Extended Strip Model for slabs subjected to load combinations

Lantsoght, Eva O.L.; van der Veen, Cor; de Boer, Ane

DOI
10.1016/j.engstruct.2017.05.012

Publication date
2017

Document Version
Accepted author manuscript

Published in
Engineering Structures

Citation (APA)
Lantsoght, E. O. L., van der Veen, C., & de Boer, A. (2017). Extended Strip Model for slabs subjected to load combinations. *Engineering Structures*, *145*, 60-69. https://doi.org/10.1016/j.engstruct.2017.05.012

Postprint of Engineering Structures
Volume145, 15 August 2017, Pages 60–69
Link to formal publication (Elsevier): http://dx.doi.org/10.1016/j.engstruct.2017.05.012

Extended Strip Model for slabs subjected to load combinations

Eva O.L. Lantsoght[a,b] (E.O.L.Lantsoght@tudelft.nl Tel: +593 2 297-1700 ext. 1186 Corresponding Author), Cor van der Veen[b] (C.vanderveen@tudelft.nl), Ane de Boer[c] (ane.de.boer@rws.nl)

[a]Universidad San Francisco de Quito, Politecnico, Diego de Robles y Vía Interoceánica, Quito, Ecuador

[b]Delft University of Technology, Concrete Structures, Stevinweg 1, 2628 CN Delft, The Netherlands

[c]Ministry of Infrastructure and the Environment, Griffioenlaan 2, 3526 LA Utrecht, The Netherlands

Abstract

The loads that are used for the assessment of existing reinforced concrete slab bridges are the self-weight, superimposed loads, and distributed and concentrated live loads. As such, the shear capacity of reinforced concrete slabs under a combination of distributed and concentrated live loads is a topic of practical relevance. For slabs subjected to a single concentrated load, a plastic model for assessment exists: the Extended Strip Model, developed based on the Strip Model for concentric punching shear. A further adaptation of the model to assess slabs subjected to distributed and concentrated loads is presented in this paper. The proposed model is compared to experiments on slabs subjected to a single concentrated load and a line load. The conclusion of this comparison is that the Extended Strip Model results in a safe estimate of the maximum concentrated load on the slab, and that the method can be used for the assessment of existing bridges subjected to heavy truck loads.

Keywords

Assessment; Extended Strip Model; Flexure; Live loads; Plasticity-based model; Punching; Reinforced concrete; Slab bridges; Shear

1. Introduction

1.1 Assessment of existing bridges in the Netherlands

As the average age of the existing bridges in many parts of the world is increasing, the importance of methods for the assessment of these existing bridges is increasing as well. A common bridge type in the Netherlands [1] is the reinforced concrete solid slab bridge. Many of these slab bridges were built between the late 1950s and the early 1980s. The loads that are used for assessment in the Netherlands are the self-weight of the structure, the superimposed load, and the live loads. The live loads are given in NEN-EN 1991-2:2003 [2] and consist of a design tandem in each lane, combined with a distributed lane load. For shear assessment, the capacity of both reinforced concrete beams and slabs is taken as the one-way shear strength given in NEN-EN 1992-1-1:2005 [3]. Typically, the evaluation is then expressed based on a Unity Check: a ratio of the resulting shear stress from the applied loads over the shear capacity. If the Unity Check is larger than 1, the evaluated bridge is considered as not fulfilling the requirements [4]. For the existing reinforced concrete slab bridges, it is often found that the shear capacity is insufficient. Therefore, the shear capacity of reinforced concrete slab bridges has been a topic of research in the Netherlands for the past decade.

1.2 Methods for one-way and two-way shear

Reinforced concrete slab bridges subjected to concentrated loads such as the design tandem failing in shear are cases that are situated at the transition between one-way shear (beam shear) and two-way shear (punching shear) [5]. Traditionally, shear models are strictly subdivided into methods for one-way shear and two-way shear. The models for one-way shear are compared with experiments on beams in three- or four-point bending [6-8], whereas the models for two-way shear are compared with experiments on slab-column connections [9]. The loading case of a reinforced concrete slab bridge subjected to the load combination used for assessment lies somewhere in between these situations.

The most commonly used models for one-way shear are semi-empirical formulas derived from analysing the existing beam shear experiments [6, 7]. The shear capacity prescribed by NEN-EN 1992-1-1:2005 [3] and ACI 318-14 [10] follows a semi-empirical formula. Another model that has a theoretical basis and that has been introduced into design codes is the Modified Compression Field Theory [11]. In this theory, cracked concrete is considered as a separate material with its own constitutive equations, derived from panel tests. A simplification of the theory [12] can be found in the AASHTO LRFD 2015 code [13] and the *fib* Model Code 2010 [14].

For two-way shear, the most commonly used models are also semi-empirical formulas derived from the results of slab-column connection tests [9]. The punching shear capacity prescribed by NEN-EN 1992-1-1:2005 [3] and ACI 318-14 [10] is described by a semi-empirical formula. Improvements to the punching shear provisions from NEN-EN 1992-1-1:2005 have been suggested [15]. Another model that has a theoretical basis is the Critical Shear Crack Theory [16, 17]. This theory is the basis for the provisions in the Swiss Code SIA 262:2003 [18] and the *fib* Model Code 2010 [14]. Recently, a simplified punching shear model has proposed that is based on the Critical Shear Crack Theory [19].

