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Agile Spacecraft Attitude Control: an Incremental Nonlinear Dynamic Inversion Approach

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Abstract: This paper presents an agile and robust spacecraft attitude tracking controller using the recently reformulated incremental nonlinear dynamic inversion (INDI). INDI is a combined model– and sensor–based control approach that only requires a control effectiveness model and measurements of the state and some of its derivatives, making a reduced dependency on exact system dynamics knowledge. The reformulated INDI allows a non-cascaded dynamic inversion control in terms of *Modified Rodrigues Parameters* (MRPs) where scheduling of the time-varying control effectiveness is done analytically. This way, the controller is only sensitive to parametric uncertainty of the augmented spacecraft inertia and its wheelset alignment. Moreover, we draw some parallels to *time-delay control* (TDC) —more familiar in the robotics community— which have been shown to be equivalent to the incremental formulation of *proportional-integral-derivative* (PID) control for second order nonlinear systems in controller canonical form. Simulation experiments for this particular problem demonstrate that INDI has similar nominal performance as TDC/PID control, but superior robust performance and stability.

Keywords: aerospace, tracking, application of nonlinear analysis and design

1. INTRODUCTION

Future small satellite systems are expected to be more performant not only for fine pointing capabilities in data acquisition but also in terms of high agility for maneuverability (Yuan, Z., Chen, Y., and He, R. (2014). This emerging field of '*agile Earth Observation*' motivated the development of a high-agility attitude control system for the the satellite platform BIROS (*Bispectral InfraRed Optical System*) while actuated with a redundant array of *'High-Torque-Wheels' (HTW)* (Acquatella B. (2018)).

The topic of optimal and agile spacecraft rotational maneuvers is quite extensive and has been studied for many decades (Junkins and Turner (1986); Ross et al. (2008); Fleming et al. (2010)). However, most of the work reported in literature relies on optimization and some form of trajectory optimization, which might be difficult to implement on-board. In this paper, we are motivated to find an agile attitude control solution in closed-loop feedback form. This is challenging because of the many nonlinearities involved.

Incremental nonlinear dynamic inversion (INDI) has been proposed as a promising sensor-based approach providing high performance and robust nonlinear control for aerospace vehicles without requiring a detailed model of the controlled plant. The INDI approach reduces its dependency on onboard or baseline models while making use of actuator output and angular acceleration measurement feedback. Theoretical development of increments of nonlinear control action date back from the late nineties by Smith (1998); Bacon and Ostroff (2000) which were further developed as 'incremental NDI' (Chen and Zhang (2008); Chu (2010); Sieberling et al. (2010); Simplício et al. (2013)) for flight control as well as for spacecraft attitude control (Acquatella B. et al. (2012)). More recently, this technique has been applied also in practice for quadrotors using adaptive control by Smeur et al. (2016), and in real flight tests by van Ekeren, W., Looye, G., Kuchar, R. O., Chu, Q. P., and Van Kampen, E. (2018); Grondman, F., Looye, G., Kuchar, R. O., Chu, Q. P., and Van Kampen, E. (2018), verifying its performance and robustness properties against aerodynamic model uncertainties and disturbance rejection.

INDI relies on the assumption that for small time increments and high sampling rates, the nonlinear system dynamics in its incremental form is simply approximated by the (linearized) control effectiveness evaluated at the current state. Recently, the INDI control in the literature has been reformulated for systems with arbitrary relative degree and without recurring to cascaded-control structures, i. e., without using a time–scale separation assumption (Wang, X., van Kampen, E., Chu, Q.P., and Lu, P. (2019b)). This reformulation allowed to extended further the incremental nonlinear control approach for Sliding Mode Control by Wang, X., van Kampen, E., Chu, Q.P., and Lu, P. (2019a). For these new reformulations and extensions, conditions for stability and robustness analyses

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have been established and analyzed using Lyapunov-based methods. Another nonlinear control method is *time–delay–control* (TDC) (Youcef-Toumi and Ito (1990); Jung et al. (2004); Chang and Jung (2009)), more commonly known in the motion control and robotics community and pioneered in the 90's by the works of Hsia, Youcef-Toumi, *et al.* (Youcef-Toumi and Ito (1990)). TDC works by estimating and compensating disturbances and system uncertainties (model and parametric) by utilizing time–delayed signals of some of the system variables.

In this paper, we present three main contributions in the context of nonlinear spacecraft attitude control system design. 1) We consider the reformulated INDI control for the spacecraft attitude control problem where input-output linearization is done without the usual time scale separation principle. 2) We revisit the reformulated INDI for the attitude control problem and introduce a time–delay explicitly in this reformulation. 3) We revisit TDC and establish the relationship and condition for equivalence between INDI and TDC. Based on previous results reported in the robotics literature showing the relationship between discrete formulations of TDC and the incremental formulation of *proportional-integral-derivative control* (PID) control, we also establish a clear relationship between INDI and nonlinear-PID control.

2. MODELING OF SPACECRAFT WITH REACTION WHEELS

First we describe the comprehensive nonlinear rotational dynamics model for spacecraft including a generic set of reaction wheels as shown in Karpenko et al. (2014); Acquatella B. (2018). In this paper, we consider the *Modified Rodrigues Parameters* (MRPs) (Tsiotras (1996); Schaub, H. and Junkins, J.L. (2003)) as they represent a well defined attitude parameterization for all Eigen-axis rotations in the large domain of $0^\circ \leq \theta < 360^\circ$ where θ is the principle angle rotation around the Euler-axis λ. The MRP attitude is a suitable kinematic parameterization given their potential advantages for spacecraft attitude control (Tsiotras (1996); Schaub, H. and Junkins, J.L. (2003)).

2.1 Kinematics

Consider first an array consisting of n reaction wheels. Introducing unit vectors $\mathbf{a}_i$ which give the orientation of the spin-axis of each reaction wheel with respect to the spacecraft coordinate system, these are collected in the configuration or alignment matrix $\boldsymbol{A} = [\,\mathbf{a}_1 \quad \dots \quad \mathbf{a}_n\,]$. In that sense, the kinematics of the $i-$th reaction wheel in terms of its spin-axis angle Φ_w and angular velocity Ω_w, is simply given by $\dot{\Phi}_{w,i} = \Omega_{w,i}$, $i = 1, \dots, n$. The MRP vector $\boldsymbol{\sigma}$ is defined in relation to the Euler-axis λ and principle angle rotation θ as $\boldsymbol{\sigma} = \lambda \tan(\theta/4)$ (Schaub, H. and Junkins, J.L. (2003)), and the kinematic differential equation relating $\boldsymbol{\sigma}$ with the spacecraft angular velocity $\boldsymbol{\omega} \in \mathcal{R}^3$ (with respect to the body fixed frame) in vector form is given by Schaub, H. and Junkins, J.L. (2003) as

$$\dot{\boldsymbol{\sigma}} = \frac{1}{4}\left[(1-\boldsymbol{\sigma}^\intercal\boldsymbol{\sigma})\boldsymbol{I}_{3\times3} + 2S(\boldsymbol{\sigma}) + 2\boldsymbol{\sigma}\boldsymbol{\sigma}^\intercal\right]\,\boldsymbol{\omega} = \frac{1}{4}\boldsymbol{B}(\boldsymbol{\sigma})\,\boldsymbol{\omega} \tag{1}$$

where $S(\cdot)$ is defined such that $S(x)y = x \times y$ for any $x, y \in \mathcal{R}^3$. Moreover, in this paper we will also interested on the exact relation (Schaub, H. and Junkins, J.L. (2003))

