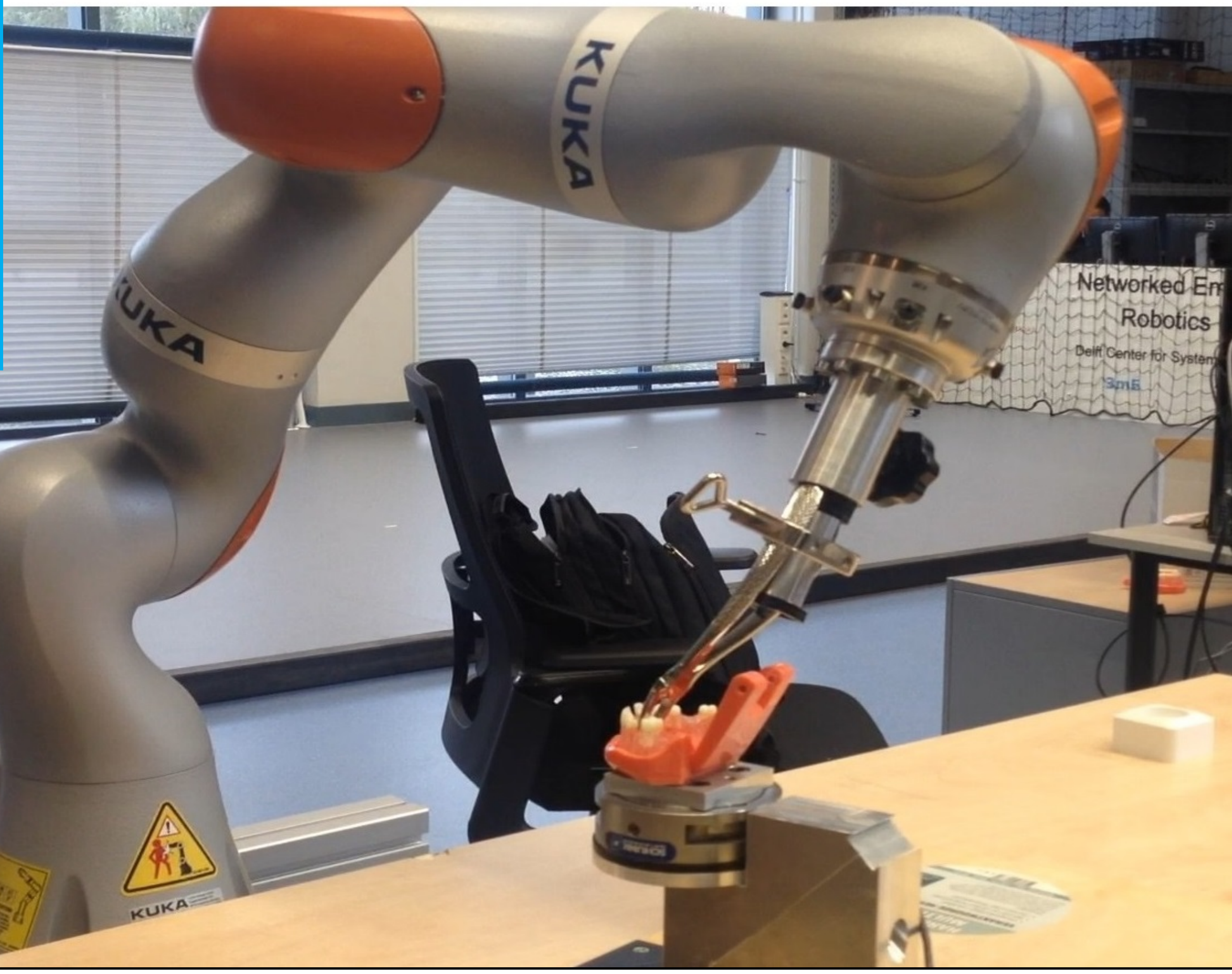
Learning Control Policies for Tooth Extraction

An Application of Programming by Demonstration

Xiang Zhang

Master of Science Thesis



Delft University of Technology

Delft Center for Systems and Control

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Master of Science Thesis

For the degree of Master of Science in Systems and Control at Delft University of Technology

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November 14, 2016

Faculty of Mechanical, Maritime and Materials Engineering (3mE) · Delft University of Technology

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Learning Control Policies for Tooth Extraction

by

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in partial fulfillment of the requirements for the degree of

Master of Science Systems and Control

Dated: November 14, 2016

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Abstract

Tooth extraction is one of the most common procedures in oral surgery. However, it is complicated as well as not fully understood. Different teeth have different structures and strengths. Even teeth of the same type differ depending on ages, genders, races, and other reasons affecting its integrity. During the procedure, the properties of the reaction force from the tooth is changing due to broken fibers and the operations are different between individuals.

The wrench and pose in the human demonstrations are collected by a recently introduced measurement setup. The purpose of the analysis is to obtain a good model for the reaction force and the effective control policy that can extract the tooth without complications such as fractures of the jaw bone.

We introduce a framework called Interactive Analysis to deal with the uncertain information in the demonstration. The framework consists of two parts, the application part and the analysis part. The model from the analysis part is used in the application for a tooth simulation algorithm. This algorithm can be applied on a robotic setup for the education and training.

As for the analysis, we propose the Factor-Controlled Experiment where the uncertain information are translated into the hypotheses and are tested on a robot reproduction system. Since the robot can reproduce the same trajectory over and over again, it can keep the components in the control policies unchanged and test the hypothesis. Both components are implemented by Programming by Demonstration (PbD) which is a machine learning algorithm that encodes the control policy from the human demonstrations.

The simulation is for the scenario where a robot extracts a tooth. Those two components in the framework can render an output that is similar to the demonstrations. The Factor-Controlled Experiment is performed based on the hypothesis that we derived for the plastic jaws by a robot. We found out in the experiment that the vertical force is important for loosening the tooth and settled some uncertainties about the reaction force of the tooth. However, all of the experiments are carried out on plastic jaws. We need to perform the experiment on more representative jaws in the future.

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Acknowledgements

I would like to thank my supervisor dr.ir. J. Kober for his assistance during my research and the writing of this thesis. The discussions that we had and his guidance not only progress my research, but also illustrate me on the critical thinking skills and the methodology for academic researches.

I also would like to thank prof.dr. R. Babuška for steering my thesis to a more realistic and achievable direction, our technician Marteen for the great design of the measurement setup and dr. T. van Riet for his valuable demonstrations and descriptions that grant me a kick-start in the project.

In addition, I would like to thank to all my friends who supported me when I was frustrated. They are always there providing help as much as they could. In the end, I thank my parents and my girlfriend for believing me and the love which encourages me even in the darkest time.

Delft, University of Technology — Xiang Zhang
November 14, 2016

“Man can do what he wills but he cannot will what he wills.”
— *Arthur Schopenhauer*

Chapter 1

Introduction

There are 32 teeth in the mouth of an adult. Most people experience problems with a tooth or the gum surrounding it over their life. Once that happens, they have to pay dentists a visit for some treatments. One of the most common and frightening operations among those treatments is tooth extraction.

Tooth extraction is performed numerous times every day. The estimated number of extracted third molars (wisdom teeth) alone in the United States each year is ten million. This costs 5 million patients over $3 billion dollars [3] as well as dentists a significant amount of time. In Britain, the average number of teeth extracted by a practitioner each week is seven [4].

However, the procedure is not as simple as what it seems but with a lot of potential risks. It may result in a breakage of the tooth or even sever complications such as the fracture of the jaw bone when a practitioner cannot apply force as accurately as planned. Moreover, teeth, when exodontia is indicated, usually are decayed with tissue loss. Some of the teeth have lost the integrity by earlier endodontic treatments and are brittle which increases the possibilities of complications. According to the information from the oral surgeons in our project, a lot of dentists refer a patient to the oral (and maxillofacial) surgeons to get a tooth removed when they expect problems. But even when they only do 'the easy ones' it can still be a difficult task that can, e.g. in case of breakage of the crown, cost a lot of time.

Furthermore, the procedure of tooth extraction is still not fully understood. The knowledge about the force is limited. Even though some force-measurement instruments are introduced [5], they are not sufficient for understanding the entire procedure. The force measured in one dimension, and we need six-dimensional data of wrench to describe what is happening during tooth extraction. Furthermore, tooth extraction has not yet been studied based on data and physics. In addition, there are other forces that may also affect the outcome or the rate of the occurrence of complications. As a result, there are only general descriptive instructions in the dental education and the actual performance of tooth extraction is based on the personal experience during the operations.

In order to understand the procedure of tooth extraction, the essential information and the data of the procedure has to be collected and then processed in an organized manner. In this project, we employ a robot to move with the dentist compliantly and take recordings of movements. It is a direct and accurate method called kinaesthetic demonstration. The wrench applied on the tooth is measured by a force sensor that is connected to the jaw.

We would like to have the model of the correlation between the reaction force of the tooth and the movements of tooth extraction. The model can be used for optimizing the efficiency of the procedure and identify the control policy that is not preferable or even dangerous. Furthermore, by directly studying the control policy in the demonstration, we can find patterns and come up with standard guidelines for practitioners. In the end, the obtained knowledge can be used for assessing and improving the skill of the students or practitioners.

1-1 Interactive Analysis

Analyzing tooth extraction is a challenging task. We have to both take forces and positions into consideration. In contrast to the position, practitioners can only 'feel' the force and the demonstration may not be the same as intended. Since the force is not visible, the skill related to force cannot transfer to students during the education. In addition, the data from the demonstration is the combination of the force applied by the dentist and the reaction force from the tooth. We need to understand the control policy and the reaction force separately, and the data alone does not include all the details of the procedure. For example, some unnecessary force that is applied to the direction along the movement may be combined with the reaction force and change the correlation between the displacement and the reaction force and therefore lead to errors in the identification. As a result, we come up with an analyzing scheme called Interactive Analysis (IA) in order to obtain the reliable model and study the control policy for tooth extraction. The diagram is as in Figure 1-1.

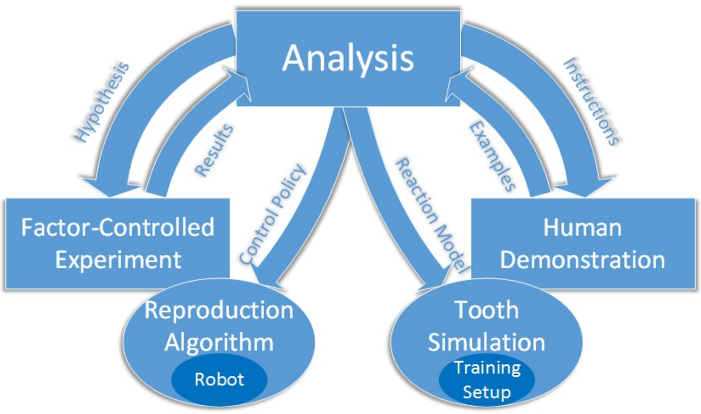

Figure 1-1: The framework of Interactive Analysis. The circles represent algorithms with physical setups. The square is the procedure performed by the human. The right cycle is the application of the result of the analysis, namely training. The left cycle is for analysis using the hypothesis testing.

The squares in the figure indicate the procedures performed by human analysts or demonstrators. The circles represent the algorithms with physical setups for the procedure above it. The right cycle is the application of the result of analysis. A robotic setup can be designed for training practitioners and meanwhile collect data from training. The left cycle is for analyzing the procedure and improving the knowledge with the robot reproduction on the real jaws.

Interactive Analysis is suitable for the procedure that shares characteristics as following which are similar to tooth extraction. Firstly, the model of the interaction force is changing due to structural decay with uncertainties during each demonstration. Secondly, the force that is applied by the human may affect the model that we are interested in. Thirdly, the technique or control policy that demonstrators apply is not consistent in different demonstrations. Therefore we need to look into them and choose the best one. And above all, there are not unlimited resources for demonstrations and reproductions of the procedure. The uncertain information which cannot be discovered by the random search.

Analysis and its application During analysis, the data from demonstrations, descriptions from practitioners and the prior knowledge are processed and correlated. By means of the analysis algorithm and human inspection on the data, we can obtain the outcome of the analysis which is the model of the reaction force from the tooth and the control policy of the human.

As illustrated in the right cycle of Figure 1-1, on the one hand, the derived control policies are used as instructions or guidelines to the practitioner. On the other hand, the model of tooth reaction is described by the tooth simulation algorithm which renders a realistic simulation system on a robotic setup for training and assessing the practitioners. In return, the collected data from the training setup, no matter if the performance is good or bad, brings us more demonstration which can be used for understanding and improving the control policy.

During the analysis, some uncertain information cannot be directly obtained from the data. Analysts can only make hypotheses about the cause and effect of those uncertainties. In addition, the hypotheses can be tested by Factor-Controlled Experiment in the left cycle in Figure 1-1. By hypothesis testing, the analysts understand the procedure better and propose a better hypothesis. In this way, we can recursively settle the uncertainties and the knowledge of tooth extraction.

Factor-Controlled Experiment In Factor-Controlled Experiment (FCE), analysts make an experiment plan corresponding to the hypothesis about effects (harm, usefulness, etc.) of a certain factor of control policy or a certain reaction force of the tooth and test them in a robot reproduction system. In other words, Factor-controlled Experiment separates the components or techniques in the control policy, reproduces the procedure and verifies the hypothesis concerning one of the components.

Obtaining a trustworthy result of hypotheses usually requires that one or some of the factors in the control policy are changed during several repeated experiment. However, humans cannot perform what they intend precisely, not to mention performing in precisely the same way over and over again. Therefore, a robot should replace the human in the analysis in order to avoid the deviations and reproduce the task as to the content of the hypothesis.

The of the experiment is compared with the prediction of the hypothesis for evaluations. After that, we can obtain additional knowledge or a better hypothesis about the control policy. For instance, by carrying out the experiments with and without a certain force, we might conclude whether the force is essential for the procedure. Furthermore, we can use a

Factor-Controlled Experiment test if the force in the demonstration for model identification involves the human-applied force. If needed, the system can also be used for reproducing the procedure and collecting data without the human-applied force for modeling the reaction.

Reproduction Factor-controlled Experiment is under the working hypothesis that the robot can extract the tooth. In order to have a control policy as the working hypothesis, we need to solve the problem that lies in the highly uncertain reaction from the tissue during the procedure. On the one hand, it is difficult to come up with the precise mathematical model describing the procedure. On the other hand, the robot should learn the precise control policy from multiple demonstrations. Moreover, the correlation between the control policies of the force and the movement should also be included in the model. One such method that fulfills these requirements is imitation learning or Programming by Demonstration (PbD) technique. Factor-controlled Experiment is a novel application of PbD.

PbD models the control policy in demonstrations via a machine learning algorithm without an analytical model. It can be seen as the mapping from states to control policies [6]. It encodes the trajectories of one or multiple demonstrations with the regression technique such as spline fitting so that the control policy can be successfully reproduced when needed. In addition, since the model of tooth extraction is changing, PbD is also applicable to modeling the reaction of the tooth, if one considers the tooth as an agent reproducing and generalizing a task within the work space that consists of the force and the position.

Most of the applications of PbD are to teach robots a new task, and let robots generalize it. Our application of PbD is for analyzing the demonstrations with force interactions. The advantages of PbD are shown in several levels. By learning from human, the control policy of removing the tooth is implicitly encoded in PbD [7]. The precise imitation is what the hypothesis testing requires. Secondly, it is convenient to encode a trajectory without explicit programming. The reaction force can be also reproduced by encoding the model of the tooth reaction in PbD in the tooth simulation, especially when we only need the specific model of the reaction such as the model of a fragile tooth for training.

However, PbD alone cannot fulfill the purpose of understanding tooth extraction. The data that we obtained is of high dimensionality and 'what to imitate' is the problem that needs to be solved before learning. Moreover, the control policy encoded in PbD is a trajectory instead of knowledge. Since the actual circumstances are changing during the procedure, we will need to come up with a high-level control policy that can change the output of the PbD algorithm according to the situations.

1-2 Thesis Objectives

In this thesis, our general goal is to investigate details and patterns of tooth extraction hidden in the raw data as much as possible and derive a model for the implementation of Interactive Analysis. The reproduction should be subject to the variable-constraint nature of the demonstration and fulfill specific requirements such as that the bone and the gum need to remain intact. The following objectives will be fulfilled in this thesis.

- We need to make the data available for analysis and find the reference frame (subspace) where the operation is performed as well as the variables that are used for analyzing the model and control policies. We also need initial segmentation that helps us understand and analyze tooth extraction conveniently.

- The information from the practitioners has to be verified by the interpretations that we extract from the data of demonstrations.
- As for the model, we are wondering how the movement during the procedure affects the strength of the teeth. Conversely, we are also interested in the reaction force (or torque) corresponding to the movements. As a result, a model should be available for the tooth simulation in the Interactive Analysis framework.
- The control policies will be studied. We will understand how the forceps (tools for tooth extraction) are moving regarding the speed and the pose. Another factor in the control policy that has an effect on tooth extraction is the pressing and pulling force in the vertical direction. Likewise, we need to implement a reproduction algorithm and Factor-Controlled Experiment in Interactive Analysis.
- We will propose hypotheses about the observations that are contradictory to the descriptions of practitioners. The missing information in the demonstration will be obtained by the hypothesis testing in Factor-Controlled Experiment.

In addition to the above objectives, a simulation system with the force/torque that is applied to the tooth as the output should be built for testing the tooth simulation and the reproduction system in the framework. Factor-Controlled Experiment will be implemented on the KUKA-iiwa robot using the Robot Operating System (ROS), programming in the C++ language. Due to the circumstances, we can now only work on plastic jaws.

1-3 Outline of the Thesis

In this thesis, we will analyze the procedure of tooth extraction based on the data that is recorded by a recently introduced measurement setup. In order to get clean and low-dimensional data which are conducive for both analysis and reproductions, we introduce a series of steps for preprocessing the raw data, according to the general procedure of preprocessing in the field of data mining. The Interactive Analysis framework is implemented by means of PbD. A general procedure for selecting the control variable will be introduced which should be performed before a PbD method learns and generates the control policies and the model. A simulation is performed to evaluate the algorithms in the framework. In the end, some hypotheses about plastic jaws are tested via Factor-Controlled Experiment employing the robot.

The following thesis is organized as follows. In Chapter 2, the background of tooth extraction and related method for analysis and control are briefly introduced. Chapter 3 describes the application of Factor-Controlled Experiment on tooth extraction.

Chapter 4, the necessary techniques for transforming the raw data for analysis and learning, and for segmentation are discussed. The analysis for the model of the tooth is carried out in Chapter 5. As a result, the tooth simulation for the Interaction Analysis is implemented by PbD. Chapter 6 includes the study of the control policy of tooth extraction. Two of the most promising PbD methods are compared, and one of them is chosen to encode the control policy.

In Chapter 7, the simulation for testing the reproduction algorithm and tooth simulation algorithm is designed. The physical setup and hypotheses for Factor-Controlled Experiments in this project are described and explained in the same chapter. After that, in Chapter 8, the results of the simulation and the designed experiment are discussed. In the end, the thesis is concluded in Chapter 9 with the recommendations for future works.

Chapter 2

Background

Tooth extraction is a subject that is not familiar to ordinary reader. In addition, we start this project from the scratch. There are a lot of concepts that need to be specified in advance.

In this chapter, the essential context of tooth extraction, analysis of data, and Programming by Demonstration (PbD) are introduced. It will give to the reader an impression of how tooth extraction is performed and provide information about the technique and context for the analysis and the implementation of Interactive Analysis.

In Section 2-1, we will introduce the concepts and background of tooth extraction. After that In Section 2-2, the preprocessing methods for analysis and reproduction are delivered to readers. In the end, the background of PbD methods and its application concerning the interaction force are included in Section 2-3 and Section 2-4.

2-1 Tooth Extraction

Before the introduction of the procedure of tooth extraction, some definitions related to the structure of teeth are shown in Figure 2-1.

As we can see from Figure 2-1, a tooth root is surrounded by bones together with some other tissue such as gum. And the connection between the root of a tooth and the bone is established by some strong tissues called parodontal fibers. In the figure, the right side of the tooth is called the palatal or lingual side (near the tongue), the other side is called the buccal side (near the cheek).

The teeth are classified into four categories, namely molar, premolar, canine and incisor, according to their structure. The molars have multiple roots while the teeth of the rest types only have one root. Moreover, different teeth in different positions are different in size. As for the same kind of teeth, the strength varies according to the age and the gender. In our project, all demonstrations are performed on the molar of the lower jaw where the structure of the lingual and buccal side are different. On the buccal side, the soft bone is thicker, and the tooth is allowed moving further without obstacle in comparison to the lingual side.

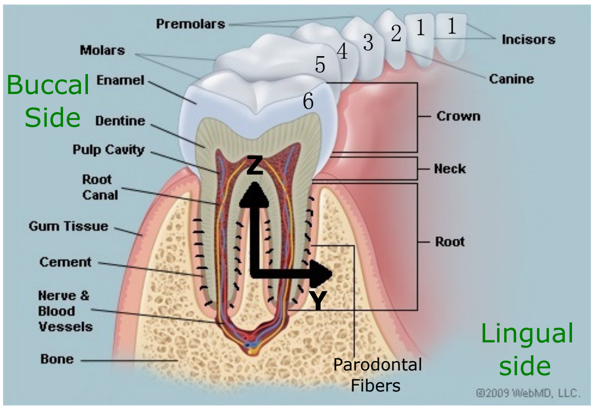

Figure 2-1: The structure of the teeth of an adult. The number represents the tooth numbering in the dental notation. The coordinate is used by the movement and the force throughout this thesis where the positive value corresponds to the lingual side (http://www.webmd.com/oral-health/picture-of-the-teeth)

Procedure of tooth extraction Most frequently, tooth extraction is performed by simple extraction, i.e. without the need to remove or operate on surrounding bones. The procedure includes two main motions; rocking and twisting. The rocking movement which is also known as lingual and buccal movement is to use forceps (as in Figure 2-2) to 'rock and move' [8] the tooth back and forth (palatal/lingual and buccal side) in order to break the fibers attached on both the root and the surrounding bone. The used technique strongly depends on the anatomy of the tooth. Single-rooted teeth with round roots can be twisted (central incisors) whilst teeth with multiple roots need to be rocked.

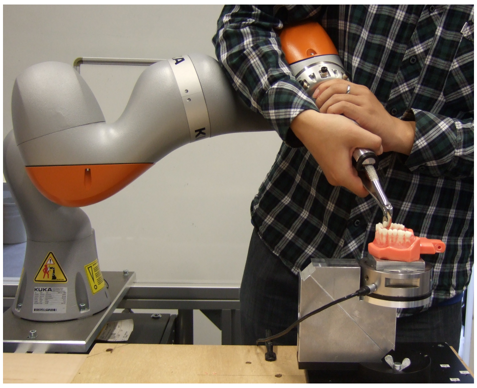

Figure 2-2: In the measurement setup. The robot is connected to the forceps and moves compliantly. The jaw is attached to the force sensor. The movement is recorded by the robot, and the interaction force is measured by the force sensor.

Reason for analysis Even though the 'simple extraction' is mostly a simple motion rocking a tooth back and forth from the buccal (cheek) side to the lingual (tongue) side, challenges and risks abound in this operation. Breaking the ligament that holds the tooth on the bone requires significantly large force. According to the research [5], the highest moment in simple extractions can reach +199 Nmm. Such forces usually are observed with tiny movement at the beginning of the procedure. It is also difficult for the dentist to find a good posture of his arms to exert such large force in the mouth. In addition, the fibers attached on a tooth vary between individuals. The huge variation in the force and movement has been found during the extractions of different teeth [8]. As to the risk, incorrect manipulations which are exerting force in wrong directions using forceps may result in the breakage of the tooth or even sever complications such as the mandibular fracture and the displacement of a root fragment [9]. The slippage of the forceps or the breakage of the crown and hitting the neighboring tooth can cause the breakage of another tooth or scarring of soft tissues.

In addition to the challenges and risk, universities are not fully capable of providing the training required for the clinical procedure. Students are obligated to perform tooth extraction on plastic phantom skulls for practices [10]. However, phantom skulls not only require less force but might also be less representative in terms of movements and/or the occurrence of complications [11]. For the lack of practices, only 40% of the graduating-year students were esteemed to be competent [12]. Moreover, Students, for now, only 'learn' this highly technical task from a book where the explanation is often a few sentences. Teachers in the clinical training can only demonstrate how to hold and move the forceps, therefore the actual work varies individually [13]. As to patients, any complications and long-lasting procedures could be annoying or even traumatizing, not mentioning the dental anxiety, which is estimated to be seen in 10% of the population [14].

Robotic applications related to tooth extraction Even if there is such demand for understanding and improving the technique of tooth extraction with the help of technology, the number of the researches related to measuring and analyzing tooth extraction by means of robotic application is very limited. Some of the applications include: X. Sun *et al.* [15] proposed a robotics system for tooth implantations; D. Wang *et al.* [16] merely designed a simulation system for tooth extractions which does not take into account the actual reaction of teeth during the extraction. None of these applications involve the analysis of tooth extraction. In our project, we have a recently introduced measurement setup for recording the whole procedure of tooth extraction, which is shown in Figure 2-2.

Dentists are asked to perform tooth extraction with the robot's end effector connected to the forceps. Concerning recordings of the movement, the robot is set to the 'gravitational compensation' mode where the robot follows the movement of the dentists compliantly and records the movement.

As to the force, we cannot use the inner force sensor in the robot since we have to measure the force from the hand. As a result, the jaw with the tooth to be removed is fixed on a force/torque sensor (Schunk FT sensor Delta). Since the forceps, the tooth and the jaw are all connected to each other, it is sensible to measure the applied force from the jaw. In this way, we don't have to integrate a force sensor into the forceps,

At last, we also make a mark on the position of the tooth and the relevant coordinate in the reference coordinate of the robot because tooth extractions are performed at different positions with different poses throughout the demonstrations.

2-2 Pre-processing for Analysis and Reproduction

Preprocess of raw data The pre-processing is quite essential for both analysis and reproduction. The time series collected from the real world are inevitably noisy. The data is also redundant for some of the inputs that are to some extent related to each other. The demonstrations received from dentists could also be faulty or could include some movements that are not relevant to tooth extraction.

For analysis purposes, the data has to be segmented according to the points where the properties of the time series change. The analyst could have a better perspective of the procedure on the basis of segments instead of the entire time series. As to the reproduction in our project, pre-processing is more than data cleaning and segmentation. Whether the control scheme is based on force or position control should be derived from the demonstrations.

2-2-1 Filtering and Selection of the Data

Pre-processing is a topic that has drawn very little attention within the field of PbD. Pre-processing is only described extremely briefly in most papers on PbD. The procedures seem to be performed arbitrarily in most of the literature. In addition, the pre-processing is always performed under the assumption that the data of the demonstrations are clean and related to the purpose of the task. Therefore only the noise needs to be taken into consideration. For example, the Gaussian filter is used in the reference [17]. However, the data from the demonstrations have more than noise within. There should be a general procedure or essential steps for preprocessing the data of demonstrations before it can be used for learning.

However, in the field of datamining, there is a complete structure for preprocessing the data. According to [18], the preprocessing procedure for datamining can be divided into following parts: firstly, the data is integrated by selecting relevant features and eliminating redundant attributes. After that, the data to be used should be free of wrong data, missing data, noise and non-standard representation after the cleaning process. For some algorithms, the data needs to be normalized properly. In addition, in some more complicated areas, the data is further transformed in order to get a better result. Finally, data reduction reduces the amount of data by means of condensation, squashing, clustering and resampling.

We will come up with the preprocessing procedure by taking the above principles in data mining as a reference and modifying them according to our requirements. In addition, one last part that should be taken into consideration as a part of preprocessing the data is how to choose a proper work space and the variables to be controlled.

Preprocessing for the reproduction For the purpose of reproduction, having clean, relevant and concise data is not enough. The work space of the robot and the variables to be controlled are also crucial for the control of the robot [19].

This topic has received very little attention in research until now. Part of the research determines the work space of the robot based on the variance among demonstrations [20]. In [21], the reference frame is chosen according to the inter- and intra-demonstration variance. Alternatively, the control variables can be selected in terms of convergent behavior in multiple demonstrations [22].

Instead of variance, there are also some other indications for choosing the reference frame or the variables to be controlled. The information about the variable to be controlled can be retrieved by the dissimilarity measurement which is usually used in time series analysis [23]. An Inverse Optimal Control approach is applied to find the relevant task spaces implicitly [24]. The authors of [25] deposit all possible task spaces into a 'task space pool' and select the appropriate task space based on prior information about the task.

A properly selected work space is essential for analysis and reproduction in robotics. Especially for the process of tooth extraction, every demonstration is performed in different positions in the mouth with inconsistent angles. If we directly employ analysis or reproduction algorithm to the raw data, different demonstrations may be treated as different procedures. Furthermore, the number of dimensions of the raw data is always high. It can be reduced by selecting right reference frame where the redundant dimensions have a constant value.