A category of models that can be used for one-way and two-way shear are plasticity-based models, which can be subdivided in lower- and upper-bound methods. While plasticity-based methods for shear [20-22] are not directly found in design codes, plasticity-based methods are the basis of engineering tools such as strut-and-tie models for D-regions [23], the strip method for flexure [24, 25], and yield line analysis [26].

1.3 Experiments on slabs under a single concentrated load

To study the behavior of reinforced concrete slabs under a single concentrated load close to the support, a number of laboratory experiments were carried out. This load configuration was chosen, as it represents the case with the design tandem close to the support, which results in the largest shear stress for assessment. The specimens were half-scale reinforced concrete slab specimens of 5 m × 2.5 m × 0.3 m with a span of 3.6 m, tested close to a simple and continuous support, to represent a continuous slab bridge. In total, 127 experiments on 18 specimens were carried out [27-30]. The parameters varied in these experiments were: the position of the load in the transverse direction, the position of the load in the longitudinal direction, the amount of transverse reinforcement, the effect of previous cracking, the size of the loading plate, the moment distribution at the support, the concrete compressive strength, the overall width (with 2.5 m as a reference), the type of reinforcement (deformed bars as compared to plain bars), and the type of support (line supports as compared to elastomeric bearing blocks). The main conclusion of these experiments was that the three-dimensional load path in a reinforced concrete slab differs significantly from the two-dimensional load path in a reinforced concrete beam, and results in a larger shear capacity. This effect was also called the transverse load distribution capacity of slabs in shear [31]. This conclusion, and the experimental results, also led to the development of recommendations [1] for the assessment of reinforced concrete slab bridges when using the Eurocode provisions NEN-EN 1992-1-1:2005 [3] and NEN-EN 1991-2:2003 [2].

2. Extended Strip Model for slabs under combinations of loads

2.1 Extended Strip Model for slabs under a single concentrated load

The Extended Strip Model for reinforced concrete slabs under a single concentrated load [32] is developed based on the Strip Model for concentric punching shear in slabs [33-35]. The Strip Model is a lower-bound plasticity-based model that describes a possible load path prior to failure. As such, it shares features with the Strip Method for designing slabs in flexure [24, 25]. In slabs under concentrated loads, a complex loading situation of one-way shear, two-way shear, and flexure develops. This situation is reflected in the Strip Model by combining beam strips that work in arching action (an element of one-way shear) together with slab quadrants that work in two-way flexure. This principle is sketched in Figure 1, which shows a column with strips branching out from the column, and the resulting quadrants. The length of the strip l_{strip} is considered from the face of the column to a position of zero shear. The load path may function until a limiting one-way shear is reached at the interface between the strip and the quadrant. This limiting one-way shear is taken as the inclined cracking load given in ACI 318-14 [10]. The maximum load is then achieved by summing the capacities of the four strips, assuming that the limiting one-way shear is achieved on the interface between the strip and the quadrant. The maximum load that can be carried in the quadrants is thus w_{ACI}, the inclined cracking load given in ACI 318-14, see Figure 1.

The Extended Strip Model [32, 36, 37] extends the concepts of the Strip Model for application to slabs of a finite size, with a single concentrated load. This load can be placed at any position on the slab, so that the Extended Strip Model can study asymmetric loading situations. The model is well-suited to combine the effects of one-way shear, two-way shear, and flexure that govern the loading case of a reinforced concrete slab subjected to a concentrated load. To take into account the finite dimensions of the slab, and possible asymmetric loading, it is necessary to take into account the geometry of the slab, the bending moment and shear diagrams, as well as the effect of torsion. The resulting Extended Strip Model is then as shown in Figure 2. The effects of the geometry and asymmetry now influence the resulting one-way shear at the intersection between the quadrants and strips. As a result, the capacity of each single strip is different. Again, the maximum concentrated load is found by summing the capacities of the strips.

Whereas the effect of torsion could be neglected in the original Strip Model that studied only symmetric loading cases, it becomes more important for asymmetric loading cases. The effect of torsion was studied in a series of linear finite element models in which the ratio between bending moment and torsional moments was analyzed [38]. The result of this analysis is a simplified expression for the relative effect of torsion:

$$\beta = 0.8\frac{a}{d_x}\frac{b_r}{b} \text{ for } 0 \leq \frac{a}{d_x} \leq 2.5 \text{ and } 0 \leq \frac{b_r}{b} \leq \frac{1}{2} \tag{1}$$

If the effect of torsion is at its largest, the value of $\beta = 0$ and it is considered that all capacity is used to resist the effects of torsion. If the effect of torsion is negligible, the value of $\beta = 1$ and it is considered that all capacity is available to develop the required load path to resist the shear effects. When $a/d_x > 2.5$, the value of a/d_x in Eq. (1) is replaced by 2.5, and only the effect of the position along the width direction on the torsional behavior remains. The strips influenced by torsion carry the factor β in Figure 2.

For loads close to the support, the effect of direct load transfer between the load and the support is taken into account by increasing the capacity of the strip between the load and the support. For loads close to the free edge, the physical length of the strip l_{edge} needs to be compared to the loaded length of the strip l_w. If the loaded length is longer than the actual strip length, then the strip length instead of the loaded length should be used. This influence of the geometry is called the edge effect.

