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RESEARCH

A novel model for corrosion-induced cracking and spalling in reinforced concrete structures

I - theoretical framework and solution techniques

J. Alfaiate · L. J. Sluys

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Abstract In this work, a new damage model for cohesive fracture is presented. The concept of damage driving mechanism is introduced, from which the damage evolution law is derived. Special attention is paid to non pure mode-I and mode-II fracture modes, such as mixed-mode fracture and mode-II fracture under compression. Conversely to the previous traction-based damage model (Alfaiate et al. 2023), where the damage variables are derived from uniaxial tensile and shear relationships, the damage variables are now obtained explicitly from *reference* traction-jump displacement relationships, leading to an integration of fracture mechanics concepts with a damage mechanics approach. In this way, it is possible to explicitly control the dissipation of energy. The model is aimed at the simulation of the behaviour of reinforced concrete structures under corrosion. Corrosion affects i) cracking, which is modelled with a discrete crack approach, ii) bond-slip degradation between steel and concrete, iii) the reduction of the sane cross section of the reinforcement steel and iv) spalling of the concrete cover. In this work some illustrative examples of the performance of the model are shown. In part II, examples of

reinforced concrete structures with and without corrosion are presented.

Keywords Damage model · Reinforced Concrete · Corrosion

List of acronyms and symbols

DSDA	Discrete Strong Discontinuity Approach
IA	Incremental Approach
TIA	Total Iterative Approach
$ \cdot $	the norm of $(\cdot)$
C	control loading function used with the Total Iterative Approach
c_0	initial cohesion
c	cohesion
d	scalar damage variable
$\mathbf{d}$	second order damage tensor
d_n	normal damage variable component
d_s	shear damage variable component
D_{nn}	normal diagonal component of tensor $\mathbf{D}_{\Gamma_d}^{el}$
D_{ss}	shear diagonal component of tensor $\mathbf{D}_{\Gamma_d}^{el}$
$\mathbf{D}_{\Gamma_d}^{el}$	second order elastic constitutive tensor
Δ	increment
f_1, f_2	limit surfaces defined in the traction space
f_{t0}	initial tensile strength
f_t	tensile strength
$\mathbf{F}$	Applied force vector
G_F	fracture energy
G_F^{II}	fracture energy under mode-II fracture

J. Alfaiate (✉)
 Instituto Superior Técnico, Universidade de Lisboa, Av.
 Rovisco Pais 1, Lisboa, Portugal
 e-mail: jorge.alfaiate@tecnico.ulisboa.pt

L. J. Sluys
 Delft University of Technology, P.O. Box 5048, 2600 GA Delft,
 The Netherlands

g_n	normal damage evolution law
g_s	shear damage evolution law
κ	monotonic increasing function of the displacement jump components
λ	loading factor
ρ	κ_s / κ_n
σ	stress tensor
ε	strain tensor
$\mathbf{t}$	traction vector acting at the discontinuity
t_n	normal component of the traction vector
t_s	shear component of the traction vector
$\mathbf{w}$	displacement jump vector
w_n	normal component of the jump displacement vector
w_s	shear component of the jump displacement vector
ϕ	internal friction angle; diameter of reinforcement bar

1 Introduction

In this work, the macroscale mechanical behaviour of reinforced concrete is studied. Special attention is given to i) the softening behaviour of concrete, taking into account both nonlinear fracture mechanics and damage concepts and ii) corrosion of the steel reinforcement.

Reinforced concrete is a composite material, of which the nonlinear response is characterized by:

- (i) localised and diffuse cracking;
- (ii) compressive nonlinear behaviour;
- (iii) compressive crushing;
- (iv) cracking under tensile stress;
- (v) nonlinear plastic behaviour of the reinforcement steel;
- (vi) nonlinear bond-slip steel-concrete relationship.

Moreover, cracking, if not properly controlled, allows for the penetration of moisture which gives rise to corrosion of the steel reinforcement. Steel corrosion is the biggest durability issue of reinforced concrete structures. Some of the most important mechanical effects due to corrosion are (Fang et al. 2004, 2006; Guzmán et al. 2011, 2012, 2014; Guzmán and Gálvez 2017; Sanz et al. 2013, 2015, 2016, 2017; P.Broomfield 1997; Jiang et al. 2018; Lin et al. 2014):

- (i) deterioration of the bond-slip relationship between concrete and steel reinforcement,

- (ii) decrease of the area of the undamaged cross section of the steel bars,
- (iii) expansion of the steel cross section due to the formation of products of corrosion, which may lead to delamination and spalling of the cover.

All these mechanisms induce damage in the structure, leading to an increase of cracking - which gives rise to the increase of corrosion leading to a vicious cycle, an increase of deformation, a decrease of strength and premature failure.

There are many factors that may influence corrosion, related to the concrete composition and the stress state (see for instance Costa and Appleton (2002), Louro (2014), Bastos (2022)), namely:

- a) environment conditions, which also leads to changes of the corrosion rate,
- b) concrete strength,
- c) stirrup confinement,
- d) stress state in the neighbourhood of the concrete-steel interface, which can lead to an increase of the bond strength under compression, or to a decrease of the bond strength under tension.

Spalling fracture is also an important issue, which depends on the level of confinement, induced by stirrups (c) or compressive stress states (b)).

Experimental research has been carried out taking into account the effects of stirrup confinement (Lin et al. 2014), concrete cover on the the bond strength and spalling (Andrade et al. 1993; Fang et al. 2004; Tepfers 1979; Moccia et al. 2021; Ajagbe et al. 2020; Mak and Lees 2022; Apostolopoulos et al. 2019), slip of the tensile bars (Sanz et al. 2018), stirrup corrosion (Zhou et al. 2017, anchorage capacity Tahershamsi et al. 2016; Zandi et al. 2010; Zandi and Coronelli 2010) etc.. Numerical analyses regarding corrosion in RC members have also been performed, namely (Molina et al. 1993; Lundgren 2005; Chernin and Val 2011; Guzmán et al. 2011, 2012; Sanz et al. 2013, 2016; Guzmán and Gálvez 2017; Ožbolt et al. 2017; Saeter and Sand 2012; Zhang et al. 2021). Although these works consist of very important contributions to the field, macro analyses which can provide a better understanding of the reinforced concrete behaviour as well as to predict the structural failure under deteriorated conditions are still lacking. This is the purpose of the present research work. This study is divided in two parts: in the former part the theoretical framework and solution techniques are dealt with and in the second part

finite element analyses of corroded reinforced concrete beams are presented.

In part I, a damage model is defined to model the behaviour of reinforced concrete. In this model, i) concrete cracking, ii) nonlinear compressive behaviour, iii) bond-slip between steel and concrete, iv) bond-slip degradation due to corrosion and v) spalling of the concrete cover are taken into account.

Cracking is modelled by means of a discrete crack approach, in which microcracking is assumed to localize at a discontinuity surface (Hillerborg et al. 1976).

The damage model is also used to describe the behaviour of the interface between the reinforcement steel and the concrete. Due to the nature of the normal traction acting at the interface, the behaviour of the interface is characterised by either mode-II fracture under compression, or mixed-mode fracture under tensile stresses.

As mentioned above, the deterioration of the bond-slip relationship between concrete and steel is an important consequence of corrosion, which also leads to the increase of damage. This is a particularly challenging issue to model, since the bond-slip relationship under corrosion evolution is continuously changing during the analysis. In Section 4.2, numerical strategies are proposed to deal with this issue.

The damage model adopted in Alfaiate et al. (2023), is summarized in appendix A. This model is analyzed in detail in Sections 2.1 and 2.2 and important modifications are introduced in Section 3, namely regarding the control of energy dissipation and the damage evolution law. The concept of damage driving mechanism is introduced, based on the evolution of the limit surface in the traction space. This is a novel aspect of the model, which enables a consistent definition of damage evolution with respect to the previous model, in particular under mixed-mode fracture.

In order to consider the nonlinear compressive concrete behaviour, an elasto-plastic relation is adopted. As a consequence, both compressive softening and crushing are neglected, in order to avoid the numerical problems associated with the continuum modelling of softening behaviour. The behaviour of steel reinforcement is also considered as elasto-plastic. In both cases, an isotropic constitutive relationship is adopted and the plastic regime is assumed whenever the maximum/ minimum principal stress component reaches the strength of the material.

In Section 4, modelling of corrosion is described. Special attention is given to the reduction of the sane cross section of the reinforcement, the modification of the bond-slip relationship and spalling of the concrete cover. All these aspects are now developed by means of the concept of damage driving mechanism.

Concrete spalling is modelled in two ways: i) by means of volume expansion of the reinforcement bars and ii) by means of explicitly enforcing the detachment of the concrete from the reinforcement steel. The latter is a novel aspect of the model, which allows for a more realistic simulation of concrete spalling in 2D.

From the above, it is clear that the nonlinear behaviour of reinforced concrete involves many aspects, namely cracking, hardening compressive behaviour, nonlinear-plastic behaviour of reinforcement steel, bond-slip between concrete and steel, bond-slip modification due to corrosion, spalling of concrete cover due to corrosion, etc.. As a consequence, the numerical modelling of concrete structures can lead to convergence difficulties using the traditional iterative Newton-Raphson algorithm. In Section 5, following the work presented in Alfaiate and Sluys (2023), a different iterative procedure, based on a total approach, is presented, in which the modifications introduced in the damage model are taken into account.

Due to the extension of the topic, here the description of the model is provided together with some illustrative examples at local level. In the work presented in part II, numerical results of reinforced concrete under corrosion are presented.

2 A traction-based damage model for cohesive fracture

In this Section the damage model presented in Alfaiate et al. (2023) is analyzed. In Sections 2.1, 2.2 and appendix B, examples are presented to show the influence of the damage evolution law adopted in the model. In particular, special attention is given to the control of the dissipation of energy. In Section 3, a new traction-based damage model for cohesive fracture is introduced, based on the concept of damage driving mechanism. A comparison between the model presented in Alfaiate et al. (2023) and the model presented herein is also presented in Section 3.

According to the earlier work presented in Alfaiate and Sluys (2017a), Alfaiate and Sluys (2017b), Alfa-

iate and Sluys (2019), Alfaiate et al. (2023), damage is introduced in the scope of a discrete crack approach. Thus, instead of a stress-strain relationship, a constitutive relation between traction $\mathbf{t}$ and displacement jump $[\![\mathbf{u}]\!]$ is adopted: upon growth of microcracking, damage increases until a stress free crack is formed, corresponding to a fully damaged state.

Furthermore, it is assumed that the damage evolution law is driven by the traction field. For this purpose, a limit surface in the traction space is introduced (see Fig. 1). The limit surface allows for the interaction between: i) the bond traction and ii) the normal traction at the interface, such that compressive confinement as well as spalling can be taken into account. In appendix A, a summary of the model introduced in Alfaiate et al. (2023) is presented.

2.1 Damage evolution

In the model presented in appendix A, the damage evolution law is defined according to Equations (68) to (71). Thus, the damage variables d_n, d_s are obtained from the evolution of the tensile strength f_t and the cohesion c , respectively. Moreover, the evolution of both f_t and c is dependent on the criterion adopted for the evolution of the limit surface. For instance, under homotetic evolution of the limit surface, we obtain $c/c_0 = f_t/f_{t0}$. Other criteria are explored in the work presented in Alfaiate and Sluys (2017a), as well as in appendix B. In any case, damage is fully defined once $\{\kappa_n, \kappa_s\}$, or $\{f_t, c\}$ are given. For instance, under proportional loading ($\alpha = t_s/t_n$, with α constant), and adopting homotetic evolution of the limit surface, the following three update strategies can be adopted:

- a. given $\{\kappa_n, \kappa_s\}$, or $\{f_t$ or $c\}$;
- b. define d_n, d_s from (70), (71);
- c. obtain t_n, t_s from the limit surface (66);
- d. define relations $t_n - w_n, t_s - w_s$ according to (67),

or

- e. given t_n, t_s ,
- f. obtain f_t, c from the limit surface (66);
- g. derive $\kappa_n, \kappa_s, d_n, d_s$ (70), (71);
- h. define relations $t_n - w_n, t_s - w_s$ according to (67),

or

- i. given w_n, w_s ,
- j. derive f_t, c, d_n, d_s from the limit surface (66);

- k. obtain t_n, t_s from (67).

For example, in item j. and for mixed-mode fracture, Equation (66) ($f = 0$) can be written as:

$$\alpha(1 - d_n)D_{nn}w_n = \frac{\phi f_{t0} - c_0 f_t / f_{t0}}{f_{t0}^2} (1 - d_n)D_{nn}w_n - \phi(1 - d_n)D_{nn}w_n + c_0 f_t / f_{t0}, \quad (1)$$

from which a unique solution for both f_t and d_n variables can be derived according to (70).

As a consequence, both mode-I and mode-II fracture are automatically taken into account for the damage evolution: under compressive normal traction only mode-II fracture is considered whereas, under mixed-mode fracture, both mode-I and mode-II fracture are involved.

In this work, the damage evolution law is based on the concept of *damage driving mechanism*, upon which the iterative procedure presented in Section 5 is also built. In reinforced concrete, the following damage driving mechanisms are taken into account: i) cracking, ii) shear between faces of a discontinuity, namely shear bands and bond-slip between concrete and reinforcement and iii) corrosion. These three cases are further explained as follows:

- i) cracking is initiated under mode-I fracture and can evolve into mixed-mode if $|t_s| > 0$. Under the assumption that the traction state remains close to mode-I fracture, the damage driving mechanism can be built upon the normal relationship $t_n - w_n$.
- ii) shear bands or bond-slip can occur for positive or negative normal tractions t_n . For mixed-mode fracture ($t_n > 0$), the driving mechanism can be either mode-I or mode-II fracture, depending on the traction state lying closer to mode-I or mode-II fracture, respectively. For compressive traction, a mode-II driving mechanism is adopted. In Section 3.3, a generalized mixed-mode driving mechanism is introduced, which is particularly useful in situations in which the traction state deviates significantly from pure mode-I or mode-II fracture.
- iii) A special damage driving mechanism is also defined for corrosion. In this case, damage is related to the evolution of the level of corrosion, which affects the bond-slip relationship as well as cracking. This issue was introduced in the work presented in Alfaiate et al. (2023) and is developed in Section 5.4, in the context of the iterative procedure.

2.2 Energy dissipation

In the model presented in Section 2, the dissipation of energy is not explicitly defined under mixed-mode fracture ($t_n^* \neq f_t$) or mode-II fracture for compression ($|t_s^*| \neq c$). In the following, the dependence of the dissipation of energy on Equations (67), (70) and (71) is exemplified. Two possibilities are analyzed separately: shear under compressive normal traction and mixed-mode fracture. In all examples, homotetic evolution of the limit surface is adopted (see Fig. 1), together with the following material properties: $f_{t0} = 3$ MPa, $c_0 = 8$ MPa, $\tan(\phi) = 0.6$, $G_F = 0.1$ N/mm, $G_F^{II} = 30$ N/mm, $D_{nn} = 10^3$ MPa/mm, $D_{ss} = 30$ MPa/mm. These material properties are typical for normal concrete and are the same, or close to the material parameters adopted in part II, as well as in [Alfaiate et al. \(2023\)](#).

2.2.1 Shear under compressive normal traction

First, assume that both the normal traction and the shear traction soften proportionally to the origin (radial red path in Fig. 1). Assume that a traction vector on the initial limit surface is obtained, designated as $\mathbf{t}^*$, with components $t_n^* = -3.91$; $t_s^* = 10$ MPa. In Fig. 2a, both the corresponding $c - \kappa_s$ and the $t_s - w_s$ relations are presented, under linear softening. Although the $t_s - w_s$ graph seems parallel to the $c - \kappa_s$ relationship, the dissipation of energy obtained from the $t_s - w_s$ relation is 49 N/mm, whereas the relationship parallel to the $c - \kappa_s$ relation would give rise to $G_F^{II} \times (10/8)^2 \approx 46.9$ N/mm¹.

Under higher compressive traction, the non-proportionality of energy dissipation with respect to the $c - \kappa_s$ relation is accentuated. In Fig. 2b, the $\kappa_s - c$ and the $t_s - w_s$ relations are presented for the initial traction vector with components $t_n^* = -13.3$; $t_s^* = t_{s,lim} = 12$ MPa, under linear softening. In this case, the dissipation of energy obtained from the $t_s - w_s$ relation is 121.1 N/mm, whereas the relationship parallel to the $c - \kappa_s$ relation would give rise to $G_F^{II} \times (10/8)^2 \approx 67.4$ N/mm. Under exponential softening the dissipation of energy increases even further. For instance, in the latter

example, we would get 144.7 N/mm under exponential softening.

As an example of non-proportional softening path, assume now that bond-slip occurs under constant compressive traction (black path in Fig. 1). Consider linear softening for $c - \kappa_s$. In order to show that non-parallel $c - \kappa_s$, $t_s - w_s$ relations are obtained with the model presented in Section 2, *reductio ad absurdum* can be applied, assuming that $t_s - w_s$ relationship is parallel to $c - \kappa_s$, as depicted in Fig. 3. Since, in this case, the traction path is not directed towards the origin, a residual shear traction is obtained, lying on the ultimate limit surface, $t_{s,res} = t_s^* - c_0$.

According to (67), (69) and (71), both the $t_s - w_s$ and the $c - \kappa_s$ damage laws depend on the same damage variable d_s . As a consequence, the absolute value of the decrease of the shear traction $|\Delta t_s|$ is larger than the absolute value of the decrease of the cohesion $|\Delta c|$ (see Fig. 3).

However, assuming a homotetic evolution of the limit surface, these differences should be equal, as depicted in Figure 1. In fact, from (66) we can obtain the difference between t_s and c :

$$|t_s| - c = \frac{\tan(\phi)}{2t_{n,lim}} t_n^2 - \tan(\phi)t_n \quad (2)$$

which remains constant under constant t_n ². As a consequence, the assumption of parallel $t_s - w_s$, $c - \kappa_s$ relationships does not hold. Inasmuch, if the $t_s - w_s$ relationship depicted in Fig. 3 is adopted, the shape of the limit surface can not be preserved, leading to non-homotetic evolution of the limit surface.