2-2-2 Segmentation of the Demonstration

Programming by Demonstration (PbD) is usually applied by means of decomposing a task into a certain number of subtasks (primitives). In most applications of the PbD, the demonstration is segmented manually. However it is preferable to automatically segment the demonstration and render a reasonable result for the segmentation. The most intuitive way of segmentation is segmenting the trajectory when the velocity reaches zero [26]. Those zero-velocity points can be interpreted as the change of direction in the movement which makes sense in terms of how humans perform a certain task.

In time series analysis, the segmentation is accomplished by detecting the change point, where the properties of the probabilistic distributions have changed. In other words, the segmentation is based on the change of the mean, the variance and the covariance in multivariate time series. Since the frequency of the demonstration of tooth extraction is relatively low, we are only interested in the change point detected by the mean and the correlation.

Unsupervised segmentation using mixture models The main challenge of the unsupervised segmentation is the choice of hyper-parameters such as the number of the kernels in a mixture. This problem can be solved by employing the Dirichlet Process (DP). In order to deal with the temporal variations and to segment the demonstrations automatically, a hierarchical clustering method called the Transition State Clustering (TSC) [27] is applied based on a switching dynamic system [28] and the Dirichlet Process. The Dirichlet Process Gaussian Mixture Model (DP-GMM) is able to automatically extract clusters without specifying the number of clusters in advance. The result from DP-GMM is steadier compared to GMM which produces wildly different results for different numbers of components. The main drawback of DP-GMM is that it could become intractable as it considers exponentially many ($O(n^n)$) partitions with n data points in the sample space [29]. There is an implicit bias in DP-GMM that may affect the result of the clustering [30].

Alternatively, the Bayesian Information Criterion (BIC) can be used to estimate the number as well [31]. The BIC score is the result of maximum evidence based on the Gaussian distribution [32]. After some simplifications, the score is only related to the length of the record and the number of free parameters. Comparing to other methods, BIC is a simple way of determining the number of clusters. The drawback of BIC is that it depends on the prior beliefs of the expectation distribution [33]. Therefore BIC renders a most likely number of clusters which could be untrustworthy. Comparing these two methods for the segmentation with the number of clusters unknown, the Dirichlet Process is more reliable with a proper concentration parameter.

Sliding window method Due to the hierarchical structure of the Hidden Markov Model (HMM), it can be used for segmenting the demonstration. A sliding window is introduced for segmenting and clustering the trajectories, and the model can be improved on-line by a hierarchical tree structure [34]. The data stream is described by a probabilistic density function, and the HMM is defined over several sliding windows in order to get the states. The states of an HMM are further clustered based on a threshold of distances between states in an HMM by a tree structure which can be improved incrementally by new demonstrations. The drawback is that the computational cost of clustering in the tree is large and thresholds of the distance are not robust.

Segmentation for time series analysis There is extensive research about change point detection in the field of applied mathematics. An auto-regressive model can be used to derive a probabilistic distribution for the change point detection [35]. In [36], Principal Component Analysis [37] is applied to reduce the dimension of the raw data and an F-test [38] is then employed for the change point detection. Reference [39] uses the recurrent plot to detect the change point in the data of human gestures. Another group of methods for change point detection is called CUmulative SUM (CUSUM) which use the cumulative statistics to find the points of interest [40].

By taking the multi-dimensional properties into consideration, Krylov subspace learning is used for change point detection [41]. Reference [42] applies the dynamic principal analysis for the same purpose. Additionally, the reconstruction error of Principal Component Analysis can be used for detection of the change of correlations [43].

The time series analysis technique is important when we are trying to understand the procedure of tooth extraction. The segmentation decomposes the data so that it is easier to identify the patterns. In some cases, when we have some assumptions that need to be confirmed, we can also apply time series analysis methods in the stage of analysis to see if it can retrieve the patterns that we have in our assumptions.

2-3 Programming by Demonstration

During the last two decades, Programming by Demonstration (PbD) has shown its advantages and is developed into a few branches for some different purposes. The algorithm aims at imitating the human demonstration and learning the control policy to avoid explicitly programming. The output of the algorithm is the high-level control policy which is illustrated in Figure 2-3.

As can be seen, the reference signal for a low-level feedback and feedforward controller is generated by the PbD algorithm. The PbD algorithm encodes the human demonstrations

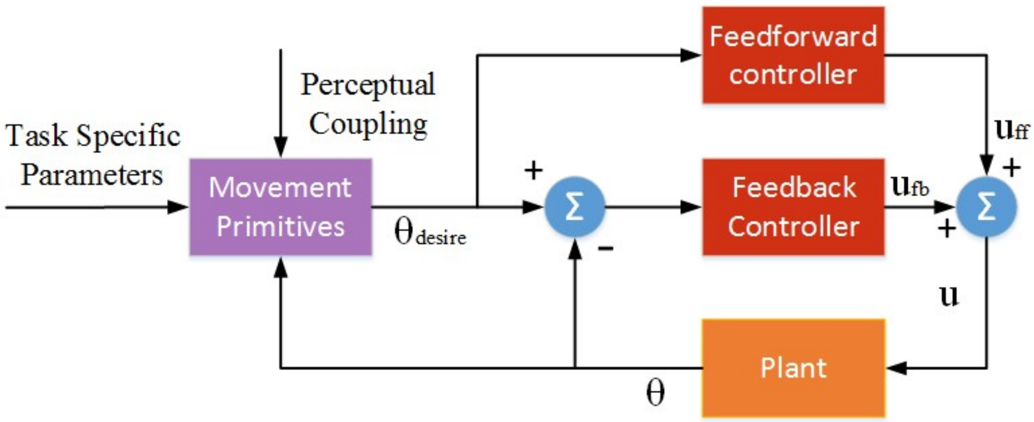

Figure 2-3: Control scheme with dynamic movement primitives [1]. The variable $\theta_{desired}$ is the reference trajectory given by DMPs. The variable θ is the sensed output of the plant. The variable $u_{ff} + u_{fb} = u$ are the control input from feedback and feedforward controller.

into a model by means of a machine learning algorithm and reproduces what is encoded. In some of the PbD frameworks, such reproduction is also robust to disturbance and with the capability of generalization.

The recent development of PbD which is based on the movement primitives has shown its advantage over methods from other branches. Similar to the motion-controlling mechanism of human beings, a trajectory is separated into multiple segments which are called primitives. This mechanism makes the reproduction of motion more intuitive and simpler. As only the primitives are modeled, a demonstration is easier to be encoded or analyzed, and the reproduction becomes flexible by combining or connecting multiple primitives. In this thesis, we will compare two of the suitable methods in PbD which are Dynamic Movement Primitives (DMPs) and the Gaussian Mixture Model (GMM).

2-3-1 Dynamic Movement Primitives

DMPs are first proposed in [44] which are inspired by the Programmable Pattern Generators [45] [46] which are modeling the central pattern generator of humans via a set of differential equations. Exploiting the coupling effect of simple first-order non-linear systems, the discrete movement primitives (point to point) are able to produce human-like and bell-shaped movements.

In the design of DMPs [44], a canonical system is initially created to implicitly encode time in a first-order differential equation. Then the non-linearity of a demonstration is encoded as an external force which is approximated by a mixture of fixed radial basis kernels (probability density functions of a Gaussian distribution are used in DMPs) that allow the learning algorithm to adjust their weights. Finally, the non-linear force is inserted into a transformation system for reproduction. The transformation system is made out of a spring-damper system that ensures the stability of the output.

Model DMPs achieves time-invariance by means of a first-order nonlinear system and encode the trajectory in a mixture of radial basis functions. The model of DMPs for point-to-point movements (discrete DMPs) is as follows: the canonical system is

$$\tau \dot{x} = -\alpha_x x \tag{2-1}$$

where the parameter τ is the time global constant. The parameter α_x controls the pace of the phase variable; The transformation system is described by

$$\tau \dot{z} = \alpha_z(\beta_z(g - y) - z) + f \tag{2-2}$$

$$\tau \dot{y} = z \tag{2-3}$$

where state variables $\dot{z},\ z,\ y$ is proportional to the desired acceleration, velocity and position respectively which are fed to the controller as commands. Parameters $\alpha_z,\ \beta_z$ are the parameters of the spring-damper system with the unique equilibrium $(y, z) = (g, 0)$. The system is critically damped when β_z is equal to $\alpha_z/4$. The forcing term in Eq. (2-1) is

$$\Psi_i = e^{-h_i(x-c_i)^2} \tag{2-4}$$

$$f(x) = \frac{\sum_{i=1}^{N} \Psi_i(x) w_i}{\sum_{i=1}^{N} \Psi_i(x)} x(g - y_0) \tag{2-5}$$

where the variable Ψ_i is the Gaussian kernel of the mixture with the mean of c_i and the width determined by h_i. The w_i are the weights that are adjusted by the algorithm. In Eq. (2-5), x ensures that the external force converges to 0 as the system reaches the goal g and $(g - y_0)$ is a scaling factor. The learning method of the parameters of DMPs are included in Appendix C.

Summary of DMPs The advantages of DMPs are as follows [47]: the model is an autonomous system without explicit time dependence. By adding a coupling terms, one can manipulate time evolution of the nonlinearity in the system. DMPs are able to coordinate multidimensional dynamical systems by introducing a reference canonical system. Incorporating coupling terms makes the system flexible for different kinds of requirements. In addition the invariance property of the dynamic system is inherited which means that the shape of output from the system is invariant under scaling of time and nonlinear force.

However, DMPs also have some drawbacks. They cannot guarantee that the trajectory stays the same in some reign because of the scaling properties. When there is a task constraint or constrained variable, the output of DMPs may damage the environment or the robot. Since DMPs are a globally asymptotically stable systems, there is alway a control action when DMPs are activated even in the wrong place. Another problem of DMPs come from the nonlinear forces, the shape of the demonstration can be preserved under the assumption that the output of the spring-damper (transformation) system is similar. However, with an uncertain interaction force from the environment, this assumption does not hold any longer.

2-3-2 Gaussian Mixture Model Based Methods

The application of Gaussian Mixture Models (GMM) in PbD originates from the idea of using a Hidden Markov Model (HMM) with a Gaussian Mixture Model (GMM) to form a state transition trajectory plan [48]. HMM-GMM obtains trajectory plans by means of statistic reasoning and probability density function regression. In the structure of HMM-GMM, the time is encoded either explicitly or inside a continuous HMM. The multiple demonstrations are encoded in a GMM which is a distribution of the density of the points in a certain reign. Gaussian Mixture Regression (GMR) is applied to the obtained GMM and an average trajectory is generated. Additionally, the reproduction from GMR is regulated by a spring-damper system. By modeling the trajectory in different reference frames and combining them with joint distribution, the Task Parameterized Gaussian Mixture Model (TP-GMM) is introduced.

Model GMM exploits the properties of joint distributions in the Gaussian Mixture Model (GMM) so that the task-related parameters can be encoded in the model. For example the distribution of a GMM in two dimensional space (x, y) is

$$p_{X,Y}(x,y) = \sum_{j=1}^{K} \pi_j \phi(x, y; \mu_j, \Sigma_j) \tag{2-6}$$

where

$$\sum_{j=1}^{K} \pi_j = 1,\ \mu_j = \begin{bmatrix} \mu_{jx} \\ \mu_{jy} \end{bmatrix},\ \Sigma = \begin{bmatrix} \Sigma_{jx} & \Sigma_{jxy} \\ \Sigma_{jyx} & \Sigma_{jy} \end{bmatrix}$$

. The function $\phi(x, y; \mu_j, \Sigma_j)$ is the joint Gaussian distribution of the j^{th} component of a GMM. The parameters μ and Σ are the parameters of the Gaussian function. The parameter π_j is the weight of an individual component in the GMM.

Gaussian Mixture Regression (GMR) is developed under the principle of nonparametric regression procedure [49] which is shown in Eq. (2-7).

$$Y = m(x) + \varepsilon \tag{2-7}$$

where

$$\begin{aligned} \varepsilon &\sim N(0, \sigma^2) && (2\text{-}8) \\ Y|X &\sim N(m(x), \sigma^2) && (2\text{-}9) \\ m(x) &= E(Y|X = x) && (2\text{-}10) \end{aligned}$$

.The above procedures can be interpreted as modeling every data point in the demonstration as a random variable which is subject to a conditional Gaussian distribution. In order to match this nonparametric regression procedure and retrieve the trajectory with respect to time, the data encoded in the GMM is divided into two parts: time and the trajectory from demonstrations. Both variables from task space and joint space can be encoded in the GMM. Without loss of generalities, we assume that Cartesian coordinates $x = \{x_1, x_2, x_3\}$ and the time t are encoded as random variables $[X,\ T]$ in a GMM. After training a GMM, we have the mean and the covariance matrix of the i^{th} component of K Gaussian kernels between each dimension as

$$\mu_i = \begin{bmatrix} \mu_{it} \\ \mu_{ix} \end{bmatrix} \quad \text{and} \quad \Sigma_i = \begin{bmatrix} \Sigma_{itt} & \Sigma_{itx} \\ \Sigma_{ixt} & \Sigma_{ixx} \end{bmatrix} \tag{2-11}$$

. Inspired by the algorithm of finding individual mean and covariance from joint Gaussian distribution [50], The mean and covariance of the conditional distribution of each component of the GMM are

$$x_i(t) = E[X|T = t] = \mu_{ix} + \Sigma_{ixt}\Sigma_{itt}{}^{-1}(t - \mu_{it}) \tag{2-12}$$

$$\Sigma_i = \Sigma_{ixx} - \Sigma_{ixt}(\Sigma_{itt})^{-1}\Sigma_{itx} \tag{2-13}$$

. The marginal density Eq. (2-10) can be derived based on the weights of the mixtures π_i, and the mean and the covariance of GMR which are

$$\hat{x}(t) = \sum_{i=1}^{K} \omega_i(t)x_i(t) \tag{2-14}$$

$$\hat{\Sigma}_x = \sum_{i=1}^{K} \omega_i^2(t)\Sigma_i \tag{2-15}$$

where the variable $\omega_i(t)$ is the mixing weights of GMR

$$\omega_i(t) = \frac{\pi_i N(t; \mu_i^{tt}, \Sigma_{itt})}{\sum_{i=1}^{K} \pi_i N(t; \mu_{itt}, \Sigma_{itt})} \tag{2-16}$$

. This model can be combined with Hidden Markov Model [51] to form a HMM-GMM [52] that could encode state-transition probabilities. In addition, another algorithm that is extended from the GMM is called TP-GMM (task-parameterized GMM). GMMs of the same task are put into different coordinate systems in order to get a systematic control policy [53]. The learning procedure for the GMM is included in Appendix C.

Summary of GMM based methods The main advantages of the learned trajectory is automatically subject to task constraints where the task can be safely and successfully performed. Since the demonstrations are statistically encoded in the GMM, the derived control policy via GMR is always subject to the constraints (inside the region covered by demonstrations). The properties of multivariate Gaussian distributions allow the derived trajectory to have dynamic features. The covariance matrix obtained from GMR denotes the variance in the task. The GMM encoding provides segmentation of the task which can be used as primitives directly. Finally, the GMMs are taken into consideration in different coordinates to generalize the control policy with different goals.

The main problem of the TP-GMM is that the selection of the number of Gaussians Mixtures is tricky. Another problem of the GMM is the scaling of different dimensions. Especially in our project, the force and position may be in very different scales. As discussed in Section 2-2-2, a slight change in the number of Gaussian could result in a significantly different outcome. This problem could be addressed by Dirichlet priors [27] In addition, the model lack interface for simple couplings and modifications as in DMPs. Since the model is based on Gaussian distributions, the details within one of the mixtures cannot be reproduced from GMR.

2-3-3 Other Methods

Probabilistic Movement Primitives Another recently proposed method is Probabilistic Movement Primitives (ProMPs)[54]. ProMPs are based on fixed basis functions similar to DMPs.

Instead of focusing on the generalization, ProMPs encode the entire trajectory in a fully probabilistic form. The weights of the basis functions and demonstrations are described by the Gaussian distributions. The regression problem is solved by evaluating the posterior distribution of weights given the demonstrations.

The main advantage of ProMPs is that they allow direct operations (blending, etc.) of different movement primitives. The properties of the Gaussian distribution enable the user to blend, combine and set via points of movement primitives. The variance of the posterior distribution can offer the evidence to set reasonable gains when the controller is designed.

In principle, ProMPs are a variation of DMPs since the linear regression and posterior distributions of weights are equivalent [32]. In order to have the operations properties, this method does not have the invariance properties of DMPs. Furthermore, in the Cartesian work space, the blending and combination of primitives make less sense since each primitive normally serves separate tasks in the Cartesian work space in imitation learning.

Simpler methods There are also some other methods which could accomplish the task of imitation learning. The spline regression algorithm is employed to achieve the trade-off between the goodness and smoothness of the regression [55]. Reference [56] encode the key points in the demonstration with HMM and reproduce the demonstration with optimization algorithm. Another attempt includes using frequency transformation and vector quantization to encode the demonstration into a HMM such that the most consistent trajectory is learned[57].

Even though these methods above by some standard could suffice the requirements of reproduction, these regression approaches lack of systematic framework and show conveniences to deal with complicated demonstrations such as the trajectory with constraints or other special requirements.

2-4 Force Application of PbD

The frameworks that we have presented in the previous Section 2-3 are all developed under the assumption that there is no contact between the end effector and the environment. However, the robot needs to learn both the interaction force and the movement in order to successfully perform tooth extraction. Since the robot cannot take force and position simultaneously as control commands, we need to first find a control strategy that aggregates force and movements into motor command. After that, DMPs or the GMM based methods can be applied to implement the control. Even though DMPs and the GMM based methods are well-developed, their application in the compliant movement still requires further studies.

Parallel force/position control The most intuitive solution to this problem is to encode force into additional primitives of PbD and apply the force and position controls by means of a hybrid or parallel force/position controller.

The parallel force/position control scheme [58] allows the force and the position of a manipulator to be controlled simultaneously. The input of the robot is the sum of the force control and the position control. The desired force and position in the controller can be generated by DMPs encoding force and position from demonstrations [59]. In this paper, the robot is controlled in a hybrid manner: the end effector applies parallel force/position control on contact with the object. Otherwise, it will apply position control. Since a position trajectory generated by DMPs has already a smooth movement with attractor properties, only the position part or the velocity part will be sufficient for the controller.

Similarly, the force/position parallel control can also be applied [58] based on the GMM. Additionally, the reference force can be encoded in the GMM in the same way as positions [60]. In the end, only force and velocity profiles in the GMM are encoded for a certain task [61]. However, the generalization capability is still debatable since only simple tasks are demonstrated in these papers.

Hybrid force/position control As an alternative to the force/position parallel control, hybrid position/force control [62] controls the system by switching between the force control and position control. The implementation of this hybrid control is demonstrated in [22]. The robot applies position control when reaching to a cube. Then force control is applied to flip the cube.

Compliant movement Tasks with interaction force can be also handled by compliant movement. PbD algorithm changes the goal of primitives or the stiffness of the controller online so that the controller reacts to the interaction compliantly.

One of the applications of DMPs employs the velocity-resolved approach [63] to learn the force of wiping table [64]. The reference trajectory can be changed in the direction of the contact force and achieves a compliant movement with respect to the surface.

Reference [65] additionally combines the force control and the coupling term. They introduce another coupling learned from the recordings of sensors from the successfully executed reproductions. Instead of simply recording such coupling terms, there is a predictive method to adapt them on-line for the purpose of leader and slave interaction tasks [66].

In the early study of TP-GMM, compliant control is achieved by an acceleration command in work space whose joint distribution is encoded in the TP-GMM [67]. It is more reasonable to calculate the stiffness matrix associated with the primitive in each dimension according to the variation in multiple demonstration [68].

The dynamics of the system can be calculated as a mixture of dynamic systems. The compliant movement can be treated as an effect of a set of virtual spring-damper systems [69]. Alternatively, the obtaining compliant movement can be formulated as an optimal control problem [70].

Summary Even though there are many references about applying parallel force/position control, this control scheme is not correct, since the force and position cannot be controlled at the same time. The reason why those algorithms work is that they only apply force or position in one dimension at the same time. This is actually force/position hybrid control by the very definition. DMPs are more suitable for force control because of its bell-shaped output. However, DMPs may also distort the original trajectory, which could result in problems, especially in force control.

2-5 Summary

In this chapter, the details of tooth extraction were introduced. We have learned that the procedure is a circular movement back and forth between lingual and buccal sides.

For the purpose of analyzing and reproducing this procedure, we further included methods for preprocessing and segmentation. The preprocessing methods render the data that is convenient and efficient for analysis and learning. Then segmentation brings us the initial knowledge about the procedure and how to analyze it.

In the end, in order to implement the Interactive Analysis framework with interaction forces, we need a PbD algorithm to reproduce the control policy or simulate to the reaction of a tooth. Therefore, we discussed the methods in PbD and its applications concerning the interaction force.

Chapter 3

Factor-Controlled Experiment

The uncertainty within the reaction force and inconsistent control policies urges us to find a new method to test and explore the cause and effect in the procedure of tooth extraction. We therefore introduce a method called Factor-Controlled Experiment for Interactive Analysis.

This chapter includes the procedures, applications, and motivation of Factor-Controlled Experiment which is the center part of the Interactive Analysis framework. It consists of a hypothesis that defines the experiment and a reproduction system that is able to carry out the experiment. We will discuss the hypothesis part in details.

In Section 3-1, we will discuss Factor-Controlled Experiment is in details in terms of components and applications. Then some essential knowledge of tooth extraction will be provided to the reader in Section 3-2-1 in order to introduce the hypothesis related to tooth extraction. In the end, the abnormal observation and hypotheses that we propose based on the observation of the pig jaw and how hypothesis testing can help us in terms of analyzing tooth extraction will be discussed in the rest of Section 3-2. The detailed hypotheses that are tested in this thesis on the plastic jaw are proposed in Section 7-3.

3-1 Content of Factor-Controlled Experiment

When studying the uncertainty, analysts need to propose and test hypotheses about the reaction of the tooth or the control policy. The experiments require most of the components in the control policy to remain the same while the component concerning the hypothesis changes so that the results from two parts of the experiment can be compared in terms of the predictions from the hypothesis. The procedure of Factor-Controlled Experiment is illustrated in Figure 3-1.

A control policy that is able to extract the tooth serves as a working hypothesis in the upper part. The lower part corresponds to the hypothesis that determines which component of the working hypothesis is to be changed. The reproduction is carried out based on

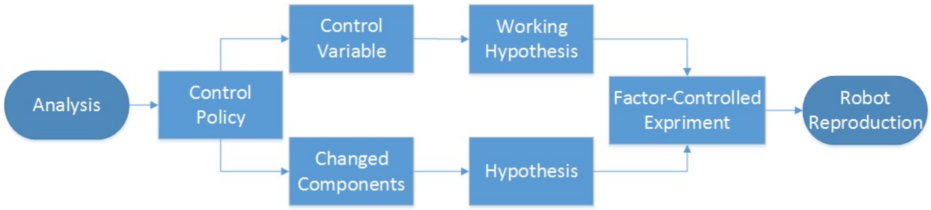

Figure 3-1: The overview of the Factor-Controlled Experiment.

Factor-Controlled Experiment with the original control policy and the modified one. Factor-Controlled Experiment can be used to study the efficiency of the control policy and the uncertain information about the model in the demonstrations.

3-1-1 Application of Factor-Controlled Experiment

The situations where the Factor-Controlled Experiment for a hypothesis is applicable can be put into the category shown in Figure 3-2.

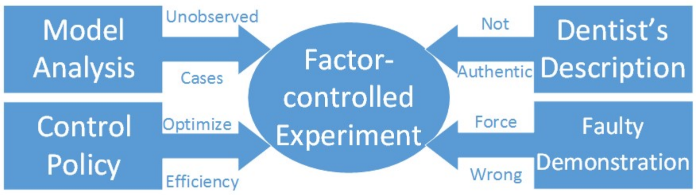

Figure 3-2: The application of the Factor-Controlled Experiment.

At the beginning of the project, the practitioner describes how the procedure is done and what is crucial for the task. However, especially for the force, the practitioner makes statement mainly in terms of what they 'feel'. In addition, the words in the descriptions themselves may be inaccurate and misleading for devising the control policy. As a result, we need to reproduce the procedure strictly according to the descriptions and verify the demonstration.

The next application of the Factor-Controlled Experiment is to identify the force that is applied to the unnecessary direction. Force control from a human is not perfect. Even if the demonstrator would like to apply force in a right direction, the force may be applied sideways and be combined with the force of the tooth reaction. This will have a negative impact on the analysis. The force changes the observed correlation between the force and the movement which is related the model of the reaction of the tooth. What is more, it is even difficult to prove the force that is observed in those demonstrations is not purely the reaction force. Nevertheless, with a hypothesis and Factor-Controlled Experiment, a few experiments can verify the origin of the force. If the data from demonstrations is not suitable for model identification, the data we collect in the Factor-Controlled Experiment can be used for identification instead.

When we study the model of the tooth, we cannot model the movement or reaction that is not in the demonstration. Since the reaction during tooth extraction is from different kinds of tissues, a full model should include the reaction from most of the tissue. Again, the human practitioner cannot precisely perform what he is asked to. The robot is a better candidate for collecting more data for modeling.

In the end, the goal of studying this procedure is to improve the control policy or technique of tooth extraction. In order to apply hypothesis testing, we need an effective control policy. We can consider that setting up the reproduction system itself is the first hypothesis that needs to be tested. Moreover, we can further study the procedure by replacing, modifying or removing a certain part from the working control policy. The situations where the Factor-Controlled Experiment for hypotheses is applicable can be put into the category shown in Figure 3-2.

3-1-2 Criteria of the Hypothesis

Hypothesis, by its very definition, is the suggested solution for an unexplained occurrence. The interaction analysis framework is actually a hypothesis testing system for the demonstrations with interaction force and uncertainties during the procedure. The hypothesis is proposed and tested recursively in order to improve the knowledge about tooth extraction.

The analysts propose hypotheses and design the experiment. The robot provides control policies that serve as a working hypothesis and preserve the other factors or independent variables that are not concerned in the hypothesis. In this way, the experiment focuses as tightly on one objective as possible.

Criteria The tricky part of the hypothesis formulation is that for different purposes, we need different experimental plans. However, no matter what the objective is, we need to make sure the experiment is good enough for analysis. The hypothesis needs to be clear and testable. The conceptual variable of the uncertainty should be converted to a measurable factor with at least one prediction. The number of measurable factors that affect the prediction should be one if it is possible. And we have to make sure other factors will not affect the result. Then the threshold which defines the observation of a certain prediction in the reproduction should be determined. During the experiment, we only change one factor of the control policy and check the effects on tooth extraction. At last, we calculate the statistics about the occurrence of the predictions.

As for the information about the model of the tooth, namely the correlation between the reaction force and the position, the criteria of the hypotheses and Factor-Controlled Experiment are how well the reproduction fits the predictions or how different the reproduction is from the demonstrations. In addition, we also have to check if the outcomes of different reproductions are similar and whether the observation in the experiment reoccurs under the same conditions.

When the hypothesis is applied for comparing the effects of control policies, the control policy should be judged by success rate, complications and how fast the tooth becomes loose. The success rate is defined by whether the control policy can break the fiber and change the model of the tooth. Complications are measured by the rate of occurrences of accidents such as fracture of the bones under the experiments of the hypothesis. The speed of the procedure is defined as how many iterations will result in a certain model change in the tooth. In general, we favor success rate of loosening the tooth over occurrences of complications over time. In other words, we first guarantee that a tooth becomes loose and then improve the control policy by reducing complications and time. Furthermore, the test is more reasonable by taking the kind of the teeth and other factors which may affect the model of the tooth into account.

However, the failed experiment does not mean that the control policies in the hypothesis do not work due to the uncertainty of the tooth. The Factor-controlled Experiment only renders results that may make the next hypothesis better. In addition, for some hypotheses which are not from yes-no questions, statistical hypotheses testing [71] may be applied to evaluate the hypothesis in terms of probabilities. In the end, if two control policies have similar outcomes, we tend to choose the one with simpler structure or the one which can be applied to more types of teeth or is able to avoid more types of complications.

3-2 The Observation Motivating Factor-Controlled Experiment

During the initial analysis on the demonstrations, we select the data based on the descriptions and the knowledge given by the oral surgeon. In this section, we will discuss the abnormal observations and what hypothesis testing can be applied to them.

3-2-1 Tooth Extraction Overview

This section includes some basic observations from demonstrations in order to justify the importance and necessity of Factor-Controlled Experiment. The detailed analysis is in Section 5-1 for model and Section 6-1 for control policy. As we know from the Section 2-1, the lingual and buccal movement is a fraction of a circle which is illustrated in the left figure of Figure 3-3.