$$\ddot{\boldsymbol{\sigma}} = \frac{1}{4}\left[\,\dot{\boldsymbol{B}}(\boldsymbol{\sigma})\cdot\boldsymbol{\omega} + \boldsymbol{B}(\boldsymbol{\sigma})\cdot\dot{\boldsymbol{\omega}}\,\right] = \frac{1}{4}\boldsymbol{C}(\boldsymbol{\sigma},\boldsymbol{\omega},\dot{\boldsymbol{\omega}}) \tag{2}$$

where

$$\dot{\boldsymbol{B}}(\boldsymbol{\sigma})\cdot\boldsymbol{\omega} = \frac{1}{2}\Big[\,2\boldsymbol{\sigma}^\intercal\boldsymbol{\omega}(1-\boldsymbol{\sigma}^\intercal\boldsymbol{\sigma})\boldsymbol{\omega} - (1+\boldsymbol{\sigma}^\intercal\boldsymbol{\sigma})\boldsymbol{\omega}^\intercal\boldsymbol{\omega}\boldsymbol{\sigma} - 4\boldsymbol{\sigma}^\intercal\boldsymbol{\omega}S(\boldsymbol{\omega})\boldsymbol{\sigma} + 4(\boldsymbol{\sigma}^\intercal\boldsymbol{\omega})^2\boldsymbol{\sigma}\,\Big]$$

which relates the MRP "acceleration" $\ddot{\boldsymbol{\sigma}}$ to the rigid body's angular velocity $\boldsymbol{\omega}$ and angular acceleration $\dot{\boldsymbol{\omega}}$. This relationship will be key for the attitude control design as it will be clear later on.

2.2 Dynamics

Following the derivations in Karpenko et al. (2014), we obtain the rotational dynamics model as follows. First, consider the angular momentum of the spacecraft equipped with the reaction wheel array in question

$$\boldsymbol{H} = \boldsymbol{I}\,\boldsymbol{\omega} + \boldsymbol{h} \tag{3}$$

where, expressed in body-fixed frame, $\boldsymbol{H} \in \mathcal{R}^3$ is the total angular momentum of the system; $\boldsymbol{I} \in \mathcal{R}^{3\times3}$ is the constant inertia matrix of the spacecraft including the reaction wheels; $\boldsymbol{\omega} \in \mathcal{R}^3$ is the spacecraft angular velocity; and $\boldsymbol{h} \in \mathcal{R}^3$ is the total angular momentum vector associated with the reaction wheel array. The angular momentum $\boldsymbol{h}$ can be expressed from individual actuator frames to body-fixed frame as

$$\boldsymbol{h} = \sum_{i=1}^{n} \mathbf{a}_i \boldsymbol{h}_{w,i} = A\,\boldsymbol{I}_{\boldsymbol{w}}\,\boldsymbol{\Omega}, \tag{4}$$

where $\boldsymbol{I}_{\boldsymbol{w}} = \mathrm{diag}[\,\boldsymbol{I}_{w,1} \quad \dots \quad \boldsymbol{I}_{w,n}\,]$ is a diagonal matrix of reaction wheel spin-axis inertia values and $\boldsymbol{\Omega} = \Omega_w + A^\intercal\boldsymbol{\omega}$ the inertial angular rate of the reaction wheel array, where the term $A^\intercal\boldsymbol{\omega}$ is the extra angular motion relative to the spacecraft. Considering the angular momentum associated with the $i-$th reaction wheel in actuator frame

$$\boldsymbol{h}_{w,i} = \boldsymbol{I}_{w,i}\,(\,\Omega_{w,i} + \mathbf{a}_i^\intercal\boldsymbol{\omega}\,)\,, \qquad i = 1, \dots, n, \tag{5}$$

we can already obtain the differential equation describing the reaction wheel dynamics in terms of reaction wheel torques $\tau_{w,i}$, which are considered as the exogenous inputs to the system provided by the wheel's powertrain

$$\dot{\Omega}_{w,i} = \boldsymbol{I}_{w,i}^{-1}\,\tau_{w,i} - \mathbf{a}_i^\intercal\dot{\boldsymbol{\omega}}, \qquad i = 1, \dots, n. \tag{6}$$

Because the angular momentum must be conserved in the absence of external perturbations, applying the transport theorem (Junkins and Turner (1986); Karpenko et al. (2014)) to Eq. (3), the following relation is obtained

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{H} + \boldsymbol{\omega} \times \boldsymbol{H} = \mathbf{0}. \tag{7}$$

Combining Eqs. (4), (6), and (7), the comprehensive nonlinear model for spacecraft dynamics equipped with reaction wheels Karpenko et al. (2014) is given by

$$\boldsymbol{\Gamma}\begin{bmatrix}\dot{\boldsymbol{\omega}}\\ \dot{\Omega}_{w,1}\\ \vdots\\ \dot{\Omega}_{w,n}\end{bmatrix} = \begin{bmatrix}-\boldsymbol{\omega}\times(\boldsymbol{I}\boldsymbol{\omega} + A\boldsymbol{I}_{\mathbf{w}}\Omega_w + A\boldsymbol{I}_{\mathbf{w}}A^\intercal\boldsymbol{\omega})\\ \tau_{w,1}\\ \vdots\\ \tau_{w,n}\end{bmatrix} \tag{8}$$

where

$$\boldsymbol{\Gamma} = \begin{bmatrix} \boldsymbol{I} + A\boldsymbol{I}_{\boldsymbol{w}}A^{\intercal} & \mathbf{a}_1\boldsymbol{I}_{w,1} & \cdots & \mathbf{a}_n\boldsymbol{I}_{w,n} \\ \boldsymbol{I}_{w,1}\mathbf{a}_1^{\intercal} & \boldsymbol{I}_{w,1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{I}_{w,n}\mathbf{a}_n^{\intercal} & 0 & \cdots & \boldsymbol{I}_{w,n} \end{bmatrix}$$

is an augmented inertia coupling matrix for the full system.

2.3 Full nonlinear spacecraft model

The augmentation of the nonlinear spacecraft dynamics model together with the MRP kinematics can be rewritten as a full model in the generic form of affine n-dimensional multivariable nonlinear system with m inputs u_i and m outputs y_i

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u} \tag{9a}$$

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}) \tag{9b}$$

where $\boldsymbol{x} \in \mathcal{R}^n$, $\boldsymbol{u} \in \mathcal{R}^m$, and $\boldsymbol{y} \in \mathcal{R}^p$. The functions $\boldsymbol{f}$, $\boldsymbol{g}$, and $\boldsymbol{h}$ are assumed to be smooth vector fields continuously differentiable on $\mathcal{R}^n$. Moreover, the system has a vector of relative degree of $[\,\rho_1 \quad \ldots \quad \rho_p\,]^{\intercal}$ which represents the number of differentiation of each output y_i ($i = 1, \ldots, p$) needed for the input to appear Slotine and Li (1990), and the total relative degree is obtained as $\rho = \rho_1 + \cdots + \rho_p$. In this paper we consider the output MRP as control variables $\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{\sigma}$, and assume to have three reaction wheels ($n_w = 3$) as actuators, hence $\boldsymbol{u} = [\,\tau_{w,1} \quad \tau_{w,2} \quad \tau_{w,3}\,]^{\intercal}$ and $p = m = 3$. Whenever $p < m$, the input-output linearization is not straightforward and some form of control allocation is required. Else, when $p > m$, the control problem is underactuated and the input-output linearization is underdetermined. These aspects are however out of the scope of this paper. Considering the vector $\boldsymbol{x} = [\,\boldsymbol{\sigma} \quad \boldsymbol{\omega} \quad \boldsymbol{\Omega}_w\,]^{\intercal}$ with, respectively, $\boldsymbol{\sigma} = [\,\sigma_1 \quad \sigma_2 \quad \sigma_3\,]^{\intercal}$, $\boldsymbol{\omega} = [\,\omega_x \quad \omega_y \quad \omega_z\,]^{\intercal}$, and $\boldsymbol{\Omega}_w = [\,\Omega_{w,1} \quad \Omega_{w,2} \quad \Omega_{w,3}\,]^{\intercal}$, the full nonlinear system dynamics in (9) is obtained with the functions given as

