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A CURRICULUM FOR ENGINEERING EDUCATION IN STRUCTURAL MECHANICS IN REACTOR TECHNOLOGY

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The paper outlines a course on structural mechanics in reactor technology taught to graduate students of mechanical engineering at Delft University of Technology. The course consists of three main parts, viz. criteria for allowable stresses, stresses and deformations in plates, shells and rings and analysis of pressure vessel components.

1. Introduction

1.1. *Purpose and aim of the course*

The course outlined in this paper is taught at Delft University of Technology to students of mechanical engineering in their 8th semester. It consists of about 15 lectures of 2×45 minutes each, completed by two afternoon sessions for solving sample problems.

In assessing the aforementioned information one must realize that, like most European universities but contrary to frequent American practice, Delft does not confer a separate degree in nuclear engineering. Rather this is one of the various possible main subjects for graduation in mechanical engineering which the student may select upon completion of his 6th semester. Other obligatory graduate courses resulting from the selection of nuclear engineering concern reactor types, reactor physics and shielding, heat removal, plant dynamics and control, component design a.o. Upon entering the present course the students' knowledge in the field of interest may be summarized as follows:

- fundamentals of mechanical behaviour of materials, including creep, fatigue and fracture;
- one-dimensional elasticity, sometimes referred to as "beam theory";
- the application of energy theorems to statically indeterminate beam structures.

The main aims of the course are:

a. to provide an understanding of the various modes of failure relevant for pressure vessels and of the types of loading or "stress categories" by which they might be caused. Such understanding is essential for establishing rational stress limits;
b. to impart knowledge of the basic elements of plate and shell theory;
c. to demonstrate the application of this theoretical knowledge to the stress and deformation analysis of selected pressure vessel components, viz. heads, flanges, nozzles and tubesheets;
d. to acquaint the students with the potential of such numerical techniques as the finite-difference and the finite-element methods for solving problems where the analytical approach fails or becomes too cumbersome.

In connection with this last point it must be emphasized that the present course does not pretend to train specialized stress analysts: both the teacher and the available time would be obviously inadequate for such a purpose. Moreover a number of courses taught by members of the Applied Mechanics group of the same Department are available for students desirous of such specialized knowledge. On the other hand it is felt that upon completion of the present course the student, in addition to being capable of solving most practical problems in pressure vessel design and analysis, should be able to find his way in the pertinent literature and know when and how to invoke the specialist's help.

This introduction would be incomplete without attempting an answer to the questions why such a course should be included at all in the curriculum for mechanical engineering and why it is presented in the context of nuclear engineering. The answer to the first question is to be found in the inadequacy of the older pressure vessel design codes, which involve only primary stress limitations, for the design of an increasing number of pressure vessels for critical applications in (petro)chemical, power, aerospace, marine and ocean engineering involving fatigue, brittle fracture or creep as possible modes of failure. The pioneering role of nuclear engineering in developing more advanced design techniques derives from a number of special circumstances prevailing for nuclear pressure vessels:

- extremely high safety and leak-tightness requirements;
- limited accessibility for periodic inspections;
- increased risk for hidden flaws due to the high wall thicknesses in PWR vessels;
- increased susceptibility to failure by fatigue or thermal ratcheting due to
  - load-dependent thermal stresses caused by neutron and γ-absorption;
  - radiation embrittlement;
  - steep thermal gradients in fuel canning and sodium-cooled equipment, particularly for loss-of-flow accidents;
  - thermal shock in the case of emergency core cooling.

1.2. *Main contents*

In accordance with the aims set forth earlier the course consists of the following three main parts

1. Criteria for allowable stresses
2. Stresses and deformations in shells, plates and rings
3. Analysis of pressure vessel components.

Of these, the first and third form the backbone of the course. The first part arrives at allowable stress values by providing the rational behind them. The third part supplies the tools for calculating the stresses and deformations actually occurring in important pressure parts such as flanges, tubesheets and nozzles under various service loads.

The second part has the auxiliary function of bridging the gap between one-dimensional elasticity and the analytical methods of part three. For most of its contents reference can be made to standard textbooks. The discussion of this part shall therefore concentrate on a few topics to which this applies to a lesser degree.

2. Criteria for allowable stresses

As explained before this chapter starts with an enumeration of the modes of failure relevant to pressure vessel design, including their physical explanation. This requires particular care for the two modes of failure caused by alternating loads because of the difficulty of presenting a succinct and yet unequivocally clear distinction between incremental collapse and low cycle fatigue, both of which may occur if the calculated elastic stress range exceeds twice the yield stress. The author has been fortunate to be able to draw from the excellent example presented by Ruiz in [A.5] on the basis of a hypothetical three-bar system loaded by a constant mechanical and a superimposed alternating thermal load, the latter being applied by cyclic heating and cooling of the outer bars. The first preparatory step in the reasoning is to show how the structure may shake down to elastic behaviour. This course of events, indicated by A in fig. 1, lies at the root of the $3S_m$ criterion of ASME III, subsequently discussed in the remainder of this chapter together with the other design criteria. The understanding of this principle will enable the interested student to take cognizance of the more refined considerations on shakedown which have been published in particular in the British literature. The second and crucial step in the reasoning is to explain the fundamental difference between the two modes of failure apt to occur if the aforementioned limit keeps being exceeded. If the structural member to which this happens forms the stiffest part of the total structure, it will force the remainder to yield too, leading to cyclic elongation of the whole and thus ultimately to failure by plastic instability: cf. part B of fig. 1. If by contrast the stiffness of the cyclically yielding member is relatively small, it will form a plastic "island" in a total structure given to elastic behaviour, absorbing per cycle an amount of

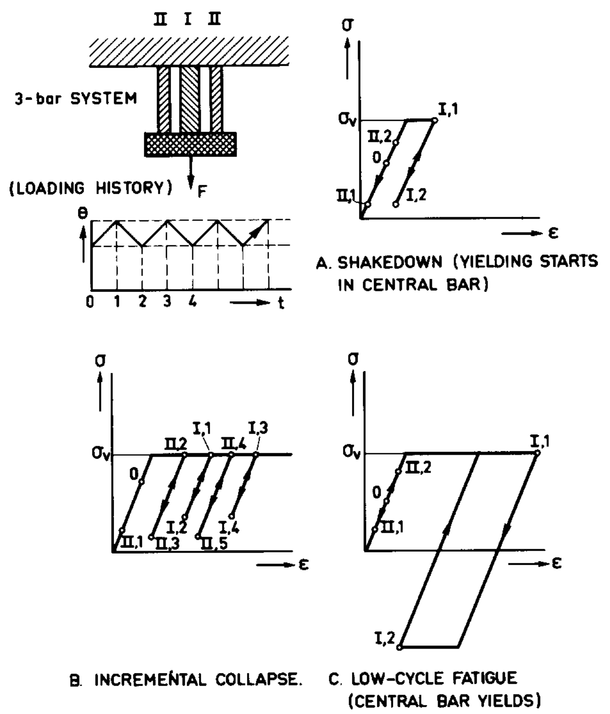

Fig. 1. Failure under alternating load, ref. [A.5].

plastic deformation energy which is relatively large due to the small area affected. This energy is indicated by the hysteresis loop of part C in fig. 1. It now remains to recall to the student how this cyclic input of energy will start to harm the structure on a molecular scale, causing pile-ups of dislocations along the slip planes and blocking further movement along these planes. It is then easy to understand how these pile-ups may in due course grow to submicroscopic, microscopic and finally visible cracks, while at the same time the blockage of slip planes will cause local embrittlement. The stage is thus set for the propagation of a crack to cause fracture. While this explanation, both in its present abbreviated form and at full length as presented in the course, may cause considerable indignation with pure-blooded metallurgists due to its gross simplifications, the author feels that it achieves the aim of visualizing fatigue and its link to fracture for mechanical engineering graduates at an acceptable price in scientific level.

