# S4. CGE Methodologies

This supplementary document contains three methodological notes supporting the CGE modelling approach used in this thesis.

In chapter 1, you can read the detailed information on the method behind Cost-Neutral Preference Shifts in CES Utility via Efficiency Augmentation. The section from 1.1 to 1.9 provides an explanation and a background of the methodology. This section can be fully contributed to Li (PBL). The last part (1.10) is developed by me, and this explains how this method can be used to create a preference change based on the output of the dMFA model for the year 2017.

In chapter 2, you can find detailed information on the method behind the decomposition of preference change, and substitution and income Effect This section can be fully contributed to Li (PBL).

In chapter 3, you can find detailed information on the method behind the decomposition of indirect emissions, based on the Shapley decomposition analysis. This section is also fully contributed to Li (PBL).

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# 1. Cost-Neutral Preference Shifts in CES Utility via Efficiency Augmentation

Developed by Li (PBL), based on Redding & Weinstein (2020), and Van der Mensbrugghe (2024).

## 1.1 Motivation

In CGE models, we often want to simulate a **consumer preference shift** — for example, households shifting consumption from new electrical and electronic equipment (EEE) toward repair services (ROS) as part of a circular economy transition.

The challenge: we want to model the *taste change itself*, not a hidden price or income effect that sneaks in through poor implementation. This note shows how to do that using an **efficiency augmenter** (also called a taste shifter) inside a nested CES utility function. This approach is used in CGE models such as the World Bank's MANAGE (van der Mensbrugghe, 2024) and formalized theoretically by Redding & Weinstein (2020).

## 1.2 The CES Nest

Consider a CES sub-utility nest **ELS** containing two goods: new EEE and repair services ROS.

### 1.2.1 Quantity Aggregator

$$Q_{ELS} = [\alpha_{EEE}(\gamma_{EEE}\, Q_{EEE})^{\rho} + \alpha_{ROS}(\gamma_{ROS}\, Q_{ROS})^{\rho}]^{1/\rho} \qquad (1)$$

Where:

- $\alpha_i$ = share parameter (calibrated from base-year expenditure shares), with $\alpha_{EEE} + \alpha_{ROS} = 1$
- $\gamma_i$ = **efficiency augmenter** (taste shifter) for good $i$
- $\rho = \frac{\sigma-1}{\sigma}$, where $\sigma$ is the elasticity of substitution between EEE and ROS

#### 1.2.2 Dual Price Index

By cost minimisation, the corresponding **price index** for the composite ELS is:

$$P_{ELS} = \left[\alpha_{EEE}\left(\frac{P_{EEE}}{\gamma_{EEE}}\right)^{1-\sigma} + \alpha_{ROS}\left(\frac{P_{ROS}}{\gamma_{ROS}}\right)^{1-\sigma}\right]^{1/(1-\sigma)} \qquad (2)$$

This is the formula that appears in the model's price-aggregator equation (EQH7). The augmenter $\gamma_i$ enters as a divisor of the price: **raising $\gamma_i$ is mathematically equivalent to lowering the effective price of good $i$**, which is exactly a taste shift toward that good.

## 1.3 Why an Efficiency Augmenter and Not a Change in the Share Parameter $\alpha$?

A naïve approach is to directly change $\alpha_{EEE}$ and $\alpha_{ROS}$. This breaks calibration: the share parameters were chosen so that, at base-year prices = 1 and quantities = 1, the model reproduces the SAM. Changing them without recalibrating $Q_0$ creates an inconsistency between the price aggregator and the demand equations, producing budget imbalances (Walras' law fails).

The augmenter $\gamma_i$, by contrast, is calibration-consistent by construction:

- At benchmark, $\gamma_i = 1$ for all $i$.
- Shifting $\gamma_i$ away from 1 directly represents a preference change without disturbing the underlying SAM calibration.

This is the Redding & Weinstein (2020) equivalence: a taste shock and a price shock affect CES demand identically. The augmenter $\gamma_i$ is just the formal way of writing this inside the model.

## 1.4 What "Cost Neutral" Means

When we shift preferences from EEE toward ROS, we want the experiment to capture only the redirection of demand between EEE and ROS. We do not want a side effect where the composite ELS becomes cheaper or more expensive — that would create an artificial income or substitution effect with other nests in the utility tree.

