DYNAMIC PACKING MODEL OF 2D FULLY-GRADED ARBITRARY SHAPED CONCRETE AGGREGATE

MU Song (1), XIE Deqing (1), LIU Jianzhong (1) and ZHANG Yunsheng (2)

(1) Jiangsu Sobute New Materials Co., Ltd., Nanjing 211100, China;

(2) College of Materials Science and Engineering, Southeast University, Nanjing 211189, China.

Abstract

The parameter equations of circle, Elliptic and arbitrary polymorphic were deduced by polar coordinates transformation in this paper, and a 2D reconstruction method of irregular particles based on cellular automata theory was introduced, briefly. The particles size and quantity information could be automatic computed by a sectional calculate and residual accumulation algorithm which was developed in this paper. The computed particles size and quantity information had a good agreement with the aggregate actual sieving curve. The corresponding dynamic packing model of 2D fully-graded arbitrary shaped concrete aggregate was further developed, and all kinds arbitrary shaped particles systems' limit accumulation states could be obtained by setting appropriate parameters. The limit packing density of circle particles system was the biggest, up to 79.59%.

Keywords: Concrete aggregate; Fully-graded; Dynamic packing model; Packing density; Numerical simulation

1. INTRODUCTION

As the most widely used civil engineering material, concrete mainly includes coarse aggregate, mortar and interface transition zone (ITZ) from microscopic perspective. The content, size, gradation, and shape of coarse aggregates have crucial effects on ion transport and mechanical properties of hardened concrete. Concrete coarse aggregate reconstruction and placement is a prerequisite for the numerical simulation of concrete. The following scholars at home and abroad have made outstanding contributions in this regard: In 1984 , Wittmann et al.[1] pioneered a multi-sides and multi-angulars random aggregate model. Z.M. Wang and A.K.H Kwan et al.[2] established another method for randomly generating polygon particles based on Monte Carlo method. Xu Wenxiang et al.[3] used the quadratic curve equation to control the production of two-dimensional elliptical particles, and established a corresponding hardened particle accumulation model. Gao Zhengguo and Liu Guangting[4] developed a two-dimensional random aggregate placement algorithm based on aggregate intrusion judgment criteria and aggregate area.

At present, the following problems exist in the numerical simulation of the coarse aggregate reconstruction and placement: (i) The particle size and quantity information cannot be automatically calculated based on the aggregate sieving curve; (ii) The maximum two-dimensional limit bulk density is only 70%[5]; (iii) There are still large discrepancies between round, elliptical, regular polygon particles and the actual crushed stone shape.

This paper develops a " sectional calculate and residual accumulation algorithm " based on C language, which could calculate the particle size and quantity information of the aggregate particles. The computed results had a good agreement with the aggregate actual sieving curve. This paper further establishes a random stacking and dynamic walking model, and all kinds arbitrary shaped particles systems' limit accumulation states could be obtained by setting appropriate parameters. For particle of a particular shape based the real screening curve, the maximum attempts number (N) and maximum walk steps(M) affect the maximum packing density of the " dynamic walking random packing model ". Setting the appropriate parameters, N = 100,000 and M = 1000, the particle system of arbitrary shaped can reach the limit maximum packing density, and the limit value of circular particle system is the highest, up to 79.59% , which is the largest value in related literature.

2. RECONSTRUCT AN ARBITRARY SHAPED AGGREGATE PARTICLE

2.1 Circular and elliptical particle

The key parameters for controlling circular particles are the center coordinate (x_i , y_i) and radius r_i. The governing equations as follows:

$$\begin{cases} x = r_i \times \cos\theta + x_i \\ y = r_i \times \sin\theta + y_i \end{cases} \quad (1)$$

Wherein, θ is the angle parameter [0,2π] of the standard circle .