The logical transition it offers towards fracture phenomena is subsequently exploited for discussing brittle fracture* as the next and last mode of failure. Again the explanation starts on the microscopic scale with the cleavage of single crystals, underscoring how this type of fracture is stress – in stead of strain-controlled and showing how the number of crystal cleavages first increases and then decreases with temperature. By proceeding from there to fracture propagation in a group of crystals the notion of a minimum crack length for instable fracture propagation is evolved, as well as the temperature dependence of the critical stress for causing instability. A graph showing the dependence of this fracture stress on temperature leads to the explanation as well as the definition of the transition temperature. Thus the student has gained an insight into the three main factors governing the susceptibility of a given material to low stress fracture: temperature, stress and flaw size. In the course of the preceding discussions on fatigue and fracture the explanation of embrittlement by fast neutron irradiation is almost self-evident. After two brief paragraphs on stress intensities and stress categories, introducing the definitions of ASME III, the design criteria are evolved for each mode of failure. The concept typical of each truly modern design code, viz. to assign separate limits to the various stress categories according to the possible modes of failure associated with each category, is thus presented in a logical and coherent manner. For the actual design stress values those of ASME III [A.27] have been retained, not because it is the only or even in each and every respect the best of its kind but because it has gained by far the widest acceptance. In the case of creep rupture Code case 1331 has been followed. Also retained is the so-called "hopper diagram" for presenting an overall view and "user's guideline".

The rationale behind the selected limits is presented on the basis of [A.26], adding some explanatory notes on the use of the modified Goodman diagram for designing against fatigue. The as yet unsolved problem of the interaction between creep and low cycle fatigue is briefly stated, referring to literature (e.g. [A.17], [A.28]) for possible further study.

* In the (Dutch) text of the course this terminology, referring to the appearance of the fracture surfaces, is avoided in favour of "low stress fracture", indicating that failure occurs well below yield stress.

The one obvious deficiency in the existing ASME III editions is of course the absence of design criteria against low stress fracture. To cover this deficiency and prepare the students for future code developments in this direction both the transition temperature and fracture mechanics approaches are briefly evaluated as possible tools for establishing design rules defining the interrelationship between temperature, stress and flaw size. In the first case this results in the presentation of the so-called "fracture analysis diagram" (sometimes referred to as "Pellini diagram") and the various definitions such as NDT, FTE and FTP associated with it, as well as the main experimental procedures, such as the Robertson and Dynamic Tear tests, for obtaining the necessary data. In the case of fracture mechanics care is taken to underscore its elastic origins – for the Westergaard equations themselves the student is referred to literature such as [A.20], [A.22], [A.23] – as the main reason for its cautious application to the tough nuclear pressure vessel steels. The requirement that the radius r_y of the plastic zone at the crack tip, always present in the case of these steels, be small in comparison to crack length and wall thickness, explains the enormous specimen sizes required for fracture toughness testing of such steels at elevated temperatures, considering that r_y will be proportional to K_{Ic}/σ_y. However, as illustrated by fig. 2 the fracture toughness K_{Ic} rises so steeply at temperatures above NDT that there seems to be little need for accurate experimentally verified values at those higher temperatures. The emergence of the fracture toughness (or critical stress intensity factor) K_{Ic} as a material property indicative of the load bearing capacity, similar to such familiar properties as yield stress or U.T.S., appears as an important advantage of the fracture mechanics approach in vessel design. For its practical application however it is further required to establish the influence of flaw shape as well as of its size upon the allowable stress for a material with a given fracture toughness. A convenient though indirect method is to take a given stress and present the corresponding critical flaw size as a function of flaw shape, as shown in fig. 3. By taking the stress equal to the yield stress, i.e. the maximum possible value, one obtains a fair impression of the size to which a fatigue crack of a certain idealized shape may grow before it becomes dangerous. Here it should be remarked that most cracks of

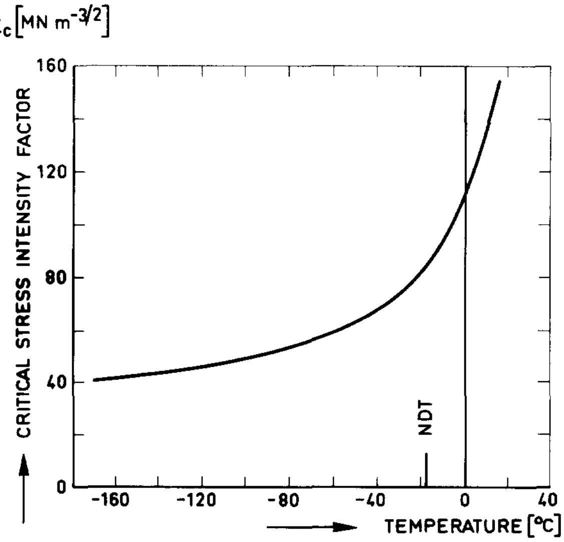

Fig. 2. Fracture: K_{Ic} as $f(\theta)$ (ref. as for fig. 3).

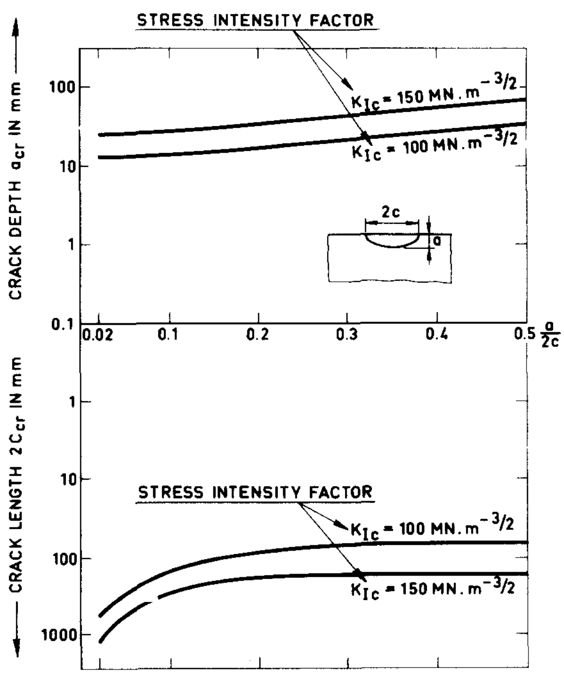

Fig. 3. Fracture: critical surface flaw size (for $\sigma_0 = \sigma_y = 400$ MN/m^2; ref.: IRS-Fachgesprach: "Die Sicherheit v. Rohrleitungen u. Druckbehältern in Kernkraftwerken", Bonn Nov. 1970 (Paper H. Spähn/H.W. Lenz).

technical significance to pressure vessel safety can be approximated between extremes of relatively simple shape, such as circular, elliptical, semi-elliptical or hairline. Furthermore elastic computations have supplied much additional information on the stress intensity factors for various crack shapes (e.g. [A.24]), some of which is included in the course in tabular form. I should not like to leave the subject without stressing that the student is cautioned against the limitations of fracture mechanics in pressure vessel design as well as being made aware of its potential. Unknowns mentioned specifically are the influence of complex stress states and of dynamic loading conditions upon the fracture toughness and the constants in Paris' law on fatigue crack growth [A.28], [A.29].

