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Channelized Reservoir Estimation Using A Low Dimensional Parameterization Based On High Order Singular Value Decomposition

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Summary

Prior to any estimation process of channelized reservoirs, in the context of an Assisted History Matching method, the parameterization of facies fields is a necessary task. The parameterization of channelized reservoirs consists of defining a numerical field (parameter field) so that a projection function recovers the facies field from the parameter field. Mostly, the dimension of parameter field is equal to the dimension of reservoir domain. The issue of dimensionality is becoming relevant when the history matching method is applied, especially due to the tremendous number of parameters involved in the estimation process of the channelized reservoirs. In addition, one of the most important issue encountered is the loss of the multi-point geostatistical properties in the updates (channel continuity). In this study, we start from an initial parameterization of the channelized fields and infer from it a low-dimensional parameterization obtained after a high order singular value decomposition of a tensor built with the parameter fields. We show how the facies fields are fully characterized by a linear combination of a small number of coefficients with "basis functions". The decomposition is followed by a truncation so that we keep the relevant information from the channel continuity perspective. This new parameterization is further introduced in the estimation process of facies fields, using the ensemble smoother with multiple data assimilation (ES-MDA), updating the coefficients of decomposition. For a fair assessment of the parameterization, we perform a comparison of the results with those obtained by applying the traditional singular value decomposition and the original parameterization. The comparison is done from the perspective of multipoint geostatistical characteristics of the updates and predictions (oil and water rates). We show that the new parameterization is able to better keep the multipoint geostatistical structure in the updates than the other two parameterizations, while the prediction capabilities are the same.

Introduction

One of the most challenging problems in the inverse modeling of subsurface flow is the estimation and uncertainty quantification of the facies distribution of channelized reservoirs. Even though the number of the facies types involved in the characterization of geology could be greater than two, the reservoirs with only two facies types (a channel and a non-channel) have been studied extensively over the past years. This type of geology is challenging because in the Assisted History Matching (AHM) process some of the geometrical and topological characteristics of the channels can be lost. At the end of the assimilation process, it raises the question whether the updated geological models have the same characteristics with the prior models(i.e. the geological plausibility is preserved or not). The first step in the AHM process is the prior characterization of the channelized reservoirs. Typically the channelized reservoirs are conceptually drawn by the geologists, but one needs a specialized software to simulate possible realization of the facies distributions. The software incorporates a so-called geological simulation model, a geostatistical based model that enables generation of a possible realization of the subsurface geology. Since now, two of the geological simulation models have proven their usefulness in the generation of channelized reservoirs: object-based simulation model (Deutsch and Wang 1996) and multi-point geostatistical simulation model (MPS, Caers and Zhang 2004). Once the geological simulation model is set, it is coupled with an AHM method in order to estimate the position of channels in the reservoir domain and reduce its uncertainty. The link between the geological simulation model and the AHM method is done with the aid of the parameterization of the reservoir properties. Since now two major techniques have been proposed. The first technique is the use of the permeability field (or a transformation of it) as the parameterization of facies fields and the second is to parameterize the facies fields with a numerical field different from the permeability. In both cases, the facies field is inferred from the values of parameter field. However, if the permeability is used as parameterization one needs to be very careful with the function used to project the numerical values of the permeability to discrete values of the facies field. For the second technique, the function is implicitly defined by the parameterization and consequently, from this perspective is much suitable for the estimation of facies fields.
Over the years have been proposed many methods for improving the updated permeability fields. Jafarpour and McLaughlin (2008) applies the discrete cosine transform (DCT) to the permeability field, defining a decomposition of the permeability field as a linear combination of some basis functions. The coefficients of decomposition are updated with the Ensemble Kalman Filter (EnKF, Evensen 2003) as the AHM method. Zhao et al. (2016) extends the DCT parameterization and the basis functions are customized chosen for a better estimation of the permeability field. In addition, the authors propose a post-processing step in order to recover the facies field from the updated permeability field. Other approaches involving the parameterization of the permeability field involve the use of the wavelet transform (Jafarpour (2011), Zhang et al. (2015)) or complex methodologies imported from machine learning (Tahmasebi et al. 2018, Golmohammadia et al. 2018). One of the most used methodology for the transformation of the permeability field as a linear combination of basis function is the principal component analysis (PCA) and its advanced form kernel principal component analysis (K-PCA). This methodology has roots in machine learning and has the advantage of extracting relevant information from the permeability field with only a few coefficients. In Sarma et al. (2008) and Sarma et al. (2009) are developed a methodology that links the K-PCA with EnKF for the estimation of permeability field of two facies model. The PCA decomposition in the kernel space was able to better capture the multi-point geostatistical properties of the permeability field. The back-transform from the kernel space into the original space is done numerically by solving an optimization procedure. Emerick (2017) presents a comparison between PCA and K-PCA for estimation and uncertainty quantification of the permeability field of channelized reservoirs using the ensemble smoother with multiple data assimilation (ES-MDA,Emerick and Reynolds 2013) as the AHM method. Tene (2013) uses the K-PCA coupled with EnKF, and he present an analytical solution for the back-transform from the kernel space using a modified characterization of the kernel space. Vo and Durlofsky (2014) propose an optimization-based PCA (O-PCA) methodology by which the PCA is viewed as an optimization problem with a regularized term that pushes the solution close to a bimodal one. The idea of the regularized term used in O-PCA is further developed in Vo and Durlofsky (2016) where the authors present a methodology to explicitly solve the pre-image problem (the back transform from the kernel feature space into the original space). Similar to the PCA parameterization is the parameterization with the coefficients provided by the singular value decomposition

(SVD). Firstly introduced in the history matching community by Tavakoli and Reynolds (2011) it was further developed by Khaninezhad et al. (2012) towards a novel methodology named K-SVD by which sparse geologic dictionary is learned from a library of prior models. The parameterization based on K-SVD is used in the inverse modeling of the facies fields in Khaninezhad et al. (2018) employing a regularization term to enforce the solution to be discrete.