In order to allow for $\Delta c = \Delta t_s$, as enforced by a homotetic evolution, we have:

$$t_s = c + t_s^* - c_0, \quad (3)$$

or,

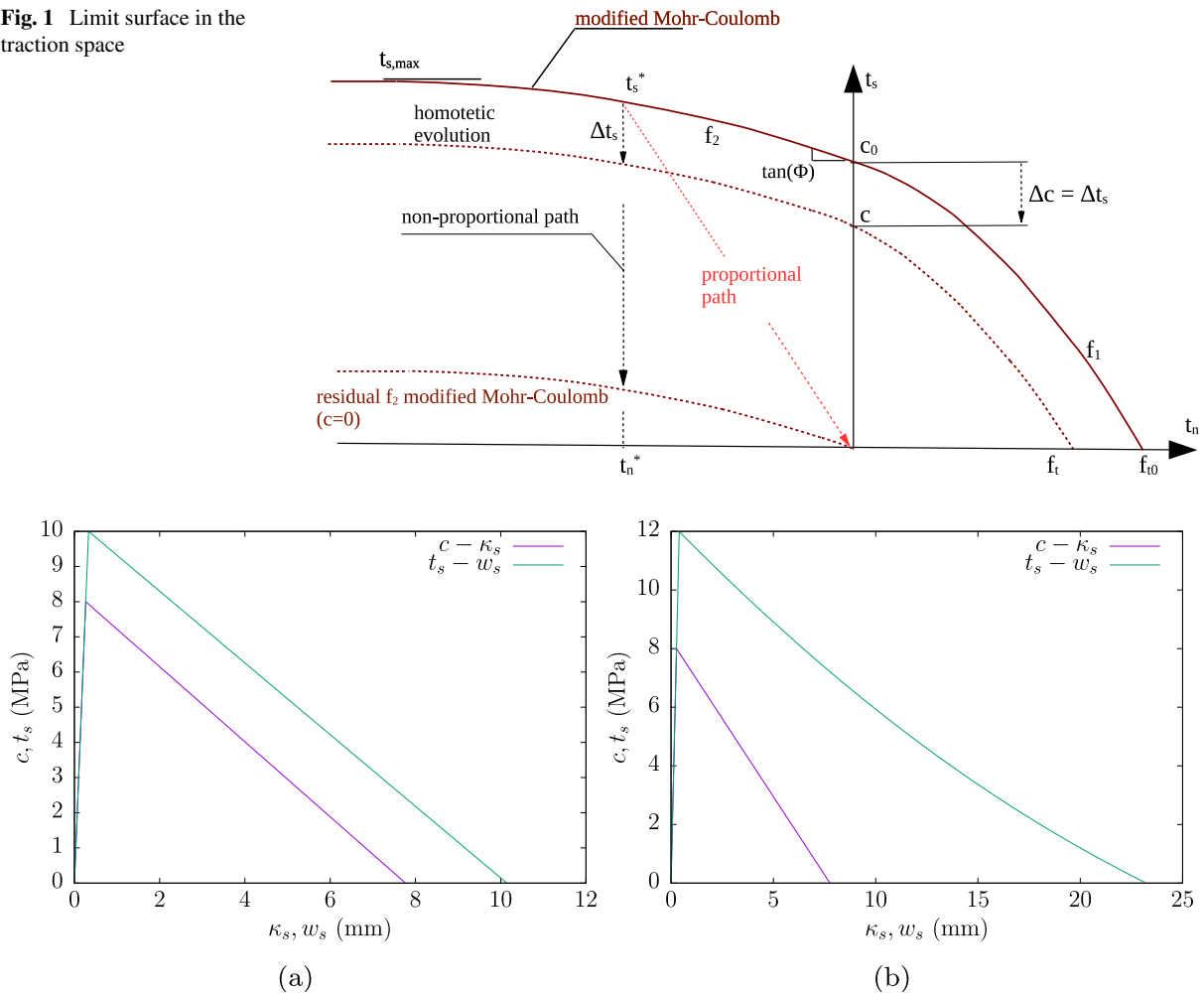
$$(1 - d_s)D_{ss}w_s = (1 - d_s)D_{ss}\kappa_s + t_s^* - c_0. \quad (4)$$

If a linear $c - \kappa_s$ relationship is adopted, we get:

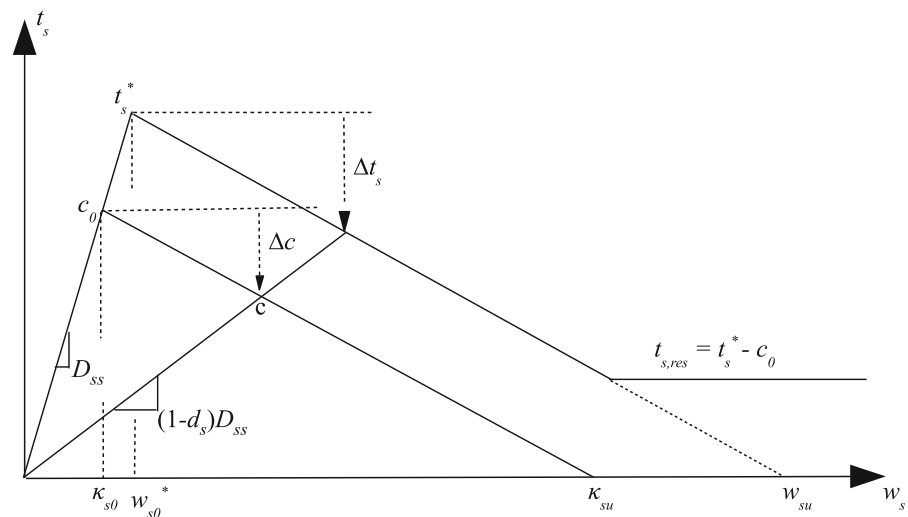
$$c = (1 - d_s)D_{ss}w_s = c_0 - c_0 \frac{\kappa_s - \kappa_{s0}}{\kappa_{su} - \kappa_{s0}}, \quad (5)$$

¹ If the $t_s - w_s$ softening branch is parallel to the $c - \kappa_s$ one, the area under the $t_s - w_s$ relationship is equal to: $\text{Area}(t_s - w_s) = (10/8)c_0 \times (10/8)\kappa_{su}/2 = (10/8)2 \times 30 \text{ N/mm} = 46.9 \text{ N/mm}$.

² Here $t_{n,lim}$ is assumed fixed, although in previous works it was allowed to vary with damage evolution ([Alfaiate and Sluys 2017b, 2019](#))

Fig. 1 Limit surface in the traction space**Fig. 2** (a) Bond-slip softening under compression: $t_s - w_s$ versus linear $c - \kappa_s$, $t_n^* = -3.91$ MPa, $t_s^* = 10$ MPa, (b) Bond-slip softening under compression: $t_s - w_s$ versus linear $c - \kappa_s$, $t_n^* = -13.3$ MPa, $t_s^* = 12$ MPa**Fig. 3** Bond-slip softening under compression:

$$|\Delta t_s| > |\Delta c|$$



which leads to

$$w_s = \kappa_s + \frac{t_s^* - c_0}{(1 - d_s)D_{ss}}. \quad (6)$$

As a consequence, under homotetic evolution of the limit surface and according to the model presented in Section 2, a linear $c - \kappa_s$ relationship leads to a non-linear $t_s - w_s$ relationship.

Let $\alpha = t_s^*/c_0$. Equation (6) becomes

$$w_s = \kappa_s + \frac{(\alpha - 1)c_0}{(1 - d_s)D_{ss}}, \quad (7)$$

or

$$w_s = \kappa_s \left[1 + (\alpha - 1) \frac{c_0}{c} \right]. \quad (8)$$

Under exponential softening we have,

$$c = c_0 e^{\left(\frac{-c_0}{G_F^{II}} (\kappa_s - \kappa_{s0}) \right)}, \quad (9)$$

and equation (8) changes into

$$w_s = \kappa_s \left[1 + (\alpha - 1) e^{\left(\frac{c_0}{G_F^{II}} (\kappa_s - \kappa_{s0}) \right)} \right]. \quad (10)$$

In Figs. 4a and 4b, equation (8), corresponding to linear softening and (10), corresponding to exponential softening, respectively, are graphically represented for two values of α : $\alpha = 1.2$ and $\alpha = 1.5$ ($c_0 = 8$ MPa, $\tan(\phi) = 0.6$, $G_F^{II} = 30$ N/mm)³

In Fig. 5, the $t_s - w_s$ relationship obtained with the model presented in Section 2 is shown, together with the linear law $c - \kappa_s$, adopting a homotetic surface evolution. In this Figure, the secant stiffness of the constitutive relation, which is common to both $c - \kappa_s$ and $t_s - w_s$, is also depicted for several values of the damage variable d_s .

It is clear that the dissipation of energy obtained under homotetic evolution of the limit surface is much larger than the one corresponding to the $t_s - w_s$ relationship parallel to $c - \kappa_s$. Furthermore, under linear

softening (6) to (8), a singularity is reached under

$$d = 1 \Leftrightarrow c = 0 \Leftrightarrow k_s \geq k_{su} = \frac{2G_F^{II}}{c_0}. \quad (11)$$

2.2.2 Mixed-mode fracture

Two examples of softening under proportional loading are presented under mixed-mode fracture. In the former case, the components of the traction vector on the initial surface are $t_n^* = 1.32$, $t_s^* = 6$ MPa. The corresponding $f_t - \kappa_n$, $t_n - w_n$ relations are presented in Fig. 6a, whereas the corresponding $c - \kappa_s$, $t_s - w_s$ graphs are presented in Fig. 6b, under linear softening. In Figs. 7a and 7b, the $f_t - \kappa_n$, $t_n - w_n$ relations and $c - \kappa_s$, $t_s - w_s$ relations, respectively, are presented under exponential softening. In this case, the traction vector components on the initial surface are $t_n^* = 2.55$, $t_s^* = 2$ MPa.

Similar to the results presented in Section 2.2.1, relations $f_t - \kappa_n$ versus $t_n - w_n$ and $c - \kappa_s$ versus $t_s - w_s$, respectively, are non-parallel. Furthermore, in Fig. 7a, it is clear that a value of dissipation of energy larger than G_F is obtained under the $t_n - w_n$ relation. Note that, with the model presented in Section 2, homotetic evolution of the limit surface does not correspond, in general, to isotropic damage softening (Alfaite and Sluys 2017a). This issue is further discussed in Section 3.

In appendix B, an example a non-homotetic evolution of the limit surface is presented adopting a variable friction coefficient. In the following, only homotetic evolution of the limit surface is considered.

3 New damage model for cohesive fracture

In the following, a new damage model for cohesive fracture is introduced. In this model, one important aspect is concerned with the control of the energy dissipation. For non-pure mode-I and mode-II fracture modes, the model in Section 2 does not allow for an explicit definition of the energy dissipation. As shown in the examples of Section 2.1, this can lead to important deviations of both G_F and G_F^{II} , specially if the traction state does not lie close to mode-I or mode-II fracture.

In order to cope with this problem, a new approach is proposed in which the damage laws $f_t - \kappa_n$ and

³ $\alpha = 1.5$ corresponds to the maximum possible relation t_s^*/c_0 , whereas $\alpha = 1.2$ corresponds to an intermediate value for this relation under compression.

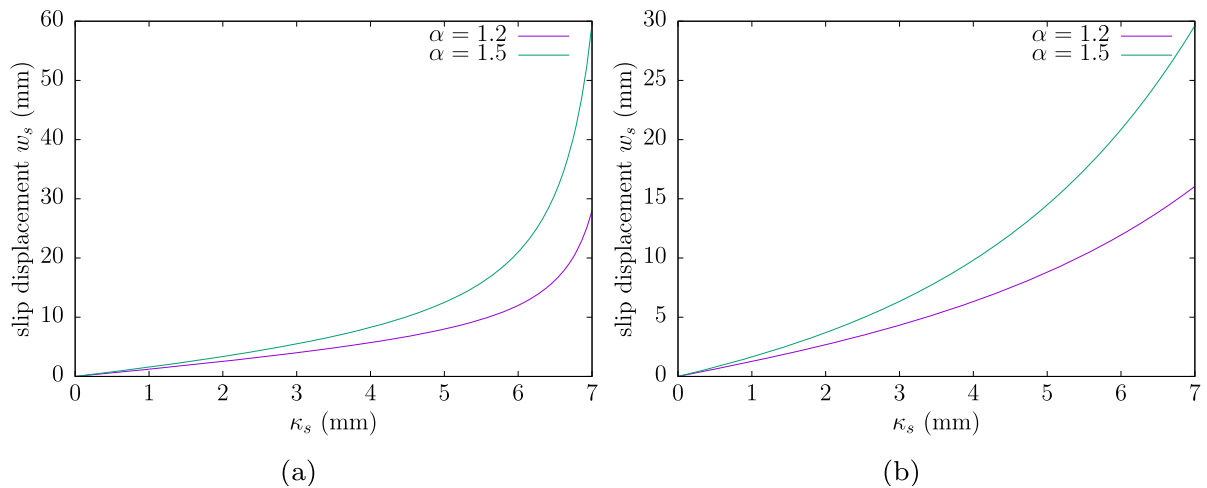


Fig. 4 Relation between w_s and κ_s , for $\alpha = 1.2$ and $\alpha = 1.5$: (a) under linear softening; (b) under exponential softening

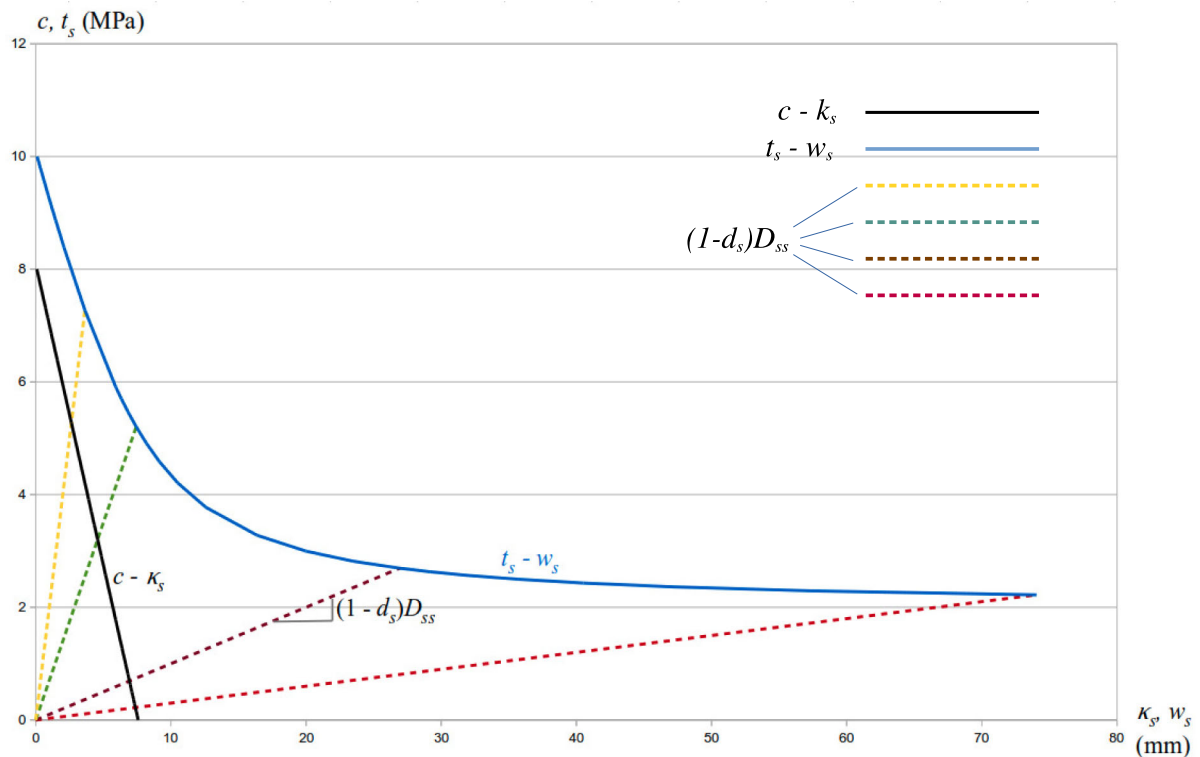


Fig. 5 Linear $c - \kappa_s$ and nonlinear $t_s - w_s$ relationships obtained with the model presented in Section 2 under constant compressive traction

$c - \kappa_s$ are replaced by *reference* $t_n^r - w_n^r$, $t_s^r - w_s^r$ relationships. These reference relationships may become the true $t_n - w_n$, $t_s - w_s$ relationships under particular circumstances, namely:

1. pure mode-I fracture;
2. pure mode-II fracture;
3. proportional loading path;
4. $t_n - w_n$ under a mode-I driving mechanism;

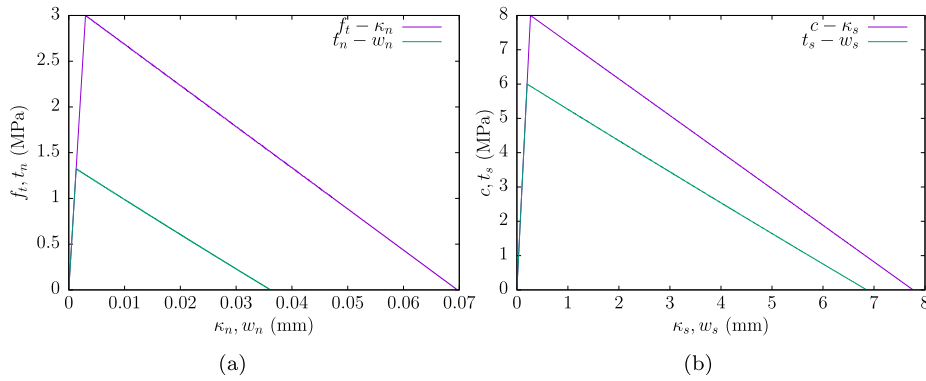


Fig. 6 Softening under mixed-mode, $t_n^* = 1.32$ MPa, $t_s^* = 6$ MPa: (a) $t_n - w_n$ versus linear $f_t - \kappa_n$; (b) $t_s - w_s$ versus linear $c - \kappa_s$

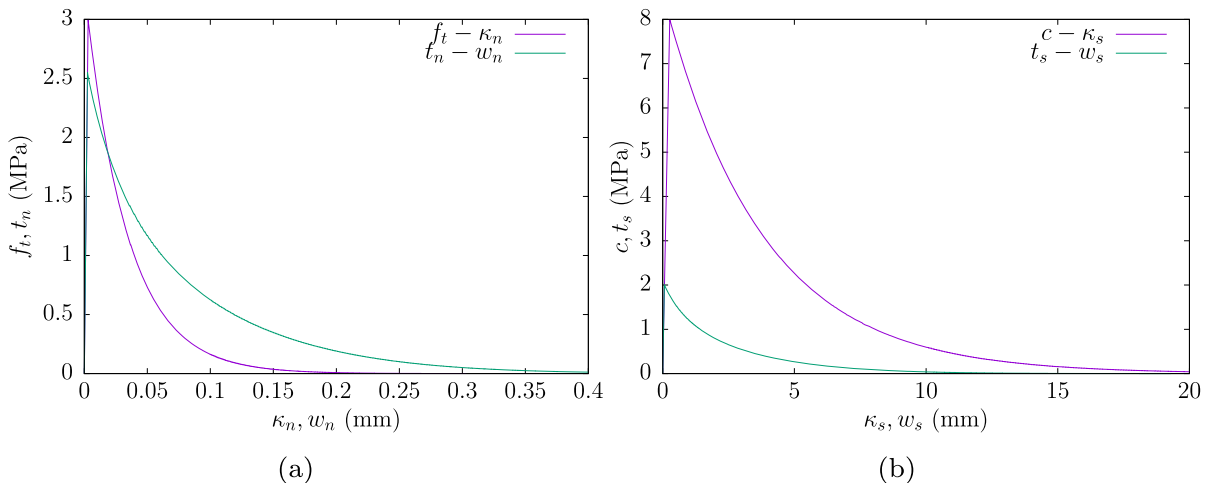


Fig. 7 Softening under mixed-mode, $t_n^* = 2.55$ MPa, $t_s^* = 2$ MPa: (a) $t_n - w_n$ versus exponential $f_t - \kappa_n$; (b) $t_s - w_s$ versus exponential $c - \kappa_s$

5. $t_s - w_s$ under a mode-II driving mechanism.

