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# Degree of similitude estimation for sub-scale flight testing

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**Sub-scaled Physical Models (SPM)** are often employed in wind-tunnel tests or in free-flight tests (physical tests) to predict flight behavior of aircraft **Full-scale Design (FD)**. However, a quality prediction of both the static and dynamic behavior is to date an open challenge. In this research, a methodology for designing SPMs is proposed for those cases where dynamic similarity between SPM and FD cannot be achieved and legacy information to compare sub-scale flight results to FD is unavailable. Instead of attempting to use just one SPM to achieve complete similarity with full scale design, this methodology enables the design and comparison of multiple SPMs to determine the **Sub-scale Design (SD)** best suited the estimation of specific aspect the FD flight behavior. To this purpose, a metric called *Degree of Similitude* (DoS) is defined, to quantify the similarity of FD and SPM based on the aerodynamic coefficients that are relevant for a given test. The DoS estimation first requires the evaluation of relevant aerodynamic coefficients, by means of Computational Fluid Dynamics (CFD). CFD analysis, requires complex geometry generation, adequate grid generation, expensive calculation and laborious post processing. To this purpose, a Knowledge Based Engineering (KBE) tool called Multi-model Generator (MMG) is developed, to automate all the labor intensive tasks in the evaluation of the DoS including the integration of CFD tool. Validation of results produced by MMG-VSAERO tool-chain is performed by means of a wind-tunnel test campaign using a 8.8% aerodynamically scaled SPM of the Cessna Citation II 550 (citation). The results of this test are compared with flight test data of full-scale aircraft (which is co-owned and operated by Delft University of Technology). Furthermore, this SPM was compared with three other Sub-scale Designs (SD) to estimate their DoS with the full-scale aircraft for two different eigenmodes, namely short period mode and phugoid mode. Of the four SDs compared, it was found that the geometrically scaled SD showed highest DoS for short period motion and one of the aerodynamically scaled SD had highest DoS for phugoid motion. From the cases studied, it can already be inferred that geometrically scaled SDs are not always preferred and in many cases, aerodynamically scaled SDs can be much more similar to FD. This case study proved the convenience of the proposed coefficient DoS which, in the next phase of the research, will be used as objective function to design optimum SPMs for a given test.

## Nomenclature

$b$	=	Wing span [m]
$c$	=	Chord [m]
$q$	=	Pitch rate [rad/s]
$\bar{c}$	=	Mean aerodynamic chord [m]
$C_i$	=	$i^{th}$ aerodynamic coefficient determining the output of a test
$C_L$	=	Aircraft lift coefficient [-]
$C_{Lq}$	=	$\frac{\partial C_L}{\partial (qc/2V)}$ [1/rad]

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$C_{L_u}$	=	$V \frac{\partial C_L}{\partial u}$ [1/rad]
$C_{L_\alpha}$	=	$\frac{\partial C_L}{\partial \alpha}$ [-]
$C_{L_{\dot{\alpha}}}$	=	$\frac{\partial C_L}{\partial \dot{\alpha}}$ [1/rad]
$C_{L_{\delta_e}}$	=	$\frac{\partial C_L}{\partial \delta_e}$ [1/rad]
$C_{L_{\dot{\delta}_e}}$	=	$\frac{\partial C_L}{\partial \dot{\delta}_e}$ [1/rad]
$C_{m_{ac}}$	=	Pitching moment coefficient about y axis at aerodynamic center [-]
$C_{m_\alpha}$	=	$\frac{\partial C_m}{\partial \alpha}$ [1/rad]
$C_{m_{\dot{\alpha}}}$	=	$\frac{\partial C_m}{\partial \dot{\alpha}}$ [1/rad]
$C_{m_q}$	=	$\frac{\partial C_m}{\partial q}$ [1/rad]
$C_p$	=	Coefficient of pressure [-]
$C_z$	=	Force coefficient in z-direction [-]
$C_{z_\alpha}$	=	$\frac{\partial C_z}{\partial \alpha}$ [1/rad]
$C_{z_{\dot{\alpha}}}$	=	$\frac{\partial C_z}{\partial \dot{\alpha}}$ [1/rad]
$C_{z_q}$	=	$\frac{\partial C_z}{\partial q}$ [1/rad]
$Fr$	=	Froude number [-]
$g$	=	Acceleration due to gravity [ $m/s^2$ ]
$K_{Fr}$	=	Froude number ratio [-]
$K_y$	=	Non-dimensional radius of gyration [-]
$K_{Re}$	=	Reynolds number ratio [-]
$L$	=	Characteristic length e.g. mean aerodynamic chord [m]
$n$	=	Number of aerodynamic coefficients selected for a test
$q$	=	Rotational velocity about y-axis
$Re$	=	Reynolds number [-]
$u$	=	Velocity in x-direction [m/s]
$V$	=	Velocity [m/s]
$W$	=	Weight [kg]
$w_i$	=	Weighting factor for a selected aerodynamic coefficient essential for the test
$\alpha$	=	Angle of attack [rad]
$\dot{\alpha}$	=	Rate of change of angle of attack [rad/s]
$\delta_e$	=	Elevator position [rad]

$\lambda$	=	Geometric scaling factor [-]
$\rho$	=	Density at flight altitude [ $kg/m^3$ ]
$\mu$	=	Dynamic viscosity [ $Pa.s$ ]
$\mu_c$	=	Relative density for symmetric motions [ $m/\rho S\bar{c}$ ]
AOA	=	Angle of Attack
CAD	=	Computer Aided Design
CFD	=	Computational Fluid Dynamics
CM	=	Computational Model
DoS	=	Degree of Similitude
FD	=	Full-scale Design
FER	=	Full-scale Experimental Response
FPM	=	Full-scale Physical Model
FVA	=	Full-scale Virtual Aircraft
FVR	=	Full-scale Virtual Response
KBE	=	Knowledge Based Engineering
MMG	=	Multi-model Generator
SD	=	Sub-scale Design
SER	=	Sub-scale Experimental Response
SPM	=	Sub-scale Physical Model
SVA	=	Sub-scale Virtual Aircraft
SVR	=	Sub-scale Virtual Response

## I. Introduction

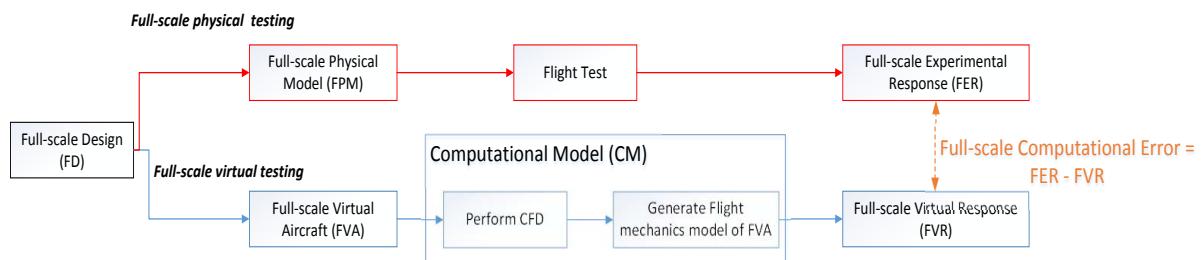
The number of passengers using air travel in 2035 is expected to be 7.8 billion, nearly double the passengers measured in 2017.[1–3] Such a growth in air traffic will strain the environment, both in terms of consumption of natural resources and in terms of environmental pollution. In order to address these problems, governmental agencies will impose stricter regulations on aircraft performance and emissions, which will render many of the conventional aircraft designs unsuitable for flight. As a consequence, novel unconventional aircraft designs will become a necessity.

Numerous conceptual designs (FD) of unconventional aircraft promising lower environmental impact can be found in the literature. [4–7] However, insufficient confidence on the flight behavior and performance of these unconventional designs has prevented manufacturers from committing to their development.

Flight testing of full-scale physical model (FPM) of an unconventional aircraft design is possibly the best way to ascertain its behavior and decide about its future development to bring it to production. However, the associated cost and risk make this impossible in early stages of design cycle (shown with red outline in figure 1). Thus, designers often

use computational models (figure 1) or sub-scale physical models 1 to predict the flight mechanics of a given design, as discussed below.

**Computational Models (CM):** Classical computational models take full-scale virtual aircraft (FVA) as input, where a FVA is a Computer-aided Design (CAD) model of FD suitable for use in a computational model such as a discretized (or meshed) model. Computational models (CM) then use FVA with CFD to generate a database of aerodynamic derivatives that can be used by flight mechanics model. These derivatives are combined with mass and inertia distribution of the FD to eventually evaluate the FVR.[8–11] (blue outline in figure 1). In an ideal scenario, the Full-scale Virtual Response (FVR) is the same as the results obtained from the full-scale flight test performed, Full-scale Experimental Response (FER). In other words, the full-scale computational error (FER-FVR) is zero.



**Fig. 1 Full-scale physical testing vs full-scale virtual testing for assessment of flight behavior of a given FD**

However, in practice, this is rarely achieved due to simplifications and assumptions in CM, for example, exclusion of viscous and compressibility effects and the turbulence model choice. Primary cause of these assumptions is the solver used to perform CFD. Depending on the time and resource available, high fidelity CFD solvers Reynolds Averaged Navier Stokes (RANS) solver or Large Eddy Simulations (LES) can be used to improve the accuracy of FVR. [11] Still, it is difficult to ensure zero full-scale computational error. While in the case of conventional designs, legacy information is available to correct the errors and account for uncertainties originating from the computational model i.e. to predict FER based on FVR, for unconventional designs, this type of correction is not possible. In such cases, sub-scale physical models (SPM) can provide convenient alternative to enhance trust in a FD evaluation.