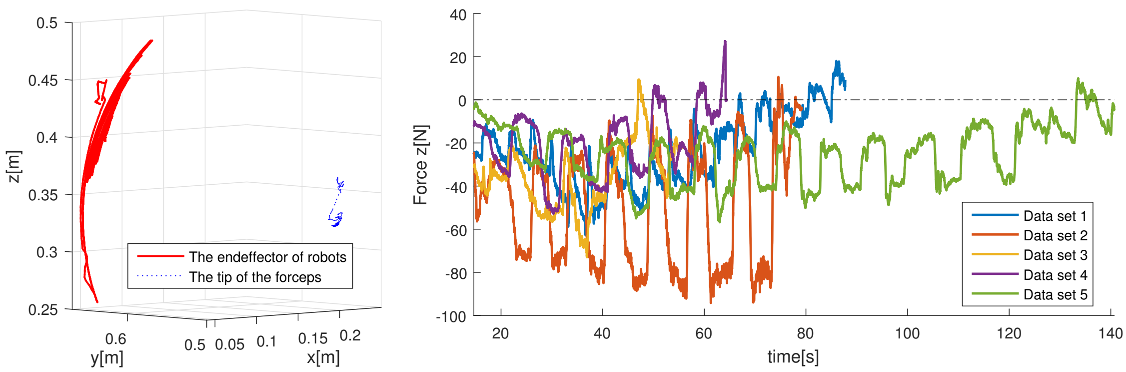

Figure 3-3: The figure one the left is the 3D trajectory of a human demonstration for tooth extraction. The figure on the right include the force in the vertical direction of multiple demonstrations

The red curve represents the raw data of the demonstration, and the blue dots indicate the estimated centers of rotation. The contour is shifting across different primitives in the direction that is perpendicular to the circle.

Another control input during tooth extraction is the force F_z in the vertical direction. The figure on the right in Figure 3-3 includes the force F_z in all demonstrations.

We can see from the figure that the force is in different trend, magnitudes and shapes. The inconsistent observations above in control policy require applying Factor-Controlled Experiment with all other factors in the control policy unchanged.

3-2-2 Abnormal Demonstrations

Intuitively, since the movement is rocking the tooth back and forth around its original position, the force F_y along the direction of the movement should be oscillating around a mean value of $0N$. This is also the description of the oral surgeon. The coordinate system for x, y and z is included in Figure 2-1.

However, some of the recordings of the force F_y are not of the same shape as our expectations. When we take a close look at F_y and F_z of one of the demonstrations which is shown in Figure 3-4.

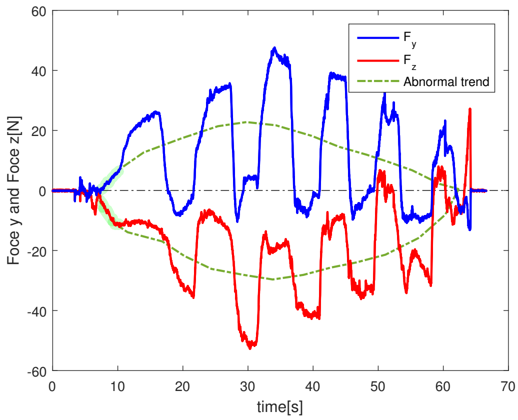

Figure 3-4: The comparison between the force in the vertical direction (red) and the force along the direction of the movement (blue). The green shade represents the start of the trend when the movement has not started. The green dash line is the estimated trend of both forces.

The force F_y in Figure 3-4 remains positive until the end of the procedure. This brings us to investigate the cause behind the observation.

During the inspection of the abnormal F_y, we notice that the F_y rises to a positive value (the green shade in Figure 3-4) before it starts to oscillate. At the same time, F_z decreases to a negative value and starts the pattern of pressure. Likewise, when the force F_z increases back to zero, the trend of force F_y returns to zero as well. Subsequently, we suspect that the trend in the force F_y is the result of applying pressure F_z in the unnecessary direction.

We have found some evidence about the origin of the trend. The detailed proof is included in Appendix B. This is a confirmation that human cannot always perform this procedure accurately and the force, which we are interested in for modeling, may change in the different positions with high uncertainty.

This abnormal data yet raises another point of the demand on using Factor-Controlled Experiment to verify if the human demonstration is good for model identification. Even though we have found the evidence through the analysis as we perform in Appendix B, we cannot conclude without physical experiments whether the force is applied to an unintended direction or it is due to the internal structure of the tooth. From the demonstration we have, only one set of data does not include this trend in F_y.

We will propose a hypothesis about this issue in Section 7-3, and test in the reproduction of the control policy as in Figure 3-2 on a robot in Section 8-2-1. If there is no such trend in force F_y, we can easily tell that this is the effect of the control policy of human instead of checking all the aspects above to draw a conclusion. Or even in some more complicated situation where we cannot judge if an observation is a mistake or not, we can use the robot to reproduce the movement with and without what seems controversy and look into the results as well as the occurrence of complications. For tooth extraction, if the reaction force to the rotation from the tooth does not involve in the trend, it will be difficult to rely on a human to collect the data for identifying the model. As a result, the Factor-Controlled Experiment can be used to collect neat data for modeling the correlation between the reaction force and the movement.

3-2-3 Comparing Inconsistent Control Policies

As we can see in the right part of Figure 3-3, the control policies of F_z are of different shapes. We do not know which one is most effective. We do not even know whether the force F_z is indispensable for tooth extraction. Furthermore, the force F_z of different shapes and magnitudes should be tested by the Factor-Controlled that is based on the hypotheses about F_z.

For a human practitioner, it is very difficult to perform different demonstrations while maintaining other factors such as the distance that he moves in every lingual and buccal movement, the shift that is observed in Figure 3-3 and so forth. With the robot reproduction, we can set those factor as fixed parameters, and the robot reproduces the force F_z with different shapes and magnitudes for comparison. We will propose hypotheses about the effects of F_z in Section 7-3 and test it in Section 8-2-2.

3-3 Summary

In this chapter, we discussed applications of the core component of Interactive Analysis in tooth extraction. Factor-Controlled Experiment is for the hypothesis about the uncertainties of tooth extraction. Hypothesis testing is a standard method in the scientific research, and Factor-Controlled Experiment allows us to test the hypothesis about correlation between the reaction force and the position, and control policies in tooth extraction.

Based on a uncertain observation in the demonstration, we proposed some vague hypotheses about the reaction of the tooth and the control policy. We also discussed the importance of the role of Factor-Controlled Experiment in hypothesis testing. We will propose hypotheses about tooth extraction in Section 7-3. Some of the hypotheses will be tested in Section 8-2 on the plastic jaw. We will further discuss the reproduction algorithm that makes Factor-Controlled Experiment feasible in the Section 6-3.

Chapter 4

Data Preparation

Most of the researches about the robotic reproduction in PbD do not concern about the preprocessing because they only concern about the Cartesian position in the demonstrations. However, in the future, the robot is about to imitate based on high-dimensional data including position, orientation, force and torque. As in our project, the raw data consist of 12 dimensions. Directly analyzing the data is not convenient and not reasonable. Moreover, a good segmentation from preprocessing would bring advantages and prior knowledge for analysis.

This chapter includes the essential procedures for obtaining proper data for analysis on model and control policy. Raw data includes a lot of redundant even misleading information that should be eliminated. The information about the procedure should be as clear as possible after the data preparation.

In Section 4-1, we will discuss the procedure of how to get clean data for analysis. Then we choose the control variables for the reproduction algorithm in Section 4-2. Finally, in Section 4-3 and 4-4, some segmentation methods will be applied to help us focus on the key factors of the procedure in the future.

4-1 Data Preprocessing

Preprocessing is a pivotal part of analysis. The noisy data and the scaling of the data should be properly disposed so that they can render a good result after the analysis. Likewise, the same procedures are also required before we apply PbD for reproduction. As we discussed in Section 2-2, it is crucial to have a general and proper procedure for preprocessing the data for both analysis and reproduction.

We propose a general procedure that is modified from the basis of reference [18].

1. Coordinate transformation

2. Estimate additional variables such as velocity and acceleration

3. Smooth the noisy data
4. Correlation check for redundant variables
5. Data Trimming for eliminating the data irrelevant to the task
6. Normalize and resample the data if needed

In this section, we will perform the preprocessing in a general way together with methods that should be used. The raw data is quite inconvenient for both looking for pattern and reproduction. For instance, we know that lingual-buccal movement is around a tooth which is in one dimension of angle. However, in the raw data of pose, there are too many variables to be concerned.

4-1-1 Coordinates Transformation

Intuitively, a task may have very different appearances in different coordinates. A properly chosen task coordinate system saves troubles for analysis and learning. Furthermore, as in our project, the data is recorded in two different coordinates which are shown in Figure 4-1. The coordinate transformation is obligatory before any further studies.

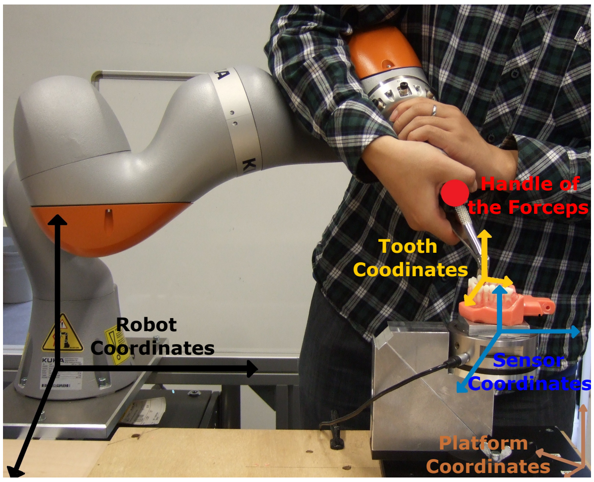

Figure 4-1: Four coordinates used in this project: robot coordinates (black), sensor coordinates (blue), tooth coordinates (yellow), platform coordinates (brown)

The trajectory of the **handle of the forceps** is recorded in the robot coordinates on the right. The wrench is based on sensors coordinates on the left. In addition, we define a third coordinate system (tooth coordinate) where tooth extraction is performed. All the coordinates above share the same z-axis which is perpendicular to the ground. According to the dentist, the circular movement is on the plane that is perpendicular to the alignment of the teeth. The tooth coordinates are obtained by rotating the sensor coordinates to the angle which is defined by the extracting tooth and its two other neighbors. This is accomplished by linear regression ($y = ax + b$) in the X-Y plane of the sensor coordinates. The regression is obtained by

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} x_{tp} & 1 \\ x_t & 1 \\ x_{tn} & 1 \end{bmatrix}^{\dagger} \cdot \begin{bmatrix} y_{tp} \\ y_t \\ y_{tn} \end{bmatrix} \tag{4-1}$$

where $[\ ]^{\dagger}$ is the notion of the pseudo-inverse of a non-square matrix and the variable $\mathbf{x}= [x_{tp}, x_t, x_{tn}]^T$, $\mathbf{y}= [y_{tp}, y_t, y_{tn}]^T$ are the positions of the three adjacent teeth in the sensor coordinate. The Homogeneous matrix from the sensor coordinates to the tooth coordinates can be obtained by

$$H_{s2t} = \begin{bmatrix} R_{s2t} & p_{s2t} \\ 0 & 1 \end{bmatrix} \tag{4-2}$$

where

$$R_{s2t} = \begin{bmatrix} \cos(-\arctan a) & \sin(-\arctan a) & 0 \\ -\sin(-\arctan a) & \cos(-\arctan a) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ p_{s2t} = \begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix} \tag{4-3}$$

. Here, the matrix R_{s2t} is the rotation matrix of the transformation between the sensor coordinates and the tooth coordinates, the vector p_{s2t} is the displacement between those coordinates. Since the raw data is represented in the coordinates of the robot, the transformation of the position is achieved by Homogeneous Matrices as

$$\begin{bmatrix} p_t \\ 1 \end{bmatrix} = H_{s2t} H_{r2s} \begin{bmatrix} p_r \\ 1 \end{bmatrix} \tag{4-4}$$

where p_t, p_r are the position data in the tooth coordinates and the robot coordinates. The matrix H_{r2s} is the Homogeneous Matrices transforming the data from the robot coordinates to the sensor coordinates, which is provided by the measurement setup.

The transformation of the Euler angle is quite different from the method for the position. Since the transformation of positions is utilizing the rotation matrix corresponding to an Euler angle, an angle is actually a matrix in the coordinate system instead of a vector. Therefore, the transformation of the Euler angle should be similar to the linear transformation of a matrix:

$$R_t = R_{s2t} R_{r2s} R_r R_{r2s}^{-1} R_{s2t}^{-1} \tag{4-5}$$

where the rotation matrices R_{s2t}, R_{r2s} are corresponding to the rotations from the sensor coordinates to the tooth coordinates and from the robot coordinates to the sensor coordinates. As to the properties of the rotation matrix, the inverse of a rotation matrix indicates the same amount of rotation in the opposite direction. The R_r, R_t are the rotation matrices corresponding to the value of Euler angle in the robot coordinates and the tooth coordinates. The position and orientation of the handle of the forceps after transformations are shown in Figure 4-2.

As can be seen from the figures on the right, the data in some of the dimensions approach to zero for positions and orientations. The number of the dimensions is reduced. However, the values in those redundant dimensions are not exactly zero. This is because the primitives in the demonstration are not strictly overlapped (as shown in the left part of Figure 3-3). Similarly, the force can be transferred from the sensor coordinates to the tooth coordinates.

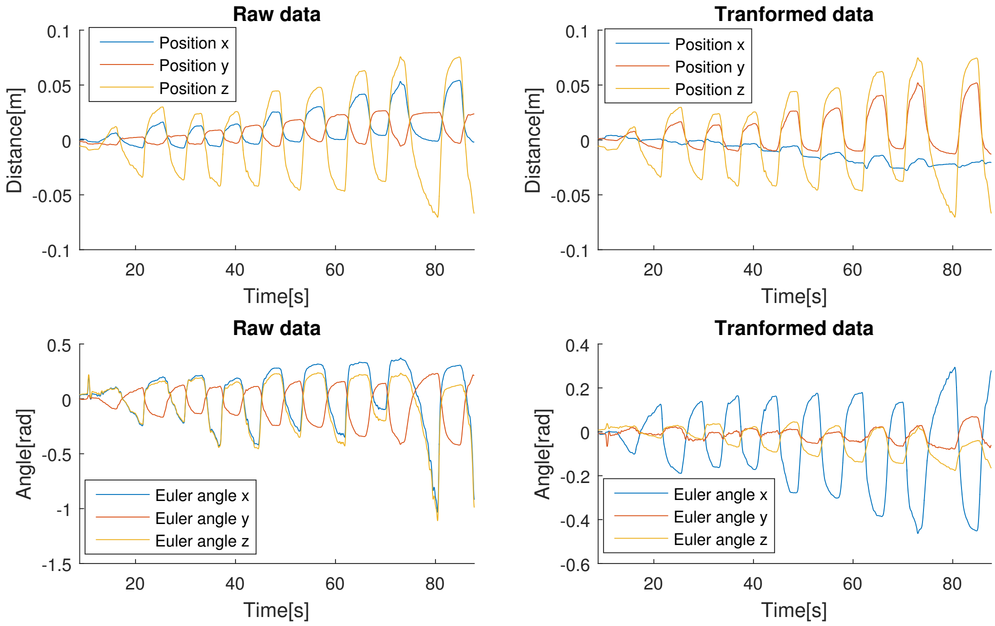

Figure 4-2: The position and orientation of the handle of the forceps. The upper part of the figure is the position data. The lower part is the orientation data. In addition, the raw data corresponds to the data on the left. The data after the coordinate transformation is on the right.

4-1-2 Additional Data Acquisition

Some of the learning algorithms for the reproduction such as DMP require the velocity and acceleration, whereas the data from measurement setup does not include the velocity and the acceleration. We hence use the finite-difference method to estimate the velocity and the acceleration:

$$v_t = \frac{p_{t+\delta t} - p_t}{\delta t} \tag{4-6}$$

where p_t is the position at a certain time instant and δt is the length of the time interval between two samples in the raw data. The acceleration is calculated in the same way by replacing the position with the velocity which is obtained from Eq. (4-6).

4-1-3 Smoothing

The raw data obtained from force/torque sensor and the velocity and acceleration that we estimated are with the noise of high frequency. This is not conducive for analysis and reproduction because the algorithms are normally sensitive to noises. Additional smoothing and filtering methods should be applied before we start learning and the analysis.

In our project, a low-pass filter is used for filtering the data from demonstration. The noise in the data of the force is white noise and we need the shape of the force unchanged. Therefore we choose the low-pass filter for the force signal. The low-pass filter that we are using is directly constructed in the frequency domain which is

$$T(s) = \frac{1}{0.2s + 1} \tag{4-7}$$

. As can be seen from the filter above, the noise whose frequency is higher than 5Hz is filtered out and therefore we can obtain a smoother trajectory.

4-1-4 Feature Selection

After the coordinate transformation as illustrated in the right part of Figure 4-2, there seems to be a correlation between the position z, y and the Euler angle around x-axis.

We assume that most of the data in the demonstration is relevant to the task and subject to this correlation which indicates that only one of those dimensions is sufficient for analysis and reproduction. The value in rest of the dimensions can be inferred from the selected one. This procedure is called Data Integration in the field of datamining [18].

One of the most well-known ways of correlation check for redundant dimensions is Pearson's product moment coefficient as in Eq. (4-8). The idea is to normalize the corvariance by the standard deviation of different variables:

$$r_{A,B} = \frac{\sum_{i=1}^{m}(A_i - m_A)(B_i - m_B)}{m\sigma_A\sigma_B} \tag{4-8}$$

where m is the number of data instances, A_i, B_i are the values of an instance of dimensions A and B. Parameters m_A, m_B are the mean values of dimensions A and B and σ_A, σ_B are the standard deviations of dimension A and B.

As can be deducted from Eq. (4-8), when the data is compared with its duplication, the result is one. By contrast, when comparing the data with its negative duplication, the outcome is -1. Thus the coefficient $r_{A,B}$ is within the interval $[-1, 1]$ and 0 implies independence. The sign of $r_{A,B}$ indicates whether the data in A and B dimensions are positive or negative correlated.

We apply the correlation check to the transformed and smoothed data and the correlations between dimensions where the data has large absolute values are included in Table 4-1.

Table 4-1: The correlation coefficients of all the data

	orientation x	position y	position z	force y	force z	torque x
orientation x	1.0000	-0.9667	-0.9979	-0.7809	0.0145	0.7908
position y	-0.9667	1.0000	0.9537	0.7242	0.0823	-0.7831
position z	-0.9979	0.9537	1.0000	0.7922	-0.0237	-0.7934
force y	-0.7809	0.7242	0.7922	1.0000	-0.2920	-0.9230
force z	0.0145	0.0823	-0.0237	-0.2920	1.0000	-0.0175
torque x	0.7908	-0.7831	-0.7934	-0.9230	-0.0175	1.0000

The elements in the table are the correlation coefficients between two dimensions specified by its column and row names. We define the coefficient of 0.95 as the indication that two variables are correlated. It can be seen from Table 4-1 that the orientation x, position y and position z are highly correlated. Since the movement is rotation, we choose the angle O_x for analysis and reproduction.

4-1-5 Data Cleaning

The demonstration usually contains some incorrect or unrelated components. In our project, the data includes the movement regarding moving towards the tooth and the recordings after the tooth was removed. These data are usually excluded manually. However, since there is a correlation between some of the dimensions in the procedure, we can take advantage of this to trim the recordings that are not relevant to the task.

We introduce a sliding window and evaluate the correlation coefficient in each window and discard the window with the coefficient lower than the overall coefficient in Table 4-1. The result of trimming is shown in Figure 4-3.

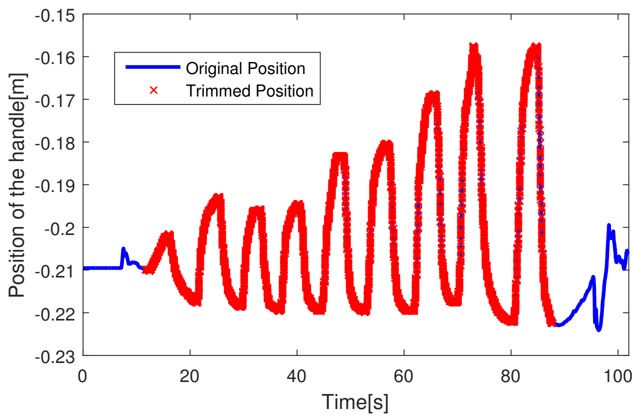

Figure 4-3: The blue line corresponds to the data with unrelated movements. The red cross represents the result of trimming based on the correlation in the demonstration.

In the figure above, the red cross indicates the data that remains after the trimming and the blue line indicates the raw data. The beginning and the end of the trajectory, which correspond to the movement of moving to the tooth and the movement after the tooth is removed, have been discarded.

4-1-6 Optional Preprocess Procedure

Some additional preprocessing procedure includes normalization and downsampling. The normalization aims at obtaining consistent scale in different dimensions of the data. Some of the algorithms especially learning algorithms may render bias outcome if the data is of different scales. In this project, we will use Min-Max Normalization to normalize different trajectories to the same minimum and maximum value for learning, which is

$$v' = \frac{v - min_v}{max_A - min_v}(max_{new} - min_{new}) + min_{new} \tag{4-9}$$

where parameters min_v and max_v are the minimum and maximum value of the original data v, and max_{new} and min_{new} the minimum and maximum value of the normalized data v'. As for analysis, Z-score Normalization is applied to normalize the data to in units of the standard deviation:

$$v' = \frac{v - \bar{v}}{\sigma_v} \tag{4-10}$$

where $\bar{v}$ is the mean and σ_v is the standard deviation of the original data.

When there are too many data instances in the recordings, some of the data instances become redundant. The redundant instances may not affect the analysis result, whereas it will slow down the learning algorithm without providing additional benefit. The raw data has the sampling frequency of about 80Hz which is more than enough for describing the procedure to which frequency is 0.06Hz on average, and we use linear interpolation for resampling the data.

4-2 Rules for Choosing Control Variables

Before we begin to segment the processed data for the purpose of control, we have to decide which dimension is of interest. The variable for the analysis should be chosen by the analyst according to the task. However, choosing the control variable for the reproduction should be a general procedure for PbD. Therefore, we summarize a general rules for the force-interaction task such as tooth extraction.

We found that there is not abundant literature about how to decide the variable to be controlled. There should be rules and procedures for selecting proper control variables for tooth extraction. It includes determining the work space of the task, choosing the control variables from force, torque, position and angle.

In principle, choosing the control variable should be a general procedure for all PbD applications. It makes tooth extraction more convenient and more intelligent to reproduce on a robot. In return, tooth extraction provides us an excellent example of thinking about rules regarding control variables with constrained task space, angle, force and torque of interactions. In this chapter, we assume that the robot is able to apply any kinds of control, which is not always true in reality and summarize and derive the following rules from the considerations of tooth extraction.

1. Discard the variable whose value is relatively small comparing to the values in other dimensions. In literature [21], this is referred to as the 'small variance with the segment/window'. As a result, all dimensions out of the subspace of extraction will be dropped.

2. The dimension with small variance across different demonstrations. This rule involves the spatial variance of the demonstrations. It implies that the trajectories from different demonstrations should be overlapped as much as possible.

3. The convergence is another key factor for choosing variables. If the position converges, it will be selected regardless whether force converges. If the position does not converge yet the force converges in the same direction, the force will be selected. There is an existing metric to check the convergence of recordings [22]. The convergence is applicable to both temporal and spatial data. The convergence indicates that all of the primitives share the same goal which comes handy for devising control policy.

4. For one tooth extraction, the same primitive is repeated several times and has different goals due to the physical limits. The end-point or goal of primitives that monotonically

increases or decreases should be chosen. This is because, for a new task with uncertainty in the maximum distance, it is more convenient for a controller to explore the goal point monotonically. This rule applies under the condition that the force or torque is uncertain and no convergence is observed in any variables to be controlled.

5. For tooth extraction, the force/angle or position/torque with more uniformed shape with respect to time should be chosen for it is very likely that the practitioners are trying to maintain a steady trajectory on such variable. It is safer to make this choice. This rule applies under the assumption that the force or torque is uncertain, no convergence is observed in any variables to be controlled and no patterns is for the observed goals.

The detailed usage of these rules is included in Section 6-1. In this chapter, we only use them to decide the which variable of the movement is worth segmenting since the segmentation in this chapter is also for the analysis.

4-3 Segmentation for Primitives

After the preprocessing, the raw data becomes more organized and cleaner. Only the meaningful data remains. In general, the data is a chunk of samples with time stamps. The control policy and the model of the reaction force are implicitly included in the recordings. As in our case, the same movement may repeat many times. According to the study [7], the movement of humans often consists of a series of smaller movements named movement primitives. Therefore the segmentation of the movement is crucial and convenient for us to understand and reproduce this procedure.

Most of the algorithms focus on how to statistically distinguish one primitive from another. In our case, every lingual-buccal movement shares similar trajectory and velocity profile. Most of the algorithms have difficulties in distinguishing from different trajectories with similarities by their distributions or cost functions. We choose the most straightforward method which segments the trajectories by the point where the movement changes its direction [26].

As we have seen in the previous sections, after the coordinate transformation and the correlation check, the data in some of the dimensions are close to zero, and we only need to consider one rotation angle of the movement in the demonstration. The outcome of segmenting one of the dimensions (orientation x) into primitives is shown in Figure 4-4.

The upper part of the figure is the segmented position and the lower one is the velocity which can be regarded as the cost function of the segmentation. Each primitive in the result is a fragment of a circle without the change of directions. As a result, the segmentation primitives groups the similar segments of demonstration together so that we can analyze and reproduce the task in a piece-wise manner.

4-4 Change Point Detection

The segmentation of movement primitives are not enough for the analysis of the demonstrations involving the interaction forces, especially as in tooth extraction, the force and the end

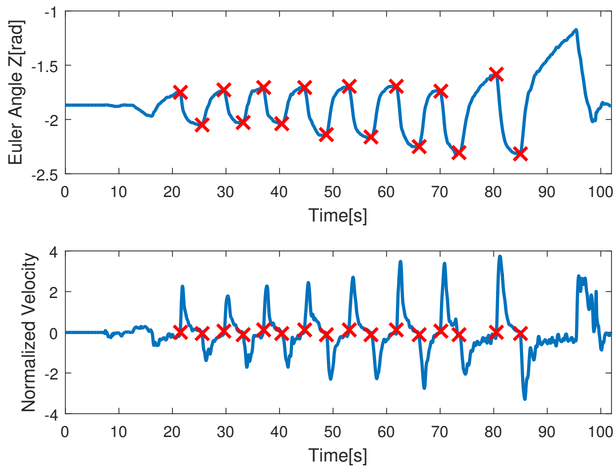

Figure 4-4: The result of the segmentation based on the change of the direction in the movement. The upper part is the movement, and the lower part is the velocity (cost function). The red cross indicates the point where the absolute velocity becomes larger than zero.

point of each primitive is changing during the procedure. We take the interaction force into consideration, when we analyze the procedure. The normalized force F_y along the direction of the movement and the angular position of the rotation are shown in the upper part of Figure 4-5.

Even though the force changes directions around the same point as in position, the situation is not entirely consistent with the position. As can be seen from Figure 4-5, the force becomes steady (the derivative with respect to time is zero) earlier than the positions. As the force represents the reaction to the movement or the control policy, we cannot help wonder if there is a correlation between the plateau of force and the control policy within each movement primitive.

Intuitively we notice a slowing down behavior in the recordings of the angular position when the force reaches its maximum value (as in Figure 4-5). It is unclear if there is a correlation between the slowing down in the position and the steady reign on the top of the force.

In order to address this problem, we try to identify the point where the movement changes the most. A time series analysis algorithm called Univariate Binary Segmentation [40] is applied. The algorithm is designed for detecting the change of mean value in a time series. This method takes the difference between the sum of the values of data points before and after a time instant. This measurement is named as CUmulative SUM (CUSUM) statistics which can be considered as measuring the change of the velocity in a more general manner.

