The Wednesday Afternoon Adventure Club

A toolkit that supports skilled 3rd and 4th grade mathematicians in developing their spatial ability with the help of design and inquiry-based learning

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"Children learn as they play. Most importantly,
in play children learn how to learn."

O. Fred Donaldson
(researcher on play for children)

Ilustration from exercise book 3rd grade (Malmberg, n.d.-a)

Preface

When I started the search for a graduation project, around the end of last summer, I did not have a perfect idea of my ideal project. However, one thing I knew for sure: I wanted to design something for children and preferably do a project that allowed me to interact with children and learn from them. Throughout my IDE career, the projects I did for children were the most inspiring and enjoyable for me, but interaction was often hard, due to covid or not having the right connections.

The word 'enjoyable' felt like a common theme throughout this project, and I will therefore highlight it in this preface and acknowledgements as well.

At the end of summer, a friend of mine introduced me to ScienHub, a company that is interested in sustainable, innovative ideas and works for and with children. After the first meeting I was given two possible graduation project, and a little later, a third. When I read the third one I felt a spark of excitement. Where I couldn't describe my perfect graduation project before, this came really close. 'Design a product to stimulate spatial ability for primary school children'. I loved mathematics as a primary school student myself and knew that a lot of children did (and do) not. My initial motivation for this project was therefore to create the same excitement I had growing up for the subject, for a new group of primary schoolers.

Throughout this project, I had the chance to visit primary schools in Delft to observe mathematics classes and later on, to conduct tests with children. These moments were what made the project most special for me. Every time I came to school I was a little nervous: would I be able to manage an entire group of 3rd and 4th graders? Will they understand my project? Will they like what I designed? What if they notice it's not perfect and will find it too boring to even interact with it? I know children can be so honest and will not hide if they do not like what they are presented. But then I reminded myself that that was exactly the reason why I wanted to design for children. Honest reactions, real excitement, and if it does not fulfil the needs: honest resentment. And the children were honest, direct even, but also honestly excited and so intelligent, they would even give me tips on how to improve my design on a pratical/technical level.

I hope this report somehow visualises and conveys the joy I had throughout the past six months. Because the goal to 'make mathematics more fun' resulted in making this project quite fun, too.

That is why, I hope you all enjoy reading it as well,

Roosje

Illustration from exercise book 3rd grade (Malmberg, n.d.-a)

Acknowledgements

Although this graduation project was individual, there are many people who helped me through the process, in various ways.

First of all, I would like to thank my client supervisor, Leonie for all the help she provided during this project and the opportunities I was given from ScienceHub to follow my own wishes during this project within the brief they provided. I was really lucky to have an IDE graduate as supervisor who understood all the design struggles I faced and had interesting ideas and tips throughout the project. Without the connections of ScienceHub, it would have been quite hard to get in contact with schools.

Next, I would like to thank my IDE supervisors, Mathieu and Mark, both of whom were excited about this project from the start and I think the excitement we shared, for designing for children and wanting to create something fun for them, is what made all the meetings enjoyable as well. I could share my excitement about the observations and tests I did and you encouraged me to think outside of the box and listen to my own design view. Thank you for the pleasent communication, insightful feedback and positive attitude towards my project and my own wishes throughout.

Furthermore, I would like to thank the schools I was able to observe and test at, especially the three teachers of the classes I joined. They took the time during their hectical mathematics mornings to help me understand their methods, systems and experiences as well as possible. I gained even more respect than I already had for you.

And to all the children who helped me by sharing their experiences and stories, tested my product and gave their honest feedback: thank you, you have all contributed to the final design and inspired me every time I visited. Your excitement made me realise that I would love to keep designing for children in the future, since you really are, the most honest target group.

Also, a huge gratitude to all the experts who took the time to help and inspire me with their knowledge and tips. It was insighful and inspiring to hear about the world of primary school education and get your different perspectives on it. Seeing everyones excitement to create the best learning experiences for children is what I would love to contribute to one day, too.

Lastly, I would like to thank my friends and family for the support during this project. Whether it was through practical help, quick feedback sessions, distraction through (short) breaks, studying alongside each other for hours or just hearing about my excitement after each time I came back from school: you too have made these six months the most enjoyable. There are many to thank but especially my close IDE friends who sat by my side throughout this entire project: Luna, Usha and Mirjam. And Pauline and Frédérique, who share my interest in designing for children and have often helped by sharing insights and helping me during the tests. To all of you and more, thank you for the previous seven years at IDE, they truly were the best, both inside and outside of our beloved faculty. To my parents and brother, thank you for the support, practical help and guidance during this project and throughout the past seven years. Thank you for inspiring and encouraging me to go to Delft.

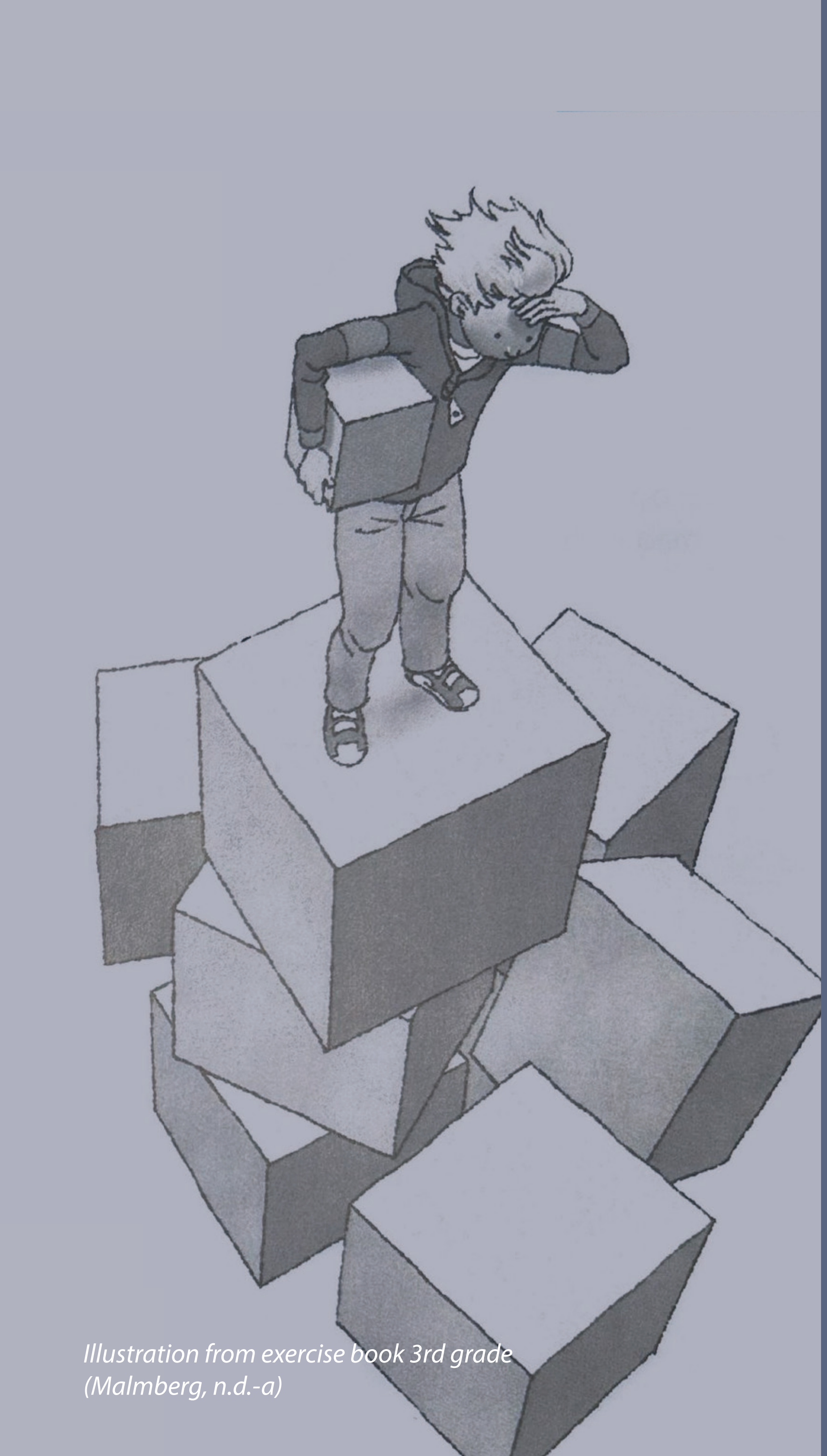


Illustration from exercise book 3rd grade (Malmberg, n.d.-a)

Reading guide

Mostly, this report is written like this, in black text on a white background, however two different styles have been used to highlight specific insights.

Requirement or Wish:

Throughout this report, research is done in order to understand the topics of primary school education, spatial ability, and children's needs in order to design a fitting product for this market. At each point where research formed an important insight that needs to be included in the designed product, a requirement or wish is formed. These requirements are highlighted throughout the research chapters of this report. Before the design chapters start, they are summarised into a list of requirements and wishes. Throughout the explanation of the final design in Chapter 6, they are mentioned again if the final design meets that requirement or wish.

At the end of most chapters, a page is marked in this blue colour. This part of the chapter states a conclusion or highlights the most important insights, that will be taken to the next part of the design phase.

How the research, design phase and outcomes are distributed per chapter is explained in a visual in Chapter 2.2: Project planning.

Illustration from exercise book 3rd grade (Malmberg, n.d.-a)

Abstract

The Netherlands is known to score well on guiding children to achieve a fundamental level in mathematics. However, they could do better in guiding the children who already score well, so called skilled mathematicians. That is why this project focused on designing a tool that would support this group of children, in the 3rd and 4th grade in learning spatial ability. This project was done in collaboration with Sciencehub TU Delft and the design goal was:

Design a tool that makes use of design and inquiry based learning in mathematics classes to improve the development of children's spatial ability for skilled mathematicians at primary school in grades 3 and 4 (groep 5 en 6) in the Netherlands.