**Method 2 - Sub-scale Physical Models (SPM):** As a substitute to the impractical FPM, Sub-scale Physical Models (SPM) are manufactured such that their behavior is similar to FPM. SPMs can then be used to perform wind-tunnel testing or flight testing. Such SPMs are considerably less expensive. Thus, feasible for manufacturing at the end of conceptual design phase. The design of SPM (called Sub-scale Design (SD)) is based on certain scaling laws and are classified into five types[12–15]:

- (a) **Geometric scaling** : Here the relationship between the SPM and the FPM is purely based on the shape . Geometric scaling can be classified into two categories:
  - 1) Isotropic scaling - It is a linear transformation that enlarges or shrinks objects by one factor that is same in all directions. This factor is commonly known as the scale factor. All geometrical scaling in this document refers to isotropic scaling unless otherwise specified.
  - 2) An-isotropic scaling - It is a non-uniform transformation where different factors are used in each axis direction. This non-uniform scaling mutates the shape of the object to be tested and thereby affects all the other relationships that depend on the shape of the object. Such an-isotropic scaling is often used in aerodynamic scaling(explained below).
- (b) **Kinematic scaling** : When the ratio of geometry and the time rate of change of fluid flow around both the SPM and the FPM are the same, thus yielding similar fluid streamlines.
- (c) **Dynamic scaling** : When the ratio of
  - 1) geometric size of FPM and SPM
  - 2) time rate of change of fluid flow around FPM and SPM
  - 3) the forces acting on the SPM and the FPM

are the same simultaneously.

(d) **Aerodynamic scaling** Since both dynamic scaling is restrictive and difficult to achieve, a variant of kinematic scaling called aerodynamic scaling has been used. [14] Aerodynamic scaling requires the modification of the SPM geometry (not necessarily geometrically scaled) to simulate the aerodynamics of the FPM to maintain the following ratios:

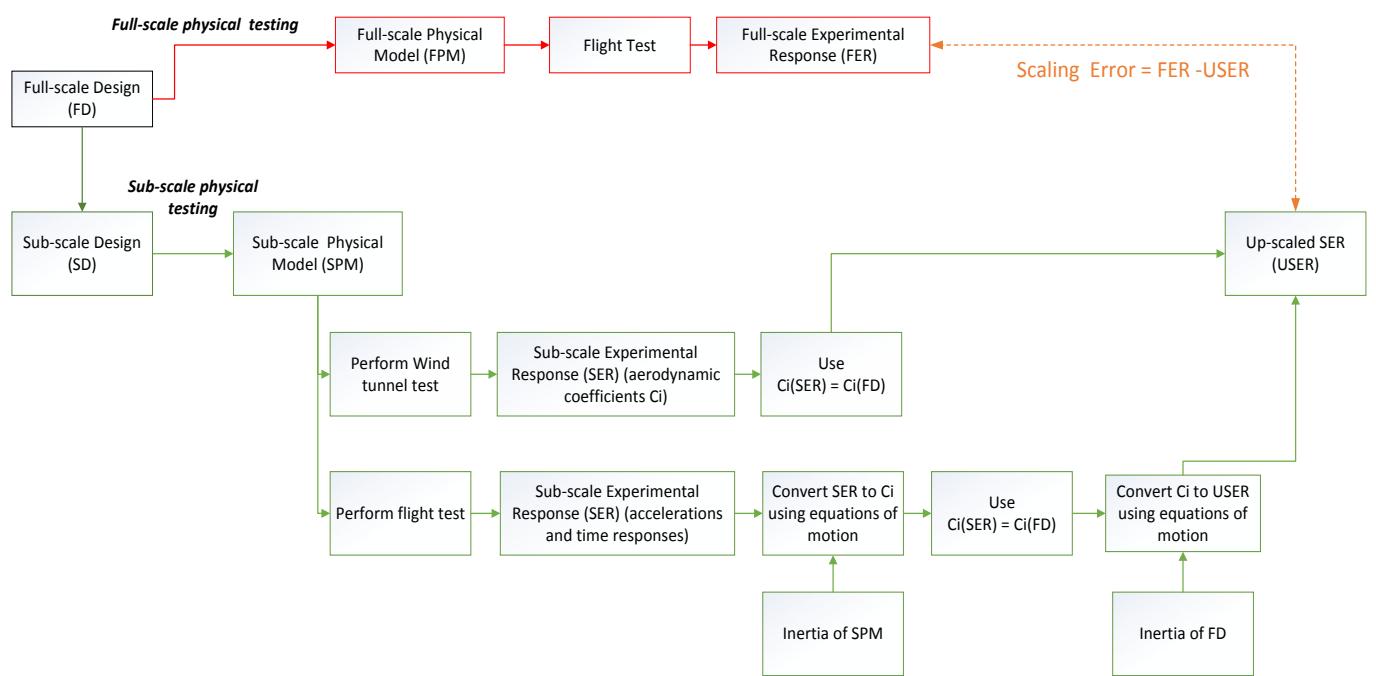
- 1) time rate of change of fluid flow around scaled-model and full-scale design for the specific phenomena being tested
- 2) of the relevant forces acting on the scaled model and full-scale design for the phenomena being tested

This is accomplished in three different ways:

- 1) using different scaling factors per axis of the FD
- 2) using different scaling factors per component of the aircraft (for example, making a 20% scaled wing while the rest of the components are 10% scaled)
- 3) using different relative distances between different components of the aircraft (for example, changing the tail volume coefficient)

(e) **Mass scaling** : Mass scaling requires the distribution of weight in the model to be scaled using a set of scaling laws, which are used to simulate aircraft motion and response. These scaling laws are an expansion of the square-cube law, with the addition of a density-scaling factor the is explained in section II using equation 4. [15]

Once SD is manufactured using one of the aforementioned scaling laws. The results of such tests, known as Sub-scaled Experimental Response (SER), are then scaled up to predict the flight mechanics of FPM. A schematic of this method is shown with green outline in figure 2.



**Fig. 2 Full-scale physical testing vs sub-scale physical testing for assessment of flight behavior of a given FD**

The process of up-scaling SER varies between sub-scale wind-tunnel response and sub-scale flight test response. In case of wind-tunnel tests, the response is scaled up by using the relationship between aerodynamic derivatives given by [12, 13, 15]:

$$C_{i_{SER}} = C_{i_{FER}} \quad (1)$$

where,

$$C_i = \text{relevant aerodynamic derivatives} \quad (2)$$

However, in case of flight tests, it is slightly more complicated because aerodynamic forces are not directly obtained from the tests and mass and inertia effects need to be included in the calculation. Thus, in case of flight tests, equations of motion are used to convert accelerations and time responses into aerodynamic derivatives using the mass and inertia properties of SPM. The aerodynamic derivatives of FD are then determined using equation 1. Equations of motion are then used again in conjunction with aerodynamic derivatives of FD to predict the flight mechanics of FPM. Schematic of scaling up flight test results is shown in figure 2.

Ideally, Up-scaled SER (USER) should be equal to FER i.e. physical scaling error is zero. However, in practice, this is not the case because of three main reasons[13, 15]:

- 1) SPM is often not completely (dynamically) similar to FPM which results in unrepresentative USER. [12, 13, 15] Because it is impossible to achieve dynamic similitude in many cases, engineers conduct test using SPM that are only geometrically or kinematically similar. This means that the forces and moments acting on these SPMs are not similar to FPM. Nevertheless, engineers can extract useful information from these tests, by using correction factors that are based on empirical data and legacy information. (Thus only available for conventional aircraft designs)
- 2) Methodology to upscale SER does not exist when complete (dynamic) similitude between SPM and FPM is impossible.
- 3) The testing conditions of SPM cannot recreate all the phenomena that is experienced by FPM due to physical limitations such as the size of the wind tunnel, the maximum speed at which a wind-tunnel can operate, the maximum allowable weight of the SPM for free-flight testing etc. [16]

In the development and testing of unconventional designs, not only is it problematic to achieve dynamic similitude but it is also not possible to apply correction factors in the absence of legacy information (as in case of conventional designs). This makes the SD and thereby the development of SPM challenging. This research aims to improve the quality of USER without using legacy correction factors by attaining the maximum possible similitude between FD and SD, subject to the physical limitations imposed by the testing environment

In order to *maximize* the similitude between a given SD and FD, different SDs must be compared and the SD with maximum similitude must be selected. To this purpose, this paper introduces a metric called Degree of Similitude(DoS) to estimate the extent of similarity between a SD and FD. In section II of this paper, the need for DoS, its definition, mathematical representation and the methodology to estimate DoS are discussed. Section III describes the implementation of the proposed methodology using a Knowledge Based Engineering (KBE) tool called Multi-Model Generator (MMG). In section IV, validation of the MMG is provided by comparing SVA, SPM, FVA and FPA of the Cessna Citation II 550. Furthermore, four sub-scale designs are compared and their DoS is estimated for short-period motion and phugoid motion in section V. Finally, Section VI of this paper the conclusions and outlook to the next steps of this research are discussed.

## II. Definition and quantification of the degree of similitude (DoS)

This section proposes and defines a new metric to determine the degree of similitude, which can be used to compare different scaled models. However, before defining the degree of similitude, the need for degree of similitude is discussed in sub-section II.A.