Suppose that there is a change point in the interval $[0, T-1]$ in a time series, this algorithm obtained the CUSUM-type statistics by the cost function shown in

$$S_{0,t,T-1} = \left(\frac{1}{T} * \sum_{u=0}^{T-1} Y_{u,T}\right)^{-1} \cdot \left| \sqrt{\frac{T-t}{T \cdot t}} \sum_{u=0}^{t-1} Y_{u,T} - \sqrt{\frac{t}{T \cdot (T-t)}} \sum_{u=t}^{T-1} Y_{u,T} \right| \tag{4-11}$$

where the variable t is the current time instant being investigated and $Y_{u,T}$ is the value at the time instant u. The normalization term in the first place makes sure that different trajectories

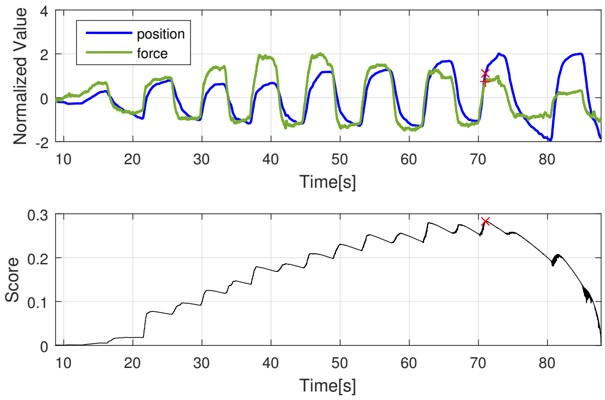

Figure 4-5: The result of one-change-point detection of the position data. The upper part is the comparison of movement and the force along its direction. The force (green) has a different shape on the top of every iteration. The cost function in the lower part of the figure shows that there are a lot of local maximums corresponding to each iteration of the movement. The red marks indicate that the positions of the change point in the position, the force and the cost function.

are normalized to the same scale. For the data point at the beginning and the end of the trajectories, the difference in the number of the data points between the parts before and after the point of interest may bias the score. Hence, the scaling terms within the absolute value are introduced to balance the effects. The result of applying Univariate Binary Segmentation to the position recording for one change point is shown in Figure 4-5.

The upper part of the figure is the result of the segmentation. The red cross specifies the segmentation on the position z. The lower part of the figure is the cost function of the CUSUM statistics. From the cost function, we can clearly see that there are multiple local maximums corresponding to all primitives of the lingual-buccal movement.

The multiple local maximums in Figure 4-5 implies that it is possible to detect all the change points in primitives. For multiple change points detection, there are in general three ways to implement it: top-down, bottom-up and sliding window. The algorithm in our case for multiple change points is based on the top-down method. The cost function Eq. (4-11) is evaluated for change point, and we split the current interval for segmentation repeatedly over all change point obtained from the previous CUSUM evaluations.

Originally, the algorithm stopped splitting current interval when the cost function is below a predefined threshold. Nevertheless, the number of the change points should be equal to the number of the primitives. All we need is where they are. Therefore we replace the threshold with a certain number of splitting that makes sure that all the change points are found and separated. In the end, we apply the CUSUM cost function to the detected change point in the interval defined by its neighbors. The point whose score is the local maximum is chosen as the final change point. Furthermore, for some intervals that are small enough, a sparse condition is introduced to prevent the algorithm from creating segment points that are too close to an existing point. The algorithm is shown as Algorithm 1.

Since the position of the first change point is unknown, we choose four times the number of the lingual-buccal movements as the number of splitting to guarantee that all of the change points can be found. For example, if we have 20 primitives, the number of the splitting should

Algorithm 1: The Change Point Detection Algorithm

```
function ChangeDetect (z, p);
Input  : filtered position data z,number of searching n
Output: the change points p
n ← number of searching;
initialize the set of segmentation points final;
p ← [1, length of the z];
initialize the set of score s;
for each n do
    l ← length of p;
    for i ← 1 to l do
        seg ← the segments of z between p(i) and p(i + 1);
        for each point in seg do
            Calculate the score in seg;
        end
        Find the point k with the largest score;
        if there is no point in p that is close to k then
            p(length of p + 1) ← k;
        else

        end
    end
    Sort the elements in p;
end
c ← length of p;
for cnt ← 2 to c − 1 do
    seg ← the segments of z between p(i − 1) and p(i + 1);
    s(length of s + 1)← the score in seg at the point p(i);
end
final ← the points in p whose s is larger than both its neighbors;
```

render more than 80 segment points which can be obtained by seven iterations of splitting over the entire trajectory. The results of the change point detection are shown in Figure 4-6.

It can be seen from the figure; that change point always lies around the time when the forces F_y stops increasing. This can be interpreted as the demonstrator tends to slow down and perform the procedure watchfully, when the reaction of the tooth becomes out of expectations. Since we have only collected data from one dentist, we are not sure if the found pattern is a personal habit or professional skill. However, the slowing-down observation in the position around the plateau of the force still is worth attention when we are deriving the control policy for the reproduction.

During the procedure of analysis, we have found out an extra use of the Univariate Binary Segmentation. Since the algorithm sums up the value at each time instant, the noise is reduced in the cost function. However, some time instants where the values change abruptly such as the point where the tooth cracked remains visible in the cost function. When the CUSUM cost function is applied to the unfiltered data of force F_y, we obtain the results as

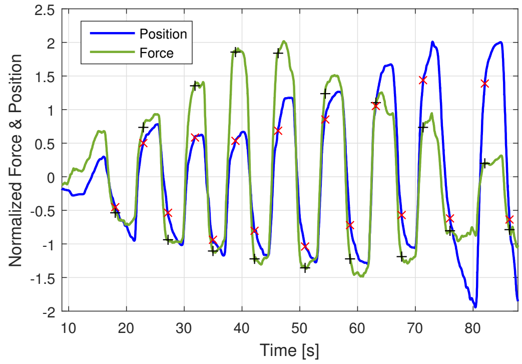

Figure 4-6: The result of the algorithm detecting all the change points. The change point in the position (red cross) corresponds to the start of the plateau of the force (black plus).

shown in Figure 4-7.

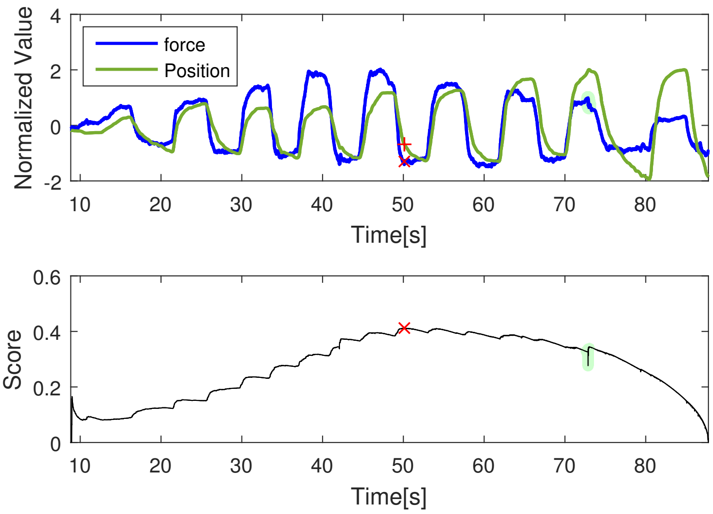

Figure 4-7: The result of cracking point detection. The lower part of the figure is the cost function of the change point detection on the force signal. The shade highlight the point the cracking point that is not obvious in the data.

It is illustrated in the figure that there is an abrupt change in the cost function when the tooth cracked. The rest parts of the cost function are represented by the smooth curve. This function can help us easily identify the point where the bones or the tooth breaks in the demonstration.

4-5 Summary

In this chapter, we performed preprocessing to reduce the dimensions of the data and improve the quality of the data. The processed data are of low dimensions and clean so that it is easier to analyze the reaction force of the tooth and the control policy of humans. After that, we summarized from literature and extended some rules for choosing control variables according to the requirement of tooth extraction.

We then segmented the demonstration in terms of movement primitives and obtained the segments that we can repeatedly perform in the reproduction. This step simplifies the analysis and the reproduction, as the movement primitives in our project are monotonically increasing with respect to time. However, the movement of the demonstration is not sufficient for understanding tooth extraction and reproducing the movement properly. The interaction force should be taken into account.

Finally, we applied the change point detection method to the force along the direction of the movement. We found out how the velocity of the movement slows down and its possible correlation with the reaction force which is related to the model of the tooth. In general, the change point detection can be applied to detect change points in the movement which are probably related to some events in the demonstrations. After that, the change points can be correlated with the events observed in the demonstration, e.g. the change of reaction force and the point when the tooth cracked.

The segmentation and the change point detection brings us information about tooth extraction and makes sure that the future analysis is performed in a more organized way that will benefit the implementations of the reproduction algorithm and the simulation of the tooth in the following chapters.

The preprocessing is crucial for the analysis on the kinesthetic demonstration with interaction force such as the demonstrations of tooth extraction in this thesis. However, we only provide the general procedure and the related methods that we used for analysis and reproducing tooth extraction. A more elaborated instruction should be created in the future that includes common methods of each step and the pros and cons of them. Moreover, the coordinate transformation in this section could be more flexible by devising an algorithm for identifying the subspace where the demonstration is operating.

The rules of choosing control variable are still vague, but it is sufficient for choosing the control variable for tooth extraction. For a more general case, it needs to be scrutinized, and some numerical score should be derived for more accurate decision. In addition, if the last rule is used and the conclusion is still not clear, more rules are required. Alternatively, as suggested in the analysis of tooth extraction, we can make a hypothesis about the control variable and test on the robot. Hypothesis testing on the robot should be the last option for this purpose, since the force or torque control is dangerous, and any testing may result in unknown consequences. Therefore it should be treated with extra care.

Chapter 5

Tooth Simulation

One of the objectives of the analysis is modeling the force and the torque from the tooth during extraction. Before we derive a control policy for robot reproduction, we should understand the model as much as possible. Good understandings of the model will result in a reasonable control policy.

In the framework of Interactive Analysis, we additionally use the model of the tooth reaction to implement tooth simulation system which can be applied for a training setup on a physical setup such as the virtual reality system in reference [16]. The physical setup for training is out of the scope of this thesis, therefore we only investigate the model of tooth reaction. In this thesis, tooth simulation will be tested in a simulation designed in Section 7-1 and then it will be used to test the reproduction algorithm (as in Figure 1-1) on the pig jaw.

In Section 5-1, we intend to investigate the movement and reaction of force and torque. It will provide essential information and insight for devising the initial control policy for robot reproduction. After we understand the model during tooth extraction, we will implement tooth simulation for Interactive Analysis by means of PbD in Section 5-2.

5-1 The Interaction Force

We choose the data that is clean, and suitable for analysis and modeling the reaction force of the tooth after the preprocessing. All in all, tooth extraction is the procedure of breaking fibers between the tooth and the bone without damaging any bones or breaking the tooth. We are therefore interested in how the lingual and buccal movement and the reaction force are correlated with each other. In this section, we are going to study the correlation between variables and understand the reaction model.

5-1-1 Force F_y Varying With Positions

Intuitively, the reaction force along the direction of the movement is increasing with the displacement during tooth extraction. It is similar to the situation where the tooth is connected with the jaw bones with two springs or rubber bands on both sides. Consequently, we should look into the correlation between the displacement that the tooth moves from the equilibrium point and the force along that direction (F_y). The correlation is included in Figure 5-1 where the force is plotted against the position.

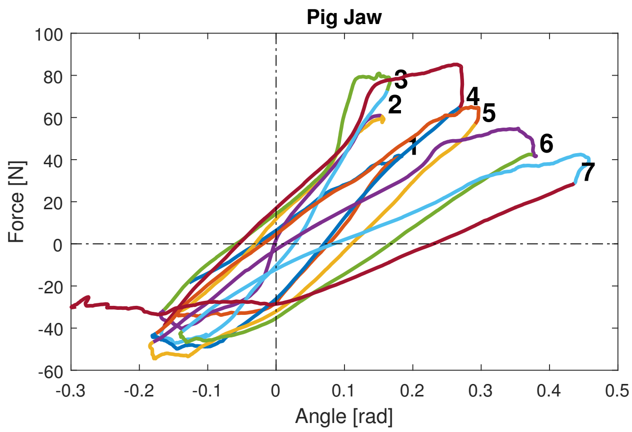

Figure 5-1: The correlation between the angle of the movement and the force F_y. The number is the order the contour is in the demonstration. The slope of the contour decreases over time. shapes are different in the positive and negative parts of the correlation.

In the figure, the negative force of the contour corresponds to the buccal side of the tooth which has a thick layer of the soft bone. The reaction force with respect to the movement looks smooth and ends up in the similar reign. When the tissue surrounding the tooth cannot provide more force, the reaction force stops increasing.

On the lingual side where the force is positive, we can see a clear and abrupt climbing before the force reaches the maximum (as in the trajectories on the left of '2' and '3' in Figure 5-1) while the position continues to increase. In addition, the movement cannot exceed a certain distance in the beginning. These observations represents that the tooth encounters the hard bone on the lingual side during the procedure. After several iterations, we can see that the climbing becomes less significant which means that the hard bone has been pushed aside. Additionally, the slope of the contour is decreasing as the procedure is performing, which indicates that the tooth is becoming loose. However, the hard bones is not the only obstacle that could result in such observations. In this thesis, we assume that these observations on the lingual side are imposed by the hard bones.

In a larger picture, each lingual and buccal movement has a shape that is very close to the strain-stress plot appearing in the research concerning material physics [2]. The comparison is shown in Figure 5-2.

The figure on the left is the strain-stress plot in the literature [2]. The reign in the beginning (green) where the force almost linearly rises with the position is called elastic reign. In this reign, the distortion of the material can be fully recovered if the force is released. The reign after elastic reign and before the fracture of the material is called plastic reign. Any distortion in this reign is permanent and reduce the strength of the material. The ultimate strength point in the middle is the indication that the material reaches its limit and the fracture will happen shortly. This kind of figures is widely used in the material physics to show the properties such as the strength and the elasticity of material.

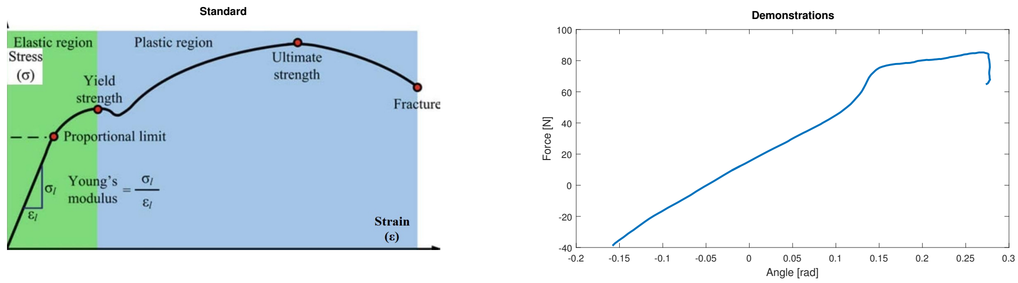

Figure 5-2: The figure on the left is an example strain-stress figure in the literature [2]. Comparing to the demonstration on the right, the plastic (green) and elastic region (blue) on the left have corresponding parts on the right. The ultimate strength is also clear which is at the end of the movement.

However, the situation here (the right part of Figure 5-2) is quite different even if the appearance of the model is similar to what is on the left of Figure 5-2. On the one hand, the reaction force recorded is not from a mix of fibers, gingiva (gum tissue) and bones of different kinds, instead of one specific kind of material. Different components are active at different points. For example, the hard bones only affect the force recorded when there is no contact with it; some of the fibers at the lower end of the root of a tooth are not fully stretched in the beginning since the movement follows a circular trajectory. The plastic reign is complicated, and it is hard to come up with a model to describe all of the behaviors.

On the other hand, the advantage that we could take from the study of material physics is limited. There is no available study of the properties of the fiber of the tooth or the tissue of the gum. However, most of the studies concerning material physics on the strain-stress plot are focused on how to staying out of the plastic reign instead of getting into it. One of the related topics could be fatigue properties of the material. Furthermore, the number of demonstrations where the force from the tooth reaction without the trend is very small. We need more data through the robot reproduction as discussed in Section 3-2. Therefore, the detailed model of the material and its fatigue behavior is a very specific topic that should be studied in another project in the future.

We also found out that the slope of the contour of the force F_y in Figure 5-2 is small in the beginning of the procedure. After a close inspection on the correlations between the slope and the vertical force F_z, we notice that the value of F_z is relatively small in the beginning. We suspect that the vertical force helps getting the tooth loose by tearing the fiber downward.

5-1-2 Torque Varying with Positions

The pig jaw Concerning the lingual and buccal movement, the movement is a circle and the input to the system is angles or torques. It will be more sensible for a physical setup to react to the movement in terms of torque according to the correlation between torques and angles.

The correlation between the torque at the position of the tooth and the angle is shown in Figure 5-3.

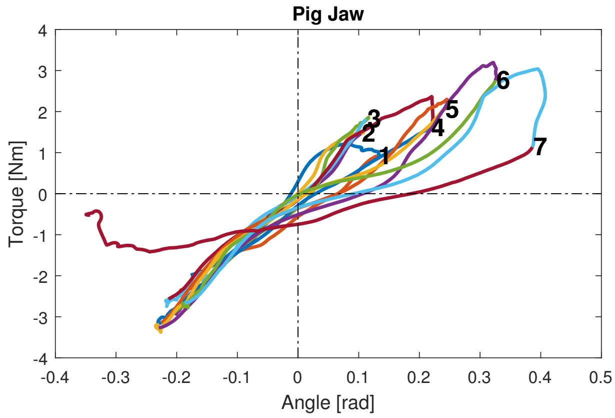

Figure 5-3: The correlation between torque and angles. The change of slope is observable on the lingual side because of the large force F_y. In addition, there is a decrease in torque with increasing position at the end of the movement on the lingual side. This corresponds to the ultimate strength in Figure 5-2.

As the figure illustrates, the torque is linearly correlated to position at the lower part of the plot, which corresponds to the buccal side. For the forces F_y in Figure 5-2 from the tooth is applying from the position near the rotation center, the reaction force from the soft bone is not large enough to distinctly influence the torque.

By contrast, the large reaction force on the lingual results in the torques with various shapes in different primitives. We can see a clear decrease of slope of the contour in one demonstration. Moreover there is an abrupt drop of torque with respect to increasing angle at the end of each primitive on the lingual side which is on the left of the marker '5','6','7' in Figure 5-3. If we correlate Figure 5-3 with the contour as in Figure 5-1, we can see that the drop at the end of primitives actually corresponds to the position of the ultimate strength. This feature will be used in the control policy for reproductions.

The plastic jaw The analysis above is based on the demonstration performing on the pig jaw. Since our testing experiment is on the plastic jaw, we also took the human demonstration on the plastic jaw. Instead of the detailed analysis on every possible aspect, we only interested in the position and torque in different demonstrations as we only consider it in a control point of view. The torque-displacement plot of a demonstration is shown in Figure 5-4.

The torque which we observe here is different than what is in the pig jaw. At the beginning of every lingual or buccal movement, the torque is not changing until the position gets pass the equilibrium namely the initial position.

5-1-3 Summary

Ideally, the reaction torque should be calculated from reaction forces F_y from the tooth. The reaction torque should be calculated from the force from F_y based on the distance from the

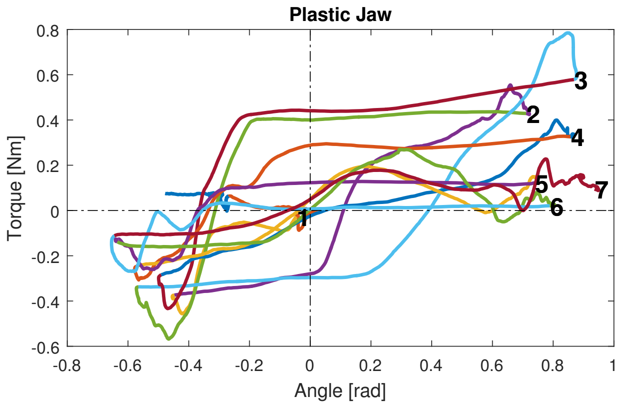

Figure 5-4: The torque-angle correlation of the plastic jaw. The shape is different from the one for the pig jaw. However, the change of the slope is similar.

point where the force applied to the rotation center. However, we do not know the exact position of the rotation center. Moreover, the exact point where F_y is applied is unclear. In some other demonstrations, the torque is even affected by the other forces. It is difficult to come up with the model for reaction torque directly from the observations in the demonstration. As a result, we choose to model the reaction of the tooth directly from the torque in demonstrations.

5-2 Learning Model

As discussed in the previous section, there is a drop after the increase in torque at the end of some primitives. Consequently, the position is not a function of torque any longer. We, therefore, choose the movement as the input of the model and the torque is generated accordingly. If we treated the tooth as a **environment**, the torque of the tooth is referred as **environment response**.

The environment response is a combination of elastic and plastic behavior as we learned from the analysis in Section 5-1. It is difficult to derive an analytical model based on our current knowledge. We choose to regard the change of slope as the change of goals of an agent. Therefore the torque with respect to position is encoded in a PbD model, and switching rules are set up as if the environment is an agent in PbD. The learning procedure in this section is under the assumption that the demonstration that we use is clean and only contains the reaction force from the tooth. We will focus on the model of the pig jaw at first and then use the same method to model the plastic jaw.

5-2-1 Model of the Reaction

Due to the difficulty of obtaining an analytically model of F_y, the missing information for calculating the torque, the scarcity of the clean and trustworthy data for model identifications, we choose to model the tooth by means of PbD.

Before setting up the model, the problem of what is related to the torque should be addressed. Regarding the observation, the torque is mainly caused by the movement. In fact, we found that a large force F_z in the vertical direction will have a seemingly static effect on the model. However, we cannot draw the detailed conclusion based on the demonstrations that we have.

For a simpler model in our project, we assume the force/torque is only affected by the angular position. In addition, we further assume that the reaction of the tooth act like a spring-damper system with varying stiffness. However, since the movement is under human control all the time during the demonstration, we could not know the damping ratio if the damping ratio on the human hand is unknown. Hence, we assume that the damping factor always results in a critical damping behavior. The damping ratio can be calculated by

$$k_v = \sqrt{2 \cdot \frac{k_p}{m}} \tag{5-1}$$

where the k_p is the stiffness of the spring, the k_v is the damping parameter of the damper, the parameter m is the mass of the load on the spring-damper system. As for modeling the stiffness, a regression method is required. Recalling what we see in Figure 5-3, the environment response in all primitives share similar shape but have ever changing slope and extremum value. We can consider the result as an agent moving in the torque-angle space. For every iteration, it reaches different goals but maintains similar shape. Inspired by this idea, we apply PbD to learn the model of interest.

We employ GMM and GMR for encoding. As we discussed in Section 5-1, lingual and buccal side have different environment responses. We, therefore, model them separately. Due to the difference in scale, we cannot directly apply the algorithm to them. We first makes the trajectories of angle and torque in the same scale by Min-Max normalization. We use GMM to encode the demonstrations and GMR to reproduce the generalized model. The generalized model is illustrated in Figure 5-5.

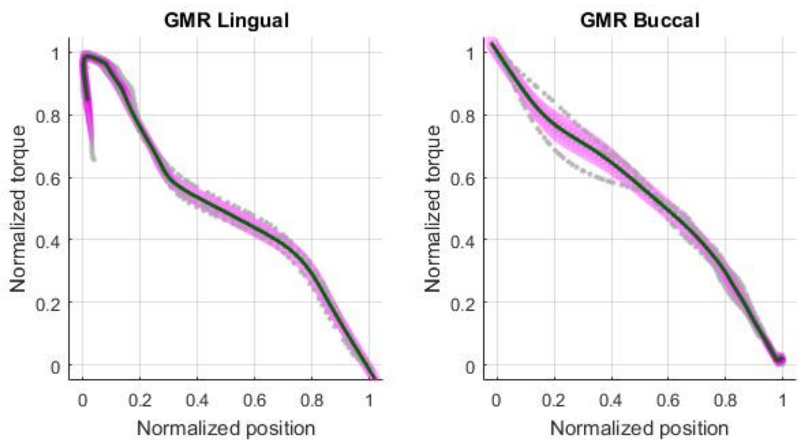

Figure 5-5: The learning result of the model of the reaction torque. The left part is the model on the lingual side and the other part is the model on the buccal side. The shade indicates the variance across different demonstrations.

The trajectory learned from demonstration has a similar shape with the observation. The figure on the left is the model on the lingual side, and the other figure includes the model on the buccal side. The learned model is in the normalized position and torque. When we have to change the slope of the model of the reaction force, we use the initial state of the current model, the desired slope and the estimated maximum torque to decide how to rescale it to our desired shape.

What is worth the attention is that the wrench recorded in the demonstration is what the practitioner applied. Hence the environment response in the simulation should be in the opposite direction to the observed torque. Another problem is extrapolation. For the position value larger than the max value in the demonstration, we have no idea about the shape of the reaction force or torque. Hence, we set the torque to a constant value indicating the tooth is broken for the unknown angle.

5-2-2 Switching Sides

We have assigned two different models for lingual and buccal sides. The model for the opposite side replaces the current one as long as the direction of the movement changes. Once the movement results in permanent damage to the fibers, the slope and the max torque on both sides should decrease. The change of the model is achieved by changing the slope of the model and setting a new target torque. Furthermore, complications, which is probably a broken tooth or bone, is very likely to occur, if the tooth moves too much within its plastic reign.

For more information about the max torque on the lingual side, we have to check if they are affected by other factors. We made a 4-dimensional figure that includes the torque, current slope, previous torque and the distance into the plastic region in the last iteration, which is shown in Figure 5-6.

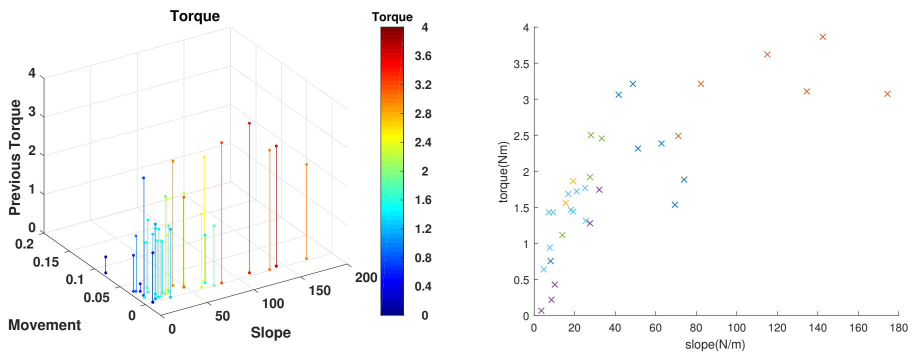

Figure 5-6: The effect of distance into the plastic region, the previous maximum torque and the slope of the correlation on the maximum value of the torque. The left part is the four-dimensional plot including all the factors. The torque is indicated by color. The right part is the plot of torque and slope after we identify a correlation between them in the left part.

We can see from the left part that there is no visible correlation between the distance by which the movement gets into the plastic reign and the slope in the model. However, a continuous trend is observed between the slope and the maximum torque (indicated by colors). We bring those two variables together into a separate plot and assign different colors to the different demonstrations as in the right part of Figure 5-6. As it turns out, the data from each demonstration stays around the overall trend, whereas each demonstration behaves differently locally. There are no patterns about how the torque of individual demonstration can be derived. They could be scattering around, along or even perpendicular to the trend.

Therefore we treat the switching of models as follows: the initial slope of the model is chosen arbitrarily, and we could estimate the mean of max torque as to the overall trend. Every time the position enters the plastic reign to a predefined amount, the slope decreases a constant proportion of the current slope. The slope change only happens at the beginning of the lingual movement. The max torque in the current model is interpolated according to the manually selected local shape from the right part of Figure 5-6. On the buccal side, the end point is constant which is the position limit. If the slope is smaller than a preset value, the tooth is deemed as removed.

As for the plastic jaw, the shape of reaction model is quite different from what is in the pig jaw. However, it is simpler than the case of the pig jaw since there are no hard bones involved. We simply set a threshold on the position beyond which we assume that the tooth will be broken. Then we apply the change of slope in a similar way comparing to what in the pig jaw.

5-3 Summary

In this chapter, we studied the correlation between the movement and the reaction force/-torque and derive the model together with how the model is changing during tooth extraction. The model is for tooth simulation part in the interactive analysis framework.

The correlation of the movement and the reaction force F_y along the direction of the movement is subject to the elastic-plastic correlations. The slope of the correlations changes when the tooth becomes loose. The effect of the bone is also visible in the correlation, and the effect becomes less and less during the extraction.

For tooth simulation, the torque is used to imitate the reaction of the tooth to the lingual and buccal movements. We modeled the tooth as a spring-damper system. The correlation between torque and angle is the stiffness. Since the analytical model is out of the scope of our thesis and the model of every iteration has a similar shape and different slope, the PbD method was used encode stiffness. In the end, we further looked into other factors that may affect the model and derive the way where the model of the tooth switches during tooth extraction.

This is not a perfect system for simulating the tooth. The model is build upon our initial analysis where a lot of information about the tooth reaction is still uncertain. The torque should be calculated from the force along the direction F_y of the movement. Nevertheless, no matter how the model improves, this switching behavior that has been derived should be the same. What is changed is the model of the stiffness and the criteria of switching.