Typically, the SVD decomposition applies to a matrix constructed with a linearization (flattening) of the permeability fields of the reservoir. However, the reservoir domain is multi-dimensional, so, with an ensemble of such prior models, one could easily build a multi-linear object called tensor. Afra and Gildin (2016) proposed a parameterization of the permeability field of two facies reservoir with the aid of the high order singular value decomposition (HOSVD, De Lathauwer et al. 2000) of the tensor built with the ensemble of permeability fields. The coefficients of the tensor decomposition were introduced in an AHM process and estimated. The same approach was used by Insuasty et al. (2017) combining the truncated HOSVD with the EnKF for estimation and uncertainty quantification of the permeability field of a channelized reservoir.

In this study, we are not using the HOSVD decomposition of the tensor defined by the ensemble of permeability fields. First, we are using the parameterization of the facies fields (different from permeability) introduced in Sebacher et al. (2015) and Sebacher et al. (2016) and second, we apply the HOSVD decomposition of the tensor defined by the ensemble of parameter fields. In this way, each parameter field is written as a linear combination of some basis functions. We truncate the decomposition (T-HOSVD), retaining the most important coefficients that are further updated with the ensemble smoother with multiple data assimilation. In this way, we define a low-dimensional parameterization of the facies fields of a channelized reservoir. We perform a comparison between this novel parameterization and the parameterization introduced by the truncated SVD (T-SVD) and the global parameterization. We show that the T-HOSVD parameterization is able to better keep the high-order statistics (multi-point) in the updates than the other two, while the data match and predictive capabilities of the updated ensembles are the same.

Tensor decomposition (HOSVD) and approximation

The tensors could be seen as the natural extension of the vector and matrices. If $A = (a_{ij})_{i,j} \in M_{m,n}(\mathbf{R})$ is a matrix with m rows and n columns having real entries, then it defines a bi-linear application (mapping)

$$\phi : \mathbf{R}^m \times \mathbf{R}^n \longrightarrow \mathbf{R}, \phi(x,y) = \sum_{i=1}^{m} \sum_{i=1}^{n} a_{ij} x_i y_j,$$

where $x = (x_1, x_2, \ldots, x_m) \in \mathbf{R}^m$ and $y = (y_1, y_2, \ldots, y_m) \in \mathbf{R}^n$. If we calculate the values of application ϕ on the elements of canonical bases $\{e_1^{(1)}, e_2^{(1)}, \ldots, e_m^{(1)}\} \subset \mathbf{R}^m$ and $\{e_1^{(2)}, e_2^{(2)}, \ldots, e_n^{(2)}\} \subset \mathbf{R}^n$ we obtain $\phi(e_i^{(1)}, e_j^{(2)}) = a_{ij}$. With this in mind, we extend this approach to multi-linear applications. We define a $(N_1 \times N_2 \times \ldots \times N_k)$ -order tensor T as a multi-dimensional array (structure) that induces a multi-linear application $\phi : \mathbf{R}^{N_1} \times \mathbf{R}^{N_2} \times \ldots \times \mathbf{R}^{N_k} \longrightarrow \mathbf{R}$. If we consider the canonical basis $\{e_1^{(i)}, e_2^{(i)}, \ldots, e_{N_i}^{(i)}\} \subset \mathbf{R}^{N_i}$ in each of the linear space $\mathbf{R}^{N_i}$, then the element $\phi(e_{i_1}^{(1)}, e_{i_2}^{(2)}, \ldots, e_{i_k}^{(k)}) = T_{i_1 i_2 \ldots i_k}$ is the entry of tensor T at the position $(i_1, i_2, \ldots, i_k)$. In addition, we have the relation

$$\phi(x^{(1)}, x^{(2)}, \ldots, x^{(k)}) = \sum_{i_1=1}^{N_1} \sum_{i_2=1}^{N_2} \cdots \sum_{i_k=1}^{N_k} T_{i_1 i_2 \ldots i_k} x_{i_1}^{(1)} x_{i_2}^{(2)} \ldots x_{i_k}^{(k)}$$

With this formulation, the arrays are (n)-order tensors and the matrices $(m \times n)$-order tensors. Similar to matrix factorization, in the mathematical literature, exits various tensor factorizations (decomposition), from which, here we use the high order singular value decomposition (HOSVD). The HOSVD is a particular case of a broader tensor factorization called Tucker decomposition (Bergqvist and Larsson 2010). The Tucker factorization of a $(N_1 \times N_2, \times \ldots \times N_k)$ -order tensor T consists of the decomposition of T as

$$T = \sum_{i_1=1}^{n_1} \sum_{i_2=1}^{n_2} \cdots \sum_{i_k=1}^{n_k} \sigma_{i_1 i_2 \ldots i_k} u_{i_1}^{(1)} \otimes u_{i_2}^{(2)} \otimes \ldots \otimes u_{i_k}^{(k)}, \tag{1}$$