The methodologies that were used throughout this project to achieve this design goal, were:

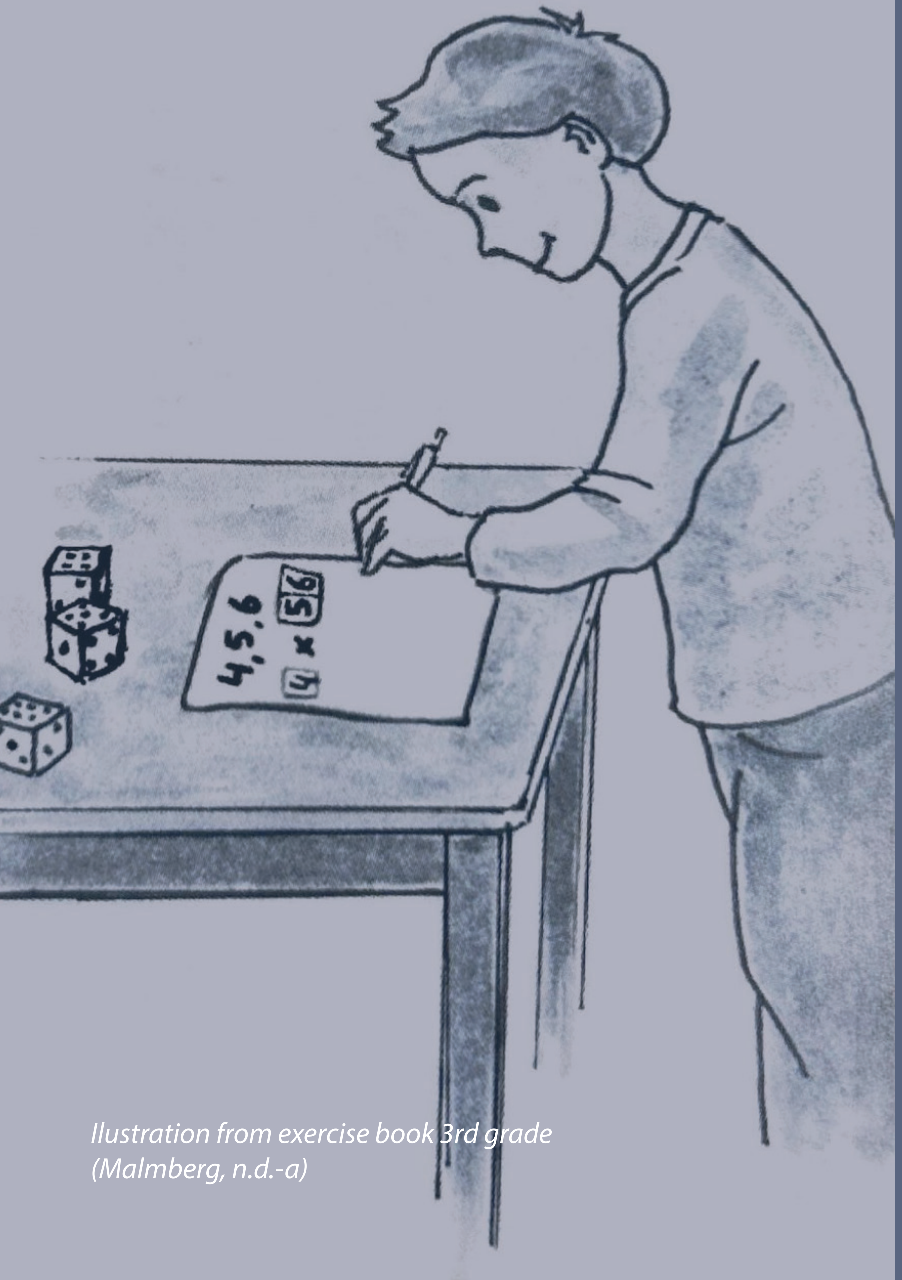
- Literature research on the fields of mathematics and spatial ability, primary school education, design- and inquiry based learning and skilled mathematicians.
- Observations during mathematics classes in the 3rd and 4th grade, both during the 'normal' classes as well as during skilled mathematician hours.
- Meetings with experts on the fields of mathematics education, spatial ability, education design and skilled mathematicians.
- Research through design and prototyping
- User tests with 3rd and 4th grade, skilled mathematicians at school.

Some of the most important findings throughout the research phase were:

- There are three types of skilled mathematicians: good, quick and creative ones. They share needs in order to be guided sufficiently, but also have their own. Furthermore there are potential skilled mathematicians, who have great potential to become skilled mathamaticians but need sufficient guidance to do so.
- Skilled mathematicians are sometimes not supported well because the education system or school has insufficient time or knowledge on how to do so. They often end up separated as a group with extra work, without instructions, guidance or interaction and reflection possibilities.
- Spatial ability can be divided into five skills that are tested at primary school: visualisation, rotation, orientation, symmetry and spatial relations. However these skills, and the word 'spatial ability' is not mentioned, nor known by all teachers, let alone children.
- Spatial ability is often tested with 2D materials and exercises, while it could be better understood if learned though physical, 3D exercises and experiences.
- Spatial exercises are sometimes not realistic or relatable, missing the possibility to let children connect to spatial situations in real life.
- Children are better of learning with an aspect of play.

The product that was designed is the Wednesday Afternoon Adventure Club toolkit. This toolkit consists of a booklet with exercises and additional physical, 3D, map, measuring tiles and characters. The booklet includes a comic story describing a story about five characters and throughout the booklet these characters phase 'issues' that result in mathematical, spatial exercises. The physical map is a direct translation of the world that the characters live in and so the story, and exercises can be directly followed, physically. The characters are based on the five spatial skills mentioned above and characterise these spatial skills with human aspects, allowing for a feeling of connection to what spatial ability means in real life situations.

The product was tested with the target group throughout different phases in the design and discussed with various experts in the field. These tests and meetings were valuable validations of the potential of the product. Recommendations for future design are provided at the end of this report.



Ilustration from exercise book 3rd grade (Malmberg, n.d.-a)

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(Vereniging van Vrijescholen, n.d.)

Glossary

3rd grade = (NL: groep 5) primary school grade for children of approximately 8-9 years old

4th grade = (NL: groep 6) primary school grade for children of approximately 9-10 years old

EDI-method = (NL: Explicite Directe Instructie) a method some schools in the Netherlands follow where they strictly follow the exercises presented by educational methods.

Fundamental level = the level of mathematical understanding that all students in the Netherlands must achieve at the end of primary school (NL: fundamenteel niveau)

SLO (Stichting Leerplan Ontwikkeling) = National organisation that develops education curricula for primary and secondary education in the Netherlands

Spatial ability = a part of mathematics that focusses on spatial skills, for example: rotation, orientation and visualisation

STEM = Science, Technology, Engineering and Mathematics

STEAM = Science, Technology, Engineering, Arts and Mathematics

Target level = the maximum level of mathematical understanding that students in the Netherlands can achieve at the end of primary school (NL: streefniveau)

Teacher training college = (NL: PABO) ; the secondary education for primary school teachers in the Netherlands

Waldorf school = (NL: Vrije school) ; a school that holds a child-centered approach to learning. Unlike traditional schools, Waldorf education focuses on experiential learning, often through arts, crafts, and nature-based activities, fostering creativity and a love of learning

Introduction

(VolgensBartjens, n.d.)

This chapter will introduce the topic of the project. It represents research done on the fields of mathematics (education), the client and spatial ability. It concludes with the research questions that form the basis for the research presented in the next chapters.

1.1 Mathematics in primary schools

In 2007, 98% of Dutch children reached the lowest level of understanding in mathematics, giving them a basic understanding of the subject. Although this number sounds promising, there are also few children who reach the highest level of understanding: 7%. In comparison, this number is 50% in Singapore. In the Dutch education system for mathematics, a lot of attention is given to under achieving students and guiding them towards a sufficient level in the subject, the country scores in the top 5% of the world of having little underachieving students. However, the amount of high achieving students in mathematics that there are in the Netherlands is quite low (Sjoers, 2025). Therefore, the report of TIMSS-research calls for action on the Netherlands to get more children to reach the advanced level of mathematical understanding (Sjoers, 2018).

In recent years, the numbers of advanced understanding kept decreasing, in 2016 only 4 percent of Dutch children in the 4th grade reached the highest level (geavanceerd niveau) in mathematics. This number was still 12% in 1996. (Mullis et al., 2016). Other numbers show that 93% of primary school children in The Netherlands reached the fundamental level of understanding in mathematics in 2022-2023, only 42% reached the target level. These numbers are significantly lower than those for Dutch language and reading (DUO, 2023).

1.2 From maths class to STEAM

With fewer children reaching a significant level in mathematics, logically the number of children who pursue a technical career path decreases. TU Delft takes interest in improving primary school education (Delta, 2023) with the intention that tackling the understanding of mathematics early on will result in a higher number of children pursuing a technical career. In collaboration with Science Hub TU Delft, the university wishes to improve the subject of spatial thinking because this seems to play an important role in overall technical understanding. Spatial thinking and its importance for technical understanding will be further explained in the next chapter.

TU Delft is part of the sell-STEM project (Spatially Enhanced Learning Linked to STEM), a collaboration between different universities holding the same wishes and conducting research on the relationship between spatial ability and success in STEM careers. Recently, the term STEM is accompanied by STEAM, adding 'arts' to describe student creativity as an important skill that should be highlighted (Morales & Ortiz-Revilla, 2021).