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} \frac{1}{4}\boldsymbol{B}(\boldsymbol{\sigma})\,\boldsymbol{\omega} \\ \boldsymbol{\Gamma}^{-1}\begin{bmatrix} -\boldsymbol{\omega} \times (\boldsymbol{I\omega} + \boldsymbol{A}\boldsymbol{I}_{\mathrm{w}}\boldsymbol{\Omega}_w + \boldsymbol{A}\boldsymbol{I}_{\mathrm{w}}\boldsymbol{A}^{\intercal}\boldsymbol{\omega}) \\ \boldsymbol{0}_{3\times1} \end{bmatrix} \end{bmatrix}, \tag{10a}$$

$$\boldsymbol{g}(\boldsymbol{x}) = \begin{bmatrix} \boldsymbol{0}_{3\times3} \\ \boldsymbol{\Gamma}^{-1}\begin{bmatrix} \boldsymbol{0}_{3\times3} \\ \boldsymbol{1}_{3\times3} \end{bmatrix} \end{bmatrix} = \boldsymbol{G}. \tag{10b}$$

3. INCREMENTAL NONLINEAR DYNAMIC INVERSION

3.1 Nonlinear Dynamic Inversion Control

Nonlinear dynamic inversion (NDI) uses an accurate model of the system to entirely or partly cancel its nonlinearities by means of feedback and exact state transformations. Finding an explicit relationship between the input and the output of the system is generally not straightforward because they are not directly related. First, recall

$$\boldsymbol{l}(\boldsymbol{x}) = \begin{bmatrix} \boldsymbol{L}_{\boldsymbol{f}}^{\rho_1} h_1(\boldsymbol{x}) \\ \vdots \\ \boldsymbol{L}_{\boldsymbol{f}}^{\rho_m} h_m(\boldsymbol{x}) \end{bmatrix} \tag{11a}$$

$$\boldsymbol{M}(\boldsymbol{x}) = \begin{bmatrix} \boldsymbol{L}_{g_1}\boldsymbol{L}_{\boldsymbol{f}}^{\rho_1-1}h_1(\boldsymbol{x}) & \cdots & \boldsymbol{L}_{g_m}\boldsymbol{L}_{\boldsymbol{f}}^{\rho_1-1}h_1(\boldsymbol{x}) \\ \vdots & \ddots & \vdots \\ \boldsymbol{L}_{g_1}\boldsymbol{L}_{\boldsymbol{f}}^{\rho_m-1}h_m(\boldsymbol{x}) & & \boldsymbol{L}_{g_m}\boldsymbol{L}_{\boldsymbol{f}}^{\rho_m-1}h_m(\boldsymbol{x}) \end{bmatrix}, \tag{11b}$$

where $\boldsymbol{L}_f^{\rho_j} h_j(\boldsymbol{x})$ and $\boldsymbol{L}_{g_i}\boldsymbol{L}_f^{\rho_j-1}h_j(\boldsymbol{x})$ are Lie derivatives of the scalar functions $h_j(\boldsymbol{x})$ with respect to the vector fields $\boldsymbol{f}(\boldsymbol{x})$ and $\boldsymbol{g}_i(\boldsymbol{x})$, with j, $i = 1$ to m. Denoting the differentiated outputs $\boldsymbol{\zeta} = [\,y_1^{\rho_1-1} \quad \ldots \quad y_m^{\rho_m-1}\,]^{\intercal}$, the following relation is obtained

$$\dot{\boldsymbol{\zeta}} = \boldsymbol{l}(\boldsymbol{x}) + \boldsymbol{M}(\boldsymbol{x})\boldsymbol{u}. \tag{12}$$

Denoting $\boldsymbol{\nu}$ as a virtual control input, the vector $\boldsymbol{\varphi}(\boldsymbol{x}) = -\boldsymbol{M}^{-1}(\boldsymbol{x})\boldsymbol{l}(\boldsymbol{x})$, and the matrix $\boldsymbol{\vartheta}(\boldsymbol{x}) = \boldsymbol{M}^{-1}(\boldsymbol{x})$, then the state feedback control law $\boldsymbol{u}$ defined as

$$\boldsymbol{u} = \boldsymbol{\varphi}(\boldsymbol{x}) + \boldsymbol{\vartheta}(\boldsymbol{x})\boldsymbol{\nu} \tag{13}$$

cancels all nonlinearities in closed-loop, and a simple linear input-output relationship between the new input $\boldsymbol{\nu}$ and the new output $\boldsymbol{\zeta}$ is obtained

$$\dot{\boldsymbol{\zeta}} = \boldsymbol{\nu} \tag{14}$$

as long as $\boldsymbol{\vartheta}(\boldsymbol{x})$ is not singular, i.e., when $\boldsymbol{M}(\boldsymbol{x})$ is invertible (which is always the case for the model in (10)). Apart from being linear, the input ν_i is decoupled from the differentiated output ζ_i. From this fact, the input transformation (13) is called a *decoupling control law* and the resulting linear system (14) is called the *single-integrator* form. The single-integrator form (14) is sought to be rendered exponentially stable with the proper design of of $\boldsymbol{\nu}$. From this typical tracking problem it can be seen that the entire control system will have two control loops (Chu (2010); Sieberling et al. (2010)): the inner linearization loop (13), and the outer control loop (14). This resulting NDI control law depends on accurate knowledge of the model ($\boldsymbol{l}(\boldsymbol{x})$ and $\boldsymbol{M}(\boldsymbol{x})$) and its parameters, hence it is susceptible to model and parametric uncertainties. For that reason we are now interested in the concept of incremental NDI.

3.2 Incremental Nonlinear Dynamic Inversion Control

The concept of incremental nonlinear dynamic inversion (INDI) amounts to the application of NDI to a system expressed in an incremental form. This improves the robustness of the closed-loop system as compared with conventional NDI since dependency on the accurate knowledge of the plant dynamics is reduced. First, we introduce a *sufficiently small* time–delay λ and define the following deviation variables $\dot{\boldsymbol{x}}_0 := \dot{\boldsymbol{x}}(t-\lambda)$, $\boldsymbol{x}_0 := \boldsymbol{x}(t-\lambda)$, and $\boldsymbol{u}_0 := \boldsymbol{u}(t-\lambda)$, which are the λ–time–delayed signals of the current state derivative $\dot{\boldsymbol{x}}(t)$, state $\boldsymbol{x}(t)$, and control $\boldsymbol{u}(t)$, respectively. Moreover, we will denote $\Delta\dot{\boldsymbol{x}} := \dot{\boldsymbol{x}} - \dot{\boldsymbol{x}}_0$, $\Delta\boldsymbol{x} := \boldsymbol{x} - \boldsymbol{x}_0$, and $\Delta\boldsymbol{u} := \boldsymbol{u} - \boldsymbol{u}_0$ as the incremental state derivative, the incremental state, and the so–called incremental control input, respectively. To obtain an incremental form of system dynamics, we consider a first-order Taylor series expansion of $\dot{\boldsymbol{x}}(t)$ (Sieberling et al. (2010); Simplício et al. (2013)), not in the geometric sense, but with respect to the newly introduced time–delay λ as