### A. Need for Degree of Similitude

To clarify the need for degree of similitude, it is necessary to observe scaling relationships currently applied to model testing.

$$C_L = C_{L_u} \frac{\Delta u}{V} + C_{L_\alpha} \Delta \alpha + C_{L_q} \frac{q \bar{C}}{2V} + C_{L_\dot{\alpha}} \frac{\dot{\alpha} \bar{C}}{2V} + C_{L_{\delta_e}} \delta_e + C_{L_{\dot{\delta_e}}} \frac{\dot{\delta}_e \bar{C}}{2V} \quad (3)$$

Equation 3 [17], for example, is the lift equation obtained from the equations of motion of an airplane. If  $C_L$  is considered an important parameter affecting a given phenomenon,  $C_L$  for FD and SD must be the same. Which

also implies, aerodynamic coefficients,  $C_{L_u}$ ,  $C_{L_\alpha}$ ,  $C_{L_q}$ ,  $C_{L_\delta}$ ,  $C_{L_{\delta_e}}$  and  $C_{L_{\delta_e}}$  should be the same for SD and FD. These aerodynamic coefficients and their derivatives depend on forces such as fluid's viscous forces, inertial forces, gravitational forces, pressure forces.[13, 15]

Reynolds number, Froude number, Mach number and Strouhal number are dimensionless numbers defined as the ratios of these forces. [13] There is an advantage of using such dimensionless numbers instead of the actual forces. If the test conditions are known and the dimensions of the SD or FD are known, the dimensionless numbers (Reynolds number, Froude number etc.) can be determined. Furthermore, it can be proven using dimensional analysis[12, 13, 15] that a phenomena can be reproduced if all the dimensionless numbers affecting the phenomena are the same, irrespective of the testing condition and the actual size of the model undergoing the test. This means that engineers do not need to go into complex calculation of different aerodynamic coefficients and their derivatives to determine if a SD is representative of FD. For this reason, engineers try to achieve similitude conditions such as certain Reynolds number similitude or Froude number similitude instead of just geometric similitude. [13] For the remainder of this paper, similitude conditions obtained from dimensionless numbers such as Froude number and Reynolds number will be defined as dimensionless similitude condition.

Due to limitations imposed by the available infrastructure, laws of physics or inhibiting cost, it is often not possible to satisfy these dimensionless similitude conditions. Furthermore, the dimensionless similitude conditions just provide binary output i.e. whether the similarity condition is satisfied or not. No clear insight is provided when an intermediate similitude situation is achieved. This is best explained with an example.

**Example:** A FD with  $34m$  span,  $4.2m$  mean aerodynamic chord and  $73000kg$  weight is considered. For the phenomenon being tested, the aircraft is flying at an approach speed of  $472km/hr$  at an altitude of  $2300m$ . The aim is to design a dynamically similar SD to study the short period motion of the aircraft. There are two constraints on the design owing to certification requirements \* for SD:

- 1) the SD must fly at an altitude of  $4000m$
- 2) its weight should not exceed  $100kg$

We start by applying geometrical scaling:

$$\frac{W_{full-scale}}{\rho_{full-scale}} = \lambda^3 \frac{W_{model}}{\rho_{model}} \quad (4)$$

Applying the values from example

$$\lambda = 8.5 \quad (5)$$

$$b_{full-scale} = \lambda * b_{scaled-model} \quad (6)$$

$$b_{scaled-model} = 4m \quad (7)$$

$$c_{full-scale} = \lambda * c_{scaled-model} \quad (8)$$

$$c_{scaled-model} = 0.5m \quad (9)$$

In order to scale for short period motion, from dimensional analysis, it is observed that it is essential to ensure Froude number scaling and Reynolds number scaling. Where,

$$Fr = \frac{V}{\sqrt{gL}} \quad (10)$$

$$Re = \frac{\rho VL}{\mu} \quad (11)$$

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\*the weight and altitude requirements vary per country. For this example, a representative value is chosen

In order to ensure dynamic similitude, we need Froude number and Reynolds number ratio to be 1. If we start by applying Froude number similarity, by substituting Eq. (12)

$$K_{Fr} = 1 \quad (12)$$

in equation 13

$$Fr_{model} = K_{Fr} * Fr_{full-scale} \quad (13)$$

and combining Eq.(10) and Eq.(13), we get the velocity of the model:

$$V_{model} = 44.44m/s \quad (14)$$

Reynolds number ratio is given by formula

$$Re_{model} = K_{Re} * Re_{full-scale} \quad (15)$$

combining Eq.(11), Eq.(14) and Eq.(15), we get:

$$K_{Re} = 0.035 \quad (16)$$

conversely, if we impose

$$K_{Re} = 1, \quad (17)$$

we get,

$$k_{Fr} = 28.90 \quad (18)$$

Clearly, Froude and Reynolds number similarity cannot be achieved simultaneously. In such cases, engineers typically choose one of the two similitude criteria and attribute variations of results with respect to full-scale aircraft to those criteria for which similitude could not be achieved.

In addition, the above example can be used to further understand two key limitations of using dimensionless similitude criteria:

- 1) In example,  $K_{Fr}$  was set to 1 by setting the test condition (i.e. velocity) and  $K_{Re}$  was accepted at 0.035. If a methodology for quantification of similitude was available,  $K_{Fr}$  could be set at 5 and the value of  $K_{Re}$  would be 0.173. After this, designs with  $K_{Fr} = 1$  and  $K_{Fr} = 5$  could be compared. The design that offers greater similitude to full-scale aircraft could then be chosen. However, this sort of quantification is not possible with dimensional analysis method as it only provides boolean results(i.e. whether similitude is achieved or not).
- 2) In the example, Reynolds number similitude and Froude number similitude were used based on dimensional analysis to study the short period motion of full-scale aircraft. [12, 13, 15]However, depending on the objective, the dimensionless similitude condition might change. A clear description of which dimensionless similitude condition must be used for a given objective does not exist. Dimensional analysis must be carried out using Buckingham Pi-Theorem to identify which dimensionless similitude conditions must be used.[12] This is not so straightforward mainly because it is difficult to determine all the parameters that affect the phenomena.

Thus, in general, it is challenging to identify the dimensionless similitude conditions that must be accounted in dimensional analysis. Even when such conditions can be defined, if dynamic similarity cannot be established, comparison of SDs is difficult. Therefore, if the dimensionless similitude conditions necessary for a test cannot be satisfied, a way to compare the results of SD and FD is to directly compare the *relevant* aerodynamic coefficients and derivatives affecting the phenomenon being tested. Comparing aerodynamic coefficients also allows us to pin point the potential cause of discrepancy between SD and FD. This determination of source of discrepancy is not possible with SD designed using dimensionless similitude condition as every dimensionless number affects multiple aerodynamic coefficients. A description of how aerodynamic coefficients and their derivatives can be used to compare SD and FD is provided in sub-section (II.B).

## B. Aerodynamic derivatives to define Degree of Similitude

In order to use aerodynamic derivatives to compare the similitude between full-scale design and a scaled model, relevant aerodynamic derivatives that affect the test must be selected. The selection of relevant derivatives can be made on the basis of governing equations in combination with dimensional analysis. For example, to study the short period motion of a design, the equations of motion can be studied and all the relevant derivatives can be selected. These derivatives can be sub-divided into three categories:

- 1) static aerodynamic derivatives: These derivatives can be computed by CFD, wind-tunnel tests and flight tests. In the specific example of short period motion,  $C_{m_\alpha}$  and  $C_{z_\alpha}$  are static aerodynamic coefficients
- 2) dynamic aerodynamic derivatives: These derivatives can be computed either using CFD or flight tests.  $C_{z_\dot{\alpha}}$ ,  $C_{m_\dot{\alpha}}$ ,  $C_{m_q}$  and  $C_{z_q}$  are examples of dynamic derivatives in short period motion
- 3) mass and inertial derivatives: These derivatives are dependent on relative density, radius of gyration and inertia of the model being tested. Examples of mass and inertia derivatives used in short period motion are  $K_y$  and  $\mu_c$ .

In addition to short-listing relevant aerodynamic coefficient, it is important to determine the *degree of influence* of each of these aerodynamic coefficients on the phenomenon being tested. Knowing the degree of influence of an aerodynamic coefficient allows engineers to maximise the similarity of those coefficients that are most influential in a test. For example, among  $C_{m_\alpha}$  and  $C_{m_q}$ , latter has greater influence on the damping and time period of short period motion than the former, thus a greater degree of influence must be attributed to  $C_{m_q}$ . The degree of influence of different aerodynamic coefficients can either be determined quantitatively or qualitatively. The physics behind the phenomenon being tested can be studied and a degree of influence factor for each coefficient can be formulated. Alternatively, governing equations can be used to determine the sensitivity of changing aerodynamic coefficients on the phenomenon being testing.

Once the relevant aerodynamic derivatives and their associated degree of influence is determined, comparison between FD and SD can be performed. However, this leads to the question: is it appropriate to compare FVR with USER? If such a comparison made, both physical scaling error and full-scale computational error will influence the result. Furthermore, there would be no way to segregate the two errors. To solve this, computational models are used to evaluate Sub-scale Virtual Response (SVR). This is shown using purple outline in figure 3. Here, Sub-scale Virtual Aircraft (SVA) is built in the same way FVA and the response of the computational model (SVR) is evaluated. SVR is then used to determine:

- 1) virtual scaling error: the difference between FVR and SVR.
- 2) sub-scale computational error: the difference between SER and SVR.