1.2.1 Science Hub TU Delft: the client

Science Hub TU Delft (Wetenschapsknooppunt) is the client for this project. Science Hub is part of the TU Delft and wishes to stimulate children and teachers in creative thinking, design and inquiry. To do so, they develop educational projects in which the design process is central. They have connections with different primary schools in the Netherlands. As mentioned, they are currently interested in improving spatial ability education for primary schools and ideally wish to create a continuous learning program (up to secondary education in mathematics).

WETENSCHAPS
KNOOPPUNTEN
ZUID-HOLLAND
partnership universiteiten
Leiden - Delft - Erasmus

Figure 1: logo of Sciencehub (wetenschapsknooppuntzh, n.d.)

1.3 Spatial ability

Spatial ability is a part of mathematics that is defined in different ways. "The ability to retain, retrieve and transform visual images is defined as spatial ability." (Marunić & Glažar, 2013). Another definition of spatial ability is given by Linn and Peterson (1985): "skill in representing, transforming, generating, and recalling symbolic, non-linguistic information". According to Barnea, three components of spatial ability have been cited: spatial visualisation ("the ability to understand accurately 3D objects through their 2D representation"), spatial orientation ("the ability to imagine what a representation will look like from a different perspective") and spatial relations ("the ability to visualise the effects of operations or to mentally manipulate objects") (2000).

Sufficient spatial ability is useful for different aspects in life, even so for (young) children, some of these aspects in life can be seen in figure 4 on the next page.

Recently, the European Union has recognised the importance of developing spatial thinking and stimulating its development at an early age (Sonneveld et al., 2024). Besides its importance for everyday life, sufficient spatial skills are strongly linked to achievement in mathematics (Hawes et al., 2022) and the chance to pursue STEM subjects and careers (Wai et al., 2009). Halpern even states that spatial understanding is crucial for success in mathematical and science course, which are the most important subjects to be successful at technical jobs (2000).Gilligan-Lee, Hawes & Mix argue that even though spatial thinking is an important aspect for reaching success in STEM fields, it is missing in current mathematical curricula and the need to therefore focus on improving this aspect in mathematical education is high (2022). Figure 1 shows different skills related to mathematical understanding, including spatial ability which again has its own skills.

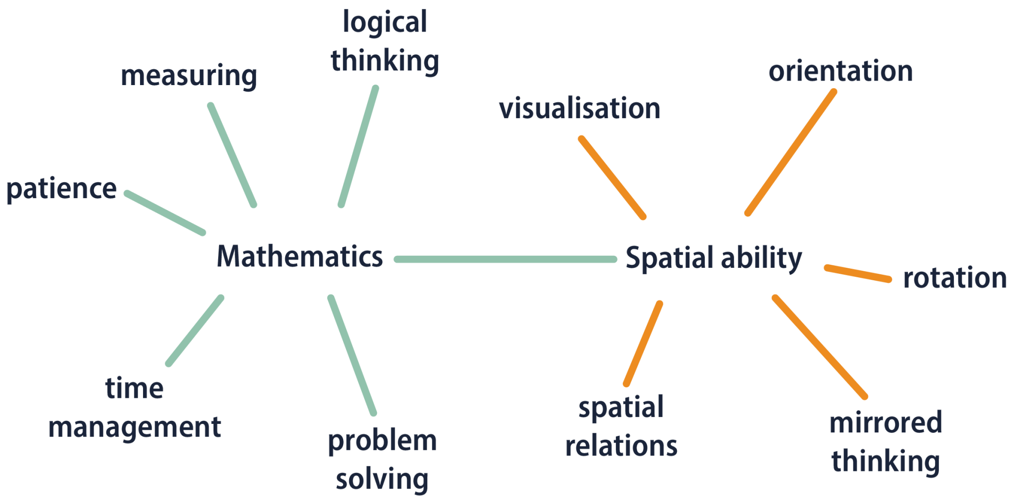
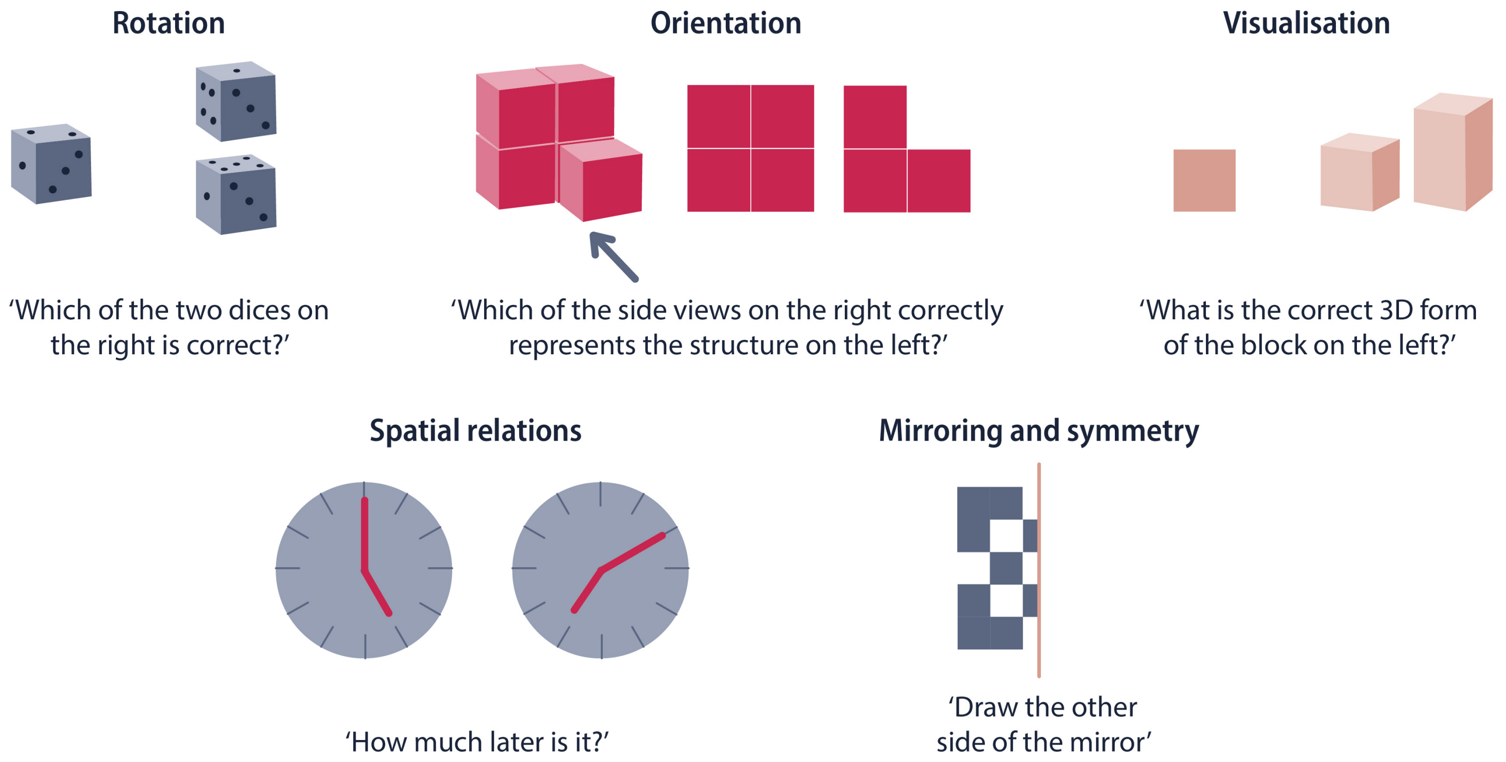


Figure 2: skills linked to mathematics and spatial ability

1.3.1 Development and adaptation of spatial ability

In practice, spatial ability is learned through a combination of exercises in mathematics classes, some examples of exercises of the 3rd and 4th grade are given in figure 3. All of these drawings are simplifications of existing exercises from the book Pluspunt, exercise book for the 3rd - and 4th grade (Pluspunt, n.d.-a), (Pluspunt, n.d.-b).

Requirement: Exercise/tool should improve children's spatial ability and test different spatial skills



Rotation

'Which of the two dices on the right is correct?'

Orientation

'Which of the side views on the right correctly represents the structure on the left?'

Visualisation

'What is the correct 3D form of the block on the left?'

Spatial relations

'How much later is it?'

Mirroring and symmetry

'Draw the other side of the mirror'

Figure 3: example exercises for different aspects of spatial ability

Besides the fact that spatial ability is necessary for different aspects in life, it is also developed through many different activities and tasks. For children, spatial ability can be learned through for example: making puzzles, playing games (f.e. Tetris or chess), making origami, building with LEGO or learning a new instrument. Although spatial ability is needed for many different aspects of life, children do not learn it that way in school nor do they realise this as they develop those skills.

Wish: Exercise/tool should show the importance of spatial ability in real life situations

In primary school, spatial ability is mostly found in the domain of geometry (personal communication, 2025). Three important aspects of spatial ability are rotation, orientation and visualisation. These are also the ones often used to test a child's spatial ability. (Zhu, 2025). Furthermore, Clements discusses how geometry is a critical component of early years mathematics and one that provides plenty of opportunities to develop spatial thinking skills (2004). However, rotation, orientation and visualisation are not the only relevant subjects of spatial ability. Nor is the skill only developed in geometry, it is used and developed in many different ways and aspects of mathematics and other fields, like design (Sonneveld et al., 2024).