Since we learned everything from the demonstrations, the phenomenon that is not in the demonstrations is unknown. For example, no information of how the system will behave when the direction of movements changes back and forth on the same side of the tooth is available. In addition, the exact model of the maximum torque on the lingual side is unclear, which can be very useful when deriving the control policy of the movement.

The PbD algorithm for imitating the tooth is not the best option. The resulting reaction of the tooth is with fewer uncertainties which is one of the characteristics of tooth extraction. This model could be partly or entirely replaced by the analytical model when the further analysis based on Factor-Controlled Experiment brings us more knowledge about the procedure.

Chapter 6

Robot Reproduction

The fundamental goal of analyzing demonstrations of tooth extraction is to understand and improve the control policy in the demonstration. In the framework of Interactive Analysis, robot reproduction plays the role of a working hypothesis that can extract the tooth and provide the support for Factor-Controlled Experiment.

This chapter includes essential consideration for the control policy and the implementation for reproduction. In this project, we initially assume that all the reactions from the tooth are handled by the control policy in the human demonstration. We investigate the details about how the practitioner is performing tooth extraction. Based on the knowledge about control policy, the problem of what to imitate is settled. Consequently, we can employ Programming by Demonstration (PbD) algorithm and train the PbD model that reproduce the procedure for Factor-Controlled Experiment.

In Section 6-1, the variable related to control policy will be studied and chosen for training a PbD model. Then we will compare and select a proper PbD algorithm for learning the control policy in Section 6-2. In the end, some additional information regarding the change of reaction model during the procedure will be added to the control policy in Section 6-3.

6-1 What to Imitate

We obtain 12 variables from the demonstrations. Concerning control, at most six of them can be chosen for reproductions[1]. This is still a relatively large number which is possible to be reduced. As a result, we should first analyze the control policy and then determine ‘what to imitate’ for PbD learning.

In this section, we will discuss the variable that is related to control. After that, a general procedure for choosing control variables for tooth extraction is introduced and applied.

[1] We cannot control the position and the force simultaneously

6-1-1 Variables Related to Control

According to the Section 3-2-1, the movement during tooth extraction is a circle. The position P_y and P_z are correlated with the orientation and can be zero if we know the center of the circle. In this context, only the angle and torque will be analyzed for control. In addition, according to the descriptions from the oral surgeon, the force in the vertical direction is essential and should also be treated as a control variable.

Orientations From the result of segmentation in Figure 4-6, we know there is a slowing down point corresponding to the force F_y in the direction of the movement. This should be taken into account when devising the control policy. The angular positions of the lingual and buccal movement of some of the demonstrations are shown in Figure 6-1

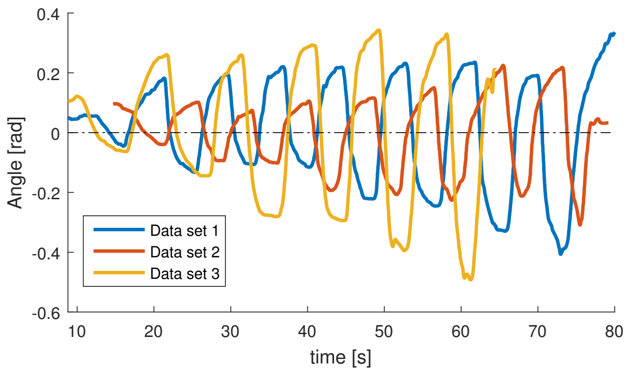

Figure 6-1: The angle of the movement with respect to time in multiple demonstrations indicated by different color.

As seen in the figure, each of the primitive includes a slowing down feature at the end. We are additionally interested in the maximum and minimum distances which the forceps move. In terms of control policy, it is the goal of the primitives. According to the information from oral surgeons, each type of tooth have its own property according to its size, number of roots and position in the mouth. Therefore we further group the demonstrations by the type of the tooth. The goal of primitives varies within one demonstration and across demonstrations. Figure 6-2 includes all the goals in all demonstrations.

From the figure, it seems that each demonstration even on the teeth of the same kind has different magnitudes of the movement. However, the goal keeps increasing with the iterations in the same demonstration. Furthermore, the positive part of the angle ends up in a similar region in the same demonstration. This is the result of soft bone mallows on the buccal side which allows the forceps moving to similar places without resistance. On the contrary, the hard bone on the lingual side prevents the forceps from moving further at the beginning of the procedure. As time passes, the bone starts to become loose, and the distance is getting larger and larger until the end of tooth extraction. These observation can be useful as we are devising the control policy.

Torque The dual variable of the angle is torque. If the position is not convenient for control, the torque may be chosen as the control variable. This motivates us to investigate the torque

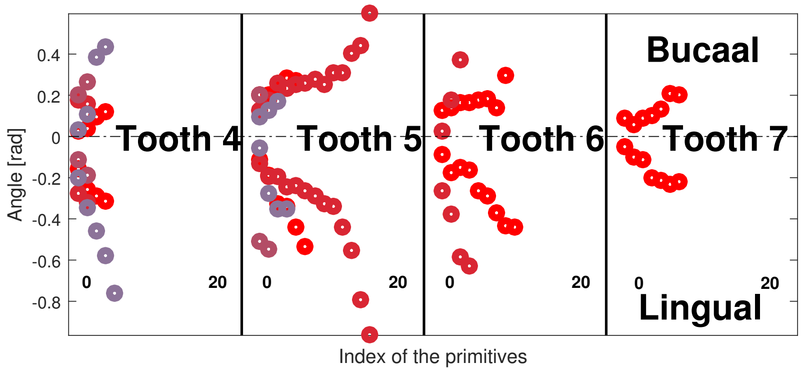

Figure 6-2: The maximum and minimum movement for every iteration grouped by the type of the tooth. Closely positioned points are the extremum for one type of the tooth. The color indicates different demonstrations. The horizontal axis in the local region is the index of iterations.

values. The torque that is recorded from force sensor is around the center of the sensor coordinates. It is corrected by compensating the torque difference from the center of the sensor to the center of the movements. The torque resulting in a rotation (T_x) is around the direction of the alignment of the teeth. The torques T_x in some of the demonstrations are included in Figure 6-3.

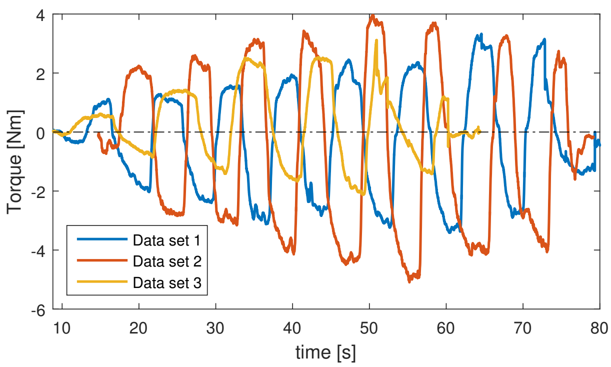

Figure 6-3: The torque with respect to time in multiple demonstrations indicated by different color

It can be observed from the figure that the magnitudes of the torque are similar to the force F_y (as in Figure 5-1) which is firstly increasing and then decreasing. Similarly, the goals of the torque are also grouped by types as it is shown in Figure 6-4.

The goals firstly increase and then decrease during each procedure. Furthermore, there is not a clear pattern for the goals for torques in the demonstration on the teeth of the same type either.

Force in the Vertical Direction The force F_z in the vertical direction does not have any physical constraints during tooth extraction. This makes it a flexible control variable. As

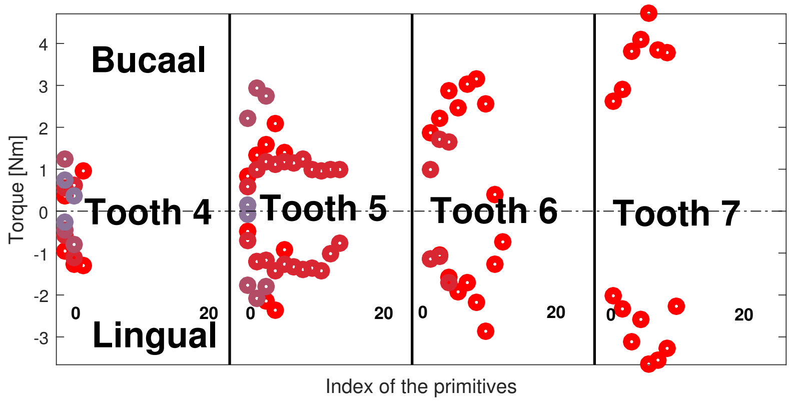

Figure 6-4: The maximum and minimum torque for every iteration grouped by the type of the tooth. Closely positioned points are the extremum for one type of the teeth. The color indicates different demonstrations. The horizontal axis in the local region is the index of iterations.

specified by the oral surgeon performing the demonstrations, the vertical force is the indispensable and the most important factor for loosening the tooth. As a control policy, it is interesting regarding its effect on the other force (e.g. F_y) or the movements.

Figure 6-5 includes the force F_z in some of the demonstrations. The force F_z also is with inconsistent magnitudes throughout all demonstrations. It is worth noticing that there is a clear trend in the each demonstration. The vertical force firstly decreases below zero which implies that the practitioner is pressing on the tooth. Then the force gradually rises to positive values until the tooth is removed.

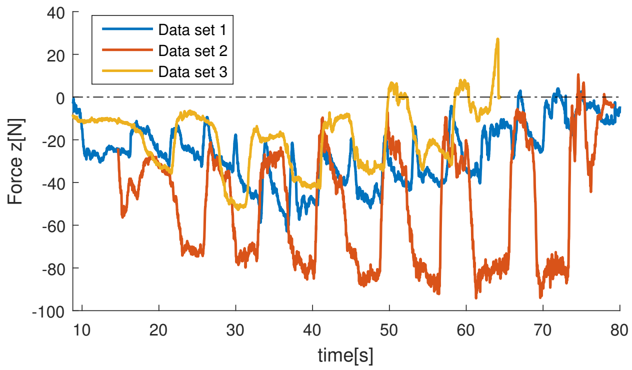

Figure 6-5: The vertical force with respect to time in multiple demonstrations indicated by different color

Comparing F_z in other demonstrations, F_z in demonstration one (blue) has the periodic peaks. Other demonstrations contain F_z of the shape that is similar to a sinusoidal signal. Those two control policies have different interpretations. The first one suggests that the oral surgeon is pressing down on both sides of the tooth and partly release the pressure when switching sides. On the contrast, the second one indicates that the oral surgeon is only pressing on the buccal side and partly release the pressure on the lingual side. Furthermore, the shape of F_z

is not regular when the dentist is pulling at the end of the procedure.

In fact, the oral surgeon states that it should be a pulling force on the lingual side. Nevertheless, the demonstration with the pressure on the lingual side succeeds in getting the tooth out and rendered relatively more readable and neater recordings in the end. We also group the largest magnitude and trend of F_z in terms of types of teeth. The magnitudes of oscillation, maximum and minimum values, of the trends are shown in Figure 6-6.

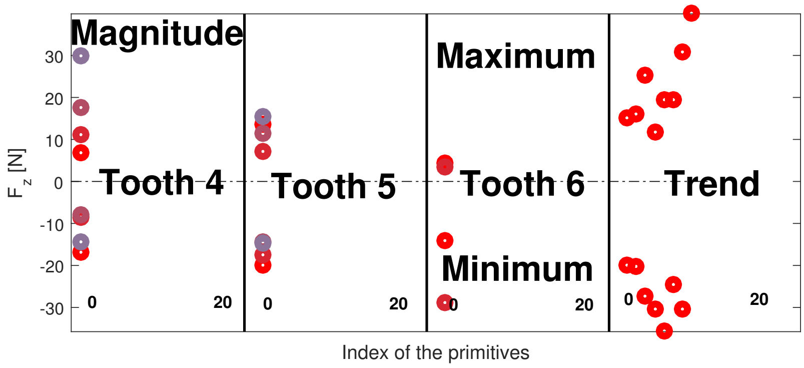

Figure 6-6: The maximum and minimum torque for every iteration grouped by the type of the tooth. Closely positioned points are the extremum for one type of the tooth. The color indicates different demonstrations. The horizontal axis in the local region is the index of iterations. The last group of point at the end of the figure is the maximum and minimum value of the trend for all demonstrations.

Even though the magnitude is of different values across demonstrations, most of the magnitude is between $5N$ and $20N$. The trend stays in the interval of $[10N,\ 30N]$ on the positive side and $[-40N,\ -20N]$ on the negative side. The description needs further investigation, for these are only the demonstrations from one oral surgeon. After all, tooth extraction is performed differently by individuals. As a result, a certain F_z should be firstly chosen for reproduction, and then other observations can be compared in terms of trends, magnitudes and shapes in the Factor-Controlled Experiment with hypotheses.

The plastic jaw In the demonstrations of the plastic jaw, the maximum position and the minimum position always stays around the value $-0.7rad$ and $0.9rad$. Since the goals of movement on both sides end up in similar regions, we do not take torque into consideration any longer.

6-1-2 Choosing Control Variables

According to the rules for choosing control variables in Section 4-2, we make the decision of which control variable we should use for the reproduction of the pig jaw and the plastic jaw.

After preprocessing in Section 4-1, we have already got some of the dimensions are either zero or correlated. We additionally found out the work space where the movement is on a circle centered at the root of the tooth. The meaningful variables for tooth extraction are the torque and the angle around x-axis in the tooth coordinates and the force F_z in the vertical direction.

Ideally, we would like to know the patterns about how the fibers are breaking and set the goal of the movement or the torque accordingly. However, we do not have this information, and the torque is unknown or uncertain as depicted in Figure 5-3. Considering both the rules in Section 4-2 and the knowledge of tooth extraction, we choose the control variables for reproduction.

The pig jaw As we have seen on the lingual side of the pig jaws, neither the position nor the force converges to a fixed goal. However, the goal of the torque is uncertain, whereas the goal of the position keeps gradually increasing all the time. Therefore, we control positions with exploratory goals whose value is gradually increasing over primitives. The exploratory goals make sure the movement slows down before the tooth is about to break. An additional algorithm that can detect the sign of breaking is used to stop the movement according to the situation. On the buccal side, as the position mostly end up in a certain reign, we choose position control with fixed goal on the buccal side.

The plastic jaw The displacement of each primitive ends up in the similar reign on both side of the tooth. Therefore, fixed goals are applied on both side of the tooth for the plastic jaws.

6-2 Programming by Demonstration

PbD is designed to encode the control policy from practitioners and imitates the demonstration. It exactly serves the purpose of reproducing the task in Factor-Controlled Experiment. Having solved the problem of what to imitate, it is necessary for us to come up with a method that can encode and reproduce the demonstrations. This can be achieved by regression algorithms. As discussed in Chpater 2, any regression methods can obtain a mean trajectory which is the trade-off between smoothness and goodness of fitting.

However, a regression algorithm is not enough for our system. Different control variables, namely components of the control policy, need to be separated, recombined, correlated. Furthermore, the control policy should be learned from multiple demonstrations, and the reproduction should be reasonable and subject to the task constraints that is observed. In addition, the algorithm should have enough capabilities of generalization for the optimization in the future. In the end, the model should be as simple as possible.

We investigated the different methods in PbD to see which is the best for reproductions. Since creating a new method or framework for PbD is not the purpose of this thesis, it narrows down our options to two well studied methods: **Dynamic Movement Primitives (DMPs)** and **Gaussian Mixture Model (GMM)** based method. In this section, we will compare these two methods and make a choice according to the requirements of tooth extraction.

6-2-1 DMPs

As discussed in Section 4-4, the slowing-down procedure at the end of the movement in the trajectory is important for the task. We first test if DMPs can render a reproduction with a similar temporal feature. We train DMPs with a randomly selected primitive in the same procedure as in Appendix C. The nonlinear force is obtained by Locally Weighted Regression.

The velocity and acceleration are estimated from the method in Section 4-1. The initial position of the reproduced trajectory is set to be the initial position of the selected movement primitive. The goal is ending positions in the movement primitive. The result of learning for the angle trajectory over time with ten radial basis functions is shown in Figure 6-7.

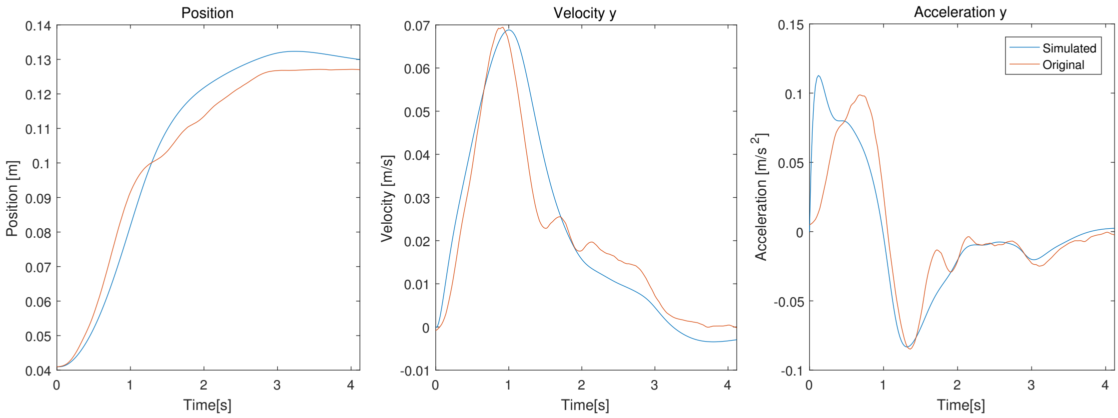

Figure 6-7: The position (left), speed (middle) and acceleration (right) of DMPs reproduction with ten radial basis functions for one demonstration. The reproduction (blue) has similar but smoother speed comparing to the demonstration (red).

As can be seen from the figure, there is an overshoot before the trajectory reaches its steady state. However, there is not such behavior in the original trajectory. It is even not preferable as any additional movements in comparison to the dentist demonstration, as it may result in complications such as the fracture of the bone or the broken tooth.

In this project, our goal is to keep the entire trajectory staying consistent with the demonstration instead of generalizing it to a different shape. However, the overshoot cannot meet this requirement. Furthermore, the nonlinearity of the forcing term is high. This also brings problems when we are interested in what has been learned or something in the reproduction goes wrong. As for the uncertainty of interaction force, the nonlinear force will probably be affected by them during the training.

In addition, the correlations between different control variables are based on the time progress. When one dimension had failed to reach its position, the other one would not stop accordingly. It is highly likely to lead to the distortion in the entire trajectory. Moreover, DMPs can be properly applied under the condition that we have accurate speed and acceleration signals which cannot be sufficed in our project. In summary, DMPs is not suitable for tooth extraction.

6-2-2 GMM-GMR Method

The GMM-GMR is designed to learn from multiple demonstrations. The demonstration can be interpreted as task constraints which the learned trajectory should be subject to. For our current requirement of reproduction, we only apply the GMM and GMR to acquire and generate the trajectory. The generalization of the GMM based on task parameters may be useful for the future application.

Unlike DMPs, the temporal alignment and the difference in scaling are not handled by the algorithm automatically. Inspired by the scaling method in DMPs, via Min-Max normalization in Eq. (4-9), we first move angular positions O_x and vertical forces F_z of all the primitives to the origin $(0, 0)$, and then scale them between zero and one. The time scale is also uniformed by scaling.

The only hyper-parameter of the GMM is the number of Gaussian mixtures. As it happens, the primitive consists of the parts of moving, slowing down and reaching the goal. We take this prior information and set the parameter to four. The additional one is for the F_z. The GMMs encodes time, angle and the force F_z in a three-dimensional space with the learning procedure as specified in Appendix C. The result of GMR on the angle of demonstrations is shown in Figure 6-8.

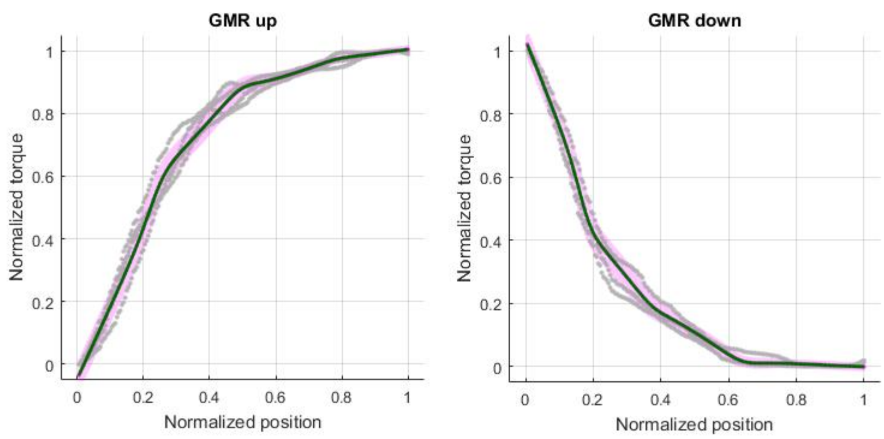

Figure 6-8: The learned lingual (right) and buccal (left) movement with respect to time. The shade indicates the variance in the demonstrations.

We can see from the figure that the trajectory is properly generated from multiple demonstrations. Both of the generated control policies of moving up and down stay within the region of demonstrations without any undesirable distortions.

Regarding the force, the vertical force F_z is the control input that is not affected by the reaction of the tooth. We are interested in F_z with respect to both the time and the angular position. The result of applying GMR on the F_z with respect to time is shown in Figure 6-10. The other regression result of F_z is shown in the next section.

Since it is a non-parametric model, the GMM is more efficient in describing data comparing to linear regression with fixed basis functions. GMR reproduces the whole trajectory of a sharp turn in first half of F_z with only two Gaussian mixtures. This is another advantage of GMR.

6-3 Control Policies for the Reproduction Algorithm

Concerning the reproduction, the entire procedure comprises two primitives: moving towards the lingual side and moving towards the buccal side. The angular position O_x is reproduced

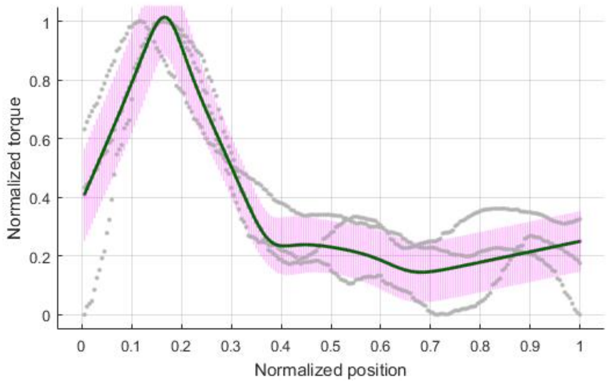

Figure 6-9: The learned vertical force with respect to time. The shade indicates the variance in the demonstrations.

by GMR with respect to time. The force F_z is encoded in a model based on O_x. Since the reaction of the tooth is changing during the procedure, additional information such as the goal of each primitive is additionally taken into consideration.

In this section, the control policies for the pig jaw and the plastic jaw are introduced. We will discuss how tooth extraction is reproduced by two primitives of angular position. Some additional control polices will be devised for the uncertainties of the tooth. In the end, we will reproduce the control policy of F_z.

6-3-1 Control Policy of Movement

As discussed in Section 4-2, we observe different convergent properties on both sides of the tooth. It is convenient to distinguish them by different primitives. In addition, we would like to study tooth extraction and improve the control policy separately on each side in the future. As a result, we set up one primitive on each side of the tooth, even though the reproduced trajectory has a similar shape.

The lingual and buccal movement result in the wrench in the opposite direction. The force in return affects the actual position during the procedure. Thus this angular movement should be generated in the first place as in Figure 6-8. Since the reaction force prevents the position from tracking the reference in the real time, the force F_z should be applied to the tooth according to the current angle of the movement. The model of F_z with respect to the angle is obtained by GMR as in Figure 6-10.

Even though the force F_z for learning has the same shape, the beginning and the end points across all of the scaled primitives are different. This is related to the trends in the demonstration. Consequently, we will implement that trend by changing the beginning and end points. We will discuss this later in the application. Having the reference trajectory for reproduction, now we have to consider the uncertainty and the change of model in the reaction of the tooth.

The pig jaw According to the rules we derive in Section 4-2, a predefined goal is set for the primitives on the buccal side. As for the lingual side, we have increasing and exploratory

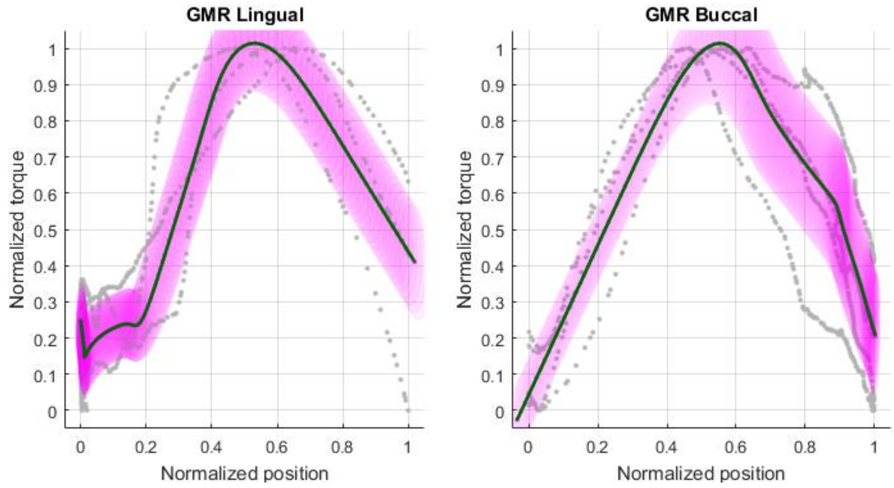

Figure 6-10: The learned vertical force with respect to time. The shade indicates the variance in the demonstrations. This is for reproduction since the actual position may be different from the reference.

goals for primitives. It is very likely that the goal is set too large for the tooth. Therefore the algorithm should be able to detect the limit of the tooth (the ultimate strength) and stop the movement.

The reproduction is performed based on the primitives that we obtained from 4-3. The algorithm also needs to decide when or where to switch to the other primitive. Once the current primitive has reached its goal and stayed at the goal for a while which guarantees that the robot reaches the reference, the control policy switches to a next primitive. The goal of the previous primitive becomes the initial position of the next one and the reference from GMR is scaled by the difference between the goal and initial position.

On the buccal side, the control policy is subject to the switching principle above with fixed goals. However, on the lingual side, there are addition criteria for goals and switch according to the situation. We divided the **lingual** movements (primitives) of tooth extraction on the pig jaw into three phases: finding the maximum wrench, pushing the bones on the lingual side, and loosening the tooth.

In phase one, the goals of the primitives gradually increase until the primitives reach the plastic-distortion reign in the force-position correlation. The switch is subject to the same criterion as on the buccal side. Phase one corresponds to the red line in the left part of Figure 6-11.

The phase two starts when the movement gets into plastic regions. The same goal is assign to the movement primitives over and over again before the bones become loose (i.e. the decrease of slope). Phase two corresponds to the blue and black curves in the left part of Figure 6-11.

In phase three, the goal starts to increase until the tooth eventually is loose enough to be removed. Phase three corresponds to the rest of the curves in the left part of Figure 6-11.

In phase two and three, the control policy makes sure that the movement is slowing down around the plastic-distortion reign and stop after the ultimate strength is reached, which is implemented by identifying the ultimate strength as observed and discussed in Section 5-1.

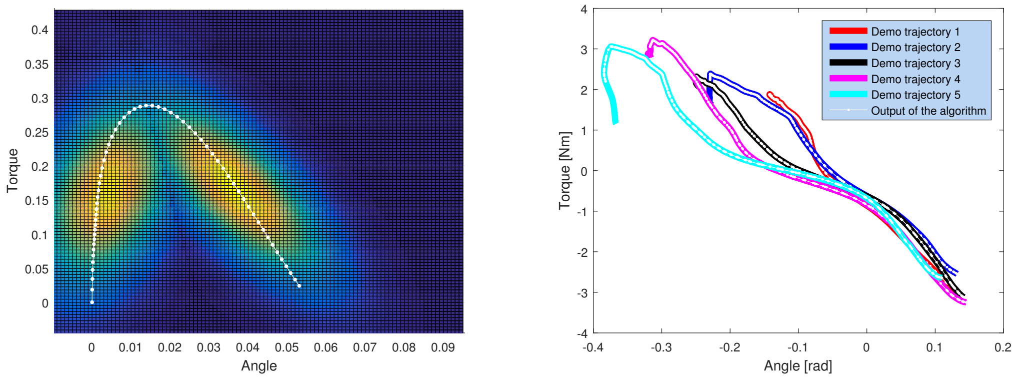

Figure 6-11: The model (left) and the result (right) of the algorithm that stop the movement after the ultimate strength. The shade color in the left part represents the probability distribution of the GMMs. The white dots are the trajectory that is used to train the model. The right part is the detection result of the demonstrations. Different colors represent different iterations. The white line indicates the region that the algorithm allows the robot to move.