The four errors namely, physical scaling error, virtual scaling error, full-scale computational error and sub-scale computational errors can be used to compare the four models. To evaluate these errors, FVR, SVR, FER and SER must be defined. In section I, the methodology of Upscaling SER was discussed. It can be seen that the aerodynamic derivatives form a key aspect of scaling for both wind-tunnel testing and flight testing (refer figure 3) Since the key link between different physical and virtual testing is formed by aerodynamic derivatives, for the remainder of this paper, the four responses namely SER, SVR, FER and FVR are considered to be a vector of all the aerodynamic coefficients affecting the phenomenon being testing. Mathematically these errors can be represented as:

$$FVR = [C_{1_{FVR}}, C_{2_{FVR}}, C_{3_{FVR}}, \dots, C_{n_{FVR}}] \quad (19)$$

$$SVR = [C_{1_{SVR}}, C_{2_{SVR}}, C_{3_{SVR}}, \dots, C_{n_{SVR}}] \quad (20)$$

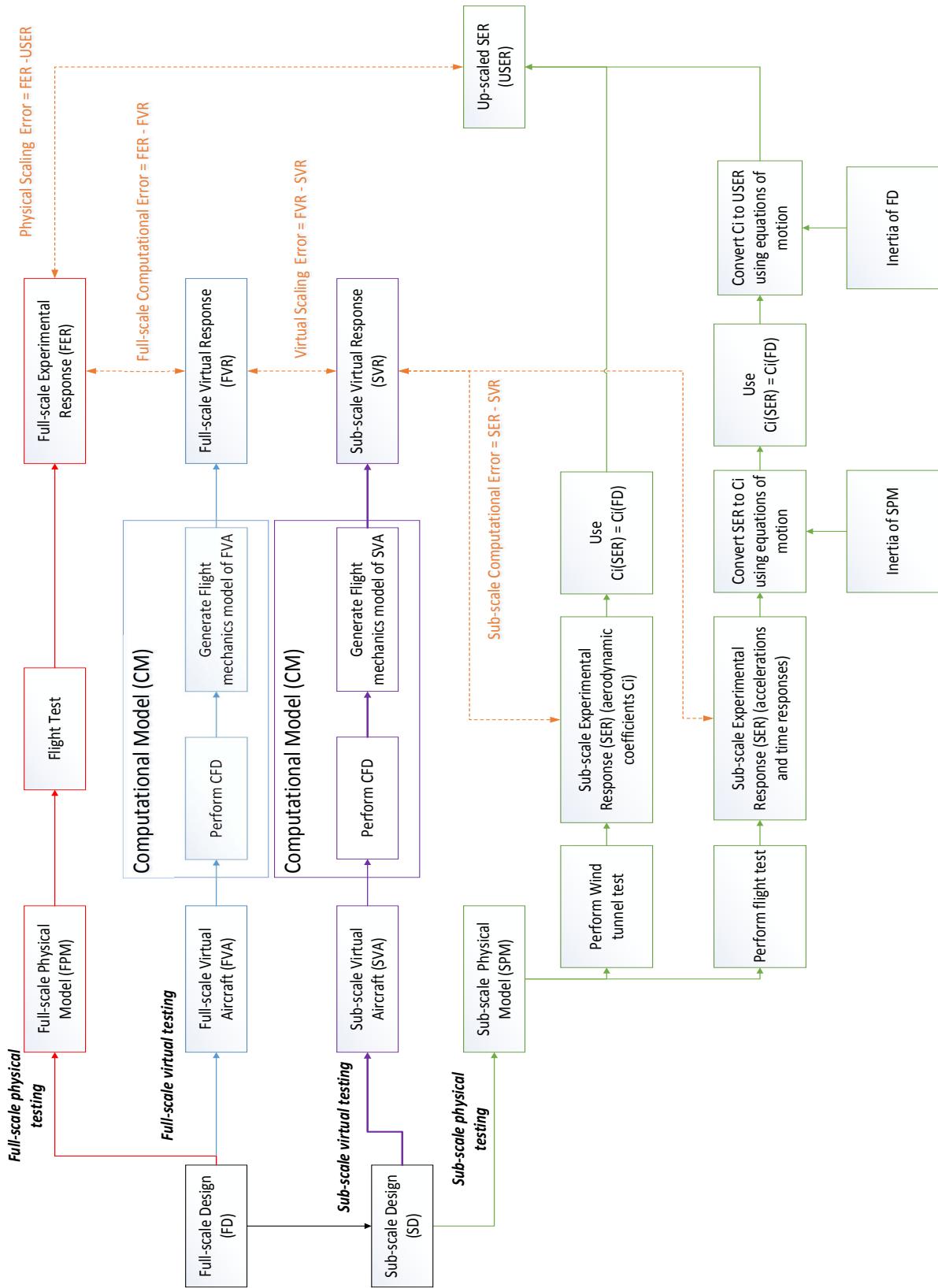
$$FER = [C_{1_{FER}}, C_{2_{FER}}, C_{3_{FER}}, \dots, C_{n_{FER}}] \quad (21)$$

$$SER = [C_{1_{SER}}, C_{2_{SER}}, C_{3_{SER}}, \dots, C_{n_{SER}}] \quad (22)$$

where,

$$n = \text{number of selected aerodynamic coefficients} \quad (23)$$

$$C_i = i^{\text{th}} \text{ relevant aerodynamic coefficient} \quad (24)$$



**Fig. 3** Overview of physical and virtual responses and their associated errors

It is important to note that FER is not available in the conceptual design phase, i.e. the aim of this study is to estimate FER. Thus, implying that full-scale computational error and physical scaling error are unavailable. However, using the other two errors i.e. virtual scaling error and sub-scale computational error, conclusions about FER can be drawn. This is explained further in II.C.

### C. Definition of Degree of Similitude

This section discusses the possibility of estimating full-scale computational error by utilizing sub-scale computational error and virtual scaling error. If we assume that the virtual scaling error is zero, i.e. the aerodynamic behavior of FD and SD are the same, we could infer that the phenomena occurring in FER and SER are the same subject to the condition that the computational model is sufficiently accurate. This condition is non-trivial and could lead to significantly wrong conclusions on non-compliance. Thus, it is important to ensure that the computational model performs accurately. For this, SVR should be compared with SER. If the phenomena occurring in FPM flight test and SPM flight test results are not different, full-scale computational error can be defined as a function of sub-scale computational error. This allows the use of SPM flight test instead of legacy information to correct FVR. However, we assumed this to be true only if virtual scaling error is zero. This is not always the case and a function of SD. Thus, multiple SDs must be designed and compared to arrive at a SD which has virtual scaling error  $\approx 0$ .

To enable this comparison, Degree of Similitude is introduced in this paper. Where, degree of similitude is a function of virtual scaling error. It can be seen that no physical test needs to be conducted to estimate the virtual scaling error and can be used to predict DoS before performing a wind-tunnel test or flight test. For the remainder of the section, we discuss the definition, advantages and disadvantages of DoS.

Once the aerodynamic derivatives affecting a phenomenon are selected and the degree of influence of each of the aerodynamic derivatives are determined, the degree of similitude for that phenomenon and sub-scale design can be defined. It is important to note that the constituent aerodynamic derivatives necessary to define DoS varies per test condition and the phenomenon being tested. In this section, a general definition of DoS is provided that can be applied to define the DoS for a given problem.

The **Degree of Similitude (DoS)** is defined as the weighted sum of normalized virtual scaling error. (Refer figure 4 for representation of different scaling and computational errors) Mathematically DoS can be expressed as:

$$DoS_{test} = 1 - \frac{1}{n} \sum_{i=1}^n w_i * \frac{|C_{iFVR} - C_{iSVR}|}{|C_{iFVR}|} \quad (25)$$

where,

$n$  = number of selected aerodynamic coefficients (26)

$C_{iSVR}$  =  $i^{th}$  relevant aerodynamic coefficient of SD obtained using a computational model (27)

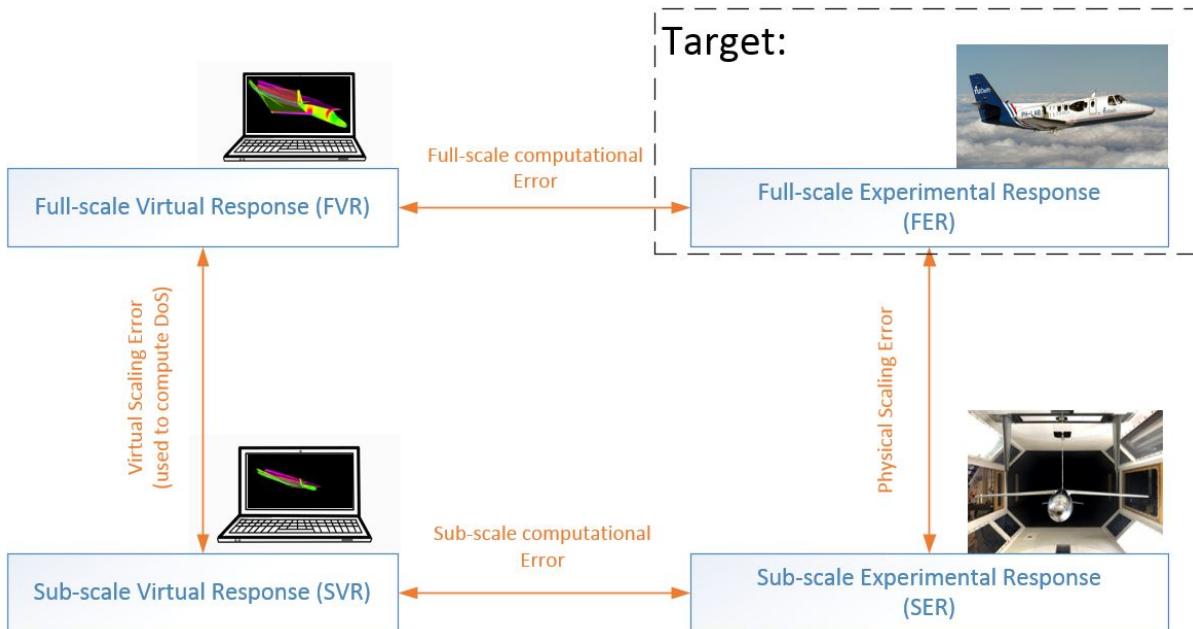
$C_{iFVR}$  =  $i^{th}$  relevant aerodynamic coefficient of the FD obtained using a computational model (28)

$w_i$  = Degree of Influence of a given aerodynamic coefficient on the phenomenon being tested (29)

subject to condition:

$$\sum_{i=1}^n w_i = 1 \quad (30)$$

In the definition provided, the coefficients are either full-scale virtual response or sub-scale virtual response. However, to obtain aerodynamic derivatives, appropriate computational model must be used. (refer figure 3) Section II.D describes how these aerodynamic coefficients can be determined.



