

Comment on "Histogram Monte Carlo Renormalization Group Method for Phase Transition Models without Critical Slowing Down"

In a recent Letter [1] Hu presented a cluster Monte Carlo method for the simulation of Potts models, using the random cluster representation [2]. Thus, graphs \mathcal{G}' have to be generated with a weight proportional to $p^{b(\mathcal{G}')} (1-p)^{E-b(\mathcal{G}')} q^{n(\mathcal{G}')}$, where $p = 1 - \exp(-K)$, with K the reduced coupling constant of the Potts model, $b(\mathcal{G}')$ the number of occupied bonds selected from a total number of E bonds, q the number of states of the Potts model, and $n(\mathcal{G}')$ the number of clusters of sites connected by occupied bonds. In Hu's method, each bond is occupied with probability p , so that \mathcal{G}' occurs with a probability $P(\mathcal{G}') = p^{b(\mathcal{G}')} (1-p)^{E-b(\mathcal{G}')} q^{n(\mathcal{G}')}$. Thus, for the calculation of canonical averages, \mathcal{G}' has to be weighed with a factor $q^{n(\mathcal{G}')}$. In this sense, Hu's method can be characterized as "crude" or "simple" cluster sampling, in analogy with simple Monte Carlo sampling as opposed to Metropolis importance sampling [3]. Since simple Monte Carlo methods generate completely independent configurations, critical slowing down is absent. In contrast with Hu's method, the importance sampling algorithms of Swendsen and Wang [4] and of Wolff [5] do include the weight factor due to the number of clusters in the probability that a graph is generated. These latter methods sacrifice the statistical independence of consecutive samples.

However, statistical independence is not a sufficient condition for the method to remain efficient for larger system sizes. A disadvantage of simple sampling methods is that they tend to generate predominantly configurations with a low statistical weight, especially when the number of possible configurations becomes high, and their range of statistical weights is large. In order to study this effect in the Potts model, consider the histogram of the number of graphs $N_{p,q}(b, n)$ with b occupied bonds and n clusters. This histogram can be determined for the percolation ($q = 1$) problem at a bond probability p' . The distribution at other p, q is obtained as $N_{p,q}(b, n) = q^n [p(1-p')/p'(1-p)]^b N_{p',1}(b, n)$. The efficiency of Hu's method hinges on the overlap of both distributions for suitably chosen p' given p . This overlap should exist for all values of b and n , where $N_{p,q}$ has significant nonzero values. Here we focus on the two-dimensional $q \leq 4$ state Potts critical point where $\langle b \rangle = E/2$ in the thermodynamic limit (this follows, e.g., from the self-duality of the random-cluster model). Thus, the distribution of b peaks near $E/2$, independent of q . Therefore we can obtain some insight into the overlap by comparing the critical one-dimensional distributions $N_q(n)$.

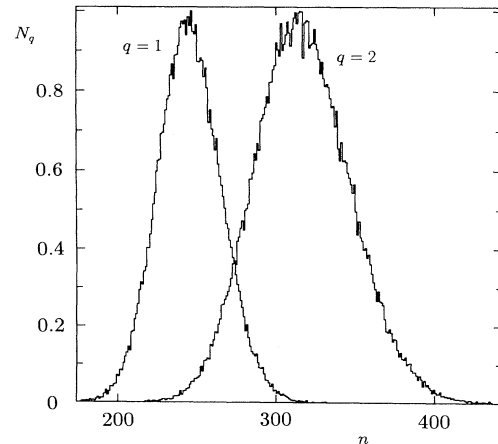


FIG. 1. Monte Carlo results for the critical distributions $N_q(n)$ of the number of clusters of q -state random-cluster models. These results apply to 50×50 systems with periodic boundaries for $q = 1$ and $q = 2$. The vertical scale for each q is normalized such that both maxima are equal. The data are averaged over 10^5 Monte Carlo configurations, taken at intervals of five sweeps for $q = 2$.

We have computed this quantity at $q = 1$ and 2 for several system sizes, using the standard Monte Carlo method. We find that, for the small sizes used by Hu, the overlap between the distributions is large, which explains why the method works well. However, for larger system sizes, the overlap decreases. This is illustrated in Fig. 1 for $q = 1$ and $q = 2$ systems of size 50×50 . The sampling procedure at $q = 1$ yields a distribution with a maximum near some 245 clusters. The fact that this number of clusters is less probable at $q = 2$ demonstrates that Hu's method becomes less efficient for larger system sizes.

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