Recall that there is an abrupt drop in Figure 5-3. Therefore we create a model for detection. We firstly extract a training trajectory around the ultimate strength and move it to the origin. And then a Gaussian mixture model consisting of two functions is trained as shown in the left part of Figure 6-11. The training procedure is the same as the EM approach that is included in Appendix C.

During reproduction, a sliding window is applied to certain numbers of the most recent data points. For every window, the torque and the angle of the data are scaled to the same one as in the model. After that the data in the window is translated so that the current point(last one in the array) is in the origin of the coordinates where the model lies. We, at last, calculate the log likelihood based on trained model as Eq. (6-1).

$$L = \log(\pi_1 N(x|\mu_1, \Sigma_1) + \pi_2 N(x|\mu_2, \Sigma_2)) \tag{6-1}$$

The log function penalizes heavily on the likelihood lower than one. Therefore, any positive likelihoods correspond to the sign of which the ultimate strength has reached. We initially apply this method to the primitives in one of the demonstration. The result is shown in the right part of Figure 6-11.

The figure on the left is a probabilistic distribution of the trained model. Moreover, all of the ultimate strength in the plastic reigns on the right part of the figure is identified by that GMM. Since the data in the window is moved to the origin, any of the trajectories without a decreasing torque in the lingual movement will be out of the first quadrant. The model is activated when enough points are in the decreasing part. Therefore the window size and the speed of the movement determines how much the robot is allowed to move beyond the ultimate strength.

The figure on the right is the result when we apply the algorithm to a demonstration. The function is activated on the last four primitives. The first primitive which has not reached the ultimate strength does not activate the function. We will further test this function in the simulation system in Section 7-1.

The plastic jaw Due to the absence of the hard bones in the plastic jaw, the tooth on the plastic jaw can be equivalent to a tooth with two buccal sides. The goals of the primitives in demonstration converge on both sides. The control policy including goals, switching, etc. of the reproduction is the same as what is used on the buccal side of the pig jaw.

We also set three phases, which are however different from the phases from the ones in the control policies of the pig jaw, for the plastic jaw. In the first phase, the movement is set to half of the desired goal. The phase is introduced only for safety. We intent to avoid abrupt movement on one side of the tooth while the other side is intact. The phase two is about moving back and forth according to preset goals and loosening the tooth. In the last phase, the movement remains the same, but the robot starts to pull the tooth in the upward direction.

6-3-2 Control Policy of Vertical Forces

Generally speaking, any control policy of F_z will do as long as the tooth can be removed without any complications. For the initial trial for reproduction or Factor-Controlled Experiment, we only need an effective control policy. The control policy of F_z will be changed , tested and generalize in Factor-Controlled Experiment based on hypotheses after the succeed of the initial trial. And we can eventually find the most efficient one.

For the magnitudes of the oscillation, we set a value of $-20N$ for the force below the trend and $20N$ for the opposite one. The minimum value in the trend is set to be $-30N$ and the maximum trend value when pulling the tooth is set to be $20N$. As for the shape, we choose the control policy with periodic peaks (the learned trajectory in Figure 6-10). The advantage of this control policy is in the stage of pulling, the robot to apply larger force in the upward direction so that it is more likely to get out upwards which is less likely to hit people or other objects in the lab.

As discussed earlier, we will implement the trend by setting different initial and end points for primitives. When the F_z start at a larger value and end with a smaller value, the trend is going down. The going-up trend is obtained in a similar way. As a result, when the robot is reproducing the F_z with the trend, the encoded primitive is separated into two parts by the maximum value. The trend is achieved by setting different scales to each part according to the situation.

We have already divided the movement primitives of tooth extraction into three phases. The force F_z will change its trend according to its own stages. In the first stage, the robot keeps pressing down until a certain value is reached. Then it maintains the current trend before the tooth becomes loose enough to be pulled. In the end, the force gradually increases to the positive values in order to remove the tooth.

6-4 Summary

In this chapter, we analyzed the details related to the control policy for both the pig jaws and the plastic jaws. The angle and the force F_z in the vertical direction were chosen to be the control variable according to the rules in Section 4-2. The shape and magnitudes of the movement and the force F_z with respect to time were extracted from demonstrations. In the end, different PbD algorithms were compared, and the suitable one to encode and reproduce the control policy were selected.

The buccal movement always ends up in a similar reign whereas the one in the lingual side keeps increasing. On the other hand, there is no pattern in terms of the goal of torque. As for the control policy of force F_z in the vertical direction, it has a trend with various of shapes and magnitudes. The effectiveness of them is unclear. It requires further studies by Factor-Controlled Experiment proposed in Section 3-1. Furthermore, since the goal of the primitive is unclear, an additional control policy was introduced to stop the movement if the exploratory goal is too large for the current state of the tooth.

The GMM-GMR method, which can encode the time model and the force-position model of the control policy altogether, was used for the reproduction. The generated trajectory is strictly within the constraints of demonstrations which is the requirement for tooth extraction. However, since the regression is based on every Gaussian mixture, the reference inside the Gaussian is close to a line. Some of the information within it may be lost. Furthermore, a Gaussian mixture for stopping the movement is able to stop the movement around the ultimate strength. However, we can never set the exact point for stopping. Moreover, the control policy should guarantee that the movement is slowing down around the limit. Otherwise, the function may fail to stop the movement.

Chapter 7

Experiment Design

The objectives of the experiment are to test and evaluate the performance of the components that we implemented for Interactive Analysis. The experiment consists of two parts: the simulation and the physical experiment.

This chapter includes the design and the implementation of the experiments. The simulation is to put tooth simulation and reproduction algorithm and simulate the situation where the robot reproduction is performed on a training setup based on tooth simulation. We assume that the reproduction is performed by an imaginary robot that is able to perform Cartesian impedance control. The purposes of the simulation is to check how realistic tooth simulation (Section 5-2) can be and whether the control policy that we derived for reproduction (Section 6-3) is effective for tooth extraction.

The physical experiment is the application of Factor-Controlled Experiment with a robot reproduction setup. We are interested in whether the robot reproduction system can render the information that is uncertain in demonstrations. In addition, some hypotheses about the plastic jaw are proposed and tested so that we can analyze the results and discover some information that is missing from the demonstration.

In Section 7-1, we firstly introduce the software structure of the simulation together with the assumptions that simplify the model. Then we will discuss how the robot will perform the reproduction physically and the software implementation in Section 7-2. Finally, in Section 7-3, we will propose the hypotheses related to the plastic jaw and experiments associated with them that will verify the effectiveness of Factor-Controlled Experiment and render more knowledge about tooth extraction.

7-1 Simulation

We aim at simulating the situation where a robot applies torques on a tooth and changes its orientation. The variables of the simulation system are the angular acceleration, velocity and position of the tooth. The entire simulation system consists of four parts: the decision part (the high-level control policy), the control part (an imaginary robot), environment part (the reaction from the tooth) and the dynamic part (the integration of the velocity and orientation). The overall flow chart is included in Figure 7-1.

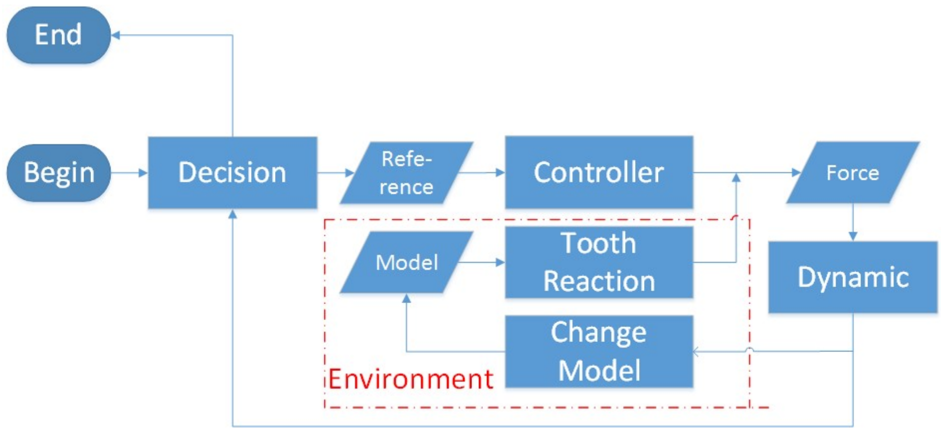

Figure 7-1: The flow chart of the entire simulation system.

The decision level generates the reference signal to a controller (of a robot) that was defined in the control level. The environment level generates the reaction force of the tooth and sends it to the dynamic level where the states of the simulation are calculated and sent back to the decision level which makes the control input for the next time instant.

In the simulation, we would like to evaluate the change of the model of the tooth reaction. As for the control policy, we would like to investigate the performance of the reference tracking and the additional criteria that switch the primitives. The uncertain part of the tooth reaction, such as the effects of the force F_z, is simplified by assumptions.

7-1-1 Decision Level

The decision level is responsible for the high-level control policy of the reproduction. It corresponds to the reproduction algorithm in Chapter 6-3. Namely, the reference trajectories of all primitives are generated, and the decision of when to switch primitives is determined in this part.

As discussed in Chapter 6-3, the control policy is based on two primitives. The control policy is adjusted or switched by the position of the ultimate strength and the states of the tooth. The control flow chart is as Figure 7-2.

The condition under which the robot switches to the next primitive is when the primitive reaches its goal or the ultimate strength is detected. The condition and mechanism for control are the same as in Section 6-3.

7-1-2 Controller Level

The controller level simulates the execution of the reference on a robot given the circumstances. During the simulation, we do not have any knowledge of what the robot is capable

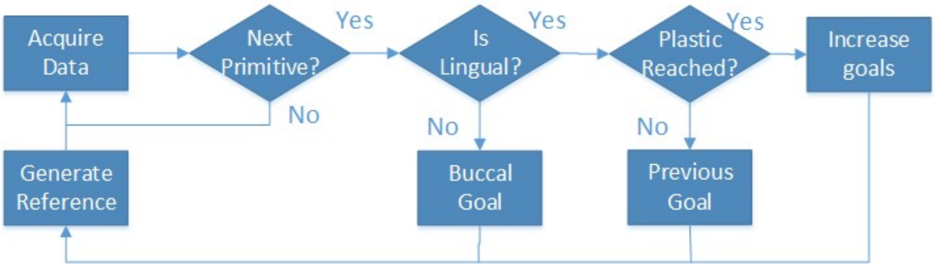

Figure 7-2: The flow chart of the decision level (High-lever controller).

of. We assume that the robot only has the very basic function. In the worse scenario, the robot should have a Cartesian impedance controller. Therefore we assume that there is an impedance controller in the robot with the desired mass of one which subjects to

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = -M(q)(K_p e_p + K_v e_v) + f_c \tag{7-1}$$

where f_c is the term that compensate the nonlinear terms $C(q,\dot{q})\dot{q}$ and $G(q)$ on the left-hand side of the equation and $M(q)$ is the inertia matrix of the robot. The parameter K_p and K_v are the gains for the controller. The variable e_p and e_v are the error of the position and the velocity from the reference signal. The flow chart of the control system is shown in Figure 7-3.

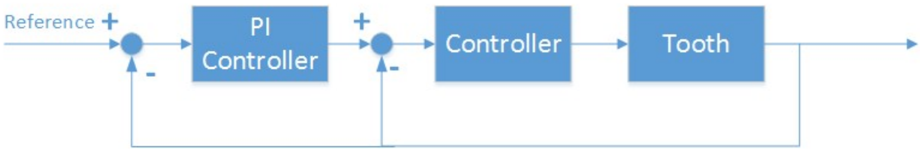

Figure 7-3: The flow chart of the controller level (robot controller).

The resulting error dynamics represent a spring-damper system. We set the desired mass to one without the loss of generality. According to the manual of the robot, the default stiffness of the control is $200Nm/rad$ and the default damping ratio is 0.7.

Since the response of the environment is also a spring-damper system but in the opposite direction, there will be a static error in the reference tracking. As a result, we further include an additional controller that is setting the virtual position to the robot with an integral term. The error dynamic of the additional controller is

$$0 = K_{ph}e_p + K_{vh}e_v + K_I \sum_{n=0}^{i} e_{pn} \tag{7-2}$$

where the stiffness, damping and integral gain is K_{ph}, K_{vh} and K_I. The variable e_{pn} is the error at the time instant n. According to our analysis in Section 5-1, the stiffness of the tooth is between $10Nm/rad$ and $20Nm/rad$. We simplify the force to be linearly relating to the position. As the model of the system is changing, and the force is decreasing over iterations, we take the largest stiffness into account. The stiffness K_{ph}, damping K_{vh} and integral gains K_I are chosen as 4, 0 and 50.

7-1-3 Environment Level

The environment level is in charge of generating torque that counteracts the output of the robot. The change of the model of the tooth reaction and switch between lingual and buccal models are also determined in this part. The details of the model and its additional information are derived from Section 5-2. In the response to the environment, we assume that a certain effective control policy of F_z is performing accordingly so that the slope of the model is what we see in the demonstrations. The flow chart of the organization of this function is shown in Figure 7-4.

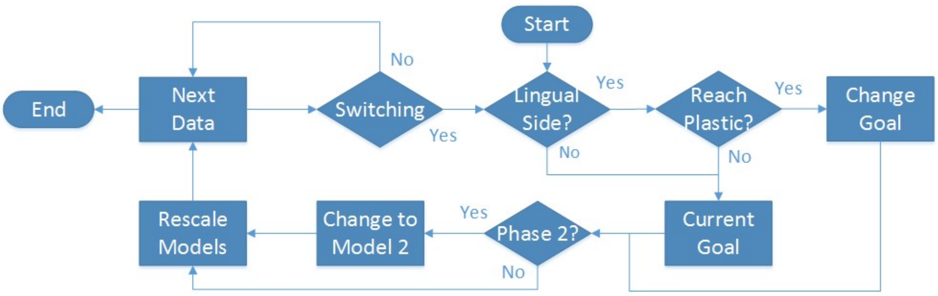

Figure 7-4: The flow chart of the environment level (reaction from the tooth).

The environment takes the position as the input and renders the corresponding torque from the tooth to the dynamic level in order to get the data of next time instant. The entire response of the pig jaw consists of three phases. Phase one has a model of its own without any plastic region at all. Every time the direction of the movement changes, the environment system decides whether to alter the model and then switches to the model of the other side.

The goal of torques during switching is subjected to a manually set model. However, considering the control, the torque is unknown to the robot as well. The key part is whether the controller can identify the plastic region. The objective of the simulation is not affected. The simulation system only needs to provide a general response which is treated unknown by the control policy.

7-1-4 Dynamic Level

This part simulates the velocity and the position based on the force that is generated from the controller and the environment. In this part, we assume that the tooth has a unit moment of inertia. We apply the symplectic Euler method [72] for calculation. This method uses the velocity from the current time step to calculate the current position which increases the numerical stability. The calculation is subject to

$$a(t) = F_{robot} - F_{env} \tag{7-3}$$
$$v(t) = v(t-1) + a(t)\Delta t \tag{7-4}$$
$$x(t) = x(t-1) + v(t)\Delta t \tag{7-5}$$

where $x(t)$, $v(t)$ and $a(t)$ are the angular position, velocity and acceleration of the tooth at the time instant t. The variable F_{robot} and F_{env} are the force of the robot and the tooth respectively. The output of this part is the variables of the simulation system. We choose the sampling time Δt to be the 0.01s.

7-1-5 Summary

The designed simulation system is sufficient for testing the components of Interactive Analysis. The simulation will be first performed on the plastic jaw and then the pig jaw. In addition, We should first test the control policy in the simulation system before starting hypothesis testing and analysis on the physical environment.

7-2 Reproduction System

In this section, we introduce the implementation of the robot reproduction system where Factor-Controlled Experiment will be performed. The reproduction system is implemented on KUKA-iiwa Robot. The control policy learned from the demonstration is sent to the robot by messages in Robotic Operating System (ROS). The experiment is based on a plastic jaw whose teeth is fixed by super glue.

Physical setup The physical setup includes the execution part and measurement part. The physical setup is shown in Figure 7-5.

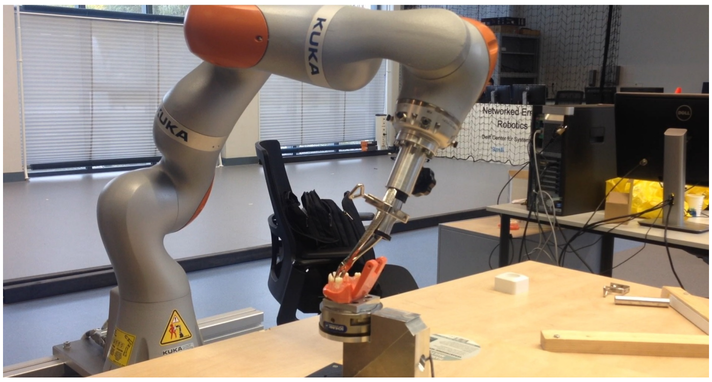

Figure 7-5: The experiment setup for Factor-Controlled Experiment

As shown in the figure, the tooth is glued to the plastic jaw and the plastic jaw is fixed on the sensor with screws. The robot uses forceps as the end effector which grab the tooth and are tightened by a clamp. The robot is performing the task under the default setting. The end effector acts like a critical damping system with the angular stiffness of a $300N/rad$.

For every reproduction, the robot starts with the same pose whose coordinates are with the same orientation as the base coordinates of the robot. The pose of the force sensor, on the other hand, is adjusted for proper lingual and buccal movements. This keeps the position and orientation recorded in the subspace of the movement. We do not have to calibrate the pose data and the robot wrench data based on the tooth position in the robot coordinates. For the force data, we only need the tooth positions with respect to the center of the sensor to calibrate it which are almost the same for different jaws.

Even though the robot has an internal force sensor on each joint and can estimate the Cartesian wrench from the joint space, the force sensor is still essential for measurement. The force sensor inside the robot is calibrated based on the mounted tool. The calibration also includes the information of the tool we provide to the robot as settings. Given the complicated shape of our end effector, we cannot guarantee that the specifications which we provide are accurate.

Software implementation The application is operating in Robot Operating System (ROS). ROS is a well-known software framework for robot software development. It allows that different kinds of the programs are cooperating together by providing efficient communications, namely publications of messages and services.

Every program in the system is referred as a node. We have four nodes, the force sensor node, the control node, the recording node and the robot node. The map which shows how different nodes communicate to each other is shown in Figure 7-6.

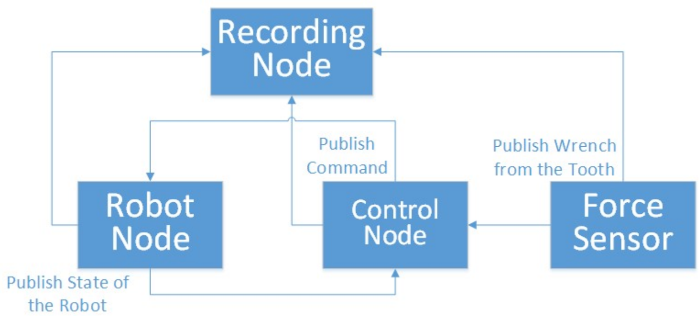

Figure 7-6: The Communication of different ROS nodes

The robot node is established by a GNU ROS stack which is available online. It collects the robot states (the Cartesian pose, wrench, etc.) and publishes to the other nodes, subscribes the control command and executes it on the robot. There are some useful functions available on the robot which turn out to be very helpful in the reproduction. We list them in the Appendix D. The force sensor only sends data through the serial connection. We implement a force sensor node to read the data from the serial port of the force sensor, convert it to the format of the ROS message and publish the data to the ROS system.

The Cartesian pose and wrench published by the robot and the force sensor node are subscribed by the control node. The control node operates based on the control policy that is derived in Section 6-3. It generates and modifies the reference during the reproduction. The control node publishes the control command of the angular reference trajectory at a rate of $10Hz$.

The command of the vertical forces is generated based on the current states of the robot. The trend of the F_z is achieved by scaling the first half and the second half of the model of the control policy with different factors. When the control node decides to switch primitives, it reads the control policy of the position and the force F_z from a file and rescales (by Max and

Min) it for the next movement. In the end, all of the massages are subscribed and recorded by an additional node.

7-3 Hypothesis Testing

In this section, hypothesis testing concerning the plastic jaw will be proposed, and the corresponding modification to the reproduction will be introduced. The number of experiments should be as many as possible to avoid the bias caused by the uncertainty of the tooth.

The reproduction Since the physical testing is performed on the plastic jaw, there are no restrictions on the lingual side. As a result, we apply the control policy on the buccal side of the pig jaw to both sides of the plastic jaw, since the demonstrations of plastic jaws are not representative. In other words, we only test a half of the pig jaw.

According to the human demonstrations on the plastic jaw, the goals for each lingual and buccal movement is set to be $0.7rad$ and $0.9rad$. The F_z is set to be the ones with periodic peak mentioned in the Section 6-3. The maximum trend of F_z is $25N$, and the minimum trend is $-30N$. The magnitude of the oscillation is $10N$ around the trend.

For the interaction analysis framework, the reproduction serves as a working hypothesis. We need an effective control policy for reproduction. The first hypothesis is that the control policy that we derived in Section 6-3 and applied by the robot is effective. The criteria are whether the tooth is getting loose during the reproduction. In addition, with the same test, we can also verify if the plastic jaw has two buccal sides.

Another factor we would like to check in the reproduction is the abnormal F_y with trend included in Section 3-2. Because the force F_y plays an important role in modeling the reaction of the tissues, we have to make sure the F_y observed in the demonstration is not affected by the force in the vertical direction. The hypothesis is that the trend of F_y is caused by the force in the vertical direction instead of the vertical force that is applied sideways. According to the previous analysis, the force F_y in some of the demonstrations has a trend that is at least half of the trend in F_z. The criteria determining whether the trend exists in F_y is that F_y has a trend or mean that is at least one-third of the trend of F_z.

The vertical force Another hypothesis raised in Section 3-2 is about the effect of the vertical force F_z. We will perform the experiment with the control policy of F_z of different shapes and inspect on the effects. Whether the tooth becomes loose is determined by the change of slope in the correlation between F_y and orientation. The change of slope can be found by some measurement such as the reconstruction error of PCA [43]. However, the slope of change is visible, so we do not need such redundant methods.

Whether the force F_z is crucial for loosening the tooth on the plastic jaw will be verified by comparing the experiment with and without controlled F_z. Additionally, in the results without controlled F_z, we can check if the recorded force F_z in the initial experiment is purely the control policy or involves the component that compensates the reaction force of the tooth. This hypothesis is measured by the value of the force F_z from the experiment. If the force F_z is less than a certain threshold, we deem that the force F_z, which is observed in the initial experiment with the control policy of F_z, as the pure human control policy.

Goals of primitives The goal of angle in the initial model seems to be relatively large comparing to the observation from the pig jaw. This may be caused by the lack of bones to restrict the movement. We would like to know if this control policy is safe to perform even on the plastic jaw.

The criteria of this experiment involve the occurrence of the complications and the success rate of the extraction. As for the speed of the extraction, the number of the experiments should be large enough to cover the uncertainties in the teeth which, however, is not possible in our project.

7-4 Summary

In this chapter, we discussed the details of implementation of the simulation and Factor-Controlled Experiment for hypothesis testing. The structure of the simulation was elaborated. It simulates the entire scenario where a robot performs tooth extraction to a tooth on a pig jaw or a plastic jaw. The system consists of a high-level controller (decision level), an executive controller (imaginary robot) and the environment (tooth simulation and dynamic simulation). The simulation aims to test if every component works properly before the physical implementation. By comparing the result of simulation with the demonstration, we will evaluate the performance of the reproduction algorithm and tooth simulation.

The focus of the simulation is tooth simulation (environment) and the control policy for reproduction (decision level). The robot is assumed to be able to implement the impedance control. Other parts of the system are not accurate, which may lead to a bad-looking result. For example, the dynamic level only uses a simple method for integration. However, as long as the control policy for reproduction and tooth simulation works appropriately, we can take those factors as the disturbances of the physical world. In tooth simulation (environment), the generalization function in PbD can be used in change of model that will render a more diverse and reasonable reaction. However, the GMM method still needs some modifications for generalizing tooth simulation.

As for Factor-Controlled Experiment, the physical and software implementation are described. The sensor, high-level controller (computer) and the robot are communicating with each other via ROS. We also include the details of the physical components and the convenient manner where the experiment is carried out. The hypotheses for plastic jaws were highlighted at the end of the chapter in terms of control policy and uncertain information that is related to the model of tooth simulation. However, the hypotheses were based on the plastic jaw. They should be tested on the pig jaw before they become general findings. In addition, the robot still has some limitations for more complicated hypotheses.

Chapter 8

Results and Discussion

Having the Interactive Analysis framework implemented and the experiment designed, we now intend to test each component and investigate how well they can perform.

This chapter includes the evaluation based on the simulation of the pig and plastic jaw and the results of Factor-Controlled Experiment on the plastic jaw. In this thesis, we only study this effect at the beginning of tooth extraction, because the reaction to pulling force from the plastic jaws is not representative.

In Section 8-1. The results of the simulation will be compared with the demonstration and justify the performance of the algorithm components of the Interactive Analysis framework. As for Factor-Controlled Experiment, we will firstly compare the reproduction (the working hypothesis) with the demonstration (Section 8-2-1), and then test and evaluate the hypothesis (Section 8-2-2) that is proposed in Section 7-3.

8-1 Simulation Result

We perform the simulation for both the plastic jaw and the pig jaw according to the design specified in Section 7-1. We will evaluate the performance of the reproduction algorithm and tooth simulation that is working with an imaginary robot. The purpose of the simulation is to test the switch of lingual and buccal models and change of the shape of the model. In addition, we will check whether the high-level controller can render a good reference, stop the movement and switch primitives as desired. As a result, the controller in this section will not stay at the goal for a while as we observed in the demonstration and will do in the physical experiments.

8-1-1 Plastic Jaw

We firstly simulate based on the plastic jaw whose reproduction algorithm and model of tooth simulation are simpler than the one for pig jaw. The purpose of the plastic jaw simulation is to test if the controller can render a good reference and work well with an ordinary robot. The simulation results of the plastic jaw are included in Figure 8-1.

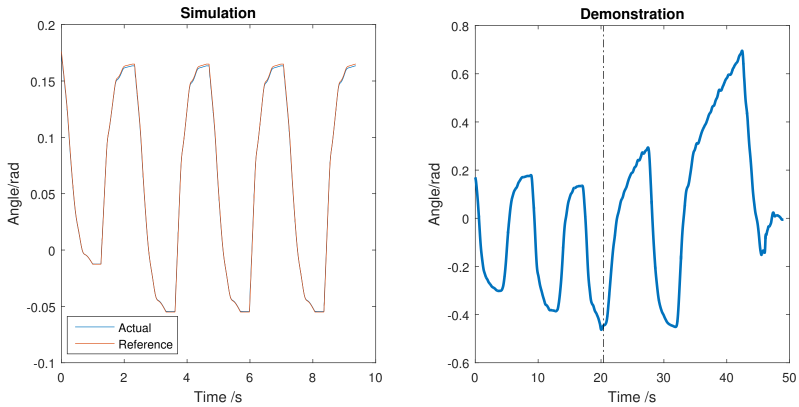

Figure 8-1: The comparison between the movement of simulation on the plastic jaw (left) and the demonstration on the pig jaw (right). In the left part, the actual position (blue) tracks the reference (red) successfully. In the right part, the first two iterations of the demonstrations are as same as most lingual and buccal movements. The last two iterations are the ones at the end of tooth extraction when the tooth is very loose.

The figure on the left is the tracking result of the angle during in the simulation. As can be seen, the reference is tracked, and the movement is stable. The goal is set according to the demonstrations in the plastic jaw which are different from the ones in the pig jaw. Comparing to the human demonstrations on the right, we can see the angular position in the simulation is in similar shape to the one in the first few observations of the demonstration. The later trajectory represents that the practitioner is pulling the tooth and the movement becomes irregular.

