector, $\bar{\mathbf{u}}(\mathbf{r}, \mathbf{k})$ is a periodic displacement vector, $e^{i(\mathbf{k} \cdot \mathbf{r})}$ is a plane wave with $i = \sqrt{-1}$ and wave vector $\mathbf{k}$ that varies through the boundaries of the irreducible Brillouin zone (Fig. 5.1b). The finite element method is used to define the following eigenvalue problem,

$$[\mathbf{K}(\mathbf{k}) - \omega_n^2 \mathbf{M}(\mathbf{k})] \mathbf{u} = \mathbf{0}, \quad (5.6)$$

where $\mathbf{K}(\mathbf{k})$ and $\mathbf{M}(\mathbf{k})$ are the stiffness and mass matrices, and $\mathbf{u}$ is the eigenvector that provides the unit cell's modal shape related to the angular frequency ω_n . The eigenvalue problem in Eq. (5.6) is solved by providing the wave vectors $\mathbf{k}$, which will give the frequencies ω_n . These two variables will then constitute the dispersion curves, which show the unit cell behavior when wave propagates through it. Figure. 5.2 displays an example of dispersion curves when solving the eigenvalue problem for an unit cell with the following geometric parameters: $a = 0.04 \text{ m}$,

$R_I = 0.008\text{ m}$, $R_F = 0.017\text{ m}$, $\theta_I = 3\pi/5$, $\theta_F = 8\pi/5$, $n_t = 0.8$, $w = 0.0012\text{ m}$. The black curves indicate eigenmodes excited in the in-plane direction. The eigenmodes related to points I ($k_x = 1$, $k_y = 0$ and $n = 1$) and III ($k_x = 0$, $k_y = 0$ and $n = 4$) are shown in the inset, where it is observed the resonator motion in the same plane of wave propagation (the colormap indicates the range between the minimum and maximum unit cell displacement). On the other hand, the gray curve represents modes excited in the out-of-plane, which it is indicated by bending motion of the unit cell from point II ($k_x = 1$, $k_y = 0$ and $n = 3$).

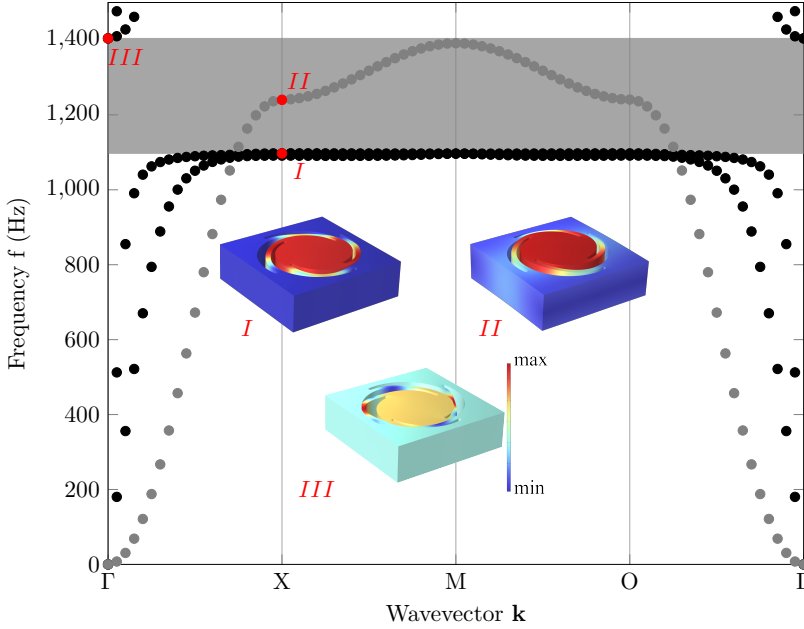


Figure 5.2.: Dispersion curve of a spiral-shaped unit cell. The gray area indicates the band gap in the in-plane direction. The inset highlights the eigenmodes at the edges of the band gap (points I and III) and at the bending related to the bending mode. The colormap indicates the variation in Von Mises stress in the unit cell.

To explore the ability of multiple frequency range attenuation of the meta-cushion, it is necessary to design unit cells generating band gaps in different frequency ranges. This is achieved through a parametric study that examines band gaps for various geometric parameter sets. In this study, we select the following parametric vector: $\theta_I = [0 : \pi/10 : \pi]$, $\theta_F = [\pi/2 : \pi/4 : 7\pi/4]$ (note that the other geometric parameters of the unit cell vary with θ_I and θ_F , therefore, they are not considered in the parametric investigation, except the width of the gap w , which remains constant due to manufacturing limitations). Figure 5.3 shows the band gap when the angles vary. It is observed that the band gap position leverages when the initial angle θ_I increases, while its position decreases when the angle θ_F varies from π to $7\pi/4$. From this analysis, three types of spiral resonators exhibiting band gaps at distinct

frequency ranges are selected, one design (B) with band gap around the attenuation zone of traditional noise mitigation systems (around 1000 Hz) and two designs with lower band gaps in potential frequency ranges for the next generation of monopiles (A and C). The selected geometries A, B, and C are indicated in Table 5.1. For all designs, $n_t = 0.8$, $R_I = 0.008\text{ m}$, and $R_F = 0.017\text{ m}$.

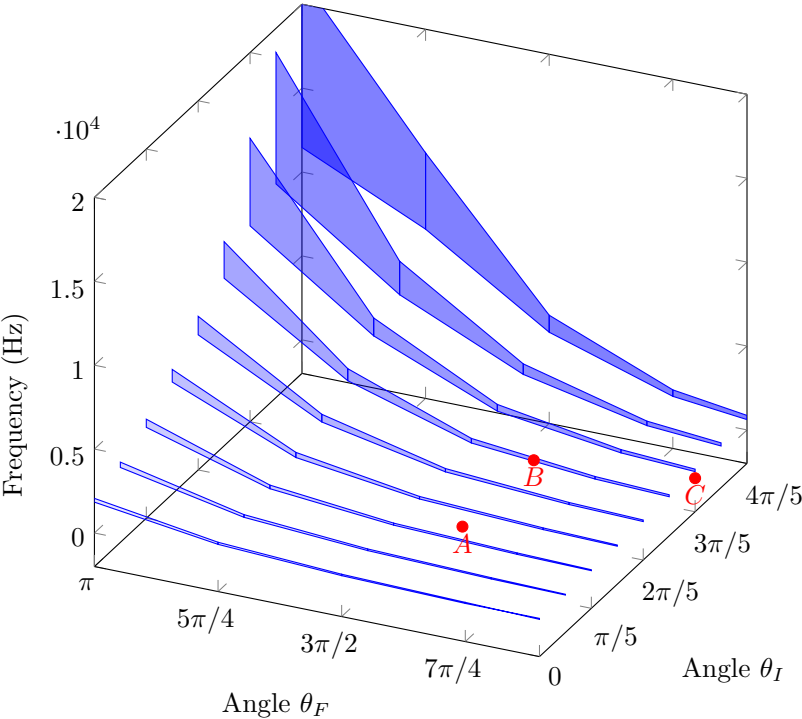


Figure 5.3.: Influence of geometric parameters on band gap formation. The blue meshes represent the bandwidth identified in the studies. The selected designs A, B, and C are highlighted by the red dots.

	Geometric parameters			
	a [mm]	θ_I [rad]	θ_F [rad]	t_b [mm]
A	40	$\pi/4$	$8\pi/5$	1.2
B	40	$\pi/2$	$8\pi/5$	1.2
C	40	$3\pi/5$	$19\pi/10$	1.2

Table 5.1.: Geometric features of spiral-shaped unit cells constituting the meta-cushion.

Figure 5.4 depicts the dispersion curves of waves propagating through an infinite domain formed by periodic spiral unit cells. Band gaps related to in-plane waves are identified by gray rectangles in frequency ranges varying from 496.87 Hz to 614.7 Hz

(type A), from 824.05 Hz to 1043.97 Hz (type B), and from 406.5 Hz to 556.9 Hz (type C). This is indicated by the unit cell deformation modes at the bottom edge of the band gap (modes A_I , B_I , and C_I). The modes A_{II} , B_{II} , and C_{II} indicate the out-of-plane deformation of the unit cell, where the squared frame bands and the spiral resonator moves perpendicularly to the wave propagation direction.

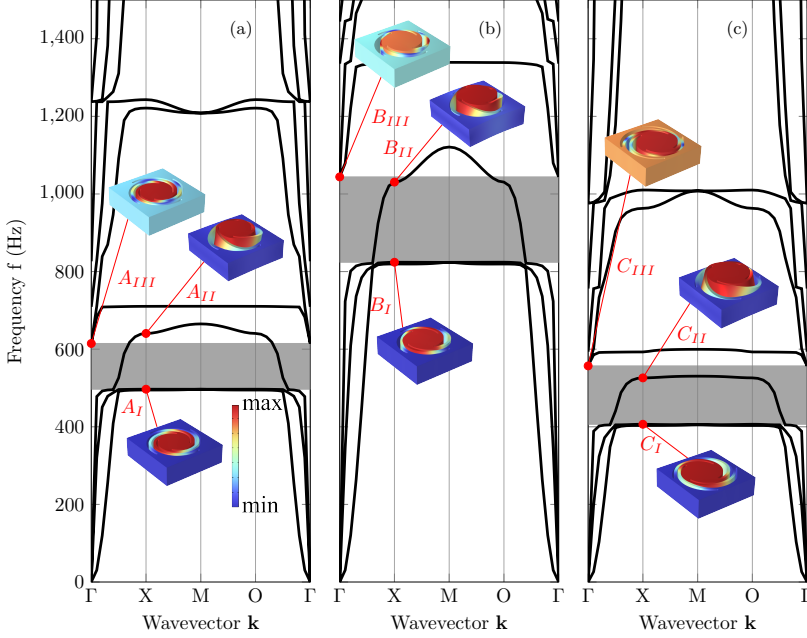


Figure 5.4.: Dispersion curves of spiral-shaped unit cells of types (a) A, (b) B, and (c) C indicated in Tab. 5.1. The gray area indicates the band gap associated to in-plane wave propagation. The curve inside the highlight band gap is related to out-of-plane modes as indicated by modes A_{II} , B_{II} , and C_{II} . The colormap indicates the Von Mises stress variation in each unit cell.

5.3.2. FREQUENCY RESPONSE FUNCTION ANALYSIS

Although the dispersion curve analysis can well indicate the vibration attenuation zones in infinite periodic domains, it is essential to verify if the attenuation performance still remains in a finite structure, such as the spiral arrays and the assembled meta-cushion. The analysis of such structures is performed through frequency response functions (FRFs), from which a time-harmonic excitation is applied in one end of the structure, while the response is measured in the opposite end. To calculate the frequency response functions (FRFs), Fourier transform is applied in the input and output signals, from which the following expression is used:

$$\text{FRF} = 20 \log \left(\left| \frac{X(\omega)}{F(\omega)} \right| \right), \quad (5.7)$$

where $X(\omega)$ and $F(\omega)$ are the frequency spectrum of the response and input signals, respectively (notice that the logarithm term in Eq. (5.7) was introduced to express the attenuation levels in terms of decibels (dB)). FRFs are commonly used to indicate the behavior of a structure due to an applied excitation. A positive infinite peak in the FRF indicates a high structural response, corresponding to a natural frequency of the structure. Conversely, a low response indicates reduced energy transmission from the input.

The spiral arrays and assembled meta-cushion are modeled as a linear PLA solid with Young's modulus $E = 3.16 \text{ GPa}$, density $\rho = 1250 \text{ kg/m}^3$, and Poisson's ratio $\nu = 0.33$. As the viscoelastic properties of PLA can change the vibration attenuation performance of the meta-cushion, viscoelasticity is integrated into the numerical model by modifying the stress tensor, which is given by

$$\sigma_v = (K' + iK'')\epsilon_e, \quad (5.8)$$

where K' storage modulus, K'' loss modulus, ϵ_e is the strain tensor of the elastic model. The storage and loss moduli are determined through dynamic mechanical analysis (DMA), where a force oscillating at a set frequency is applied at PLA samples under varying temperatures [325]. Extensive literature exists on using DMA to obtain viscoelastic properties of neat and composite polymers [326–328]. For PLA samples manufactured under conditions similar to the ones presented in this work, the storage and loss moduli are, respectively, $K' = 2220 \text{ MPa}$ and $K'' = 112.9 \text{ MPa}$.

Regarding the boundary conditions, the external excitation is represented by a time-harmonic point load applied at the center of the cross-section area of one array end (of the meta-cushion's top plate), while acceleration is measured in the middle point at the opposite end (at the meta-cushion's bottom plate). The load frequency varies from 1 Hz to 1500 Hz, covering the attenuation range of interest. Stress-free boundary conditions are applied at the remaining array and meta-cushion surfaces. The excitation load and the measured response are then replaced in Eq. (5.7) to obtain the numerical FRF, from which the results will be further compared to the experimental data.

5.4. EXPERIMENTAL ANALYSIS

The experimental analysis of the meta-cushion starts with evaluating the FRFs of the arrays containing the unit cells from Table 5.1. The arrays are made of PLA (Fig. 5.5a) and are fabricated using a FDM 3D manufacturing process. Once they have indicated the desired band gap, the arrays are inserted to the top and bottom plates, also made of PLA and manufactured via the same 3D printing process, finalizing the meta-cushion assemble. Initially the meta-cushion is analyzed for arrays of the same type (Fig. 5.5b) and then for combined types. The setup used to calculate the FRFs is presented in Fig. 5.6, where the metamaterial structures are suspended by fishing wires. An excitation signal is applied in one structure end by a modal impact hammer (Type DJB of model IH/02) with sensitivity 2.58 mV/N, while an accelerometer (ICP[®] accelerometer model 35C22 with sensitivity 1 mV/m/s²) measures the response in the opposite end. The signals are sent from the

data acquisition system (DAQ) Sirius-HD to a laptop, where the FRF is processed in the software DewesoftX. The FRF is evaluated from 1 Hz to 10000 Hz, from which a frequency resolution of 5 Hz was chosen, but for sake of visualization the FRFs are shown until 1500 Hz, where the band gaps have been identified in the numerical simulations.

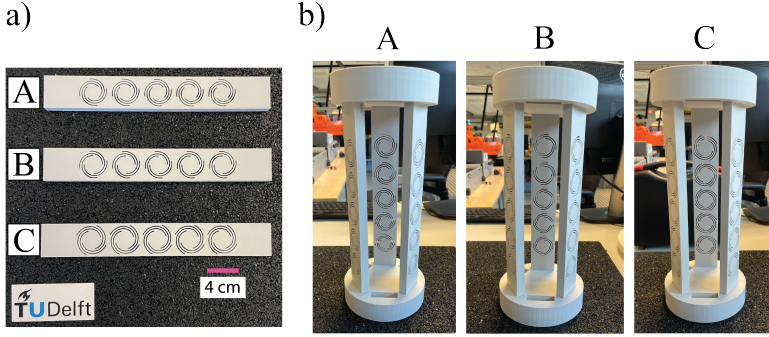


Figure 5.5.: a) Printed spiral arrays for assembling the meta-cushion. b) Meta-cushions each containing three identical arrays of each type for experimental validation.

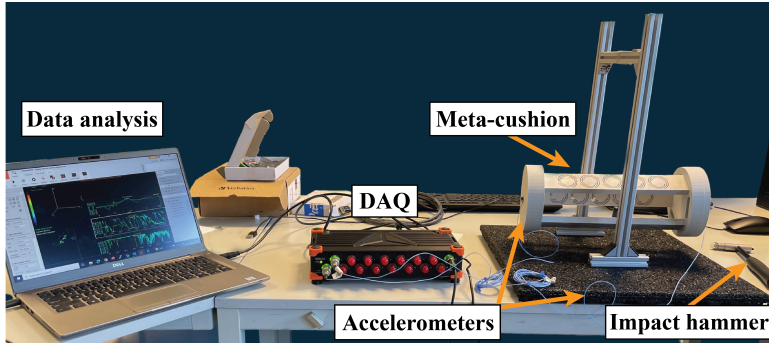


Figure 5.6.: Experimental setup used to measure the frequency response functions (FRFs) of arrays and assembled meta-cushions.