**Cost neutrality requirement:** at base-year prices, the composite price $P_{ELS}$ must remain equal to 1 after the shift.

This isolates two phenomena cleanly:

| Channel | What it captures |
|---|---|
| The taste shift ($\gamma$ change) | Redirection of demand from EEE to ROS within ELS |
| GE feedback (endogenous prices) | How relative prices respond as resources reallocate |

Without cost neutrality, you would also pick up a third (spurious) channel: an income effect through a mechanical shift in $P_{ELS}$.

## 1.5 The Cost-Neutrality Condition

At base prices $P_{EEE} = P_{ROS} = 1$, the price index becomes:

$$P_{ELS}^{1-\sigma} = \alpha_{EEE}\,\gamma_{EEE}^{\sigma-1} + \alpha_{ROS}\,\gamma_{ROS}^{\sigma-1} \qquad (3)$$

(using $(1/\gamma)^{1-\sigma} = \gamma^{\sigma-1}$).

Setting $P_{ELS} = 1$ gives the cost-neutrality condition:

$$\alpha_{EEE}\,\gamma_{EEE}^{\sigma-1} + \alpha_{ROS}\,\gamma_{ROS}^{\sigma-1} = 1 \qquad (4)$$

One equation in two unknowns — so we have **one degree of freedom**: we choose the strength of the shift on one side, and solve for the other side.

## 1.6 Solving the Condition

### 1.6.1 Exact Form (use this in code)

Pick the size of the boost to ROS: $\gamma_{ROS} = 1 + s$, where $s$ is the "shift strength" (e.g., $s = 0.20$ for a 20% taste boost).

Solve for $\gamma_{EEE}$:

$$\gamma_{EEE} = \left[\frac{1 - \alpha_{ROS}(1+s)^{\sigma-1}}{\alpha_{EEE}}\right]^{1/(\sigma-1)} \qquad (5)$$

**Existence condition:** the bracket must be positive, i.e.,

$$\alpha_{ROS}(1+s)^{\sigma-1} < 1 \qquad (6)$$

For realistic $\alpha_{ROS}$ and moderate $\sigma$ and $s$, this is usually satisfied. For very large $s$ or very high $\sigma$ on the dominant good, the bracket may become negative — physically, this means the shift you have asked for is too aggressive to be cost-neutralised.

### 1.6.2 Linearised Form (intuition only)

For small shifts and $\sigma$ near 1, take logs and use $\gamma_i \approx 1 + \hat{\gamma}_i$:

$$\alpha_{EEE}\,\hat{\gamma}_{EEE} + \alpha_{ROS}\,\hat{\gamma}_{ROS} \approx 0 \qquad (7)$$

So if $\hat{\gamma}_{ROS} = s$, then

$$\hat{\gamma}_{EEE} \approx -\frac{\alpha_{ROS}}{\alpha_{EEE}}\,s \qquad (8)$$

**Intuition:** the share-weighted average of the log taste shifts must be zero. You are rotating preferences in the simplex without changing the overall cost of the composite.

This form is useful for intuition and back-of-envelope checks, but in code use the exact form, since real CGE work routinely uses $\sigma = 2$ to 4 where the linearisation drifts noticeably.

#### 1.6.3 The Degenerate $\sigma = 1$ Case (Cobb–Douglas)

If $\sigma = 1$, the exponent $\sigma - 1 = 0$ makes the formula degenerate. The cost-neutrality condition becomes (in log form):

$$\alpha_{EEE}\ln\gamma_{EEE} + \alpha_{ROS}\ln\gamma_{ROS} = 0 \qquad (9)$$

So:

$$\gamma_{EEE} = \gamma_{ROS}^{-\alpha_{ROS}/\alpha_{EEE}} \qquad (10)$$

## 1.7 Worked Example

Suppose base-year shares are $\alpha_{EEE} = 0.258087$, $\alpha_{ROS} = 0.741913$, with $\sigma = 1.5$ and $s = 0.20$. **Check the existence condition:**

$$\alpha_{ROS}(1+s)^{\sigma-1} = 0.741913 \times 1.20^{0.5} = 0.741913 \times 1.09545 = 0.81282 < 1 \quad \checkmark \qquad (11)$$

The condition is comfortably satisfied — in fact, the shift strength could go as high as $s \approx 0.817$ before the cost-neutral counterpart becomes undefined.