The effect of the overall bending moment diagram is reflected in Figure 2 by using the distance between the points of contraflexure L and the distance a_M, which is the smallest of the distance between the load and the support, or the distance between the load and the point of contraflexure. The effect of the self-weight of the slab, which becomes important for the assessment of slab bridges, is taken into account on the shear diagram by considering the stress v_{DL} of the dead load caused at the position of the concentrated load. Additionally, the Extended Strip Model includes the size effect in shear on the limiting shear stress w_{ACI}. This limiting shear stress is calculated differently for the x- and y-directions of the slab, to take into account the different value of the effective depth depending on the layer of reinforcement that is considered. Therefore, Figure 2 uses $w_{ACI,x}$ and $w_{ACI,y}$ for the different directions.

In the Extended Strip Model, the total maximum concentrated load P_{ESM} is calculated as:

$$P_{ESM} = P_x + P_{sup} + P_y + P_{edge} \tag{2}$$

$$P_x = \sqrt{2\left(1+\beta\right) M_{sag,x} w_{ACI,x}} \tag{3}$$

$$P_{sup} = \frac{2d_x}{a_v}\sqrt{2\left(1+\beta\right) M_{s,x} w_{ACI,x}} \tag{4}$$

$$P_y = \sqrt{2\left(\frac{L}{L-a_M}\right) M_{s,y}\left(w_{ACI,y} - v_{DL}\right)} \tag{5}$$

$$P_{edge} = \begin{cases} \sqrt{2\beta\left(\frac{L}{L-a_M}\right)M_{s,y}\left(w_{ACI,y}-v_{DL}\right)} \text{ for } l_w < l_{edge} \\ \beta\left(\frac{L}{L-a_M}\right)\left(w_{ACI,y}-v_{DL}\right)l_{edge} \text{ for } l_w \geq l_{edge} \end{cases} \quad (6)$$

The loaded length of the strip is determined as:

$$l_w = \sqrt{\frac{2M_{s,y}}{\beta\left(w_{ACI,y}-v_{DL}\right)\frac{L}{L-a_M}}} \quad (7)$$

The moment capacities are determined as:

$$M_{s,x} = M_{sag,x} + \lambda_{moment} M_{hog,x} \quad (8)$$

$$M_{s,y} = M_{sag,y} + \lambda_{moment} M_{hog,y} \quad (9)$$

with:

$$\lambda_{moment} = \frac{M_{sup}}{M_{span}} \quad (10)$$

and M_{sup} and M_{span} follow from the moment diagram of the slab subjected to all loads. At a simple support, the value of λ_{moment} becomes 0, and the moment capacities from Eqs. (8) and (9) become the sagging moment capacities $M_{sag,x}$ and $M_{sag,y}$.

The one-way shear capacity is calculated based on ACI 318-14 [10], but a correction for the size effect has been added [39]:

$$w_{ACI,x} = 0.166 d_y \sqrt{f_{ck}} \left(\frac{100\text{mm}}{d}\right)^{\frac{1}{3}} \quad (11)$$

$$w_{ACI,y} = 0.166 d_x \sqrt{f_{ck}} \left(\frac{100\text{mm}}{d}\right)^{\frac{1}{3}} \quad (12)$$

In Figure 2, the resulting loads are shown when the effects of the geometry, torsion, the acting dead load, the static equilibrium, the position of the point of contraflexure, and the size effect are taken into account.

2.2 Application to slabs under combinations of loads

When a slab is subjected to a combination of loads the Extended Strip Model can be used as well. When only a single tandem is used, the Extended Strip Model can be used by taking the perimeter of the four considered wheel prints, and considering this area as one large concentrated load from which the strips and quadrants are developed. Based on a field experiment on the Ruytenschildt Bridge, which was tested to failure [40], it was shown that this application of the Extended Strip Model results in a safe prediction of the maximum load in the test [36].

When a slab is subjected to a combination of concentrated and distributed loads, for example as used in the live load model from NEN-EN 1991-2:2003 [2], the Extended Strip Model can be used as well. The effect of the distributed load can now be taken into account in the span direction as a reduction of the shear capacity. This effect of the distributed load is represented by the shear stress caused by the distributed load at the position of the concentrated load, v_{dist}. As a result, the loading on the quadrants and strips becomes as shown in Figure 3. Since the effect of the distributed load is only considered in the span direction, only the values of P_y and P_{edge} from Eqs. (5) and (6) are changed for this application of the Extended Strip Model:

$$P_y = \sqrt{2\left(\frac{L}{L-a_M}\right)M_{s,y}\left(w_{ACI,y} - v_{DL} - v_{dist}\right)} \tag{13}$$

$$P_{edge} = \begin{cases} \sqrt{2\beta\left(\dfrac{L}{L-a_M}\right)M_{s,y}\left(w_{ACI,y}-v_{DL}-v_{dist}\right)} \text{ for } l_w < l_{edge} \\ \beta\left(\dfrac{L}{L-a_M}\right)\left(w_{ACI,y}-v_{DL}-v_{dist}\right)l_{edge} \text{ for } l_w \geq l_{edge} \end{cases} \tag{14}$$

As a result, the loaded length of the strip between the load and the support is now determined as:

$$l_w = \sqrt{\frac{2M_{s,y}}{\beta\left(w_{ACI,y}-v_{DL}-v_{dist}\right)\dfrac{L}{L-a_M}}} \tag{15}$$

An overview of these changes to the model is represented by the loads on the strips and quadrants shown in Figure 3.