**Fig. 4 Overview of errors in responses of different models**

#### D. Methodology to calculate Degree of Similitude

Once the DoS for a given test is defined on the basis of equation 25, the values of different aerodynamic derivatives must be filled in the equation. By definition of DoS, computational models must be used for the evaluation of degree of similitude.

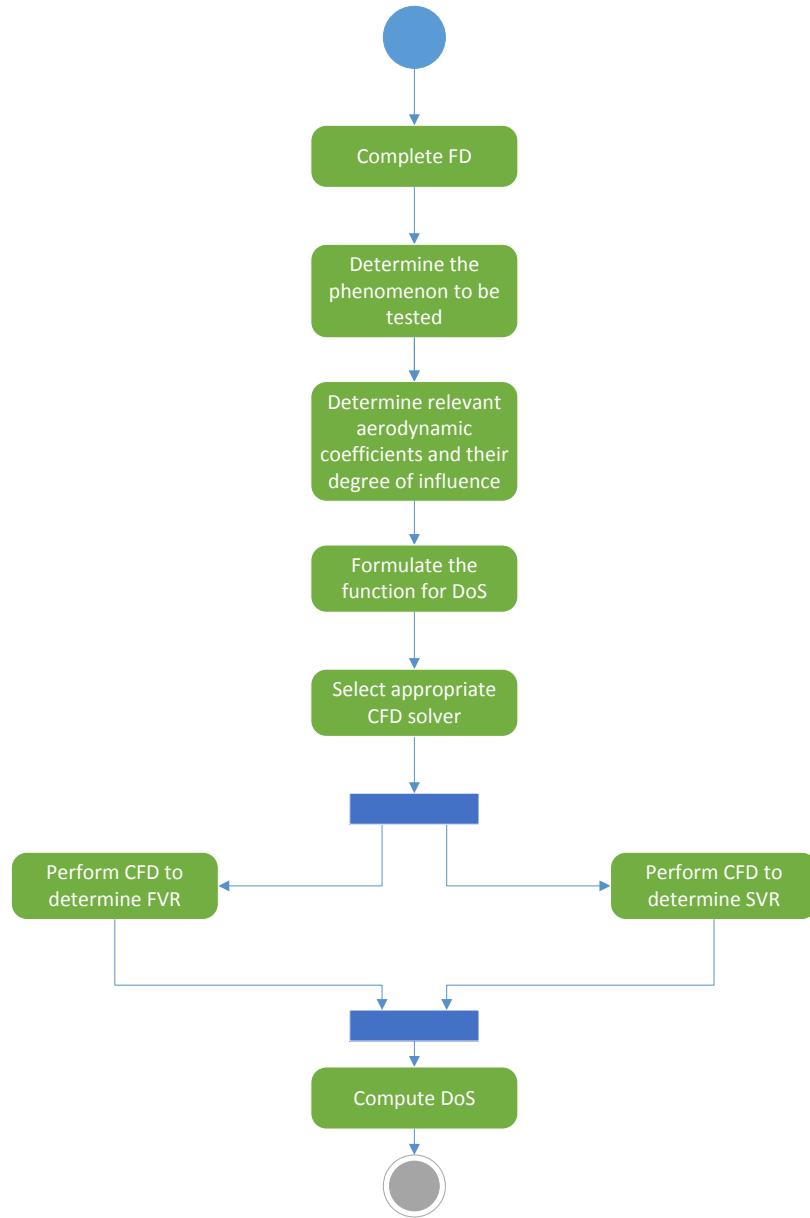
There are several ways of computing these aerodynamic derivatives. Classically, empirical data (or legacy experimental data) were used to determine these aerodynamic coefficients. However, this is not possible with unconventional design. Thus, the proposed way ahead is to use computational fluid dynamics (CFD). Several computational methods exist for this, such as, vortex lattice methods, panel methods, Euler equation solvers, Navier-Stokes equation solvers. Any of these methods can be selected by designers to evaluate the relevant aerodynamic coefficients. However, two important points must be borne in mind while making a selection of such a computational method:

- Computational time : If the selected computational method takes too long to converge, designers cannot compare sufficient SVAs. In addition to the computational method, the computational time also depends on the type of test being simulated and the complexity of SD, which should be taken into account when selecting a test method.
- Accuracy of result : If the results obtained from the computation is inaccurate, the discrepancy between physical test (e.g. wind-tunnel test) and CFD analysis using the same SD can be quite high, making the prediction of full-scale design performance difficult.

After computing aerodynamic derivatives for the sub-scale design and full-scale design, these derivatives must be compared by substituting appropriate values in equation 25. Figure 5 schematically depicts the steps that must be carried out to estimate DoS. This can be used for three different purposes:

- compare one or more SDs to select the best suited SD for testing
- to flag unsuitable test cases and/or designs when DoS is too low so as to prevent the loss of time and resources in testing unsuitable SD
- optimize scaled model design by maximizing DoS for one or more test cases.

Employing the degree of similitude in the design of SPM has some risks and limitations which must be minimized to obtain maximum benefit from this methodology. Firstly, in its current definition, DoS only accounts for aerodynamic coefficients. However, mass, frequency of response and the structural rigidity also affect the the performance of SPM.



**Fig. 5 Flowchart describing the methodology to calculate DoS**

These must also be included in the evaluation of DoS in the future. Secondly, when considering aerodynamic scaling, the calculation of aerodynamic coefficients at different geometric and aerodynamic scales can be

- 1) error-prone: due to approximations and assumptions in the computational method
- 2) time-consuming: due to the time needed for the CFD solver to converge. This is in addition to the time needed for pre-processing and post-processing the models for CFD,
- 3) labour-intensive: due to all the pre-processing and post-processing effort that needs to be done when using CFD

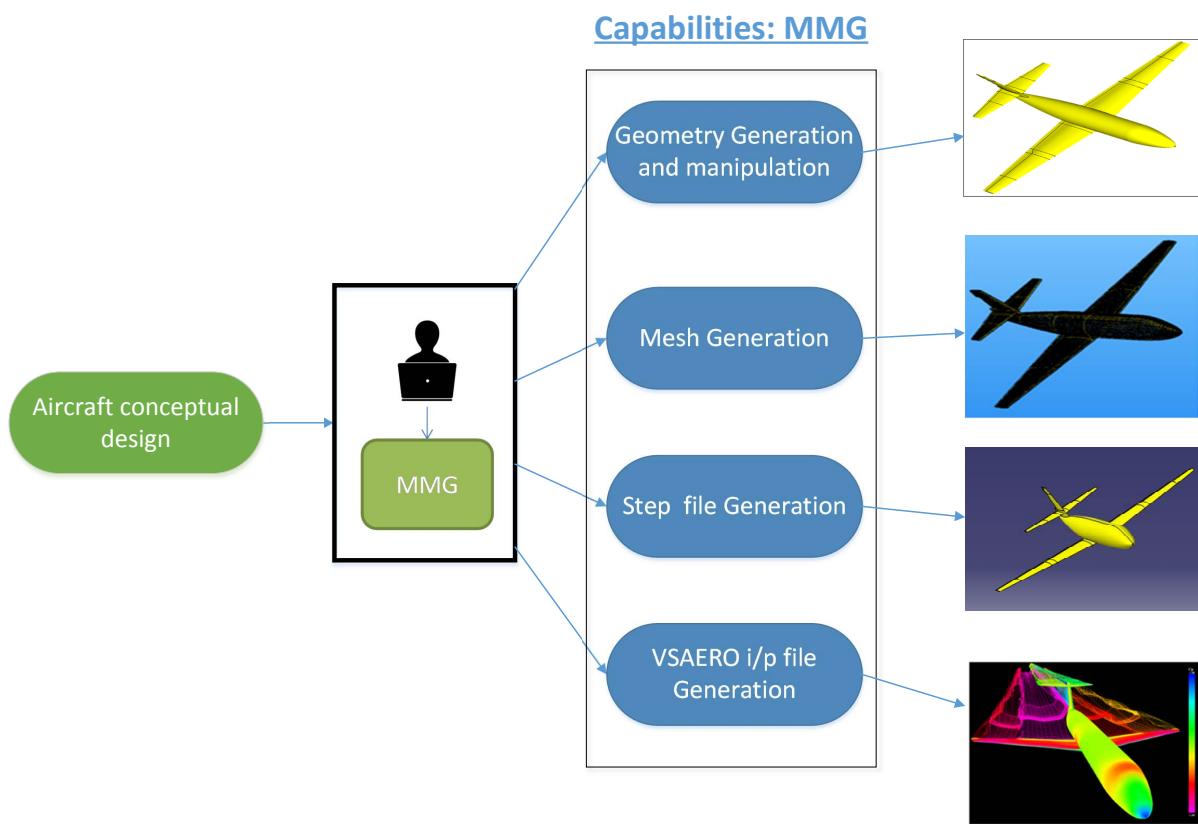
The problem of computational error (i.e. approximation or simplification in computational model) can be resolved by validating the computational models with appropriate wind-tunnel/flight tests. This allows the quantification of sub-scale computational error. The problem of convergence time of these computational models can be solved by using multi-fidelity tools i.e. use of low fidelity CFD to restrict SPM design space and then using higher fidelity CFD to arrive

at a suitable SPM. Furthermore, surrogate models can be used when optimization of SD for maximum DoS is necessary. Finally the challenge of repetitive labor intensive tasks can be solved by design automation. To this end, a KBE tool is developed to aid engineers in quickly determining DoS. This is discussed in section III.