The result of the simulation indicates that we can apply this reference trajectory safely on a robot. If the goals are properly set, the control policy is less likely to result in complications. The switch of the lingual and buccal model is shown in Figure 8-2

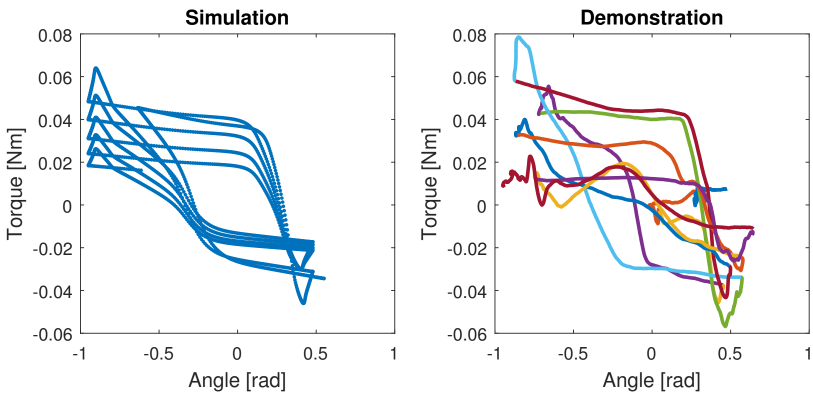

Figure 8-2: The response of the simulated plastic jaw (left). It has a similar shape and the change of the slope to what in the demonstrations of the plastic jaw (right). Different colors indicate different iterations of the movement.

The result of the simulation is in the left part of the figure. The slope does not change in the beginning when the movement fails to reach the plastic region. Comparing to what we observed in the demonstrations as in the right part of the figure, the resulting contour is similar, and the differences between the individual primitives are missing in the simulation.

8-1-2 Pig Jaw

Having a control policy that can render a good reference, we investigate the performance of tooth simulation in the scenario of the pig jaw which is a complete system. This simulation includes the switch of lingual and buccal models and the change of the shape of the model in tooth simulation. In addition, the detection of the ultimate strength in the control policy is also tested by assuming that the robot has no idea where the goal of the primitive is. Similar to the study of the plastic jaw, the reference tracking results of simulation on the pig jaw is firstly included in Figure 8-3.

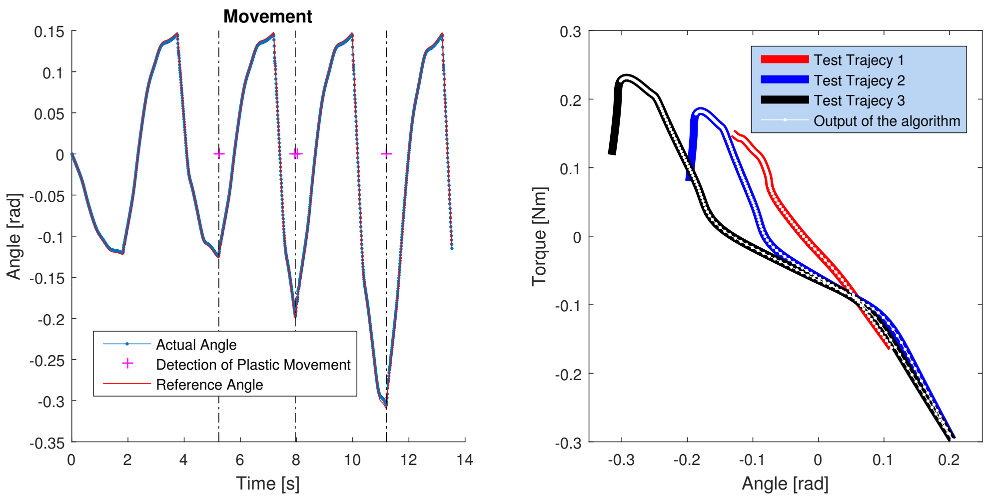

Figure 8-3: The left part is the tracking result of the simulation on the pig jaw. The actual position (blue) track the reference (red) successfully with the stopping point identified (magenta plus). The right part of the figure is the result of applying the algorithm that stops the movement after the ultimate strength. Different colors represent different iterations of the movement. The white line indicates the region that the algorithm allows the robot to move.

We can see from the figure on the left that the goals are reached, and there is not a large static error between the reference and the actual angle. We should notice that the buccal side where the goal of movement stays steady corresponds to the positive angle in the simulation.

The buccal side of the pig-jaw simulation is similar to the one in the plastic-jaw simulation, whilst the ones on the lingual side are different. Comparing to the primitives on the lingual side of the demonstration, the later parts in demonstrations and the pig-jaw simulation both have a sharp turn. In the pig-jaw simulation, it is a result of the function that identifies the ultimate strength. However, it is not clear if the sharp turn can be interpreted in the same way in the demonstration. The magenta plus sign in the figure indicates the time instant when the function of identifying the ultimate strength and stopping the movement is activated. We can tell that the movement slows down before the angle reaches the ultimate strength. In addition, the figure on the right shows some examples of ultimate-strength identifications in the simulation.

As for the reaction of the tooth, the relationship between angles and torques is included in Figure 8-4.

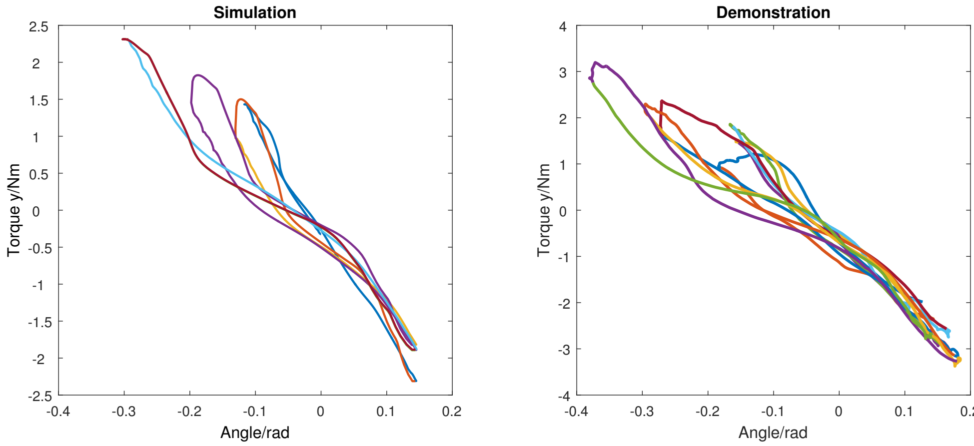

Figure 8-4: The comparison of the torque-angle relationship between simulation (left) and the demonstration (right). Different colors are used to distinguish different iterations of the movement. The overall trends are similar however, the details are different.

As can be seen in the figure, the switch of model happens when the direction of the movement changes. At the beginning of the procedure, the response is subject to a different model which indicates that the bone has not been pushed away. In the later part of the trajectory, the reaction torque switches between two models of lingual and buccal movements and gradually decrease the slope with respect to the angle.

The switch of the lingual and buccal model works properly. However, the shapes of different primitives are too similar comparing to the demonstration. The change of the shape of the model requires a better mechanism than scaling. Perhaps the generalization function of the TP-GMM can render a better result. But, in the end, the model should be replaced by an analytical model which has the mathematical descriptions of the switch and change of the model.

8-2 Results of Factor-Controlled Experiment

In this section, the hypotheses proposed in Section 7-3 are tested on plastic jaws by the robot. We will first reproduce tooth extraction with the initial control policy and obtain the uncertain information about the model. After that, more Factor-Controlled Experiments will be carried out for understanding and comparing control policies.

8-2-1 Reproduction

According to the outcome of our experiments, the robot has successfully removed the tooth with the initial control policy. We will inspect on the results as our hypothesis about the control policy of the initial reproduction (or working hypothesis) that is proposed in Section 6-3. The reference tracking is illustrated in Figure 8-5.

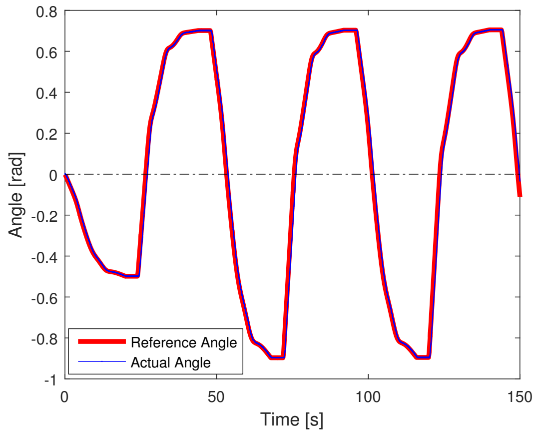

Figure 8-5: The reference tracking of the initial experiment on the robot. The red trajectory is the reference. The blue trajectory is the actual position of the robot

The reference is tracked nicely with a little delay. The delay can be ignored due to the time scale of the procedure. The goals of the primitives are reached. However, we found during the experiments that if the reference changes too fast, the robot cannot follow that reference. We suspect that the reason is that the maximum speed is not large enough to reach the reference in time. For the safety reason, we do not prefer to increase the maximum speed.

Now we start to verify our hypotheses and consider the force during the procedure. The force that is recorded by the force sensor, and the correlation between angular positions and the force F_y are included in the left and right part of Figure 8-6.

We will first test whether the trend in the force F_y is the natural reaction to the vertical force F_z. Both forces are shown in the left figure in Figure 8-6. The force F_y has no visible trend that is larger than $10N$ which is one-third of the trend of F_z. We can conclude that the F_y that we observed in the demonstration is not a natural reaction to the force in the vertical direction. However, where the trend really comes from needs additional hypotheses. Moreover, this can only explain the trend in the demonstration on the plastic jaw. The situation of the pig jaw is likely to be the same but needs to be verified by experiments.

Now we investigate whether the tooth on the plastic jaw gets loose from the beginning of the movement as what we have seen in the demonstrations. It is reflected in the relationship between the force F_y and angle shown in the right figure in Figure 8-6. We found out that the slope of the contour decreases over primitives. The change of the slope indicates that the plastic jaw with super glue is representative in terms of loosening the tooth. However, the contour has a quite different shape compared to the one in the pig jaw. Furthermore, The plastic jaw cannot reflect the behavior of the bone and the tissue around the tooth.

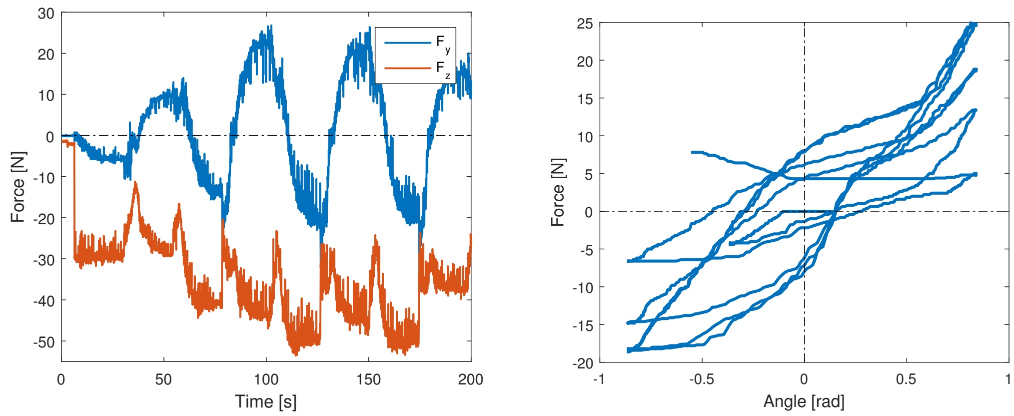

Figure 8-6: The left part of the figure is the comparison of the force F_y (blue) and F_z (red). There is no trend observed in F_y. The right part of the figure is the relationship between F_y and the angle. We can see a drop of the slope indicating the tooth is getting loose.

8-2-2 Hypothesis Testing

Goals of primitives Having a control policy that works, we can modify it and perform experiments about our hypotheses. First and foremost, we need to inspect the large goals of the initial control policy. Some of the extractions with large goals which are $[0.7rad,\ 0.9rad]$ on the lingual side and buccal side, show that the tooth becomes loose, but the gum is destroyed. Consequently, it is not a preferable control policy. Therefore we change the goals to $[0.5rad,\ 0.4rad]$ and perform tooth extraction. The results show that the success rate of loosening the tooth decreases for the small goals however the gum is intact, and the control policy is still effective. The detailed statistical summary of all experiments is included in Section 8-2-3. The correlation between the force and the angular position in the experiment is included in the right part of Figure 8-8. We can see that the control policy is able to loosen the tooth.

The vertical force We also would like to study the effect of F_z. We eliminate the F_z from the control policy with the smaller goals and perform tooth extraction again. The hypothesis about whether there is a reaction force in the vertical direction is justified by the force F_z which is shown in Figure 8-7.

As we can see from the figure, there is F_z oscillating with a relatively high magnitude. According to the description from the practitioner, they are pressing down on the buccal side. However, as it turns out in the result of Factor-Controlled Experiment, it requires larger vertical pressure to reach the goal of the buccal side.

We also notice that the force F_y is less regulated than the testing when there is the control policy for F_z. We suspect that the F_z also fully stretches the tissue so that it is easier to be broken. This has to be tested on the pig jaw. However, this needs improvement on the experiment setup since the table is not entirely steady. Therefore our suspicion is not the only possible cause of the irregular force.

After that, we compare results in terms of breaking of the tooth which is indicated by the decrease of slope in the correlation with or without F_z. The result is included in Figure 8-8.

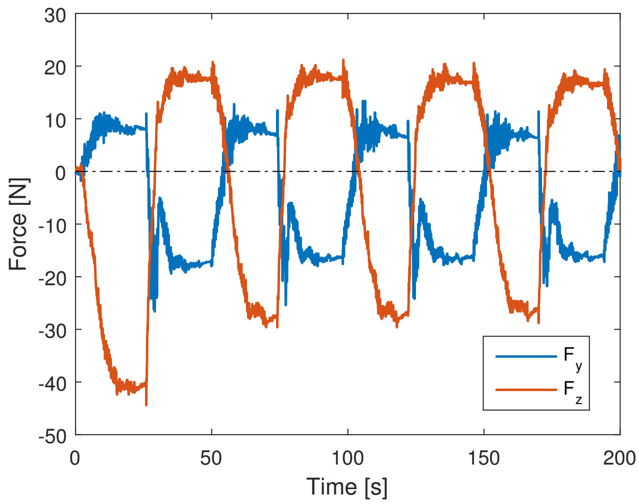

Figure 8-7: The force F_z (red) and F_y (blue) experiment without F_z. There is still a force F_z with relatively large value even though the robot apply no force in the vertical direction.

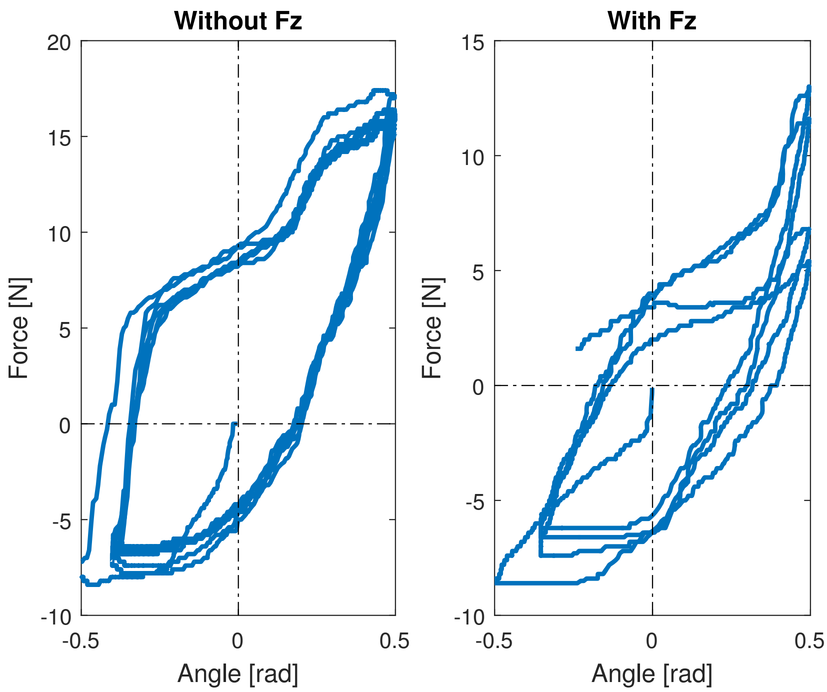

Figure 8-8: The comparison between the experiment with (left) and without F_z (right). The experiment with F_z results in a slope decrease in the model which means that the tooth becomes loose.

In the figure on the left without F_z control policy fails to decrease in slope. By contrast, the one with F_z on the right successfully decreases the slope. Not all of the experiments with F_z succeed in the task. However, none of the experiments without F_z manage to loosen the tooth. We performed four experiments for each situation. The comparison shows that it is very likely that the force F_z plays an important role in loosening the tooth.

The control policy we have is rather complicated for human. We simplify the control policy of F_z to a constant value as shown in Figure 8-9.

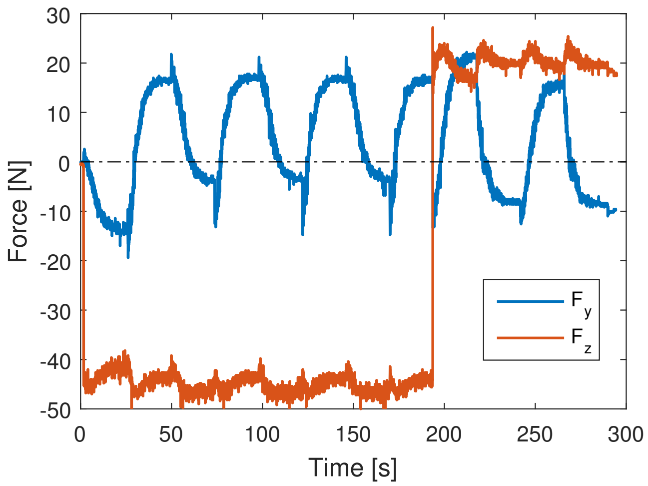

Figure 8-9: The force F_z (red) and F_y (blue) experiment of constant F_z.

It turns out that the tooth does not get loose. However, we only performed 2 experiments on this policy. It is not sufficient to conclude that the control policy is not effective due to the uncertainty of the tooth. In addition, the hypothesis about the control policy needs more experiments so that we can draw the conclusion statistically based on the criteria for hypotheses that are mentioned in Section 7-3.

8-2-3 Summary of all Experiments

In total, we carried out ten experiments for the initial control policy. One of them fails to get the tooth loose. Three of them manage to get the teeth loose, but some unexpected occurrences happened such as the clamp falls or the plastic jaw is torn off the base. Five of them pulled the tooth out. As to the complications, at least four of the experiments result in damaged gums. Surprisingly, with the movement so large, only one of the experiments broke the root of the tooth whereas it is quite common in the human demonstration. The initial experiments help us get familiar with the procedure. For example, in the later hypothesis testing, we manage to avoid the clamp falls.

For hypothesis testing, we performed 1 experiment with the same goals as in initial control policy but without F_z. The tooth is extracted out, but the complications are more visible. In addition, we perform five experiments with F_z but with a small movement. Three of them manage to loosen the tooth. All the gums and roots are intact. Another five experiments are carried out without F_z and large movements. None of them tooth becomes loose. For more reliable comparison, we choose the teeth in the opposite direction on the same jaw. It is more likely that the pair of teeth have a similar structure and the similar extend of how much they are glued.

8-3 Summary

In this chapter, we evaluated the components of Interactive Analysis in a simulation based on both the pig jaw and the plastic jaw. The simulation results show that tooth simulation responds to the movement properly in a way similar to what is observed in the demonstrations. The control policy for the reproduction can be implemented on an ordinary robot and perform tooth extraction on the pig jaw or the plastic jaw.

After the simulation, we performed reproductions for tooth extraction. The reproduction with the initial control policy removed the tooth with a very high success rate. Based on the reproduction and Factor-Control Experiment, we found out that the trend force of F_y is not the natural reaction to the vertical force. In terms of the control policy, the goals of the lingual and buccal movement should be small enough to keep the gingiva (gum) intact. In addition, we discovered that the force F_z in the vertical direction is crucial for loosening the tooth. In the end, we tested a simpler control policy of F_z that failed to loosen the tooth. However, the ineffectiveness of this control policy requires more experiments for confirmations.

The Factor-Control Experiment that we performed is only a trial on the plastic jaw. The observation from the plastic jaw needs to be confirmed by the experiments on the pig jaw. The force F_y without the trend is an important result regarding understanding and modeling the reaction force of the tooth, since most of the demonstrations have the trend in the force F_y. This means we have to model which is the correlation between movement and force based on the data collected from the robot. In the end, we need a better place for the force sensor where fewer factors are affecting the procedure such as the moving table.

Tooth simulation still needs a lot of improvements. The shape of the correlation is simple and without the uncertain change of shapes. The scaling method for the change of slopes may distort the correlation and results in an unrealistic response. Furthermore, how to set goals to primitives in the control policy is still unclear. The discovered effect of the F_z on the model of tooth simulation is too general. The model of tooth simulation can be improved after we have the neater data through Factor-Controlled Experiment and settle the uncertainties by hypothesis testing. In the meantime, the knowledge about the model would help us devise more efficient control policies.

Chapter 9

Conclusion and Future Work

In this thesis, we preprocessed, segmented and analyzed the human control policy and the reaction force from the tooth based on the data of human demonstrations of tooth extraction. Motivated by the knowledge that cannot be obtained from the analysis on human demonstrations, we implemented the Interactive Analysis framework which is a new application of Programming by Demonstration (PbD). The application is inspired by the analysis of tooth extraction and used for settling uncertain observations, collecting missing information, and studying, improving the control policy of tooth extraction. In addition, the framework can also transfer the improved control policy to students and practitioners by a training setup.

The Interactive Analysis consists of three parts: tooth simulation for training, the reproduction algorithm and Factor-Controlled Experiment for a hypothesis. Factor-Controlled Experiment based on the reproduction algorithm is for the analysis, and tooth simulation is for training. As the result of this thesis, all of the components are functioning, and some uncertain information of the plastic jaw was obtained by Factor-Control Experiment in the framework.

9-1 Summary and Conclusion

In this thesis, we firstly summarized a general procedure for preprocessing. This part is indispensable for both analysis and PbD. The high-dimensional recordings with irrelevant information were filtered and trimmed in order to get a set of clean and low-dimensional data. We further segmented the data in terms of primitives so that similar movement can be grouped together and studied. In the last step of preprocessing, we applied a time series analysis method investigating the CUmulative SUM statistics of the demonstrations and found out a pattern in the control policy. Furthermore, in order to encode the control policy properly, we summarized and came up with some principles for choosing control variables for tooth extraction.

As a result of the analysis, a model should be derived for describing the reaction of the tooth. We found out that loosening the tooth could be described as the change of slope in the correlation between the force along the direction of the movement and the angular position. However, the model is unclear, and the reliable data for the force are scarce. Hence, before the procedure can be described by an analytical model, PbD is used to model the reaction force for the training setup in the framework.

Factor-Controlled Experiment is the core component of Interactive Analysis. The uncertain information about the control policy and the tooth reaction are obtained from this component by means of hypothesis testing. A robot is used to reproduce tooth extraction. By only changing a certain factor concerning the hypothesis in the control policy and comparing the result with the prediction of the hypothesis, the hypothesis is tested.

The robot reproduction is the basis of Factor-Controlled Experiment. As we have the demonstrations from the practitioners, PbD is applied for precisely reproducing the observed control policy.

Tooth simulation and the reproduction algorithm were tested in a simulation system and proved to be functional under the model of the pig jaw. The hypothesis testing was used to retrieve the information about the plastic jaw by means of Factor-Controlled Experiment. The conclusions of this thesis are as following:

- An Interactive Analysis framework which consists of tooth simulation, the reproduction algorithm and Factor-controlled Experiment was proposed for the analysis of tooth extraction. The framework is suitable for analyzing the control procedure where the model of the environment is changing with uncertainties, and the interaction force is involved in the procedure. The framework obtains the uncertain information by testing the hypothesis which a standard scientific method that can be used by any researchers studying tooth extraction.

  However, there are also some limitations of the framework. The robot that reproduces the procedure should be advanced enough for the requirement of the hypothesis. The force should be measurable by the instrument. However, in the PbD application, measuring force is not always convenient [59].

- The reaction force of the tissue is related to the force F_y along the direction of the movement. The correlation between the force F_y and the position of the movement is of an elastic-plastic shape. The loosening of the tooth occurs when the movement reaches the plastic region. The reaction force that is applied to the root of the tooth results in the torque for the rotational movement. In the current model of tooth simulation, all of the effects of tissue are regarded as a spring-damper system. The torque is used to represent the reaction to the movement in tooth simulation. Tooth simulation renders a similar correlation in terms of the switch of the model comparing to what is shown in the demonstration.

  However, the data of the F_y without trend is very limited, and they are not sufficient to derive the model not mentioning the uncertainties of the tooth. Furthermore, the vertical force has some effects on F_y, but the details are unknown. Only the correlation between the average maximum torque and the slope was discovered. How the slope is decreasing and how the maximum torque is changing during the decrease of the slope is unknown. Therefore tooth simulation requires more information to make it more realistic.

- As for the control policy, the movement slows down around the plastic region in the demonstrations. In addition, the movement on the lingual side is with an increasing goal whereas on the buccal side the movement always stops in a similar region. Furthermore, the force F_z is crucial for tooth extraction, and the control policy of F_z has several kinds of shapes, magnitudes and the trend. The angle of the lingual-buccal movements and the force F_z in the vertical direction were chosen for reproduction.

  However, the effectiveness of different control policies of F_z is still unknown not to mention determining the most effective one. Furthermore, some other factors that is ignored in the analysis such as the shift in the direction perpendicular to the movement and the trend in the force F_y may be meaningful regarding the control policy.

- The GMM-GMR algorithm in PbD was applied to encode the control policies and the model of the reaction force. The advantage is that the reproduced trajectory always stays within the constraints in the observations. The control policy of human is implicitly in the model. The trajectories of multiple variables can be reproduced individually or with correlation, Therefore, the control policies in different dimensions can be applied separately or recombined according to the requirements. In addition, the robot learns the procedure directly from humans. Therefore the explicit programming is avoided, and the system is easier to use.

  However, the uncertainty of the model in tooth simulation is lost in the model of PbD. The trajectory inside GMM-GMR is regressed as linear which results in loss of details. Furthermore, this algorithm cannot set the trend in F_z as well. The trend is set by the human in our project. In addition, PbD can only imitate what has been seen in the demonstrations. The extrapolation of the control policy and the model is not reasonable and even dangerous

- The general steps of preprocessing for PbD were proposed, which include coordinate transformations, smoothing, feature selections, data cleaning, normalization and segmentation. Furthermore, some rules for choosing control variables for reproduction was summarized as well.

  As for what is missing in the procedure, the abnormal observations or inconsistent control policies are identified by humans. In addition, the coordinate transformation is now between several user-defined coordinates. Furthermore, the rules for choosing the control variables are still descriptive. The result of choosing control variables may be biased if there is no metrics for the rules. In addition, the rules cannot decide what to choose when none of the patterns that are used in the rules are in the demonstrations.

- Factor-Controlled Experiment for the hypothesis testing was applied to the plastic jaw, which has proved that the abnormal trend of F_y is not the natural reaction force to the movement or to the vertical force. There is a reaction force in the vertical direction corresponding to the movement without the control policies of the vertical force. Moreover, Factor-Controlled Experiment is able to obtain the proper data for the identification of the correlation between the force F_y and the position.

  The applied hypotheses in this thesis are very general. It is only sufficient for a question whose answer is yes or no. It is not good enough to get more detailed information. In addition, the plastic jaw is not representative for the actual tooth extraction.

9-2 Future Work

This thesis highlighted some problems of analysis of tooth extraction and introduced a systematic framework for analyzing the model, improving the control policy and transferring the improved control policies to students or practitioners. There are many aspects that can be improved.

- The Interactive Analysis framework can be improved to make it more convenient for analysis. It is only one of the solution for analyzing the demonstration where some information is missing because of the interaction force. There may be better methods that could render us better data involving interaction forces. This is an interesting topic that is worth exploring.
- As we can obtain the data of F_y without trends easily now, an analytical model can be derived by system identification based on Factor-Controlled Experiment. Consequently, the torque of the reaction should be calculated from F_y in tooth simulation. The detailed effects of the vertical force F_z on the correlation should be put into hypotheses in the Interactive Analysis framework. In addition, other uncertainties, such as the ones in the slope and the maximum of reaction wrench when fibers are breaking, may be unveiled to us. Those factors are closely related to the goals of the primitives during tooth extraction.
- The force F_z of different shapes, trends and magnitudes should be tested and evaluated by hypotheses in Factor-Controlled Experiment. Another factor about both the control policy and model is how the interaction force changes when the tooth only moves towards one side of the tooth. We do not have demonstrations of them, and we are curious to see if the control policy is better than the ordinary ones. In addition, all of the analysis shall be under the same conditions that relate to uncertainties (type of the tooth, age, etc.).
- For preprocessing, we should develop an algorithm that is able to identify the abnormal observations and the inconsistent control policies. In addition, for the variable-constrained procedure such as tooth extraction, an algorithm could be devised to extract the subspace of the coordinate frame where the procedure is performing. Finally, the rules for choosing control variables is an interesting and important topic for PbD. The generality of the rules should be further studied. Some numerical metrics for those rules are required for an accurate decision for humans or even combining with the algorithm of PbD in the robot. If the rules are not sufficient for an application, maybe more rules are needed or, in the worst case, we could apply Factor-Controlled Experiment based on hypotheses of the control variables.
- In order to obtain more details that are missing, we need more elaborating hypotheses and to control different factors more carefully. The statistical hypothesis testing should be used for the results of Factor-Controlled Experiment. And above all, since the Factor-Controlled Experiment is working, the experiments should be performed on more representative jaws such as pig jaws to confirm the findings in this thesis.