3. Experiments on slabs under combinations of loads

3.1 Test setup

To assess the behavior of slabs under a combination of loads, representative of the load combination used for the assessment of reinforced concrete slab bridges, experiments were carried out [41]. The tested specimens were eight slabs in total, each with the same size of 5 m × 2.5 m × 0.3 m. In total, 23 experiments were carried out on these slabs, with two or four tests carried out per slab depending on the loading configuration. The load combination used for the assessment of reinforced concrete slab bridges consists of the self-weight, the superimposed dead load, and distributed and concentrated live loads. Since the application of a uniformly distributed load in a laboratory setting in combination with concentrated loads becomes complex, a simplified loading scheme was used for these experiments. A single concentrated load close to the support (as used in the first series of experiments described in §1.3) was combined with a line load acting over the full width of the slab, as can be seen in Figure 4.

In the experiments, the line load was applied in force-controlled manner first. Then, the concentrated load was increased in a displacement-controller manner until failure of the slab. The maximum value of line load was 240 kN/m. This load was calculated as the load causing 50% of the failure shear stress at the support as determined in experiments on wide beams [28]. The basic assumption here was that the behavior of a slab subjected to a line load would be similar to the behavior of a beam subjected to a concentrated load [42]. However, the behavior of a slab subjected to a line load and a concentrated load was unknown when preparing these experiments.

Two types of supports were used for the experiments: steel bearings or elastomeric bearings. For some specimens, a steel strip of 100 mm wide was used. As a result, the value of the support width b_{sup} changes, see Table 1.

A test was carried out at the simple support (sup 1 in Figure 4) as well as at the continuous support (sup 2 in Figure 4) when the load was placed in the middle (b_r = 1250 mm). Two tests were carried out at each support when the load was placed close to the edge (b_r= 438 mm). Whereas the slab specimen only had one span, it was built to represent continuous slab bridges. Therefore, prestressing bars coupled to the strong floor of the laboratory were used to create a moment over support 2, creating the moment distribution of a continuous slab, as shown in Figure 5. The moment diagram in Figure 5 is also used to show the difference between the distances a, a_M, L and l_{span}.

The standard span length is 3.6 m, as shown in Figure 4. For a limited number of experiments, a temporary support was used to test at the continuous support, as testing at the simple support had resulted in large damage to the slab.

3.2 Specimens

The concrete used in the specimens was delivered by truck mixer. The concrete quality C28/35 was used. Glacial river aggregates with a maximum aggregate size of 16 mm were used. The concrete compressive strength was measured in the laboratory on cubes. For the conversion to the cylinder compressive strength, a factor 0.82 was used [43], as recommended for the assessment of reinforced concrete slab bridges in the Netherlands. The resulting concrete compressive strengths of the individual specimens can be found in Table 1.

The reinforcement layout of the slabs is shown in Figure 6. All bars were deformed bars of steel quality S500. The measured yield strength of the **Ø** = 20 mm bars was 542 MPa and of the **Ø** = 10 mm bars f_{ym} = 537 MPa. For all specimens, the longitudinal reinforcement ratio was $\rho_{x,sag}$ = 0.996% and the transverse reinforcement ratio was $\rho_{y,sag}$ = 0.258%.

3.3 Results

The results of the 20 experiments are given in Table 1. In this table, the position of the load is indicated with CS/SS (testing at the continuous or simple support), a, the center-to-center distance between the load and the support, and b_r, which equals 1.25 m when the concentrated load is applied in the middle of the width, or 0.438 m when the concentrated load is applied close to the free edge - see Figure 4 for the two positions of the load. The result of the experiment is expressed as P_{conc}, the maximum value of the concentrated load, and v_{line}, the distributed load applied by the line load. The failure mode is either “B”, a beam shear failure with a clear shear crack on the side face of the slab, or “WB”, a wide beam shear failure for which the crack is inside the slab, and inclined cracks indicating shear stress can be observed on the bottom face of the slab. These failure modes are shown in Figure 7. For all experiments, a loading plate of 300 mm × 300 mm was used, except for S20T2b, where a loading plate of 200 mm × 200 mm was used.

4. Comparison between experiments and Extended Strip Model

To verify the proposed Extended Strip Model and its application to slabs subjected to concentrated and distributed loads, the maximum concentrated load P_{conc} from experiments from Table 1 are calculated with the Extended Strip Model, P_{ESM}. The value of P_{ESM} is determined as given in Eq. (2), with P_y and P_{edge} as given in Eqs. (13) and (14). The results of all calculations, with the formulas as outlined in §2.2, are given in Table 2. A beam diagram is used to find the moment and shear diagrams along the span direction of the slab. Based on this moment diagram, the value of λ is determined. For example, for S24T2 the support moment is 188 kNm and the span moment at the position of the concentrated load is 695 kNm, as can be seen in Figure 5. As a result, λ = 188kNm/695kNm = 0.27. The effect of torsion is taken into account with the factor β, see Eq. (1), which equals 1 if the effect of torsion is negligible and which approaches 0 as the effect of torsion increases. The value of the loaded length of the strip l_w is determined as given in Eq. (15). The capacity of the x-direction strip between the load and the support is determined as P_{sup}, according to Eq. (4). The capacity for the x-direction strip between the load and the position of zero shear, P_x is not affected by the formation of a direct strut, and is determined according to Eq. (3). The capacity of the y-direction strip between the edge and the load is affected by torsion and the edge effect, and is determined as given in Eq. (14). The capacity of the y-direction strip between the load and the far side of the slab is determined as given in Eq. (13). Then, the capacity of the four strips is determined, and summed to find P_{ESM}, see Eq. (2). It can be seen that, as a result of the direct strut that forms between the load and the support for concentrated loads close to the support, the value of P_{sup} is larger than the value of P_x. For the experiments with a concentrated load close to the free edge, the value of P_{edge} becomes significantly smaller than the value of P_y.