$$\begin{aligned} \dot{\boldsymbol{\zeta}} = {} & \dot{\boldsymbol{\zeta}}_0 + \frac{\partial}{\partial \boldsymbol{x}}\left[\boldsymbol{l}(\boldsymbol{x}) + \boldsymbol{M}(\boldsymbol{x})\boldsymbol{u}\right]\Big|_{\substack{\boldsymbol{x}=\boldsymbol{x}_0 \\ \boldsymbol{u}=\boldsymbol{u}_0}} \Delta\boldsymbol{x} \\ & + \boldsymbol{M}(\boldsymbol{x}_0)\Delta\boldsymbol{u} + \mathcal{O}(\Delta\boldsymbol{x}^2) \\ \cong {} & \dot{\boldsymbol{\zeta}}_0 + \boldsymbol{L}_0(\boldsymbol{x}_0)\Delta\boldsymbol{x} + \boldsymbol{M}(\boldsymbol{x}_0)\Delta\boldsymbol{u} \end{aligned}$$

with

$$\dot{\boldsymbol{\zeta}}_0 := \dot{\boldsymbol{\zeta}}(t-\lambda) = \boldsymbol{l}(\boldsymbol{x}_0) + \boldsymbol{M}(\boldsymbol{x}_0)\boldsymbol{u}_0 \tag{15a}$$

and

$$\boldsymbol{L}_0(\boldsymbol{x}_0) = \frac{\partial}{\partial \boldsymbol{x}}\Big[\boldsymbol{l}(\boldsymbol{x}) + \boldsymbol{M}(\boldsymbol{x})\boldsymbol{u}\Big]\Big|_{\substack{\boldsymbol{x}=\boldsymbol{x}_0\\ \boldsymbol{u}=\boldsymbol{u}_0}} \tag{16a}$$

which represents the Jacobian linearization of the onboard model. This means an approximate linearization about the $\lambda-$delayed signals is performed *incrementally*, and not with respect to a particular equilibrium or operational point of interest. We will refer to the decoupling matrix $\boldsymbol{M}(\boldsymbol{x})$ evaluated at $\boldsymbol{x}_0$ as the *instantaneous control effectiveness (ICE)*, i.e., the control effectiveness evaluated at the current state, $\boldsymbol{M}(\boldsymbol{x}_0)$.

Time-scale separation (TSS) assumption: *For a sufficiently small time-delay λ and for any incremental control input, it is assumed that $\Delta \boldsymbol{x}$ does not vary significantly during λ. In other words, the input rate of change is much faster than the state rate of change:*

$$\epsilon_{INDI_{\text{TSS}}}(t) \equiv \Delta \boldsymbol{x} := \boldsymbol{x} - \boldsymbol{x}_0 \cong 0,\ \forall\ \Delta \boldsymbol{u} \tag{17}$$

which leads to

$$\dot{\boldsymbol{\zeta}} \cong \ \dot{\boldsymbol{\zeta}}_0 + \boldsymbol{L}_0 \underbrace{(\boldsymbol{x}-\boldsymbol{x}_0)}_{\cong 0} + \boldsymbol{M}(\boldsymbol{x}_0)\cdot(\boldsymbol{u}-\boldsymbol{u}_0)$$

or simply

$$\Delta\dot{\boldsymbol{\zeta}} \cong \boldsymbol{M}(\boldsymbol{x}_0)\cdot \Delta \boldsymbol{u} \tag{18}$$

This assumption shows that for high sampling rates the nonlinear system dynamics in its incremental form is simply approximated by the ICE matrix $\boldsymbol{M}(\boldsymbol{x}_0)$, and that for the development of control laws, it is required the availability of $\dot{\boldsymbol{\zeta}}_0$ and $\boldsymbol{u}_0$ that are implicit in (18). For the obtained approximation $\Delta\dot{\boldsymbol{\zeta}} \cong \boldsymbol{M}(\boldsymbol{x}_0)\cdot\Delta\boldsymbol{u}$, NDI is applied to obtain a relation between the incremental control input and the output of the system

$$\boldsymbol{u} = \boldsymbol{u}_0 + \boldsymbol{M}(\boldsymbol{x}_0)^{-1}(\boldsymbol{\nu} - \dot{\boldsymbol{\zeta}}_0). \tag{19}$$

Note that the incremental input $\boldsymbol{u}_0$ that corresponds to $\dot{\boldsymbol{\zeta}}_0$ is obtained from the output of the actuators, and it has been assumed that a commanded control is achieved *sufficiently fast* in regards to the actuator dynamics. The total control command along with the obtained linearizing control $\Delta\boldsymbol{u} = \boldsymbol{u}(t-\lambda)$ can be rewritten as

$$\boldsymbol{u}(t) = \boldsymbol{u}(t-\lambda) + \boldsymbol{M}(\boldsymbol{x}_0)^{-1}[\boldsymbol{\nu} - \dot{\boldsymbol{\zeta}}(t-\lambda)]. \tag{20}$$

The dependency of the closed-loop system on accurate knowledge of the dynamic model in $\boldsymbol{l}(\boldsymbol{x})$ is largely decreased, improving robustness against model uncertainties contained therein. Therefore, this implicit control law design is more dependent on accurate measurements or estimates of the state derivatives $\dot{\boldsymbol{\zeta}}_0$ and on the incremental control input $\boldsymbol{u}_0$, but is still largely dependent on the model reflected in $\boldsymbol{M}(\boldsymbol{x})$.

3.3 NDI attitude control

Since the output of the system has been selected to be the MRP vector $\boldsymbol{y} = \boldsymbol{\sigma}$ the system has a vector of relative degree $[\,\rho_1 \quad \rho_2 \quad \rho_3\,]^\intercal = [\,2 \quad 2 \quad 2\,]^\intercal$ and total relative degree $\rho = 6$. Since $\rho < n$, there are internal states $\boldsymbol{\eta}$ which can be easily proven to lead to marginally stable zero dynamics. Denoting the differentiated outputs $\boldsymbol{\zeta} = [\,\sigma_1^{\rho_1-1} \quad \sigma_2^{\rho_2-1} \quad \sigma_3^{\rho_3-1}\,]^\intercal = [\,\dot{\sigma}_1 \quad \dot{\sigma}_2 \quad \dot{\sigma}_3\,]^\intercal$, the relation (12) is obtained, where $\boldsymbol{l}(\boldsymbol{x}) = \boldsymbol{L}_f^2\boldsymbol{\sigma}$ and $\boldsymbol{M}(\boldsymbol{x}) = \boldsymbol{L}_g\boldsymbol{L}_f^1\boldsymbol{\sigma}$. The NDI control law (13) cancels all nonlinearities in closed-loop and the nominal closed-loop system (external states) is obtained as

$$\dot{\boldsymbol{\xi}}^{(6)} = \boldsymbol{A}^{(6\times6)}\boldsymbol{\xi}^{(6)} + \boldsymbol{B}^{(6\times3)}\boldsymbol{\nu}^{(3)} \tag{21}$$

$$\boldsymbol{y}^{(3)} = \boldsymbol{C}^{(3\times6)}\boldsymbol{\xi}^{(6)} \tag{22}$$

where the upper indices indicate the dimensions of the vectors and matrices and the new state vector $\boldsymbol{\xi}$ is defined in terms of the original state $\boldsymbol{x}$ as $\boldsymbol{\xi} = [\,\sigma_1 \quad \dot{\sigma}_1 \quad \sigma_2 \quad \dot{\sigma}_2 \quad \sigma_3 \quad \dot{\sigma}_3\,]^\intercal$ and $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ are in Brunovsky block canonical form (Wang, X., van Kampen, E., Chu, Q.P., and Lu, P. (2019b)).