Appendix A

Tools for Analysis

In this project, we introduce a tool based on the Matlab figure for analysts to look into the data conveniently. The screen shot of the tool is included in Figure A-1.

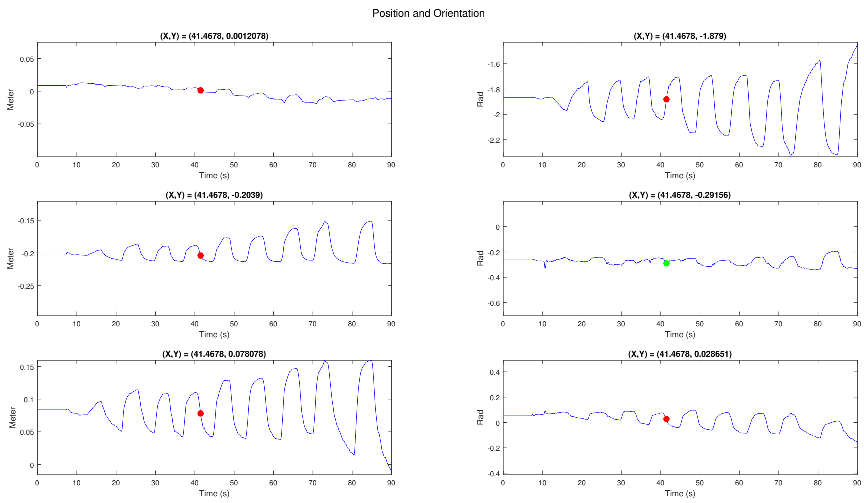

Figure A-1: The assisting tool for analysis

All of the data for pose is included in a Matlab figure above. In the figure, there is a dot that is moving with the mouse cursor for each of the subplot.

The green dot indicates that the mouse cursor is closest to its subplot and it is moving with the mouse cursor. All the red dots are moving with the green dot according to its time value.

The title of each subplot is displaying the value of its own dot in real time. This tool is very convenient for handling multiple time series in the real time. We could check the correlation between different dimensions by simply moving the mouse cursor. This tool brings a good insight before further analysis.

Appendix B

The Abnormal Trend

We further investigate the movement in details, which is discussed in Section 3-2, to see if there is other evidences. Since the trajectory should be a circle, we fit a circle on the trajectory of the end effector of the robot with both the demonstration with and without trend in the force F_y. Suppose that the circle is subject to

$$x_n^2 + y_n^2 + ax_n + by_n + c = 0 \tag{B-1}$$

where

$$c = \frac{a^2}{4} + \frac{b^2}{4} - R^2$$

where the variable R is the radius of the movement, x_n, y_n are the coordinate of each data point from recordings after transformations and $a/2$, $b/2$ is the coordinate of the center of the circle. As a result, we can apply least square method on all data points in the same primitive in order to obtain the parameter a,b and c:

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = - \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ \ldots & \ldots & \ldots \\ x_n & y_n & 1 \end{bmatrix}^{\dagger} \begin{bmatrix} x_1^2 + y_1^2 \\ x_2^2 + y_2^2 \\ \ldots \\ x_n^2 + y_n^2 \end{bmatrix} \tag{B-2}$$

where $[\ ]^{\dagger}$ is the notion of the pseudo-inverse of a non-square matrix. Consequently we can obtain the parameter of the circle. The result of fitting a circle in one of the demonstration is included in Figure B-1.

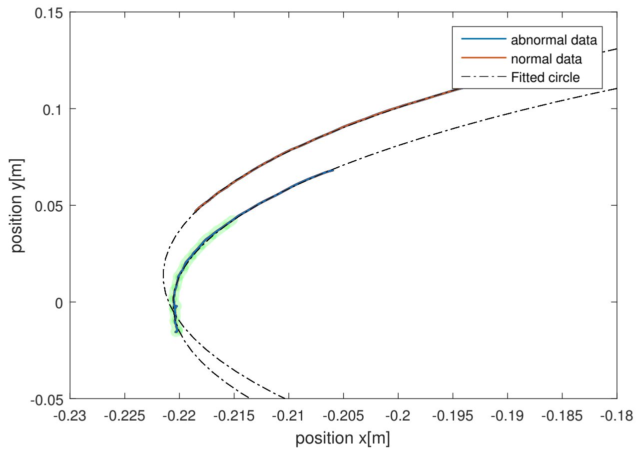

Figure B-1: The movement of the forceps in the demonstration with trend in F_y (red) and without the rend (blue). The shape highlights the area where the movement is no longer on the circle.

We can see from the figure that at the end of the lower trajectory, the data fails to stay on the circle that is fitting the data in other parts of the trajectories. However, the rest of data point are as described on the circle which is centered on the point around the root of the tooth.

Appendix C

Learning Methods for DMPs and the GMM

C-1 Learning of DMPs

In the DMPs model as in Section 6-2-1, the non-linearity is only encoded in a set of weighted Gaussian functions. Therefore the output of the system is a sequence of desired positions, velocities and accelerations. The forcing term can be computed from the formulas in the transformation system directly. Now we assume that we retrieve the position, velocity and acceleration $[\ddot{y}_d(t), \dot{y}_d(t), y_d(t)]$ from a demonstration where $t \in [1, 2, ..., N]$ is the duration of the demonstration. The weights of the Gaussian kernels can be learned in a supervised learning manner. Given the data of demonstrations of the forcing term calculated from

$$f_{\text{target}} = \tau^2 \ddot{y} - \alpha_z(\beta_z(g - y_d) - \tau \dot{y}_d) \tag{C-1}$$

In order to obtain the value of f_{target}, the constant parameters α_x, α_z and β_z in the canonical and the transformation system should be predefined by the user. The goal g is the position at the end of the demonstration, i.e. $g = y_d(N)$. Intuitively, y_0 equals the position at the beginning of the demonstration ($y_d(0)$). The parameter τ is related to the duration of the demonstration which is recommended in [47] to be 1.05 times the duration of the demonstration. Since the duration time is not always accurate, we need some redundancies to make sure the system finishes the desired task.

In the forcing term of the discrete DMPs, the value c_i can be set equal to the pacing of canonical system $x(t)$, and h_i is normally set to be identical values. The weights w_i are learned by Locally Weight Regression (LWR) [73] in most of the literature about DMPs for LWR is a fast learning procedure for this regression problem. LWR aims to find the weights w_i that minimize the error between f_{target} and force f calculated from the weighted sum of Gaussian functions:

$$J_i = \sum_{t=1}^{P} \Psi_i(t)(f_{\text{target}} - w_i \xi(t)) \tag{C-2}$$

where $\xi(t) = x(t)(g - y_0)$ for the DMPs system. Since it is a linear regression problem, the solution can be found analytically by

$$w_i = \frac{s^T \Gamma_i f_{\text{target}}}{s^T \Gamma_i s} \tag{C-3}$$

where

$$s = \begin{pmatrix} \xi(1) \\ \xi(2) \\ \vdots \\ \xi(N)) \end{pmatrix} \quad \Gamma_i = \begin{pmatrix} \Psi_i(1) & 0 & \cdots & 0 \\ 0 & \Psi_i(2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Psi_i(N) \end{pmatrix} \quad f_{\text{target}} = \begin{pmatrix} f_{\text{target}}(1) \\ f_{\text{target}}(2) \\ \vdots \\ f_{\text{target}}(N)) \end{pmatrix}$$

. We can see from Eq. (C-3) that, for different scales of time and $\xi(t)$, the weights w_i remain the same due to the invariance property of the DMPss. Parameters w_i can also be learned incrementally on-line using the recursive least squares [73] or the Locally Weighted Projection Regression which is an on-line regression with varying shapes of Gaussian kernels [74]. Another method worth mentioning for learning movement primitives is reinforcement learning [75]. Since on-line learning methods cannot be applied in this project, we skip the details of these methods.

C-2 Learning of the GMM

In the GMM as in Section 6-2-2, the spatial constraints or task parameters are encoded in the GMM. The learning procedure of the GMM consists of 2 parts: determining the center of Gaussian functions and estimating the weights and shapes of Gaussian functions.

The number of the Gaussian functions in the mixture is predefined by users. Then an unsupervised learning method such as K-means can be applied to decide the initial parameters of the mixture iteratively [76]:

1. For a given cluster C, the total variance is minimized by a set of means $\{m_i, m_2, ..., m_k\}$ of current assigned clusters

$$x_C = \arg\min_m \sum_{x \in C} \|x_i - m\|^2 \tag{C-4}$$

.

2. Given the current set of $\{m_i, m_2, ..., m_k\}$, assign each observation to the closest cluster mean

$$C(i) = \arg\min_{1 \le k \le K} \|x_i - m_k\|^2 \tag{C-5}$$

.

3. Iterate step 1 and step 2 until the assignment stays still. Calculate covariance matrix and exit.

Having the initial values of the Gaussian functions, an Expectation-Maximum (EM) [76] algorithm is applied to determine the shape of Gaussian functions and weights of the mixture. For simplicity we take a mixture of 2 Gaussian functions ϕ_{θ_2} and ϕ_{θ_1}:

1. Expectation Step: Compute the responsibilities for each Gaussian component. i is sampling index which is $\{1, 2, ..., N\}$:

$$\hat{\gamma}_i = \frac{w_2 \phi_{\theta_2}(x_i)}{(1 - w_2)\phi_{\theta_1}(x_i) + w_2 \phi_{\theta_2}(x_i)}$$

.

2. Maximization Step: Compute the weighted means and variances:

$$\begin{aligned}
\hat{\mu}_1 &= \frac{\sum_{i=1}^{N}(1 - \hat{\gamma}_i)x_i}{\sum_{i=1}^{N}(1 - \hat{\gamma}_i)} \\
\hat{\sigma}_1 &= \frac{\sum_{i=1}^{N}(1 - \hat{\gamma}_i)(x_i - \hat{\mu}_1)^2}{\sum_{i=1}^{N}(1 - \hat{\gamma}_i)} \\
\hat{\mu}_2 &= \frac{\sum_{i=1}^{N} \hat{\gamma}_i x_i}{\sum_{i=1}^{N} \hat{\gamma}_i} \\
\hat{\sigma}_2 &= \frac{\sum_{i=1}^{N} \hat{\gamma}_i (x_i - \hat{\mu}_2)^2}{\sum_{i=1}^{N} \hat{\gamma}_i} \\
\hat{w}_2 &= \frac{\sum_{i=1}^{N} \hat{\gamma}_i}{N}
\end{aligned}$$

where $\theta = (\mu, \sigma^2)$ is parameters of Gaussian distribution ϕ.

Appendix D

Function of the Robot

The robot can operate under two control modes: Cartesian mode and Joint mode. Given the requirements of the reproduction, it is sufficient for the robot to work with the Cartesian mode.

One of the helpful functions of the robot is registering the tool which allows the robot rotating around any given point. The rotation command is sent in terms of the quaternion which does not contain singularities. We cannot set a desire velocity as command to the robot, but the velocity can be implicitly set by sending continuous reference signals.

Regarding the controller, the robot has an internal integral controller in the Cartesian control mode with **default setting**. We discover the integral controller by contacting the endeffector with a tooth and setting the reference position to $1mm$. Given the stiffness of $5000N/m$, the contact force should be $5N$. However, as we observed, the contact force start with $5N$ and increases its value every 3-5 seconds. It reached $40N$ in the end before we stopped the robot.

The internal integral controller gets grid of the inconvenience of building an additional high-level controller. On the contrary, it also prevents us implementing the vertical force by the virtual position in the impedance control. Fortunately, there is another mode name 'constant force over lay' that could work with Cartesian control mode with **default setting** and apply an additional force while the robot is operated by the impedance controller.

There is a flaw in the 'constant force over lay' mode. Every time the robot receives a new force command, it at first switches off the current force and then applies the new force. This results in a large noise with a nonzero mean.

Bibliography

[1] S. Schaal, J. Peters, J. Nakanishi, and A. Ijspeert, "Control, Planning, Learning, and Imitation with Dynamic Movement Primitives," *Workshop on Bilateral Paradigms on Humans and Humanoids, 2003 IEEE International Conference on Intelligent Robots and Systems IROS*, pp. 1–21, 2003.

[2] T. J. Teo, G. Yang, and I.-M. Chen, "Compliant manipulators," in *Handbook of Manufacturing Engineering and Technology*, pp. 2229–2300, Springer, 2015.

[3] J. W. Friedman, "The prophylactic extraction of third molars: A public health hazard," *American Journal of Public Health*, vol. 97, no. 9, pp. 1554–1559, 2007.

[4] L. K. McCaul, W. M. Jenkins, and E. J. Kay, "The reasons for extraction of permanent teeth in Scotland: a 15-year follow-up study.," *British dental journal*, vol. 190, no. 12, pp. 658–662, 2001.

[5] M. Cicciù, E. Bramanti, F. Signorino, A. Cicciù, and F. Sortino, "Experimental study on strength evaluation applied for teeth extraction: an in vivo study.," *The open dentistry journal*, vol. 7, pp. 20–6, 2013.

[6] B. D. Argall, B. D. Argall, S. Chernova, S. Chernova, M. Veloso, M. Veloso, B. Browning, and B. Browning, "A survey of robot learning from demonstration," *Robotics and Autonomous Systems*, vol. 57, no. 5, pp. 469–483, 2009.

[7] A. Billard, S. Calinon, R. Dillmann, and S. Schaal, "Robot programming by demonstration," pp. 1371–1394, 2008.

[8] R. Lehtinen and T. Ojala, "Rocking and twisting moments in extraction of teeth in the upper jaw.," *International journal of oral surgery*, vol. 9, no. 5, pp. 377–382, 1980.

[9] L. Bodner, P. a. Brennan, and N. M. McLeod, "Characteristics of iatrogenic mandibular fractures associated with tooth removal: review and analysis of 189 cases.," *The British journal of oral & maxillofacial surgery*, vol. 49, no. 7, pp. 567–72, 2011.

[10] M. Macluskey and J. Durham, "Oral surgery undergraduate teaching and experience in the United Kingdom: a national survey.," *European journal of dental education : official journal of the Association for Dental Education in Europe*, vol. 13, no. 1, pp. 52–57, 2009.

[11] R. Lehtinen and T. Ojala, "Apparatus for simulating extraction forces.," *International journal of oral and maxillofacial surgery*, vol. 15, no. 4, pp. 444–445, 1986.

[12] J. Durham, U. Moore, I. Corbett, and P. Thomson, "Assessing competency in dentoalveolar surgery: a 3-year study of cumulative experience in the undergraduate curriculum," *European Journal of Dental Education*, vol. 11, no. 4, pp. 200–207, 2007.

[13] A. MacGregor and G. Tomlinson, "An Apparatus for Measuing The Forces of Dental Extraction," vol. 17, no. December 1977, pp. 71–76, 1979.

[14] D. Locker, R. Poulton, and W. M. Thomson, "Psychological disorders and dental anxiety in a young adult population," *Community Dent Oral Epidemiol*, vol. 29, no. 6, pp. 456–463, 2001.

[15] X. Sun, F. D. McKenzie, S. Bawab, J. Li, Y. Yoon, and J.-K. Huang, "Automated dental implantation using image-guided robotics: registration results," *International journal of computer assisted radiology and surgery*, vol. 6, no. 5, pp. 627–634, 2011.

[16] D. Wang, H. Tong, Y. Shi, and Y. Zhang, "Interactive haptic simulation of tooth extraction by a constraint-based haptic rendering approach," in *Robotics and Automation (ICRA), 2015 IEEE International Conference on*, pp. 278–284, May 2015.

[17] K. Grave and S. Behnke, "Incremental action recognition and generalizing motion generation based on goal-directed features," *2012 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pp. 751–757, 2012.

[18] S. García, J. Luengo, and F. Herrera, *Data preprocessing in data mining*. Springer, 2015.

[19] H. Bruyninckx and J. De Schutter, "Specification of force-controlled actions in the âĂIJtask frame formalismâĂİ-a synthesis," *IEEE transactions on robotics and automation*, vol. 12, no. 4, pp. 581–589, 1996.

[20] S. Dong and B. Williams, "Motion learning in variable environments using probabilistic flow tubes," in *Robotics and Automation (ICRA), 2011 IEEE International Conference on*, pp. 1976–1981, IEEE, 2011.

[21] L. Pais, K. Umezawa, Y. Nakamura, and A. Billard, "Learning robot skills through motion segmentation and constraints extraction," in *HRI Workshop on Collaborative Manipulation*, 2013.

[22] J. Kober, M. Gienger, and J. J. Steil, "Learning Movement Primitives for Force Interaction Tasks," *International Conference on Robotics and Automation*, vol. 3, pp. 3192–3199, 2015.

[23] S. Morante, J. G. Victores, and C. Balaguer, "Automatic demonstration and feature selection for robot learning," in *Humanoid Robots (Humanoids), 2015 IEEE-RAS 15th International Conference on*, pp. 428–433, IEEE, 2015.

[24] N. Jetchev and M. Toussaint, "Discovering relevant task spaces using inverse feedback control," *Autonomous Robots*, vol. 37, no. 2, pp. 169–189, 2014.

[25] M. Mühlig, M. Gienger, J. J. Steil, and C. Goerick, "Automatic selection of task spaces for imitation learning," in *2009 IEEE/RSJ international conference on intelligent robots and systems*, pp. 4996–5002, IEEE, 2009.

[26] T. Asfour, P. Azad, F. Gyarfas, and R. Dillmann, "Imitation learning of dual-arm manipulation tasks in humanoid robots," *International Journal of Humanoid Robotics*, vol. 5, no. 02, pp. 183–202, 2008.

[27] S. Krishnan, A. Garg, S. Patil, C. Lea, and G. Hager, "Transition State Clustering : Unsupervised Surgical Trajectory Segmentation For Robot Learning," *international Symposium on Robotics Research (ISRR)*, pp. 1–16, 2015.

[28] V. Pavlovic, J. Rehg, and J. MacCormick, "Learning switching linear models of human motion," *Nips*, 2001.

[29] Y. Xu, K. A. Heller, and Z. Ghahramani, "Tree-based inference for dirichlet process mixtures," pp. 623–630, 2009.

[30] F. Pedregosa, G. Varoquaux, A. Gramfort, V. Michel, B. Thirion, O. Grisel, M. Blondel, P. Prettenhofer, R. Weiss, V. Dubourg, J. Vanderplas, A. Passos, D. Cournapeau, M. Brucher, M. Perrot, and E. Duchesnay, "Scikit-learn: Machine learning in Python," *Journal of Machine Learning Research*, vol. 12, pp. 2825–2830, 2011.

[31] S. H. Lee, I. H. Suh, S. Calinon, and R. Johansson, "Autonomous framework for segmenting robot trajectories of manipulation task," *Autonomous Robots*, vol. 38, no. 2, pp. 107–141, 2015.

[32] C. M. Bishop, *Pattern Recognition and Machine Learning*. Springer, 2006.

[33] D. L. Weakliem, "A critique of the bayesian information criterion for model selection," *Sociological Methods & Research*, vol. 27, no. 3, pp. 359–397, 1999.

[34] D. Kulić, W. Takano, and Y. Nakamura, "Online segmentation and clustering from continuous observation of whole body motions," *IEEE Transactions on Robotics*, vol. 25, no. 5, pp. 1158–1166, 2009.

[35] E. Gombay, "Change detection in autoregressive time series," *Journal of Multivariate Analysis*, vol. 99, no. 3, pp. 451–464, 2008.

[36] L. Zhao, Q. Liu, P. Du, G. Fu, and W. Cao, "A novel change-point detection approach for monitoring high-dimensional traffics in distributed systems," in *Applied Mechanics and Materials*, vol. 536, pp. 499–511, Trans Tech Publ, 2014.

[37] I. Jolliffe, *Principal component analysis*. Wiley Online Library, 2002.

[38] W. Mendenhall, T. Sincich, and N. S. Boudreau, *A second course in statistics: regression analysis*, vol. 5. Prentice Hall Upper Saddle Riverˆ eNew Jersey New Jersey, 1996.

[39] Y. Kawahara, T. Yairi, and K. Machida, "Change-point detection in time-series data based on subspace identification," in *Seventh IEEE International Conference on Data Mining (ICDM 2007)*, pp. 559–564, IEEE, 2007.

[40] H. Cho and P. Fryzlewicz, "Multiple-change-point detection for high dimensional time series via sparsified binary segmentation," *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, vol. 77, no. 2, pp. 475–507, 2015.

[41] V. Jurdjevic, "Change-point detection using krylov subspace learning," 2007.

[42] Z. Bankó, L. Dobos, and J. Abonyi, "Dynamic principal component analysis in multivariate time-series segmentation," *Conservation, Information, Evolution-towards a sustainable engineering and economy*, vol. 1, no. 1, pp. 11–24, 2011.

[43] A. Singhal and D. E. Seborg, "Clustering multivariate time-series data," *Journal of chemometrics*, vol. 19, no. 8, pp. 427–438, 2005.

[44] A. Ijspeert, J. Nakanishi, and S. Schaal, "Movement imitation with nonlinear dynamical systems in humanoid robots," *Proceedings 2002 IEEE International Conference on Robotics and Automation (Cat. No.02CH37292)*, vol. 2, no. May, pp. 1–6, 2002.

[45] S. Schaal and D. Sternad, "Programmable Pattern Generators," *International Conference on Computational Intelligence in Neuroscience*, pp. 48–51, 1998.

[46] H. Hoffmann, P. Pastor, D.-H. Park, and S. Schaal, "Biologically-inspired dynamical systems for movement generation: Automatic real-time goal adaptation and obstacle avoidance," *2009 IEEE International Conference on Robotics and Automation*, pp. 2587–2592, 2009.

[47] A. J. Ijspeert, "Dynamical Movement Primitives : Learning Attractor Models for Motor Behaviors," vol. 373, pp. 328–373, 2013.

[48] S. Calinon and A. Billard, "Recognition and reproduction of gestures using a probabilistic framework combining PCA, ICA and HMM," *Proceedings of the 22nd international conference on Machine learning ICML 05*, pp. 105–112, 2005.

[49] H. G. Sung, *Gaussian mixture regression and classification.* PhD thesis, Rice University, 2004.

[50] K. V. Mardia, J. T. Kent, and J. M. Bibby, *Multivariate analysis.* Academic press, 1979.

[51] L. R. Rabiner, "A tutorial on hidden Markov models and selected applications in speech recognition," 1989.

[52] S. Calinon, F. D'Halluin, E. L. Sauser, D. G. Caldwell, and A. G. Billard, "Learning and reproduction of gestures by imitation," *IEEE Robotics and Automation Magazine*, vol. 17, no. 2, pp. 44–54, 2010.

[53] S. Calinon, "A tutorial on task-parameterized movement learning and retrieval," *Intelligent Service Robotics*, 2015.

[54] A. Paraschos, C. Daniel, J. R. Peters, and G. Neumann, "Probabilistic movement primitives," in *Advances in neural information processing systems*, pp. 2616–2624, 2013.

[55] A. Ude, "Trajectory generation from noisy positions of object features for teaching robot paths," *Robotics and Autonomous Systems*, vol. 11, no. 2, pp. 113–127, 1993.

[56] S. Calinon, F. Guenter, and A. Billard, "Goal-directed imitation in a humanoid robot," pp. 299–304, 2005.

[57] S. Tso and K. Liu, "Hidden markov model for intelligent extraction of robot trajectory command from demonstrated trajectories," in *Industrial Technology, 1996.(ICIT'96), Proceedings of The IEEE International Conference on*, pp. 294–298, IEEE, 1996.

[58] S. Chiaverini and L. Sciavicco, "The Parallel Approach to Force/Position Control of Robotic Manipulators," *IEEE Transactions on Robotics*, vol. 9, no. 4, pp. 361—-373, 1993.

[59] A. Montebelli, F. Steinmetz, and V. Kyrki, "On Handing Down Our Tools to Robots : Single-Phase Kinesthetic Teaching for Dynamic In-Contact Tasks," pp. 5628–5634, 2015.

[60] P. Kormushev, S. Calinon, and D. G. Caldwell, "Imitation Learning of Positional and Force Skills Demonstrated via Kinesthetic Teaching and Haptic Input," *Advanced Robotics*, vol. 25, no. 5, pp. 581–603, 2011.

[61] L. Rozo, P. Jiménez, and C. Torras, "Force-based robot learning of pouring skills using parametric hidden Markov models," *9th International Workshop on Robot Motion and Control, RoMoCo 2013 - Workshop Proceedings*, pp. 227–232, 2013.

[62] J. J. Craig, "Hybrid Position / Force Control of," *Journal of Dynamic Systems, Measurement, and Control*, vol. 102, no. June 1981, pp. 126–133, 1981.

[63] L. Villani and J. De Schutter, "Force control," in *Springer handbook of robotics*, pp. 161–185, Springer, 2008.

[64] A. Gams, M. Do, A. Ude, T. Asfour, and R. Dillmann, "On-line periodic movement and force-profile learning for adaptation to new surfaces," *2010 10th IEEE-RAS International Conference on Humanoid Robots, Humanoids 2010*, pp. 560–565, 2010.

[65] P. Pastor, L. Righetti, M. Kalakrishnan, and S. Schaal, "Online movement adaptation based on previous sensor experiences," *IEEE International Conference on Intelligent Robots and Systems*, pp. 365–371, 2011.

[66] T. Kulvicius, M. Biehl, M. J. Aein, M. Tamosiunaite, and F. Wörgötter, "Interaction learning for dynamic movement primitives used in cooperative robotic tasks," *Robotics and Autonomous Systems*, vol. 61, no. 12, pp. 1450–1459, 2013.

[67] S. Calinon, P. Evrard, E. Gribovskaya, A. Billard, and A. Kheddar, "Learning collaborative manipulation tasks by demonstration using a haptic interface," *International Conference on Advanced Robotics*, pp. 1–6, 2009.

[68] S. Calinon, I. Sardellitti, and D. G. Caldwell, "Learning-based control strategy for safe human-robot interaction exploiting task and robot redundancies," *IEEE/RSJ 2010 International Conference on Intelligent Robots and Systems, IROS 2010 - Conference Proceedings*, pp. 249–254, 2010.

[69] L. Rozo, S. Calinon, D. Caldwell, P. Jimenez, C. Torras, and P. Jiménez, "Learning Collaborative Impedance-based Robot Behaviors," *AAAI Conference on Artificial Intelligence*, pp. 1422–1428, 2013.

[70] L. Rozo, D. Bruno, S. Calinon, and D. G. Caldwell, "Learning Optimal Controllers in Human-robot Cooperative Transportation Tasks with Position and Force Constraints," pp. 1024–1030, 2015.

[71] E. L. Lehmann and J. P. Romano, *Testing statistical hypotheses.* Springer Science & Business Media, 2006.

[72] E. Hairer, C. Lubich, G. Wanner, *et al.*, "Geometric numerical integration illustrated by the stormer-verlet method," *Acta numerica*, vol. 12, no. 12, pp. 399–450, 2003.

[73] S. Schaal and C. Atkeson, "Constructive incremental learning from only local information," *Neural computation*, vol. 10, no. 8, pp. 2047–84, 1998.

[74] S. Vijayakumar and S. Schaal, "Locally Weighted Projection Regression: An {O}(n) Algorithm for Incremental Real Time Learning in High Dimensional Space," *Proceedings of the Seventeenth International Conference on Machine Learning*, vol. 1086, pp. 1079–1086, 2000.

[75] J. Kober and J. Peters, "Learning motor primitives for robotics," *Robotics and Automation, 2009. ICRA '09. IEEE International Conference on*, pp. 2112–2118, 2009.

[76] T. Hastie, R. Tibshirani, and J. Friedman, *The Elements of Statistical Learning: Data Mining, Inference, and Prediction, Second Edition.* Springer Series in Statistics, Springer, 2009.

Acronyms

List of Acronyms

IA	Interactive Analysis
FCE	Factor-Controlled Experiment
ROS	Robot Operating System
GMR	Gaussian Mixture Regression
PbD	Programming by Demonstration
GMM	Gaussian Mixture Model
GMR	Gaussian Mixture Regression
DMPs	Dynamic Movement Primitives
CUSUM	CUmulative SUM
EM	Expectation-Maximum
LWR	Locally Weight Regression