5.4.1. VALIDATION OF SPIRAL ARRAYS ATTENUATION

Figure 5.7 presents the FRFs obtained numerically and experimentally for the three array designs indicated in Table 5.1. The numerical elastic results indicate a reduction in the FRF within the frequency range near the band gaps predicted by the dispersion curve analysis (gray rectangles), confirming that the attenuation properties identified for infinite domains are applicable to the spiral arrays (finite domains). However, the numerical results are not aligned with the experimental measurements, which show the attenuation zone shifting towards higher frequencies.

This discrepancy suggests that damping effects should be incorporated into the numerical model [329], particularly for high-frequency band gap design – notice that the elastic and experimental results of arrays A and C shows better agreement than results of array B. The numerical results with incorporated viscoelastic effects as described in Eq. (5.3.2) are shown in blue in Fig. 5.7, from which an improved agreement with experimental data is observed. Regarding the amplitude of attenuation, the experimental results reveal a reduction in attenuation level and a larger attenuation frequency range at higher frequencies in comparison to the elastic model, highlighting the frequency-dependent nature of viscoelastic materials [330–332].

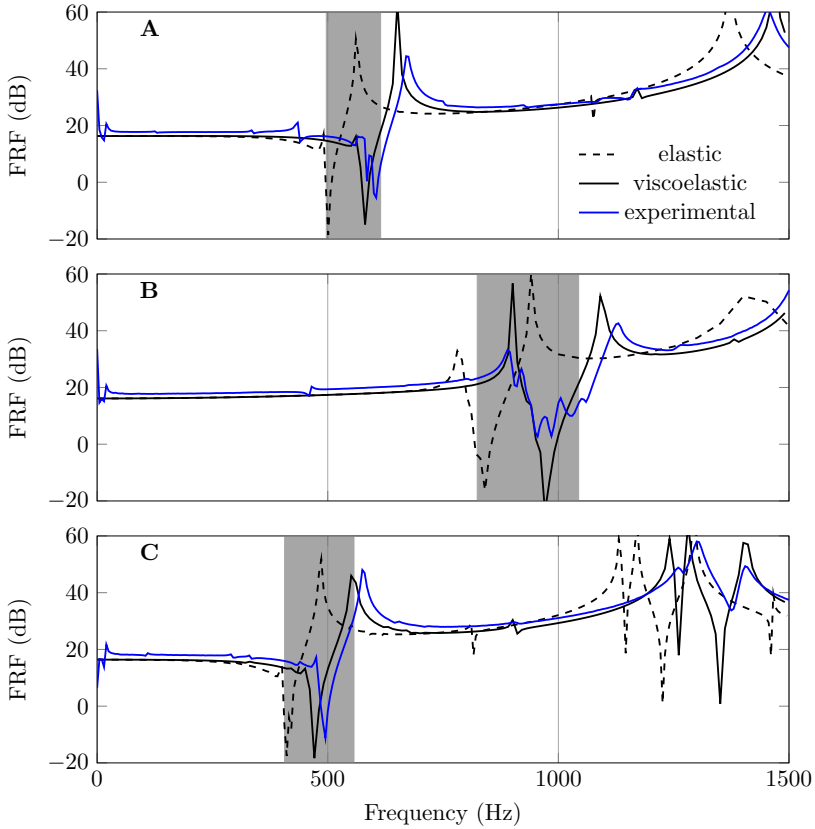


Figure 5.7.: Comparison between the FRFs obtained through numerical and experimental analyses of spiral arrays. The numerical analysis was performed for both elastic and viscoelastic model of the arrays. The gray areas indicate the band gaps obtained via the dispersion curve analysis.

5.4.2. VERIFICATION OF META-CUSHION PRINCIPLE

After demonstrating that the spiral arrays can filter waves within desired frequency ranges, it is crucial to assess whether this mechanism still remains when the meta-cushion is assembled. To verify this, three identical spiral arrays (as indicated in Fig. 5.5b) are inserted in the top and bottom plates with 130 mm of diameter and 30 mm of height H . The modal impact testing is conducted where the excitation load is applied in the center of the top plate, with the response measured at the center of the bottom plate. The experimental measurements are then compared with the attenuation zone predicted by the viscoelastic model of the arrays (as indicated in Fig. 5.7) and with a numerical model of the meta-cushion with same boundary conditions. Figure 5.8 displays the FRFs for each meta-cushion, with the gray areas indicating attenuation zones of the array model and the blue curves indicating the FRFs of the numerical models of the meta-cushion. Although peaks are observed in other frequency zones, for both numerical and physical model of the meta-cushion, the attenuation zones remain consistent with those identified in the spiral array analysis. Despite that, both models indicate a key feature of locally-resonant metamaterials, where the vibration attenuation performance relies on the resonators instead of the periodicity. The resonant behavior of each single meta-cushion model is illustrated in Fig. 5.9. The colormap indicates the acceleration field due to a harmonic point load with frequency associated to the highest attenuation value identified in the numerical simulation. Notice that the highest acceleration value occurs in the spiral resonators near the point load and the acceleration reduces along the array length.

In the final step of the experimental analysis, the principle of multiple attenuation zones of a graded meta-cushion is verified. With the three types of spiral arrays and six cavities available in the top/bottom plates, four graded meta-cushion configurations are investigated, as indicated in Table 5.2, from which two attenuation zones are expected in configurations AB, AC and BC, and three attenuation zones are expected in configuration ABC.

Array type	Meta-cushion configurations			
	AB	AC	BC	ABC
A	2	2	-	2
B	2	-	2	2
C	-	2	2	2

Table 5.2.: Meta-cushion configurations for verification of multiple attenuation zones. The number two indicates the quantity of arrays inserted in the cavities of the top/bottom plates.

The FRFs for the graded meta-cushion configurations are presented in Fig. 5.10, where gray rectangles highlight the attenuation ranges observed in the experimental investigation of the single arrays. In Figure 5.10, two main drops for configuration AB are identified at frequencies ranging from 620 Hz to 640 Hz and from 940 Hz to 1045 Hz, aligning well with the attenuation zones seen in the single array analysis.

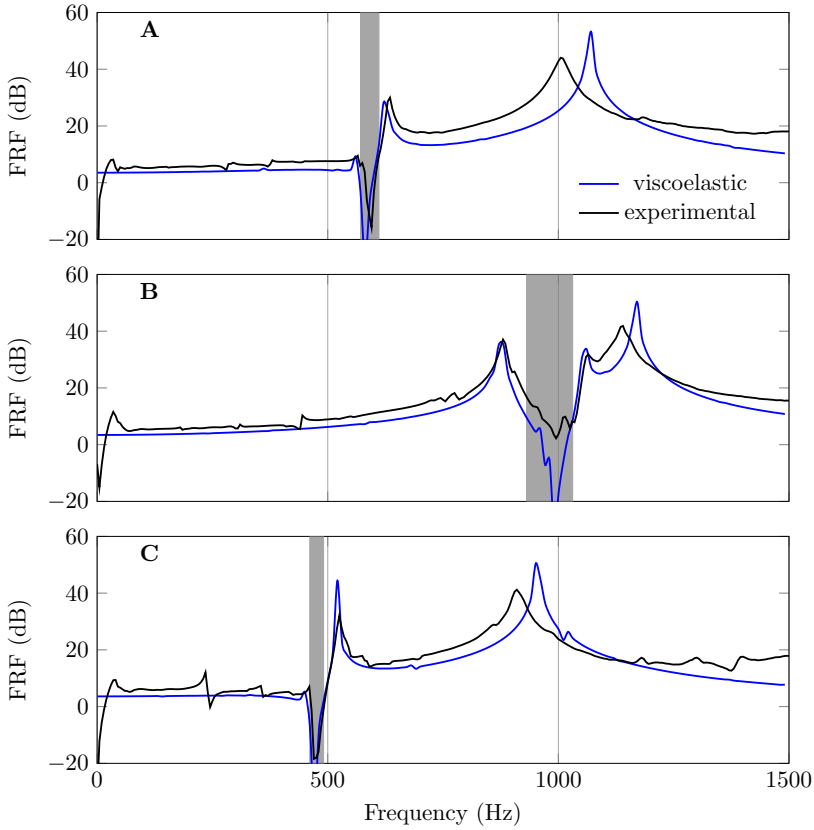


Figure 5.8.: FRFs of meta-cushions formed by three spiral arrays of type A, B, and C. The gray areas indicate the attenuation zones identified in the viscoelastic numerical model displayed in Fig. 5.7.

Similar patterns are observed for the other configurations; 470-520 Hz and 625-645 Hz for configuration AC, 470-525 Hz and 935-955 Hz for configuration BC, and 485-515 Hz, 620-640 Hz and 955-1040 Hz for configuration ABC. The blue curves indicate the numerical results of the combined cushions, where a reduction in FRF is also observed in the band gap regions. The extra peaks identified for frequencies higher than 1000 Hz, particularly for configurations BC and ABC, can be attributed to the increase of wave reflections due to the addition of more arrays in comparison to the case of a single-cushion. The deformation pattern of each configuration is obtained by numerical models illustrated in Fig. 5.11. The patterns are identified in each frequency related to the highest attenuation for each configuration, which are highlighted by the red vertical lines in Fig. 5.10. Notice that the spiral resonators are mostly excited at their corresponding frequencies, while outside those, the resonators move slightly. This confirms the ability of the meta-cushion in filtering waves from distinct frequencies.

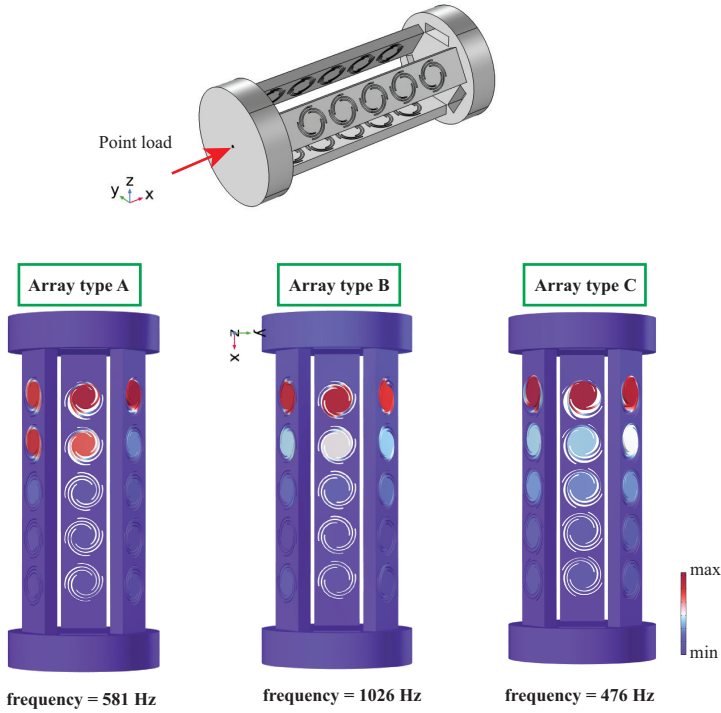


Figure 5.9.: Acceleration field in each single meta-cushion due to point load with frequency at corresponding highest attenuation value. The acceleration decreases as the wave caused by the excitation point load propagates through the spiral resonators.

Interestingly, the experimental responses of configurations AC and ABC, with band gaps in close frequency ranges, show a reduction of the resonant peaks presented between the attenuation zones, which can be related to band gap merging and damping effects [332, 333]. The merging principle can be applied to increase the width of band gaps in low-frequency ranges, which have demonstrated to be narrower for arrays type A and C than for array type B. In configuration ABC, the shift of the highest peak above 1000 Hz is related to an increase in stiffness in comparison to the other configurations, which only contain four arrays. The increase in stiffness of configuration ABC is also indicated by the low FRF amplitude for frequencies near zero, indicating then low acceleration of the structure. The analysis presented here confirms the meta-cushion capability to attenuate vibrations across multiple frequency zones, demonstrating its potential in reducing multiple pile vibrations during its installation in an offshore environment.

5.5. CONCLUSIONS

This study demonstrated the effectiveness of the meta-cushion in attenuating vibrations across multiple frequency ranges, as shown by both experimental and

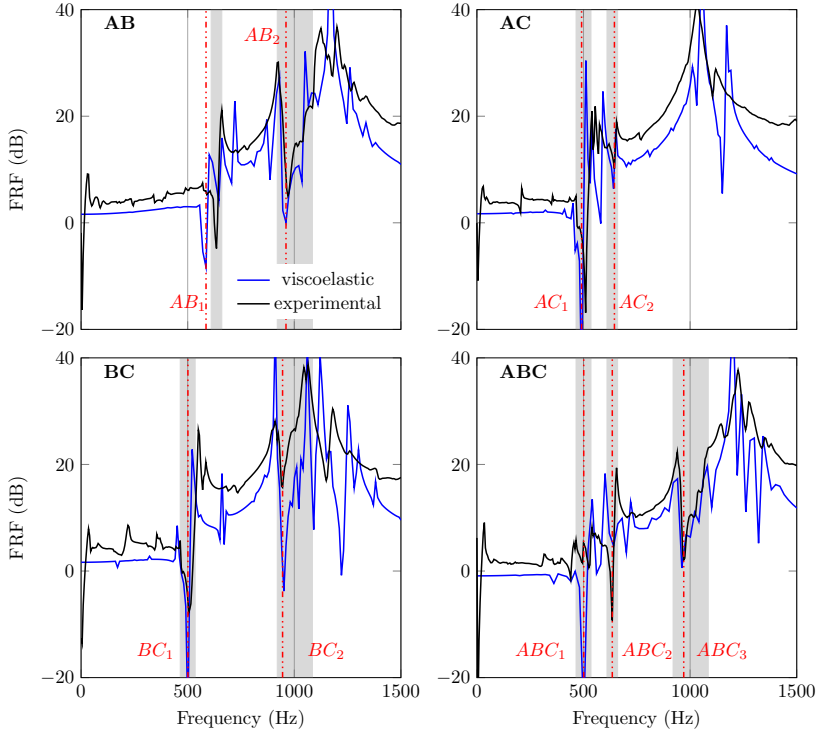


Figure 5.10.: Numerical and experimental investigation of meta-cushion principle for combined configurations as indicated in Tab. 5.2. The FRFs indicate multiple attenuation zones, which confirm the attenuation principle of the meta-cushions. The gray areas highlight the attenuation zones obtained in the experimental analysis of each array type, while the vertical red lines indicate the lowest response given by the numerical simulations.

numerical analyses. Experimental results for individual arrays targeting specific frequencies aligned well with numerical models, particularly when viscoelastic effects were considered. Similar behavior was observed for single and combined meta-cushions. While the meta-cushion provided attenuation zones near 1000 Hz broader than the numerical model, the performance was slightly reduced due to frequency-dependent damping. The experimental validation of combined meta-cushions confirmed the principle of multiple vibration attenuation. Although the low-frequency band gaps were narrow, combining adjacent gaps with different spiral shapes showed potential for improving low-frequency attenuation. Some extra peaks and shifts in the band gaps suggest the need for a refined viscoelastic model, but the numerical models remain reliable for assessing attenuation performance and can serve as a basis for developing new designs without requiring experiments.