The last three items correspond to novel features of the model, which, conversely to the previous model, allow for the definition of the energy dissipation under non-pure mode-I or mode-II fracture modes.

Thus, damage is now defined explicitly from the traction-jump displacement relation, disregarding the need for introducing damage internal kinematic variables κ_n, κ_s . From the authors point of view, this leads to an integration of fracture mechanics concept into the present damage mechanic approach (see also [Alfaite and Sluys \(2019\)](#)).

The following strategy is adopted:

1. First, depending on the traction state, a traction vector $\mathbf{t}^*$ is defined on the initial limit traction surface ($f = 0|_{\mathbf{d}=0}$). This traction state defines the initial,

undamaged, normal and shear strength of the discontinuity. This traction vector $\mathbf{t}^*$ may vary in time, depending on the evolution of the traction state.

2. Next, the corresponding modified mode-I and mode-II fracture energies G_F^* , G_F^{II*} , are explicitly defined as functions of G_F and G_F^{II} , respectively:

$$\begin{aligned} G_F^* &= G_F^*(G_F, t_n^*/f_{t0}) \\ G_F^{II*} &= G_F^{II*}(G_F, t_s^*/c_0), \end{aligned} \quad (12)$$

where t_n^* and t_s^* are the normal and shear components of the traction vector $\mathbf{t}^*$.

3. Then, the reference traction-jump displacement relationships $\mathbf{t}^r - \mathbf{w}^r$ are introduced, in which $\mathbf{t}^r$ depends on the initial traction state on the limit sur-

face and on the modified fracture energy:

$$t_n^r = t_n^r(t_n^*, G_F^*, w_n^r) \quad (13)$$

$$t_s^r = t_s^r(t_s^*, G_F^{II*}, w_s^r). \quad (14)$$

As stated above, this $\mathbf{t}^r - \mathbf{w}^r$ relationship is defined assuming that the traction evolves under proportional loading, following a radial softening path.

4. According to the damage driving mechanism, the traction $\mathbf{t}$ is derived. For instance, under mode-II driving, $t_s = t_s^r$; other examples are given below.
5. Given the traction $\mathbf{t}$, both the tensile strength f_t and the cohesion c are obtained directly from the limit surface $f = 0$ (66).
6. From Equations (12) to (14), depending on the damage driving mechanism adopted and according to the damage relation (67), the damage variables $\mathbf{d}$ are obtained:

$$\mathbf{t} = \begin{bmatrix} (1 - d_n)D_{nn} & 0 \\ 0 & (1 - d_s)D_{ss} \end{bmatrix} \mathbf{w}. \quad (15)$$

Note that, conversely to the previous model, in which damage evolution is based on internal kinematic variables κ_n, κ_s ⁴, damage is now derived directly from the traction state. In this way, a truly traction-based damage model is obtained. Moreover, damage evolution is now controlled by the dissipation of energy and not the other way around.

There is no restriction on the definition of the reference constitutive relations in 3.. Here, linear, bilinear/multilinear or exponential relations are adopted. Since relations (13), (14) are given explicitly, there is no need to define internal variables κ_n, κ_s , as well as relations (68) and (69). In sum, the main differences between this new model and the model in Section 2 are the following:

- i) dissipation of energy is explicitly controlled, namely under non-pure mode-I and mode-II fracture modes;
- ii) damage is derived from the traction state, not from internal kinematic variables;
- iii) equations (68) to (71) are left out ;
- iv) due to ii), there is no need to define internal variables κ_n, κ_s ;

- v) evolution of f_t and c is obtained directly from the limit surface;
- vi) due to iii), relations $f_t - d_n$ or $c - d_s$ are no longer needed;
- vii) relations $\mathbf{t} - \mathbf{w}$ are derived explicitly from reference relationships.

In the following, a more detailed description of the strategy (items 1.. to 6.) is presented. It will be shown that, with this approach and conversely to the previous model, it is possible to enforce homotetic evolution of the limit surface with explicit control of energy dissipation.

3.1 Definition of the traction state $\mathbf{t}^*$ on the initial limit surface

As a simple rule, $\mathbf{t}^*$ can be defined as the traction reaching the initial limit surface, either under mixed-mode fracture (f_1) or under mode-II fracture under compression (f_2). Under the evolution of the traction state, this discontinuity strength may change. For instance, if the traction state evolves from mode-II under compression to mixed-mode fracture, the shear strength can decrease significantly, and vice-versa.

In general, $\mathbf{t}^*$ must be defined for three different cases:

- i) the traction state lies inside the limit surface ($|\mathbf{t}| < |\mathbf{t}^*|$), in which case the definition of $\mathbf{t}^*$ only matters if the material is either unloading from or reloading to the limit surface;
- ii) the traction state lies on the limit surface, $\mathbf{t} = \mathbf{t}^*$ and
- iii) the traction state lies outside the limit surface, which is allowed in the iterative procedure described in Section 5. This topic is revisited in Section 5.3, in relation to the iterative procedure.

3.2 Energy dissipation G_F^* and G_F^{II*}

In the model presented in 2, the fracture energy dissipated in non-pure mode-I or mode-II fracture is implicitly dependent on the damage evolution law (Alfaiate and Sluys 2017a). This is not a clear issue since there is no consensus whether mode-II fracture energy of concrete is a real material parameter⁵.

⁴ Recall that $\kappa_n = w_n$ under pure mode-I fracture and $\kappa_s = w_s$ under pure mode-II fracture.

⁵ see, for instance, Carpinteri et al. (1993)

Here, as well as in the works presented in [Alfaia et al. \(1998\)](#), [Alfaia et al. \(2002\)](#), [Neto et al. \(2007\)](#), [Neto et al. \(2004b\)](#), [Alfaia et al. \(2005\)](#), [Neto et al. \(2004a\)](#), [Neto et al. \(2014\)](#), [Neto et al. \(2015\)](#), [Neto et al. \(2016\)](#), [Neto and Alfaia \(2016\)](#), [Alfaia and Sluys \(2017a\)](#), [Alfaia and Sluys \(2017b\)](#), [Alfaia and Sluys \(2019\)](#), mode-II fracture energies for both concrete and bond-slip between concrete and steel are assumed to be material properties. In particular, for corrosion, the mode-II fracture energy is defined as the area below the adopted bond-slip curves, obtained experimentally.

In this work, two possibilities are adopted for the dependence of fracture energy on the traction state. In the former case, it is assumed that the modified fracture energy is given by

$$G_F^* = G_F \left(\frac{\langle t_n^* \rangle^+}{f_{t0}} \right)^2, \quad (16)$$

$$G_F^{II*} = G_F^{II} \left(\frac{t_s^*}{c_0} \right)^2, \quad (17)$$

where $\langle \cdot \rangle$ are the McAuley brackets, so that modification of G_F only takes place under mixed-mode fracture (positive normal traction t_n^*). In the case of linear softening, this leads to a $t_s - w_s$ softening law parallel to the $c - \kappa_s$ law, as depicted in [Figs. 3 and 8](#) corresponding to the model in [Section 2](#). In these Figures, it is assumed that $|t_s^*| > c_0$, corresponding to compressive normal traction. For radial softening path the shear traction goes to zero and dissipation of energy is G_F^{II*} defined in (17). If the non-proportional path is followed under t_n constant, a residual shear traction is obtained as shown in [Figs. 3 and 8](#), equal to $t_{s,res} = t_s^* - c_0$.

In the second case, a more moderate change of the fracture energy is adopted: it is assumed that the modified fracture energy is proportional to the relation between $\langle t_n^* \rangle^+$ and f_{t0} and $|t_s^*|$ and c_0 :

$$G_F^* = G_F \frac{\langle t_n^* \rangle^+}{f_{t0}}, \quad (18)$$

$$G_F^{II*} = G_F^{II} \frac{|t_s^*|}{c_0}. \quad (19)$$

In the case of linear softening, the ultimate jump displacement value is kept constant, $w_{su} = w_{su}^r$ ([Fig. 8](#)). In [Fig. 9](#), both cases are illustrated for relation $t_s - w_s$, under exponential softening and mixed-mode fracture ($|t_s^*| < c_0$).

Consider first the case of mode-II under compression. Assume exponential softening and (16). In this

case the mode-II damage driving mechanism is adopted since $d_n^- = 0$. Upon damage evolution, at time t , and given $\mathbf{u}|_t$, $\mathbf{w}|_t$, we obtain the new value of $t_s|_t$, either incrementally, from $t_s|_{t-1}$, or under a total approach. From (67), d_s is determined and from surface f_2 we evaluate the cohesion:

$$c = |t_s|_t - \frac{\tan(\phi)}{2t_{n,lim}} t_n^2 + \tan(\phi) t_n. \quad (20)$$

Under mixed-mode fracture ($t_n > 0$), we need to define the relation between f_t and c or d_n and d_s . Under homotetic surface evolution, we get $f_t/f_{t0} = c/c_0$. Moreover, the damage driving mechanism must also be defined. From surface f_1 we can obtain c from equation,

$$\mu^2 c^2 - \mu^2 (\tan(\phi) t_n + |t_s|) c + (\mu \tan(\phi) - 1) t_n^2 = 0, \quad (21)$$

where $\mu = f_{t0}/c_0$.

In [Fig. 10](#), different $t_s - w_s$ curves are drawn, under proportional loading, corresponding to different t_n values, namely: $t_n = 0$, $t_n = f_{t0}/4$, $t_n = f_{t0}/2$, $t_n = 3f_{t0}/4$. In these Figures, the mode-II damage driving mechanism is adopted with (17), together with the same material parameters adopted in [Section 2.2](#).

A similar description could be made for crack evolution under mixed-mode fracture adopting mode-I damage driving mechanism, through the $t_n - w_n$ relation.

Thus, conversely to the model presented in [Section 2](#), the dissipation of energy in non-pure mode-I and mode-II fracture modes is defined *a priori*. This is a major advantage of the new model, in which experimental relationships can be adopted in a straightforward manner.

3.3 General mixed-mode driving mechanism

Although, in many cases, the damage driving mechanism can be defined *a priori* as previously stated, it can happen that, due to the rotation of the principal stress directions, mode-I crack opening can evolve into mixed-mode fracture⁶. Similarly, under bond-slip,

⁶ Here, conversely to the rotating crack model presented in [Rots \(1988\)](#), [Bresser et al. \(2023\)](#), the direction of the discontinuity is kept fixed, namely for fictitious cracks.

Fig. 8 Bond-slip softening under compression: linear (blue) and quadratic (brown) dependence of G_F^{II*} on $(|t_s|^*/c_0)$

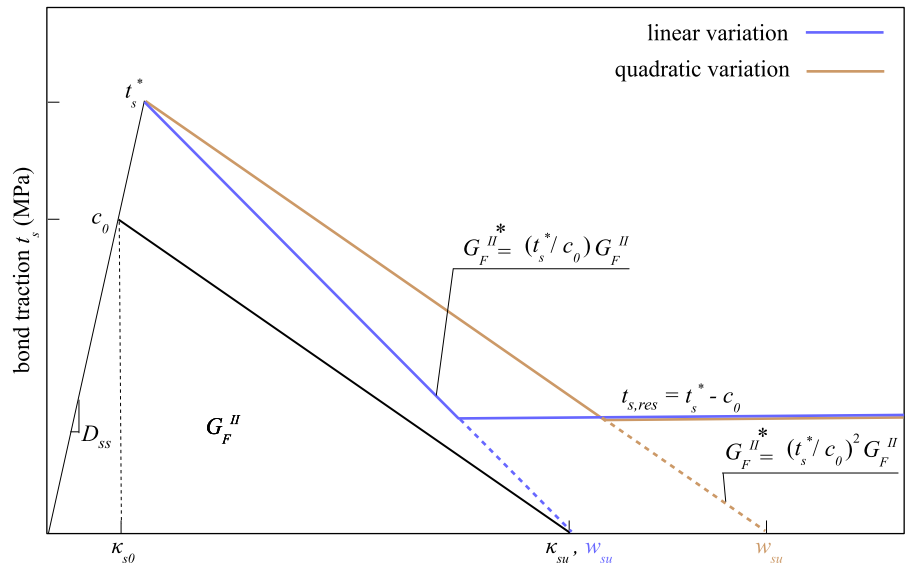
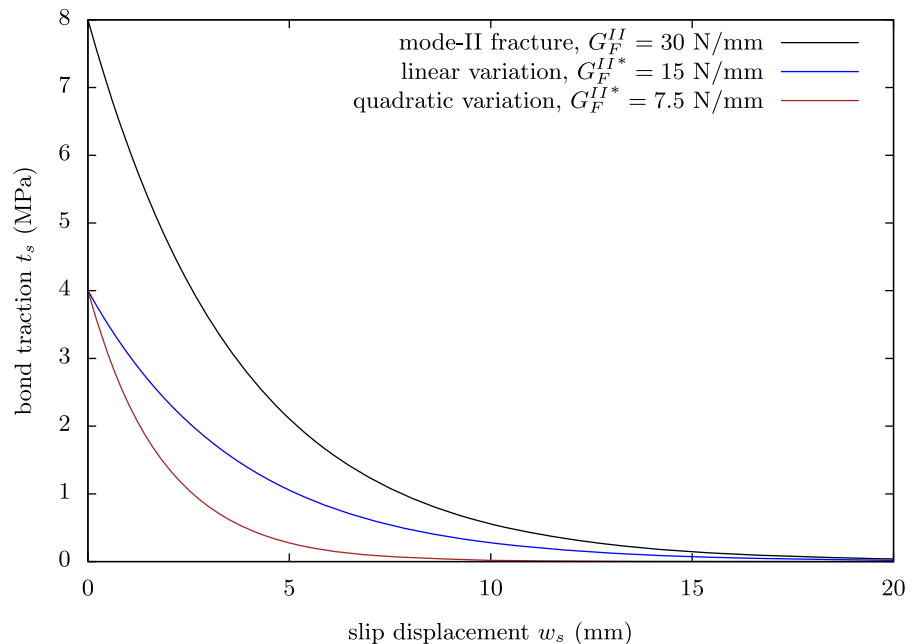


Fig. 9 Linear (blue) and quadratic (brown) variation of fracture energy with relation $t_s^*/c_0 = 0.5$



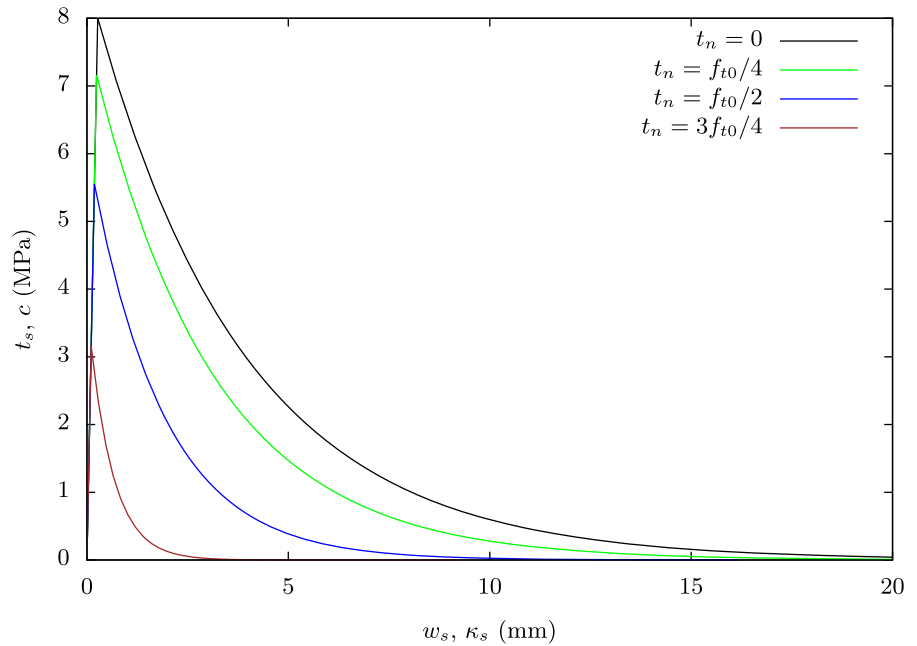
considerable tensile normal stresses can develop at the interface, namely due to corrosion of the reinforcement steel and spalling. In both cases, a pure mode-I or mode-II driving mechanism is no longer adequate, leading to incorrect values of dissipation of energy, as in the example corresponding to Fig. 7a. This is why a general, non-prescribed mixed-mode driving mechanism is also introduced, which is defined continuously between mode-I and mode-II fracture, according to the traction state.