As can be seen in Table 2, all predicted values of the maximum concentrated load are conservative estimates; all values of P_{conc}/P_{ESM} are larger than one. The mean value (AVG) of P_{conc}/P_{ESM} equals 1.47. The standard deviation (STD) is 0.18, which results in a coefficient of variation (COV) of 12.5%. Given the complexity of the problem, which is a combination of one-way shear, two-way shear, and two-way flexure, the obtained value of the coefficient of variation is acceptable, especially since the presented method allows for a quick estimate of the maximum load with a hand calculation. The characteristic value (5% lower bound, assuming a normal distribution) equals 1.17, as would be expected from a lower-bound method. It can thus be concluded that the method is suitable for design and assessment purposes.

The comparison between the tested and predicted results is shown graphically in Figure 8. From this figure, it can be seen that the general trend of the data follows a line that is parallel to the 45^o line that is drawn in Figure 8. From Figure 8, it can be concluded as well that the Extended Strip Model provides a safe lower bound estimate of the maximum concentrated load on a reinforced concrete slab subjected to a combination of a concentrated load and a distributed line load. The actual distribution of the tested to predicted results is shown in a histogram in Figure 9. From the cumulative distribution, it can be found that the 5% lower bound of P_{conc}/P_{ESM}

equals 1.12, which is similar to the value that was found based on the assumption of a normal distribution.

5. Discussion

Previous research [36] has shown that the Extended Strip Model can be used for reinforced concrete slab bridges subjected to a single tandem. The current research shows that the Extended Strip Model can be used for reinforced concrete slab bridges subjected to a concentrated load and a distributed load. Extrapolating the results from the previous research makes it likely that the Extended Strip Model can be applied to reinforced concrete slab bridges subjected to a single tandem and the distributed loads. For these distributed loads, the effect of the load on the strips would be taken into account for the *y*-direction strips in the same way v_{DL} is accounted for in Figure 2. As such, the proposed method can be used for the assessment of bridges with a limited width, for estimating the maximum load that can be used in proof load testing, and for the assessment of superloads. For bridges with a limited width of a single lane, the loading combination of a single tandem and the distributed loads is the load combination required for assessment. For proof load testing [44], a single tandem is applied during the proof load test, and the distributed loads of the self-weight and the superimposed dead loads remain acting on the structure. Similarly, for the assessment of superloads, the superload can be simplified into a large surface of a concentrated load. The bridge then is subjected to this concentrated load, and the distributed loads of the self-weight of the bridge and the superimposed dead load.

The currently proposed method gives a lower bound of the maximum concentrated load. Since the method is based on the lower-bound theorem of plasticity, conservative results are expected. Moreover, in the derivation of the effect of torsion and other loads, conservative approaches were used. The goal of the developed method is to be able to estimate a maximum load with a quick hand calculation. For more precise results, it is recommended to use more advanced methods, such as nonlinear finite element models.

Currently, the proposed method cannot yet be extended to the use of multiple tandems staggered in different lanes. For this application, further research is required to evaluate how the tandems can be joined in the Extended Strip Model. However, no experimental results are available to compare the Extended Strip Model to this loading type.

6. Summary and conclusions

For the shear assessment of reinforced concrete slab bridges, a load combination consisting of permanent loads and live loads is used. The permanent loads are distributed loads, whereas the live loads are a combination of distributed lane loads, sometimes with different values for the distributed load for each lane, and concentrated loads that represent concentrated truck loads. This loading case represents a complex case, combining one-way shear, two-way shear, and two-way flexure.

To safely estimate the maximum concentrated load that can be applied to a reinforced concrete slab, representing a reinforced concrete slab bridge, the Extended Strip Model was developed. The Extended Strip Model combines strips working in arching action (one-way shear) with quadrants working in two-way flexure, and shows a possible load path prior to the collapse state of the slab. It is a lower-bound plasticity-based method.

In the presented research, the Extended Strip Model is extended further to estimate the maximum concentrated load for the case of a reinforced concrete slab subjected to a concentrated load and distributed loads. This loading situation was used, as experiments on reinforced concrete slabs, representing reinforced concrete slab bridges, subjected to a concentrated load close to the support and a line load acting over the full slab width are available for comparison. The main features of the test setup, properties of the eight specimens, and results of the twenty experiments are repeated in this paper for convenience.

To evaluate the performance of the proposed changes to the Extended Strip Model for the application to a combination of a concentrated load and a distributed load, the experimental results were compared to the predicted values with the Extended Strip Model. This comparison showed that the Extended Strip Model leads to conservative estimates for the maximum concentrated load. Given that the proposed method is an easy-to-use hand calculation, it can be used to have a quick estimate of the maximum concentrated load for bridges with a single lane, in the case of proof load testing, and for the passing of a superload.

Acknowledgements

The authors wish to express their gratitude and sincere appreciation to the Dutch Ministry of Infrastructure and the Environment (Rijkswaterstaat) for financing this research work. The discussions with Dr. S. Alexander are gratefully acknowledged.