Future research will focus on refining numerical models by incorporating viscoelastic effects more accurately, particularly in scenarios involving multiple

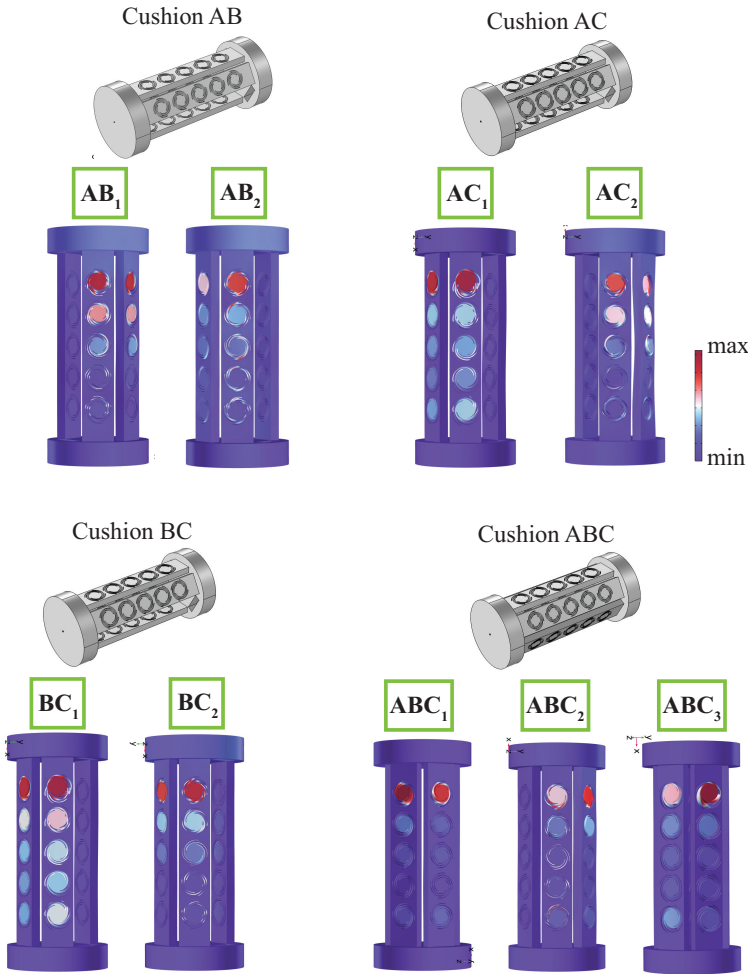


Figure 5.11.: Acceleration field in each combined meta-cushion due to point load with frequency at corresponding highest attenuation value. The acceleration decreases as the wave caused by the excitation point load propagates through the spiral resonators.

resonant effects. This will enhance the prediction of band gap location and attenuation performance. Additionally, to bring the meta-cushion closer to real-world application, its principle will be tested in an offshore environment. Here, the effectiveness of the meta-cushion in mitigating monopile vibrations and reducing underwater noise during pile installation will be measured against baseline data from installations without any noise mitigation system. By advancing these investigations, the meta-cushion will be well-positioned for full-scale implementation in offshore pile installations, marking a significant step forward in vibration

mitigation technology.

6

DESIGN METHODOLOGY OF A META-CUSHION FOR NOISE ATTENUATION

Previous chapters outlined the development of the meta-cushion, including its numerically verified and experimentally validated attenuation performance, as well as a design optimization strategy for high-stress scenarios that could affect its functionality. This chapter concludes this development by providing detailed guidelines for designing meta-cushions adaptable to different pile hammering operations, paving the way for practical implementation. This is a transcription of the journal paper in preparation for *Applied Acoustics*⁵.

6.1. INTRODUCTION

With only half of the planned energy capacity currently installed in Europe, a rapid expansion of offshore wind turbines (OWTs) is expected to meet 2030 targets. Newer OWTs are larger and more efficient [1], and are typically supported by monopile foundations, which offer a cost-effective manufacturing and installation solution. These monopiles, made from steel pipes ranging from 25 to 40 m in length and 4 to 8 m in diameter [179, 217, 323], are expected to increase in size to support next generation of OWTs. The installation process involves driving the pile into the seabed with an impact hammer, which generates longitudinal waves that propagate along the pile, converting to radial motion due to Poisson's effect [11]. This pile motion produces underwater sound waves capable of traveling over kilometers and reaching intensities high enough to harm marine life, potentially causing hearing loss and even death [13, 334].

To protect marine life during monopile installation, several noise mitigation systems (NMSs) have been developed, such as noise mitigation screens [335], bubble

⁵Vasconcelos, A.C.A., van Zijl, C., Schott, D., Jovanova, J., Design methodology of a meta-cushion for reducing underwater noise during offshore pile hammering, *Applied Acoustics* (*under review*).

curtains [282], and hydro damping screens [29]. These systems utilize impedance mismatch principle – either between water and screen material or between water and bubbles (free or encapsulated). Effective NMS design requires targeting the frequency ranges of peak noise emissions higher than threshold noise values [18]. Tsouvalas (2020) [23] stated that the frequencies of propagating waves are related to the geometric and material parameters of the monopile. His work indicated that the underwater pressure levels are dependent on the ring frequency of the cylindrical shell (f_r) and it concentrates at frequencies between $0.5f_r \leq f \leq 0.8f_r$. For a 4 m diameter monopile, this translates to noise radiating between 200 and 350 Hz. The traditional NMS are effective at frequency ranges around 1000 Hz, however, for lower frequencies, bubbles from 8 mm to 50 mm of radius are required. The generation of such bubble sizes is extremely complex, mostly when released from harsh environment, as turbulent waters. As future monopiles increase in size [336], addressing these lower frequencies will demand innovative solutions.

As a new solution to reduce the low-frequency underwater noise, we proposed in our previous work [273] a redesigned based on locally resonant metamaterials [53]. This metamaterial-based cushion, or meta-cushion, is placed between the impact hammer and the monopile, filtering out longitudinal waves tied to the monopile's radial resonance. The attenuation principle of the meta-cushion relies on the resonance of its internal elements, which are distributed along the propagation direction. This reduces underwater noise, protecting marine life while allowing essential installation vibrations to pass through. The principle of the meta-cushion was verified in a pile-water-soil interaction model using finite element simulations, which showed noise reduction at resonance-related frequencies. To target even lower frequencies, the meta-cushion employs spiral-shaped unit cells with curved beams for reduced stiffness. A follow-up study presented in Chapter 5 experimentally validated this filtering effect, exploring combinations of multiple resonator designs to broaden the attenuation range across varied frequencies. With the meta-cushion principle verified and validated, a framework can now be established to guide designers in systematically selecting resonator types and array distributions for a meta-cushion tailored to any monopile hammering procedure.

This chapter presents a design methodology for a meta-cushion aimed at reducing underwater noise from offshore monopile hammering. Key input parameters for critical noise levels are identified and applied to the design of the meta-cushion's resonant elements. These elements are assembled in various configurations, from which structural features critical for high-impact loads are evaluated. Numerical models are developed to examine the attenuation principle and mechanical features of the meta-cushion, along with underwater noise measurements. Selected designs are manufactured and experimentally tested in a small-scale monopile setup, from which resonators exhibiting the best attenuation properties and configurations with highest mechanical performance are selected. Finally, a nondimensional analysis of the selected resonator designs demonstrates their potential for noise attenuation across multiple frequency ranges by adjusting unit cell size and material properties. This design methodology provides a systematic approach to developing meta-cushions for effective underwater noise reduction during general monopile

installation.

6.2. DESIGN METHODOLOGY FOR A META-CUSHION

This section introduces the design methodology employed for developing a meta-cushion for reducing noise levels during pile hammering process, from which input parameters are defined and the procedure for selecting and investigating the noise attenuation and structural performance is presented. A schematic of the proposed methodology is illustrated in Figure 6.1.

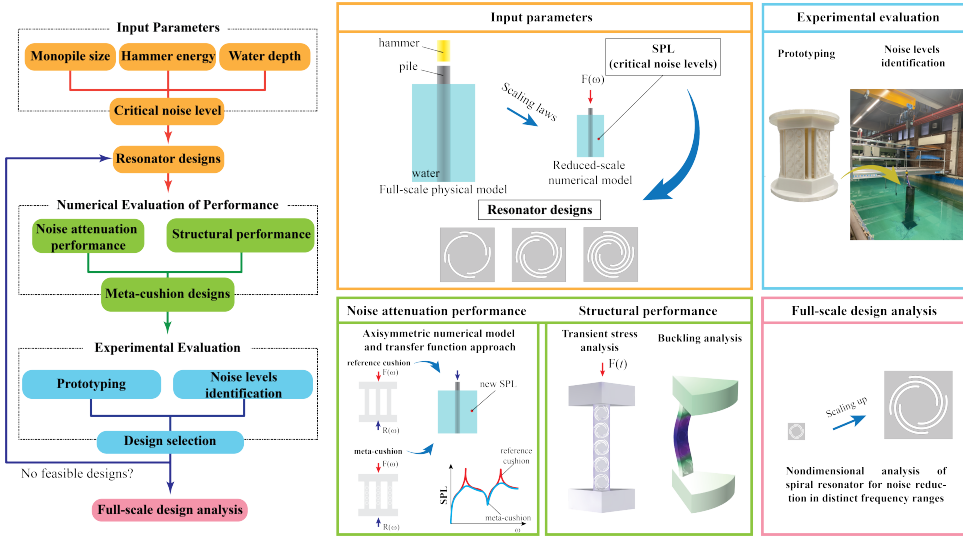


Figure 6.1.: Schematic of design methodology for a meta-cushion for reducing underwater noise during monopile driving process.

In the initial step, monopile dimensions, impact hammer energy, and the seawater depth [293, 336] are extracted from a full-scale monopile installation. Given the high costs of full-scale experimental evaluations, a scaled-down model with adjusted input parameters is defined using appropriate scaling laws. These updated parameters establish the critical noise levels, which are then used to design resonators capable of reducing noise below the threshold.

After selecting the designs, the resonator are arranged in arrays that will be assembled to constitute several meta-cushion configurations. Numerical studies will assess the noise attenuation performance, focusing on attenuation range and amount of resonators. Then, structural analysis is performed to evaluate transient stresses and critical buckling loads that will give the most suitable array distribution for the applied hammer energy. From such studies, final meta-cushion designs are selected.

In the final step, meta-cushion prototypes are fabricated and tested in a small-scale setup to evaluate noise attenuation performance. If the resonator designs

demonstrate sufficient performance, they can advance to further investigations for potential full-scale monopile applications. Otherwise, new resonator designs will be explored and tested again. The following sections discuss each design step through a scaled-down case study.

6.3. INPUT PARAMETERS

Designing the meta-cushion begins by identifying the frequencies associated with high noise levels, which depend on monopile dimensions, water depth, and impact hammer characteristics (energy and frequency spectrum content). Since full-scale offshore impact tests are extremely expensive, case studies are often scaled down for performing the vibro-acoustic analysis. Due to limited access to hammer and soil data, this work demonstrates how to determine impact load when scaled hammer energy is known. Then, the scaling laws related to the monopile are introduced.

The strike hammer energy E is numerically modeled as the vertical pile head velocity profile $v(t)$, which is given by [337]

$$v(t) = v_0 \exp(-Z/m_r t), \quad (6.1)$$

where v_0 is the initial velocity of the ram, derived from the strike energy and expressed as $v_0 = \sqrt{2E/m_r}$, m_r is the ram weight, and Z is the pile head impedance, expressed as $Z = E_p A_p / c_p$, being E_p , A_p , and c_p the Young's modulus, cross sectional area of the pile head, and the longitudinal wave speed of the pile, respectively.

The monopile is designed through scaling laws to keep the dimensions consistent with full-scale monopiles. Typical offshore monopiles are made of steel and have embedded length L (below sea surface level) of 25-40 m and diameter D of 4-8 m, which correspond to an aspect ratio (L/D) in the range of 3-8 [179, 217, 323]. In this work, an aspect ratio of 7.3 was chosen, which has been defined according to water depth of the available testing tank. Further information about the scaled monopile is given in Table 6.1.

Parameter	Pile	Parameter	Tank
Length L	2.4 m	Water depth	2.2 m
Diameter D	0.3 m	Width	3.4 m
Thickness t	3 mm	Density ρ_w	1000 kg/m ³
Young's modulus E_p	205.8 GPa	Sound speed in water c_w	1500 m/s
Density ρ_p	7850 kg/m ³	-	-
Poisson's ratio ν_p	0.3	-	-

Table 6.1.: Geometric and material parameters of the scaled monopile.

Once defined the monopile dimensions, a vibro-acoustic model is developed to predict the potential noise frequency ranges. To that end, an axisymmetric numerical

model containing a pile partially submerged is proposed, as shown in Fig. 6.2a. The monopile is modeled as a shell structure, with water represented as an acoustic medium. Perfectly matched layers (PMLs) are used to minimize wave reflections, and a zero-pressure condition is applied at the water surface. To focus on the primary noise propagation path—directly from the pile to the water—the soil effect is neglected and replaced by a rigid boundary, though soil contributions can be incorporated as demonstrated in our previous work [273]. A zero-displacement condition is imposed at the bottom edge of the pile, and an acoustic-structure boundary couples the pile vibrations with acoustic emissions. To capture the physics of waves propagating at the interest frequency range, the maximum element size h is defined as

$$h = \frac{\lambda}{5} = \frac{1}{5} \frac{c}{f_{\max}}, \quad (6.2)$$

where $\lambda = c/f_{\max}$ is the minimum wavelength, determined by the minimum wave speed c in the domain and the maximum frequency $f_{\max}$ of interest. The factor of five represents the number of finite elements per wavelength.

Due to limited access to full-scale hammer data, the excitation load is modeled as a time-harmonic ring load with a 1 N amplitude, applied to the top shell edge (indicated by the red ring in Fig. 6.2a). This load spans a frequency range of 1 to 1500 Hz, aligning with the tested hammer's frequency range, as detailed later. In case of full-scale impact load data is available, it could be scaled and used in this model. The output of the numerical model includes pile wall acceleration measured at $1.5D$ from the top, according to the American Society for Testing and Materials (ASTM) D4945 standard [338], and sound pressure levels in water at 1 m and 2 m from the pile wall. Figure 6.3 shows the radial and axial accelerations in the pile wall and the sound pressure levels at the selected points. The observed peaks correspond to the monopile's natural frequencies, identified through an eigenfrequency analysis of the axisymmetric model and indicated by vertical lines. The highest peak occurs at the monopile's third natural frequency (546 Hz), requiring a resonator design that targets this range. Additionally, alternative designs are proposed to attenuate multiple frequency ranges.

Four spiral resonator designs have been selected, with their geometric parameters and attenuation zones – or band gaps – summarized in Table 6.2. PLA was chosen as the constituent material due to its availability for fabrication via FDM. Because PLA properties have frequency-dependent behavior, viscoelasticity is included in the numerical model. Two designs show band gaps at zones near the natural frequency of the monopile, while designs II and IV present, respectively, high and low frequency attenuation zones. In the next section, the selected resonator designs are assembled into meta-cushion designs, from which their attenuation and mechanical performance are evaluated.

6.4. NUMERICAL ANALYSIS OF META-CUSHION PRINCIPLE

The numerical analysis of the meta-cushion principle will address two design aspects. First, the noise attenuation capacity of each resonator design is verified. The

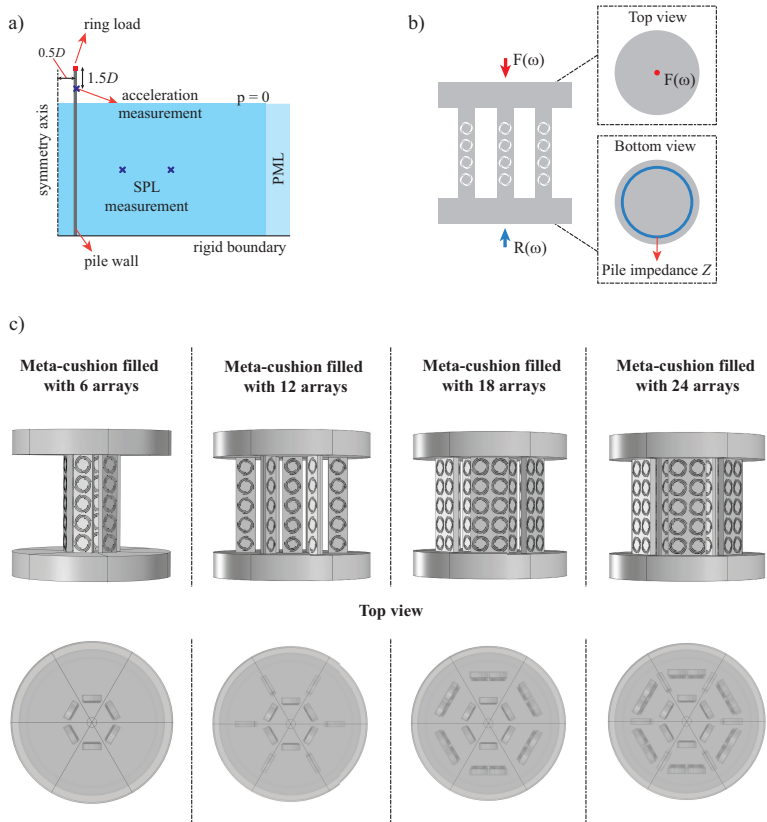


Figure 6.2.: a) Vibro-acoustic numerical model for calculating the responses in the pile wall and underwater; b) Transfer function approach for calculating the reaction load, used to update the vibro-acoustic model; c) Numerical designs of the meta-cushions containing 6, 12, 18, and 24 arrays.