Denoting $\boldsymbol{e} = \boldsymbol{\sigma} - \boldsymbol{\sigma}_{ref}$ (valid for small deviations), this single-integrator form can be rendered exponentially stable with

$$\boldsymbol{\nu} = \ddot{\boldsymbol{y}}_d + \mathbf{k}_D\dot{\boldsymbol{e}} + \mathbf{k}_P\boldsymbol{e} \tag{23}$$

where $\ddot{\boldsymbol{y}}_d$ is the feedforward term for tracking tasks, and $\mathbf{k}_D$ and $\mathbf{k}_P$ being 3×3 constant diagonal matrices whose $i-$th diagonal elements $\mathbf{k}_{D_i}$ and $\mathbf{k}_{P_i}$, respectively, are chosen so that the polynomials $s^2 + \mathbf{k}_{D_i}s + \mathbf{k}_{P_i}$ $i = 1,\ldots,n = 3$ may become Hurtwiz. This results in the exponentially stable and decoupled error dynamics

$$\ddot{\boldsymbol{e}} + \mathbf{k}_D\dot{\boldsymbol{e}} + \mathbf{k}_P\boldsymbol{e} = \boldsymbol{0} \tag{24}$$

which implies that $\boldsymbol{\sigma}(t) \to \boldsymbol{\sigma}_{ref}(t)$ exponentially.

3.4 INDI attitude control

Since we will consider the dynamics in its incremental form for the control design

$$\dot{\boldsymbol{\zeta}}(t) - \dot{\boldsymbol{\zeta}}(t-\lambda) \cong \ \boldsymbol{M}(\boldsymbol{x}_0)\left[\boldsymbol{u}(t) - \boldsymbol{u}(t-\lambda)\right], \tag{25}$$

the incremental nonlinear dynamic inversion results in a control law that is only depending on the uncertainties contained within the ICE matrix

$$\boldsymbol{u}(t) = \boldsymbol{u}(t-\lambda) + \boldsymbol{M}(\boldsymbol{x}_0)^{-1}\Big[\boldsymbol{\nu} - \dot{\boldsymbol{\zeta}}(t-\lambda)\Big]. \tag{26}$$

however, notice that

$$\boldsymbol{M}(\boldsymbol{x}_0) = \underbrace{\frac{\partial\big[\boldsymbol{L}_f^1\boldsymbol{h}(\boldsymbol{x}_0)\big]}{\partial\boldsymbol{x}}}_{\text{purely kinematic}} \cdot \underbrace{\boldsymbol{G}}_{\text{purely parametric}} . \tag{27}$$

This means that in the particular case of this plant, namely a rigid body spacecraft actuated with a non-redundant set of orthogonal reaction wheels and parameterized by MRPs, the incremental nonlinear dynamic inversion is robust since the control law is only exposed to uncertainty in the parametric matrix $\boldsymbol{G}$ which contains information about inertia values (of the rigid body and of the reaction wheels). The term which is purely kinematic in this control law is fully known and contains no uncertainties other than the ones contained within the measured state $\boldsymbol{x}_0$. To conclude the INDI attitude control design, we have made use of the fact that

$$\dot{\boldsymbol{\zeta}}(t-\lambda) = \dot{\boldsymbol{\zeta}}_0 = \ddot{\boldsymbol{\sigma}}_0 = \frac{1}{4}\Big[\ \dot{\boldsymbol{B}}(\boldsymbol{\sigma}_0)\cdot\boldsymbol{\omega}_0 + \boldsymbol{B}(\boldsymbol{\sigma}_0)\cdot\dot{\boldsymbol{\omega}}_0\ \Big] \tag{28}$$

where the relationship

$$\dot{\boldsymbol{B}}(\boldsymbol{\sigma})\cdot\boldsymbol{\omega} = \frac{1}{2}\big[\ 2\boldsymbol{\sigma}^\intercal\boldsymbol{\omega}(1-\boldsymbol{\sigma}^\intercal\boldsymbol{\sigma})\boldsymbol{\omega} - (1+\boldsymbol{\sigma}^\intercal\boldsymbol{\sigma})\boldsymbol{\omega}^\intercal\boldsymbol{\omega}\boldsymbol{\sigma} \tag{29}$$

$$- 4\boldsymbol{\sigma}^\intercal\boldsymbol{\omega}S(\boldsymbol{\omega})\boldsymbol{\sigma} + 4(\boldsymbol{\sigma}^\intercal\boldsymbol{\omega})^2\boldsymbol{\sigma}\ \big] \tag{30}$$

is highly beneficial to compute $\ddot{\boldsymbol{\sigma}}_0$ which is otherwise very hard to estimate because of the noise contained in the measurements. By using the measured $\dot{\boldsymbol{\zeta}}(t-\lambda)$ and commanded $\boldsymbol{u}(t-\lambda)$ incrementally, we practically obtain a nonlinear 'self-scheduling' NDI control law that is robust to model and parametric uncertainties. The use of $\boldsymbol{M}(\boldsymbol{x}_0)$ in INDI is one of the key differences with respect to *time-delay control*, where the control effectiveness is substituted with a constant gain matrix instead. This method is briefly presented next.

3.5 Time-Delay Control and relationship to INDI

Consider the following transformation as in Chang and Jung (2009)

$$\dot{\boldsymbol{\zeta}} = \boldsymbol{H}(\boldsymbol{x},\boldsymbol{u}) + \bar{\boldsymbol{M}}\boldsymbol{u} \tag{31}$$

with

$$\boldsymbol{H}(\boldsymbol{x},\boldsymbol{u}) = \boldsymbol{l}(\boldsymbol{x}) + \left[\boldsymbol{M}(\boldsymbol{x}) - \bar{\boldsymbol{M}}\right]\boldsymbol{u}, \tag{32}$$

and with $\bar{\boldsymbol{M}}$, an scalar-valued and invertible gain matrix referred to as the *incremental gain effectiveness (IGE)* matrix from now on. Defining the vector $\boldsymbol{\alpha}(\boldsymbol{x})$ and matrix $\boldsymbol{\beta}$ as

$$\boldsymbol{\alpha}(\boldsymbol{x}) = -\bar{\boldsymbol{M}}^{-1}\boldsymbol{H}(\boldsymbol{x},\boldsymbol{u}) \tag{33a}$$

$$\boldsymbol{\beta} = \bar{\boldsymbol{M}}^{-1} \tag{33b}$$

then, the state feedback control law $\boldsymbol{u}$ defined as

$$\boldsymbol{u} = \boldsymbol{\alpha}(\boldsymbol{x}) + \boldsymbol{\beta}\cdot\boldsymbol{\nu} = \bar{\boldsymbol{M}}^{-1}\left[\boldsymbol{\nu} - \boldsymbol{H}(\boldsymbol{x},\boldsymbol{u})\right] \tag{34}$$

cancels all nonlinearities in the nominal closed-loop case, as shown before, where we have used the virtual control input as $\boldsymbol{\nu} = \dot{\boldsymbol{\zeta}}_{\text{des}}$. Notice however, that still a full model of $\boldsymbol{H}(\boldsymbol{x},\boldsymbol{u})$ is needed. Because this reformulated NDI control law is nevertheless *still* depending on the model represented by $\boldsymbol{H}(\boldsymbol{x},\boldsymbol{u})$, this controller is again susceptible to uncertainties in this term.