Assume that the position on the initial limit surface is known, given by the traction vector $\mathbf{t}^*$. Following a proportional loading path, the new traction state, lying on the new surface, is given by,

$$\mathbf{t} = \alpha \mathbf{t}^*, \quad (22)$$

where α is a reduction factor in case of softening ($\alpha \leq 1$). Let α be a weighted sum of $\alpha_I(G_F^*, w_n)$ and $\alpha_{II}(G_F^{II*}, w_s)$, where α_I and α_{II} are the reduction fac-

Fig. 10 Exponential $t_s - w_s$ curves under mixed-mode fracture



tors obtained from the corresponding reference softening laws under mode-I fracture and mode-II fracture, respectively:

$$\alpha_I = \frac{t_n^r(G_F^*, w_n)}{t_n^*} \quad (23)$$

and

$$\alpha_{II} = \frac{t_s^r(G_F^{II*}, w_s)}{t_s^*}. \quad (24)$$

Adopting a linear weighted sum of α_I and α_{II} , we have:

$$\alpha = \frac{\alpha_I \pi_I + \alpha_{II} \pi_{II}}{\pi_I + \pi_{II}}, \quad (25)$$

in which $\pi_I = t_n^*/f_{t0}$ and $\pi_{II} = t_s^*/c_0$. According to the definition of the modified Mohr Coulomb limit surface, we obtain:

$$\lim_{\pi_I \rightarrow 1} \alpha = \alpha_I, \quad (26)$$

$$\lim_{\pi_{II} \rightarrow 1} \alpha = \alpha_{II}, \quad (27)$$

which correspond to a mode-I fracture damage driving mechanism and mode-II fracture damage driving mechanism, respectively.

A different weight function is also introduced, such that the influence of pure mode-I and mode-II is more persistent for values of t_n^* close to f_{t0} and t_s^* close to c_0 , respectively:

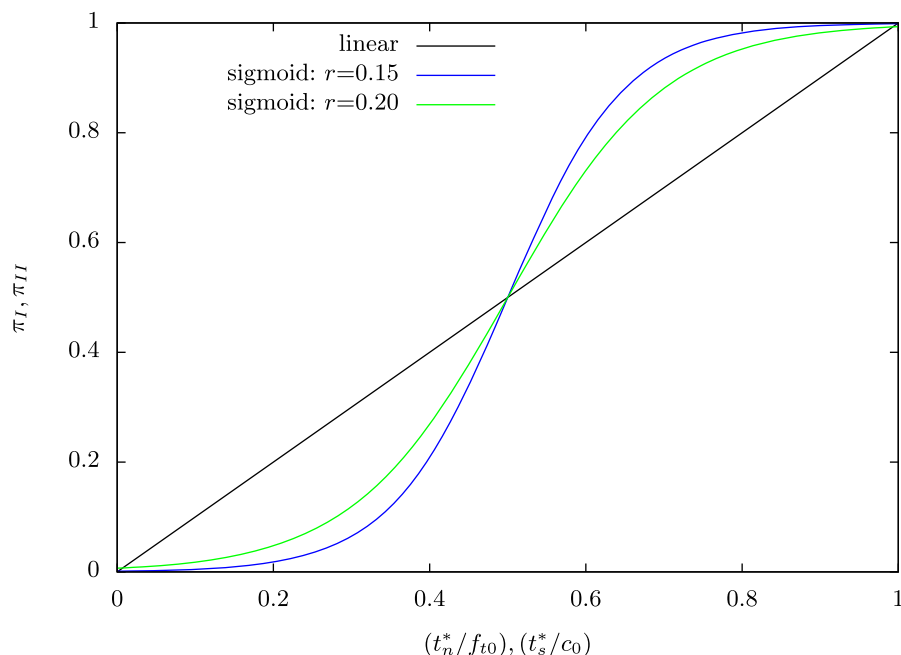
$$\pi_I = \frac{1}{1.0 + e^{(-x_I/r)}}, \quad \pi_{II} = \frac{1}{1.0 + e^{(-x_{II}/r)}}, \quad (28)$$

where

$$x_I = 2 \frac{t_n^*}{f_{t0}} - 1, \quad x_{II} = 2 \frac{t_s^*}{c_0} - 1. \quad (29)$$

In (28), π_I and π_{II} are sigmoid functions of t_n^*/f_{t0} and t_s^*/c_0 , respectively, which tend to the Heaviside function when r tends to zero. In Fig. 11, the linear functions $\pi_I = t_n^*/f_{t0}$, $\pi_{II} = t_s^*/c_0$ as well as sigmoid weight functions π_I , π_{II} from (28) are represented adopting $r = 0.15$ and $r = 0.2$. Note that, for $r > 0.2$, values of π_I and π_{II} tend to deviate further from zero for $t_s = 0$ and $t_n = 0$, respectively. As a consequence, values of $r \in [0; 0.2]$ should be adopted.

In Figs. 12a to 14b the traction-jump displacement curves obtained under proportional loading, for three

Fig. 11 Linear and sigmoid weight functions π_I and π_{II} 

different relations between t_s and t_n are shown, namely $t_s = 0.5t_n$, $t_s = 3t_n$ and $t_s = 8t_n$. In all cases, the solutions obtained with a mode-I driving mechanism, a mode-II driving mechanism, the linear function (25) and the sigmoid function (28) with $r = 0.2$ are represented. The adopted material parameters are: $f_{t0} = 3$ MPa, $c_0 = 8$ MPa, $\tan(\phi_0) = 0.6$, $G_F = 0.1$ N/mm, $G_F^{II} = 1.0$ N/mm, with exponential softening.

In the former case, the solution obtained with the sigmoid function is closer to the solution obtained with mode-I driving mechanism, in the intermediate case, both the linear function and the sigmoid function give rise to similar solutions and, in the latter case, the solution obtained with the sigmoid function is closer to the solution obtained with mode-II driving mechanism (see appendix). It can also be observed that, in the former case, the solution obtained with mode-II driving mechanism proves inadequate, whereas, in the latter case, the solution obtained with mode-I driving mechanism also proves inadequate (see appendix C). Moreover, in the intermediate case, either mode-I or mode-II driving mechanisms lead to distant solutions from the ones obtained with the weighted functions. 13

In Figs. 15a, 15b, the results obtained with the previous model (presented in Figs. 7a and 7b) are compared to the results obtained with the new model. In the latter case, a generalized mixed-mode driving mechanism is

adopted with the sigmoid weighted function ($r = 0.15$) and quadratic variation of G_F , G_F^{II} (16), (17), respectively.

In Figs. 16a, 16b, the results obtained with the previous model (presented in Figs. 7a and 7b) are compared to the results obtained with the new model. In the latter case, the generalized mixed-mode driving mechanism is adopted with the sigmoid weighted function ($r = 0.15$), but now with a linear variation of G_F , G_F^{II} (18), (19), respectively.

It is clear that the dissipation of energy is now controlled with the new model. Conversely to the previous model, the dissipation of energy under mode-I is smaller than G_F as expected, as can be seen in in Figs. 15a and 16a.

3.4 Definition of the damage variables

Note that, although the limit surface (66) does not depend explicitly on the damage variables d_n , d_s , they are uniquely defined for each position of the limit surface: under $f = 0$ and given f_t , c , it is possible to obtain the traction vector lying on the surface and the corresponding damage variables.

The updated damage variables are obtained according to the driving damage mechanism: i) mode-I fracture driving mechanism, ii) mode-II fracture driving

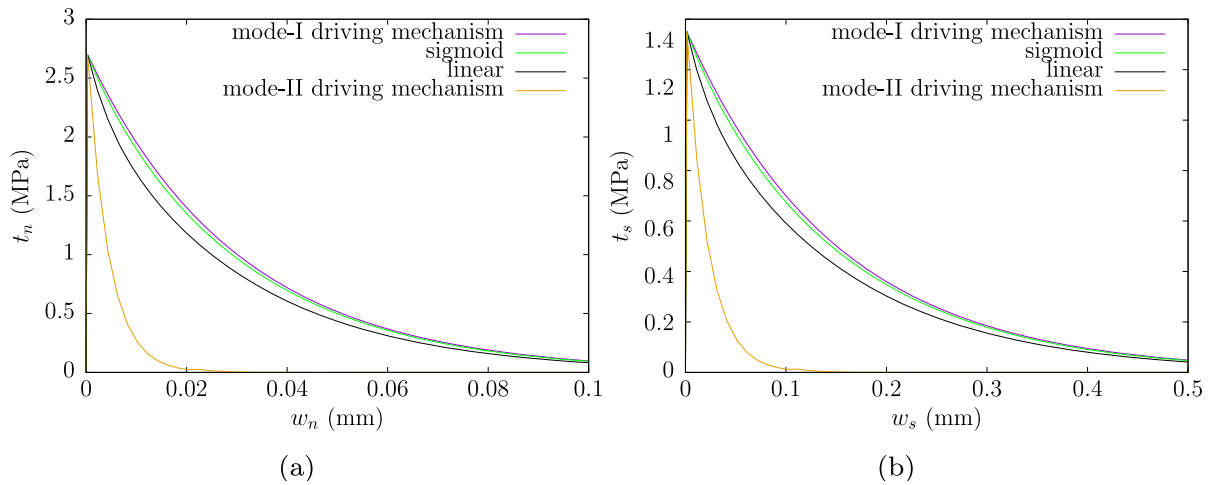


Fig. 12 (a) $t_n - w_n$ and (b) $t_s - w_s$ curves obtained for $t_s = 0.5t_n$

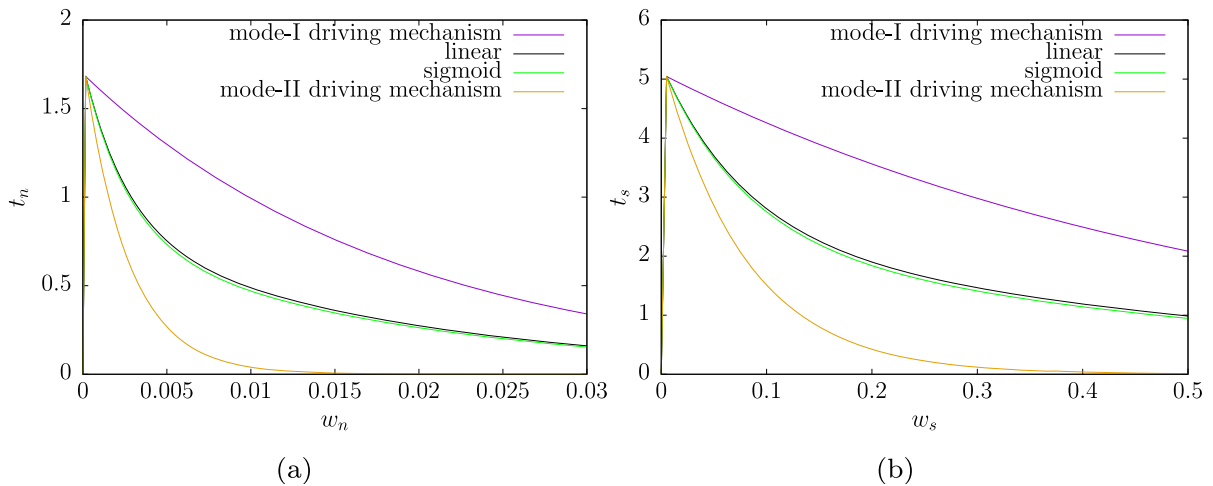


Fig. 13 (a) $t_n - w_n$ and (b) $t_s - w_s$ curves obtained for $t_s = 3t_n$

mechanism, iii) generalized mixed-mode fracture driving mechanism or iv) corrosion-induced damage driving mechanism. The first three modes were already discussed, the latter will be dealt with in Section 5.

In D, two examples of homotetic evolution of the limit surface under generalized mixed-mode driving mechanism are shown. Other examples of both proportional and non-proportional loading are presented in Section 5.3.

Under any of these damage driving mechanisms (i) to iv)), it is possible to enforce isotropic damage. Some particular cases are analyzed in the following, regarding isotropy.

Under a traction radial path, we have:

$$\mathbf{t} = \alpha \mathbf{t}^* = \alpha (\beta \mathbf{t}^{out}), \quad (30)$$

or,

$$t_n = \alpha t_n^* = \alpha D_{nn} w_n^* = (1 - d_n) D_{nn} w_n, \quad (31)$$

$$t_s = \alpha t_s^* = \alpha D_{ss} w_s^* = (1 - d_s) D_{ss} w_s. \quad (32)$$

When isotropic damage is enforced through $d_n = d_s$ we obtain

$$w_s/w_n = w_s^*/w_n^*. \quad (33)$$

The solutions from the new model presented in Figs. 15a to 16b are obtained with isotropic damage evo-

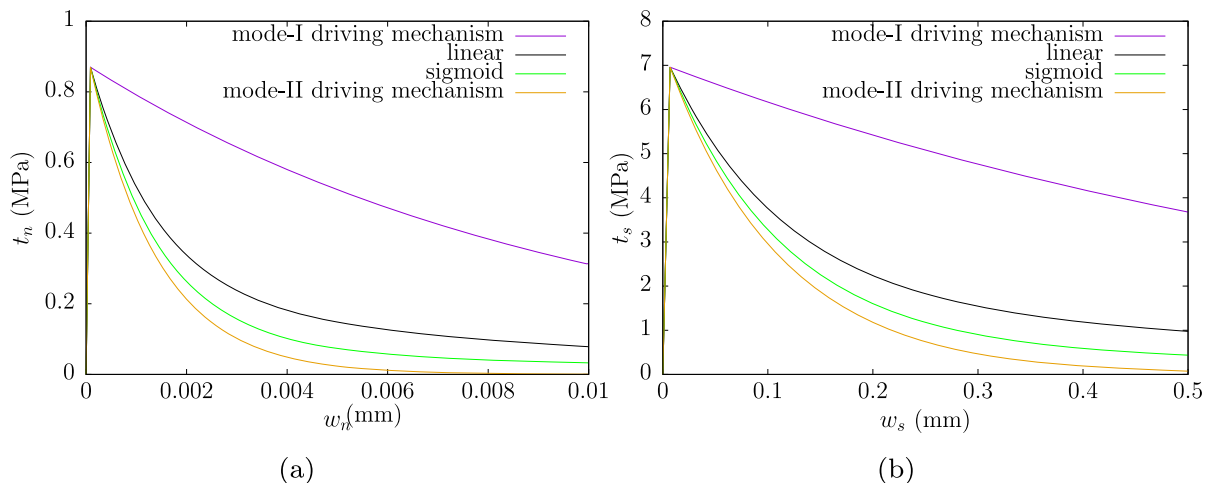


Fig. 14 (a) $t_n - w_n$ and (b) $t_s - w_s$ obtained for $t_s = 8t_n$

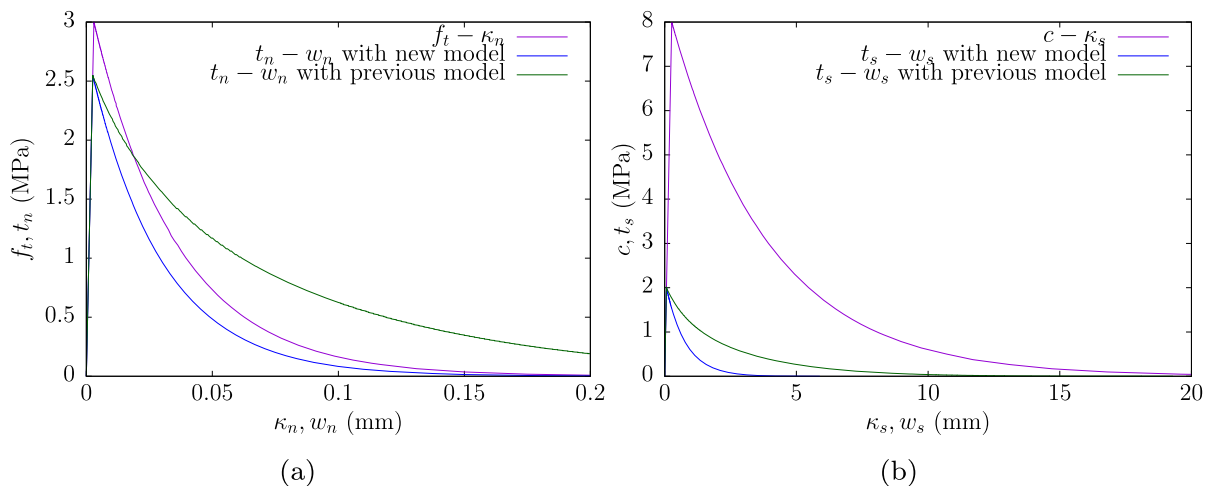


Fig. 15 (a) $t_n - w_n$ and (b) $t_s - w_s$ obtained for $t_n^* = 2.55$, $t_s^* = 2$ MPa. Comparison between the previous model and the new model, with generalized damage driving mechanism and quadratic variation of G_F , G_F^{II}

lution. This was achieved under displacement control, given w_n , w_s according to (33).