Spiral design	Geometric parameters				Attenuation zone
	a [mm]	θ_I [rad]	θ_F [rad]	t_b [mm]	
I	40	$\pi/4$	$8\pi/5$	1.2	[580, 605]
II	40	$\pi/2$	$8\pi/5$	1.2	[895, 1070]
III	40	$3\pi/5$	$19\pi/10$	1.2	[475, 525]
IV	40	$\pi/4$	$7\pi/4$	2.0	[265, 285]

Table 6.2.: Geometric parameters of selected spiral resonator designs. The last column indicates the attenuation zones identified through a transmission loss analysis.

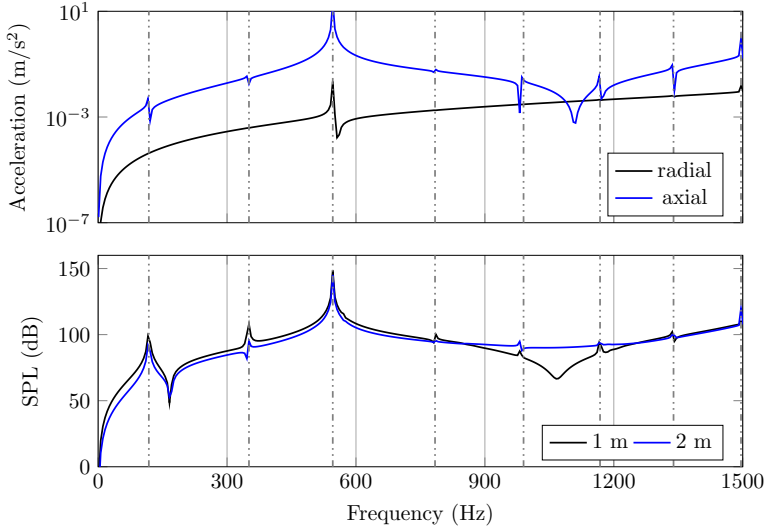


Figure 6.3.: Acceleration and sound pressure levels measured at the axisymmetric vibro-acoustic model for evaluating potential noise frequency ranges. The vertical lines indicate the natural frequencies of the monopile obtained through the same model.

resonators are assembled into various meta-cushion configurations with different numbers and types of resonators, from which their attenuation performance is compared to reference cushions without resonators. Second, to ensure that the meta-cushion remains in the elastic regime, transient stress levels and critical buckling loads are evaluated, from which the suitable configurations are selected.

For the first design aspect, the resonator designs in Table 6.2 are arranged into arrays that will form single and combined meta-cushion configurations. The single configurations consist of six arrays in a hexagonal arrangement (see Fig. 6.2c) and are labeled as configurations A, B, C, and D (Tab. 6.3). To enable multiple-frequency filtering, six arrays of different types are combined, which give the remaining configurations listed in Tab. 6.3. The arrays of combined cushions are distributed according to the configurations indicated in Fig. 6.2c.

Directly modeling the meta-cushion design within the vibro-acoustic model would significantly increase computational costs. Instead, a transfer function approach is employed, where the ring load in the original model is replaced by the meta-cushion's contribution as a reaction force (see Fig. 6.2b). To compute this reaction force, a separate numerical model of the meta-cushion is developed (Fig. 6.2c). To reduce computational demands, only one-sixth of the geometry is analyzed using cyclic symmetry. A time-harmonic point load of 1 N amplitude is applied to the top plate of the meta-cushion. The monopile is replaced by a dashpot (total damping constant of $1.1251\text{E}5 \text{ N.s/m}$ based on the pile head impedance Z) and the reaction load is calculated at the cushion-monopile interface and transferred to the vibro-acoustic

Array	Meta-cushion configurations								
	A	B	C	D	Comb1	Comb2	Comb3	Comb4	Comb5
I	6	-	-	-	6	-	-	6	6
II	-	6	-	-	6	6	6	6	6
III	-	-	6	-	-	6	-	6	6
IV	-	-	-	6	-	-	6	-	6

Table 6.3.: Meta-cushion configurations for verification of single and multiple attenuation zones. The number six indicates the amount of arrays involved in the assembling.

model, where the response of the model is calculated. Figure 6.4 presents the results of the radial acceleration and sound pressure level at 1 m from the pile wall for cushion designs A, B, C, and D, with arrays containing five, seven, and ten spiral resonators. The gray areas indicate the band gaps as described in Tab. 6.2. The meta-cushion results are compared with reference cushions to assess the resonant effects, revealing reduced monopile acceleration in band gap areas, which also lowers sound pressure levels. This effect enhances by adding more resonators to the meta-cushion, where reductions up to 40 dB in SPL are observed. Among the designs, cushion B gives the broadest band gap, particularly when the maximum number of resonators, while cushion D provides minimum attenuation even with the maximum capacity of resonators.

The same analysis is conducted for the combined meta-cushion configurations, with results shown in Figs. 6.5 and 6.6 for arrays containing 10 resonators. While the inclusion of different resonator types introduces more dynamic behaviors compared to the simple structure of the reference cushion, reductions in measurements are still observed, with resonator type II achieving the most significant attenuation. Notably, some designs exhibit limited attenuation zones due to the boundary conditions applied. When arrays are positioned far from the loading area, wave behavior may become more complex, deviating from the purely longitudinal propagation assumed for attenuation zone determination (Tab. 6.2). Another contributing factor is the narrow band gap generated, as seen with the meta-cushion containing type IV resonators, where only slight attenuation is noted in the FRF of combination 3 in Fig. 6.5 and combination 5 in Fig. 6.6. Thus, when designing the meta-cushion, arrays that attenuate less noise could either be placed farther from the impact load or use fewer resonators.

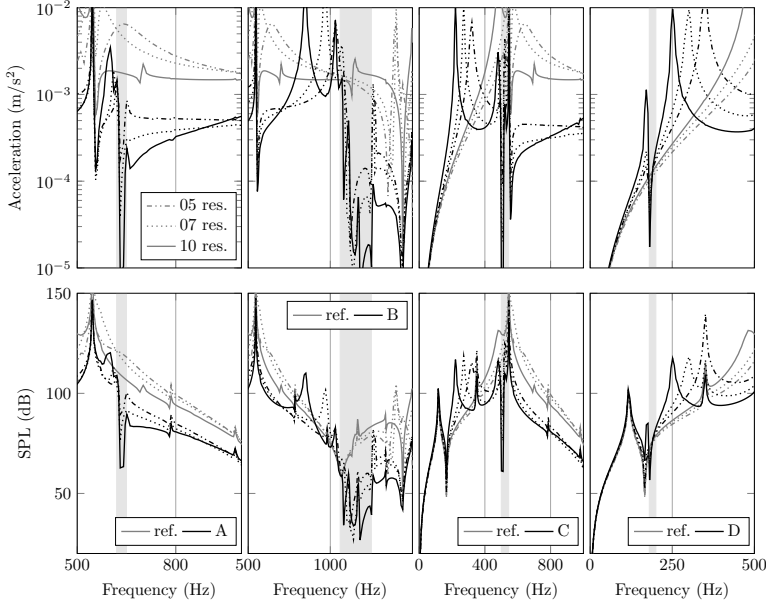


Figure 6.4.: Acceleration and sound pressure levels measured at the axisymmetric vibro-acoustic model when including the contribution of the single meta-cushion designs with distinct number of resonators per array. The gray areas indicate the attenuation zones of the designed resonators. The gray and black curves indicate the response for the reference cushion and meta-cushion configurations A to D, respectively.

After determining the meta-cushion's attenuation performance, transient stress levels are evaluated for each configuration subjected to a transient excitation given by the pressure expression $20e^{(-t/0.01)}$ kPa—notice that the stress amplitude can be replaced when hammer data is accessible. Figure 6.7 displays the time history of maximum Von Mises stress for each configuration and varying array sizes. The maximum von Mises stress reduces as the number of arrays increases and develops more rapidly for shorter cushions. Snapshots at the peak stress moment, shown in Fig. 6.8, reveal stress concentrations near the initial points of the curved beams and in the narrow regions between the spiral shapes and the array edge. Further studies are required to optimize array geometry for improved stress distribution under impact loads. Additionally, arranging spiral arrays through the meta-cushion plates helps reduce stress concentration, as reflected by the maximum stress values in Fig. 6.8.

Since the meta-cushion is a cylindrical structure with relatively thin beams under axial compressive load, evaluating the critical buckling loads for each configuration and array size is essential. To that end, a linear buckling analysis is performed and the critical loads are exhibited in Tab. 6.4, while the eigenmodes are indicated in Fig. 6.9. Adding more arrays increases the meta-cushion's load capacity before

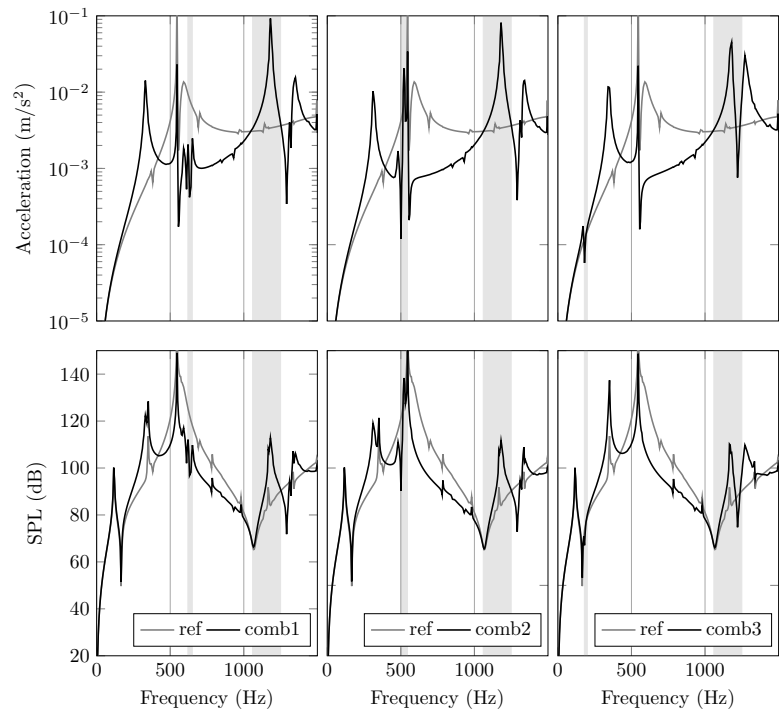


Figure 6.5.: Acceleration and sound pressure levels measured at the axisymmetric vibro-acoustic model when including the contribution of the reference and combined meta-cushion designs 1 to 3 containing 10 resonators. The gray areas indicate the identified attenuation zones.

buckling, however, longer arrays decrease the critical load by almost half. Therefore, if additional resonators are needed to attenuate pile vibration, designs with more arrays are preferable.

Amount of resonators	Amount of arrays			
	6 arrays	12 arrays	18 arrays	24 arrays
5	2988 N	4537 N	9409 N	9271 N
7	1580 N	2246 N	5003 N	4581 N
10	789 N	1053 N	2482 N	2145 N

Table 6.4.: Critical loads in Newton of each meta-cushion configuration with varied array sizes.

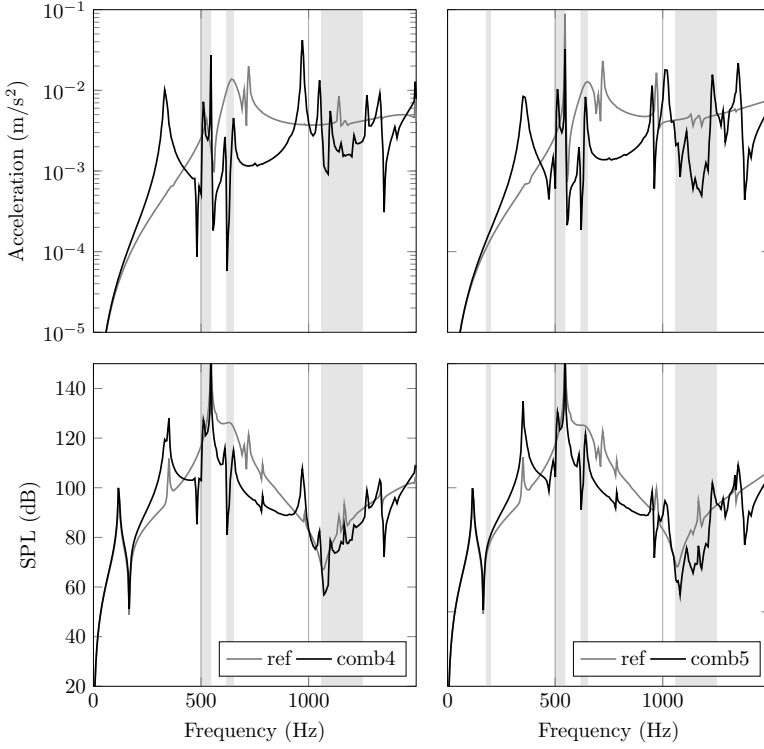


Figure 6.6.: Acceleration and sound pressure levels measured at the axisymmetric vibro-acoustic model when including the contribution of the reference and combined meta-cushion designs 4 and 5 containing 10 resonators. The gray areas indicate the identified attenuation zones.

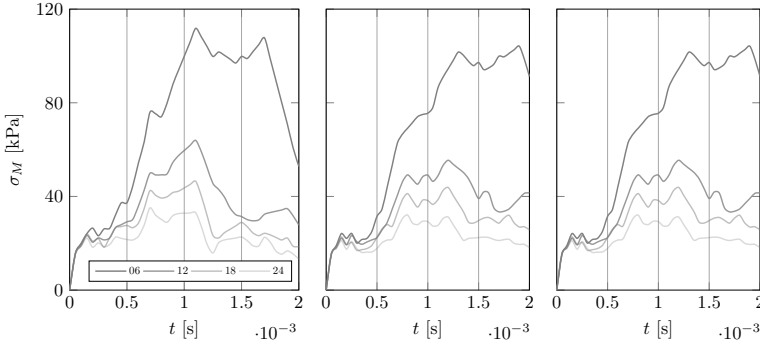


Figure 6.7.: Time history of maximum von Mises stress for each configuration of meta-cushion containing 5 (left), 7 (middle), and 10 (right) resonators per array.

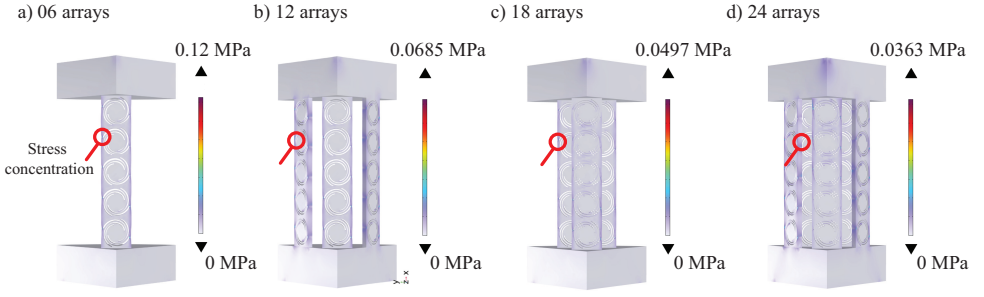


Figure 6.8.: Snapshot of maximum Von Mises stress field for meta-cushions containing a) 6, b) 12, c) 18, and d) 24 arrays. The colormap indicates the maximum Von Mises value, while the red circles indicate the stress concentration.