To cope with the uncertainty issue, we will consider an estimation of $\boldsymbol{H}$ denoted by $\bar{\boldsymbol{H}}$ along the lines of *time delay control* (TDC) (Chang and Jung (2009)), and therefore we will consider the usual dynamic inversion input transformation of (31) but with the $\bar{\boldsymbol{H}}$ estimate instead

$$\boldsymbol{u} = \bar{\boldsymbol{M}}^{-1}\left[\boldsymbol{\nu} - \bar{\boldsymbol{H}}(\boldsymbol{x},\boldsymbol{u})\right] \tag{35}$$

being the nominal case when $\bar{\boldsymbol{H}} = \boldsymbol{H}$ which results in perfect inversion. Our remaining task is therefore to find a suitable $\bar{\boldsymbol{H}}$ estimate such that, in combination with $\boldsymbol{\nu}$, the closed-loop system converges exponentially fast to Eq. (14) while avoiding the uncertain terms to grow unbounded. This means that, ultimately, the control law given by Eq. (35) is able to obtain the desired closed-loop dynamics defined by the nominal single integrator form while rejecting the perturbation due to the uncertainties in $\Delta\boldsymbol{H}$. For the sufficiently small time-delay λ already introduced, we consider the following approximation to hold (Chang and Jung (2009)) such that $\boldsymbol{H}$ does not vary significantly during λ

$$\boldsymbol{\epsilon}_{\text{TDE}_{error}}(t) \equiv \boldsymbol{H}(\boldsymbol{x},\boldsymbol{u},t) - \boldsymbol{H}(\boldsymbol{x},\boldsymbol{u},t-\lambda) \cong 0 \tag{36}$$

which is called *time-delay estimation error* at time t. If we write the following current, and delayed dynamics, respectively

$$\dot{\boldsymbol{\zeta}} = \boldsymbol{H}(\boldsymbol{x},\boldsymbol{u}) + \bar{\boldsymbol{M}}\cdot\boldsymbol{u}, \quad \dot{\boldsymbol{\zeta}}_{\mathbf{0}} = \boldsymbol{H}(\boldsymbol{x}_0,\boldsymbol{u}_0) + \bar{\boldsymbol{M}}\cdot\boldsymbol{u}_0$$

it is clear that

$$\boldsymbol{H}(\boldsymbol{x},\boldsymbol{u}) - \boldsymbol{H}(\boldsymbol{x}_0,\boldsymbol{u}_0) = (\dot{\boldsymbol{\zeta}} - \dot{\boldsymbol{\zeta}}_{\mathbf{0}}) - \bar{\boldsymbol{M}}\left(\boldsymbol{u} - \boldsymbol{u}_0\right) \cong 0.$$

or simply

$$\Delta\dot{\boldsymbol{\zeta}} \cong \bar{\boldsymbol{M}}\cdot\Delta\boldsymbol{u}. \tag{37}$$

This relationship is used together with Eq. (31) to obtain what is called *time-delay estimation* (TDE) as the following

$$\bar{\boldsymbol{H}} = \boldsymbol{H}(t-\lambda) = \dot{\boldsymbol{\zeta}}(t-\lambda) - \bar{\boldsymbol{M}}\cdot\boldsymbol{u}(t-\lambda) \tag{38}$$

therefore we can rewrite in our usual notation as

$$\bar{\boldsymbol{H}} = \boldsymbol{H}_0 = \dot{\boldsymbol{\zeta}}_0 - \bar{\boldsymbol{M}}\cdot\boldsymbol{u}_0 \tag{39}$$

3.6 Parallels between INDI and TDC

With the TDE, the incremental counterpart of Eq. (20) results in a control law that is not depending on the dynamics model in $\boldsymbol{H}$ which contains $\boldsymbol{l}(\boldsymbol{x})$ and the control effectiveness $\boldsymbol{M}(\boldsymbol{x})$, but instead on the IGE matrix $\bar{\boldsymbol{M}}$ as

$$\boldsymbol{u} = \boldsymbol{u}_0 + \bar{\boldsymbol{M}}^{-1}\left[\boldsymbol{\nu} - \dot{\boldsymbol{\zeta}}_0\right]. \tag{40}$$

in other words

$$\boldsymbol{u}(t) = \boldsymbol{u}(t-\lambda) + \bar{\boldsymbol{M}}^{-1}\left[\boldsymbol{\nu} - \dot{\boldsymbol{\zeta}}(t-\lambda)\right]. \tag{41}$$

This TDC law can be interpreted as an INDI control whenever

$$\bar{\boldsymbol{M}} = \boldsymbol{M}(\boldsymbol{x}_0), \tag{42}$$

however, we had taken from the literature of TDC as the IGE being a time-invariant gain matrix, which is the main distinction with regards to INDI control laws. In that regard, we can conclude that the INDI control laws are combined model– and sensor–based control laws which are promising for high-performance nonlinear and robust attitude control because of this self–scheduling property of the ICE matrix $\boldsymbol{M}(\boldsymbol{x}_0)$. Note that the self-scheduling properties of INDI in Eq. (20) due to the ICE term $\boldsymbol{M}(\boldsymbol{x}_0)$ were lost in the TDC law of Eq. (40), suggesting that $\bar{\boldsymbol{M}}$ should be an scheduling variable as in INDI by imposing the equivalence $\bar{\boldsymbol{M}} = \boldsymbol{M}(\boldsymbol{x}_0)$.

3.7 Discrete formulations of INDI, TDC, and PID control and their relationships

For practical implementations, sampled-time formulations involving continuous and discrete quantities as in Chang and Jung (2009) are more convenient and restated here. For that, the smallest λ one can consider is the equivalent of the sampling period of the on-board computer. The sampled formulation of (41) may be expressed as

$$\boldsymbol{u}(k) = \boldsymbol{u}(k-1) + \bar{\boldsymbol{M}}^{-1}\left[\boldsymbol{\nu}(k-1) - \dot{\boldsymbol{\zeta}}(k-1)\right] \tag{43}$$

where it has been necessary to consider $\boldsymbol{\nu}$ at sample $k-1$ for causality reasons (see Chang and Jung (2009), Fig. 1). Replacing the sampled virtual control $\boldsymbol{\nu}$ accordingly, we have

$$\boldsymbol{u}(k) = \boldsymbol{u}(k-1) + \bar{\boldsymbol{M}}^{-1}\Big[\dot{\boldsymbol{\zeta}}_d(k-1) + \mathbf{k}_D\dot{\boldsymbol{e}}(k-1) + \mathbf{k}_P\boldsymbol{e}(k-1) - \dot{\boldsymbol{\zeta}}(k-1)\Big] \tag{44}$$

which results in

$$\boldsymbol{u}(k) = \boldsymbol{u}(k-1) + \bar{\boldsymbol{M}}^{-1}\big[\ddot{\boldsymbol{e}}(k-1) + \mathbf{k}_D\dot{\boldsymbol{e}}(k-1) + \mathbf{k}_P\boldsymbol{e}(k-1)\big] \tag{45}$$

Previous results reported in the robotics literature (Chang and Jung (2009)) show the relationship between this discrete formulation of TDC and *proportional-integral-derivative control* (PID). Acquatella B. et al. (2017) showed that INDI is equivalent to TDC but only under the consideration when the ICE matrix was constant. This in turn suggested a meaningful and systematic method for PI(D)-control tuning of robust nonlinear flight control systems via INDI as originally suggested in the systematic method for gain selection of robust *proportional-integral-derivative* (PID) controllers for nonlinear plants by Chang and Jung (2009). Chang and Jung (2009) showed this relationship first by considering the discrete implementation of a PID control

$$\boldsymbol{u}(k) = \boldsymbol{K}\left[\boldsymbol{e}(k-1) + \boldsymbol{T}_I^{-1}\sum_{i=0}^{k-1} t_s \boldsymbol{e}(i) + \boldsymbol{T}_D \dot{\boldsymbol{e}}(k-1)\right] + \boldsymbol{u}_B. \tag{46}$$

where $\boldsymbol{K}$ denotes a diagonal proportional gain matrix, $\boldsymbol{T}_I$ a constant diagonal matrix representing a (reset) integral time, $\boldsymbol{T}_D$ a constant diagonal matrix representing derivative time, and $\boldsymbol{u}_B$ denotes a constant vector representing a trim-bias from initial conditions. When subtracting two consecutive terms of a discrete formulation, the integral sum can be removed and thus the so-called PID controller in incremental form can be obtained

$$\begin{aligned}\boldsymbol{u}(k) &= \boldsymbol{u}(k-1)\\ &+ \boldsymbol{K}\cdot t_s \cdot \left[\boldsymbol{T}_D\,\ddot{\boldsymbol{e}}(k-1) + \dot{\boldsymbol{e}}(k-1) + \boldsymbol{T}_I^{-1}\cdot \boldsymbol{e}(k-1)\right]\end{aligned} \tag{47}$$