The generalized damage driving mechanism can also be formulated to enforce isotropic damage under stress control. For this purpose, given the traction $\mathbf{t} = \alpha \mathbf{t}^*$, function (25) can be modified in order to take into account the relative weight of the damage variables d_n , d_s . The isotropic damage variable d is defined as:

$$d = \frac{d_n \pi_I + d_s \pi_{II}}{\pi_I + \pi_{II}}. \quad (34)$$

In (34), the weighting functions π_I and π_{II} are defined in Section 3.3, and the damage variables d_n , d_s are obtained from the reference laws:

$$\begin{aligned} d_n &= 1 - t_n / (D_{nn} w_n^r), \\ d_s &= 1 - t_s / (D_{ss} w_s^r). \end{aligned} \quad (35)$$

In the following chart, the adopted strategy under stress control is presented:

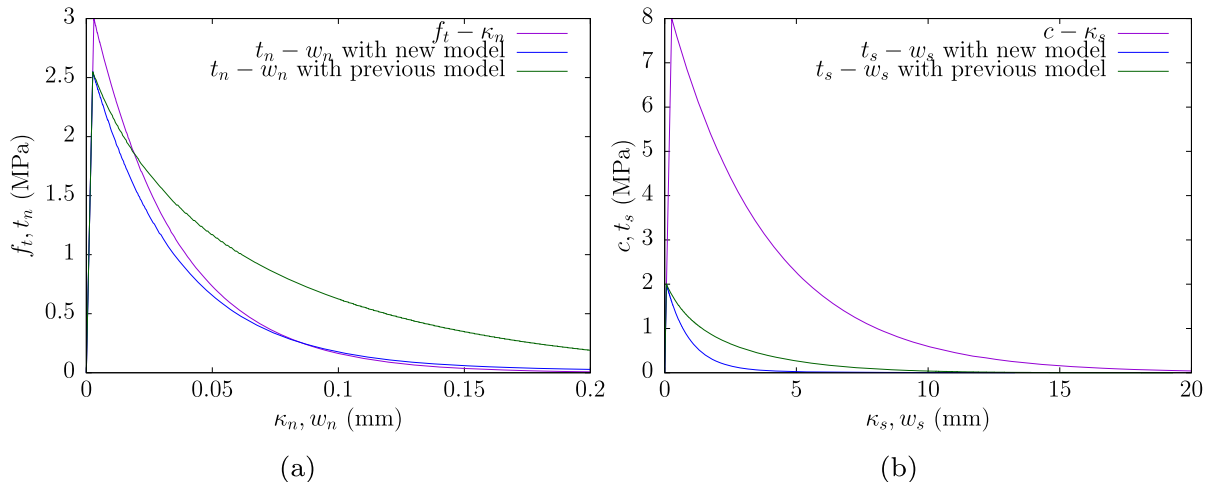


Fig. 16 (a) $t_n - w_n$ and (b) $t_s - w_s$ obtained for $t_n^* = 2.55$, $t_s^* = 2$ MPa. Comparison between the previous model with generalized damage driving mechanism and linear variation of G_F , G_F^{II}

1. define the traction on the initial limit surface, $\mathbf{t}^*$;
2. define the corresponding modified fracture energies under both mode-I and mode-II fracture, G_F^* , G_F^{II*} ;
3. given reduction factor α ,
4. define the traction vector:
 $t_n = \alpha t_n^*$, $t_s = \alpha t_s^*$;
5. from the reference relationships $\mathbf{t}^r - \mathbf{w}^r$ obtain the jump displacements: w_n^r, w_s^r ;
6. derive d_n, d_s from (35);
7. define isotropic damage variable d by means of (34);
8. obtain the jump displacements from (67):

$$\begin{aligned} w_n &= \frac{t_n}{(1-d)D_{nn}}, \\ w_s &= \frac{t_s}{(1-d)D_{ss}}. \end{aligned} \quad (36)$$

In Figs. 17a, 17b, the results obtained with the new model under generalized damage driving mechanism and displacement control are compared to the results obtained under generalized damage driving mechanism and stress control (please refer to Section 5.1 for the definition of the control loading function). In both

cases, the sigmoid weighted function ($r = 0.15$) and quadratic variation of G_F , G_F^{II} (16), (17), respectively, are adopted.

In Figs. 18a, 18b, the results obtained under a generalized damage driving mechanism and displacement control are compared to the results obtained under generalized damage driving mechanism and force control. In both cases, the sigmoid weighted function ($r = 0.15$) and linear variation of G_F , G_F^{II} (18), (19), respectively, are adopted.

It can be concluded that the generalized driving damage mechanism gives rise to similar solutions either under displacement control or force/stress control. The conclusions obtained with quadratic variation of the fracture energy are similar. Adopting a linear variation of the fracture energy, minor differences can be observed in the tail of the traction-jump displacement curves, in which the solutions obtained under displacement control soften slightly later than the solutions obtained under force control. This is due to the fact that, under linear variation of the fracture energy and adopting relation (33) between the jump displacement components, the traction path obtained from the reference curves deviates considerably from the radial path. As a consequence, the differences obtained in weighting either the tractions (25) or the damage variables (34) become more evident in this case. In Fig. 19a, the traction path obtained from the reference curves under linear variation of the fracture energy is compared to the traction path obtained from the reference curves under

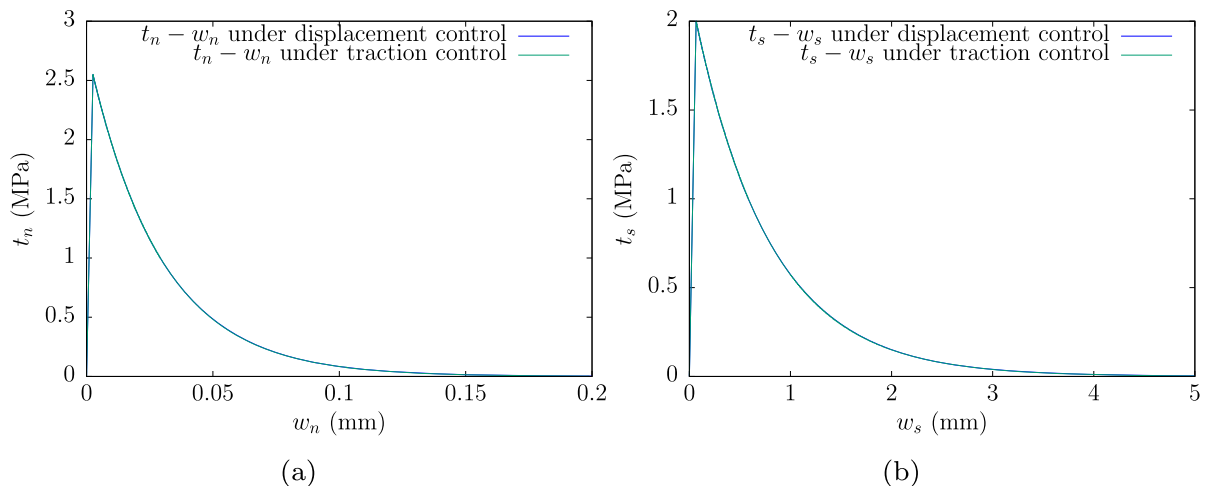


Fig. 17 (a) $t_n - w_n$ and (b) $t_s - w_s$ for $t_n^* = 2.55$, $t_s^* = 2$ MPa. Comparison between the solutions obtained with the generalized damage driving mechanism under displacement control and force control, adopting (16), (17)

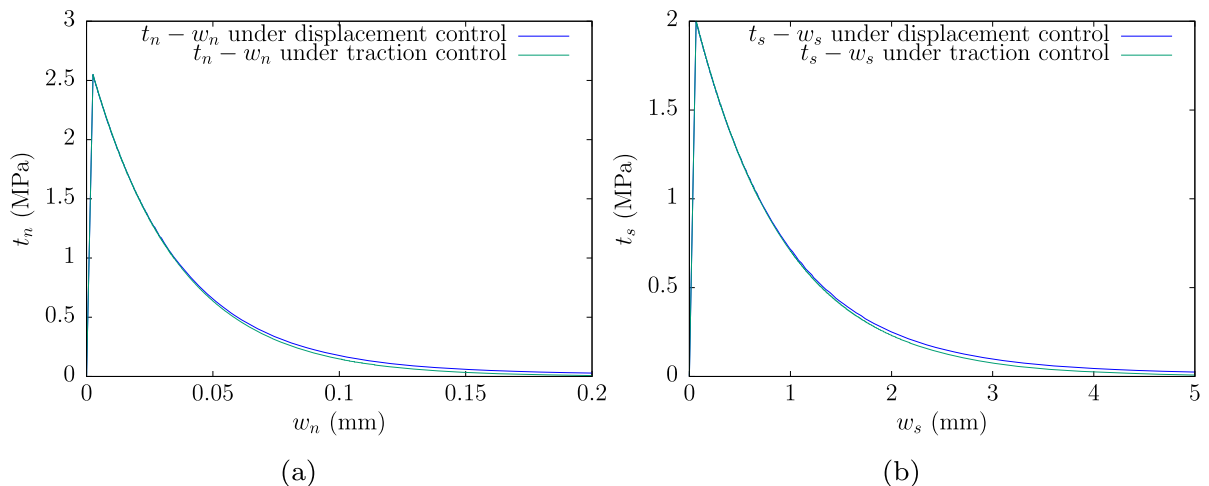


Fig. 18 (a) $t_n - w_n$ and (b) $t_s - w_s$ for $t_n^* = 2.55$, $t_s^* = 2$ MPa. Comparison between the solutions obtained with the generalized damage driving mechanism under displacement control and force control, adopting (18), (19)

quadratic variation. In both cases displacement control is adopted. It can be seen that, under quadratic variation, the traction path is much closer to the radial path than the traction path obtained with the linear variation of the fracture energy.

Notwithstanding, with the new model, isotropic damage is now possible under a radial softening loading path and homotetic evolution of the limit surface. Note that, with the model reviewed in Section 2, homotetic evolution of the limit surface is not compatible, in general, with isotropic damage. This is due to the fact that the damage variables are obtained from Equations (68)

to (71). In fact, given κ_n , we get f_t from (68), d_n from (70), $c = f_t/f_{t0} \times c_0$ from homotetic evolution of the limit surface and d_s from (71). As shown in the work presented in Alfaiate and Sluys (2017a), isotropic damage leads to a particular, non-realistic relation between G_F and G_F^{II} , which is dependent on the elastic stiffness coefficients as well as on the material strength.

In Fig. 19b, evolution of the damage variables $d_n - d_s$ is shown for the examples presented in Figs. 15a to 16b. From this figure it is clear that, conversely to the new model, damage evolution under the previous model is not isotropic.

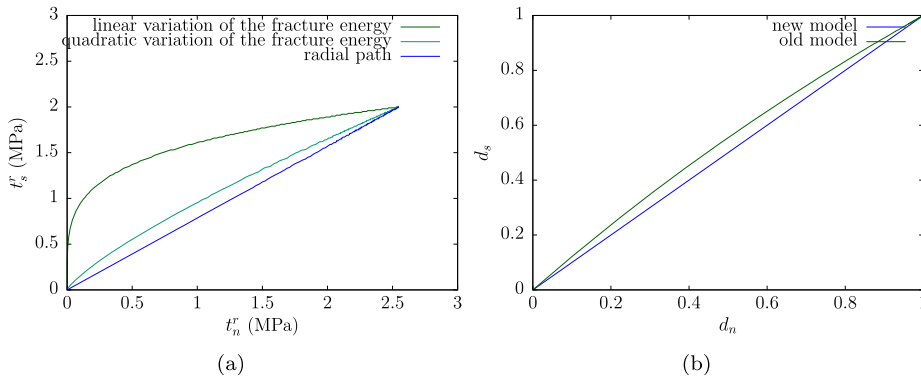


Fig. 19 (a) Evolution of the traction path obtained from the reference laws given relation (33) between jump displacement components, (b) Evolution of relation $d_n - d_s$ corresponding to the examples presented in Figs. 15a to 16b

4 Corrosion

Corrosion induces damage in the structure, both directly and indirectly. In the former case three main mechanisms are considered: the reduction of the sane cross section of the reinforced steel, the degradation of steel to concrete bond and spalling due to the formation of corrosion products around the steel cross section. Indirectly, corrosion affects cracking behaviour and tension stiffening, giving rise to increased deformation. These indirect effects are also accounted for in this paper.

In the present work damage mechanics is used to model corrosion. Special attention is given to spalling of concrete in Section 4.3.

4.1 Changes in the steel bar cross section

Similar to the work presented in Alfaiate et al. (2023), the corrosion model proposed by Fang et al. (2004) is adopted (see also Chernin and Val (2011)).

The corrosion level η is given by (see Fig. 20):

$$\eta = \frac{A_0 - A}{A_0}, \quad (37)$$

where A_0 is the initial cross section area of the bar and A is the uncorroded cross section area of the bar, after removing the corrosion sub-products. The corrosion level evolves from 0 to η_f , the latter corresponding to the final corrosion stage.

In Fig. 20, r_0 corresponds to A_0 , x corresponds to the reduction of the sane cross section of the bar and

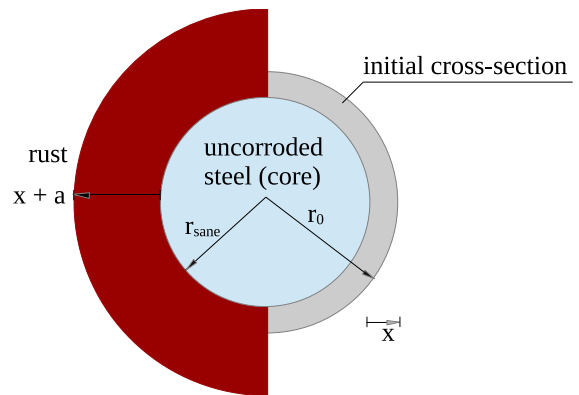


Fig. 20 Evolution of corrosion in the steel cross section

a is the increase of the radius due to the formation of rust.

The expansion of the bar cross section gives rise to the strain value:

$$\varepsilon^{corr} = \sqrt{1 + \eta} - 1. \quad (38)$$

The core reduction x is given by

$$x = r_0 \left(1 - \sqrt{1 - \eta} \right), \quad (39)$$

and

$$r_{sane} = r_0 - x = r_0 \sqrt{1 - \eta}. \quad (40)$$

4.2 Bond-slip degradation

Bond-slip degradation due to corrosion gives rise to the increase of damage. However, the corresponding damage evolution no longer obeys the rules described in Sections 2.1 and 3. As a consequence, an extension of the damage model is considered, in which damage evolution is based on experimental evidence. In this work, these experimental curves are taken from Silva (2018), Wu and Zhao (2013), Jiang et al. (2018).

As assumed in Alfaiate et al. (2023), corrosion is introduced at a pre-defined fixed load level, such that the material point should lie on a *plateau* defined between the initial bond-slip law ($\eta = 0$) and the final bond-slip law ($\eta = \eta_f$).

In Fig. 21, the bond-slip path corresponding to the transition between bond-slip laws defined at different time steps, from $\eta = \eta_{t-1}$ to $\eta = \eta_f$ (point *a* to *f*) is shown.

4.3 Spalling

Apart from the reduction of the steel bar cross section (Section 4.1) and the bond-slip modification due to corrosion, with the corresponding shear strength decrease as illustrated in Section 4.2, there is another important phenomenon which must be taken into account, namely spalling of the concrete cover.

There are different types of spalling which may be caused by, e.g., sudden impact, fire or thermal shock, giving rise to explosive spalling, freeze-thaw cycles, when water infiltrates the concrete and freezes, etc. However, only spalling due to corrosion of the reinforcement steel is taken into account here. In this case, spalling develops due to volume increase of the steel bars which accumulate products of corrosion (see Fig. 20). Cracking transversal to the steel bar develops, and full separation between the cover and the reinforcement bar may occur, which may lead to premature local or total failure of the structure (see for instance Coronelli et al. (2013), Zhang et al. (2021), Chang et al. (2024)). Moreover, spalling can also occur in the compression region, leading to a decrease of the internal lever arm and a decrease of the bending moment capacity. This effect is not taken into account in this work.

Two main effects are considered due to spalling:

1. volume expansion of the steel reinforcement,

2. development of longitudinal cracking at the reinforcement level.

In order to take into account both effects, concrete spalling is modelled in two ways: i) by means of volume expansion of the reinforcement bars and ii) by means of steel-concrete interface expansion.

4.3.1 Spalling modelled by means of volume expansion

In the numerical tests, the volume expansion due to corrosion is enforced by means of an equivalent temperature increase. Thus, the expansion strain (38) is evaluated as:

$$\varepsilon^{\Delta T_{eq}} = \sqrt{1 + \eta} - 1 = \alpha^T \Delta T_{eq}. \quad (41)$$

where α^T is the thermal coefficient of linear expansion and ΔT_{eq} is the equivalent temperature variation which gives rise to the expansion strain. Since the variation of temperature induces an isotropic strain state, $\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon^{\Delta T_{eq}}$, the corresponding strain component along the steel direction is eliminated. In the present study, we define:

$$\varepsilon_{xx}^{\Delta T_{eq}} = 0, \quad \varepsilon_{yy}^{\Delta T_{eq}} = \sqrt{1 + \eta} - 1 = \varepsilon_{yy}^{corrosion}, \quad (42)$$

where direction *x* is along the length of the beam (see Fig. 22).

4.3.2 Spalling modelled by means of interface expansion

The volume expansion gives rise to radial cracking around the bar (Guzmán and Gálvez 2017; Bazán et al. 2018), in particular to the formation of longitudinal cracking at the reinforcement level (item 2.), as schematically depicted in Fig. 23.

This is a 3D effect which can not be fully taken into account in this work (see also Stanish (1997), Ajagbe et al. (2020), Moccia et al. (2021), Zhang et al. (2021)). However, in the simulations presented in part II, this effect is considered in an approximate way. In Fig. 23, a schematic representation of the cross section of the beam is presented, with longitudinal cracking at reinforcement level. The horizontal crack opening in the neighbourhood of the bar can be defined as:

$$w_n^{spalling} = \varepsilon_{yy}^{corrosion} \times \phi, \quad (43)$$

Fig. 21 Transition between bond-slip laws with increasing corrosion: bond-slip path under constant t_s

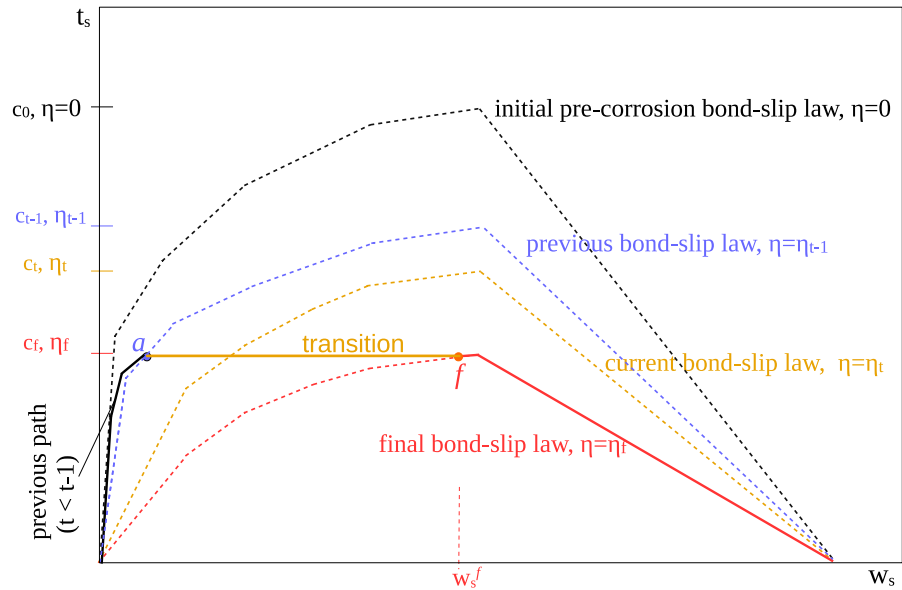
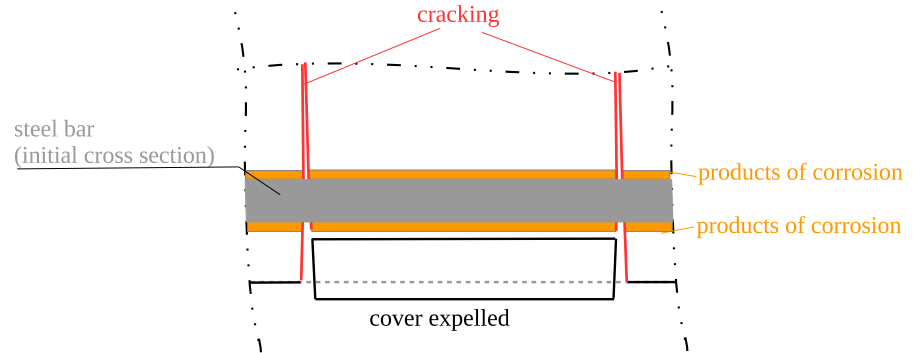


Fig. 22 Schematic representation of spalling of the concrete cover



where $\phi = 2r_0$ is the initial bar diameter (see Fig. 20). This crack opening $w_n(\eta)$, which varies from $w_n(0) = 0$ to $w_n(\eta_f) = 2r_0(\sqrt{1 + \eta_f} - 1)$, gives rise to a damage increase in the neighbourhood of the bar. In the simulations, an equivalent crack opening w_n^{eq} is defined as (see Fig. 23):

$$w_n^{eq} = w_n^{spalling} \cdot c_w, \quad (44)$$

where c_w is a coefficient which accounts for the variation of the crack opening w_n across the width: the larger the beam width the smaller the value of c_w is. Coefficient c_w can be defined as a function of the width of the steel bars:

$$c_w = f\left(\frac{n\phi}{b}\right), \quad (45)$$

where n is the number of steel bars, ϕ is the diameter of the steel bar⁷ and b is the width of the beam. In the simulations, this normal opening at the steel-concrete interface is introduced by enforcing splitting forces directly at the interface (see Fig. 24). These self-balanced forces are introduced at each step, during the corrosion stage, using a control loading function, according to Section 5.1. Thus, the volume expansion corresponding to spalling is transferred to the steel-concrete interface.