List of notation

a center-to-center distance between load and support

a_M	center-to-center distance between load and support or between load and point of contraflexure, whichever is smaller
a_v	face-to-face distance between load and support
b	slab width
b_r	distance between free edge and center of load along the width direction
b_{sup}	width of the support
d	average of d_x and d_y
d_x	effective depth to the x-direction reinforcement
d_y	effective depth to the y-direction reinforcement
f_{ck}	characteristic concrete compressive strength
f_{cm}	average concrete compressive cylinder strength
f_{ym}	average steel yield strength
l_{edge}	length of the strip between the load and the edge
l_{span}	span length
l_w	loaded length of the strip
mode	failure mode
q_{self}	distributed load caused by self-weight
v_{dist}	shear stress caused by the distributed load
v_{DL}	shear stress caused by the dead load
v_{line}	applied line load over the width of the slab
w_{ACI}	one-way shear capacity given by ACI 318-14
$w_{ACI,x}$	one-way shear capacity based on d_x given by ACI 318-14
$w_{ACI,y}$	one-way shear capacity based on d_y given by ACI 318-14

x	position along span length
B	beam shear failure
CS	continuous support
F_{pres}	load caused by prestressing bars coupling the slab to the strong floor of the laboratory
L	distance between points of contraflexure
M	bending moment
$M_{hog,x}$	hogging moment capacity in the x-direction
$M_{hog,y}$	hogging moment capacity in the y-direction
$M_{s,x}$	moment capacity in the x-direction
$M_{s,y}$	moment capacity in the y-direction
$M_{sag,x}$	sagging moment capacity in the x-direction
$M_{sag,y}$	sagging moment capacity in the y-direction
M_{span}	sagging moment in the span caused by all loads on the slab
M_{sup}	hogging moment over the support caused by all loads on the slab
P_{conc}	maximum load at the concentrated load in the experiments
P_{edge}	capacity of strip between load and free edge
P_{ESM}	maximum load according to the Extended Strip Model
P_{line}	resultant of line load, maximum value
P_{sup}	capacity of strip between load and support
P_x	capacity of a strip in the x-direction
P_y	capacity of a strip in the y-direction
SS	simple support
WB	wide beam shear failure

β effect of torsion

$\rho_{x,sag}$ reinforcement ratio of the main flexural sagging moment reinforcement

$\rho_{y,sag}$ reinforcement ratio of the transverse flexural sagging moment reinforcement

References

[1] Lantsoght EOL, van der Veen C, de Boer A, Walraven JC. Recommendations for the Shear Assessment of Reinforced Concrete Slab Bridges from Experiments Structural Engineering International. 2013;23:418-26.

[2] CEN. Eurocode 1: Actions on structures - Part 2: Traffic loads on bridges, NEN-EN 1991-2:2003. Brussels, Belgium: Comité Européen de Normalisation; 2003. p. 168.

[3] CEN. Eurocode 2: Design of Concrete Structures - Part 1-1 General Rules and Rules for Buildings. NEN-EN 1992-1-1:2005. Brussels, Belgium: Comité Européen de Normalisation; 2005. p. 229.

[4] Lantsoght EOL, van der Veen C, de Boer A, Walraven J. Using Eurocodes and AASHTO for assessing shear in slab bridges. Proceedings of the Institution of Civil Engineers – Bridge Engineering. 2016;169:285-97.

[5] Lantsoght EOL, Van der Veen C, Walraven JC, De Boer A. Transition from one-way to two-way shear in slabs under concentrated loads. Magazine of Concrete Research. 2015;67:909-22.

[6] Regan PE. Shear resistance of members without shear reinforcement; proposal for CEB Model Code MC90. London, UK: Polytechnic of Central London; 1987. p. 28.

[7] König G, Fischer J. Model Uncertainties concerning Design Equations for the Shear Capacity of Concrete Members without Shear Reinforcement. CEB Bulletin 224, Model Uncertainties and Concrete Barrier for Environmental Protection. 1995:49-100.

[8] Collins MR, Bentz EC, Sherwood EG. Where is shear reinforcement required? Review of research results and design procedures. ACI Structural Journal. 2008;105:590-600.

[9] ASCE-ACI Task Committee 426. Shear-Strength of Reinforced-Concrete Members - Slabs. Journal of the Structural Division-ASCE. 1974;100:1543-91.

[10] ACI Committee 318. Building code requirements for structural concrete (ACI 318-14) and commentary. Farmington Hills, MI: American Concrete Institute; 2014.

[11] Vecchio FJ, Collins MP. The Modified Compression-Field Theory for Reinforced-Concrete Elements Subjected to Shear. Journal of the American Concrete Institute. 1986;83:219-31.

[12] Bentz EC, Vecchio FJ, Collins MR. Simplified modified compression field theory for calculating shear strength of reinforced concrete elements. ACI Structural Journal. 2006;103:614-24.

[13] AASHTO. AASHTO LRFD bridge design specifications, 7th edition with 2015 interim specifications. 7th ed. Washington, DC: American Association of State Highway and Transportation Officials; 2015.

[14] fib. Model code 2010: final draft. Lausanne: International Federation for Structural Concrete; 2012.

