Evolutionary Dynamics for Designing Multi-Period Auctions[1]

Tomas Klos Gerrit Jan van Ahee

Delft University of Technology, P.O. Box 5031, 2600 GA, Delft, The Netherlands

Mechanism design (MD) has recently become a very popular approach in the design of distributed systems of autonomous agents. Also called 'inverse game theory' [4], MD is concerned with designing the games or systems in which agents interact, and to do this in such a way that rational agent behavior in those games leads to certain desirable properties for the system as a whole. A key assumption in MD is that *agents behave rationally*, since this provides the predictability of agent behavior required for optimizing the mechanism's design. In many practical circumstances, however, agents don't behave rationally since, in general, finding Nash equilibrium strategies to play is intractable [2].

Because of such negative results, many have resorted to heuristic approaches to these problems. Here, we propose studying the interaction between the mechanism designer and the game participants as a higher level, 'meta-game,' in which the designer chooses among alternative mechanism designs, while the agents choose among alternative strategies to play. We solve this game 'heuristically' using evolutionary game theory techniques, specifically, the (coupled) replicator dynamics (RD) [3]. To illustrate, we adopt the multi-period auction scenario developed by Pardoe and Stone (PS) [5].

1 Analyzing Meta-Games

Pardoe and Stone [5] studied a setting in which a seller wants to sell a large number of identical goods, and chooses to do so by repeatedly auctioning off batches of 60 items in a sequence of uniform-price sealed-bid auctions. In each auction (of 60 goods), all winning bidders pay the price of the highest losing bid. There are 120 bidders for each auction, and each receives a 'signal,' indicating the value of each of the (identical) goods, drawn uniformly at random from $[0, 1]$. Bidders are not assumed to bid strictly rationally, but to probabilistically choose one of a limited set of 5 'heuristic' bidding strategies (which we don't detail here, see [5]), $\{$EQ, OE, UE, DO, AV$\}$. Given such bidders, the seller in turn, is unsure whether it is most profitable to auction off all 60 items at once, or to distribute them evenly over $2, 3$, or 4 'periods,' thereby revealing the winning price in between periods, and allowing the remaining bidders to update their bids.

Whereas PS need to perform extensive simulation experiments, we have designed an efficient algorithm which allows us to exactly calculate (1) expected revenues for the seller's choices of 'number of periods,' and (2) the bidders' utilities of using each of the 5 heuristic strategies. This algorithm allows us to study the interaction between the seller and the bidders as a meta-game, which we do using coupled replicator equations [1, 3, 6, 7] for the two sets of strategies. The RD are a popular and intuitive way of modeling deterministic evolutionary dynamics in games [3]. With RD, the state of a population is represented as a vector of relative frequencies of different strategies. In our case, there are two 'populations' of strategies, viz. the seller's vector $\mathbf{x} = (x_1, \ldots, x_n)$ with $n = 4$ choices for number of periods, and the bidders' vector $\mathbf{y} = (y_1, \ldots, y_m)$ with $m = 5$ choices of heuristic strategies for the bidders. In each of a sequence of 'generations,' the states of these populations are evolved using replicator equations, which express that above (below) average strategies become more (less) prevalent in their population:

$$\frac{\dot{x}_i}{x_i} = ((\mathbf{A}\mathbf{y})_i - \mathbf{x} \cdot \mathbf{A}\mathbf{y}) \qquad \text{and} \qquad \frac{\dot{y}_j}{y_j} = ((\mathbf{B}\mathbf{x})_j - \mathbf{y} \cdot \mathbf{B}\mathbf{x}).$$

Here, x_i and y_i are the densities of pure strategies for the seller and the bidders, respectively, a dot indicates derivative wrt time, $\mathbf{A}$ is the payoff (revenue) matrix of the seller, and $\mathbf{B}$ the payoff (utility) matrix of the

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bidders. With *coupled* RD, the growth rate for each strategy in each population depends on the distribution of strategies in the other population.

2 Experiments

We performed experiments in which the seller's and the bidders' strategy distributions are evolved using RD based on the strategies' expected average revenues and utilities, respectively. Figure 1 shows an example where the initial bidding strategy distribution is drawn from the 4-simplex uniformly at random. The seller's initial probability distribution is always uniform.

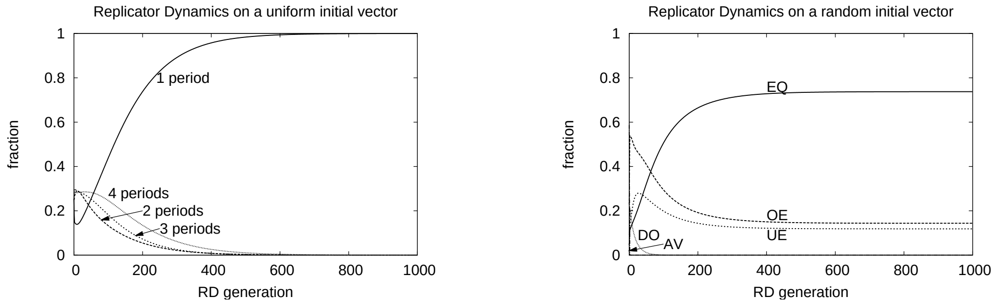

Figure 1: Coupled replicator dynamics.

In Figure 2 (left) we plot the number of samples in which each of the numbers of periods maximizes revenue—before and after RD. Choosing 1 or 2 periods generates the highest performance in 82% of all

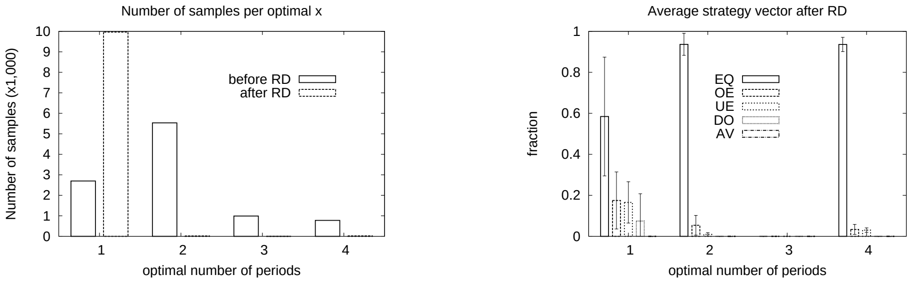

Figure 2: Vectors after RD.

randomly generated distributions (before RD). After RD, choosing 1 period is optimal virtually always: 2 (4) periods is optimal in only 19 (21) out of 10,000 samples. The graph on the right shows the average strategy distributions occuring in those cases, showing a high prominence of the EQ bidding strategy, although when 1 period is optimal, the high standard deviation (errorbar) suggests maybe we're averaging over several qualitatively distinct attractors. Further research is in order here.

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