As previously mentioned, the longitudinal crack opening at the reinforcement level is not homogeneous. Apart from the dependence on the width of the beam and the distribution of the reinforcement bars, introduced with coefficient c_w , it also seems reasonable to assume that crack opening w_n is affected by the dam-

⁷ in case of different diameters of the steel bars an equivalent diameter ϕ_{eq} is adopted, according to Eurocode 2 (CEN 2004)

age state, i.e., w_n is expected to be larger at the most damaged locations of the beam, namely close to cracking. In the simulations presented in part II, a uniform stress state is enforced by the splitting forces along the steel-concrete interface. As a consequence, larger values of equivalent crack opening w_n^{eq} are automatically obtained near the most damaged zones along the length of the beam.

The bond-slip relationship defined in Section 4.2 is used when modelling spalling with splitting forces. In fact, the deterioration introduced by the corrosion level η is already taken into account experimentally. Nevertheless, if the split of the cover occurs, the steel-concrete bond is lost at the steel-concrete interface. As a consequence, the bond strength is also considered as a function of the interface normal opening w_n : beyond a certain level of w_n^{lim} , the bond strength is reduced to a residual value, c_{res} . Note that, conversely to the case of dilatancy (Alfaiate and Sluys 2017b), the total constitutive relation remains symmetric according to (15). The dependence of the bond strength on the interface normal opening is introduced at the transition process defined in Section 4.2 where, in addition to the corrosion level, the bond strength also decreases with w_n . The definition of w_n^{lim} depends on the normal softening law. For instance, when considering bilinear softening and taking the normal fracture energy equal to the concrete fracture energy under mode-I, G_F , the normal traction decays to zero for $w_n^{lim} = \frac{54}{15} G_F / f_t$. It is assumed that, under such a value of the normal opening of the interface, the bond traction is reduced to c_{res} . For values of the normal opening $w_n \in [0, w_n^{lim}]$, the cohesion decreases linearly from c to c_{res} .

This is a novel aspect of the model, in which both damage evolution and splitting of the concrete cover due to spalling are taken into account in a 2D simulation.

5 Iterative procedure

Similar to the work presented in Alfaiate and Sluys (2023), Alfaiate et al. (2023), a total iterative approach (TIA) is adopted in which damage is iteratively updated, on several material points simultaneously. The method is reviewed in this work, taking into account the new approach introduced in Section 3. In particular, in Section 5.3 return mapping is included as a novel feature.

5.1 Control loading function

In the general case of non-proportional loading, the current external force vector $\mathbf{F}_2$ is applied on top of the previous ones, accumulated in $\mathbf{F}_1$. At time step $t - 1$ we have: $\mathbf{F}_1 + \lambda_{t-1} \mathbf{F}_2$, where λ_{t-1} is the load factor adopted in step $t - 1$. At the end of step $t - 1$ all material points either lie on or stay inside the surface, $f \leq 0$ (72). In the current step t , a new load level is applied $\mathbf{F}_1 + \lambda_t \mathbf{F}_2$. At this stage the internal variables from the previous step remain unchanged, namely damage $\mathbf{d} = \mathbf{d}|_{t-1}$. At each material point a check is performed regarding the position on the surface:

$$f \leq 0 \vee f > 0. \quad (46)$$

Assume that one particular material point now lies outside surface f_2 ,

$$|t_{s1} + \lambda_t t_{s2}| > \frac{\tan(\phi)}{2t_{n,lim}} (t_{n1} + \lambda_t t_{n2})^2 - \tan(\phi)(t_{n1} + \lambda_t t_{n2}) + c|_{t-1}. \quad (47)$$

The purpose of the iterative procedure is to ensure that, for all points lying outside the surface ($f > 0$), damage is increased until $f = 0$. Equation (47) therefore becomes:

$$|t_{s1} + \lambda_t t_{s2}| = \frac{\tan(\phi)}{2t_{n,lim}} (t_{n1} + \lambda_t t_{n2})^2 - \tan(\phi)(t_{n1} + \lambda_t t_{n2}) + c|_t, \quad (48)$$

Note that the only difference between equations (47) and (48) is due to the increase of damage, which applies to the cohesion value: $c|_{t-1} \rightarrow c|_t$. Similarly, if mixed-mode fracture is active ($f_1 > 0$), we should get

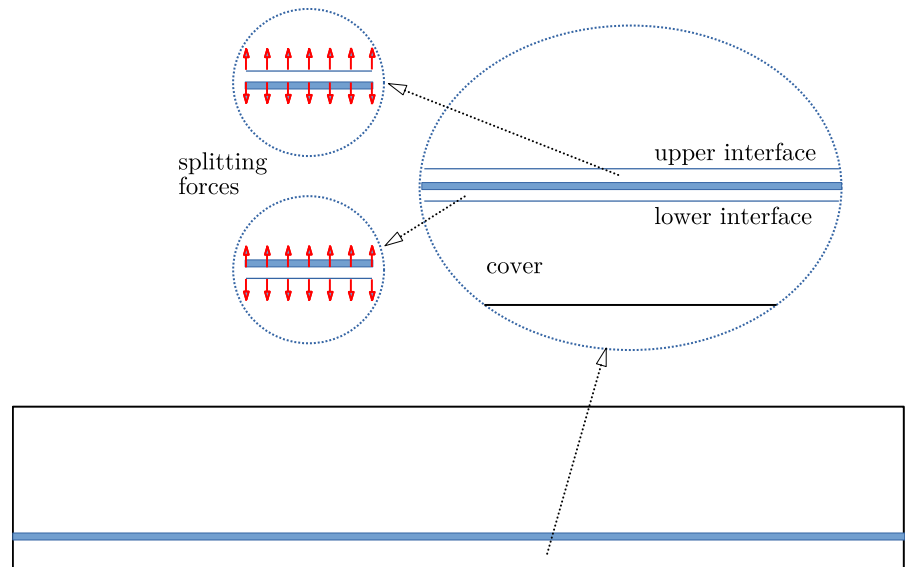
$$|t_{s1} + \lambda_t t_{s2}| = \frac{f_t|_t \tan(\phi) - c|_t}{f_t^2|_t} (t_{n1} + \lambda_t t_{n2})^2 - \tan(\phi)(t_{n1} + \lambda_t t_{n2}) + c|_t, \quad (49)$$

where $f_t|_t$ and $c|_t$ denote new tensile and cohesion values, respectively, corresponding to the increase of damage.

In fact, the load step factor λ_t may or may not remain fixed during the iterative process. Step λ_t is defined in each iteration, according to a control loading function C . This function can be made dependent on the displacements, forces, displacement jumps, as well as damage or corrosion level. Thus, at each time step t and each iteration j , step size λ_t^j is defined such that

Fig. 23 Schematic representation of the cross section of the beam with longitudinal cracking at reinforcement level

Fig. 24 Splitting forces at steel-concrete interface in a beam



$C = 0$, where

$$C = C(\mathbf{u}_1 + \lambda_t^j \mathbf{u}_2, \mathbf{F}_1 + \lambda_t^j \mathbf{F}_2, \mathbf{w}_1 + \lambda_t^j \mathbf{w}_2, \mathbf{d}_1 + \lambda_t^j \mathbf{d}_2, \dots)$$

Here, C is defined as a monotonic increasing or decreasing function of some parameter, similar to the constrained functions used for the arc length method.

For instance, if a monotonic variation of the force vector $\Delta \mathbf{F}_2^*$ is enforced, as done with the splitting forces defined in Section 4.3.2, we obtain

$$(50) \quad C = \lambda_t^j \mathbf{F}_2 - (\lambda_{t-1} \mathbf{F}_2 + \Delta \mathbf{F}_2^*). \quad (51)$$

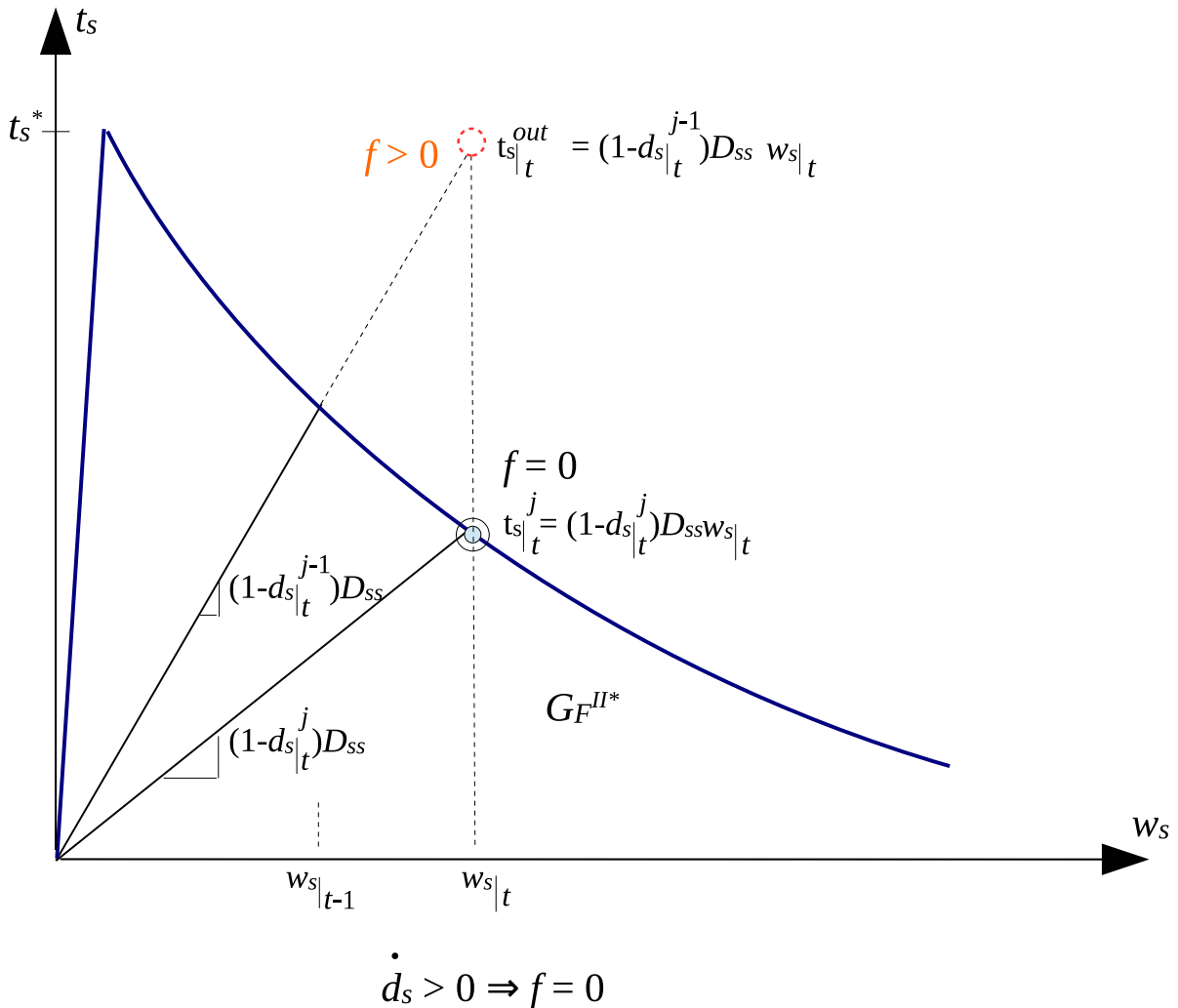


Fig. 25 Uniaxial representation of iteration scheme for bond-slip softening behaviour

In the corrosion stage and assuming that corrosion occurs under constant load, as is usually the case in reinforced concrete structures, function C is defined as:

$$C = \lambda_t^j - (\lambda_{t-1} + \frac{\Delta\eta^*}{\eta_f}), \quad (52)$$

where $\Delta\eta^*$ is the adopted monotonic increase of the corrosion level. In this case, the step factor is given by the relative corrosion level:

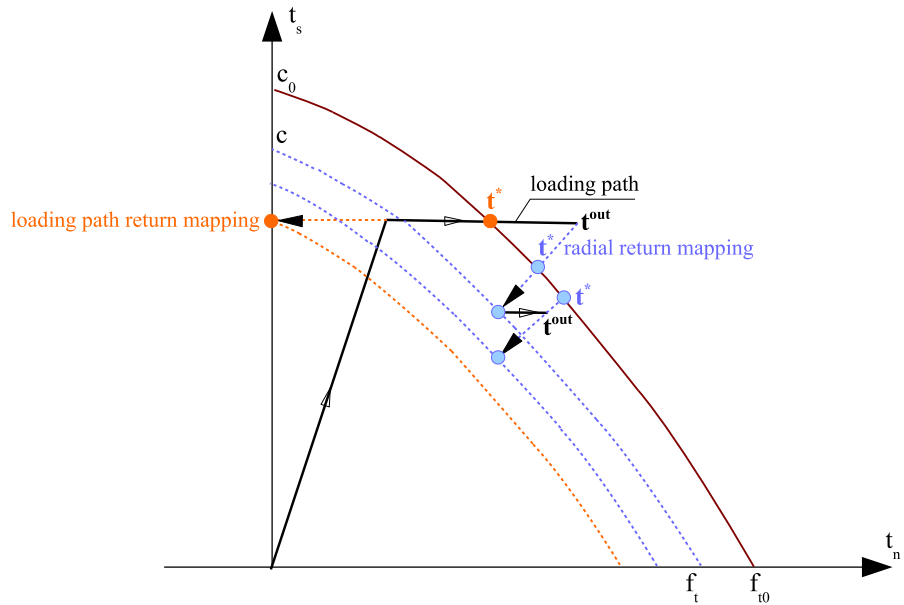
$$\lambda(t) = \frac{\eta_t}{\eta_f}. \quad (53)$$

5.2 Damage evolution under the iterative procedure

In this Section, damage evolution under the iterative procedure is defined. For this purpose, recall the definition of *damage driving mechanism* introduced in Section 2.1, namely: i) mode-I driving mechanism, which is usually adopted under cracking, ii) mode-II driving mechanism, usually adopted under bond-slip or iii) the generalized mixed-mode driving mechanism, adopted under mixed-mode fracture.

At time t , under a given damage state reached at the previous iteration $j-1$ ($d_n|_t^{j-1}$, $d_s|_t^{j-1}$) and after solving the system of equations corresponding to iteration j with load factor λ_t^j , we obtain the total kinematic

Fig. 26 Return mapping under non-proportional loading



variables: displacements $\mathbf{u}|_t$, jump displacements $\mathbf{w}|_t$, strains $\boldsymbol{\varepsilon}|_t$. From these, we evaluate the actual stress and traction states, $\boldsymbol{\sigma}|_t$ and $\mathbf{t}|_t$, respectively.

Consider, for instance, mode-II fracture under compression. Assume that, at a certain material point, we obtain the shear traction lying outside the surface ($f > 0$), as illustrated in Fig. 25:

$$t_s|_t^{out} = (1 - d_s|_t^{j-1})D_{ss}w_s|_t. \quad (54)$$

According to the mode-II damage driving mechanism, the bond traction is updated to the new position such that the bond-slip law is obeyed, i.e., $f = 0$:

$$t_s|_t^j = t_s^r(w_s|_t, G_F^{II*}) = (1 - d_s|_t^j)D_{ss}w_s|_t, \quad (55)$$

where $t_s^r(w_s|_t, G_F^{II*})$ denotes the reference mode-II softening law as defined in Section 3. From (55), the damage variable is updated from $d_s|_t^{j-1}$ to $d_s|_t^j$. A similar procedure is adopted for mode-I fracture or strain hardening, as described in Alfaiate and Sluys (2023).

In this example, it is assumed that the traction follows a proportional or radial path towards the origin of the limit surface. Different traction paths can be adopted, as assumed in Section 2.2, for instance under constant normal traction. Under mixed-mode fracture, the generalized damage driving mechanism is designed to enforce a proportional traction path, although other possibilities can take place. Since, during the iterative

process, the traction lies outside the surface, the concept of return mapping, well known to the plasticity theory, is taken into account in the next Section, to cope with the definition of the traction evolution.

5.3 Return mapping

Consider first that either a mode-I or mode-II damage driving mechanism is adopted. After evaluating the driving traction component on the surface, the other traction component is defined according to the adopted return mapping scheme.