### III. KBE tool to support estimation of degree of similitude

In order to apply the concept of DoS, tools must be made available to engineers to quickly and automatically evaluate the degree of similitude for different SDs. Without such a tool, the use Degree of Similitude will be impractical, as engineers will have to spend as much or more time computing DoS than actually carrying out tests using SPMs. The tool to estimate DoS should be able to help engineers evaluate all the aerodynamic coefficients relevant to the test, determine the degree of influence of each of the relevant coefficients and calculate the degree of similitude. In this section, description of the tool necessary to compute the DoS is provided.

The conceptual FD of the aircraft should be ready in order to perform SD. This FD information is used to generate FVA using Multi-Model Generator (MMG)<sup>†</sup>, a Knowledge Based Engineering tool.[18] MMG facilitates designers in modelling diverse aircraft configurations and configuration variants and preparing dedicated models to feed to various analysis tools involved in the design process (see figure 6). In this research, the capabilities of MMG are extended to quickly and automatically compute the DoS for a given test using VSAERO<sup>‡</sup>, a 3D panel code. This CFD solver was selected because it provides a convenient compromise between computation time and accuracy of the results.

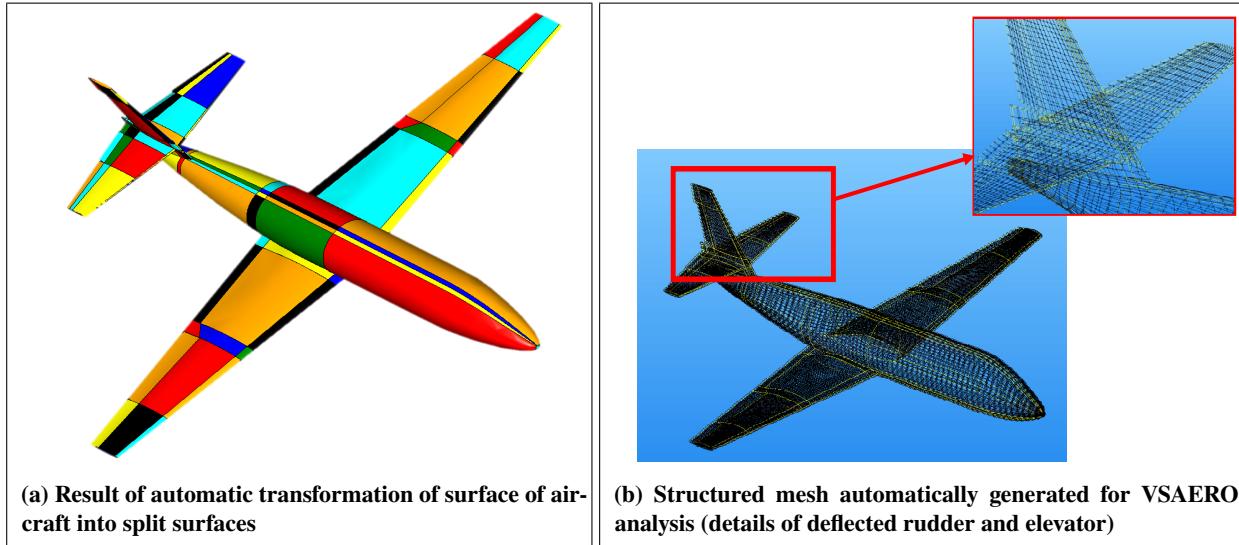


**Fig. 6 An overview of MMG capabilities and supported input formats necessary for estimation of DoS**

<sup>†</sup>The current version of MMG, which is under development at TU Delft, uses the KBE software, ParaPy, <https://www.parapy.nl/>

<sup>‡</sup><https://www.ami.aero/software-computing/amis-computational-fluid-dynamics-tools/vsaero/>

VSAERO requires information of the geometry in the form of a mesh (preferably structured mesh [19]). Generating a structured mesh for an aircraft geometry which has faces with more than 4 edges is difficult. Thus the first step is to modify the geometry into a shape that can be meshed. To do so, an algorithm was developed and incorporated in MMG, using which, the geometry of the aircraft is split into four sided faces (see fig. 7a). This split aircraft can be used to automatically generate structured mesh by placing equal number of nodes on all the opposite faces of a quadrilateral. A screenshot of complete mesh of an aircraft generated automatically for VSAERO analysis is shown in fig.7b.



**Fig. 7 Screen-shot of different models necessary for the generation of FVA**

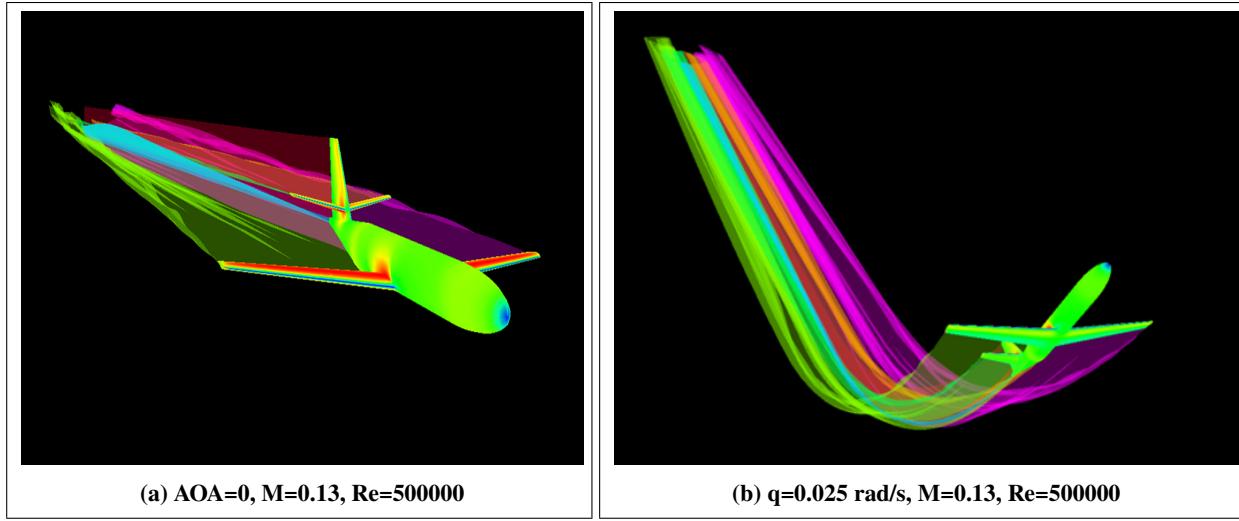
The mesh is then used by MMG to automatically write an input file for VSAERO. This input file can then be run by VSAERO solver to generate aerodynamic coefficients and derivatives. The results of analysis carried out by VSAERO are shown in figure 8. These results are used to then compute the DoS. In section IV, results generated by VSAERO are reported and compared with wind-tunnel tests. Furthermore, in its current form, MMG can generate VSAERO input file in 3 minutes. Without a tool such as MMG, the manual generation of geometry and mesh would easily take a few days or even weeks. Thus, such a reduction in lead time will also make application of DoS to multiple designs and test cases possible thereby allowing design of experiments and optimization studies (this is discussed in section V).

#### IV. Validation of Multi-model Generator

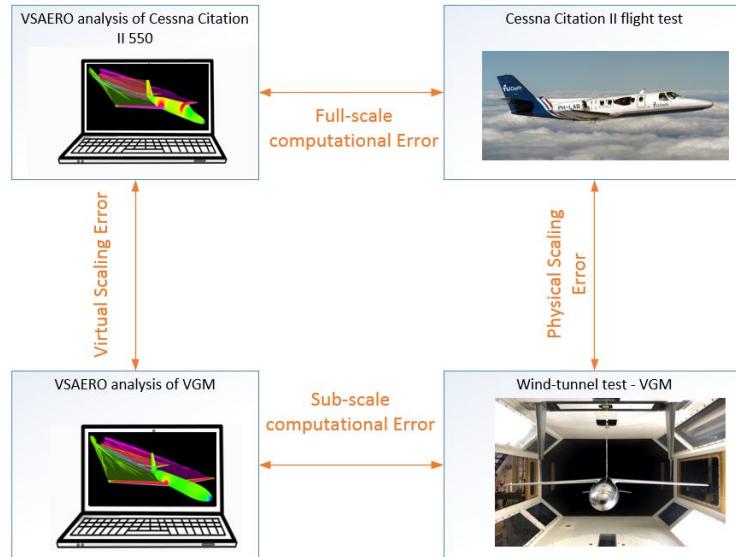
In section III, the development of new capability module for MMG to support the estimation of DoS was described. In order to use this tool effectively, accuracy of computation carried out by MMG+VSAERO tool-chain must be ascertained. This will help quantify uncertainties introduced by a tool like VSAERO. To validate MMG+VSAERO tool-chain, four models were built and compared (see figure 9), namely:

- 1) FVA of Cessna Citation II 550 using MMG-VSAERO tool-chain
- 2) SVA of 8.8% aerodynamically scaled model of Cessna Citation II 550 (called VGM)
- 3) SPM of VGM was used in wind-tunnel test
- 4) FPM of Cessna Citation II 550 used in flight test

To validate the accuracy of VSAERO, a wind-tunnel test campaign was performed with a 8.8% aerodynamically scaled model of the Cessna Citation II 550. This wind-tunnel test is used to evaluate the static aerodynamic derivatives which can be compared with the results generated by VSAERO. The Cessna Citation II 550, is chosen because TU Delft owns and operates it and thus extensive flight test data for full-scale aircraft is available. In section IV.A, the test setup and the VGM model are described. In section IV.B, the different errors shown in figure 9 are quantified.