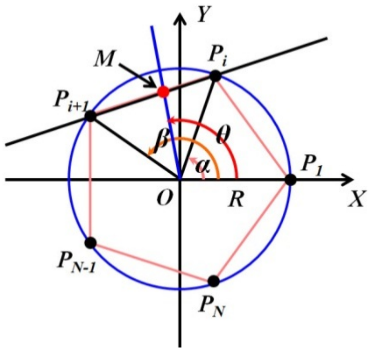
The key parameters for controlling the ellipse particles are the central coordinates (x_i , y_i), the long semi-axis length F_b, the short semi-axis length F_b, the ratio of long semi-axis to minor semi-axis ratio R_{ab} and the rotated angle β. The governing equations as follows:

$$\begin{cases} x = F_a \times \cos\beta \times \cos\theta - F_b \times \sin\beta \times \sin\theta + x_i \\ y = F_a \times \sin\beta \times \cos\theta + F_b \times \cos\beta \times \sin\theta + y_i \end{cases} \quad (2)$$

Wherein, θ is the angle parameter standard elliptical [0,2π].

2.2. Regular polygon particles

The key parameters of regular polygon particles are the center coordinates (x_i, y_i), the radius of circumscribed circle R, and the number of sides N. The regular polygon particle should be rotated randomly by an angle β as a standard regular polygon has a certain orientation.



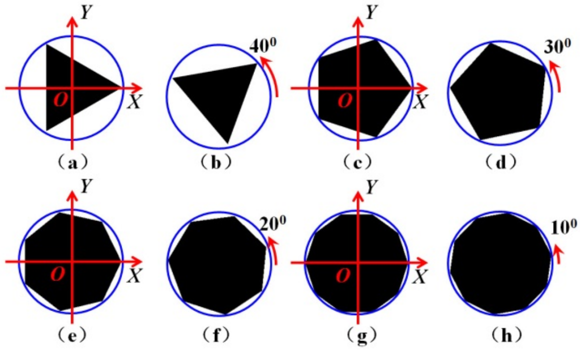


Figure 1: Regular polygon key parameters **Figure 2: Reconstruction results**

However, the inclination of the regular polygon equations could not descripted by a simple formula. The key is to find the parametric equation of point M in Fig. 1, its polar coordinates are (ρ , θ). Based on that the three points including point P_i , point P_{i+1} and point M on the same line, the polar coordinates of point M could be calculated as follows;

$$\begin{cases} x_M = \rho * \cos\theta = \cos\theta * \dfrac{R^2 * \cos\alpha * \sin\beta - R^2 * \cos\beta * \sin\alpha}{(R*\sin\beta - R*\sin\alpha)\cos\theta + (R*\cos\alpha - R*\cos\beta)\sin\theta} \\ y_M = \rho * \sin\theta = \sin\theta * \dfrac{R^2 * \cos\alpha * \sin\beta - R^2 * \cos\beta * \sin\alpha}{(R*\sin\beta - R*\sin\alpha)\cos\theta + (R*\cos\alpha - R*\cos\beta)\sin\theta} \end{cases} \qquad (3)$$

Then, the parametric equation of a regular N-shaped particle with center coordinates (x_i, y_i), a circumscribed circle radius R, and an inclination angle γ is:

$$\begin{cases} x = x_M * \cos\gamma - y_M * \sin\gamma + x_i \\ y = x_M * \sin\gamma - y_M * \cos\gamma + y_i \end{cases} \qquad (4)$$

In formulas (3, 4), the only variable is $\theta \in [0,2\pi]$; $\alpha = (i-1)*2\pi/N$、$\beta = i*2\pi/N$, which is controlled by θ .

Fig. 2 shows the regular polygon particles reconstructed by this algorithm. The reconstructed results show that the algorithm can be used to reconstruct rotation any regular polygon rotated by any angle.