Assume the non-proportional traction path shown in Fig. 26, under mixed-mode fracture. First the increase of the shear traction is predominant ($t_s > t_n$) but, due to the second loading, mode-I becomes the driving mechanism. Once outside the surface and as described in Section 3, a traction state on the initial surface $\mathbf{t}^*$ is defined such that the reference traction law $t_n^r(t_n^*, w_n, G_F^*)$ can be defined. In Fig. 26, two possibilities are illustrated: i) the traction state marked in blue, giving rise to the radial return mapping and ii) the traction state marked in orange, giving rise to the designated loading path return mapping.

In the former case, which is the natural choice for proportional loading, the new limit surface is sought such that the traction state decreases proportionally, with increasing damage under both the normal and the

shear directions, $\dot{d}_n > 0$, $\dot{d}_s > 0$. Upon subsequent loading, a new radial return mapping path is defined, and so forth. This can be achieved with the generalized damage driving mechanism introduced in Section 3.3, since a radial traction path is enforced under this criterion.

In the latter case, the new limit surface is sought on the actual loading path which, in this example, leads solely to the increase of normal damage, $\dot{d}_n > 0$, $\dot{d}_s = 0$. Upon full dissipation of the modified mode-I fracture energy G_F^* , the final position is marked on the vertical axis, keeping the shear traction constant. The corresponding numerical solutions are presented below.

The definition of the shear traction depends on the adopted return mapping. For instance, after evaluating the normal traction $t_n|_t^j$, under radial return mapping we obtain

$$t_s|_t^j = \frac{t_n|_t^j}{t_n^*} t_s^*, \quad (56)$$

where $t_n|_t^j$ and $t_s|_t^j$ are the actual tractions on the surface (iteration j) and t_n^* and t_s^* are the traction components on the initial limit surface. From $t_n|_t^j$ and $t_s|_t^j$ we derive the damage variables, whereas the cohesion and the tensile strength are defined according to (21) and (65), respectively. In any case, the radial return mapping direction is defined by $\mathbf{t}^{out}$, lying outside the surface, which is obtained with the previous secant stiffness.

In general, radial return mapping gives rise to a decrease in both the normal and the shear traction, which seems to be physically acceptable: upon crack opening, an increase in damage under shear also takes place; conversely, if shear damage increases under mixed-mode, the interface normal strength is expected to decrease. This also seems to be a more reliable procedure under general non-proportional loading paths, with unpredictable changes in the loading directions, as long as the material point stays close to the limit surface.

Conversely, non-radial return mapping is performed by evaluating damage evolution independently from normal and shear laws. As a consequence, radial return mapping can lead to more severe damage. In part II, both radial and non-radial return mapping are adopted and compared.

For instance, assuming radial return mapping and a mode-I damage driving mechanism, the adopted strategy can be summarized in the following:

1. solve the system of equations and evaluate both the total displacements and the jump displacements $\mathbf{u}, \mathbf{w}$;
2. if $f > 0$:
 - i. determine the traction lying on the initial limit surface, $\mathbf{t}^*$, corresponding to the actual position outside the limit surface, $\mathbf{t}^{out}$;
 - ii. define both the modified fracture energy G_F^* and the corresponding law, $t_n = t_n(G_F^*, w_n)$;
 - iii. evaluate d_n from $t_n = t_n(G_F^*, w_s) = (1 - d_n)D_{nn}w_n$;
 - iv. evaluate t_s according to the radial return mapping path (56);
 - v. evaluate material strength parameters f_t and c , according to the predefined surface evolution criterion;
 - vi. update the stiffness matrix and go back to 1, until $f \leq 0$ everywhere.

If radial return mapping and generalized damage driving mechanism are adopted, the following strategy is summarized:

1. solve the system of equations and evaluate both the total displacements and the jump displacements $\mathbf{u}, \mathbf{w}$;
2. if $f > 0$:
 - i. determine the traction lying on the initial limit surface, $\mathbf{t}^*$, corresponding to the actual position outside the limit surface, $\mathbf{t}^{out}$;
 - ii. define the corresponding modified fracture energies under both mode-I and mode-II fracture, G_F^* , G_F^{II*} and reference laws $\mathbf{t}^r - \mathbf{w}^r$;
 - iii. evaluate d_n, d_s from the reference laws;
 - iv. evaluate the traction $\mathbf{t} = \alpha \mathbf{t}^*$ taking α according to (25);
 - v. define the new surface and material strength parameters f_t and c corresponding to $\mathbf{t}$;
 - vi. update the stiffness matrix and go back to 1, until $f \leq 0$ everywhere.

Note that, although homotetic evolution of the limit surface is adopted in this work, the model also allows for non-homotetic evolution. In fact, the return mapping criterion is independent of the surface evolution criterion, i.e., it is possible to enforce radial return mapping and non-homotetic surface evolution, such as, for instance, in the case of no shear damage evolution (Alfaite and Sluys 2017a).

In Figs. 27a to 29b, the traction paths and the traction-jump displacement curves corresponding to the non-proportional loading example shown in Fig. 26 are presented.

For non-proportional loading, it is assumed that the load factor is applied to the last loading case only, as described in Section 5.1.

In this example, the second load is applied adopting two different procedures:

- i) constant shear traction return mapping for a mode-I driving mechanism, leading to the traction path marked in orange in Fig. 26;
- ii) radial return mapping, leading to the blue traction path in Fig. 26. In this case, the linear weighted function (25) is adopted under the general mixed-mode driving mechanism.

In the former case load control is enforced, whereas in the second case displacement control is enforced such that, in the second loading, $\dot{w}_n > 0$, $\dot{w}_s = 0$. In both cases the control loading function is the monotonic variation of normal displacement jump.

Figs. 27a and 27b correspond to case i) and Figs. 28a to 29b to correspond case ii). In both cases, the first traction state is obtained elastically, $t_n = 0.5$ MPa and $t_s = 6.0$ MPa. From that point on, the results differ significantly. Upon enforcing the second load, the initial limit surface is reached with $\mathbf{t}^*$, which depends on the return mapping criterion adopted.

For load control, only the normal traction decreases, since t_s is kept constant. Although non-isotropic damage is enforced, a homotetic surface evolution criterion is adopted, with the final limit surface represented in Fig. 27a.

In case ii), both the normal traction and the shear traction vary under radial return mapping. First, the normal traction increases significantly to $t_n = 2.59$ MPa, with a decrease of the shear traction. In fact, the initial traction path lies close to the initial limit surface, almost as if neutral loading is taking place ($\dot{f} \approx 0$). Next, both t_n and t_s decrease to zero. An intermediate limit surface corresponding to $t_n = 0.93$ MPa is presented, with the final limit surface contracting to the origin.

In Fig. 28b the radial return mapping procedure is illustrated with larger step sizes than the ones adopted in Fig. 28a. At two specific points ($t_n = 2.42$, $t_s = 2.04$) and ($t_n = 2.27$, $t_s = 0.166$), the load factor was increased even further, only to show that return mapping is always performed to the current limit surface, independently of the step size.

5.4 Damage driving mechanism under corrosion

The transition laws defined in Section 4.2 for corrosion are examples of driving relationships under mode-II fracture. However, these laws are taken from experimental pullout tests, under $t_n = 0$. Thus, they also

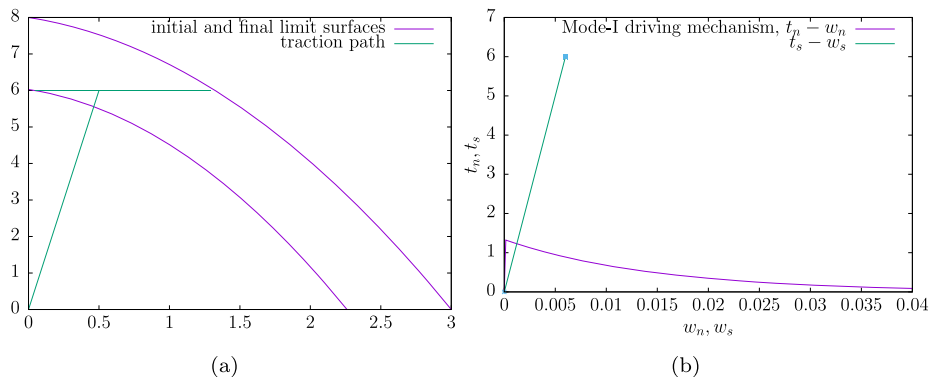


Fig. 27 Load control under constant t_s (a) Traction path and limit surfaces, (b) Traction-jump displacement curves

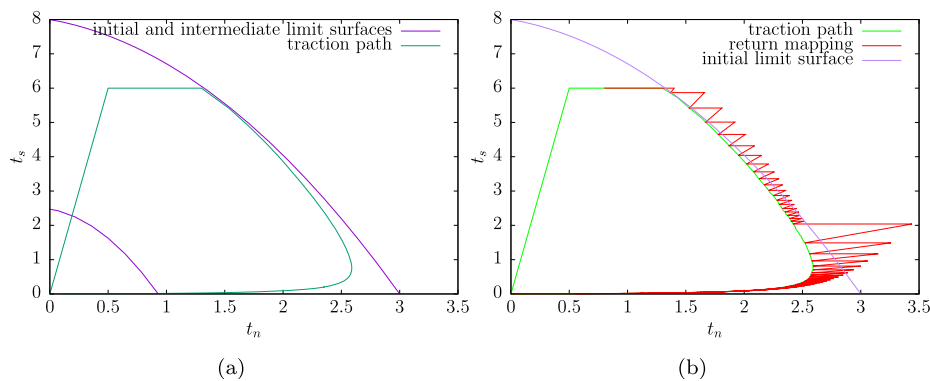


Fig. 28 (a) Traction path and limit surfaces for displacement control (constant w_s), (b) Return mapping scheme corresponding to (a)

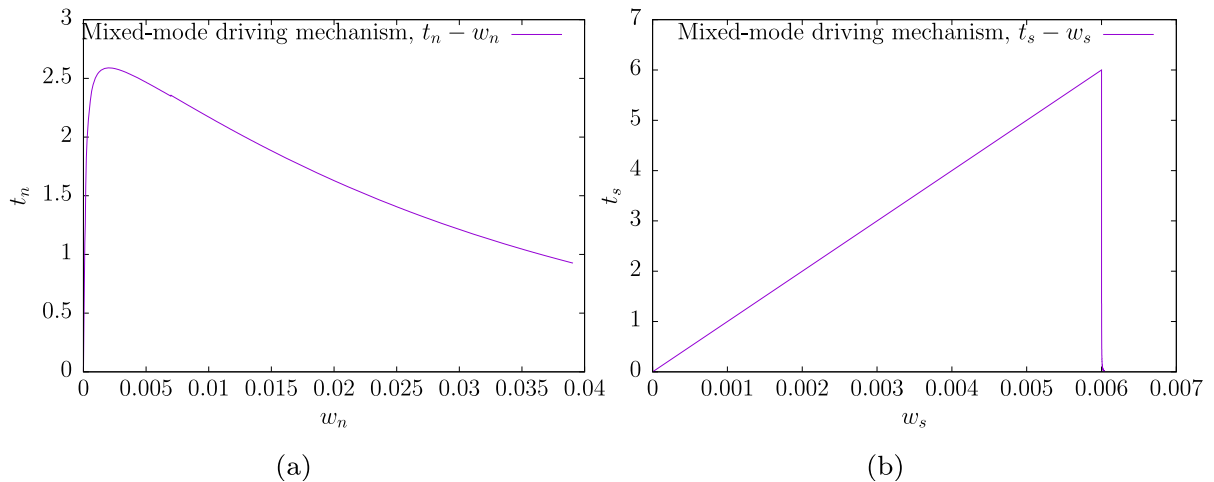


Fig. 29 Traction-jump displacement normal (a) and shear curve (b) for displacement control (constant w_s)

need to be adjusted for mixed-mode fracture or mode-II under compression. This is done according to the procedure described in Section 3.2, with the transition bond-slip law defined by

$$t_{s|t}(w_s) = \hat{t}_{s0}(w_s) + [t_{sf}(w_s) - \hat{t}_{s0}(w_s)] \times \frac{\eta_t}{\eta_f}. \quad (57)$$

In (57),

$$\begin{aligned} \hat{t}_{s0} &= t_{s0} \frac{t_s^*}{c_0}, \\ \hat{t}_{sf} &= t_{sf} \frac{t_s^*}{c_0}, \end{aligned} \quad (58)$$

where t_s^* is the bond traction on the initial limit surface. As a rule, c_0 and $\hat{t}_{s0}$ lie on functions $t_{s0}(w_s)$ and $t_s^*(w_s)$, respectively (see examples in [Alfaite et al. \(2023\)](#)). Thus, in this case, t_s^* also depends on the corrosion level η :

$$t_s^* = t_s^*(f, \eta). \quad (59)$$

Regarding the control of energy dissipation, adopting the linear relation (19) for G_F^{II*} we have

$$w_{s,n} = \left[\frac{w_{su} - w_{s0} \frac{t_s^*}{c_0}}{w_{su} - w_{s0}} \right] w_{s,n} + w_{su} \left(1 - \left[\frac{w_{su} - w_{s0} \frac{t_s^*}{c_0}}{w_{su} - w_{s0}} \right] \right), \quad (60)$$

whereas in case of the quadratic variation of G_F^{II*} (17), we get

$$w_{s,n} = w_{s,n} \frac{t_s^*}{c_0}. \quad (61)$$

In Equations (60) and (61), $w_{s,n}$ is the abscissa of the n^{th} point of the multilinear relationships corresponding to t_{s0} and t_{sf} (see Fig. 21) and $w_{s,n}$ is the abscissa of the n^{th} point of the multilinear relationships corresponding to $\hat{t}_{s0}$ and $\hat{t}_{sf}$. From Equation (60), we obtain (see Fig. 8),

$$\begin{aligned} w_{s,0} &= w_{s,0} \frac{t_s^*}{c_0}, \\ w_{s,u} &= w_{s,u}. \end{aligned} \quad (62)$$

Assume that a control loading function given by (52) is adopted, corresponding to the monotonic increase of corrosion. At each iteration j of each time step t , the transition law in each material point m is defined

according to (57). The damage d_s^j is increased and the corresponding stiffness is defined. If radial return mapping is adopted, the procedure introduced in Section 5.3 is followed.

Finally, the iterative procedure is summarized in Table 1. In Table 2, the possible combinations among fracture mode, traction on initial limit surface, damage driving mechanism and return mapping are listed. The three groups are defined according to the priority criterion marked in red. For instance, in the second group, if the generalized damage driving mechanism is adopted as priority criterion (iii), the admissible combinations are: mixed-mode fracture (4), traction $\mathbf{t}^*$ defined according to radial return mapping (t3), linear or quadratic variation of the fracture energy (e1, e2) and radial return mapping (a).

In Table 1.d. (no prescribed return mapping), the traction state on the initial surface is defined such that damage evolution is obtained independently from the normal and the shear law.

5.5 Spalling

As previously mentioned in Section 4.3, the volume expansion due to corrosion gives rise to an additional strain ε^{corr} , which is enforced by means of an equivalent temperature increase. Since the iterative procedure is based upon a total approach, for each iteration the system of equations is solved twice, for both the accumulated load $\mathbf{F}_1$ from previous loading cases and for the actual load $\mathbf{F}_2$ (see Section 5.1). Due to the reduction of the effective cross section of the steel bar, the equivalent nodal forces necessary to enforce ε^{corr} , which are obtained by integration of the steel elements, must be corrected in $\mathbf{F}_2$. Moreover, these equivalent thermal nodal forces must also be corrected in $\mathbf{F}_1$, whenever the iterative procedure leads to the nonlinear behaviour of steel or concrete, for instance due to the introduction of the plastic regime. As a consequence, the thermal equivalent nodal forces are reevaluated at each iteration and corrected in either $\mathbf{F}_2$, during the evolution of corrosion with core reduction, or in $\mathbf{F}_1$, due to nonlinear bulk behaviour.

6 Final remarks

In this work a new damage model for cohesive fracture and corrosion is introduced. This model allows for the

Table 1 Summary of iterative procedure involving fracture mode, traction on the initial limit surface, energy dissipation, damage driving mechanism and return mapping

1.	MODE-I fracture: pure mode-I fracture, keeping the initial tensile strength f_{t0} and fracture energy G_F unchanged. Dependence on the shear traction is neglected;
2.	MODE-II fracture: pure mode-II fracture, keeping the initial cohesion c_0 and fracture energy G_F^{II} unchanged. Dependence on the normal traction is neglected ;
3.	MODE-II fracture under compression ($t_n \leq 0$): dependence on the limit surface is taken into account;
4.	MIXED-MODE fracture ($t_n \geq 0$): dependence on the limit surface is taken into account.
TRACTION ON INITIAL LIMIT SURFACE:	
t1.	traction $\mathbf{t}^*$ on constant t_n load path
t2.	traction $\mathbf{t}^*$ on constant t_s load path;
t3.	traction $\mathbf{t}^*$ defined according to radial return mapping;
t4.	traction $\mathbf{t}^*$ is defined according to no prescribed return mapping;
ENERGY DISSIPATION:	
e1.	linear variation of fracture energy:
$G_F^* = G_F (t_n^*/f_{t0}), \quad G_F^{II*} = G_F^{II} (t_s^*/c_0);$	
e2.	quadratic variation of fracture energy:
$G_F^* = G_F (t_n^*/f_{t0})^2, \quad G_F^{II*} = G_F^{II} (t_s^*/c_0)^2.$	
DAMAGE DRIVING MECHANISM:	
i.	mode-I - damage evolution is defined according to the modified fracture energy value G_F^* and the corresponding normal relationship;
ii.	mode-II - damage evolution is defined according to the modified mode-II fracture energy value G_F^{II*} and the corresponding shear relationship;
iii.	general mixed-mode - both fracture energies are modified to G_F^* and G_F^{II*} , and the final tractions are obtained with the linear or sigmoid weight functions;
iv.	damage driving mechanism under corrosion - damage (and stiffness) is evaluated from the transition mode between evolving corrosion states.
RETURN MAPPING:	
a.	radial - both normal and shear tractions are decreased proportionally with the same reduction factor. In case of the mode-I driving mechanism, the shear traction t_s is reduced after defining $t_n(t_n^*, w_n, G_F^*)$, whereas in case of the mode-II driving mechanism, the normal traction t_n is reduced after defining $t_s(t_s^*, w_s, G_F^{II*})$. In case of mixed-mode driving mechanism both traction components are reduced according to the adopted weight function;
b.	constant t_n - return mapping is performed keeping the normal traction constant;
c.	constant t_s - return mapping is performed keeping the shear traction constant;
d.	no prescribed return mapping - damage evolution is obtained independently from the normal and the shear law.