[15] Kueres D, Siburg C, Herbrand M, Classen M, Hegger J. Uniform Design Method for punching shear in flat slabs and column bases. Engineering Structures. 2017;136:149-64.
[16] Muttoni A, Ruiz MF. MC2010: The Critical Shear Crack Theory as a mechanical model for punching shear design and its application to code provisions. Shear and punching shear in RC and FRC elements – Proceedings of a workshop held on 15-16 October 2010 in Salò, Lake Garda, Italy. 2010;fib bulletin 57:31-59.
[17] Muttoni A, Fernández Ruiz M. Shear strength in one- and two-way slabs according to the Critical Shear Crack Theory. International FIB Symposium 2008. Amsterdam, The Netherlands: Fédération Internationale du Béton 2008. p. 559-63.
[18] SIA. Concrete Structures SIA 262:2003. Zurich2003.
[19] Muttoni A. The Critical Shear Crack Theory for Punching Design: From a Mechanical Model to Closed-Form Design Expressions. ACI SP International Punching Symposium. 2016:20.
[20] Nielsen MP, Hoang LC. Limit analysis and concrete plasticity. 3rd ed. ed. Boca Raton, Fla.: CRC; 2011.
[21] Cho SH. Shear strength-prediction by modified plasticity theory for short beams. ACI Structural Journal. 2003;100:105-12.
[22] Salim W, Sebastian WM. Plasticity model for predicting punching shear strengths of reinforced concrete slabs. ACI Structural Journal. 2002;99:827-35.
[23] Schlaich J, Schafer K, Jennewein M. Toward a Consistent Design of Structural Concrete. Journal Prestressed Concrete Institute. 1987;32:74-150.
[24] Hillerborg A. Strip method design handbook. London ; New York: E & FN Spon; 1996.
[25] Hillerborg A. Strip Method of Design. Wexham Springs, Slough, England: Viewpoint Publications, Cement and Concrete Association; 1975.
[26] Park R, Gamble WL. Reinforced concrete slabs. 2nd ed. New York: Wiley; 2000.
[27] Lantsoght EOL, van der Veen C, Walraven JC. Shear in One-way Slabs under a Concentrated Load close to the support. ACI Structural Journal. 2013;110:275-84.
[28] Lantsoght EOL, van der Veen C, De Boer A, Walraven J. Influence of Width on Shear Capacity of Reinforced Concrete Members. ACI Structural Journal. 2014;111:1441-50.
[29] Lantsoght EOL, van der Veen C, Walraven JC. Shear capacity of slabs and slab strips loaded close to the support. ACI SP-287, Recent Development in Reinforced Concrete Slab Analysis, Design and Serviceability. 2012:5.1-5.18.
[30] Lantsoght EOL, van der Veen C, Walraven J, de Boer A. Experimental investigation on shear capacity of reinforced concrete slabs with plain bars and slabs on elastomeric bearings. Engineering Structures. 2015;103:1-14.
[31] Lantsoght EOL, van der Veen C, de Boer A, Walraven J. Transverse Load Redistribution and Effective Shear Width in Reinforced Concrete Slabs. Heron. 2015;60:145-80.
[32] Lantsoght EOL, van der Veen C, de Boer A, Alexander SDB. Extended Strip Model for Slabs under Concentrated Loads. ACI Structural Journal. 2017;114:565-74.
[33] Alexander SDB, Simmonds SH. Bond Model for Concentric Punching Shear. ACI Structural Journal. 1992;89:325-34.
[34] Ospina CE, Alexander SDB, Cheng JJR. Punching of Two-Way Concrete Slabs with Fiber-Reinforced Polymer Reinforcing Bars or Grids. ACI Structural Journal. 2003;100:589-98.
[35] Afhami S. Strip model for capacity of flat plate-column connections [Thesis (Ph D)]: University of Alberta, 1997; 1997.

[36] Lantsoght EOL, van der Veen C, de Boer A, Alexander S. Bridging the gap between one-way and two-way shear in slabs. ACI SP International Punching Symposium. 2016:20.
[37] Lantsoght EOL, van der Veen C, de Boer A. Plastic model for asymmetrically loaded reinforced concrete slabs. ACI SP Recent developments on slabs. in press:21.
[38] Valdivieso D, Lantsoght EOL, Sanchez TA. Effect of torsion on shear capacity of slabs. SEMC 2016. Cape Town, South Africa2016. p. 6.
[39] Alexander SDB. Shear and Moment Transfer at Slab Column Connections. ACI SP of the International Punching Symposium. 2016.
[40] Lantsoght E, Yang Y, van der Veen C, de Boer A, Hordijk D. Ruytenschildt Bridge: field and laboratory testing. Engineering Structures. 2016;128:111-23.
[41] Lantsoght EOL, van der Veen C, de Boer A, Walraven J. One-way slabs subjected to combination of loads failing in shear. ACI Structural Journal. 2015;112:417-26.
[42] Sherwood EG, Lubell AS, Bentz EC, Collins MR. One-way shear strength of thick slabs and wide beams. ACI Structural Journal. 2006;103:794-802.
[43] van der Veen C, Gijsbers FBJ. Working set factors existing concrete bridges - Memo shear assessment existing bridges. 2011. p. 6.
[44] Lantsoght EOL, Van der Veen C, De Boer A, Hordijk DA. Proof load testing of reinforced concrete slab bridges in the Netherlands. Structural Concrete. in press:29.