If we consider a nonlinear-PID control in the form

$$\boldsymbol{u}(k) = \boldsymbol{K}(\boldsymbol{x})\left[\boldsymbol{e}(k-1) + \boldsymbol{T}_I^{-1}\sum_{i=0}^{k-1} t_s \boldsymbol{e}(i) + \boldsymbol{T}_D \dot{\boldsymbol{e}}(k-1)\right], \tag{48}$$

comparing terms from Eqs. (45)-(47)-(48), we have the following relationships as originally found by Chang and Jung (2009) which are the relationship between the discrete formulations of TDC and PID in incremental form

$$\boldsymbol{K}(\boldsymbol{x}) = \bar{\boldsymbol{K}} = \mathbf{k}_D \cdot (\bar{\boldsymbol{M}} \cdot t_s)^{-1}, \tag{49a}$$

$$\boldsymbol{T}_I = \mathbf{k}_D \cdot \mathbf{k}_P^{-1}, \tag{49b}$$

$$\boldsymbol{T}_D = \mathbf{k}_D^{-1}, \tag{49c}$$

Referring back to the Eqs. (42)-(48) which shows the relationship between INDI and TDC, considering the state-dependent (and therefore scheduled) nonlinear–PID proportional gain matrix $\boldsymbol{K}(\boldsymbol{x})$, it is related to the ICE matrix $\boldsymbol{M}(\boldsymbol{x}_0)$ via the relationship

$$\boldsymbol{K}(\boldsymbol{x}) = \boldsymbol{K}(\boldsymbol{x}_0) = \mathbf{k}_D \cdot \left[\boldsymbol{M}(\boldsymbol{x}_0)\cdot t_s\right]^{-1}, \tag{50}$$

which then clearly suggests not only that an equivalent discrete and incremental PID controller with gains $< \boldsymbol{K},\ \boldsymbol{T}_i,\ \boldsymbol{T}_d >$ can be obtained in relationship to TDC but also in relationship to INDI when considering an incremental and self-scheduled nonlinear-PID controller with gains $< \boldsymbol{K}(\boldsymbol{x}_0),\ \boldsymbol{T}_i,\ \boldsymbol{T}_d >$. Moreover, the tuning of these (nonlinear-)PIDs proves to be more meaningful and systematic than heuristic methods as already pointed out in Chang and Jung (2009); Acquatella B. et al. (2017). This is because the design starts from prescribing desired error dynamics $\ddot{\boldsymbol{e}} + \mathbf{k}_D\dot{\boldsymbol{e}} + \mathbf{k}_P\boldsymbol{e} = \mathbf{0}$ by tuning the Hurwitz gains $< \mathbf{k}_P,\ \mathbf{k}_D, >$ and what follows is finding the remaining IGE matrix $\bar{\boldsymbol{M}}$ by the TDC approach, or with the ICE matrix $\boldsymbol{M}(\boldsymbol{x}_0)$ with the INDI approach. In essence, this procedure is more efficient and much less cumbersome than designing a whole set of PID gains iteratively. Moreover, for attitude control systems, the self-scheduling properties of inversion-based controllers have suggested superior advantages with respect to PID controls since these are, in general, not gain-scheduled according to the nonlinear motion of the plant (Smeur et al. (2016)). The relationships here outlined suggests that scheduling of incremental PID control shall be done at the level of the proportional gain $\boldsymbol{K}(\boldsymbol{x})$ via the IGE matrix $\bar{\boldsymbol{M}}$ or ICE matrix $\boldsymbol{M}(\boldsymbol{x}_0$, and *not* over the whole set of gains $< \boldsymbol{K}(\boldsymbol{x}),\ \boldsymbol{T}_i,\ \boldsymbol{T}_d >$.

3.8 Stability and Robustness Analysis

INDI relies on the assumption that for small time increments and high sampling rates, the nonlinear system dynamics in its incremental form is simply approximated by the (linearized) control effectiveness evaluated at the current state. However, and owing to the finite time delay one can achieve in digital devices, there exists an error $\boldsymbol{\epsilon}(t)$ (Wang, X., van Kampen, E., Chu, Q.P., and Lu, P. (2019b)), called the TDE error in the TDC literature (Chang and Jung (2009)), for which the error dynamics can be regarded as

$$\ddot{\boldsymbol{e}} + \mathbf{k}_D\dot{\boldsymbol{e}} + \mathbf{k}_P\boldsymbol{e} = \boldsymbol{\epsilon}(t). \tag{51}$$

Previous theoretical stability and robustness proofs for INDI controllers had the problem of not having considered this important residual error as pointed out by Wang, X., van Kampen, E., Chu, Q.P., and Lu, P. (2019b). Recently, the INDI control in the literature has been reformulated for systems with arbitrary relative degree and without recurring to cascaded-control structures, i.e., without using a time–scale separation assumption Wang, X., van Kampen, E., Chu, Q.P., and Lu, P. (2019b). This reformulation allowed to extended further the incremental nonlinear control approach for Sliding Mode Control Wang, X., van Kampen, E., Chu, Q.P., and Lu, P. (2019a). For these new reformulations and extensions, conditions for stability and robustness analyses of incremental nonlinear control have been finally established and analyzed using Lyapunov-based methods. Details on the sufficient conditions for closed–loop stability under INDI and discrete TDC, and therefore applicable to this problem can be found in Wang, X., van Kampen, E., Chu, Q.P., and Lu, P. (2019b,a); Jung et al. (2004); Chang and Jung (2009).

The existing sufficient condition for closed-loop stability of INDI (Wang, X., van Kampen, E., Chu, Q.P., and Lu, P. (2019a)) for input–output linearizable plants have been proposed as follows, which is similar to the one proposed for TDC Youcef-Toumi and Ito (1990); Jung et al. (2004); Chang and Jung (2009), and under the condition that zero dynamics of the plant is exponentially stable and the desired trajectory and its derivatives are bounded

$$\left\|\boldsymbol{I}_n - \boldsymbol{M}(\boldsymbol{x})\cdot\bar{\boldsymbol{M}}^{-1}\right\| \le \bar{b} < 1 \tag{52}$$