Other distinctions that can be made of spatial ability are that of intrinsic/extrinsic and of dynamic/static. Hodgkiss et al. (2018) have examined these components of spatial ability and their relationship to the science fields of biology, chemistry and physics for 7-11 year old children. They concluded that particularly mental folding, spatial scaling and disembedding are predictive of these children's science achievements.

Figure 4: the importance of spatial ability for different aspects in life

1.4 Design goal and research questions

The design goal for this project is to:

Design a tool that makes use of design and inquiry based learning in mathematics classes to improve the development of children's spatial ability for skilled mathematicians at primary school in grades 3 and 4 (groep 5 en 6) in the Netherlands.

The choice to design specifically for skilled mathematicians and for children of the 3rd and 4th grade is further explained in chapter 3.

The research questions to achieve this design goal and to get a better understanding of the scope and subject, are:

RQ1: Which characteristics in children lead to them not achieving their full potential in mathematics because of the current education methods?

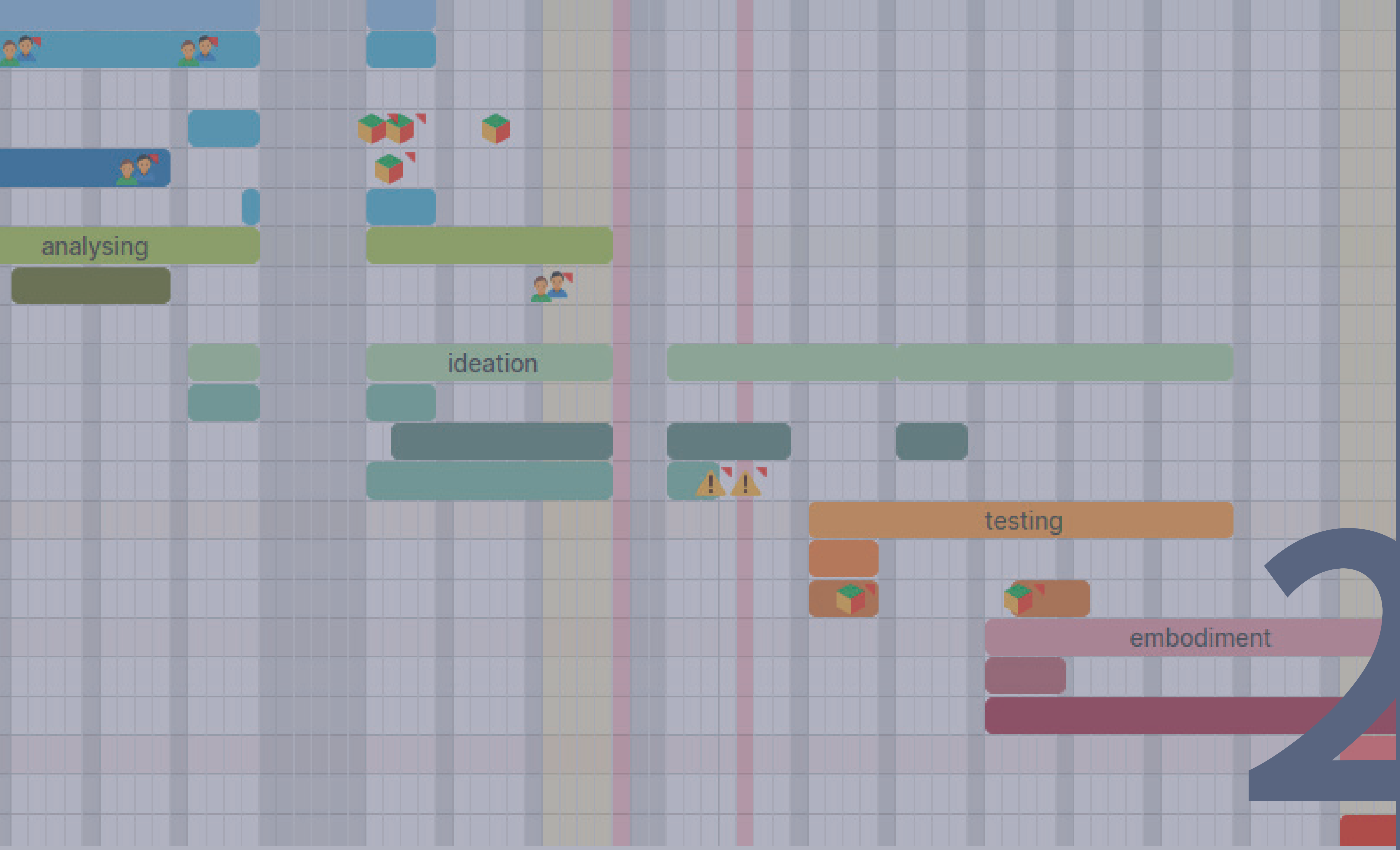
RQ2: Which characteristics of the current mathematics classes and methods lead to children not achieving their full potential and how could these be improved?

RQ3: How can a new tool help to stimulate children to developing spatial skills in a creative and interactive way?

RQ4: What is needed for a good interaction between children and teachers in class in order to let the children achieve their full mathematical potential?

Furthermore, the design question that will be investigated is:

DQ: How would a new tool inspire innovation in the current mathematics classes?



Methodology

2

In this chapter, the methods that were used to conduct research are explained, as well as the plan to answer the research questions and the overall design approach. Lastly, the stakeholders of the project are presented.

2.1 Design- and research methods

For this project the basic design cycle model is used, where after the research phase, five stages are described for a designer to follow when solving a design problem. These stages can be repeated as much as possible or needed. In between each stage extra research is done where needed. The basic design cycle is shown in figure 5 (Van Boeijen & Daalhuizen, 2013).

In the Delft Design Guide the cycle starts with values, needs and functions followed by 'Analyse' and the rest of the circle. For my process I replaced 'values, needs and functions' by 'Initial Research' since that is what my analysis is based on.

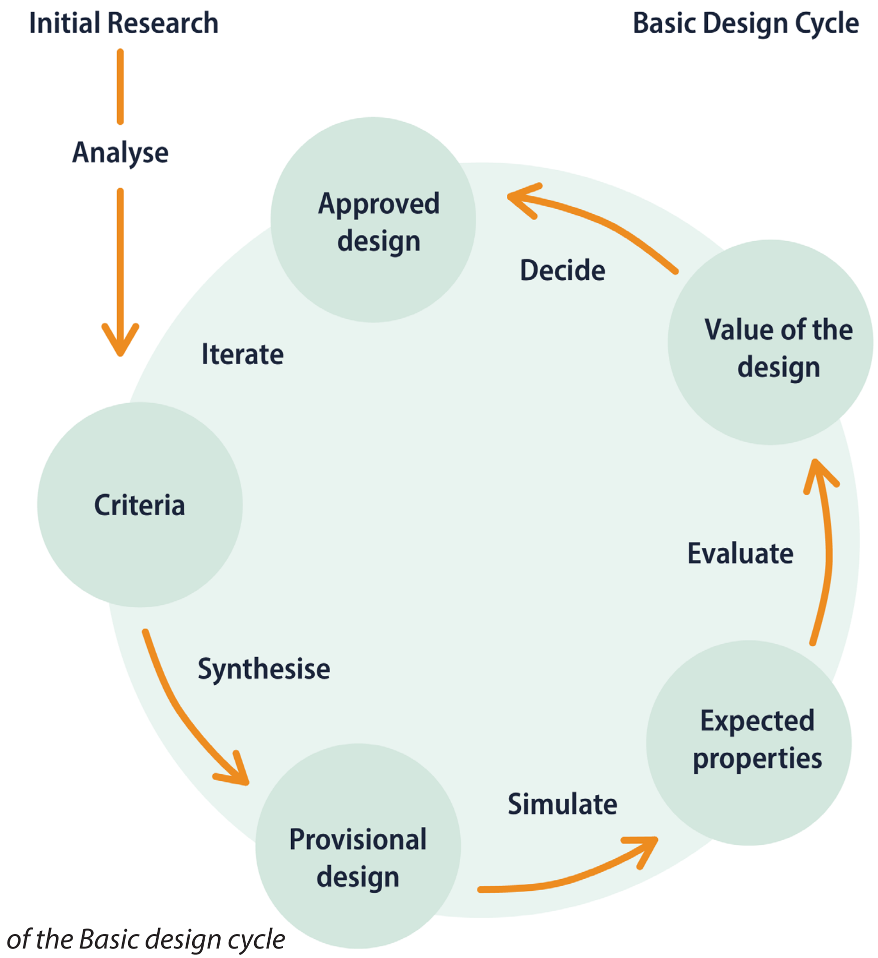


Figure 5: Interpretation of the Basic design cycle

2.2 Project planning

In figure 6 the overall planning of this project is presented. Prior to the kickoff, the first meeting with the client was head and some initial research was done, after the kickoff, the project brief was written including an introduction to the subject and initial design goal, see Appendix A. In the first part of the project research was done, alongside of analysis of that research and the start of the ideation phase. At the midterm point, five design directions were presented. After this point, the directions were transformed into a provisional design, which was then iterated on and improved through more ideation, tests and embodiment. During each iteration, the design was improved based on the requirements and wishes that it should meet. Before the green light point, the final design was created, this design is explained in this report in Chapter 6. The value of this final design will be tested with the target group. Improvements that follow after this test will be implemented and presented after the green light point, followed by the approved design in the final report.

In order to answer the research questions and understand the important fields for this project, different forms of research were conducted. Firstly, desktop research was done on the fields of primary school education in the Netherlands, mathematical domains and topics, spatial ability, design and inquiry based learning. Next to desk research, I read some books on design and inquiry based learning (in mathematics classes), and looked through existing mathematics of the 3rd and 4th grade.