3. Plates, shells and rings

With so many outstanding textbooks (e.g. [B.1–5]) the only problem in a course like the present one is to condense the available information to fit the restricted aims of the course. Therefore this chapter starts by enumerating the following limitations to which its contents will be subject:

- *Shape*: the text covers only cylindrical and spherical shell elements and circular plates, all of uniform thickness, and rings of rectangular cross section with large radius-to-thickness ratios. Fig. 4 illustrates how all the important parts of nuclear pressure vessels can be schematized into the aforementioned elements.
- *Load*: only axisymmetric loads are to be considered. Nuclear and other comparable pressure vessels are subject to the following loads
  - internal pressure, causing axisymmetric load patterns except at nozzles on the cylindrical shell or hill-side nozzles on bottom or top head. Such non-axisymmetric cases, while beyond the scope of this course as regards their true solution, will be briefly discussed at the end of section 4;
  - temperature gradients across the wall or between adjacent vessel parts: here the same applies as for the pressure loading;
  - pipe, control rod scram and support pad reactions: with a few exceptions these are not axisymmetric and therefore excluded from discussion in this course. While this admittedly constitutes an incompleteness, it should be remarked that such local loads are hardly amenable to analytical treatment by non-specialists, who are usually referred to existing charts deriving from Bylaard's work [C.15–17].
- *Deformations*: only "small" deformations are considered, excluding feedback effects between deformation and load. This restriction is generally accepted in pressure vessel analysis and requires no further amplification.
- *Stress state*: only "thin" shells ($h/R_{av} \leqslant 0.1$) and plates ($h/R \leqslant 0.2$) are considered, i.e. stresses perpendicular to the middle plane and shear stresses are neglected. In order to explain the full consequences of this limitation for subsequent analyses this part of the course contains a concise but rigorous treatment of the basic assumptions of thin shell theory in analogy to those of beam theory with which the students are familiar. While this restriction does not impose any practical limitation as regards nuclear components except for the tubesheets of large steam generators and for reinforced parts of nozzles, both of which are discussed in section 4, it does exclude vessels for a number of high pressure applications in chemical engineering. Again it is felt that this is not too high a price to pay for the considerable simplification obtained through this restriction in the various formulae and their derivations, the more so because the student interested in thick-walled shells will be well prepared for the use of pertinent literature, such as Faupel's contribution in [G.2].

In summarizing it may be said that the above restrictions constitute a deliberate choice for clarity over completeness.

Turning now to the actual contents of this chapter two introductory remarks on the arrangement and sequence of the subject-matter seem in order. First it should be pointed out that the main subdivision according to stress category has been deliberately chosen with the intention to further familiarize the students with this concept, aiding them to integrate their subsequent stress analyses into code procedures. Secondly it should be explained that in the paragraph on primary stresses shells – comprising both cylinders and spheres – are treated prior to plates because in their case a separation between membrane and bending stresses does make practical sense.

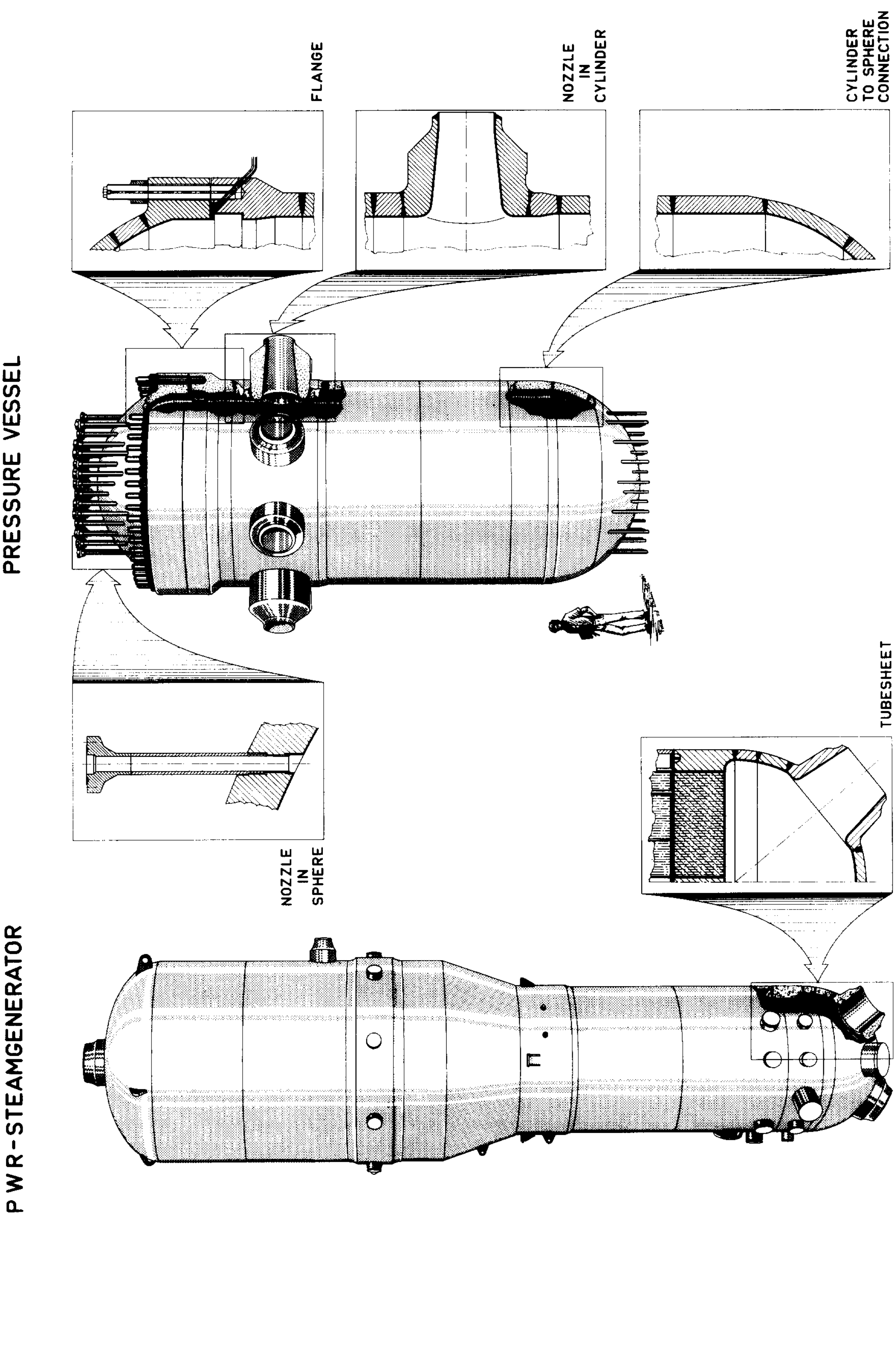

Fig. 4. Schematization of p.v. components into basic elements.

This paragraph on primary stresses derives almost totally from standard textbooks, viz. [B.2] and [B.3] for spheres, [B.1] and [B.3] for plates and [B.4] and [B.5] for rings. Therefore the discussion here may be limited to two features concerning the bending of shells loaded by edge forces and moments whose treatment differs slightly from that given in the textbooks. In the case of cylindrical shells the well-known fourth order differential equation is derived by considering the cylinder to consist of staves which can be treated as beams on an elastic foundation; apart from being an alternative to the usual procedure based upon equilibrium and compatibility considerations of a shell element, this approach has the added advantage of being reminiscent of the beam problems with which the students are thoroughly familiar. In the case of (hemi)spheres the differential equations

$$\frac{d^2 Q_\phi}{d\phi^2} + \frac{dQ_\phi}{d\phi}\cot\phi - Q_\phi \cot\phi \pm 2\,i\kappa^2 Q_\phi = 0$$

are derived in the classical manner described e.g. in [B.2], Q_ϕ being the radial stress resultant on the shell element considered. However, for the solution of these equations only two approximations are discussed because of their suitability for the two kinds of edge loading of interest, i.e. near a meridian and near the apex of the (hemi)sphere. For the first of these ($\phi \geqslant 30°$) the Geckeler approximation of discarding the terms containing Q_ϕ and $dQ_\phi/d\phi$ gives satisfactory results, while for the second case – relating to radial nozzles – the Reissner–Esslinger approach substituting $1/\phi$ for $\cot\phi$ is recommended.