**Fig. 8 Visualization of  $C_p$  distribution and wake deformation for an aircraft flying in steady flight and pitch-up rotation simulated using VSAERO**

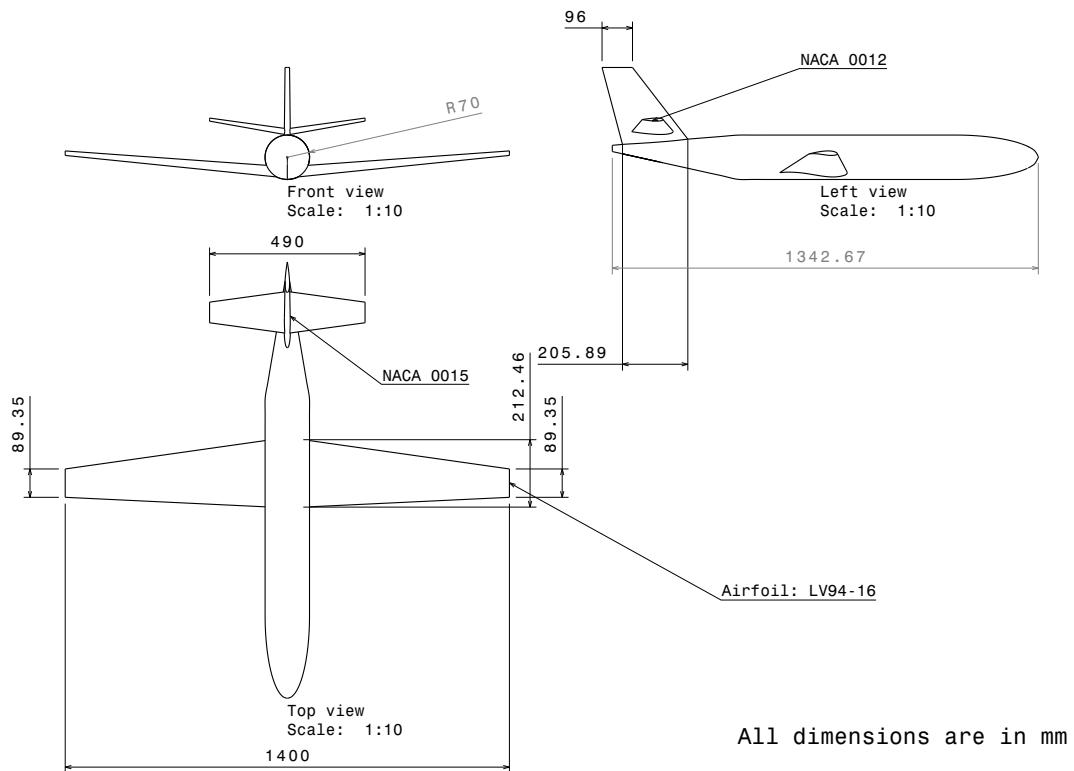


**Fig. 9 Different models used for validating VSAERO tool-chain and their associated errors**

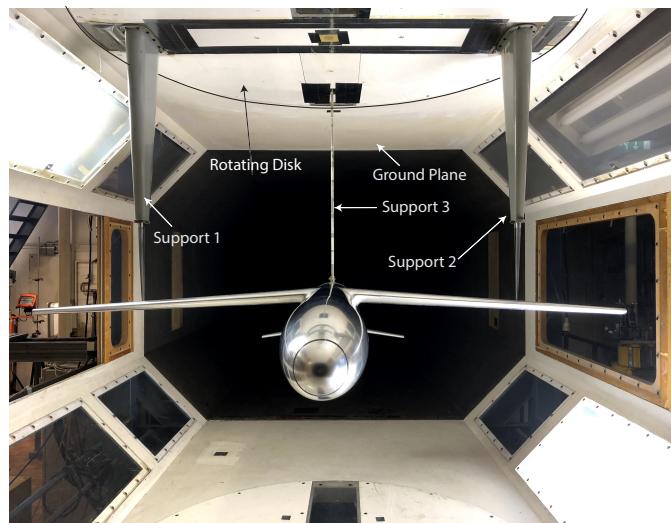
#### A. Wind-tunnel test campaign

The test was conducted at the Low Turbulence Tunnel (LTT) of Delft University of Technology. It is a low-speed, closed return wind-tunnel. At the test Reynolds number of 300000 and 500000, the turbulence is less than 0.1%. The test-section has the dimension of 1.80 X 1.25m. The model used for the test was an aerodynamically scaled 8.8% model of Cessna Citation II 550 called Variable Geometry Model (VGM). The rationale of using this SD was its availability from previous research work and its close similarity to the dimensions of Cessna Citation II. The key dimensions of the model being tested are shown in figure 10. A photograph of the setup used in LTT is shown in figure 11.

VGM was tested for angle of attack sweep from  $-5$  deg to  $14$  deg with a step size of 1 deg. Testing beyond  $14$  deg was not possible due to physical limits of the wind-tunnel setup. For each of these test points, balance readings for forces and moments on the VGM model were acquired using an external six-component balance. Some results from the tests are described in section IV.B.



**Fig. 10 Key dimensions of VGM**



**Fig. 11 Setup of VGM in LTT**

## B. Evaluation of errors - $\alpha$ sweep

The wind tunnel tests conducted using VGM only yield static aerodynamic derivatives. Thus, in this current comparison, the responses used for estimation of various errors shown in figure 9 are as follow:

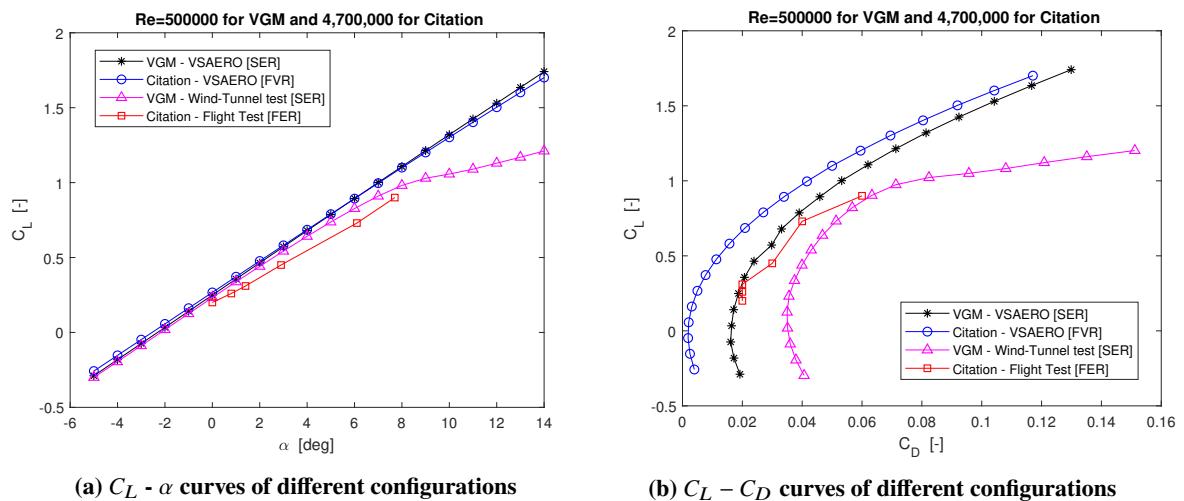
$$FVR = [C_{LFVR}, C_{DFVR}, C_{L\alpha FVR}] \quad (31)$$

$$SVR = [C_{LSVR}, C_{DSVR}, C_{L\alpha SVR}] \quad (32)$$

$$FER = [C_{LFER}, C_{DFER}, C_{L\alpha FER}] \quad (33)$$

$$SER = [C_{LSER}, C_{DSER}, C_{L\alpha SER}] \quad (34)$$

The graphical representation of FER, SER, FVR and SVR for  $C_L - \alpha$  and  $C_L - C_D$  are shown in figures 12a and 12b. The mean absolute error percentages of different aerodynamic coefficients and derivatives over the complete angle of attack range are shown in table 1.



**Fig. 12** Graphical representation of FVR (Citation flight test), FER(Citation VSAERO analysis), SVR(VGM VSAERO analysis) and SER(VGM Wind-tunnel test)

**Physical Scaling Errors:** It can be seen that the  $C_L - \alpha$  curve slope of FER and SER match well (refer 12a). This fit is incidental as the Reynolds number at which the flight test was performed was highly variable whereas for both VSAERO analysis and wind-tunnel test, it was constant. Furthermore, the indicated Reynolds number of 4.7 million was the average of all the measurements performed in the flight test. This Reynolds number discrepancy causes the difference in drag polar of FER and SER. (refer 12b) It is important to note that VGM was not designed for maximum similitude with Citation for all the aerodynamic coefficients used in this comparison (equations 31 - 34). VGM was only designed to match the lift coefficient to full-scale citation. Nevertheless, since it has a close similitude to full-scale Citation and was readily available for test, it is used in these measurements.

**Computational errors:** The VSAERO was set to use potential flow solver with integral boundary layer equations. However, this solver is incapable of modelling non-linear aerodynamic effects. Thus, as the angle of attack increases, the FVR and SVR deviate further from FER and SER respectively. Thus, the computational errors that are shown in 1 are mostly the deviations at high angle of attack. Moreover, the 3D panel code is generally not good in estimating interference drag that is accounted for in FER and SER. This causes a large error in drag estimation. In the specific case of full-scale computational error, discrepancies in measurements due to constantly changing flight conditions could also add to the error.

**Virtual scaling errors:** The virtual scaling error is seen mainly due to the differences in physics at sub-scale testing conditions and full-scale testing conditions as modelled by the computational model. In this particular case, the difference in VGM simulation and Citation simulation occur due to two main causes:

1) The effect of differing operating conditions of the two models.

2) The difference in the geometry of the two models used for comparison

While in this case, the virtual scaling error was evaluated after the test, in general it is recommended to quantify this error before performing the physical test. Since the rationale of VGM design was to match lift coefficient of full-scale citation, the virtual scaling errors seen are low. However, if other aerodynamic coefficients were considered for this comparison, the virtual scaling error might be high.