**Compute $\gamma_{EEE}$ (exact):**

$$\gamma_{EEE} = \left[\frac{1 - 0.741913 \times 1.20^{0.5}}{0.258087}\right]^{1/0.5} = \left[\frac{1 - 0.81282}{0.258087}\right]^{2} = 0.72527^2 \approx 0.5260 \qquad (12)$$

So $\gamma_{ROS} = 1.200$, $\gamma_{EEE} \approx 0.5260$: ROS taste up 20%, EEE taste down ~47.4%.

**Why the asymmetry?** Because ROS already accounts for 74% of ELS expenditure, a modest taste boost on the dominant good must be offset by a much larger taste decline on the smaller good to keep the composite price unchanged. The share-weighted log shifts must net to zero, and a small share carries a larger per-unit weight.

**Verify cost neutrality:**

$$P_{ELS}^{1-\sigma} = 0.258087 \times 0.5260^{0.5} + 0.741913 \times 1.20^{0.5} = 0.18718 + 0.81282$$
$$= 1.00000 \quad \checkmark \qquad (13)$$

**New expenditure shares at base prices:**

$$s_i^{new} = \alpha_i \gamma_i^{\sigma-1} \qquad (14)$$

| Good | Old share | New share |
|---|---|---|
| EEE | 0.2581 | $0.258087 \times 0.5260^{0.5} = 0.258087 \times 0.72527 = 0.1872$ |
| ROS | 0.7419 | $0.741913 \times 1.20^{0.5} = 0.741913 \times 1.09545 = 0.8128$ |

The ROS expenditure share rises from 74.2% to 81.3%, and the EEE share falls from 25.8% to 18.7% — a ~9.5% relative increase in ROS expenditure (and a ~27.5% relative decrease in EEE expenditure), consistent with a substantial taste rotation away from new electronics and toward repair services.

**Alternative framing.** Because the shares are so asymmetric, you may prefer to pick the shift on the smaller side — for example, $\gamma_{EEE} = 1 - s$ — and solve for $\gamma_{ROS}$. With $s = 0.20$ (a 20% taste decline for new EEE):

$$\gamma_{ROS} = \left[\frac{1 - 0.258087 \times 0.80^{0.5}}{0.741913}\right]^{1/0.5} = \left[\frac{1 - 0.23085}{0.741913}\right]^{2} = 1.03671^2 \approx 1.0748 \qquad (15)$$

This yields a milder reciprocal change: EEE taste −20% paired with ROS taste +7.5%. Either framing is valid; the choice depends on which side of the story you want to anchor the shift magnitude to.

## 1.8 What General-Equilibrium Prices Do Afterwards

Cost neutrality holds **at base prices**. In equilibrium, $P_{EEE}$ and $P_{ROS}$ will move endogenously because demand reallocates resources:

- ROS demand rises → ROS price rises slightly
- EEE demand falls → EEE price falls slightly

The new equilibrium $P_{ELS}$ will not be exactly 1, but it will deviate only via second-order GE effects, not via a mechanical taste-induced wedge. That is the point of cost neutrality: it isolates the preference channel from the price-aggregator channel so you can attribute results to the right mechanism.

## 1.9 Summary Recipe

To implement a cost-neutral preference shift from good $A$ to good $B$ inside a CES nest:

1. **Do not change $\alpha$ (the share parameter).** It breaks calibration.

2. **Use the efficiency augmenter $\gamma$ instead.** It is calibration-consistent.
3. **Pick the shift strength on one side:** $\gamma_B = 1 + s$.
4. **Solve the cost-neutrality condition for the other side:**

$$\gamma_A = \left[\frac{1 - \alpha_B(1+s)^{\sigma-1}}{\alpha_A}\right]^{1/(\sigma-1)}$$

5. **Verify** that $P_{ELS}$ is unchanged at base prices, and that the new expenditure shares match $s_i^{new} = \alpha_i \gamma_i^{\sigma-1}$.

This procedure cleanly captures the taste shift, preserves benchmark calibration, and lets general-equilibrium price adjustments do their work without contamination from a spurious income effect.

