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Optimizing a maintenance strategy by combining age-based maintenance and an imperfect fault detection model

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Abstract In predictive maintenance, fault detection models are often imperfect, with both false positives and false negatives. By solely planning maintenance based on an imperfect fault detection model, the number of failures, maintenance tasks and maintenance costs might increase compared to age-based maintenance. In this paper, we therefore combine age-based maintenance with an imperfect fault detection model. First, we derive the probability of a false positive given the age of the component, using Bayes theorem. We subsequently use the renewal reward theory to optimize the age at which to preventively replace components, if we also replace components based on the alarms of the fault detection model. Here, we may also ignore alarms when the component is just installed. The optimal parameters of this approach indicate whether it is optimal to perform preventive maintenance based on i) only the age (age-based maintenance), ii) only the alarms of the fault detection model, iii) based on the alarms and the age or iv) to have no preventive replacements. We illustrate this approach in a small case study, where we show that even fault detection models with many false positives and false negatives can lower the maintenance costs, if combined with age-based replacements.

1. Introduction

A popular maintenance strategy in industry is age-based, also called time-based, maintenance. In age-based maintenance, components are always preventively replaced after a fixed amount of time. The age-replacement model is often used to find the optimal preventive replacement age with the renewal-reward theorem (Barlow and Proschan (1996)). In recent years, the abundant availability of sensors installed around components has incentivized the development of predictive maintenance instead. In predictive maintenance, the measurements of these sensors are used to develop prognostic models. Components often degrade according to the delay time model, exhibiting three health states: Healthy, degraded and failed. In this case, predictive maintenance models are often divided into two type of models (Lei et al. (2018)). First, the measurements are used to generate an alarm when a component goes from the healthy state to a degraded state. This is called a fault detection model. Once a component is identified as degraded, the Remaining Useful Life (RUL, the time left until failure) is optionally predicted. For instance, neural networks (Fink et al. (2020)) and random forests (Nunes et al. (2023)) are popular methods to develop prognostic models.

In literature, prognostic models are often developed with simulated or lab data, which gives good results. When optimizing the maintenance planning, it is assumed that these good results will also be obtained in practice. For instance, in de Pater et al. (2022) and Wang et al. (2024), accurate RUL predictions with small errors are obtained for a simulated aircraft engine data set. The components are consequently maintained just before the predicted failure time, with a safety margin. With real-life data, however, training a prognostic model is difficult due to noisy data, a lack of failure data, changing operating conditions, multiple failure modes, etcetera. For instance, in Bieber et al. (2023), the authors obtain an F1-score of approximately 60% for anomaly detection with real data from satellites reaction wheels.

Prognostic models trained on real data thus often contain errors, i.e., are imperfect. Specifically, an imperfect fault detection model has false negatives (the model identifies a component as healthy, while it is degraded) and false positives (the model identifies a component as degraded, while it is healthy). When planning maintenance solely based on the outcomes of such an imperfect fault detection model, the number of maintenance tasks might actually increase, compared to age-based maintenance, due to false positives. At the same time, the number of failures might increase as well due to false negatives. Some studies therefore combine health information of the component with age-based replacements. Specifically, in Berrade et al. (2013); Driessen et al. (2017) and Zhang et al. (2021), it is assumed that imperfect inspections reveal whether a component is

degraded or healthy. To handle the errors of the inspections, preventive replacements are performed based on both the estimated degradation and the age of the component.

In this paper, we combine an imperfect fault detection model with age-based replacements. Instead of inspections, we assume that the health state of a component is directly assessed from the sensor measurements gathered during operations. Updates on the health state thus become available frequently. We first derive the probability of a false positive alarm given the age of the component using Bayes theorem. We subsequently optimize a maintenance strategy where we combine preventive replacements based on both the age of the component, and on the alarms of the fault detection model. We formulate the average maintenance costs using the renewal-reward theorem considering two variables: i) from what age of the components onward to act upon alarms of the fault detection model (to handle false positives) and ii) the age to preventively replace the component (to handle false negatives). We illustrate our approach for a component of which the lifetime follows a Weibull distribution and with a fault detection model with varying false positive and false negative rates. Here, the combination of age-based replacements with a fault detection model with a true positive rate of 0.5 (50% of the failures is detected beforehand) and a false positive rate of 0.001 (we expect one false positive per 1000 alarm moments) lowers the maintenance costs with 23.8% compared to only age-based replacements.

In the rest of this paper, we first present the methodology in Section 2, and we illustrate the maintenance optimization strategy in Section 3. Last, we present our conclusions in Section 4.