2.3 Irregular particles

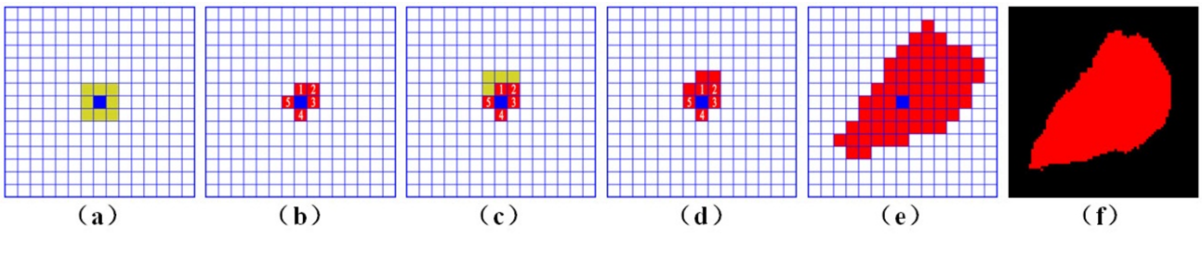


Figure 3: Using "central growth method" to reconstruct an irregular particle

Based on the principle of cellular automata[6] , this paper develops a two-dimensional irregular particle reconstruction rule, which is named "central growth method". The core steps are: Firstly, the coordinates of the first node in the list (i.e., central point) are read and the corresponding pixel is turned into a blue particle pixel, as shown in Fig. 3a. Its eight neighbouring pixels highlighted in yellow colour are then activated and a set of eight probability values between 0 and 100 are generated for them sequentially based on a Monte Carlo simulation. The eight probability values are then compared with the corresponding characteristic values of the special irregular particles, respectively. If the pointed cell has a probability value smaller than its corresponding characteristic one, the cell would be turned into a part of the particle. As seen in Fig. 3b, five neighbours among the eight neighbouring pixels are turned into cement particle and their coordinates are delivered into the list one by one. Repeating the steps shown in Fig. 3a and b for these five neighbouring pixels, more neighbouring pixels can be classified into the cluster of cement particle, as shown in Fig. 3c and d. These steps are continuously operated on the activated neighbouring pixels until the target area of this particle is achieved. Fig. 3e illustrates an obtained irregular cement particle with 67 pixels, while Fig. 3f illustrates an obtained irregular cement particle with 2021 pixels.

2.4 Periodic boundary conditions

When particles are put into the sample space, the boundary conditions of the sample space need to be set. In this paper, periodic boundary conditions are selected[7]. The periodic boundary means that when the particles intersect with the boundary of the sample space, the particles outside the space need to be compensated to the opposite edge.

3. FULLY GRADED AGGREGATE DYNAMIC STACKING MODEL

3.1 Automatic calculation of aggregate particle information

This paper develops a "sectional calculate and residual accumulation algorithm" based on C language. This algorithm can calculate the particle size and quantity information of the aggregate particles by the real screening curve automatically. The core algorithm as follows: (a) Dividing the coarse aggregate screening curve into different intervals from big to small, as shown in Fig. 4. (b) Using linear screening method to calculate the interval sieve residual percentage. (c) Calculating the quantity of particle in this interval based on the aggregate size and the area of simulated space. (d) The less than one part is accumulated in the next interval.