Table 2 Possible combinations of fracture mode, traction on the initial limit surface, energy dissipation, damage driving mechanism and return mapping. Three groups defined according to the

priority criterion marked in red: fracture mode, damage driving mechanism and return mapping

FRACT. MODE	TRACTION t^*	ENERGY DISS.	DAM. DRIV. MECH.	RETURN MAP.
1	-	-	i	-
2	-	-	ii, iv	-
3	t1, t3	e1, e2	ii, iv	a, b
4	t1, t2, t3, t4	e1, e2	i, ii, iii, iv	a, b, c
1, 4	t2	e1, e2	i	a, c, d
2, 3, 4	t1	e1, e2	ii	a, b, d
4	t3	e1, e2	iii	a
2, 3, 4	t1, t3, t4	e1, e2	iv	a, b, d
3, 4	t3	e1, e2	i, ii, iii, iv	a
3, 4	t1	e1, e2	ii	b
4	t2	e1, e2	i	c
1, 2	t4	e1, e2	i, ii, iv	d

simulation of the softening behaviour of concrete, due to fracture and the bond-slip relationship between steel and concrete.

Special focus is paid to the control of energy dissipation under mixed-mode fracture and mode-II fracture under compression. This is achieved by integration fracture mechanics concepts into the damage mechanics model, namely by:

1. introducing the new concept of *damage driving mechanism*, from which the damage evolution law is derived, for:
 - (a) a mode-I fracture driving mechanism,
 - (b) a mode-II fracture driving mechanism,
 - (c) a generalized mixed-mode fracture driving mechanism, or
 - (d) a corrosion-induced damage driving mechanism,
2. defining reference traction-jump displacement laws under mixed-mode fracture as well as mode-II fracture under compression;
3. determining the damage variables from the reference traction-jump displacement laws and not from uniaxial tension and shear relationships.

As a consequence, a pure traction based damage model for cohesive fracture is obtained: a limit surface is defined in the traction space and all internal variables are obtained from the limit surface and the traction laws, without the need to introduce the usual kinematic variables.

Special attention is paid to the modelling of concrete spalling, which is performed in two ways: i) by enforcing the volume expansion of the reinforcement bars and ii) by a new method introducing splitting forces at the steel concrete interface. In the latter case, both

the detachment of the concrete from the reinforcement steel and the damage at the steel concrete interface due to spalling, are captured in a 2D simulation, in a more realistic way.

The model is implemented in a finite element program making use of a total approach and an iterative procedure. The iterative algorithm defined in the work presented in [Alfaite and Sluys \(2023\)](#) is modified, taking into account the new aspects introduced in the damage model, and by introducing return mapping. Illustrative examples at local level are presented. In the work presented in part II, numerical examples of reinforced concrete beams under corrosion are presented.

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All authors reviewed the manuscript

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Declarations

Competing interests The authors declare no competing interests.

A Summary of Damage Model introduced in [Alfaite et al. \(2023\)](#)

In Figure 1, t_n and t_s are the normal and shear traction components, respectively, f_{t0} is the initial tensile strength, f_t is the tensile strength under damage evolution, c_0 is the initial cohesion and c is the cohesion under damage evolution. Moreover, f_1 is the limit surface under mixed-mode, i.e., $t_n > 0$, whereas f_2 corre-

sponds to the limit surface under compression, $t_n < 0$. In this modified Mohr-Coulomb surface, the maximum shear value $t_{s,max}$, defines a horizontal plateau such that $|t_s| = t_{s,max}$ for $t_n \leq t_{n,lim}$. Furthermore, this surface evolves towards the origin according to a damage evolution law, induced by softening behaviour. In Figure 1, homotetic evolution of the limit surface is represented, such that

$$c/c_0 = f_t/f_{t0}. \quad (65)$$

The modified Mohr-Coulomb surface f is defined by:

$$f = 0 \Leftrightarrow \begin{cases} |t_s| = \frac{f_t \tan(\phi) - c}{f_t^2} t_n^2 - \tan(\phi) t_n + c, & \text{if } t_n > 0 \quad (f_1) \\ |t_s| = \frac{\tan(\phi)}{2t_{n,lim}} t_n^2 - \tan(\phi) t_n + c, & \text{if } t_{n,lim} \leq t_n \leq 0 \quad (f_2) \\ |t_s| = t_s(t_{n,lim}) = t_{s,max}, & \text{if } t_n < t_{n,lim} \quad (f_2). \end{cases} \quad (66)$$

In equations (66), ϕ is the internal friction angle of the discontinuity (see Fig. 1).

The following relationship is adopted:

$$\begin{aligned} t_n &= (1 - d_n) D_{nn} w_n \\ t_s &= (1 - d_s) D_{ss} w_s, \end{aligned} \quad (67)$$

where w_n is the normal jump displacement, w_s is the sliding jump displacement or slip displacement, d_n is the damage under normal traction, d_s is the damage variable under bond-slip and D_{nn} , D_{ss} are the normal and shear elastic stiffness coefficients, respectively, corresponding to the diagonal components of the elastic constitutive tensor at discontinuity Γ_d , $\mathbf{D}_{\Gamma_d}^{el}$.

The limit surface (Fig. 1) is dependent on material strength parameters, namely:

$$f_t = f_t(g_n(G_F, \kappa_n)) = g_n(G_F, \kappa_n) D_{nn} \kappa_n, \quad (68)$$

$$c = c(g_s(G_F^{II}, \kappa_s)) = g_s(G_F^{II}, \kappa_s) D_{ss} \kappa_s, \quad (69)$$

where f_t is dependent on the fracture energy G_F and a scalar variable κ_n , c is dependent on the mode-II fracture energy G_F^{II} and a scalar variable κ_s and g_n and g_s are damage evolution laws under mode-I and mode-II, respectively. In the works presented in Alfaiate and Sluys (2017a), Alfaiate and Sluys (2017b), Alfaiate and

Sluys (2019), Alfaiate et al. (2023), it is assumed that:

$$g_n = 1 - d_n = \frac{f_t}{D_{nn} \kappa_n}, \quad \text{under mode-I fracture} \quad (70)$$

$$g_s = 1 - d_s = \frac{c}{D_{ss} \kappa_s}, \quad \text{under mode-II fracture} \quad (71)$$

The scalar variables κ_n and κ_s are defined such that: $\kappa_n = w_n$ under pure mode-I fracture and $\kappa_s = |w_s|$ under pure mode-II fracture.

Mixed-mode fracture is defined under $w_n > 0$, whereas mode-II fracture due to compressive states is defined under $w_n = 0 \wedge t_n < 0$.

Different d_n^+ and d_n^- damage variables are defined, for tensile normal traction and for compressive normal traction, respectively.

Both the Kuhn-Tucker conditions and the consistency condition are satisfied:

$$\dot{d}_n \geq 0, \quad \dot{d}_s \geq 0, \quad f \leq 0, \quad \dot{d}_n f = 0, \quad \dot{d}_s f = 0, \quad \dot{d}_n \dot{f} = 0. \quad (72)$$

B Non-homotetic evolution of the limit surface adopting a variable friction coefficient

Recall the non-proportional example presented in Section 2.2.1. In order to allow for inequality $|\Delta t_s| > |\Delta c|$, as depicted in Fig. 3, a decreasing friction coefficient with increasing damage can be introduced. Assume, for the sake of simplicity, that: $t_n = t_{n,lim} \Rightarrow |t_s^*| = t_{s,lim}$. Under $d_s = 0$ we get:

$$t_{s,lim} = \frac{\tan(\phi_0)}{2} t_{n,lim} - \tan(\phi_0) t_{n,lim} + c_0, \quad (73)$$

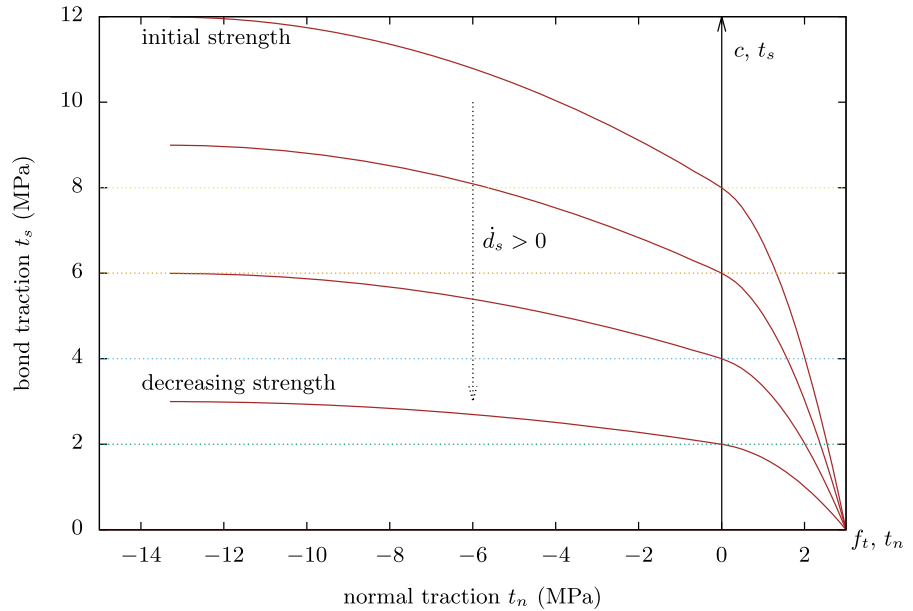
where ϕ_0 is the *initial* friction coefficient angle. Equation (73) can be written as:

$$\tan(\phi_0) = -2 \times \frac{t_{s,lim} - c_0}{t_{n,lim}}. \quad (74)$$

Upon damage evolution we get:

$$\begin{aligned} |t_s| &= \frac{\tan(\phi)}{2} t_{n,lim} - \tan(\phi) t_{n,lim} + c \Leftrightarrow \tan(\phi) \\ &= -2 \times \frac{|t_s| - c}{t_{n,lim}}. \end{aligned} \quad (75)$$

Fig. 30 Example of non-homotetic evolution of the limit surface allowing $\Delta c < \Delta t_s$ according to Fig. 3, with variable friction angle and constant tensile strength



Assume the (non-homotetic) particular case illustrated in Fig. 3, in which the traction on the surface remains proportional to the cohesion, i.e.,

$$\frac{|t_s|}{c} = \frac{t_s^*}{c_0}, \quad (76)$$

we get

$$\tan(\phi) = \frac{c}{c_0} \tan(\phi_0).^8 \quad (77)$$

Under mixed-mode fracture ($t_n > 0$), we have:

$$|t_s^*| = \frac{f_{t0} \tan \phi_0 - c_0}{f_{t0}^2} t_n^2 - \tan(\phi_0) t_n + c_0, \quad (78)$$

where $|t_s^*|$ is the shear traction on the initial surface ($d_n = d_s = 0$), and

$$|t_s| = \frac{f_t \tan \phi - c}{f_t^2} t_n^2 - \tan(\phi) t_n + c. \quad (79)$$

Assuming similar conditions as above for mode-II under compression:

- t_n constant,
- (77),

⁸ Note that equation (77) still holds for $t_s^* \neq t_{s,lim}$.

$$\cdot |t_s|/c = |t_s^*|/c_0,$$

we get

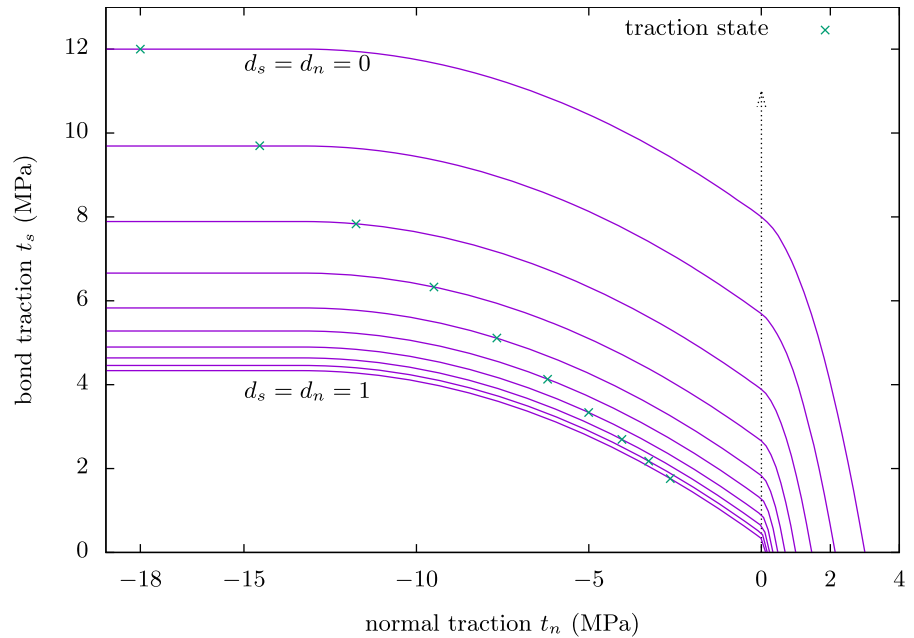
$$\cdot f_t = f_{t0}.$$

In Fig. 30, the corresponding representation of the evolution of the limit surface is depicted ($f_{t0} = 3$ MPa, $c_0 = 8$ MPa, $\tan(\phi_0) = 0.6$, $t_{n,lim} = -13.3$ MPa, $t_{s,lim} = 12$ MPa).

C More on energy dissipation under generalized mixed mode driving mechanism

For example, in Figs. 14, we get $G_F^{II*} = 0.78$ N/mm ($G_F^{II*}/G_F^{II} = 0.78$) and $G_F^* = 8.4 \times 10^{-3}$ N/mm ($G_F^*/G_F = 8.4 \times 10^{-2}$). Under mode II driving mechanism, the area below the $t_s - w_s$ curve is equal to G_F^{II*} , whereas the area under the $t_n - w_n$ curve is equal to 1.22×10^{-3} N/mm ($0.15 G_F^*$). Under mode I driving mechanism, we obtain the area under the $t_n - w_n$ curve equal to $G_F^* = 8.4 \times 10^{-3}$ N/mm, whereas the area under the $t_s - w_s$ curve is equal to 5.433 N/mm, ($7 G_F^{II*}$). Under generalized mixed mode driving mechanism using the sigmoid function, we obtain the area under the $t_n - w_n$ curve equal to 1.95×10^{-3} N/mm ($0.23 G_F^*$) and the area under the $t_s - w_s$ curve equal to 1.25 N/mm ($1.6 G_F^{II*}$). It is clear that, in this case, adequate dissipation of energy is obtained from

Fig. 31 Evolution of the traction state and the limit surface under $t_n = -1.5|t_s|$.



the generalized mixed mode driving mechanism, followed by mode II driving mechanism, whereas mode I damage driving mechanism is totally inappropriate. Similar conclusions can be obtained for Figs. 12, where mode II driving mechanism deviates considerably from the right solution. Finally, in the intermediate case, it is clear that only the generalized mixed mode driving mechanism provides an appropriate dissipation of energy.

D Examples of homotetic evolution under generalized mixed-mode driving mechanism

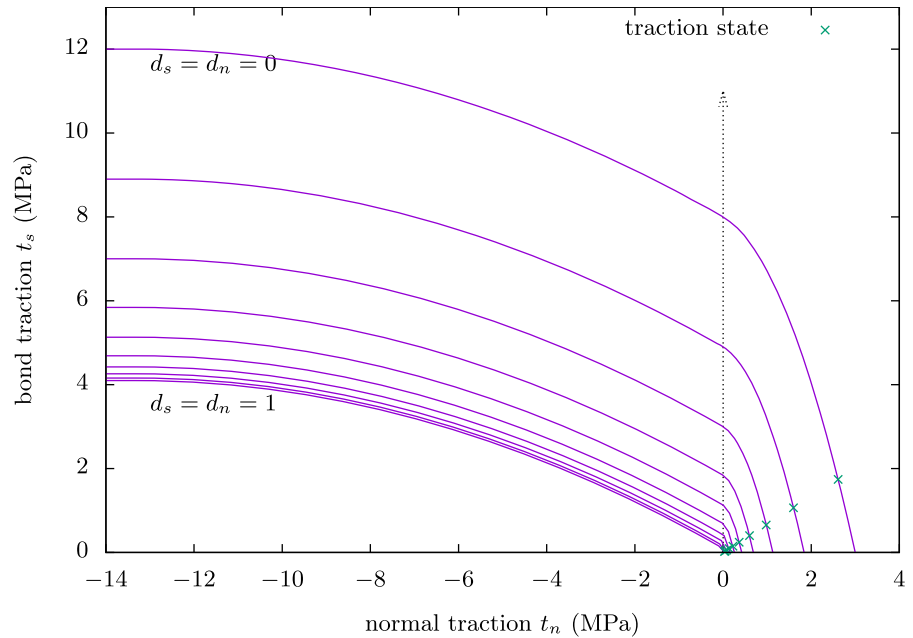
In Figs. 31 and 32 the homotetic evolution of the limit surface and of the corresponding traction state is shown,

under generalized mixed-mode driving mechanism, for two different loading cases: $t_n = -1.5|t_s|$ and $t_n = 1.5|t_s|$, respectively. In the former case ($d_n^- = 0, \dot{d}_s > 0$), the traction state on the initial limit surface lies on the plateau corresponding to $|t_s| = t_{s,lim}$. In the second case, $\dot{d}_n^+ > 0 \wedge \dot{d}_s > 0$.

The corresponding traction values are presented in the Table below.

traction state (x) in Fig. 31		traction state (x) in Fig. 32	
t_n	t_s	t_n	t_s (MPa)
-18.00	12.00	2.61	1.74
-14.54	9.70	1.60	1.07
-11.75	7.83	0.98	0.65
-9.49	6.33	0.60	0.40
-7.67	5.11	0.37	0.24
-6.20	4.13	0.22	0.15
-5.00	3.34	0.14	0.09
-4.04	2.70	0.08	0.06
-3.27	2.18	0.05	0.03
-2.64	1.76	0.03	0.02

Fig. 32 Evolution of the traction state and the limit surface under $t_n = 1.5|t_s|$



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