List of tables and figures

List of Tables

Table 1 – Overview of experimental results

Test		**l_{span} (m)**	**f_{cm} (MPa)**	**a (m)**	**b_r (m)**	**b_{sup} (m)**	**mode**	**P_{conc} (kN)**	**v_{line} (kN/m)**
S20T1	SS	3.6	49.62	0.60	1.250	0.28	B	1542	241.2
S20T2b	CS	2.4	49.62	0.60	1.250	0.28	WB	1552	240.4
S20T3	CS	2.4	49.62	0.60	0.438	0.28	WB + B	1337	240.4
S20T4	CS	2.4	49.62	0.60	0.438	0.28	WB + B	1449	240.4
S21T1	CS	3.6	46.54	0.60	1.250	0.10	WB + B	1165	240.8
S21T2	SS	3.6	46.54	0.60	1.250	0.10	WB + B	1386	241.2
S22T1	CS	3.6	47.54	0.60	0.438	0.10	WB + B	984	240.8
S22T2	CS	3.6	47.54	0.60	0.438	0.10	WB + B	961	240.8
S22T3	SS	3.6	47.54	0.60	0.438	0.10	WB + B	978	241.2
S22T4	SS	3.6	47.54	0.60	0.438	0.10	WB + B	895	241.6
S23T1	CS	3.6	48.27	0.60	1.250	0.28	WB + B	1386	240.4
S23T2	SS	3.6	48.27	0.60	1.250	0.28	WB + B	1132	240.8
S24T1	CS	3.6	48.27	0.60	0.438	0.28	WB + B	1358	240.4
S24T2	CS	3.6	48.27	0.60	0.438	0.28	WB + B	1182	240.4
S24T3	SS	3.6	48.27	0.60	0.438	0.28	WB + B	995	240.8
S24T4	SS	3.6	48.27	0.60	0.438	0.28	WB + B	784	240.8
S25T2	CS	3.6	48.03	0.40	1.250	0.10	WB + B	1620	240.4
S25T3	CS	3.6	48.03	0.40	0.438	0.10	WB + B	1563	240.8
S26T1	SS	3.6	48.03	0.42	0.438	0.10	WB + B	1448	240.8
S26T2	SS	3.6	48.03	0.42	0.438	0.10	B	1324	240.8
S26T3	CS	3.6	48.03	0.40	1.250	0.10	WB + B	1555	240.8
S26T4	CS	3.6	48.03	0.40	0.438	0.10	B	1363	240.8
S26T5	CS	3.6	48.03	0.40	0.438	0.10	WB + B	1451	240.8

Table 2 – Comparison between test results and maximum load predicted with the Extended Strip Model

Test	P_{conc} **(kN)**	**λ**	**β**	l_w **m**	P_x **kN**	P_{sup} **kN**	P_y **kN**	P_{edge} **kN**	P_{ESM} **kN**	P_{conc}/P_{ESM}
S20T1	1542	0.00	0.91	0.877	294	503	61	58	917	1.682
S20T2b	1552	0.73	0.91	0.728	240	465	85	81	872	1.781
S20T3	1337	0.81	0.32	1.545	245	562	106	11	924	1.447
S20T4	1449	0.72	0.32	1.502	245	549	104	11	909	1.595
S21T1	1165	0.33	0.91	1.161	289	441	61	55	847	1.376
S21T2	1386	0.00	0.91	0.955	289	383	56	53	781	1.774
S22T1	984	0.37	0.32	1.949	242	375	64	5	685	1.436
S22T2	961	0.36	0.32	1.942	242	373	63	5	684	1.406
S22T3	978	0.00	0.32	1.582	242	320	57	6	625	1.565
S22T4	895	0.00	0.32	1.581	242	320	57	6	625	1.432
S23T1	1386	0.27	0.91	1.085	292	562	63	60	977	1.419
S23T2	1132	0.00	0.91	0.918	292	499	58	56	905	1.251
S24T1	1358	0.27	0.32	1.833	243	468	63	6	779	1.744
S24T2	1182	0.27	0.32	1.834	243	468	63	6	779	1.518
S24T3	995	0.00	0.32	1.553	243	415	58	6	722	1.378
S24T4	784	0.00	0.32	1.547	243	415	59	6	722	1.085
S25T2	1620	0.43	0.60	1.486	267	848	63	36	1215	1.333
S25T3	1563	0.43	0.21	2.512	232	736	63	3	1035	1.511
S26T1	1448	0.00	0.22	1.952	233	562	56	4	855	1.693
S26T2	1324	0.00	0.22	1.949	233	562	56	4	855	1.548
S26T3	1555	0.53	0.60	1.544	267	877	65	36	1245	1.249
S26T4	1363	0.62	0.21	2.685	232	783	67	3	1085	1.256
S26T5	1451	0.58	0.21	2.653	232	774	66	3	1076	1.349
									AVG	1.471
									STD	0.184
									COV	0.125

List of Figures

Figure 1 – Overview of strips and quadrants [33].

Figure 2 – Load in quadrants and resulting loads on strips for the Extended Strip Model.

Figure 3 – Load in quadrants and resulting loads on strips for the Extended Strip Model for the case of a concentrated load and one or more distributed loads.

Figure 4 – Overview of test setup used in the laboratory to study the combination of a concentrated and distributed load.

Figure 5 – Detail at continuous support: (a) coupling slab to strong floor of laboratory with prestressing bars; (b) beam scheme of applied loads, with values for S24T2; (c) resulting bending moment diagram for S24T2.

Figure 6 – Reinforcement layout of slabs, top view of slab.

Figure 7 – Observed failure modes: (a) WB – bottom view of slab, S20T2b; (b) B – side view of slab, S26T2.

Figure 8 – Graphical comparison between the maximum concentrated load as obtained from the experiment P_{conc} and the predicted maximum concentrated load with the Extended Strip Model P_{ESM}.

Figure 9 – Histogram of P_{conc}/P_{ESM}.