where $\otimes$ is the outer (tensor) product of the arrays (i.e. $x \otimes y = xy^T$). The $(N_1 \times N_2 \times \ldots \times N_k)$ -order tensor σ is called core tensor and the sets $\{u^{(1)}_{i_1}, u^{(2)}_{i_2}, \ldots, u^{(r)}_{i_r}\} \subset \mathbf{R}^{N_r}, r \in \overline{1,k}$ are bases of the linear spaces $\{\mathbf{R}^{N_r}, r \in \overline{1,k}\}$. In addition, if all the bases are orthonormal (with respect with the canonical inner product in $\mathbf{R}^{N_r}, r \in \overline{1,k}$) and any two different "slices" of the core tensor σ (taken along the same mode) are orthogonal we say that Eq.1 is the HOSVD decomposition of tensor T. The core tensor is defined so that the Frobenius norms of "the slices" (taken along the same mode) decreases. This last property is a generalization of the SVD decomposition of matrices where the rows and columns of the singular matrix are mutually orthogonal and the singular values decreases on the diagonal. Just like in SVD decomposition, the summations from eq.1 is done for indices $i_r \in \{1, \ldots, n_r\}$, with $n_r \leq N_r$ which means that the $(n_1 \times n_2 \times \ldots \times n_k)$ -order tensor σ could be seen as a compression of the $(N_1 \times N_2 \times \ldots \times N_k)$ -order tensor T. In addition, because the core tensor σ has the "slices" in decreasing order with respect to Frobenius norm, one could truncate even more Eq.1 obtaining a higher compression (or low-rank approximation) of the tensor T. Consequently, taking the indices $i_r \in \overline{1, \overline{n_r}}, r \in \overline{1,k}$ with $\overline{n_r} < n_r$ for each $r \in \overline{1,k}$ we obtain a so called approximation of the tensor T with the tensor

$$\overline{T} = \sum_{i_1=1}^{\overline{n_1}} \sum_{i_2=1}^{\overline{n_2}} \ldots \sum_{i_k=1}^{\overline{n_k}} \sigma_{i_1 i_2 \ldots i_k} u^{(1)}_{i_1} \otimes u^{(2)}_{i_2} \otimes \ldots \otimes u^{(k)}_{i_k}, \tag{2}$$

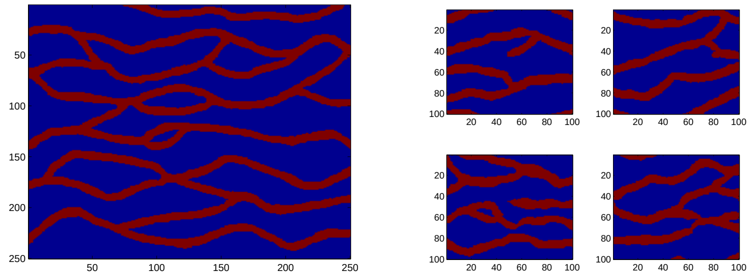
Knowing that the bases $u^{(1)}_{i_1}, u^{(2)}_{i_2}, \ldots, u^{(r)}_{i_r} \in \mathbf{R}^{N_r}, r \in \overline{1,k}\}$ are orthonormal, the Frobenius norm of tensor T, $\| T \|_F = \sqrt{< T, T >}$ becomes

$$\| T \|_F = \sqrt{\sum_{i_1=1}^{N_1} \sum_{i_2=1}^{N_2} \ldots \sum_{i_k=1}^{N_k} \sigma^2_{i_1 i_2 \ldots i_k}}.$$

The level of approximation of the tensor could be quantified by the ratio $\frac{\|T - \overline{T}\|_F}{\|T\|_F}$, but in this paper we define a different criterion for truncation of the sum from Eq.1.

Low dimensional parameterizations

In this section we present the parameterization of channelized reservoirs introduced in Sebacher et al. 2015 followed by the definition of the tensor and its HOSVD decomposition and approximation. Using the tensor approximation we define a low-dimensional parameterization of the facies fields. We continue with the introduction of the truncated SVD parameterization of the facies fields.
We present the methodology for a channelized reservoir with a rectangular domain of 100 grid cells in each direction. The geological simulation model used for the generation of the prior models is a multi-point geostatistical one, called the Single Normal Equation Simulation (SNESIM, Strebelle 2002). We use its implementation from S-GeMS software (Remy 2005).



(a) Training Image (b) Members

Figure 1 *The training image from Strebelle (2002) (a) and the first 4 members (b).*

The MPS models use a training image (TI, Fig.1, (a)) from which are simulated facies fields with similar multi-point geostatistical characteristics as the training image (Fig.1,(b)). We start by generating of an ensemble of N facies fields from the training image. In machine learning this ensemble is called the training set and is used to create the library of basis functions. Here, we want to estimate a facies fields with an ensemble based method and we use a single ensemble for the training set and for the AHM method. Each ensemble member is an image with two values, 0 for the background (non-channel facies type) and 1 for the channel. The target is to define a numerical field on the reservoir domain and a function (rule) so that, we reconstruct the facies field by applying the function to the numerical field. For each grid cell j of the reservoir domain we calculate, from the ensemble, the probability of occurrence of the channel at that location and we denote it with $p_j = \sum_{i=1}^{N} Ind_i(j)$, where $Ind_i(j) = 1$ if at location j in ensemble member i is a channel and $Ind_i(j) = 0$ otherwise. Thus, we define a discrete variable denoted $facies_j$ with the distribution

$$facies^j \sim \begin{pmatrix} Channel & Non-channel \\ p^j & 1-p^j \end{pmatrix} \tag{3}$$