Figure 7 on the next page visualises this graduation project in a chronological order by placing the basic design cycle and project planning next to each other. Furthermore, it provides an overview of how research, observations, expert meetings, analysis, prototyping an testing had an impact on important outcomes of the project. Lastly, it shows which parts of the research and design are explained in which chapter of this thesis.

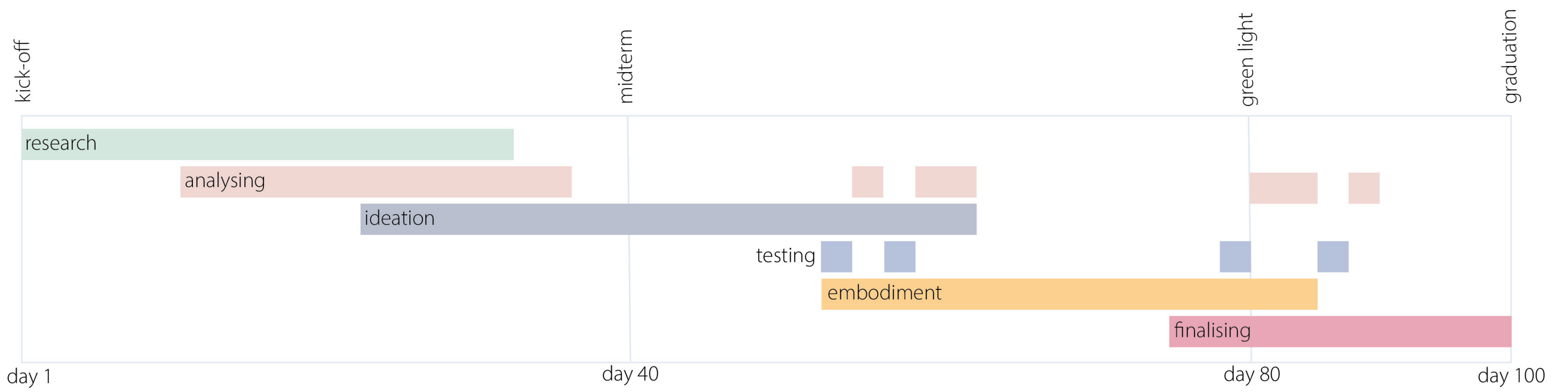


Figure 6: Project planning in phases

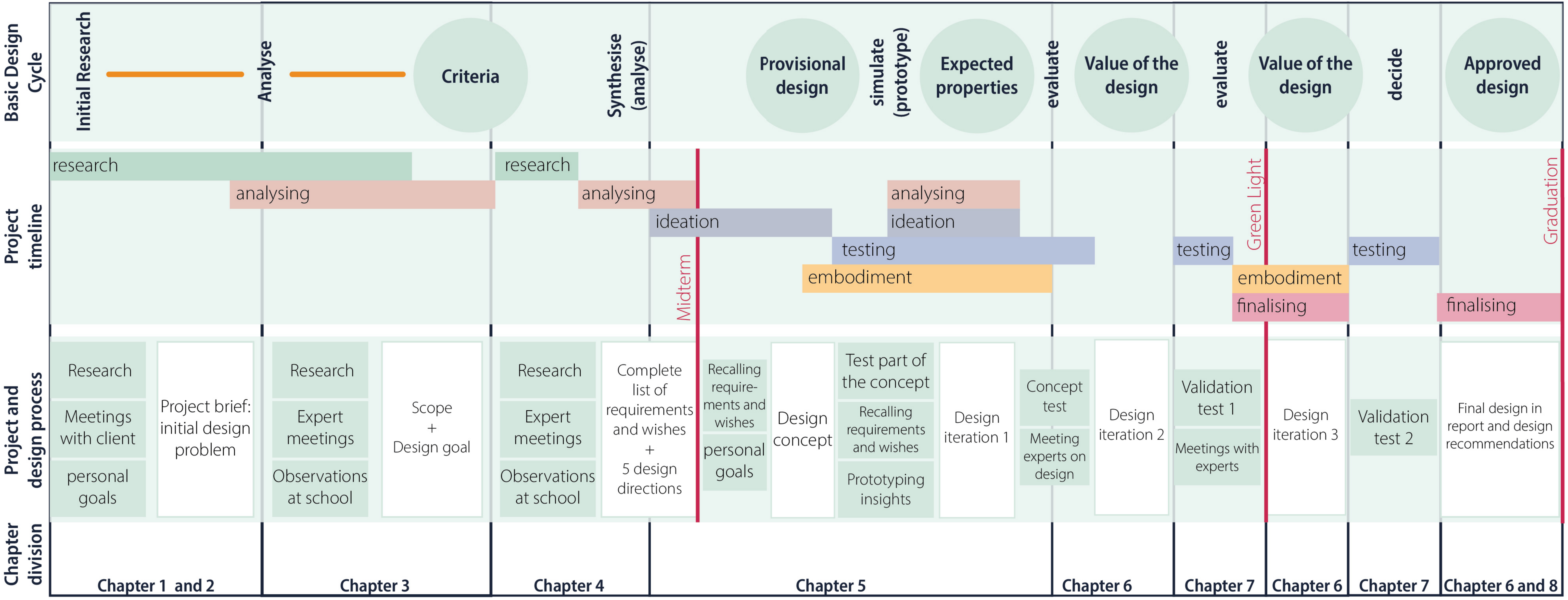


Figure 7: Basic Design Cylce + Project planning + Research and Design Process + Chapter division

Furthermore, I met up with my client L. Sonneveld many times during this project, as well as with other experts in the field to get a deeper understanding of experiences in primary school education and mathematics classes. An overview of the experts I spoke to is given in the table on the next page.

The experts that are mentioned by name have agreed for this personal reference, others are anonymised. For these expert meetings I prepared questions beforehand. At the start of the meetings I asked to briefly explain their expertise/research, the remaining of the meetings had an interview and discussion approach.

Lastly, I had the possibility to observe at two primary schools in Delft. At the first school I observed one normal mathematics class of the 4th grade and at the second school I observed one normal mathematics class (combined class of 3rd and 4th grade) and two instruction/individual work moments especially for skilled mathematicians of this class. The first school follows a strict way of teaching; the EDI-method (Expliciete Directe Instructie) and the second school has a more creative and inquiry based learning method. For the two observations in class I wrote down some topics I was interested in understanding and after class I had a short conversation with the teachers, the notes of all observations can be seen in appendix B1, B2 and B3.

Expert	Expertise and experience	Topics discussed
Dr. R. Keijzer	Professor mathematics (rekenen en wiskunde) at teacher training college iPabo	Meeting on children's experience in maths classes, maths anxiety, the interaction between students-teachers and on possible improvements of the current education system.
Anonymous	Mathematics teacher at teacher training college Thomas More Hogeschool	Meeting on the experience of teacher training college students, spatial ability in the existing methods and possible improvements of the current education system.
Dr. H. Kekkonen	Researcher and assistant professor at TU Delft – Department of Applied Mathematics	Meeting on spatial understanding of students and geometry applications for mathematics education.
C. Zhu	PHD researcher at TU Delft (research includes understanding children's spatial thinking)	One meeting on spatial ability and how to test it with children. Later on in the project she provided feedback on the exercise booklet regarding questioning language and approach and if it sufficiently tested spatial ability.
S. Sjoers	Researcher and writer on skilled mathematicians, as well as curriculum developer for Dutch education materials for mathematics	Meeting about skilled mathematicians; how they are currently treated and what could be improved in order for them to receive sufficient guidance. She also showed and discussed examples current education methods and mathematics exercises, as well as which of them do or do not sufficiently stimulate skilled mathematicians.
Dr. A. Hotze	Previous teacher, lectorate in science and technology education	Meeting towards the end of the project to review the designed product and discuss its opportunities for implementation in the 3th and 4th grade, as well as discussing its strengths/weaknesses and giving design improvements.
Anonymous	Teacher of the 3rd/4th grade and education material designer	Meeting after a test was conducted with her class, about the product, its opportunities for implementation in the 3th and 4th grade, as well as discussing its strengths/weaknesses and giving design improvements.

Table 1: Overview of experts

2.2.1 Research questions

The research questions are answered throughout the report as follows:

Table 2: Where and how each research question is answered

Research Question (RQ) or Design Question (DQ)	How the question was researched/answered	Answered in chapter
RQ1: Which characteristics in children lead to them not achieving their full potential in mathematics because of the current education methods?	I learned about the different children in mathematics classes by talking to R. Keijzer (professor mathematics at teacher training college iPabo), a mathematics teachers of the teacher training college Thomas Moore Hogeschool and S. Sjoers (researcher and writer on skilled mathematicians). They told me about (type of) children that have a hard time understanding and answering specific questions. Furthermore I observed mathematics classes at two different primary schools to get some understanding of how children respond to the subject.	**3.3 and 3.4**
RQ2: Which characteristics of the current mathematics classes and methods lead to children not achieving their full potential and how could these be improved?	For this question I could also receive information from the experts mentioned for the previous question. S. Sjoers could especially explain many lacks of the current education curricula, since she also helps create exercises for primary school mathematics education. Besides, the teacher of one of the schools I visited could explain how she adds exercises to her classes to compliment the methods of the books. Lastly, desktop research was done to understand mathematical education in the Netherlands and its lacks and opportunities.	**4.2**
RQ3: How can a new tool help to stimulate children to developing spatial skills in a creative and interactive way?	This question was answered after I conducted most of the research and is presented as the list of requirements and wishes, summarising most of the research.	**4.3**
RQ4: What is needed for a good interaction between children and teachers in class in order to let the children achieve their full mathematical potential?	To answer this question I have again learned from the experts I spoke to, alongside of doing desktop research.	**4.1 and 4.2**
DQ: How would a new tool inspire innovation in the current mathematics classes?	The answer to the design question can follow after reading the list of requirements and wishes. Through research through design, testing the product with children and validating it with expert, the solution was formed in the form of a toolkit.	**6 and 7**

2.3 Stakeholders

For the duration of this project, it is important to keep in mind who have interest in the tool that is to be designed and what the specific needs and wishes of these parties are. Therefore, a stakeholder map is created showing all parties involved. As can be seen in figure 8, the most important stakeholders are the children, teachers and the designer, me. Since Science Hub is the client, I design with their wishes in mind. Science Centre TU Delft is a possible future client if interested in the outcome of this project.