## 1.10 Method to Calculate Efficiency Augmenter Based on dMFA Reduction

### 1.10.1 Step 1: Target New Expenditure Share

The MFA provides a relative reduction in EEE expenditure of $x\%$. This relative reduction is applied to the old share of EEE to calculate the new share based on the dMFA calculated reduction:

$$s_{EEE}^{new} = \alpha_{EEE} \cdot (1 - x)$$

*Example for 23% reduction:*

$$s_{EEE}^{new} = 0.2581 \times 0.77 = 0.198737$$

### 1.10.2 Step 2: Calculate a New Efficiency Augmenter for EEE

Based on equation (14), we can calculate the required $\gamma_{EEE}$ to achieve this reduction of $x\%$:

$$s_i^{new} = \alpha_i \cdot \gamma_i^{\sigma-1}$$

Given $\sigma = 1.5$ and $s_{EEE}^{new} = 0.198737$:

$$0.198737 = 0.2581 \cdot \gamma_{EEE}^{0.5}$$

$$\gamma_{EEE} = 0.5929$$

### 1.10.3 Step 3: Calculate the Efficiency Augmenter for ROS to Uphold Cost Neutrality

$$\gamma_{ROS} = \left(\frac{1 - s_{EEE}^{new}}{\alpha_{ROS}}\right)^{1/(\sigma-1)}$$

Based on original share of ROS:

$$\gamma_{ROS} = \left(\frac{1 - 0.198737}{0.7419}\right)^{1/0.5} = 1.1663$$

# 2. Decomposition of Preference Change, and Substitution and Income Effect

This section is developed by Li (PBL), based on standard microeconomic theory using Slutsky decomposition.

## 2.1 Alternative to Utility Maximisation

We have examined how individuals maximise their welfare by maximising their utility. Diagrammatically, it is like moderating the individual's indifference curve until it is just tangent to the budget constraint. The individual's choice thus selected gives us a demand for the goods in terms of their price and income. We call such a demand function a **Marshallian Demand function**. In general mathematical form, letting $x$ be the quantity of a good demanded, we write the Marshallian Demand as $x_i^M \equiv x_i^M(p, y)$.

Thinking about the process, we can reverse the intuition about how individuals maximise their utility. Consider the following: what if we fix the utility value at the above level, but instead vary the budget constraint? Would we attain the same choices? We should — the demand thus achieved is, however, in terms of prices and utility, not income. We call such a demand function a **Hicksian Demand**, $x_i^H \equiv x_i^H(p, u)$. This method means that the individual's problem is instead framed as minimising expenditure subject to a particular level of utility. The problem is framed as:

$$\min_{x_1, x_2} p_1 x_1 + p_2 x_2$$

subject to $u(x_1, x_2) = u$.

We refer to this as expenditure minimisation. This problem generates a parallel demand function which we refer to as Hicksian Demand, also commonly referred to as **Compensated Demand**.

## 2.2 Decomposition of Changes in Choices Induced by a Price Change

Let us examine the decomposition of a change in consumer choice as a result of a price change. Assuming a Cobb–Douglas utility function for the problem of:

$$\max_{x_A, x_B} A\, x_A^{\alpha} x_B^{\beta}$$

subject to $y = p_A x_A + p_B x_B$,

the consumer's choice may be characterised by the following demand functions:

$$x_A^M = \frac{\alpha}{\alpha + \beta} \frac{y}{p_A} \qquad \text{and} \qquad x_B^M = \frac{\beta}{\alpha + \beta} \frac{y}{p_B}$$

Note that income is actually a function of prices. Let the choices at the current prices and income level be $\bar{x}_A$ and $\bar{x}_B$. Then the fixed level of income yielding the choices can be written as $y = p_A\bar{x}_A + p_B\bar{x}_B$. We can rewrite both demand functions as **Slutsky Demand functions**:

$$x_A^S = \frac{\alpha}{\alpha+\beta}\frac{p_A\bar{x}_A + p_B\bar{x}_B}{p_A} \qquad \text{and} \qquad x_B^S = \frac{\beta}{\alpha+\beta}\frac{p_A\bar{x}_A + p_B\bar{x}_B}{p_B}$$

The question is how a price change alters choices (comparative statics). Let the price of good $A$ change. Differentiating with respect to $p_A$:

$$\frac{\partial x_A^S}{\partial p_A} = \frac{\alpha}{\alpha+\beta}\left[\frac{1}{p_A} - \frac{p_A\bar{x}_A + p_B\bar{x}_B}{p_A^2}\right] = \frac{\alpha}{\alpha+\beta}\cdot\frac{1}{p_A} - \frac{\alpha}{\alpha+\beta}\frac{y}{p_A^2}$$

From this expression, the first term $\frac{\alpha}{(\alpha+\beta)\,p_A}$ is the **substitution effect**, and the second term $\frac{\alpha}{(\alpha+\beta)}\frac{y}{p_A^2}$ is the **income effect**.