We link this random variable with standard normal variables with the normal score transform and define a threshold $\alpha^j \in \mathbf{R}$ so that for any random variable $X \sim N(0;1)$ we have the conditions $P(X \leq \alpha^j) = p^j$ and $P(X > \alpha^j) = 1 - p^j$. For each ensemble member $i \in \{1, \ldots, N\}$ we define a parameter field denoted

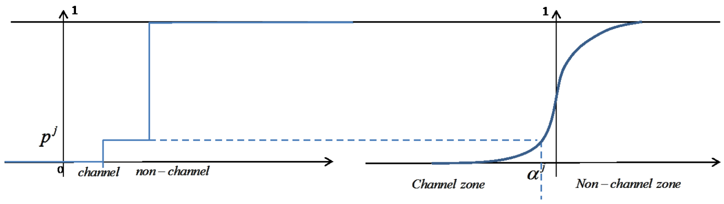


***Figure 2** The normal score transform.*

θ_i that has the value $\theta_i(j)$ in the grid cell j

$$\theta_i(j) = \begin{cases} \mathbf{E}(X|X \leq \alpha^j) & if \quad j \in channel \\ \mathbf{E}(X|X > \alpha^j) & if \quad j \in nonchannel \end{cases} \tag{4}$$

The *truncation rule* that reconstructs the facies field from the values of parameter field consists of comparing the values of θ with the thresholds α; for each ensemble member i if $\theta_i(j) \leq \alpha^j$ then we assign a channel at location j and if $\theta_i(j) > \alpha^j$ we assign non-channel.
In this way we end up with N parameter fields defined on the reservoir domain that parameterize the facies fields.

Low dimensional truncated HOSVD (T-HOSVD) parameterization

With the ensemble of parameter fields $(\theta_i)_{i\in\overline{1,N}}$ we built a $(100 \times 100 \times N)$ order tensor T, by stacking the parameter fields. Thus, the slice (layer) $k \in \{1, \ldots, N\}$ of the third mode of tensor T is defined as $\theta_k = T(:,:,k)$ and is the parameter field of the k^{th} ensemble member. Applying the HOSVD decomposition to tensor T we obtain

$$T = \sum_{i=1}^{100} \sum_{j=1}^{100} \sum_{k=1}^{N} \sigma_{ijk} u_i^{(1)} \otimes u_j^{(2)} \otimes u_k^{(3)}, \tag{5}$$

where $(\sigma_{ijk})_{i,j,k}$ is the core tensor and all the sets $(u_i^{(1)})_{i=\overline{1,100}}$, $(u_j^{(2)})_{i=\overline{1,100}}$, $(u_k^{(3)})_{i=\overline{1,N}}$ are orthonomal basis. We truncate Eq.5 keeping only the first n_x and n_y elements from the first two sets, obtaining a $(n_x \times n_y \times N)$-order tensor

$$\overline{T} = \sum_{i=1}^{n_x} \sum_{j=1}^{n_y} \sum_{k=1}^{N} \sigma_{ijk} u_i^{(1)} \otimes u_j^{(2)} \otimes u_k^{(3)}. \tag{6}$$

In the linear space $\mathbf{R}^N$ we consider the canonical basis $B_3 = \{e_3^{(1)}, e_3^{(2)}, \ldots, e_3^{(N)}\}$ and applying to each vector of the basis B_3 the multi-linear application defined by the tensor $\overline{T}$ we can define the $(n_x \times n_y)$-

order tensor (a matrix)

$$\overline{\theta_r} = \overline{T}(:,:,e_3^{(r)}) = \sum_{i=1}^{n_x}\sum_{j=1}^{n_y}\sum_{k=1}^{N}\sigma_{ijk}u_i^{(1)}\otimes u_j^{(2)} < u_k^{(3)}, e_r^{(3)} >= \sum_{i=1}^{n_x}\sum_{j=1}^{n_y}(\sum_{k=1}^{N}\sigma_{ijk} < u_k^{(3)}, e_r^{(3)} >)u_i^{(1)}\otimes u_j^{(2)}.$$

By denoting $\alpha_{ij}^r = \sum_{k=1}^N \sigma_{ijk} < u_k^{(3)}, e_r^{(3)} >$, we obtain the decomposition

$$\overline{\theta_r} = \sum_{i=1}^{n_x}\sum_{j=1}^{n_y}\alpha_{ij}^r u_i^{(1)}\otimes u_j^{(2)}. \tag{7}$$

Eq.7 could be seen as the writing of the $(n_x \times n_y)$-order tensor $\overline{\theta_r}$ with respect to the $(n_x \times n_y)$-order tensors $(u_i^{(1)}\otimes u_j^{(2)})_{i,j}$. We then consider the set $(u_i^{(1)}\otimes u_j^{(2)})_{i,j}$ as the basis functions and the coefficients $(\alpha_{ij}^r)_{i,j}$ the low-dimensional parameterization of the parameter field θ_r (denoted T-HOSVD).