Since teachers are the once who will decide if, when and how the tool is used in the end, they are placed in the middle. Teachers do, however, follow methods learned at teacher training college and regulations decided on by the school board, which at its place is decided by the government. SLO, the creator of most educational tools in the Netherlands is placed in the stakeholder map because they create exercises the tool will be an addition to.

Stakeholder map

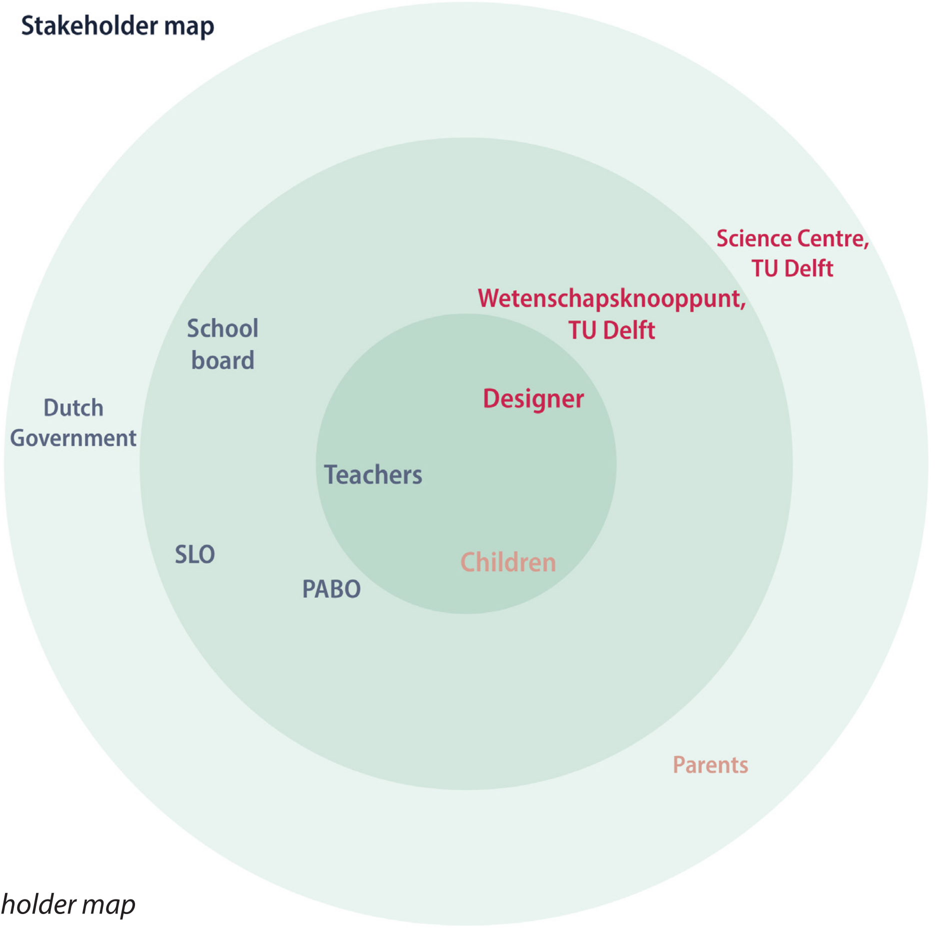


Figure 8: Stakeholder map

(medilexonderwijs, n.d.)

3

Scope:
Skilled Mathematicians

This chapter will explain for which children will be designed, in other words: the target group of the project will be presented.

3.1 One of the three

The more children a class has, the more different learning styles there are. All children are unique and so are their educational needs. Because it is difficult for teachers to fulfil the different needs of every student, a simplified distinction is made. In mathematics classes in the Netherlands this distinction consists of 'weak', 'average' and 'strong' students (personal communication, 2025).

3.1.1 Being labelled by a test

The distinction between the three levels can be seen as labels given to children. These labels are the results of reference tests, given once or twice a year to children of all primary schools in the Netherlands, the most commonly known example is the Cito test. (personal communication, 2025). Based on the results of these tests a reference level is linked to each student, ranging from 1F (fundamental level) to 1S (target level) at primary schools, these levels are continued into the Dutch secondary education (taal en rekenen, n.d.). Such a level indicates how well a student masters the subject and is used by teachers to give students advice on which secondary education they should follow (Mottart & Vanhooren, 2015). Besides holding schools accountable for their students to reach the fundamental level, the reference levels also ensure that the autonomy of schools is limited (Expertgroep Doorlopende leerlijnen Taal en Rekenen, 2008).

Reference tests for mathematics are divided in four domains (figure 9): numbers (getallen); proportions (verhoudingen); measurement and geometry (meten en meetkunde); connections (verbanden).

These four domains, build up by different subdomains, are taught throughout the entirety of primary school with different learning objectives in each grade.

Every primary school in the Netherlands is obliged to implement the same learning objectives, however, it differs per school which education method they follow and how they guide students to achieve these objectives. The reference tests do not distinguish in different education methods and are the same for all children in the Netherlands. (personal communication, 2025).

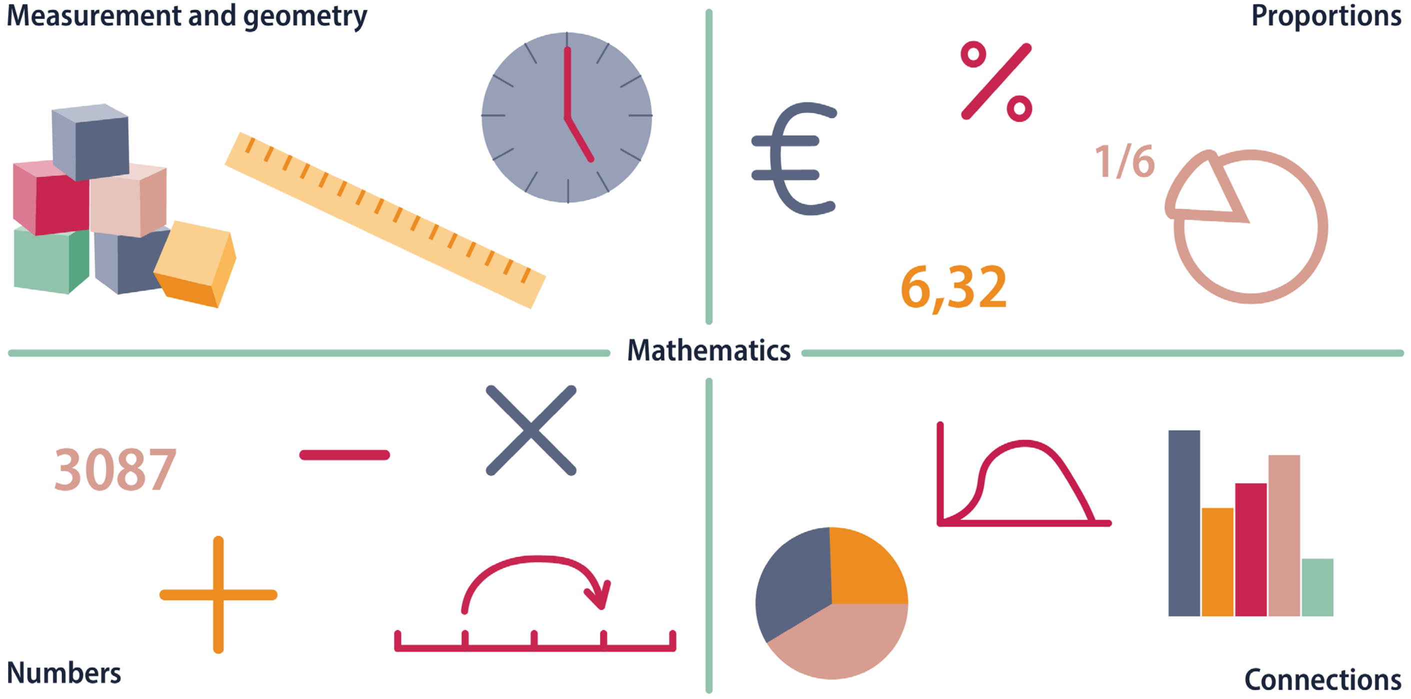


Figure 9: the four domains of mathematics in primary school

3.1.2 Maths anxiety

For children who are labelled as 'weak' as a result of reference tests, it can be hard to believe that this label will change for the better. The feeling of being labelled as bad in mathematics is one of the triggers that can result in having maths anxiety (Keijzer, 2024). Almost one fourth of all adults have maths anxiety and for many people this feeling starts or is triggered as early as during primary school (TED-Ed, 2017).

However, receiving a discouraging label is not the only reason maths anxiety can occur. People who do not lack skills in mathematics can also experience this feeling. For example, because they misinterpret questions due to unclear phrasing or language or because they are unsure on what the method wants them to do in order to answer properly. Furthermore, some existing exercises simplify situations of real life which confuses students because they know the real life situation that is being simplified and are unsure of the level of realism in the exercise and whether or not they should skip their own knowledge on the situation (Sjoers, 20205). Another issue leading to maths anxiety is mentioned in Chapter 4.