2. Methodology: Combining preventive replacements based on age and a fault detection model

2.1. Considered fault detection model

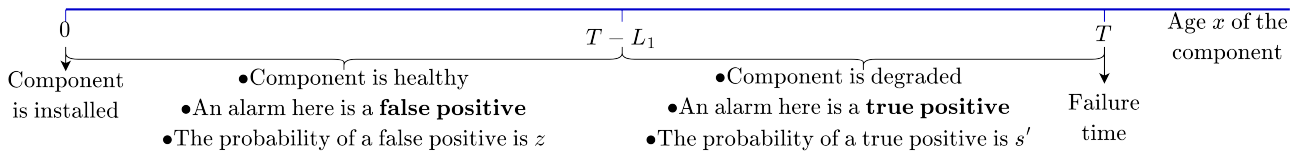


Figure 1. An overview of the fault detection model.

We consider a component that degrades according to the delay-time model, with three health states: Healthy, degraded and failed. The aim of the fault detection model is to raise alarm(s) when the component is degraded. We assume that the fault detection model can only raise an alarm at *discrete* time alarm moments, for instance after each flight. Let $x(i)$ denote the age of a component at alarm moment i , and let T denote the failure time of a component. We have two types of alarms. First, we assume that the component is degraded in the time-window $[T - L_1, T)$, with $L_1 > 0$ a constant. An alarm raised if the age of the component is in $[T - L_1, T)$ is therefore a true positive. Let $0 \leq s' < 1$ denote the true positive rate, i.e., probability that we raise a true alarm at any alarm moment i with $x(i) \in [T - L_1, T)$. Second, we assume that the component is healthy before time $T - L_1$. An alarm raised when the age of the component is smaller than $T - L_1$ is therefore a false positive. Let z denote the false positive rate, i.e., the probability that we generate a false positive at an alarm moment i with $x(i) < T - L_1$. We assume that the alarms are independent of each other. An overview of this fault detection model is in Figure 1.

After a single alarm, we employ a RUL prediction model. We assume that this RUL prediction model perfectly predicts the failure time T if the alarm is a true positive. Let $0 \leq s < 1$ denote the probability that at least one alarm is generated between $[T - L_1, T)$:

$$\begin{aligned} s &= p(\text{At least one alarm is raised between } [T - L_1, T)) \\ &= 1 - p(\text{No alarm is raised between } [T - L_1, T)) \\ &= 1 - (1 - s')^m. \end{aligned} \tag{1}$$

where m denotes the number of alarm moments in $[T - L_1, T)$ and $p(\cdot)$ denotes the probability of an event. Last, we assume that the lifetime distribution of the component is known. Let $f(\cdot)$ and $F(\cdot)$ denote the PDF and CDF of the lifetime distribution of the component, respectively.

2.2. Probability of a false positive alarm given the age of a component

Let A_i denote the event that the fault detection model raises an alarm at alarm moment i , when the age of the component is $x(i)$. Using the law of total probability, we derive the probability of A_i :

$$\begin{aligned} P(A_i) &= P(A_i | T \leq x(i)) P(T \leq x(i)) + P(A_i | x(i) < T \leq x(i) + L_1) P(x(i) < T \leq x(i) + L_1) + \\ &\quad P(A_i | T > x(i) + L_1) P(T > x(i) + L_1) \\ &= 0 \cdot F(x(i)) + s' [F(x(i) + L_1) - F(x(i))] + z [1 - F(x(i) + L_1)]. \end{aligned} \quad (2)$$

With Bayes theorem, we derive the probability that an alarm A_i at alarm moment i is a false positive:

$$\begin{aligned} P(T > x(i) + L_1 | A_i) &= \frac{P(A_i | T > x(i) + L_1) P(T > x(i) + L_1)}{P(A_i)} \\ &= \frac{z \cdot [1 - F(x(i) + L_1)]}{s' [F(x(i) + L_1) - F(x(i))] + z [1 - F(x(i) + L_1)]}. \end{aligned} \quad (3)$$

The probability that an alarm is a false positive thus depends on the age $x(i)$ of the component, s and z .

2.3. Combining preventive replacements based on the fault detection model and on the age

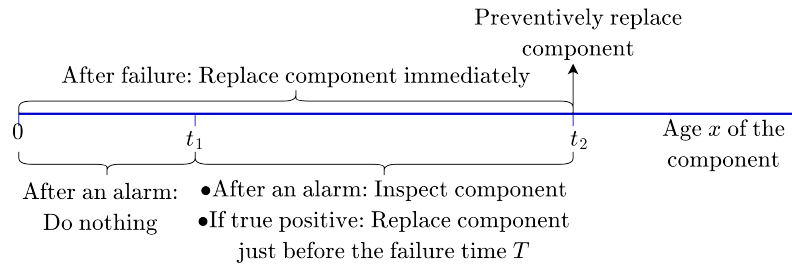


Figure 2. Overview of the considered actions when combining age-based replacements with a fault detection model.