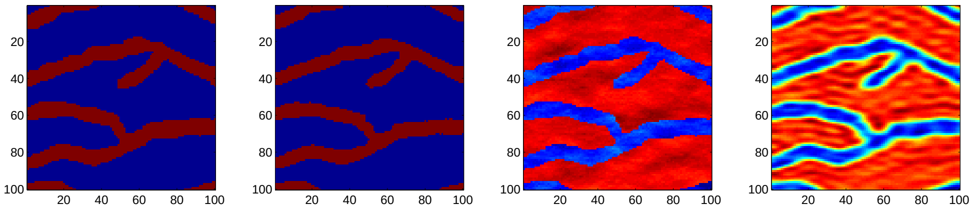


Figure 3 *From left to right: original field, field after truncation of tensor approximation, original parameter field, parameter field after tensor approximation.*

The question that remains is how to truncate the expansion from Eq.5. The channelized reservoirs were sampled from the training image and consequently,and due to the stochastic nature of the MPS algorithm, have about the same multi-point geostatistical properties with the training image. By truncating the expansion from Eq.5 and apply the truncation rule to each field θ_r we obtain facies fields that are different than the original ones. Higher values of the numbers n_x and $\overline{n_y}$ from Eq.6 render more accurate approximation of the prior parameter fields. As we mentioned before, a prior facies field approximately respects the multi-point geostatistics of TI so, a field that little differs from it could be a candidate. We then could choose n_x and n_y so that the difference of values (in grid cells or pixels) between the original facies fields and the approximated facies fields is in mean less than 3%.
In Fig.3 we present an example with $N = 120$, $n_x = 30$, $n_y = 15$. Here is shown an example for the first member. We have from left to right: the original facies field, the facies field obtained after the truncation of parameter field θ_1, the parameter field θ_1 and the parameter field $\overline{\theta_1}$. The facies fields obtained after the truncation of $\overline{\theta_1}$ have about 97% percent of the values of the original facies fields and could be considered that have similar geostatistical properties as the training image. The field θ_1 is defined by 10000 values while after the tensor truncation the new field $\overline{\theta_1}$ depends only on $n_x \times n_y$ parameters (Eq.7). For the case presented we obtained a reduction from 10000 parameters to $n_x \times n_y = 450$ parameters. A visual representation of equation 7 is presented in Fig.4 where $\overline{\theta_1}$ is approximated with a linear combination of the basis functions $(u_i^{(1)}\otimes u_j^{(2)})_{i,j}$.

Low dimensional truncated SVD (T-SVD) parameterization

The SVD decomposition is applied to a matrix build with the liniarization (flattening) of the bi-dimensional fields θ_i, $i \in \overline{1,N}$. We define a matrix denoted A that has the column i equal to the flattening of θ_i (denoted here by θ^i). Then, the matrix A has dimension $10000 \times N$. We apply the SVD decomposition to matrix A obtaining the orthonormal sets of vectors $\{U_k\}_{k=\overline{1,N}} \subset \mathbf{R}^{10000}$, $\{V_k\}_{k=\overline{1,N}} \subset \mathbf{R}^N$ and the decreasing non-negative singular values $\{\sigma_k\}_{k=\overline{1,N}} \subset \mathbf{R}$ so that

$$A = \sum_{k=1}^{N}\sigma_k U_k \otimes V_k = USV^T. \tag{8}$$

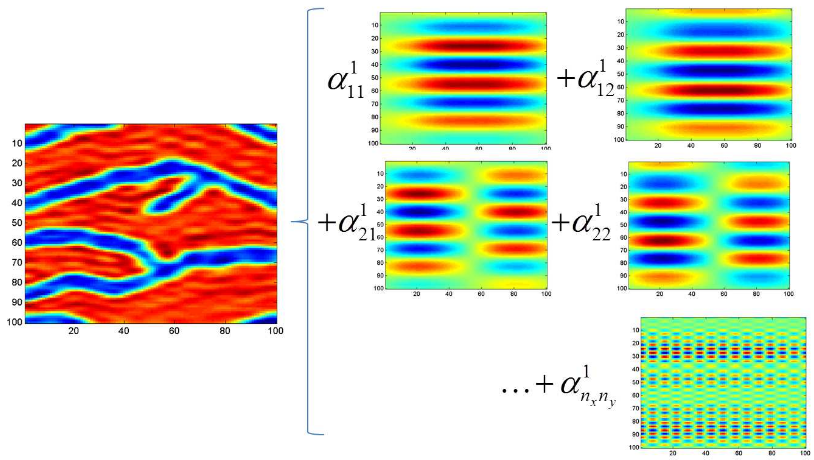


Figure 4 *Parameter field expansion with HOSVD*

In Eq. 8, U is the $10000\times N$ matrix built with the columns $\{U_k\}_{k=\overline{1,N}}$, V is the $N\times N$ matrix built with the columns $\{V_k\}_{k=\overline{1,N}}$ and S is a $N\times N$ diagonal matrix with entries $\{\sigma_k\}_{k=\overline{1,N}}$. If we denote by $\beta = SV^T$ then the SVD decomposition of the matrix A becomes $A = U\beta$. We write this equation for each column of matrix A, so for each $i\in\overline{1,N}$

$$\theta^i = A^i = \sum_{j=1}^{N} \beta^i_j U^i, \tag{9}$$

where β^i_j is the entry from row j and column i of matrix β. We truncate Eq.9 retaining only the first $m<N$ members defining new columns $\widetilde{\theta}$ as

$$\widetilde{\theta}^i = \sum_{j=1}^{m} \beta^i_j U^j, \tag{10}$$

All the columns $\widetilde{\theta}^i$ have dimension 10000 and we can reshape them to 100×100 matrices (denoted here $\widetilde{\theta}_i$) that approximate θ_i. If we apply the truncation rule to $\widetilde{\theta}_i$ we obtain a facies field (binary) that approximate the prior one. The difference between them depends on the value of m from Eq.10. We choose m with the same condition as for the truncated HOSVD parameterization i.e. the difference between facies fields (original and transformed after truncation) in average to be less than 3%. We then consider the set $(U^i)_{i=\overline{1,m}}$ as the basis functions and the coefficients (β^j_i) the low-dimensional truncated SVD (T-SVD) parameterization of the parameter field θ_i.