A feature discussed under the heading of primary stresses which is not found in the aforementioned textbooks concerns the bursting pressure of shells. The main reason for its inclusion is to give the student, whose view of strength problems prior to this course has been restricted to homogeneous materials obeying Hooke's law, a first glimpse beyond the elastic range, subsequent to the introduction of inhomogeneities under the subjects of fatigue and fracture. The fact that this is done within the framework of nuclear technology appears justified by the unique phenomenon of a multi-million dollar pressure vessel – the containment – being designed and built for a load it will have to withstand, if ever, once in its lifetime. It should be obvious that its design should incorporate plasticity considerations. Of course the aim of such a first glimpse can only be to indicate the way in which such an *engineering* problem is tackled beyond the realm of Hooke's law. This implies the introduction and utilization of an alternative relationship between loads and strains – for strainhardening materials – in a rigorous step-by-step procedure, the main steps being:

- assumption of constant volume;
- introduction of the yield surface described by $\phi = S_t^2 + S_{ax}^2 + S_r^2 = \text{constant}$, where $S_t = \sigma_t - \frac{1}{3}(\sigma_t + \sigma_{ax} + \sigma_r)$ etc.;
- derivation of the replacement for Hooke's law

  $\delta\epsilon_t = \frac{2}{3}\mu|\sigma_t - \frac{1}{2}(\sigma_{ax} + \sigma_r)|,$

  $\delta\epsilon_{ax} = \frac{2}{3}\mu|\sigma_{ax} - \frac{1}{2}(\sigma_r + \sigma_t)|.$

  $\delta\epsilon_r = \frac{2}{3}\mu|\sigma_r - \frac{1}{2}(\sigma_t + \sigma_{ax})|,$

  where μ will be a function of S_t, S_{ax} and S_r;
- introduction of an equivalent stress, based upon the Von Mises criterion

  $\bar{\sigma}_M = \frac{1}{2}\sqrt{3}(R/h)p$

  and an equivalent strain

  $\bar{\epsilon}_M = \frac{2}{3}\sqrt{3}\,\epsilon_t;$
- determination of the point of instability from $\delta p = 0$, giving

  $\delta\bar{\sigma}_M/\delta\bar{\epsilon}_M = \sqrt{3}\bar{\sigma}_M;$
- definition of the behaviour of a tensile specimen of the material in the plastic range by

  $\bar{\sigma}_M = K\,\bar{\epsilon}_M^n,$

  where K = strength coefficient and n = strain hardening exponent;
- derivation of the bursting pressure

  $p_B = \frac{2}{(\sqrt{3})^{n+1}} S_B h_o/R_o$ for cylinders,

  $p_B = 2\,(\frac{2}{3})^n S_B\, h_o/R_o$ for spheres,

  where S_B = U.T.S. of the material and h_o and R_o indicate the initial wall thickness and radius of the shell, respectively.

These results, corresponding to those published by Svensson [B.9], are instructive both through their similarity to those obtaining for the yield pressure from elastic considerations and because they can be used to show the decreasing "plastic reserve", i.e. the decreasing margin between yielding and bursting, of high-strength steels with their lower $S_B : S_y$ ratios.

The next subject of this section is that of secondary mechanical stresses as they occur near discontinuities. The example of the junction between a pressurized cylinder and its hemispherical bottom, forming a direct application of the shell bending problems covered in the preceding paragraph, serves to illustrate the general method:

- dividing the structure into its constituent elements by a cut at the junction;
- expressing the individual deformations (radial displacement and edge rotation) in terms of the redundant shear forces and moments at the junction;
- applying the conditions of equilibrium and compatibility at the junction;
- solving the resulting set of equations for the redundant forces and moments;
- determining the deformations and stresses in the shell elements caused by these forces and moments.

Results of such a sample calculation are shown in fig. 5. They serve to illustrate two points concerning these secondary stresses. The first is their local character due to the quick decay of the sinh and cosh functions present in both the exact solution of the cylinder and the Geckeler solution for the hemisphere this helps the students to visualize the "relaxation length" concept. The second point is the relative smallness of these stresses compared to the primary membrane stresses due to the same pressure loading;

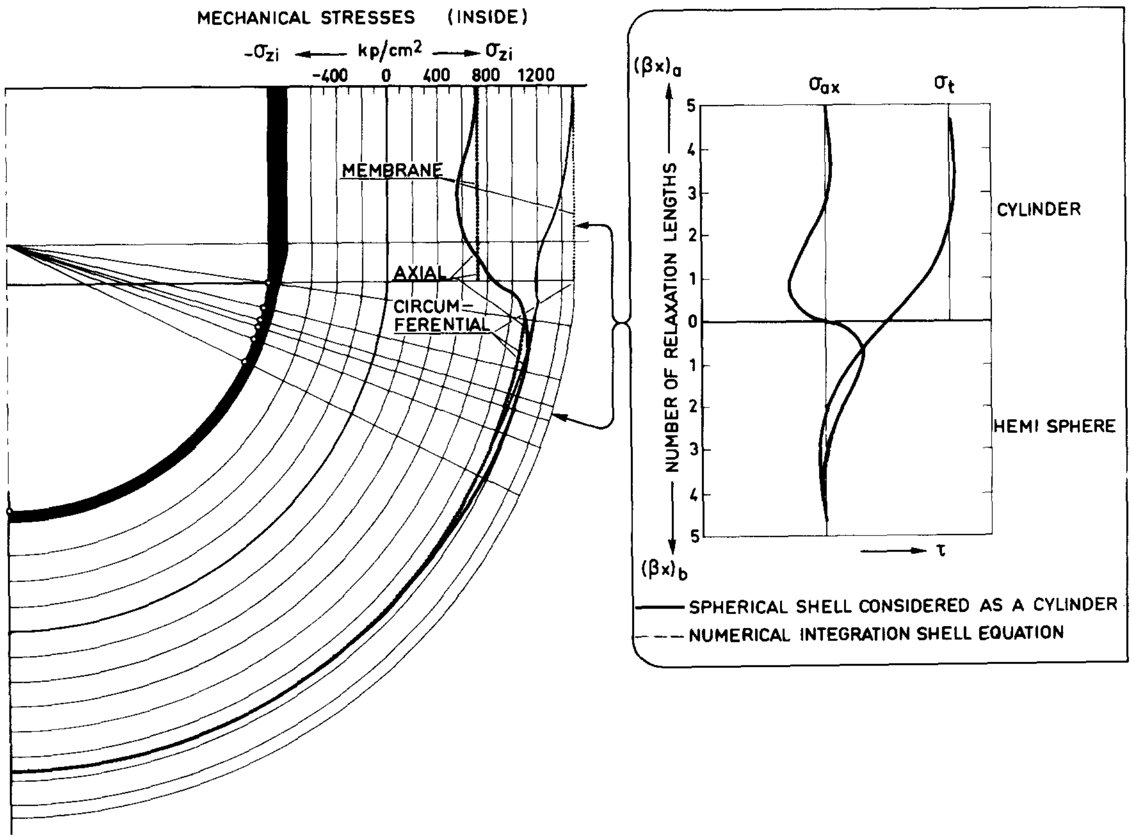

Fig. 5. Application to discontinuity stresses, ref. [G.7].

even the peak value of the axial discontinuity stress does not exceed 30% of the corresponding membrane stress.

A relatively much greater attention has to be paid to secondary stresses of thermal origin, not only because of their importance to nuclear and conventional power engineering but also because the vast amount of literature available on the subject has comparatively little to offer to the student looking for a basic general introduction leading to the solution of practical problems. Many high-quality books on thermo-elasticity such as e.g. [B.13] offer far too much in the field of theory, while on the other hand many useful articles on the solution of particular problems presuppose a basic knowledge which has yet to be imparted to the students. The approach taken in the present course is to limit the discussion in the first instance to the temperature distribution in thin-walled cylinders. The solution of temperature distribution problems forms the key to all practical thermal stress problems, the vast majority of which – both for nuclear and non-nuclear applications – refer to the cylindrical geometry in the form of drums, vessels, piping, thermal shields, turbine casings etc. It is then easily shown that as far as the temperature distribution is concerned the thin-walled cylinder geometry can be further simplified to that of the semi-infinite flat plate.