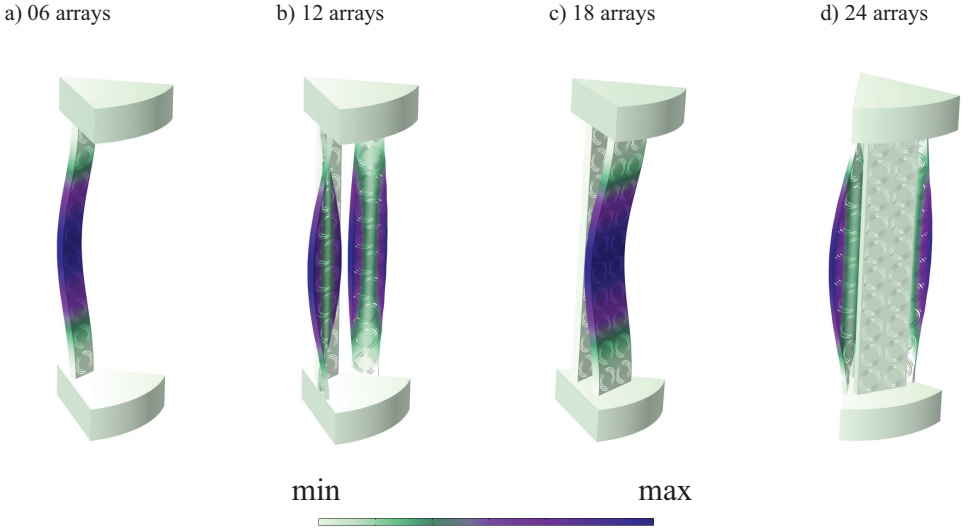


Figure 6.9.: First buckling mode of meta-cushions containing a) 6, b) 12, c) 18, and d) 24 arrays. The colormap indicates the deformation field.

6.5. EXPERIMENTAL EVALUATION OF THE META-CUSHION DESIGNS

In the next phase of the meta-cushion design, the proposed designs from the previous section will be experimentally evaluated for their effectiveness in reducing monopile vibration at specific frequency ranges, and consequently, underwater noise. Physical prototypes of the meta-cushion, shown in Figure 6.10, are developed. These consist of arrays containing resonant elements (Fig. 6.10a) and plates with 24 cavities for inserting the arrays (Fig. 6.10b). All meta-cushion components are made of PLA and manufactured using the FDM 3D printing process. The arrays have 100 % infill and the plates have 15 % infill to minimize damping effects. Due to printer bed size

limitations, the arrays contain only five resonators.

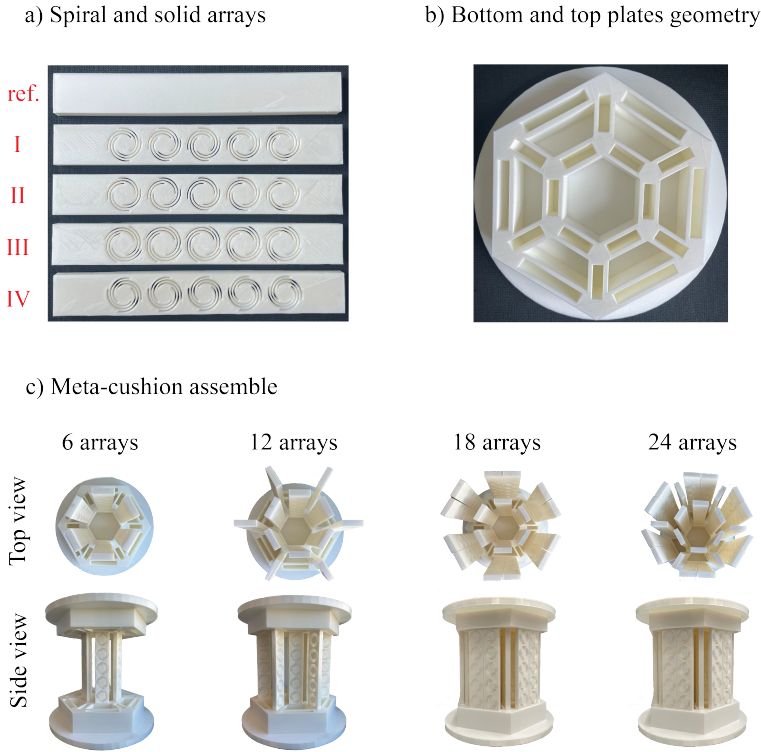


Figure 6.10.: Printed cushion parts: a) spiral and solid arrays; b) top and bottom plates. c) Assembled meta-cushion configurations containing 6, 12, 18, and 24 arrays.

To verify the presence of band gaps in the physical models, a modal impact analysis is conducted on the printed arrays (Fig. 6.10a). In this test, a modal impact hammer applies an impact load to one end of the arrays, and acceleration is measured at the opposite end using accelerometers. The modal impact hammer (Type DJB of model IH/02) features a force transducer with sensitivity 2.58 mV/N. Similarly to the numerical analysis, the load frequency varies from 1 to 1500 Hz, where the load spectrum energy is concentrated. Transfer functions are obtained through the ratio between the acceleration signal $a(x, y, z, \omega)$ at a point (x, y, z) and the excitation signal, $F(\omega)$, which are given as

$$T_a(x, y, z, \omega) = \frac{a(x, y, z, \omega)}{F(\omega)} \quad (6.3)$$

Figure 6.11 highlights the transfer functions T_a for each designed array obtained by the viscoelastic and experimental analysis. Notice that the viscoelastic model agrees well with the experimental measurements. As also indicated by the numerical

models, the band gaps of resonator designs I, III, and IV, although located in low-frequency ranges, are narrow, which may not give significant contributions for the noise attenuation. This could be overcome by introducing more resonators in the arrays as observed in Fig. 6.4.

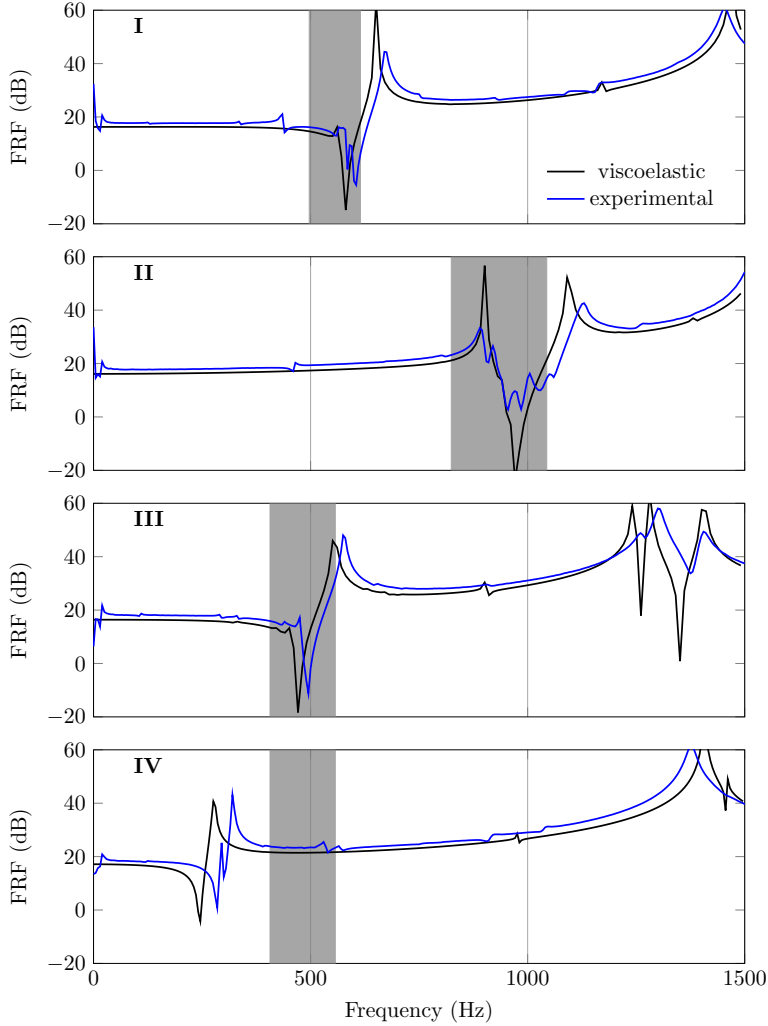


Figure 6.11.: Comparison between the FRFs obtained through numerical and experimental analyses of spiral arrays. The numerical analysis was performed for both elastic and viscoelastic model of the arrays. The gray areas indicate the band gaps obtained via the dispersion curve analysis.

Once the arrays exhibit the desired attenuation zones, the meta-cushion is assembled according to the configurations defined in Tab 6.3. To assess the ability of the meta-cushion in reducing the underwater noise, we replicated the hammering of

an offshore monopile in the towing tank of the lab facilities in TU Delft. Figure 6.12 shows the experimental setup for performing the hammering test. The tank has a water depth of 2.2 m and width of 3.4 m. The monopile is placed inside the tank with the aid of a crane. To guarantee that the monopile is placed in the middle of the tank, a rubber plate is used to adjust the pile position. Two hydrophones (Fig. 6.12b) are placed in the middle of the tank, so that they are aligned with the center of the monopile. The closest hydrophone of model RB9-ETH (6370) is placed 1 m away from the pile wall, while the second hydrophone of model SC2-ETH (6083) is placed 2 m from the pile wall. Because of the limited amount of hydrophones, a pressure sensor was placed at the same height of the hydrophones, but closer to the tank wall (one quarter away from tank wall). Transfer functions are obtained through the ratio between the pressure signal $p(x, y, z, \omega)$ at a point (x, y, z) and the excitation signal, $F(\omega)$, which are given as

$$T_p(x, y, z, \omega) = \frac{p(x, y, z, \omega)}{F(\omega)} \quad (6.4)$$

To express the force to pressure transfer functions in terms of the reference value in water $T_w = 1 \mu \text{ Pa N}^{-1}$, Eq. (6.5) is modified by $20\log(|T_p|/T_w)$.

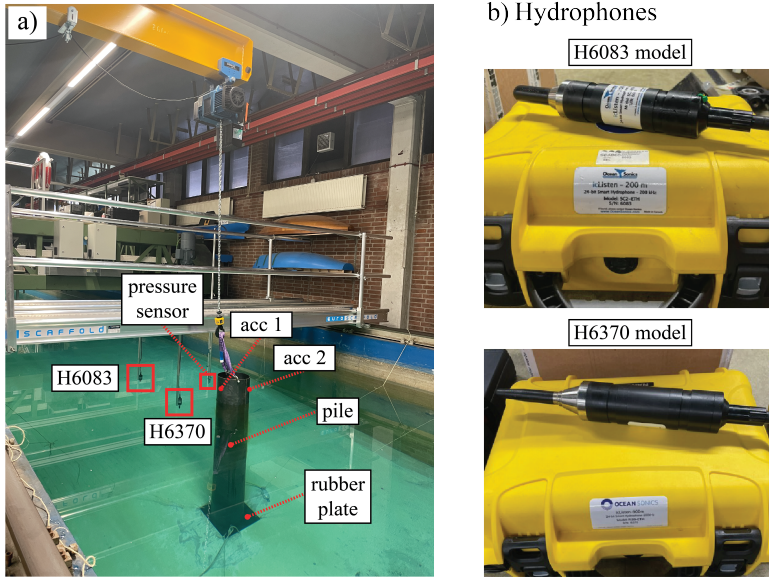


Figure 6.12.: a) Experimental setup for noise emission during pile hammering. The picture shows the monopile positioning inside the tank with the aid of a crane. The hydrophones are aligned with the monopile. b) The hydrophones 6083 and 6370.

The impact load is generated by the same modal impact hammer used for obtaining the arrays' transfer functions. To measure the pile acceleration in the

radial direction, two accelerometers are installed on the pile wall from a distance of $1.5D$ from the pile top surface, according to the American Society for Testing and Materials (ASTM) D4945 standard [338]. Transfer functions related to the monopile acceleration are also obtained through Eq. (6.5).

As performed in the numerical evaluation, the monopile vibration is initially investigated to identify the natural frequencies related to the critical noise levels. Figure 6.13 shows the frequency response function of the averaged accelerometer data from the monopile wall when the pile is axially hit fifteen times by a modal impact hammer without a cushion. The experimental result indicated by the black dotted curve shows a large number of peaks that could indicate the existence of background noise and noise from the cables. Since this will difficult the process of analyzing the data, the Savitzky-Golay digital filter is applied, once it acts as a low-frequency band pass and smooths the data, indicating the trend of the results [339]. Two parameters are used to set the options of the filter, the window size, and the polynomial order of the fitting curve. A study of such parameters is performed before hand, so that important data information is not lost.

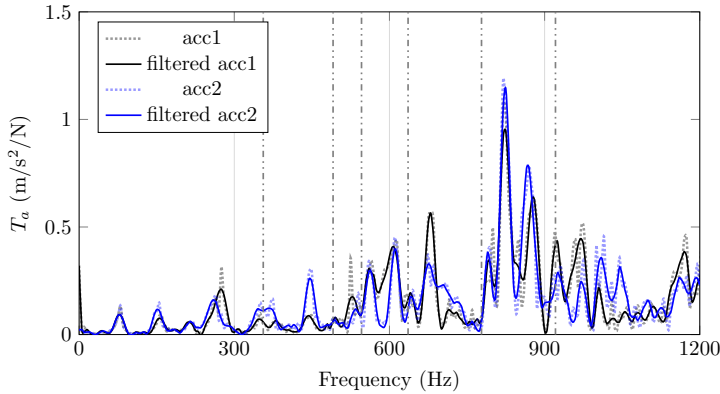


Figure 6.13.: FRF of radial acceleration of a monopile without cushion for numerical and physical models. The black dashed curve indicates the FRF without filtering. The vertical lines indicate the natural frequencies extracted from the numerical model, also displayed in Fig. 6.3.

Despite the existence of peaks that may be associated with background noise and manufacturing imperfections, some of them are related to the natural frequencies of the monopile as they are near the values predicted by the numerical model (indicated by the dashed vertical lines). However, unlike the numerical model, the critical vibration occurs in higher frequency ranges, which can be attributed to the boundary condition at the bottom of the physical monopile, how the load is applied at the top surface, and the monopile is not completely cylindrical. In the further analyses, the ability of reducing vibrations at tailored frequency ranges and consequently the underwater pressure waves of the meta-cushions is investigated.

The transfer functions from the hammering pile containing the single meta-

cushions are initially evaluated (cushions A, B, C, and D as indicated in Tab. 6.3). To observe the attenuation principle of the meta-cushion, the results are compared with the transfer functions from hammering pile containing a reference cushion formed by six solid arrays.

Figure 6.14 shows the acceleration and sound pressure levels measured during hammering of the monopile with cushion A. The gray areas represent the predicted attenuation frequency zones from the experimental analysis of the printed arrays (Fig. 6.11) and the red dotted curves indicate the response of the reference cushion. Notice that in the pressure sensor response (Figure 6.14b), a 6 dB drop occurs at the left edge of the predicted attenuation zone, coinciding with a decrease in vibration on the monopile wall (Figure 6.14a). As predicted by the numerical model, cushion A shows limited attenuation, with a narrow band gap. However, this could be improved by increasing the number of resonators.