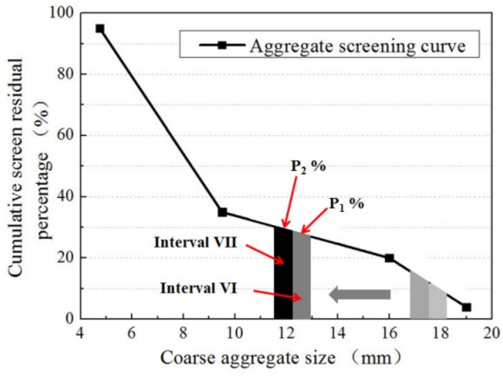


Figure 4: Automatic calculation diagram

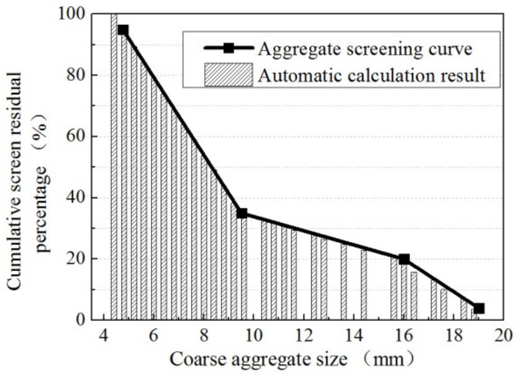


Figure 5: Automatic calculation result

The two-dimensional placement space used in this paper is 500 pixels × 500 pixels, corresponding to an actual size of 100mm × 100mm , each pixel represents 200μm. The specific information of particle size and number using the above algorithm to calculate, when the aggregate content is 80%, the width of the scanning interval is 0.4mm. The cumulative sieving curve of the calculation results is plotted in Fig. 5, which shows that the particles information agrees well with the actual sieving curve.

3.2 Random stacking and Dynamic walking model

This paper further established a two-dimensional random stacking and dynamic walking model, Fig. 6 shows the flow chart of this model. The specific operation steps are as follows:

(1) Determining the size of the sample space based on the " triple principle " and the " minimum particle principle "[8]. For the coarse aggregate particle size is 4.4 ~ 18.8 mm, the final setting space size is 500 pixels × 500 pixels, corresponding to the actual size of 100 mm × 100 mm with a single pixel size is 200 μm;

(2) Calculating the particle size and quantity information of the aggregate particles based on the "regional calculation and residual accumulation algorithm".

(3) Developing a virtual space of appropriate size and reconstructing a particle of arbitrary shape in the virtual space, from big to small.

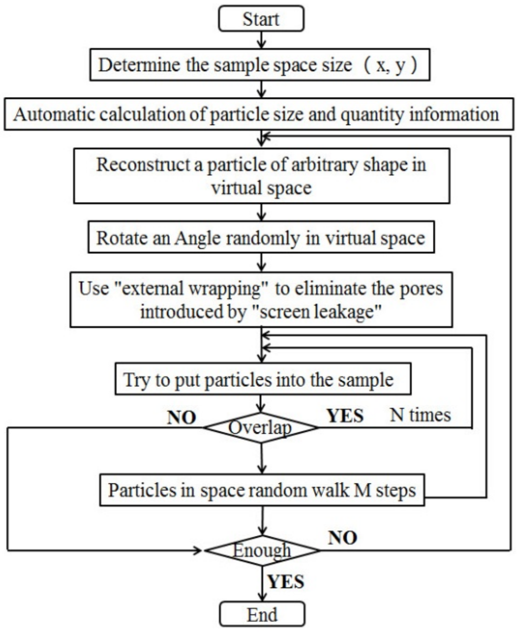


Figure 6: Flow chart of random stacking and dynamic walking model

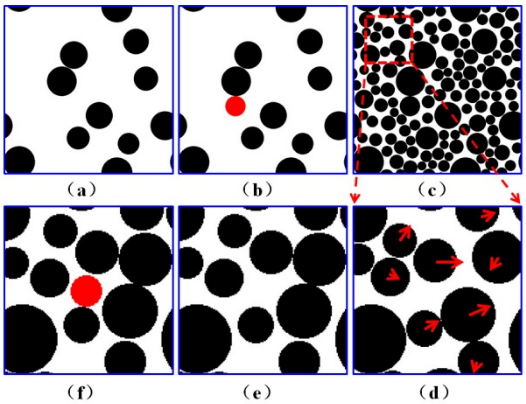


Figure 7: Schematic diagram of key steps

(4) In order to simulate the random distribution of aggregate particle orientation,the particle in the virtual space need to rotate a random angle. Using "out wrapping method" to eliminate internal pores generated during rotation by“ hourglass phenomenon ”[9].

(6) Randomly selecting a coordinate in sample space and throwing the particle at that position. If the particle overlaps with the particle that has been cast, then randomly select a coordinate again and try again. Fig. 7(b) shows that the 12th particle was successfully thrown , and Fig. 7(c) shows that 121particles have been thrown.