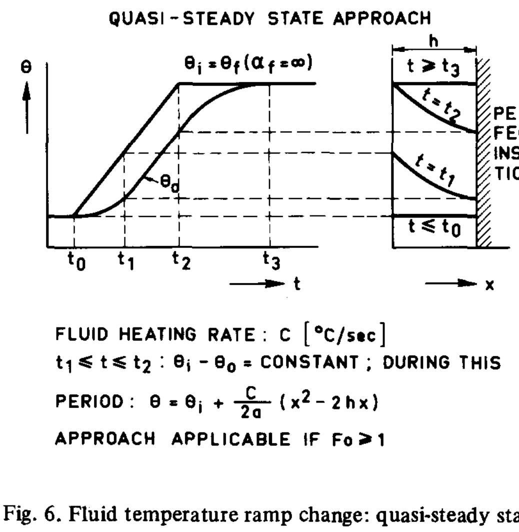

Fig. 6. Fluid temperature ramp change: quasi-steady state approach.

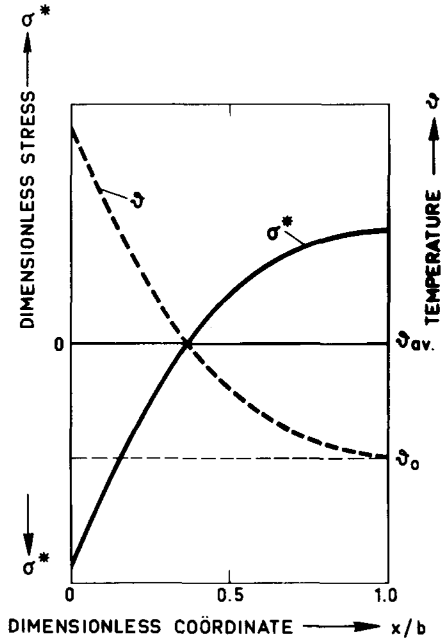

Fig. 7. Wall temperature stress distribution for quasi-steady state heating.

Within this geometrical limitation the first subject to be emphasized is that of the temperature distribution in an externally insulated wall resulting from a ramp- or stepwise temperature change of the fluid flowing past its inside surface. This problem statement, being typical for temperature fluctuations in power and process plants including their startup and shutdown and thereby for the majority of cases involving important thermal stresses, would entail solution of the Fourier equation in its partial differential form. Fortunately the great majority of transients in power and chemical engineering originate from fluid temperature ramp changes whose slope and duration permit a quasi-steady state approach (fig. 6), which only considers the period during which the temperature difference across the wall is constant and maximum. This latter fact implies that the results will be conservative, but calculations performed by Junghanss [B.20] have shown them to be within 10% of the exact analytical solution for cylinders provided the Fo number is at least equal to 1. This condition, implying that the ramp temperature change must neither be too steep nor too small, is met by most transients even in sodium-cooled plants. The temperature distribution in the wall during this period will be non-linear, as described by the equation $\theta = \theta_i + (C/2a)(x^2 - 2hx)$, where θ_i represents the temperature at the inside surface and C [°/min] the slope of the temperature ramp [B.18]. This is plotted in fig. 7, which also shows the resulting stress in order to remind the student that the thermal stress at a given location in the wall will be proportional to the difference between the local and average temperatures. Such a reminder has been found necessary because this simple fact tends to be obscured by the bulky formulae for thermal stresses derived subsequently.

In the above sketch illustrating the principle of this approach the simplifying assumption of an infinite heat-transfer coefficient α_f from fluid to wall did not affect the wall temperature distribution. By contrast in the rare case where a technically important problem can not be adequately solved by the quasi-steady state approach because the Fo number is too low, it will no longer be possible to ignore the effect of this convective heat transfer. On the contrary the ratio between this heat-tranfer resistance and that of the wall represented by the reciprocal value of the dimensionless Biot number $\mathrm{Bi} = \alpha h/\lambda_w (\lambda_w$ = wall thermal conductivity) is highly indicative of the seriousness of the thermal stress problem: e.g. Bi values much below 1, often found for gaseous coolants, mean that transient thermal stresses will be negligible.

Where this is not the case and solution of the Fourier equation in its partial differential form is required it seems advisable to present the results in graphical form available from literature [B.22] [B.27] rather than using lecturing time for their analytical derivation [B.21]. Fig. 8. shows such a graph [B.25],

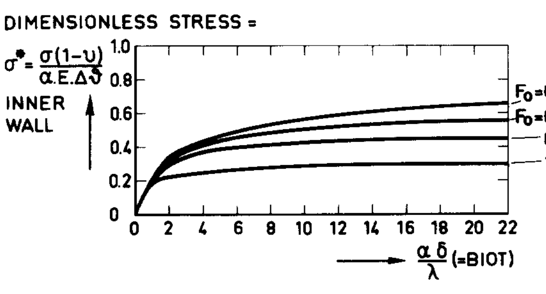

Fig. 8. Maximum stress for transient heating, ref. [B.25].

indicating the influence of the Fo and Bi numbers; the stresses are conveniently presented in a dimensionless form through dividing them by $[E\alpha/(1-\nu)]\ \Delta\theta_f$. This latter expression represents the thermal stress in a cylinder for the case of pure thermal shock, i.e. a step change of $\Delta\theta_f$ in fluid temperature instantly communicated to the inside wall surface (i.e. $\alpha_f = \infty$). While this represents an unrealistic extreme it forms a useful yardstick for the severity of actual temperature transients.

The second feature to be emphasized within the continuing restriction of cylindrical geometry is the temperature distribution resulting from volumetric heat sources inside the wall. In nuclear reactor engineering such sources will generate from nuclear radiation. It is therefore essential that the students be first reminded of the types of radiation emanating from the nuclear fission process, in order to understand that the final consequence of interest for the present problem is the conversion of γ-ray energy to heat. They are thus prepared for the reintroduction of the energy absorption coefficient μ, which, together with the relative wall thickness, determines the temperature distribution. The schematic presentation of these types of radiation shown on fig. 9 and originating from [B.27] seems a convenient tool for this purpose.

In deriving the formulae for the temperature distributions due to γ-ray capture excessive complication is avoided by limitation to steady-state conditions. This limitation appears justified by the operating characteristics of central station power reactors. The integration constants in the general solution for the temperature distribution in a flat plate with exponential volumetric heat source $(H = H_0 e^{-\mu x})$:

$$\theta = -(H_0/\lambda\mu^2)e^{-\mu x} + C_1 x + C_2$$

are determined for two sets of boundary conditions, corresponding to the reactor vessel wall and thermal shield, respectively. Results for these two cases including the resulting thermal stresses for cylindrical geometry, are presented in fig. 10 [B.29].