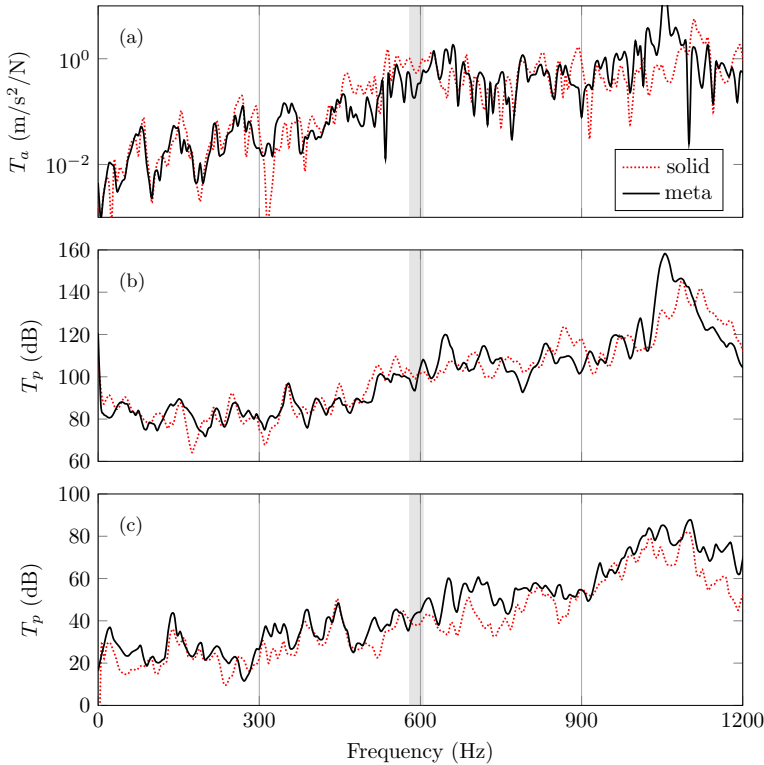


Figure 6.14.: Comparison between transfer functions obtained by hammering a monopile containing a reference cushion and cushion A. a) Response at pile wall, b) response from pressure sensor c) response from hydrophone sensor.

Unlike the previous cushion, the measurements of the monopile containing a cushion B show insightful remarks, as indicated in Figure 6.15. It may be assumed the attenuation performance of cushion B is not significant when comparing the

response with the reference cushion. However, as observed in the case of cushion A, the meta-cushion introduces a natural frequency which also falls at the same region of attenuation of array type II, causing a reduction in FRE. This can be better observed when comparing the frequency responses of cushion A and B, as shown in Fig. 6.18, where the pressure transfer function drops significantly in the higher edge of the attenuation zone. Similarly to the previous case, the attenuation is more pronounced at the lower edge of the band gap—where a drop of 7 dB is observed—and it decreases along the bandwidth. This highlights the asymmetric feature of local-resonant metamaterials. However, the bandwidth is larger than that of the previous configuration, especially due to the frequency-dependent characteristic of viscoelastic materials. Similar behavior is also observed in the hydrophone measurements, as shown in Fig. 6.15c.

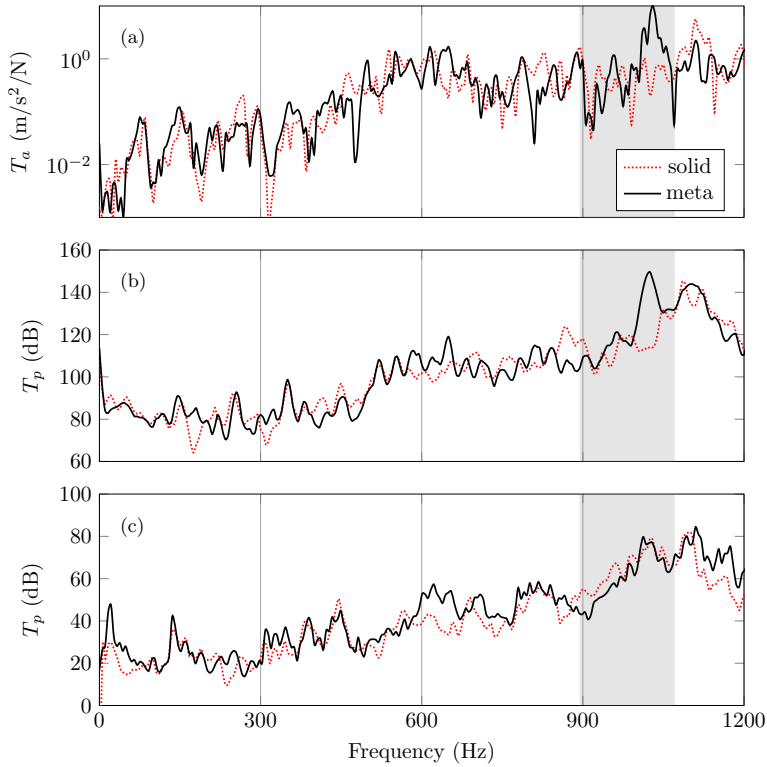


Figure 6.15.: Measurement from accelerometer and pressure sensors for hammering a monopile containing a cushion B. a) Response at pile wall, b) response at the pressure, and c) response measured by the hydrophone. The band gap of the spiral unit cell is indicated by the gray area.

Figure 6.16 shows the measurements for the case of a monopile with a cushion of configuration C. Although a significant reduction of sound levels is not observed in the band gap range, a drop is clearly observed in the vibration of the monopile wall,

as indicated in Fig. 6.16a. However, a clear increase of sound level and monopile vibration at frequencies higher than 900 Hz is observed, which can indicate that this array is not suitable for the noise attenuation in case of monopiles generating critical noise levels at such range. Similar behavior is also observed in the hydrophone measurements, as shown in Fig. 6.16c.

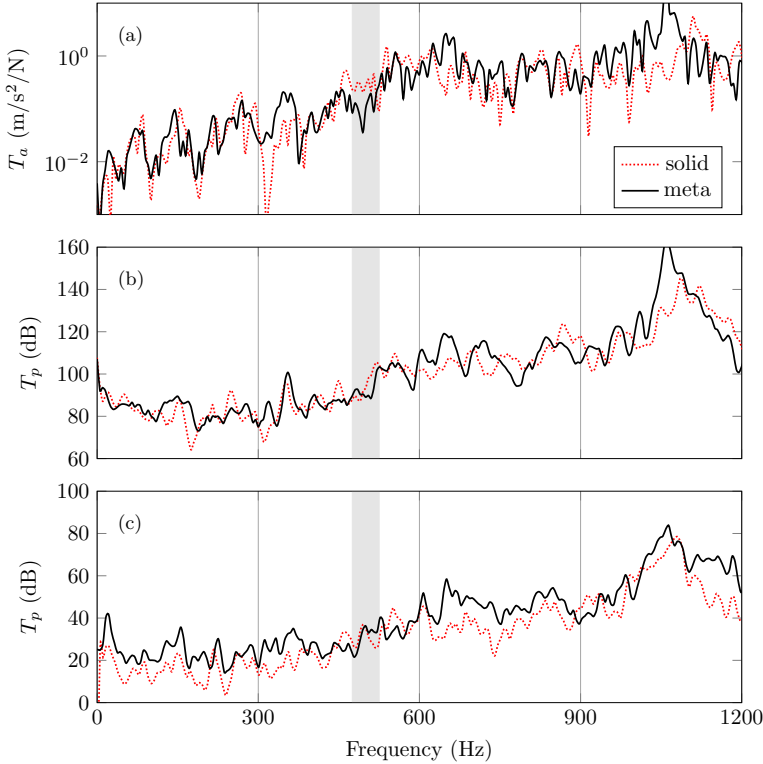


Figure 6.16.: Measurement from accelerometer and pressure sensors for hammering a monopile containing a cushion C. a) Response at pile wall, b) response at the cushion, and c) response measured by pressure sensor inside the tank. The band gap of the spiral unit cell is indicated by the gray area.

Figure 6.17 shows the measurements for the case of a monopile with a cushion of configuration D. As observed in the numerical evaluation, this design does not show significant contributions to attenuate noise, since the pile vibration is only reduced at the lower edge of the band gap, which has not been enough to reduce the noise levels, as indicated by the pressure sensor 6.17b and the hydrophone measurement 6.17c. However, unlike the previous designs, this cushion has a much lower frequency response function at frequencies higher than 900 Hz.

The noise attenuation performance is also evaluated for meta-cushions with combined configurations, where the results are presented in figs. 6.18 to 6.21. In Fig. 6.18, the pressure transfer functions of the single configurations A (array I) and

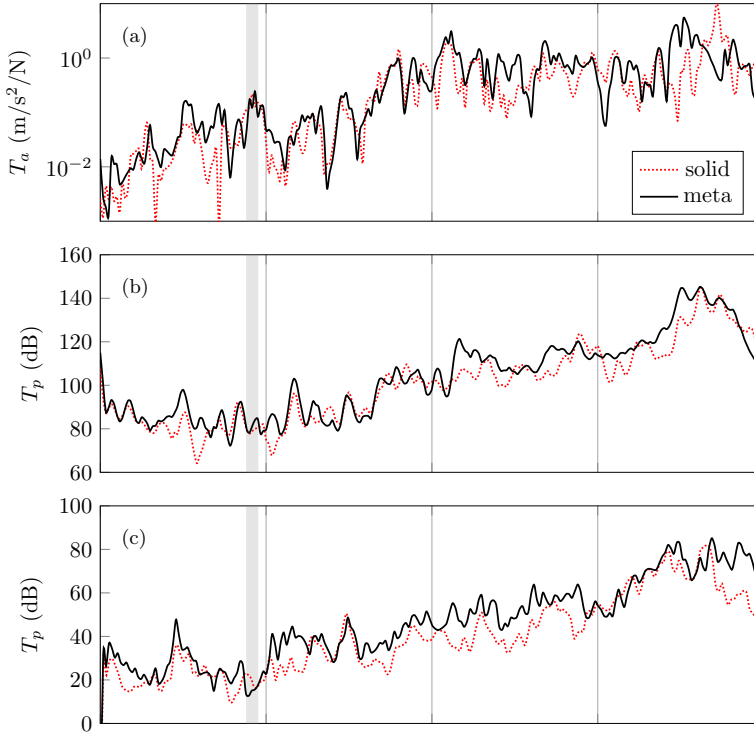


Figure 6.17.: Measurement from accelerometer and pressure sensors for hammering a monopile containing a cushion D. a) Response at pile wall, b) response at the cushion, and c) response measured by pressure sensor inside the tank.

B (array II) are repeated and compared with combined configuration 1 (arrays I and II). The single cushions show reduced response at their respective band gaps, while the combined cushion reflects an average of each individual cushion's contribution. Although array I does not exhibit a pronounced band gap, it can still be used to attenuate potential natural frequencies of the designed cushion, preventing their propagation to the monopile vibration. Similar behavior is observed in Fig. 6.19, where it can also be highlighted that the high peak induced by cushion C (array III) reduces in about 30 dB when combined with cushion B (array II), as shown by the curve of combined cushion 2 (arrays II and III). Interestingly, the combined configuration 3 has shown the lowest pressure levels at the frequency range around 900 Hz, in comparison to the previous configurations. Because array IV does not induce peaks at this range and since array II induces band gap at this range, the attenuation is amplified, being the lowest drop below 100 dB (see Fig. 6.20). Finally, figs. 6.21 and 6.22 indicate the pressure levels induced by combined cushions 4 and 5, respectively, where the superposition effect is reinforced when different resonators are combined. In particular for combined cushion 5, it can be highlighted that pressure levels are below 140 dB for the full frequency range under investigation.

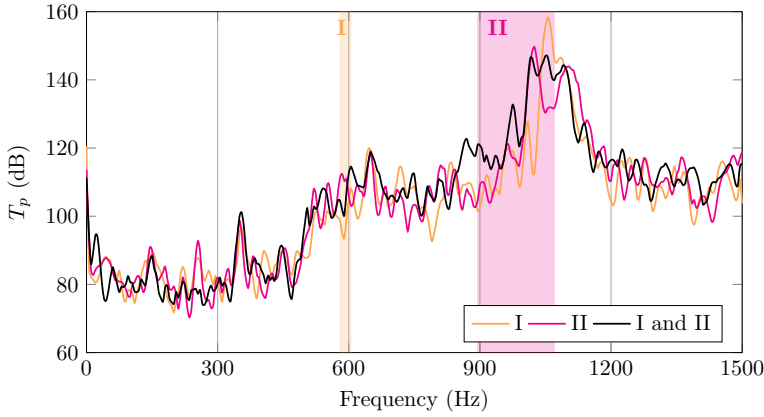


Figure 6.18.: Comparison of pressure sensor measurements among impact tests with cushion A, B, and comb1 from Table 6.3. Band gaps have been highlighted according to the resonator type.

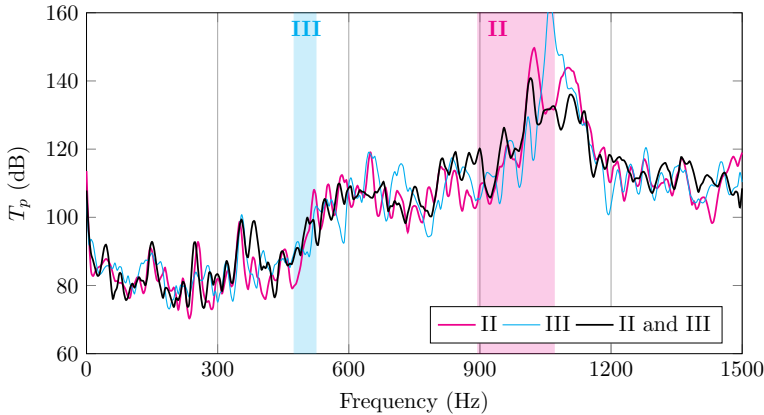


Figure 6.19.: Comparison of pressure sensor measurements among impact tests with cushion B, C, and comb2 from Table 6.3. Band gaps have been highlighted according to the resonator type.

6.6. NONDIMENSIONAL INVESTIGATION OF SPIRAL RESONATOR

The previous studies have demonstrated that meta-cushions formed by arrays of type II give the best attenuation performance in a wider band gap range. Thus, this design has been selected to perform a nondimensional investigation and predict the attenuation zones for different unit cell sizes and materials. To that end, nondimensional dispersion curves are extracted by considering an infinite periodic system formed by resonators II. Such problem can be reduced by applying periodic

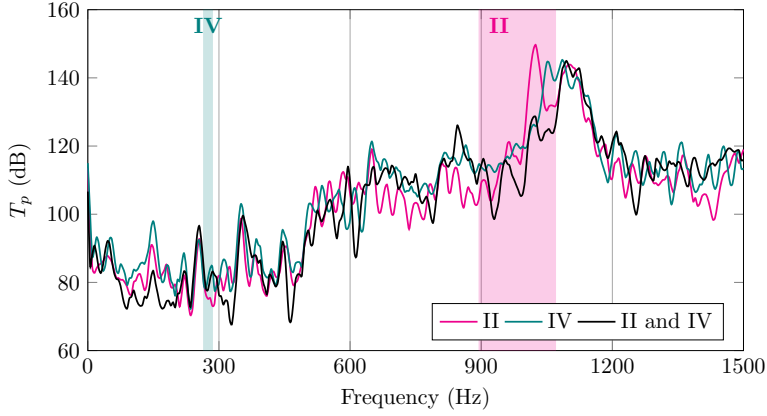


Figure 6.20.: Comparison of pressure sensor measurements among impact tests with cushion B, D, and comb3 from Table 6.3. Band gaps have been highlighted according to the resonator type.