More generally, the decomposition of a price change in good $i$ is:

$$\frac{\partial x_i^M(P,y)}{\partial p_i} = \frac{\partial x_i^S(P,PX)}{\partial p_i} - \bar{x}_i\frac{\partial x_i^M(P,y)}{\partial y}$$

This can be rearranged to:

$$\frac{\partial x_i^S(P,PX)}{\partial p_i} = \frac{\partial x_i^M(P,y)}{\partial p_i} + \bar{x}_i\frac{\partial x_i^M(P,y)}{\partial y}$$

This is the more general version of the decomposition. It also tells us why we typically refer to the substitution effect as the **Slutsky substitution effect**, where income is held at the original choice level. From the above we can make several points about the sign and direction of impact:

1. The substitution effect is always negative. That is, as price increases (falls) the change in quantity demanded falls (rises).

2. The income effect can be either positive or negative.

    a. When the income effect of a good is positive, we call it a **normal good**. The greater (lower) one's income, the greater (lower) one's demand for the good. For a normal good, the direction of both income and substitution effects reinforce each other.

    b. When the income effect is negative, we say a good is an **inferior good**. For such a good, the greater the income, the lower the demand, and the net impact of a price change is ambiguous.

*Note: Diagrams of indifference curves and budget constraints illustrating the substitution and income effects are not reproduced here. Please refer to the original source.*

## 2.3 Decomposition of the Hicksian Demand

The Hicksian Demand expresses the individual's choice in terms of the price of the good and the utility level attained, $x_i^H \equiv x_i^H(p_1, p_2, u)$. The Slutsky substitution and the Hicksian substitution effect are the same:

$$\frac{\partial x_i^H(P,u)}{\partial p_i} = \frac{\partial x_i^S(P,PX,y)}{\partial p_i}$$

The entire decomposition can be written as:

$$\frac{\partial x_i^M(P,y)}{\partial p_i} = \frac{\partial x_i^H(P,PX)}{\partial p_i} - \bar{x}_i \frac{\partial x_i^M(P,y)}{\partial y}$$

Note the slight difference in the income effect in this decomposition, since we are not considering the income effect from the original level of income — the substitution effect holds utility, not income, constant.

## 2.4 Types of Goods and Their Characterisation

There are essentially three types of goods:

1. **Normal Goods** — Both the substitution and income effects reinforce each other, leading to a fall in quantity demanded when price rises.

2. **Inferior Goods** — The income effect is negative (opposite sign to substitution effect). Under a price increase, substitution reduces demand, but the income effect (from lower real income) increases it. If the substitution effect is dominant, demand still falls.

3. **Giffen Goods** — A special case of inferior goods where the income effect is stronger than the substitution effect, so that demand rises when price rises. A Giffen good must be an inferior good, but an inferior good is not necessarily a Giffen good.

## 2.5 Substitution and Income Effects Under Special Indifference Curve Forms

**Leontief (perfect complements) utility:** Because this utility function assumes goods are consumed in fixed proportions, there is only one consumption combination for every income level. Consequently, there is **no substitution effect** — only an income effect.

**Perfectly linear utility (perfect substitutes):** There is **no income effect**, only a substitution effect. The consumer switches entirely between goods depending on relative prices.

**Quasilinear utility:** When the price of the numéraire good changes, the effect is generated **purely by the substitution effect**, with no income effect. This arises because the marginal utility of income is constant in quasilinear preferences.

*Note: Diagrams illustrating these special cases are not reproduced here. Please refer to the original source.*

# 3. Shapley Structural Decomposition Analysis of Emission Changes

This section is developed by Li (PBL), based on Shapley (1953), Dietzenbacher & Los (1998), Lenzen (2006), Owen (2014), Sun (1998), and Wood & Lenzen (2006).

## 3.1 Methodological Framework

Indirect emissions are represented by the environmentally extended input–output model

$$M = \mathbf{ELF} \qquad (1)$$

where

- $\mathbf{E}$ is the vector of emission intensities,
- $\mathbf{L}$ is the Leontief inverse matrix,
- $\mathbf{F}$ is the final demand matrix,
- $M$ is the vector of consumption-based emissions.