Requirement: Exercise/tool should be linguistically clear and not allow for misinterpretation

Wish: Exercises should simplify real life situations as little as possible and stimulate students' knowledge

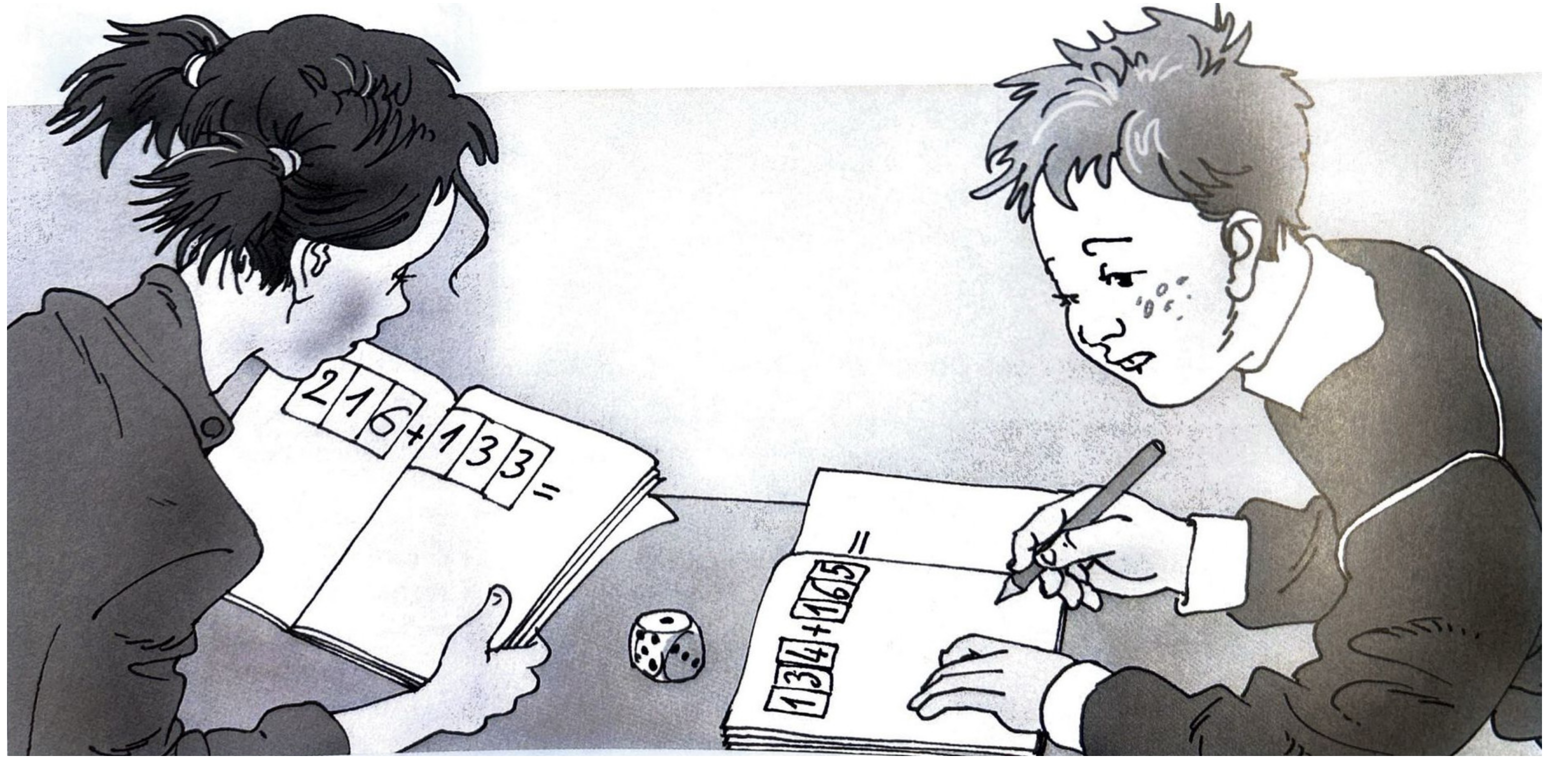


Illustration from exercise book 3rd grade (Malmberg, n.d.-a)

3.2 Differentiation: different students have different needs

Teachers are educated on treating students with different levels in a subject differently, when a teacher acts upon those differences, in any education situation, we speak of differentiation. There are different forms to implement differentiation in class. The first distinction is made between convergent and divergent differentiation, where for the first a minimum goal is set which the entire group works towards. For the latter, learning objectives are adapted to the individual needs of students.

A second distinction is made between didactic and organisational differentiation. During didactic differentiation the teacher exploits the differences between students to learn from and with each other. In this case the teacher is the director of how all children learn. Examples are creating groups of students who have a good understanding with those who do not, asking different questions or giving different homework exercises to different students. Organisational differentiation is used when a teacher organises the class in such a way that different groups of students can all work at their own pace. Creating learning modules is a way of doing so, allowing students to work in modules in which they can make different choices based on pace, approach, level, interest, whether or not to participate in the class explanation or if they want extra explanation. (Van Ast et al., 2020). Further on in the report will be referred to differentiation and to ways of using it in different class situations.

3.2.1 Differentiation in the mathematics classroom

It depends on the teacher to what extent and what type of differentiation is used. Primary school teachers are all taught different possible ways to implement differentiation at the teacher training college (PABO). Worth mentioning is that there is significantly more focus on how to support weaker students in mathematics than how to support the skilled ones (personal communication, 2025). This is not surprising since the weaker students need to improve their skills in order to achieve the fundamental level needed to pass an exam or to proceed to the next grade. The 'best' students of the class already achieve such high grades that logically the teachers focus on helping the weaker students (personal communication, 2025).

It depends per school how stronger students are 'treated' in class, however, most schools use extra materials (booklets) specifically designed for these students. These materials include extra, challenging matter that is connected to the existing methods and classes. Often, stronger students can practice with these extra exercises during self study moments at the end of class, or they are separated from the rest of the class and placed outside of the classroom to work on these exercises (which can be done individually, collaboratively or with the help of the teacher). However, it would be helpful for stronger students to collaborate with the rest of the class as well. This commonly used way of guiding stronger students has its disadvantages, explained in chapter 3.3 and 3.4.

Wish: Exercise/tool could allow for interaction between students of different levels

The lack of right support for stronger mathematicians was the motive for conducting research about these children, which resulted in the need to invest in improvement of education for stronger mathematicians (Expertgroep Doorlopende Leerlijnen, 2008). By focussing more on how to support stronger mathematicians, the chance to develop an interest within these children to choose for a technical career path could increase, since these children already have the fundamental understanding, interest and invested attitude towards the subject.

Wish: Exercise/tool should inspire children to develop technical interest

3.3 Skilled mathematicians: good, quick and creative

Taking into account that teachers are sufficiently implementing differentiation in their classes, S. Sjoers (developer of mathematics curricula for SLO and writer of the book 'Sterke rekenaars in het basisonderwijs' (Skilled mathematicians in primary school education)) describes that there are still points of improvement to be made to sufficiently support the stronger mathematicians (2017). First of all, the group of skilled mathematicians should not be seen and supported as one group (as described previously and as most schools currently do), since there are different types of skilled mathematicians who all have different needs.

Sjoers identifies three groups: good, quick and creative students ('goede -', 'snelle -' and 'creatieve rekenaar') (2017), see figure 10. There are no numbers on the average amount of skilled mathematicians per class, nor on the division between the good, quick and creative ones (Sjoers, 2025).

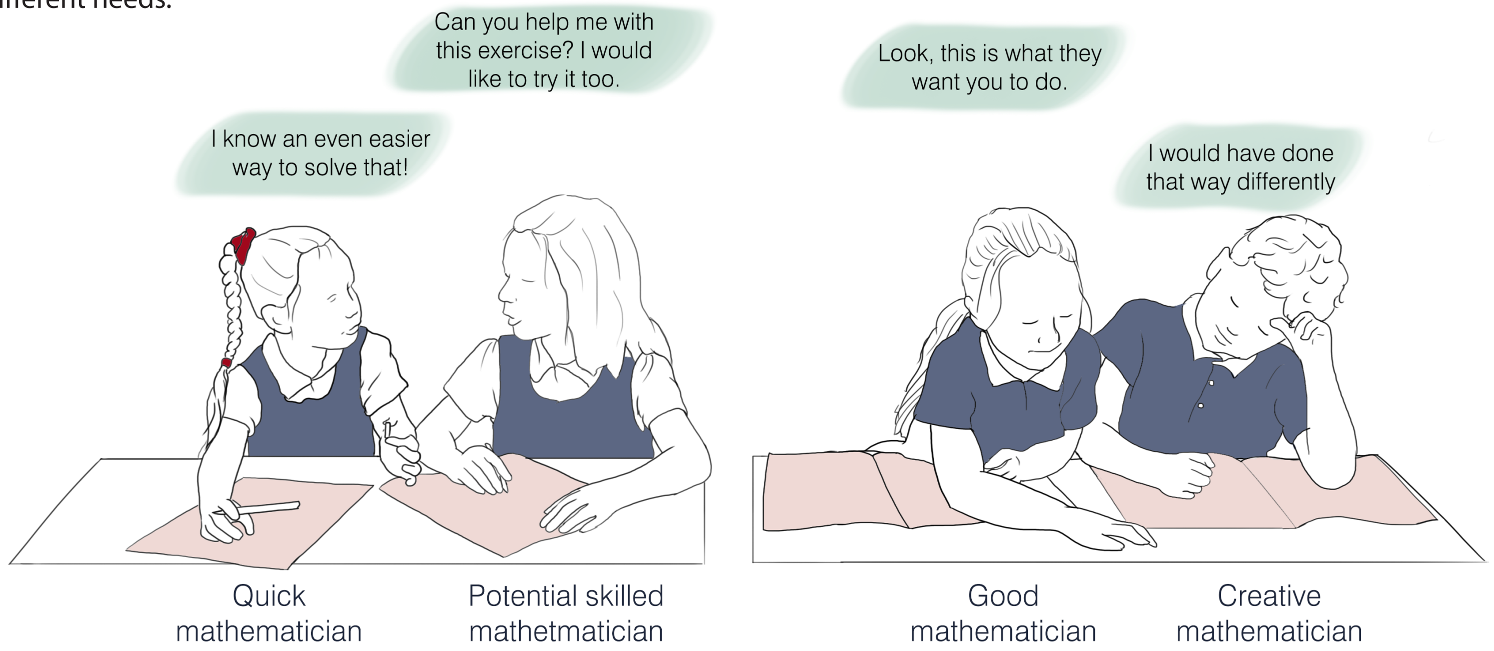


Figure 10: Different types of skilled mathematicians

Good mathematician (Goede rekenaar)

The good mathematician is a student who quickly understands the exercises and easily solves them the way that it is intended to by the education method. These students therefore get high grades, however are not supported sufficiently since they lack challenges and deepening exercises. Often these students show predisposition, interest and persistence.