After this fairly extensive discussion of temperature distribution in thin cylinders approximated by flat plates a few words are in order on the temperature distribution in rings, required for obtaining the thermal stresses in flanges and in the unperforated rims of tubesheets (cf. section 4). In these cases the radius-to-thickness ratio for reactor vessels and PWR

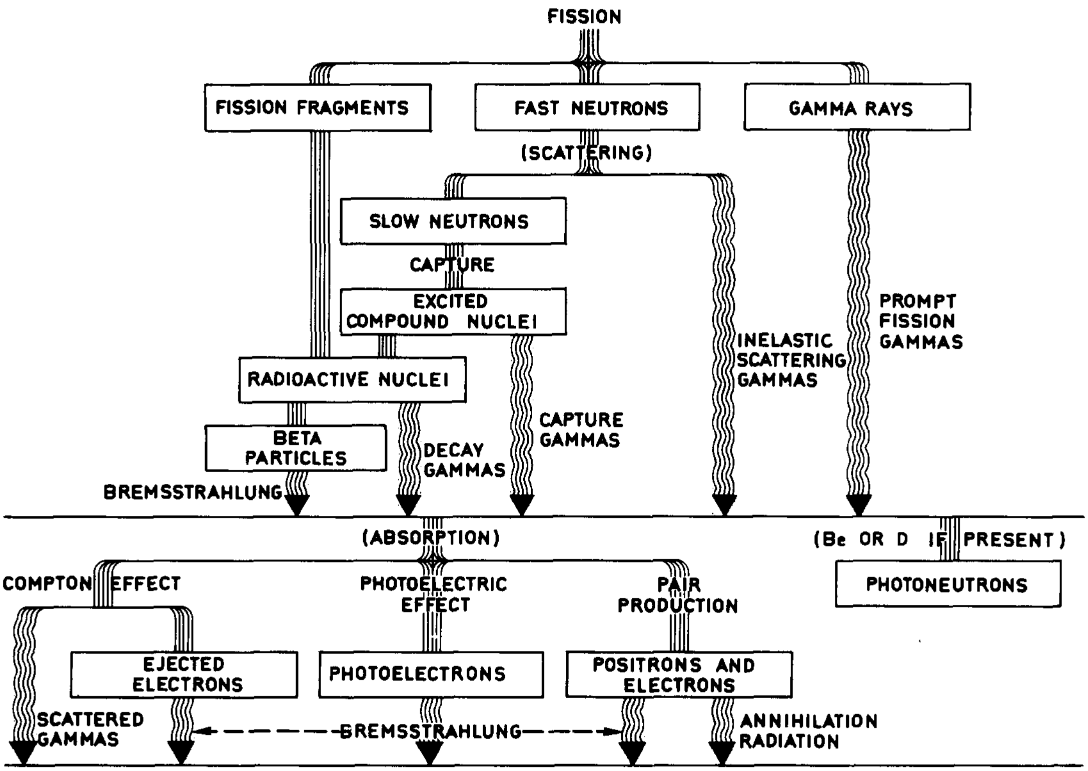

Fig. 9. Types of radiation from a nuclear reactor core, Ref. [B.27].

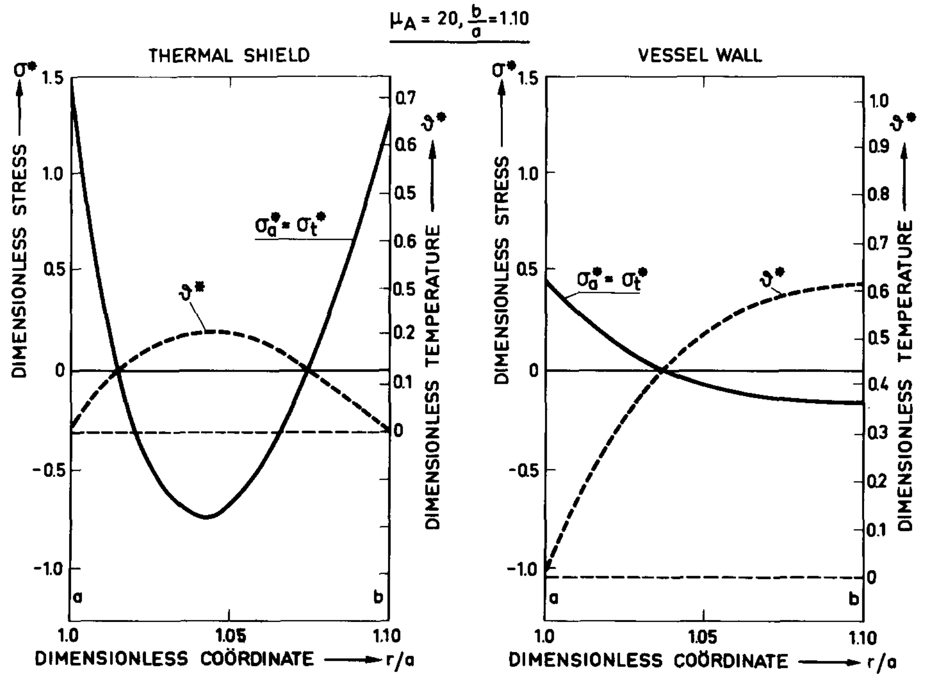

Fig. 10. Temperature and stress distribution in vessel wall and thermal shield due to γ-heating, ref. [B.29].

steam generators is sufficiently large to permit the approximation $R = \infty$, i.e. the assumption of straight bars. This results in a steady-state temperature described by

$$\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} = 0,$$

where y and z represent the "radial" and "axial" coordinates, respectively. For perfect outward insulation and assuming the same in axial direction – a rather crude assumption for reactor vessel flanges – one obtains rapidly converging series expressions for the average ring temperatures in the inner and end face temperatures, containing the ring width and height as constants [B.21], [C.23].

Once the temperature distributions have been obtained little remains to be said on the determination of the associated stresses, bearing in mind their abovementioned proportionality to the difference between the local and average temperatures. Formulae for the geometries of interest together with their derivations are obtained from literature sources, e.g. [B.15] or [B.17].

So far we have only been concerned with free body temperature distributions and thermal stesses. However, with the information presented in the preceding text of this paragraph it requires little imagination to proceed with the determination of discontinuity thermal stresses along the lines shown earlier for discontinuity stresses of mechanical origin. In the course this is illustrated by the example of a fuel cladding tube-to-end plug connection, where for the sake of simplicity the plug is considered perfectly rigid.

Turning now to the final paragraph of this section on peak stresses, its main point and the only one to be discussed here concerns the desirability of a clear distinction between these stresses and those of the secondary category discussed before. For this purpose let us describe the essential difference between these two stress categories defined in ASME III as follows: secondary stresses extend through all or at least a sizeable part of the load-bearing section concerned, while peak stresses do not directly affect the load-bearing capacity of a structure at all. Peak stresses occur e.g. at nozzle corners and are usually computed by multiplying the membrane stresses in the undisturbed vessel part by experimentally obtained stress concentration factors. In this way discontinuity stresses are lumped together with true peak stresses. As only the former should be taken into account for assessing the adequacy of reinforcement, this approach tends to overestimate the amount of reinforcement required. While the desirable distinction between the two stress categories at nozzle penetrations requires refined methods such as those presented by Van Campen e.a. in Session G.2 of this

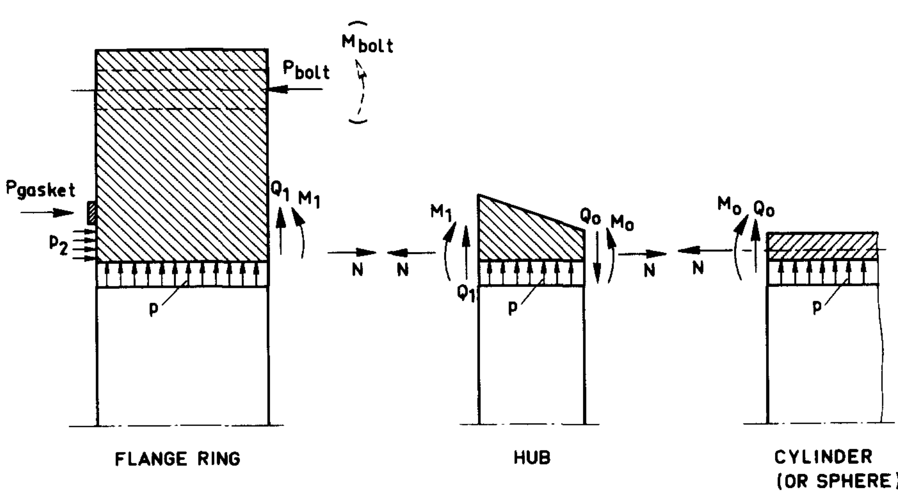

Fig. 11. Schematization of flange.

conference, which are beyond the scope of this course, it seems important to focus the student's attention on the existing problem. This and some references to relevant literature [B.31–33], [C.19] form the gist of this last paragraph.