In Fig.5 we present an example taking $N = 120$, $m = 70$, values that fulfilled the condition that the

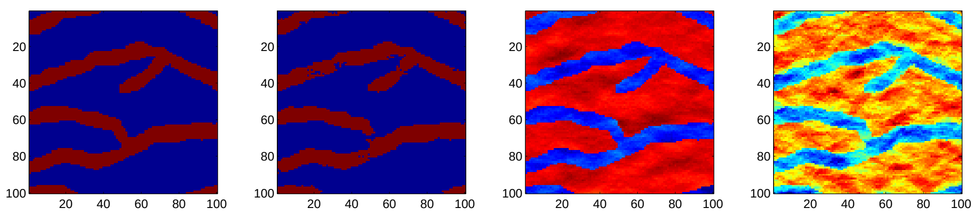


Figure 5 *From left to right: original field, field after truncation of tensor approximation, original parameter field, parameter field after tensor approximation*

transformed facies fields have in average 97% of the grid cells similar with the original facies fields. In this figure are show from left to right: original facies field, facies field after truncation of SVD decomposition, original parameter field, parameter field after SVD decomposition. Comparing with a visual inspection Fig.3 and Fig.5 One can see that the truncated SVD parameterization takes out pixels from inside of the channels while the truncated HOSVD keeps better the channel geometry, taking out pixels mostly from the borders. For this particular case, the difference of pixels compared with the original is for HOSVD 3% (i.e. 300 pixels) and for SVD 1% (i.e. 100 pixels). In Fig.6 is shown the

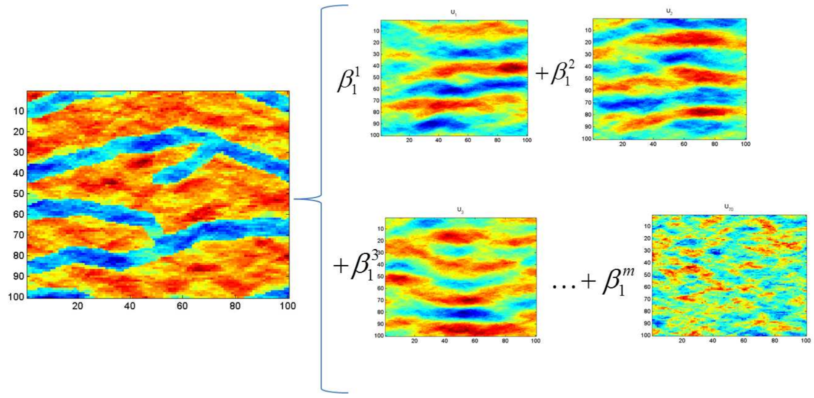


Figure 6 *Parameter field expansion with SVD*

linear decomposition $\widetilde{\theta}_i$ with respect to the basis function $(U^i)_{i=\overline{1,m}}$.

Ensemble smoother with multiple data assimilation (ES-MDA)

The ensemble smoother with multiple data assimilation (ES-MDA, Emerick and Reynolds 2013) is an ensemble based data assimilation method designed to improve the results obtained with a standard smoother. Being a smoother, it assimilates all the observations at the same time. The difference with a standard smoother consists of the fact that the available data are assimilated multiple times (not a single time as in the standard procedure). We call each assimilation cycle an iteration. However, to preserve a mathematical consistency, the error covariance matrix of all measurements C_D is inflated (multiplied) at each iteration l ($l \in \overline{1,N_a}$) with factors α_l taken with the condition $\sum_{l=1}^{N_a} \frac{1}{\alpha_l} = 1$ (N_a is the number of times the data are assimilated). In this study, the ES-MDA is used for parameter estimation purpose and we denote by m_i the model parameters involved in estimation and by $\mathscr{G}$, the function that maps the parameters to simulated observations. The measurements (observations) used in this study are the production data (i.e. the bottom hole pressures taken at the injection wells, the oil and water rates taken at the production wells). Then, the state vector X for the i ensemble member is defined as:

$$X_i = [m_i^T \quad \mathscr{G}(m_i)]^T, i = 1,\ldots,N, \tag{11}$$

where N is the number of ensemble members. Based on this augmentation, we construct a binary matrix H that linearly maps the state vector X_i on the observation space $HX_i = \mathscr{G}(m_i)$. At each iteration $l \in \overline{1,N_a}$, the forecast step does not modifies the values of the parameters, but calculates the simulated measurements (Eq.11) based on the values of parameters from previous iteration (or prior at the first iteration). The values of parameters modify in the update step when all the observations are assimilated. The equation of the update step is

$$X_i^{l,a} = X_i^{l,f} + C_{X^{l,f}} H^T (H C_{X^{l,f}} H^T + \alpha_l C_D)^{-1} (d_{obs,i} - H X_i^{l,f}), \tag{12}$$