If a fault detection model has a small true positive rate, then only replacing components based on alarms might lead to more failures than with age-based maintenance. If a fault detection model has a high false positive rate, then always acting upon alarms might lead to more maintenance tasks than in the age-based maintenance. To prevent this, we combine preventive replacements based on the age of the component, with replacements based on the alarms of the fault detection model. Specifically, we consider the following four maintenance actions, depending on two variables t_2 and $t_1 \leq t_2$ (see Figure 2):

- We replace a component immediately upon failure. A replacement after failure costs c_f .
- We replace a component immediately once it reaches the age t_2 . Such a preventive replacement costs $c_p < c_f$.
- If the fault detection model raises an alarm when the age is smaller than t_1 , we do nothing.
- If the fault detection model raises an alarm when the age is between t_1 and t_2 , we employ the RUL prediction model. If the predicted failure time is smaller than t_2 , we immediately inspect the component. This inspection reveals whether the alarm is a false positive or a true positive. If the alarm is a false positive, we do nothing. We only incur the inspection costs c_i . However, if the alarm is a true positive, we preventively replace the component just before the failure time T . We incur the costs $c_i + c_p < c_f$. We assume that all false positives between t_1 and t_2 lead to an inspection.

With these actions, we model four different maintenance strategies at once. Depending on the values of the variables t_1 and t_2 , we have preventive replacements based on i) only the age of the component, ii) the age of the component and the alarms, iii) only on the alarms or iv) no preventive replacements. The optimal maintenance strategy for preventive replacements depends on the optimal value of t_1 and t_2 .

According to the renewal-reward theorem (see Arts et al. (2024)), the long-term average maintenance costs per time unit equal the expected costs of one replacement, divided between the expected time between two replacements. We use this theorem to derive the long-term average cost per time-unit $g(t_1, t_2)$ with our maintenance

nance strategy:

$$\begin{aligned}
 g(t_1, t_2) &= \frac{c_f F(t_1) + s(c_p + c_i) [F(t_2) - F(t_1)] + (1-s)c_f [F(t_2) - F(t_1)] \\
 &\quad + c_p [1 - F(t_2)] + zc_i \sum_{i=m(t_1)+1}^{m(t_2)} [1 - F(x(i) + L_1)]}{t_2 [1 - F(t_2)] + \int_0^{t_2} u f(u) du} \\
 &= \frac{s [c_f - c_p - c_i] F(t_1) + c' F(t_2) + c_p + zc_i \sum_{i=m(t_1)+1}^{m(t_2)} [1 - F(x(i) + L_1)]}{\int_0^{t_2} [1 - F(u)] du}, \quad (4)
 \end{aligned}$$

where $c' = (1-s)c_f + (s-1)c_p + sc_i$, while $m(t)$ denotes the number of alarm moments up to, but not including, time t . We find the optimal value of t_1 and t_2 with a grid search.

3. Illustration of combining age-replacements with a fault detection model

We illustrate the combination of the age-replacements with a fault detection model with an example. Here, we assume that the lifetime distribution of a component follows a Weibull distribution with PDF $f(x)$:

$$f(x) = \frac{kx^{k-1}}{\lambda^k} \exp\left(-\frac{x^k}{\lambda^k}\right), \quad (5)$$

where k is the shape parameter and λ is the scale parameter. We use $k = 2$ and $\lambda = 1000$ (see Figure 3), which gives a mean lifetime of 886 hours with a standard deviation of 463 hours. The Weibull distribution is often used in literature to model the lifetime of a component. However, our proposed approach works with other lifetime distributions as well. Note that a prognostic fault detection model purely raises alarms based on sensor measurements and/or on a degradation model, and not on the lifetime distribution or the age of the component.

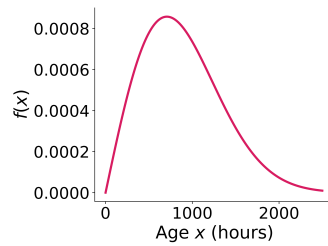


Figure 3. The pdf $f(x)$ of the considered lifetime distribution.

3.1. Probability of a false positive alarm given the age of a component

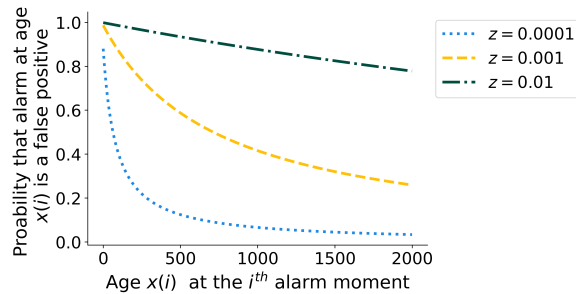


Figure 4. The probability that an alarm at the i^{th} alarm moment, when the age of the component is $x(i)$ hours, is a false positive for $s = 0.5$ and several values of z .