4. Analysis of pressure vessel components

Like in the preceding parts no attempt at completeness is made in this chapter either. However the three components included not only cover the most critical areas in reactor primary circuits, but also represent applications of all the information contained in section 3. This is so because thermal as well as mechanical loads are considered and determined in two extensive sample calculations on flanges and radial nozzles. Finally the non-specialized audience is offered a few guidelines for the solution of problems falling beyond the scope of this course.

4.1. *Flanges*

The course is limited to flanges for large H.P. vessels such as those of LWR's and requiring extremely high standards of leaktightness. The latter circumstance implies that the functional failure by excessive deformation causing leakage must precede structural failure for this component, meaning that the primary aim of the designer will be to determine deformations under operating conditions rather than stresses. It furthermore implies high gasket pressures, to be obtained by a combination of narrow gasket faces and a great number of closely spaced bolts. The well-known narrow, high, tapered-hub geometry resulting from these requirements entails an analytical approach considerably different from that followed for conventional L.P. flanges.

Prior to such an analysis the external loads must be known as input data. These comprise internal pressure (may be absent), bolt forces, gasket reaction and gasket face friction (during thermal transients only). The bolt forces are "smeared" uniformly over the pitch circle to maintain the assumption of axisymmetry, a procedure found acceptable in analytical studies for the close pitch prevailing here. A much tougher problem is posed by the gasket reaction: as a consequence of the compact flange geometry the difference between the bolt pitch radius R_b and that of the centroid of the flange ring cross section R_c is very small. This increases the relative importance of the gasket reaction moment M_g as compared to the bolting moment M_b, thereby posing the problem of determining as exactly as possible the location of the radius of application for the gasket reaction force. The course suggests to use the formula given in Appendix I-12 of ASME III to obtain this radius, while pointing out at the same time the need for experimental information on the plastic behaviour of LWR vessel gasket faces for verification of the formula's applicability.

Another unknown even more difficult to obtain is the gasket face friction factor needed for computing the additional moment caused by the temperature lag between the vessel and head flanges during startup and shutdown. Here the course suggests following Dorner and Ruf's approach of assuming zero and infinite friction factors to obtain the extreme values for the required moment [G.7]. Other effects requiring experimental evidence, such as the influence of the bolt holes on flange ring stiffness and possible long-term changes in bolt elasticity are mentioned in the text but ignored in the analysis. The three-piece schematization used for this analysis and shown in fig. 11 has been widely used in publications on this type of flange and does not require further comment. The shell and head behaviour will be described by that of the edge-loaded cylinder or hemisphere, while for the dimensions typical for LWR vessels it is equally obvious that the flange rings are to be represented by rings of undeformable cross-section. All of these elements have been covered in section 3. By contrast the elastic behaviour of the tapered hub may be treated by at least three different analytical methods, viz. as a cylindrical shell with varying thickness [B.3], [C.6], as a ring with undeformable cross-section or by the finite element method. The first two represent direct extensions of section 3 and are covered in detail, thereby exposing the computational simplification resulting from the ring approach. The finite element analysis is not discussed, as this technique will only be briefly touched upon at the end of the course. However its results, being by far the most exact ones, are used as a reference standard for establishing the limits of taper and relative wall thickness which define the applicability of the two aforementioned methods. This subject is dealt with in a separate paper

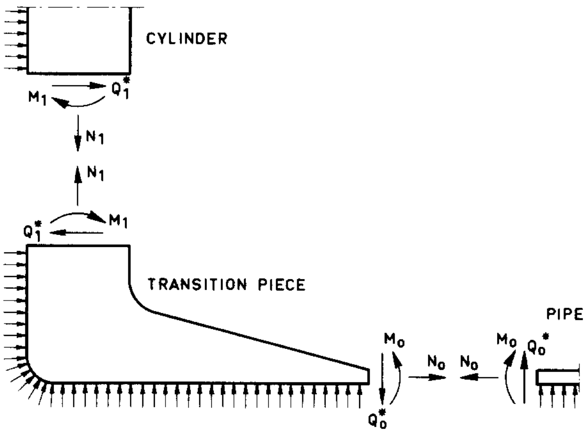

Fig. 12. Schematization of nozzle.

presented by Van Campen in Session G. 3 of this conference.

4.2. *Nozzles*

This section focuses on forged nozzles for LWR and similar H.P. vessels, characterized by d/D ratios not exceeding 0.25 and by heavily reinforced and therefore definitely thick-walled transition pieces. Only spherical shells will be considered for the analysis in order to comply with the requirement for axisymmetry. The three-piece schematization shown in fig. 12, while reminiscent of that for the flange shown earlier, is thus seen to differ by at least one essential feature, i.e. the thick-walled shape of the transition piece. This precludes the correct application of thin shell theory both regarding the elastic behaviour of this element and the compatibility conditions at the shell junction. With respect to the former it is noted that the inaccuracy resulting from the schematization is at least as important as that obtaining from neglect of the radial stress and strain components; it is therefore recommended to ignore this error in the present three-piece analysis. The conical shape of the transition piece middle plane can be taken care of by treating it as a cylinder of variable thickness similar to the tapered hub discussed in the preceding section. The compatibility problem at the nozzle-to-shell junction results from the fact that thin shell theory formulates the compatibility conditions with reference to the shell middle planes, i.e. for points A and D, whereas in reality these are separated by a ring of

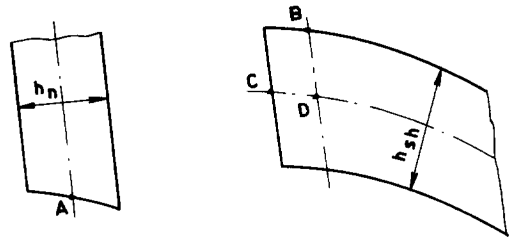

Fig. 13.

material of significant stiffness. The approach suggested to overcome this problem is shown in the next sketch: ring KLEC is considered part of the nozzle, ring KBGF part of the shell. In referring the compatibility requirements to D the stiffness of ring KBDE is counted twice while that of ring CDGH is ignored; these two counteracting effects are assumed to approximately neutralize each other.

Where thermal stresses are concerned it is recommended to use at least a four-piece analysis, dividing the transition piece into two rings with undeformable cross-section, one tapered and the other – adjacent to the shell – rectangular.

Although the above three- or four-piece analyses are already quite laborious due i.a. to the Kelvin and Bessel functions contained in the Reissner–Esslinger solution for the spherical shell, they are of course inadequate where peak stresses are required for a fatigue analysis. Unless one follows the stress concentration factor approach with its inherent drawback mentioned before the finite element technique becomes mandatory.

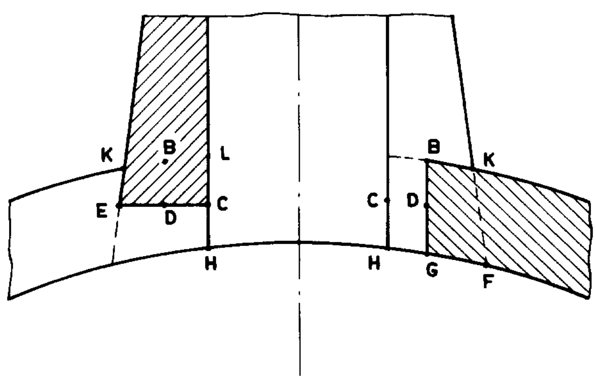

Fig. 14.