We compare a baseline scenario (0) and a policy scenario (1):

$$M_0 = E^0 L^0 F^0 \qquad (2)$$

$$M_1 = E^1 L^1 F^1 \qquad (3)$$

The total change in emissions is therefore

$$\Delta M = M_1 - M_0 \qquad (4)$$

## 3.2 Decomposition Factors

The total change is decomposed into five drivers:

$$N = \{E,\ L,\ F_{EEE},\ F_{ROS},\ F_{OTHER}\} \qquad (5)$$

where

- $E$ represents changes in emission intensities;
- $L$ represents changes in production technology and inter-industry linkages captured by the Leontief inverse;
- $F_{EEE}$ represents changes in European final demand for electrical and electronic equipment (EEE);

- $F_{ROS}$ represents changes in European final demand for repair and other services (ROS);
- $F_{OTHER}$ represents all remaining changes in final demand.

The objective is to allocate the total emission change among these five factors:

$$\Delta M = \phi_E + \phi_L + \phi_{EEE} + \phi_{ROS} + \phi_{OTHER} \qquad (6)$$

where $\phi_i$ denotes the contribution of factor $i$.

## 3.3 Need for Shapley Decomposition

Because emissions are determined by the multiplicative relationship

$$M = \mathbf{ELF} \qquad (7)$$

the contribution of one factor depends on the values taken by the other factors. For example, the effect of changing emission intensities depends on whether technology and demand remain at baseline values or have already been changed to their policy values:

$$E^1L^0F^0 - E^0L^0F^0 \neq E^1L^1F^1 - E^0L^1F^1 \qquad (8)$$

Consequently, the decomposition results depend on the order in which factors are introduced. To avoid this path-dependency problem, the Shapley decomposition averages contributions across all possible factor orderings (Shapley, 1953; Dietzenbacher and Los, 1998).

## 3.4 Shapley Structural Decomposition Analysis

Let $\Pi(N)$ denote the set of all permutations of the five factors $(9)$.

Since there are five factors,

$$|\Pi(N)| = 5! = 120 \qquad (10)$$

Each permutation represents a possible sequence for transforming the baseline economy into the policy economy.

For a given permutation $\pi$, let $S^\pi(i)$ denote the set of factors that precede factor $i$ in that permutation $(11)$.

Define the value function $v(S)$ as the emissions obtained when all factors contained in subset $S$ take their policy values, while all remaining factors remain at baseline values $(12)$.

The marginal contribution of factor $i$ in permutation $\pi$ is

$$\Delta_i^\pi = v(S^\pi(i) \cup \{i\}) - v\big(S^\pi(i)\big) \qquad (13)$$

The Shapley contribution of factor $i$ is the average marginal contribution across all permutations:

$$\phi_i = \frac{1}{5!} \sum_{\pi \in \Pi(N)} \Delta_i^{\pi} \qquad (14)$$

Since $5! = 120$,

$$\phi_i = \frac{1}{120} \sum_{\pi \in \Pi(N)} \Delta_i^{\pi} \qquad (15)$$

This calculation is performed for each factor $i \in \{E,\ L,\ F_{EEE},\ F_{ROS},\ F_{OTHER}\}$ (16).

## 3.5 Final Decomposition

The total emission change is decomposed as

$$\Delta M = \phi_E + \phi_L + \phi_{EEE} + \phi_{ROS} + \phi_{OTHER} \qquad (17)$$

Substituting the definition of total change yields

$$E^1 L^1 F^1 - E^0 L^0 F^0 = \phi_E + \phi_L + \phi_{EEE} + \phi_{ROS} + \phi_{OTHER} \qquad (18)$$

## 3.6 Additivity Property

A key property of the Shapley decomposition is perfect additivity:

$$\sum_{i \in N} \phi_i = \Delta M \qquad (19)$$

Therefore,

$$\phi_E + \phi_L + \phi_{EEE} + \phi_{ROS} + \phi_{OTHER} = E^1 L^1 F^1 - E^0 L^0 F^0 \qquad (20)$$

This property guarantees that the decomposition exactly reproduces the observed change in emissions, leaving no unexplained residual apart from numerical rounding errors.

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