(7) When the number of attempts is greater than 100,000 times, it is still impossible to choose a suitable space to accommodate the particles to be dropped. The particles that have been dropped in the sample space need to be randomly moved *N* steps as shown in Fig. 7(d). Then, try to drop the particle again.

(8) When all the particles have been thrown into the space, or the next particles cannot be thrown after the moved particles have moved N steps, the program terminates.

(a) (b) (c) (d) (e)

Figure 8: Reconstruction results of fully-graded aggregate particles of various shapes

The aggregate particle systems of different shapes are reconstructed by random stacking and dynamic walking model developed in this paper. The total aggregate content is 50% , which is consistent with the sieving curve in Fig. 4. The reconstruction results are shown in Fig. 8 , (a) circular particle, (b) elliptical particle (the ratio of the long axis to short axis is 5.0), (c) regular triangle, (d) Regular pentagon; (e) irregularly shaped particle. The particle information and the areas of small color particles in different figures are the same. Only the shape of the particles is different. As can be seen from the figure, the random stacking and dynamic walking model can be used to reconstruct arbitrary shape particle systems.

3.3 Maximum packing density

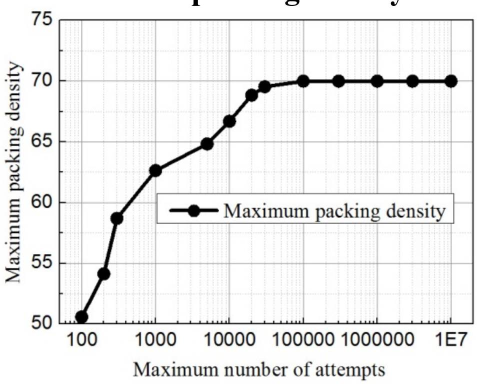


Figure 9: The impact of the maximum attempts number

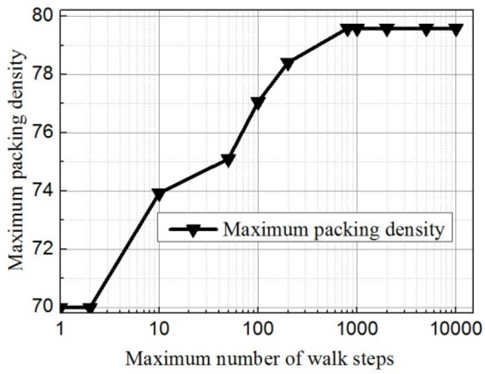


Figure 10: The impact of the maximum walk steps

For circular particle based the real screening curve shown in Fig. 4. The maximum attempts number and maximum walk steps affect the maximum packing density of the " dynamic walking random packing model ". Fig. 9 shows the impact of the maximum attempts number on the maximum packing density. As the maximum attempts number increasing, the maximum packing density increases, but the growth rate increase gradually decreases and there is a limit value. The maximum packing density is stable at the limit value, which is 70.00%, when the maximum attempts number is greater than 100,000. The impact of the maximum walk steps on the maximum packing density show the same pattern. The maximum packing density is stable at the limit value, which is 79.59% shown in Fig. 10, when maximum walk steps are greater than 1,000. Compared with the "static random stacking model", the maximum packing density of "Random stacking and Dynamic walking model" reached nearly 80% , with the increase was nearly 10% , which is the largest value in related literature.

4. CONCLUSIONS

1. The governing equations of circular, elliptical and regular polygon particle were deduced in this paper. A "central growth method" based on the principle of cellular automata was developed in this paper to reconstruct irregular particle.
2. A sectional calculate and residual accumulation algorithm was developed in this paper was used to calculate the particle size and quantity information of the aggregate particles by the real screening curve automatically. The computed results had a good agreement with the aggregate actual sieving curve.
3. The corresponding dynamic packing model of 2D fully-graded arbitrary shaped concrete aggregate was further developed, and all kinds arbitrary shaped particles systems' limit accumulation states could be obtained by setting appropriate parameters. The limit packing density of circle particles system was the biggest, up to 79.59%.

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