Unfortunately the restriction of axisymmetry adopted early in the course ignores the fact that by far the most important nozzle in LWR and LMFBR vessels are attached to cylindrical parts of the shell. The techniques required for even an approximate stress analysis of such configurations are undoubtedly beyond the scope of the present course. Fortunately for the designer faced with this problem the large D/d-ratio of 5 or more permits the approximation of the cylindrical shell by either a flat plate or a sphere of different radius. The first approach, discussed in a paper presented by Van Campen e.a. in Session G.2 of this conference, still entails considerable mathematics due to the non-axisymmetric part of the free-edge displacement of the hole in the plate. Calculations deriving from the aforementioned flat plate analysis suggest that the replacement of the cylinder by a spherical shell with a diameter 2.5 × the original value would permit the use of the axisymmetric model within an accuracy of abt. 20% as compared to the truly cylindrical description of the shell. It will be obvious that such rough and debatable rules-of-thumb cannot be considered as alternatives to a proper stress analysis for nozzle-to-cylinder connections. However, as pointed out e.g. in Lekkerkerker's invited paper presented to Session G.2 of this conference, the most promising approaches to this difficult problem have not yet reached the state where their results can be presented in a form accessible to the students for which the present course is intended. For the sake of completeness it should be mentioned that one approximative method coming very close to this point is the one presented by Engelke e.a. to session G.2 of this conference [C.19B]; it is based upon the flat plate approach mentioned above.

4.3. *Tubesheets*

The discussion is limited to tubesheets of uniform thickness. The following loads are taken into account:
- pressure difference between primary and secondary surfaces → primary mechanical stresses;
- pressure inside the tube perforations → primary mechanical stresses;
- edge loads at the transition between perforated plate and solid, clamped rim → secondary mechanical and thermal stresses;
- stress concentrations at the holes → mechanical peak stresses;
- temperature difference between primary and secondary fluids → thermal (peak) stresses.

Local or ring loads transverse to the plane of the plate are not considered because the analysis is primarily intended for PWR steam generators using U-tubes. However, their subsequent inclusion would not present any problems. Also excluded, with no better justification than the pursuit of simplicity, are the loads caused by temperature differences between the in- and outlet sides of the primary (tube-side) fluid and by the presence of a partition in the primary fluid channel.

Different approaches are followed for the analysis of the mechanical and thermal stresses. The basic notion underlying the analytical approach for the *mechanical* stresses is that of the equivalent solid plate showing the same elastic behaviour as the perforated area of the tubesheet which it replaces. The analysis comprises the following main steps:
- replacement of the perforated part of the plate by an equivalent solid plate through the use of the so-called effective elasticity constants E^* and υ^*;
- computation of the fictitious stresses σ_r^* and σ_t^* occurring in this equivalent plate;
- reduction of σ_r^* and σ_t^* to actual primary and secondary stresses;
- determination of peak stresses caused by stress concentrations around the holes.

These steps are taken using the information contained in the relevant Article I-9 of ASME III [C.20] and the reasoning behind it supplied in O'Donnell and Langer's paper [C.21]. (In the case of the stress due to the pressure inside the perforations the derivation for the proposed formula had to be derived anew by the staff of the author's laboratory.) The only true intellectual effort required from the students in following this approach is contained in the third step, which is necessitated by the angle between the directions of the principal stresses and those of the triangular hole pattern. It can of course be argued whether the experimental information contained in the 1968 edition of ASME III in particular on the effective elasticity constants E^* and υ^* is the best available at present, but such arguments are considered irrelevant by the author in connection with the aim pursued in the course to impart a line of thought, which is not at all affected by the experimental data used.

The equivalent solid-plate approach is not suitable for determining the temperature distribution prerequisite for the thermal stress analysis. From a number of excellent papers on this subject the approximative method proposed by De Pater [C.23] was selected for its relative simplicity. This simplicity is mainly due to two assumptions, the first of which concerns the replacement of the solid rim by a straight bar as already suggested in section 3. The second considers the material around each hole as a separate tube exchanging no heat with its neighbours, i.e. having perfect outside insulation. This assumption that the perforated part of the plate consists of an assembly of such hollow cylinders appears justified for cases where the plate thickness is large compared to the ligament width. It reduces the problem to a simple one-dimensional case with the axial (plate tickness) co-ordinate z remaining the only independent variable. Introducing the boundary conditions:

$z = 0 \rightarrow \theta = \theta_1$ (primary fluid temperature),

$z = \mathrm{h} \rightarrow \theta = \theta_2$ (secondary fluid temperature),

the following simple solution is obtained for the axial temperature distribution in the ligaments:

$$\frac{\theta - \theta_1}{\theta_2 - \theta_1} = \frac{\sinh Az}{\sinh Ah}$$

with

$$A = \sqrt{\frac{2R_{\mathrm{i}}}{R_{\mathrm{o}}^2 - R_{\mathrm{i}}^2} \frac{h}{\lambda}},$$

where $R_{\mathrm{i(o)}}$ = inside (outside) diameter of fictitious tube, h = plate thickness = length of fictitious tube, λ = thermal conductivity of tubesheet material.

If the latter differs significantly from that of the tubes a weighted average may be taken.

The temperature distribution shown in fig. 15 was obtained by the aforementtioned method for a tubesheet of much lower height-to-bore ratio than usual for PWR steam generators. It nevertheless supports the approach suggested by ASME III to consider the resulting thermal stress as a "skin effect" or peak stress limited to the shell-side surface. The stress obtained from this figure is in good agreement with that resulting from the use of ASME III's "skin stress intensity factor". It may be concluded that the use of this approximative method is warranted for not too thin tubesheets.

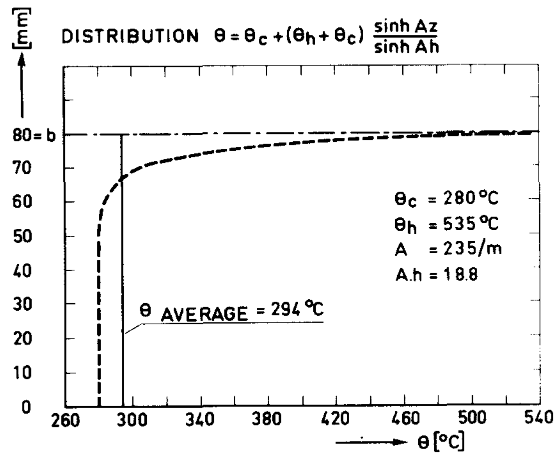

Fig. 15. Temperature distribution in tubesheet.

4.4. *Hints on numerical techniques*

Several times in the above text the need for more advanced techniques has been mentioned, while at the same time referring to the limited scope of this course. The method followed to at least partly reconcile these conflicting requirements has been to include in the last part of the course a brief outline of the basic principles of the finite-difference and finite-element techniques and make the students aware of their possibilities for cases not easily solvable by the methods discussed before. Care is taken to explain that whereas the former method is a *mathematical* tool to solve differential equations, the latter is based on a different *physical* approach. Knowing that the finite-element method is suitable for temperature as well as stress analysis, it may be asked why the other technique is mentioned at all. This is partly because its principle is already known to the students from courses on numerical analysis, but mainly because it is the obvious choice in many applications outside the stress analysis field because of greater simplicity of operation and interpretation of results. The latter argument is particularly true for the application to temperature distributions because in this case the finite element method lacks a clear physical principle such as the minimum virtual work or potential energy which underlies its application to stress and deformation analysis.

In order to enliven the explanation c.q. repetition of the basic principles for the students a worked sample application is presented for each of them, viz. determination of the transient temperature distribution in a flange assembly by finite differences and of the stress distribution in a reinforced radial nozzle-to-sphere connection by the finite-element method. Interested students are urged to familiarize themselves with the relevant literature, e.g. [D.1–5], and/or to follow a special couse on finite-element techniques taught by the Applied Mechanics group of our Department.

5. Conclusion

The above text has tried to summarize the main aspects of the course of which it bears the title, referring to background literature wherever this was thought possible. The author will be happy to supply further information upon request.

Appendix

Full contents

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The literature on the subject covered in the course has grown so vast that even from the author's limited knowledge only publications considered by him most directly useful to course participants were selected.

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