**Table 1 Scaling and computational errors of static derivatives**

	$\Delta C_L [\%]$	$\delta C_D [\%]$	$\Delta C_{L\alpha} [\%]$
Virtual Scaling Error	8.33	5.23	1.34
Physical Scaling Error	9.56	76.72	9.21
Sub-scale computational error	19.63	30.80	4.21
Full-scale computational error	11.31	25.64	19.84

## V. Results

In the previous section, an aerodynamically scaled SPM and SVA of Citation aircraft was used. It was seen that VGM had low virtual scaling error for the coefficients considered. The question remains, if a flight test to study longitudinal flight mechanics would be performed, would VGM still be the SPM of choice? To answer this, four different SVAs were studied for similitude in short-period motion and phugoid motion;

- 1) VGM : Since wind-tunnel tests were already performed on this SPM, VGM was used as a baseline to compare with other SPMs.
- 2) VGM modified: Citation uses a NACA 5 series airfoil whereas an in-house developed airfoil is used for VGM. In addition, the fuselage of VGM has been simplified into ellipsoid. In this model, the whole SPM was kept the same except the main-wing airfoils, which were modified to NACA 23014 and 23012 for the root and tip respectively
- 3) 8.8% Citation : An 8.8% isotropically scaled model of citation is used as it helps eliminate all errors due to model differences and thus only the differences in test conditions are reflected by the computational model.
- 4) 17.6% Citation : A 17.6% isotropically scaled model of citation is used to study the effect of increased size of the model on the aerodynamic derivatives affecting short-period motion and phugoid motion.

The FVR of Citation will be compared to the four SVRs for short-period motion in section V.A and for phugoid motion in section V.B by evaluating their DoS.

### A. Comparison of SDs for short-period motion case

In this study, simplified equation of motion was used to determine four relevant aerodynamic coefficients namely,  $C_{M_q}$ ,  $C_{Z_q}$ ,  $C_{M\alpha}$  and  $C_{Z\alpha}$ . The derivative  $C_{M\alpha}$  is ignored in this analysis as its effect on the short period motion can be considered negligible. Furthermore, the degree of influence of each of the aerodynamic derivative is assumed to be equal. The FVR and SVR are shown in table 2.

On comparison of different models, it can be seen that the 17.6% geometrically scaled SVA of Citation is most similar to full-scale citation. It is important to note that with a marginally small difference, both VGM and modified VGM are almost as good as its larger counterpart. Thus, if one were to pick a design for flight-test, VGM or its modified version would be a better option as it could have a multiple usage i.e. it can be used in a small closed wind-tunnel section (as the one at TU Delft) and in the flight test. This could mean reduction of cost and effort in manufacturing.

**Table 2 DoS of different designs for Short period motion**

SD description	$C_{M_q}$	$C_{Z_q}$	$C_{M_\alpha}$	$C_{Z_\alpha}$	$Fr$	$Re$	$DoS$
VGM	-8.454	-5.83	-1.03	-6.055	34.4798	500000	0.9367
8.8% Citation	-3.76	-0.506	-2.329	-4.830	34.4798	500000	0.8913
VGM modified	-8.53	-5.811	-1.065	-6.007	34.4798	500000	0.9384
17.6% Citation	-8.996	-9.4119	-2.586	-4.995	24.3809	1000000	0.9731
Full-scale Citation	-8.82	-7.239	-2.75	-5.26	20.6059	5000000	1

**B. Comparison of SDs for phugoid**

From the approximate equations of motion, it can be seen that  $C_{X_u}$ ,  $C_{Z_u}$  and  $C_{Z_0}$  are the relevant aerodynamic derivatives affecting phugoid motion. In this case too, all the derivatives were assumed to have equal degree of influence. The SVR and FVR of the different SVAs that were compared are reported in table 3.

It can be seen that the VGM modified had the highest DoS. Furthermore, its DoS much higher than the second best SVA (17.6% Citation). In addition, if the two cases i.e. short period motion and phugoid are considered together, the modified VGM works out the best. This is in addition to the advantage of being able to rest in small closed wind-tunnels. It is important to note that some of the DoS values are negative. This is not surprising because in the case of  $C_{X_u}$ , the full-scale design values were much lower than the sub-scale designs. This is analogous to margin of safety in structural design and can be seen from equation 25.

**Table 3 DoS of different designs for phugoid motion**

SD description	$C_{X_u}$	$C_{Z_u}$	$C_{Z_0}$	$Fr$	$Re$	$DoS$
VGM	0.001987	-0.711	-0.3541	34.4798	500000	-0.2784
8.8% Citation	-0.00388	-0.4877	-0.2385	34.4798	500000	-1.0442
VGM modified	0.00001	-0.256	-0.1279	34.4798	500000	0.7690
17.6% Citation	0.0015	-0.4968	-0.2436	24.3809	1000000	0.0488
Full-scale Citation	-0.0002	-0.5368	-0.262	20.6059	5000000	1

**C. Time studies**

The time needed to estimate DoS can be an important factor at the end of conceptual design phase, where, limited time is available to support testing. In this research design automation for estimation of DoS was performed using MMG. The important question here is, does this automation make the use of DoS viable?

To answer this question, the time needed to calculate DoS was studied. In general, this time can be segregated into two parts:

- 1) time needed to pre-process the sub-scale design such that it can be used by the computational model and then post-process the results obtained from the computational model
- 2) time needed to run the computational model

The latter is the choice of the user and a function of the accuracy. In general, this time can only be reduced by improvements in computational power. The former can be reduced significantly using MMG. The time needed to preprocess and post process one sub-scale design manually is about 70 hours. The same tasks can be accomplished in 4 hours if the input file for MMG must be generated manually and in case this file is provided by the conceptual design toolbox like Initiator, the pre-processing and post-processing takes about 30 mins. Which means a over 90% reduction in time to perform pre-processing and post processing activities in the estimation of DoS. This time improvement does not include the development time of this tool which was about 1000 hours full-time effort. Thus, DoS of 15 different sub-scale designs must be estimated to off-set the time needed for the development of DoS features in MMG.

## VI. Conclusions and recommendations

### A. Conclusions

The concept of degree of similitude was introduced in this paper and a specific figure of merit was defined to quantify it. This DoS can be used to compare different SDs, flag unsuitable designs and optimize DoS for one or more tests. Moreover, in cases where DoS is equal to 1, it can be seen from figure 4 that only full-scale computational error needs to be evaluated for a given design to predict the behavior of FD. Since all the aerodynamic derivatives will be equal in this specific test, one could reason that the phenomenon occurring in the full-scale tests and the sub-scale tests are the same. This means that sub-scale computational error is the same as full-scale computational error and therefore, computational models in combination with SPMs can be used to predict flight mechanics of FD in such cases.

Furthermore, developments made in MMG to rapidly estimate the DoS to select adequate scaled models for testing were also discussed. The implemented design automation showed an improvement of over 90% in terms of time needed for pre-processing and post-processing models. This capability of MMG can help designers estimate the degree of similitude before the start of the test and can make scaled flight testing a viable alternative to legacy information that was used in the design of conventional aircraft.

In the particular case-study that was performed, of the four sub-scale designs that were compared, the geometrically scaled model (17.6% citation) performed better than the aerodynamically scaled models in case of short period motion. On the other hand, aerodynamically scaled model (modified VGM) performed much better than the geometrically scaled model for phugoid motion. Furthermore, if both the results were combined, the modified VGM would be the chosen SD. This goes to show that geometrically scaled models are not always the best option for physical testing. Furthermore, these SDs were chosen on the basis of the available SPM i.e. VGM. A detailed design space exploration and optimization for maximization of DoS can yield much better designs that can be used for multiple tests and test-setups (wind-tunnel, flight tests etc.)

An important take-away from the studies presented in this paper was that dimensionless scaling parameters alone are not sufficient to scale the models. Despite the low accuracy of computational models, in the design of SPM for short period motion, it could be seen that doubling the Reynolds number and reducing the Froude number only had a minor effect on the DoS. Whereas in case of Phugoid motion, a 70% increase in DoS can be seen. More such studies and design space exploration will allow us to better understand the relationships between the dimensionless scaling parameters and the DoS and thereby improve the estimation of full-scale design behavior.

### B. Recommendations

Some recommendation for further research are as follows:

**Fidelity of tools to estimate DoS:** In the current study, the errors relating to the computational model namely the virtual scaling error, the sub-scale computational error and full-scale computational errors were quite high when the 3D panel code was used. In order to make DoS a credible method to compare SDs, higher fidelity tools must be used to ascertain the quality of DoS predicted by computational models.

**Design space exploration:** Higher fidelity tools also imply higher time requirements which makes them unfeasible for optimization studies that has been concluded to be essential in the section VI.A. Thus, investing resources in developing multi-fidelity DoE and optimization would be very useful. In addition, development of surrogate models to speed up the optimization process is recommended.

**Uncertainty of up-scaled results** When DoS is not equal to 1, up scaling the sub-scale experimental response should include the uncertainty of the up-scaled results. This will allow engineers to better understand the flight mechanics of the FD especially when it is at the limits of acceptable handling qualities.

**Flyability of SD:** In the current study, some aerodynamic derivatives were compared however, a complete flight mechanics analysis to check the flyability of a SD was not performed. In addition to checking the flyability, it should become a capability module that can be used to constrain the aforementioned DOE/optimization.

**Sub-scale flight test:** In this research, static derivatives were compared with a wind-tunnel model. A similar study to compare dynamic derivatives must be performed to study the associated errors. Furthermore, results from such a test must be scaled-up to predict the full-scale performance and compared with full-scale flight test.

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