However, this condition does not have the sampling time explicitly considered and it has been found that even with a very small sampling time this condition might be violated Jung et al. (2004). A sufficient condition for closed-loop stability for discrete TDC systems is presented by Jung et al. (2004); Chang and Jung (2009) as the following (taking λ as the sampling):

$$\left\| \boldsymbol{I}_n - \boldsymbol{M}(\boldsymbol{x}) \cdot \bar{\boldsymbol{M}}^{-1} \right\| < \frac{1}{1 + \left[(1+\beta_1\gamma_P)\gamma_D + \beta_2\gamma_{PD}\right]\lambda} \tag{53}$$

where β_1, β_2, γ_D, γ_P, and γ_{PD} are tunable gains. To conclude, the influence model uncertainties to the reformulated system can be regarded as

$$\dot{\boldsymbol{\zeta}} = \boldsymbol{H}(\boldsymbol{x}, \boldsymbol{u}) + \Delta\boldsymbol{H}(\boldsymbol{x}, \lambda) + \bar{\boldsymbol{M}} \cdot \boldsymbol{u} \tag{54a}$$

and therefore, application of the control law $\boldsymbol{u} = \bar{\boldsymbol{M}}^{-1}\left[\boldsymbol{\nu} - \boldsymbol{H}(\boldsymbol{x}, \boldsymbol{u})\right]$ to this uncertain dynamics actually gives $\dot{\boldsymbol{\zeta}} = \boldsymbol{\nu} + \Delta\boldsymbol{H}(\boldsymbol{x}, \lambda)$ which is not linearizing as expected because of the extra uncertain term. This major flaw of NDI-based control systems is well known and also previously demonstrated by Sieberling et al. (2010) among others. Wang, X., van Kampen, E., Chu, Q.P., and Lu, P. (2019b,a) proved that

$$\lim_{\lambda \to 0} \|\Delta\boldsymbol{H}(\boldsymbol{x}, \lambda)\| = \boldsymbol{0}, \quad \forall \boldsymbol{x} \in \mathcal{R}^n \tag{55}$$

which implies that the term $\Delta\boldsymbol{H}$ becomes negligible for sufficiently high sampling rates, which has been the common assumption behind INDI control laws, and furthermore, asymptotic stability of the nominal system is proven as the closed-loop system can be ultimately bounded by a class $\mathcal{K}$ function of the perturbation bounds.

4. ATTITUDE CONTROL SIMULATIONS

For numerical simulations to demonstrate the high-agility attitude control system as derived in Sections 3.4 and 3.5, we use the comprehensive analytical nonlinear model of Section 2 for a small satellite with an inertia matrix of

$$\boldsymbol{I} = \begin{bmatrix} 10 & 1 & 0.5 \\ 1 & 7 & 0.2 \\ 0.5 & 0.2 & 9 \end{bmatrix} \ \mathrm{Kg \cdot m^2},$$

and as main torque actuators, an array of three *'High-Torque-Wheels'* (*HTW*) in orthogonal configuration (and aligned with the principal axes). Wheel characteristics for these *HTW*s are presented in (Acquatella B. (2018)), where the most important ones are their max. torque of 0.23 [Nm] and moment of inertia of 5×10^{-3} [$\mathrm{Kg \cdot m^2}$].

The initial *HTW* wheel speeds are zero; normally during operation, initial wheel speeds represent the angular momentum stored in the satellite. The MRP tracking reference commands are designed smooth up to a second order with a simple reference trajectory generator. The second derivative of these reference commands will act as feed-forward acceleration commands. We restrict these maneuvers according to the actuator limits in order to avoid the case of actuator saturation. For all simulations we consider the virtual controller $\boldsymbol{\nu} = \ddot{\boldsymbol{y}}_d + \mathbf{k}_D\dot{\boldsymbol{e}} + \mathbf{k}_P\boldsymbol{e}$ so that the error dynamics are equivalent across different scenarios. This is a classical second order dynamics where considering a natural frequency $\omega_n = 3$ rad/s and damping coefficient of $\zeta = 0.707$ we can obtain the gains $\mathbf{k}_{D_i} = 2 \cdot \zeta \cdot \omega_n = 4.242$ and $\mathbf{k}_{P_i} = \omega_n^2 = 9, \ i = 1, 2, 3.$

Simulation results in nominal condition verifies that INDI and TDC/PID control perform quite similarly. To study the performance under realistic conditions, we apply uncertainty in the inertia matrix of the satellite platform and perform Monte-Carlo simulations. Figure 1 presents the performance of the INDI attitude control under the uncertainty considered by showcasing the attitude tracking for the MRP reference maneuver commanded and the respective tracking error. Further simulations showcase a similar performance of the TDC/PID attitude control under the same uncertainty in the inertia parameters, not shown here because of the paper length limitation.

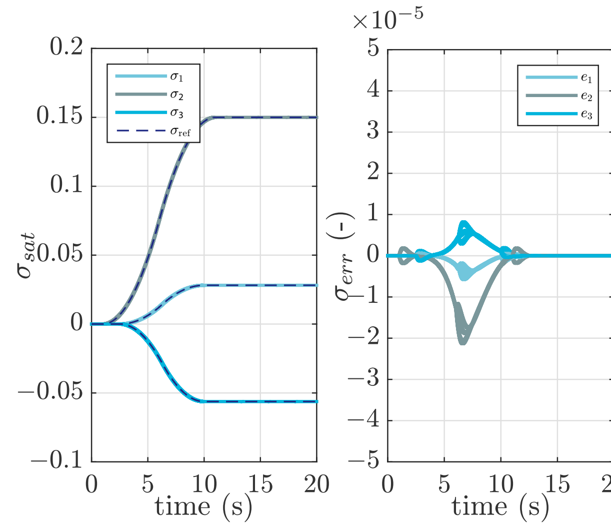

Fig. 1. INDI control: MRP attitude tracking and tracking error during a fast slew maneuver under uncertainty.

However, the nonlinear control laws perform differently in terms of robust performance and stability according to the metric in Eq. (52). This result is shown in Figure 2 for both INDI and TDC/PID. At this stage it becomes evident that the self–scheduling property of the INDI controller as compared to the TDC/PID controllers makes the attitude control system to guarantee a better stability margin as compared to TDC/PID; in the latter case, their static control effectiveness hinders the stability margin as it is proportional to both the maneuver and the size of the uncertainty. In summary, simulation results verified similar nonlinear performance of agile attitude control using both INDI and TDC/PID control. The robustness and stability properties have been shown to be superior for INDI in comparison to TDC/PID control for this particular case.

5. CONCLUSIONS

In this paper an agile and robust nonlinear spacecraft attitude controller is developed based on the recent incremental nonlinear dynamic inversion (INDI) reformulation. This controller is an improvement over the previously INDI approach for spacecraft atittude control in that it considers a non-cascaded dynamic inversion control where scheduling of the time-varying control effectiveness is done analytically. This results in a nonlinear controller scheduled only by kinematic (fully known) and parametric terms, making it robust to model uncertainties. Finally, a relationship between INDI, time-delay control, and nonlinear-PID control is established. The systematic gain tuning and self scheduling property of our INDI controller can be scaled and readily applied to attitude control of rigid spacecraft for agile maneuvers that do not saturate the actuators; this issue will be addressed in future research. Simulations results shows the effectiveness of our method.

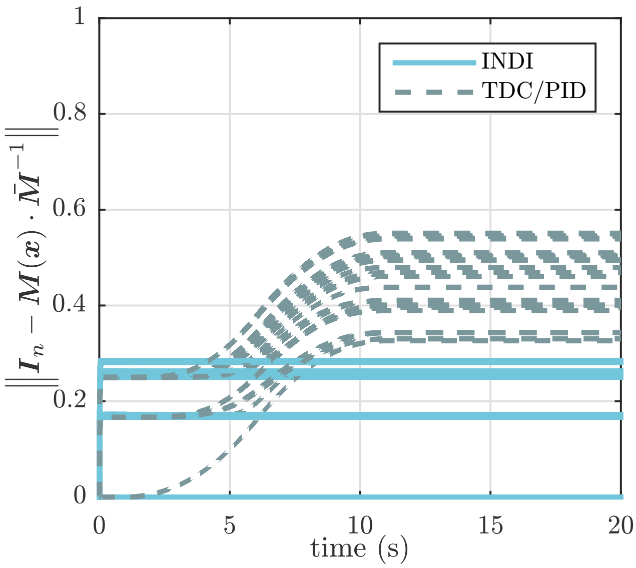

Fig. 2. INDI and TDC/PID criterion for closed-loop stability under uncertainty.

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