where $X^{l,a}$ is the updated (analyzed) state vector, $C_{X^{l,f}}$ is the covariance matrix of the forecasted state vector calculated from the ensemble and $d_{obs,i} = d_{obs} + \varepsilon_i$ are the perturbed observations for the ensemble member i at the l-iteration (d_{obs} are the available observations and ε_i is a random sampling from a Gaussian distribution with 0 mean and covariance matrix $\alpha_l C_D$).
We perform and compare three experiments. The first experiment consists of the AHM method applied to the low-dimensional parameterization introduced by the truncated HOSVD (T-HOSVD). In the second experiment we estimate the parameters that come out from the truncated SVD (T-SVD) parameterization and, in the last experiment, we estimate all 10000 parameters of the global parameterization introduced

in Sebacher et al. (2015) (entire field θ). In the first experiment, the model parameters are α^r_{ij}, $i \in \overline{1,n_x}$, $j \in \overline{1,n_y}$, $r \in \overline{1,N}$ from Eq.6, in the second experiment we introduce in the state vector the parameters β^r_i, $i \in \overline{1,m}$, $r \in \overline{1,N}$ from Eq.10 and in the last experiment we estimate the entire field θ^r, $r \in \overline{1,N}$ (Eq.4). Then, the first experiment has 450 parameters, the second 70 parameters and the third 10000 parameters.

Case study

The reservoir model used for testing has a square shape with 10000 grid cells having dimension of each grid cell $30 \times 30 \times 20$ ft. We design the reservoir as a 13-spot water flooding black oil model, having four injection wells and nine production wells (Fig.7). The reservoir is initially filled with oil at a constant uniform saturation of 0.8 (the connate water saturation is 0.2) and with a uniform pressure of 5000 psi in every grid cell. The producers work under constant bottom hole pressure (BHP) with a value of 3000 psi and the injectors operate at 3500 STB/D constrained by a maximum BHP of 100000 psi. The measurements were obtained through forward simulation of a synthetic model presented as the "reference" which was randomly sampled from the same training image using SNESIM (Fig.7). The

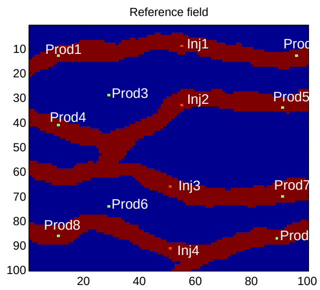


Figure 7 *The reference field.*

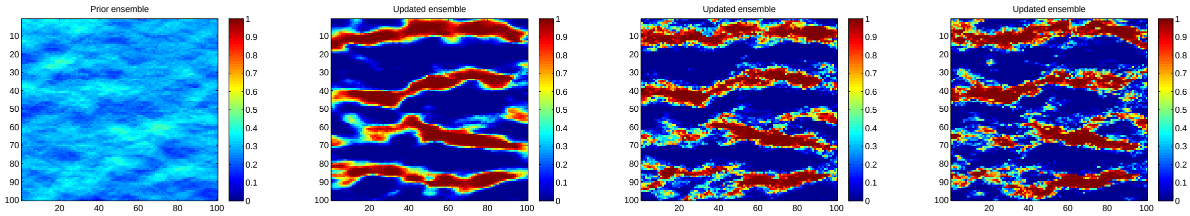
measurement errors of the production data are considered having Gaussian distribution with 0 mean and standard deviations of 70 STB/D for water rates (WR) and oil rates (OR) at the producers, and 200 psi for BHP at the injectors. We use these values for generation of noisy observations from the reference model. In addition, the distribution is used to perturb the observations of production data in the analysis step of the HM process. Water injection starts from the first day and continues thereafter for a period of 351 days of production. We assimilate data at 60-day intervals resulting in a total of 6 assimilation steps. The permeability values were set at 9 mD and 1 mD for the channel facies type and for the non-channel facies type, respectively, while the porosity of both facies types is set to 0.2 and considered as known. During the HM process the permeability and porosity are kept constant, although they could be considered uncertain within each facies and estimated together with facies positions (Hanea et al. 2015). The ensemble size is set to 120. We use the ES-MDA method with four data assimilations (iterations) with decreasing inflation factors of $(9.333 : 7 : 4 : 2)$.
We check if this new low-dimensional parameterization performs better than the other two parameteri-zations from the following perspectives:

- At the end of the assimilation process is able to provide an ensemble of facies fields with better geostatistical properties.
- Estimates better the channel positions.
- The updated ensemble has better data match and predictions.

Results

Figure 8 presents the probability fields of the channel, figures that should be compared with the truth (Fig.7). The first picture represents the probability field of the channel calculated from the prior ensemble and, from it, can be seen that we have started with a high prior uncertainty of the channel positions.