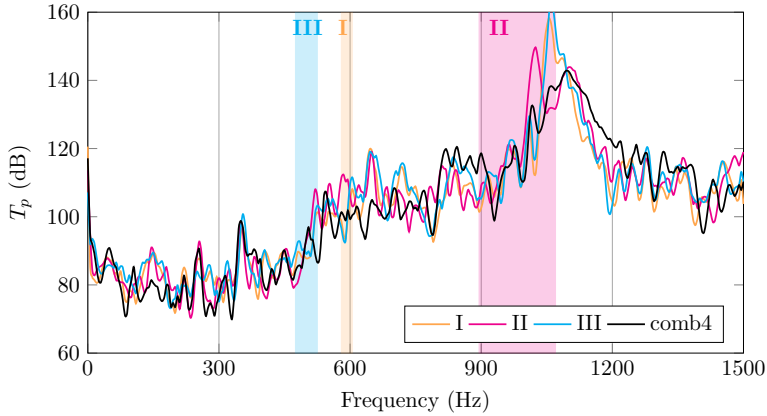


Figure 6.21.: Comparison of pressure sensor measurements among impact tests with cushion A, B, C, and comb4 from Table 6.3. Band gaps have been highlighted according to the resonator type.

boundary conditions on a single unit cell [53]. Fig. 6.23 shows the dispersion relation curve of the proposed unit cell. The nondimensional frequency is defined as $\Omega = fa/c$, where f is the eigenfrequency obtained for each wave vector $\mathbf{k}$, a is the unit cell size, and c is the wave speed, given as $c = \sqrt{E/\rho}$. Two band gaps are observed between nondimensional frequency ranges $[0.0206, 0.0263]$ and $[0.0337, 0.0363]$. By adjusting either the unit cell size a or the material parameters defining c , the band gap can be tuned to match required critical noise levels, as shown in Tabs. 6.5 and 6.6. This allows the meta-cushion design to be adapted for monopiles of varying dimensions or those with multiple critical noise zones.

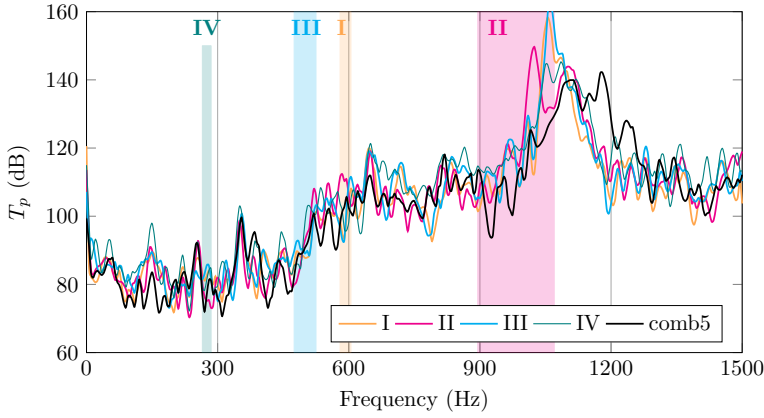


Figure 6.22.: Comparison of pressure sensor measurements among impact tests with cushion A, B, C, D, and comb5 from Table 6.3. Band gaps have been highlighted according to the resonator type.

Different material choices also enable the design of meta-cushions for both low- and high-energy impact hammers.

6

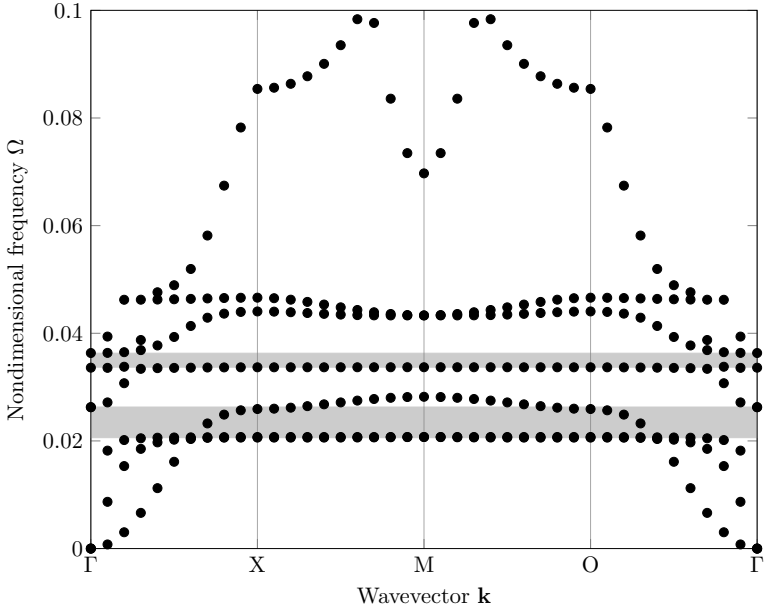


Figure 6.23.: Nondimensional dispersion curve of spiral resonator type II. The gray area indicates the band gap in the in-plane direction.

Material	Unit cell size a			
	0.01 m	0.04 m	0.1 m	0.4 m
PLA	[3275, 4181]	[818.83, 1045]	[327.5, 418.2]	[81.88, 104.5]
Aluminium	[10489, 13391]	[2622, 3347]	[1048, 1339]	[262.6, 334.8]
Steel	[10781, 13764]	[2695, 3441]	[1078, 1376]	[269.5, 344.1]

Table 6.5.: First band gap range, in Hertz, for unit cells of different sizes and materials.

Material	Unit cell size a			
	0.01 m	0.04 m	0.1 m	0.4 m
PLA	[5358, 5771]	[1339, 1045]	[535.8, 577.1]	[133.9, 144.3]
Aluminium	[17159, 18483]	[4289, 3347]	[1715, 1848]	[429, 462]
Steel	[17637, 18997]	[4409, 3441]	[1763, 1899]	[440.9, 474.9]

Table 6.6.: Second band gap range, in Hertz, for unit cells of different sizes and materials.

6

6.7. CONCLUSIONS

This chapter presents a methodology for designing meta-cushions to reduce underwater noise during monopile installations. The approach is adaptable to any hammering process by considering key design variables, including installation energy, monopile dimensions, and water depth. Moreover, the interaction between the impact load and the meta-cushion enhances not only noise attenuation—achievable by adjusting resonator types and quantities—but also mechanical performance, as array distribution influences stress concentration and critical buckling loads. Additionally, nondimensional analysis of high-performing resonator designs allows for targeted noise reduction across tunable frequency zones, supporting meta-cushion customization for diverse monopile characteristics. The proposed methodology can be extended to other noise mitigation challenges, for instance installation of different foundation types or underwater operations such as deep-water drilling.

7

CONCLUSIONS AND RECOMMENDATIONS

As the global effort for green energy intensifies, achieving the 2030 targets requires the deployment of high-capacity offshore wind farms. This demand leads to a corresponding increase in the size of monopiles, which are currently the first choice of foundations due to their low fabrication and operation costs. However, the vibration of larger monopiles generate higher levels of underwater noise shifting towards low-frequency ranges, more specifically, lower than 1000 Hz. This raises significant concerns about the effectiveness of traditional noise mitigation systems, which have shown limitations in reducing noise from such frequencies, posing a growing threat to marine life.

This thesis introduces a design methodology of a novel noise mitigation system able to attenuate noise generated by the pile hammering process. This noise mitigation system consists of a systematically engineered cushion placed between the impact hammer and pile head. The cushion employs the principle of local resonance obtained in architected structures known as metamaterials. These metamaterials are formed by resonators that vibrate at specific frequencies, blocking wave propagation in those ranges. Thus, the metamaterial-based cushion, or meta-cushion, filters partially the impact load, eliminating waves that cause excessive monopile vibrations and underwater noise, while allowing the remaining waves to propagate for the installation process.

7.1. CONCLUSIONS

A review of the state-of-the-art in mechanical metamaterials revealed that low-frequency wave attenuation from impact loads is achieved through local resonance. Based on the monopile installation problem, the meta-cushion was designed to attain this principle. Using resonant elements made of single material, the attenuation principle of the meta-cushion relies mostly on the resonator's geometry, instead of the constituent material. This simplification not only enhances the dynamic performance of the cushion, where multiple vibration modes can be explored to attenuate vibrations, but also allows for easier manufacturing. In terms of mechanical response to impact loads, distinct resonator arrangement inspired on lattice structures were chosen when designing the

meta-cushion, while optimization procedures were applied to improve the mechanical features of the resonant elements.

A numerical model of the meta-cushion was developed to verify its effectiveness in reducing underwater noise. The verification demonstrated several key advantages of the meta-cushion as a novel noise mitigation system. Compared to a standard cushion without resonators, the meta-cushion significantly attenuates noise in the critical frequency range, where peak levels are observed. While adding more unit cells can enhance attenuation, the meta-cushion is designed to reduce impact load energy sufficient to maintain acceptable sound pressure levels (SPL) while still transmitting energy to drive the monopile. Moreover, this approach mitigates noise directly at the source, unlike traditional techniques that attempt to reduce noise further from the impact zone. By controlling sound at the source, the meta-cushion eliminates the need of large encasing structures around the monopile, reducing both costs and installation complexity.

As the meta-cushion is subjected to high impact loads, assessing its structural integrity and vibration attenuation performance under these load conditions is essential. Therefore, a design optimization strategy was introduced to bridge the gap between attenuation performance and structural integrity. This approach incorporates stress analysis for both the meta-cushion and any acoustic/elastic metamaterial intended to mitigate impact-induced vibrations and evaluates the maximum Von Mises stress of several designs exhibiting distinct dynamic properties. The evaluations revealed that neglecting stress analysis could lead to structurally unfeasible designs under such loads. The findings include unit cell designs that not only achieve comparable vibration attenuation but also offer improved load resistance, enhancing the mechanical and vibration performance of metamaterials. This optimization strategy offers a systematic way for designing meta-cushions, instead of relying on straightforward parametric study.

After verifying the meta-cushion's principle, its effectiveness was validated across single and multiple frequency zones. The experimental studies showed strong agreement with numerical models, particularly when accounting for viscoelastic effects. The results confirm the meta-cushion's potential as an effective tool for filtering unwanted vibrations. While the meta-cushion produces relatively narrow low-frequency band gaps, the merging of adjacent band gaps using two spiral shapes presents a promising approach for improved low-frequency attenuation. Additionally, the meta-cushion demonstrated broader high-frequency attenuation compared to numerical predictions, though with slightly reduced performance due to frequency-dependent damping effects.

Once the attenuation principle of the meta-cushion was verified and validated, the design methodology for the meta-cushion to reduce underwater noise from pile installation (scale of 1:18) was proposed. Input parameters were initially defined based on the pile system variables associated with underwater noise, which were further used to characterize the meta-cushion – resonator types, amount of resonators per array, and arrays distribution. Since the meta-cushion is designed according to the features of the pile installation problem, the methodology can be applied to hammering processes of scale 1:25 to 1:1. Moreover, the in-depth investigation of the interaction between the impact load and the meta-cushion not only allows enhanced noise attenuation performance – achievable by adjusting amount and type of resonators – but also improves mechanical performance, as stress concentration and critical buckling loads can be improved

by changing the arrays' arrangement. Additionally, nondimensional analysis of effective resonator designs enables targeted noise reduction across tunable frequency zones, allowing customization for varying monopile characteristics. The proposed methodology can be extended to other noise mitigation challenges, for instance installation of different foundation types or underwater operations such as deep-water drilling.

In closing, the meta-cushion's design methodology provides a novel, systematic approach to mitigating low-frequency underwater noise by targeting pile system variables associated to noise generation and mechanical integrity of the cushion. This approach solves the noise problem from its source, unlike traditional noise mitigation systems, which predominantly focus on reducing noise along its propagation path. This research establishes the meta-cushion's feasibility and potential to reduce the negative effects of noise in marine life, setting the foundations for future advancements in noise control.

7.2. RECOMMENDATIONS FOR FUTURE WORK

The implementation of the meta-cushion in full scale monopile installations can be achieved by addressing the recommendations outlined in this section.

Metamaterials are evolving towards intelligent and adaptable designs that improve performance across various applications. Thus, incorporating active elements into the meta-cushion to dynamically adjust resonators' band gaps could further improve adaptability, particularly as the natural frequencies of the monopile change during driving process, potentially shifting the noise frequency range.

In the numerical studies, noise transmitted from soil to water was excluded due to the complexity of modeling pile-soil interaction, which was beyond the scope of this project. Future research should explore this interaction in greater depth to identify potential sources of noise and ways to integrate them into the meta-cushion design. Moreover, improving numerical models by more accurately incorporating viscoelastic effects will enhance the prediction of band gap locations and attenuation, specially when using polymers as the meta-cushion material. To address the computational demands of the multi-objective optimization approach, future work should combine this strategy with reduced order models (ROMs) to accelerate the calculation of metamaterial dynamic properties. Although some damping effects were, further studies should investigate material damping and its influence on both vibration and mechanical performance.

While the experimental study validated the meta-cushion's principle, further investigations are required due to limitations in the experimental setup. Firstly, dynamic mechanical analysis of the spiral arrays can give a better prediction of storage and loss moduli, improving the viscoelastic parameters incorporated in the numerical models. Before full-scale testing, in-field trials using the tested 1:18 scale piles or larger, combined with numerical simulations, will provide deeper insights, particularly regarding wave reflection reduction and incorporation of soil effects. Developing a scaled-down impact hammer capable of generating stress levels comparable to full-scale monopile installation will also allow more accurate prediction of sound pressure and attenuation levels. Finally, since the meta-cushion removes partially the energy from the impact hammer, assessing the amount of energy required to drive the monopile is essential to ensure that sufficient energy remains to complete the installation process.

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A

SUPPLEMENTARY INFORMATION FOR CHAPTER 3

This chapter provides additional details about the numerical modeling of the meta-cushion presented in Chapter 3. This is a transcription of the the supplementary information of the journal paper published in *Engineering Structures* [273].

A.1. EFFECT OF GEOMETRIC PARAMETERS ON THE RESONANT BAND GAP

The resonant BG presented in Figure 4 was obtained by a specific set of geometric parameters for the periodic unit cell (PUC). To visualize their effect on the BG we conducted three parametric studies; these investigate separately the beam's width w_b , the beam's length l_b , and the mass' radius R_2 . For theses studies, we considered an unit cell with the following parameters: $a = 0.4\text{ m}$, $R_1 = 0.42a$, $R_2 = 0.4a$, $w_b = 0.02a$, $l_b = 0.99a$, $l_g = R_2 - 0.08a$, $w_g = 0.05a$, and $h = 0.23a$. Fig. A.1 shows the band structure and the transmissibility for such unit cell, where a complete resonant BG is observed at a nondimensionalized frequency range $\Omega = [0.051, 0.056]$. Notice that this unit cell has a thicker and shorter beam and a smaller mass in comparison to that presented in Fig. 4, which results in a higher frequency BG, with a lower BG frequency of 164.65 Hz and an upper frequency of 188.48 Hz. The parametric studies were conducted such that:

1. The beam's width was varied in the range $w_b(\text{m}) = [0.004, 0.01]$ while fixing the beam's length and mass' radius at $l_b = 0.136\text{ m}$ and $R_2 = 0.16\text{ m}$, respectively;
2. The beam's length was varied in the range $w_b(\text{m}) = [0.128, 0.138]$ while fixing the beam's width and mass' radius at $w_b = 0.008\text{ m}$ and $R_2 = 0.16\text{ m}$, respectively; and
3. The mass' radius was varied in the range $w_b(\text{m}) = [0.14, 0.16]$ while fixing the beam's width and length at $w_b = 0.008\text{ m}$ and $l_b = 0.136\text{ m}$, respectively.