Figure 4 shows the probability that an alarm at age $x(i)$ is a false positive alarm for $s = 0.5$ and several values of z . As expected with this lifetime distribution, the probability that an alarm is a false positive decreases when the component gets older. We might thus indeed save costs by ignoring alarms when the component is just installed, as these alarms have a higher probability of being a false positives.

Table 1. The optimal t_1 , t_1^* , and t_2 , t_2^* and the corresponding minimum average costs per hour $g(t_1^*, t_2^*)$ for several values of s and z . Strategy “Age only” - Only preventive replacements based on age. Strategy “Age & alarms” - Preventive replacements based on both age and alarms.

z	0	0.0001	0.001	0.01
$s = 0.1$				
t_1^*	0	14	142	336
t_2^*	354	354	351	336
$g(t_1^*, t_2^*)$	5.76	5.78	5.96	6.06
Strategy	Age & alarms	Age & alarms	Age & alarms	Age only
$s = 0.5$				
t_1^*	0	3	29	282
t_2^*	474	474	473	382
$g(t_1^*, t_2^*)$	4.38	4.40	4.62	5.99
Strategy	Age & alarms	Age & alarms	Age & alarms	Age & alarms
$s = 0.9$				
t_1^*	0	0	16	158
t_2^*	1018	1018	1020	921
$g(t_1^*, t_2^*)$	2.29	2.32	2.53	4.48
Strategy	Age & alarms	Age & alarms	Age & alarms	Age & alarms

3.2. Combining the fault detection model with age-based replacements

In this case study, a corrective replacement costs $c_f = 10,000$, a preventive replacement costs $c_p = 1000$, an inspection costs $c_i = 250$, $L_1 = 20$ hours, and we have one alarm moment per hour. Without a fault detection model, it is optimal to preventively replace the components at an age of $t^* = 336$ hours, with the average maintenance costs per hour of 6.06.

Table 1 shows the optimal value of t_1 , t_1^* , and of t_2 , t_2^* , and the corresponding minimum costs per hour $g(t_1^*, t_2^*)$ for several values of z and s . We find these values with a grid search, which takes less than two minutes for all cases. As expected, if $z = 0$, $t_1^* = 0$: If there are no false positives, it is optimal to perform a preventive replacement after each alarm. When z increases, the probability of a false positive increases as well (see Figure 4). As expected, t_1^* therefore increases as well, i.e., we ignore more alarms when z increases. If s increases, we detect more failures before they occur, and the optimal preventive replacement time t_2^* therefore increases.

Moreover, the optimal solution also shows the optimal maintenance strategy. If $s = 0.1$ while $z = 0.01$, it is optimal to only have age-based preventive replacements and to ignore all alarms of the fault detection model. In all other considered cases, it is optimal to mix preventive replacements based on the age and on the alarms. For instance, if $s = 0.5$ (50% of the failures is detected beforehand) while $z = 0.001$ (we expect one false positive per 1000 alarm moments), the maintenance costs are 23.8% lower with our strategy compared to only age-based replacements.

4. Conclusions

In predictive maintenance, fault detection models are usually not perfect. If we only plan maintenance based on the outcomes of such an imperfect model, the number of maintenance tasks and the number of failures might actually increase, compared to age-based maintenance. In this paper, we therefore combine age-based maintenance with an imperfect fault detection model. We use the renewal-reward theorem to find an expression for the average maintenance costs per time unit. This formula contains two variables, i) from which age onwards to act upon alarms of the imperfect fault detection model and ii) at which age to preventively replace the component. These variables are optimized with a grid search. The optimal solution shows if it is optimal to have preventive replacements based on i) only the age of the component, ii) the age of the component and alarms, iii) only on the alarms or iv) no preventive replacements. We illustrate our approach with an example, where the lifetime distribution of a components follows a Weibull distribution. Here, even combining a fault detection model with a high false positive rate and a low true positive rate with age-based replacements leads to lower maintenance costs, than having only age-based replacements.

This research is a first step in combining fault detection models with the lifetime distribution of a component and/or age-based maintenance strategies. In future research, we will analyse our approach on a real life case study, where we can fit a lifetime distribution and develop a real fault detection model. For this, we will have to make more realistic assumptions, such as having an imperfect or no RUL prognostic model, dependent alarms, no possibility for inspections and a varying value for L_1 .

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