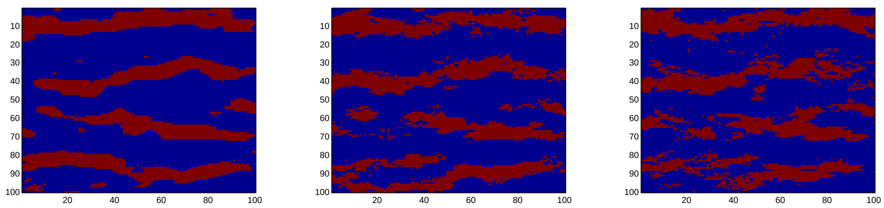
The next pictures depict the probability field of the channel calculated from the updated ensemble ((b) from T-HOSVD, (c) from T-SVD and (d) from estimation of the entire field). The assessment of the



(a) Prior (b) T-HOSVD (c) T-SVD (d) Field

Figure 8 *The probability fields.*

estimation of channel positions in all experiment is done by comparison the updated probability fields (Fig.8) with "the truth" (Fig.7). Here, by a visual inspection of the Figure 8 one can conclude that the continuity of the channel position in the T-HOSVD parameterization is clearly better than the other two. In addition, the channel position seems to be better estimated with T-HOSVD parameterization than with the other two parameterizations.
One of the most important query of the updated ensemble of facies fields is the existence of the geological plausibility (realism) of the updated facies fields. The geological realism means that the posterior ensemble of facies fields and prior ensemble should have similar multi-point geostatistical properties (i.e. similar with the training image). This is a very hard task for ensemble-based methods and has been fulfilled only in few particular cases (e.g. small reservoir cases with non complicated geometry and many wells). Our reservoir model has 10000 grid cells, four channels of which two are intersecting and only 13 wells. In Fig.9 is shown the first ensemble member of facies fields in updated ensemble ((a) from T-HOSVD, (b) from T-SVD and (c) from estimation of the entire field). We are not claiming that the updated ensemble of facies fields have exactly the same multi-point geostatistical characteristics with the prior, but is clearly from Fig.9 that the low-dimensional parameterization with T-HOSVD keeps much better multi-point characteristics of the prior in the updates than the other two. From the picture one can see a good continuity of the channels and only few regions with small channel areas. The first

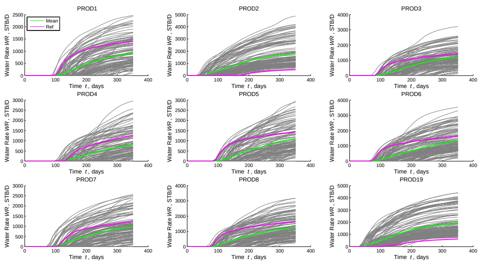


(a) T-HOSVD (b) T-SVD (c) Field

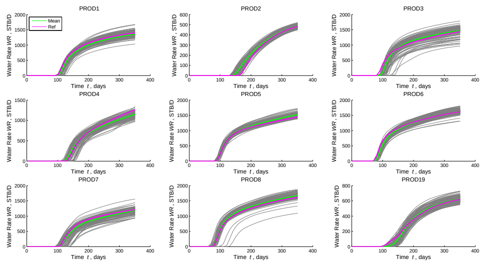
Figure 9 *The first member in updated ensemble.*

$n_x \times n_y$ elements of basis $(u_{i_1}^{(1)} \times u_{i_2}^{(2)})_{i,j}$ were able to keep important multi-point characteristics from the prior during data assimilation and the linear combination from Eq.7 conserves many more aspects from the prior during history matching than the linear combination of T-SVD. Is clear that the AHM-method applied to the entire field θ destroys parts of the multi-point geostatistics. At this moment, in Sebacher et al. (2015) is proposed a re-sampling step for regaining the continuity of the channel. Unfortunately, this can be done only if a MPS model is used and is not possible for object based simulation models.

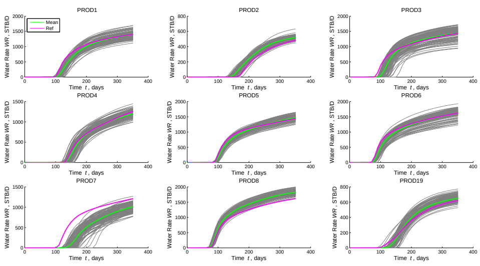
In Fig.10 are shown the water production rates in the initial ensemble (a), in the updated ensemble of the T-HOSVD experiment (b), T-SVD (c) and in the experiment with the estimation of the entire field θ (d). From here one can see a comparable reduction in variability for all cases and, as expected, there is no clear difference between the experiments; all the experiments behave equally well from the data match perspective. Even though is not shown here, the predictive capabilities of the updated models are



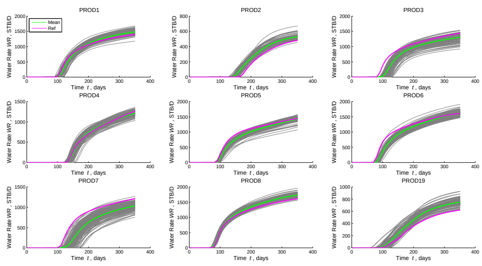
(a) Initial ensemble



(b) T-HOSVD



(c) T-SVD



(d) Field

Figure 10 *Water production rates.*

similar.

Conclusions

In this paper, we have introduced a novel low dimensional parameterization of the channelized reservoirs based on the high order singular decomposition of tensors. The estimation and uncertainty quantification of the channel positions is a complicated task and with the new parametrization, we wanted to ask at two challenges: to obtain updated facies fields realistic as possible and to have a very good data match. The geological realism of the updates means that the updated facies fields have similar multi-point geostatistical characteristics as the prior. We have not reached completely this target, but the updates show a good channelized structure, close to the prior and without the need of re-sampling. The AHM method used was the ES-MDA with four data assimilations (iterations), a method that does not have a resampling step between iterations. We have performed a comparison of this parameterization with other two parameterizations: a global parameterization introduced in a previous study and a low dimensional parameterization based on the singular value decomposition. We have proven that even though the data match is similar between parameterizations, the novel HOSVD parameterization outperforms the other two when speaking on the geological structure of the updates.

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