The bandwidth and the central frequency of the BG for these parameter studies are presented in Fig. A.2. As apparent from the figure, resonators with thinner beams generate BGs with lower central frequency. However, their BGs are quite narrow in relation

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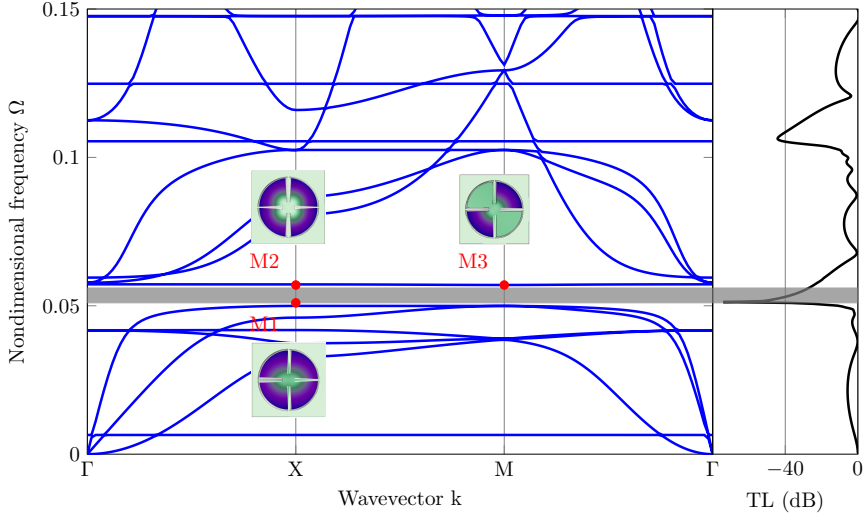


Figure A.1.: Band structure and transmission loss diagrams for unit cell with BG range $\Omega = [0.051, 0.056]$. The inset in each diagram highlights the eigenmodes at the frequencies defining the band gaps and the finite array of unit cells. In the unit cell's vibration modes, the color scale indicates the displacement magnitude, which increases from light green to dark purple.

to their counterparts with thicker beams. Increasing the beam's length results in decreasing the central frequency of the BG, while the bandwidth increases up to $0.135m$. Finally, varying the mass' radius has a similar effect as that of the beam's length. While an increase in beam's width results in higher natural frequencies, an increase of the beam's length or the mass' radius yields lower natural frequencies. The investigation of the influence of those parameters in the dynamic characteristic of unit cell can be applied to design BGs for the same application but with different dimensions.

A.2. TRANSFER FUNCTIONS FOR COMMON CUSHION AND META-INTERFACE

As explained in Section 4.3, the common cushion and the meta-interface have been replaced by a transfer forcing function, so that both structures are not explicitly modeled to determine SPL, since the finite element size that would be required to model the details of the meta-interface is prohibitively expensive. Fig. A.3 shows the model used to calculate the vertical reaction forces at the bottom of both structures. The frequency spectrum of the Gaussian pulse is applied at the top surface, whereas a spring representing the axial stiffness of the pile k_p is inserted at the bottom boundary. Due to the cylindrical shape of both structures, a cyclic symmetric condition is considered at the side boundaries, so that the analysis is restricted to a section containing one array of unit cells. Damping is introduced in the model by means of an isotropic loss factor η (for

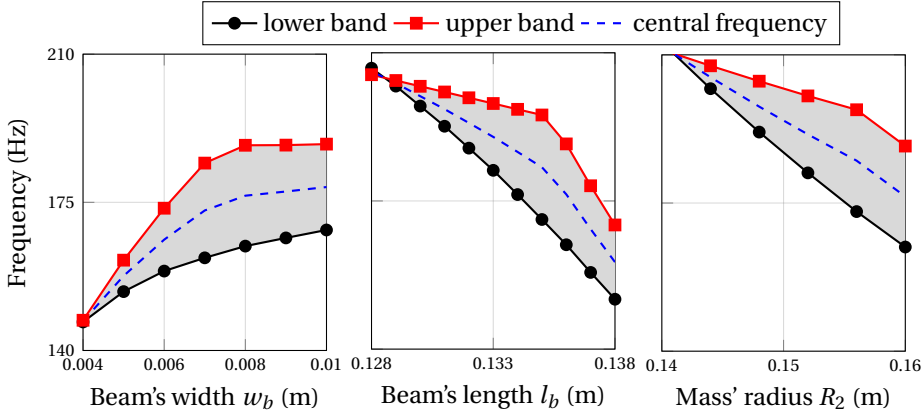


Figure A.2.: Variation of the bandwidth and central frequency of the BG for different ranges of the beam's width w_b (left), the beam's length l_b (middle), and the mass' radius R_2 (right).

Nylon, $\eta_n = 0.024$).

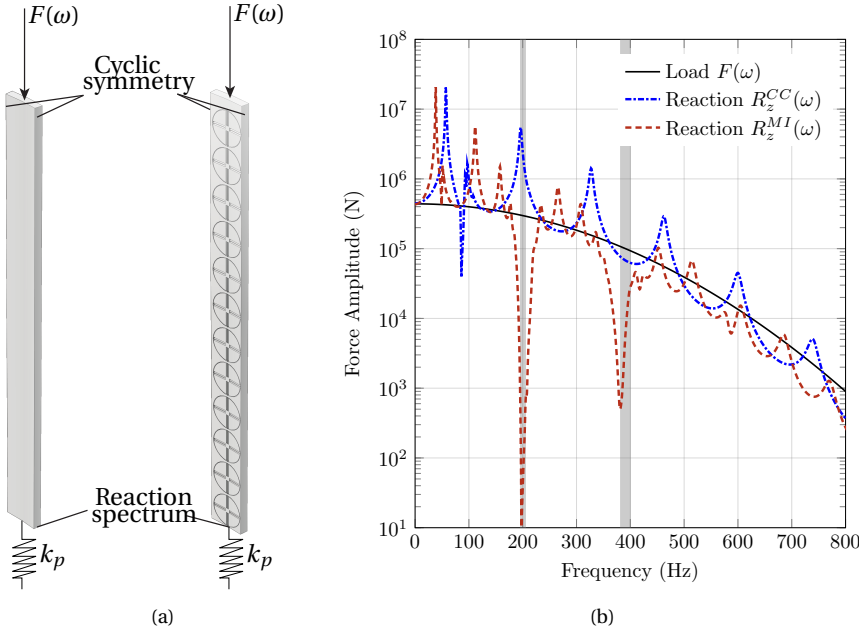


Figure A.3.: (a) Schematic representation of the reaction force calculation on the bottom boundaries of the common cushion (CC) and meta-interface (MI). (b) The curves present the comparison among the load spectrum $F(\omega)$ and vertical reaction spectra of both common cushion $R_z^{CC}(\omega)$ and meta-interface $R_z^{MI}(\omega)$. The shaded rectangles show the noise reduction performance due to the resonant effect of the meta-interface.

The comparison among the load spectrum and the reaction at the bottom of both interfaces is shown in Fig. A.3a. The high peaks in both reaction curves represent the structures' eigenmodes excited by the external load, as shown in Tab. A.1. In Fig. A.3b we observe that high-frequency peaks are slightly attenuated due to the presence of damping. Regarding the meta-interface functionality, the two shaded areas indicate the frequency regions corresponding to the local resonant modes of the unit cell. Associated to those regions, we observe a reduction in the amplitude of the reaction force after the unit cell's arrangement, while the remaining reaction amplitudes follow the decay of the load spectrum. This is an important feature for the noise reduction system, since only the energy related to high SPLs should be removed and the remaining energy should be transmitted to drive the monopile.

Model	Natural frequencies identified in forcing function of Figure A.3				
	f_1	f_2	f_3	f_4	f_5
CC	57.255 Hz	196.21 Hz	327.23 Hz	462.55 Hz	600.27 Hz
MI	38.501 Hz	111.65 Hz	157.85 Hz	265.10 Hz	308.00 Hz

Table A.1.: Natural frequencies of the common cushion and meta-interface.

Due to the presence of prominent peaks in the reaction spectrum of the meta-interface, it is important to verify the monopile strength under the new load condition. To that end, an inverse fast Fourier transform (IFFT) of the meta-interface reaction is applied to obtain the variation of the reaction signal in the time domain. The resulting time signal is then introduced as a boundary condition of the monopile's top surface, with a prescribed zero displacement applied at the bottom surface. Additionally, to model the energy dissipation to the surrounding environment, mechanical impedance Z is introduced ($Z_W = 1.47 \text{ MNs/m}^3$ for pile-water interface and $Z_S = 1.40 \text{ MNs/m}^3$ for pile-soil interface). The transient analysis is conducted with a time step of $\Delta t = 0.01 \text{ ms}$.

Figure A.4 illustrates both the time signal of the meta-interface's reaction force and the maximum Von Mises stress identified at each time step. The observed decay behavior is attributed to the energy dissipation into the surrounding water and soil. Specifically, a maximum stress of 248 MPa is identified at approximately 2 ms, which results from wave reflection at the monopile's bottom surface. It is noteworthy that this maximum stress value is lower than the yield stress of steel ($\sigma_Y = 250 \text{ MPa}$). With increasing pile penetration, the contact area among the pile, soil, and water increases, leading to greater energy dissipation and mitigating stress levels.

A.3. META-INTERFACE RESPONSE AT NOISE GAPS

From Figure 4, it was observed that the noise reduction at frequencies around 200 Hz was associated to the complete band gap highlighted in the shaded rectangles. However, for the noise gap at frequencies around 382 Hz, a complete bandgap is not clear, although some attenuation is confirmed in the transmission loss diagram. To visualize the behavior of the meta-interface at such frequency range, a frequency response analysis is carried out, whose results are shown in Fig. A.5. The unit cell's deformation in Fig. A.5a

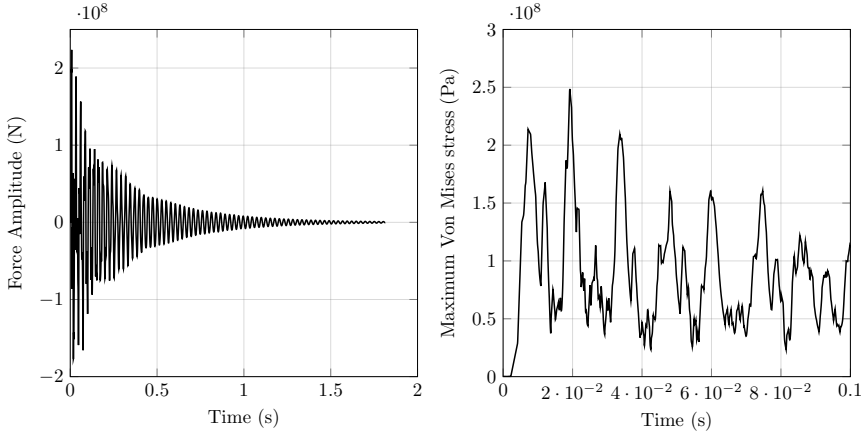


Figure A.4.: Reaction force of the meta-interface in the time domain obtained through an inverse fast Fourier transform of its spectrum showed in Fig. A.3b. The maximum Von Mises stress at each time step of the transient analysis is depicted in right. A maximum value of 248 MPa is observed, which is lower than the yield stress of steel.

has a similar configuration as that of mode M_1 (see Fig. 4). The unit cell's mode shape at the bottom indicates in a different perspective its particular motion – the resonator's mass moves in-plane with the unit cell's frame. Notice that through the meta-interface's depth, such motion reduces due to the wave attenuation effect caused by the resonators' out-of-phase motion. For the second noise gap around 382.77 Hz, the resonant mode M_2 is identified. The frequency response of the meta-interface at such frequency also highlights the same deformation of the unit cell. Again, wave attenuation occurs through the meta-interface's depth, since we observe a reduction of the resonant motion through the meta-interface's depth due to the attenuation of the propagating wave (Fig. A.5b).

A.4. SPL INVESTIGATION AT DIFFERENT WATER HEIGHTS

To ensure that the noise reduction is also achieved at different water heights, the SPL analysis is conducted at measuring points close to the monopile (25 m away from the pile wall) at three underwater levels: 5 m below the searface, 25 m below the searface (middle of water domain), and 1 m above the seabed. Fig. A.6 shows that at those locations, a high SPL is also identified at frequencies around 200 Hz. A peak near 100 Hz occurs due to the shift of natural frequencies of the meta-interface, which can be controlled by modifying the number of layers or the unit cell distribution over the meta-interface.

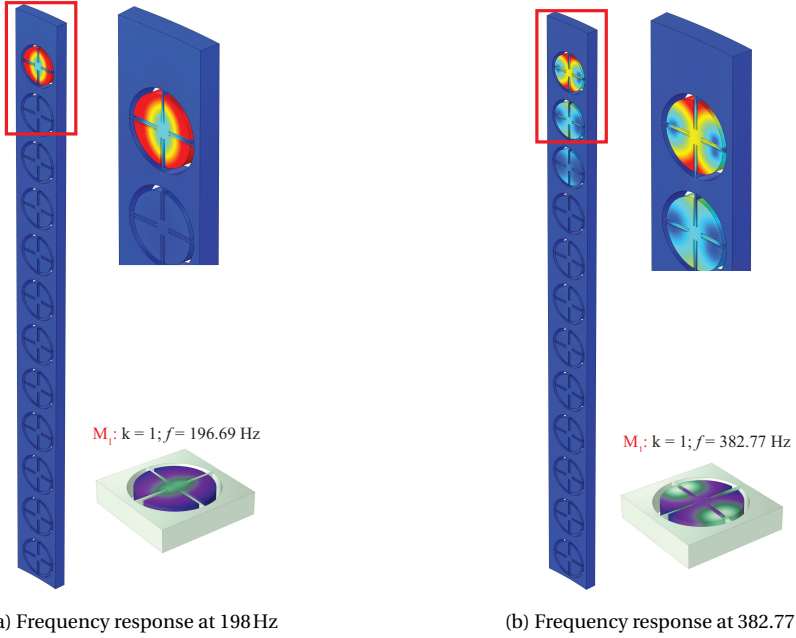


Figure A.5.: Investigation of the meta-interface's frequency response corresponding to frequencies with high noise levels. The color scale indicates the displacement magnitude, which increases from blue to red (for the meta-interface) and from light green to dark purple (for the unit cell's vibration modes). The bottom unit cells at (a) and (b) represent resonant mode shapes at 196.69 Hz and 382.77 Hz, respectively.

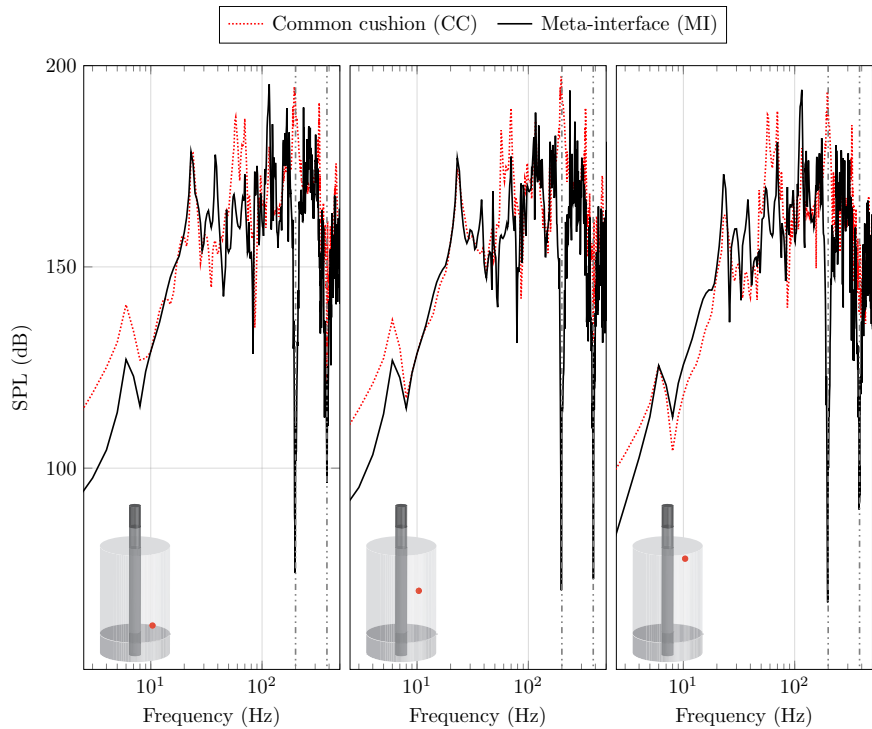


Figure A.6.: SPL comparison between the common cushion (red dotted curve) and the meta-interface (solid black curve) at measuring points close to seabed (left), at the middle of underwater column (center), and close to seasurface (right). The device contains 12 layers of periodically distributed unit cells. A reduction of about 86 dB is observed in the region with the highest SPL (around 200 Hz) for all measuring points. At 382.77 Hz the noise attenuation is 51 dB.

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LIST OF PUBLICATIONS

Journal publications

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