Quick mathematician (Snelle rekenaar)

Quick mathematicians are students who quickly write down solutions to an exercise but sometimes miss the correct route to achieve their answers because they often do not understand the presented method and therefore write down incorrect solutions. This results in them also not achieving high grades.

Creative mathematician (Createve rekenaar)

The third type is the creative mathematician, these are the students who can show a remarkable understanding of mathematics. However, their ways of reaching the right answer to an exercise often differ from those presented in school and they are sometimes quick in creating connections that are not logical. They therefore also do not get high grades and are often confronted with misunderstanding from the teacher and other students, making them believe that they are 'wrong'. These students can also misunderstand questions that are simplified from reality, because the student has a better understanding of the situation but the question seems to exclude part of that information they do not know how to tackle it and can even experience maths anxiety (Sjoers, 2025).

Potential skilled mathematician

Beside the three types of identifiable stronger mathematicians, S. Sjoers describes the children who are not yet part of this group but have great potential to become one of them: the potential skilled mathematicians, see figure 7. Similar to the good, quick and creative mathematicians, these students are in need of fitted guidance and challenges to support their personal development. With the correct guidance and exercises these students too can reach more than is currently possible. As Gardner describes in his multiple intelligence theory, people can be intelligent in more than one way and the potential of these students might not be supported by the current ways of teaching and testing.

Wish: Exercise/tool should provide fitting guidance for potential skilled mathematicians

In the following chapter, other points of improvement to correctly guide skilled mathematicians are explained.

3.4 Reaching their potential

Since there are different types of skilled mathematicians, logically there are different ways in which they all should be guided and challenged, since all three of these students could achieve more if the right support is given. (Sjoers, 2017). As mentioned above, the good mathematicians are in need of more challenging and deepening exercises. For quick mathematicians, a focus on process instead of results would be beneficial, along with guidance on learning and work strategies. Lastly, creative mathematicians are most helped with guidance on understanding the existing method and challenges in which they can use their gained knowledge.

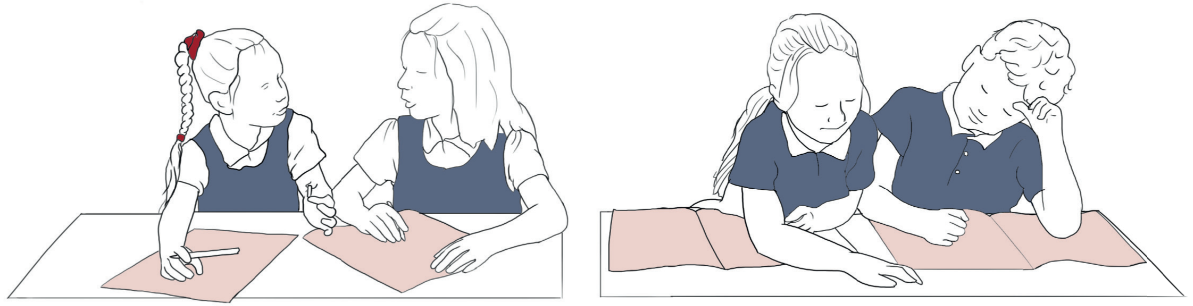
Children who have good mathematical understanding might have the same skills in other subjects. Many children then choose a career direction based on which subjects or fields they are not only good in, but also enjoy and get pleasure from doing. Unfortunately, in relation to other subjects, mathematics does not score well on the enjoyable aspects. Thus many children, even though they are good in mathematics, do not choose to continue their careers in this direction. Taking this consideration into account it is not only relevant to challenge and guide strong mathematicians, but to also increase the fun that could be derived from the subject.

Wish: Exercise/tool should create an enjoyable interaction with mathematics

Besides their individual needs there are two strategies that would help all three student types. Firstly, mathematics classes should have extra challenges since all groups need practice to understand new subjects but they do not need too much of it. Therefore the existing amount of practice materials should be decreased. Furthermore, existing exercises should be reformulated or extended and lastly existing content should be replaced by more challenging versions. Second, skilled mathematicians would all benefit from scheduled instruction moments since they are in need of clear instructions, guidance and space for reflection but are currently often separated from the rest of the class which makes them miss the normal instructions. These instruction moments should include discussions about broadening and deepening exercises, explanations of work and learning strategies and self-thought solution strategies. The students should be appreciated and rewarded more for having the latter and inspired to use these more often if applicable to other exercises. Figure 11 gives an overview of the needs of each type of student individually as well as those that account for all four types.

Requirement: Exercise/tool should be challenging and stimulating for quick and creative mathematicians, alongside that of the good ones

Requirement: Exercise/tool should allow for reflection



Quick mathematician
- Focus on process
- Explanation on working strategies

Potential skilled mathematician
- Guidance
- Exercises that support personal development

Good mathematician
- Deepening exercises
- Extra challenges

Creative mathematician
- Logical real life exercises
- Guidance to understand the existing methods
- Challenges to use their gained knowledge

All
- Scheduled instruction moments
- Discussion and reflection moments
- Guidance from teacher
- Extra challnges
- Interaction with each other

Figure 11: Needs of different types of skilled mathematicians

3.5 Start young

The scope of this project is stronger mathematicians of the third and fourth grade of primary school, there are multiple reasons why this age group is chosen.

Firstly, the best way to identify stronger mathematicians is as early as possible in order to guide them sufficiently to develop their potential (Sjoers, n.d.). Also, currently, stronger mathematicians are labelled as such from the second grade onwards, focusing on the two grades above ensures that most stronger mathematicians in primary school will have been identified by then.

Thirdly, this age group allows for sufficient support in spatial ability development. In addition to the recognition of the European Union to develop spatial ability at an early age mentioned before, Yang et al., also state that spatial skills should ideally be taught early (2020). According to Clements, the domain of geometry is often not emphasised enough in the early years curriculum (2004). Furthermore, Keijzer mentions that during the second half of primary school, exercises concerning spatial ability start to require more effort from students than previously. Alongside this, maths anxiety commences to develop during this age. That could partly be because exercises increase in difficulty from the second grade onwards (Keijzer, 2024).

Requirement: Exercise/tool should be ergonomically designed for 3rd and 4th grade students

Requirement: Exercise/tool should provide opportunities for exercises that fit 3rd and 4th grade learning objectives of Dutch educational materials

3.6 Conclusion

In conclusion this chapter explains that children in mathematics class are labelled as three different levels: weak, average and strong. This is done based on reference tests, which are based on the four mathematical domains of numbers; proportions; measurement and geometry; connections and that these tests are the same in the entire country. Getting a low label can be one of the reasons for children to experience maths anxiety. Other reasons are misinterpretation of the wording of questions or misunderstanding of methods that need to be used. These misinterpretations and misunderstandings are important aspects to keep in mind for the final design as it should avoid them.

What teachers are doing to guide students of different levels of understand accordingly, is differentiation. However, sufficient guidance for stronger students is still lacking. S. Sjoers conducted research these students and divided the group into three different skilled mathematicians: good, quick and creative ones. Besides, there are also potentially skilled mathematicians, who could become stronger scoring students with sufficient guidance. All four groups have specific needs on sufficient guidance in the classroom. The final product of this project will be designed for these four groups with their specific needs in mind. This will be done for the 3rd and 4th grade.

4

Design Challenges and Opportunities

(Freepik, n.d.)

This chapter will give an overview of the ways in which the current system is lacking in sufficient guidance for skilled mathematicians. This chapter dives deeper into specific shortcomings from either teacher, mathematics books and education methods. These ways can be reformed into opportunities for the project and seen as design challenges. Later on, the chapter lists the requirements and wishes of the product to be designed, summarising the previously presented research. Furthermore, it introduces five design directions, followed by a comparative analysis and ending with a conclusion towards the final design.

Currently, classes around spatial thinking are mainly theoretical and do not allow for much collaboration between students or interaction between students and physical objects or real life situations. For example, it is difficult to understand the dimensions of a (complex) geometric shape without having the shape in front of you and only seeing 2D sketches or words describing it. Tools can inspire interactive and creative learning and are needed to improve children's experience in mathematics classes (TED-Ed, 2017). Besides, referring to real life situations can also help students to create a feeling of connection. However, teachers are hesitant to implement creative learning in their classes themselves due to the lack of these supporting tools (Schoevers, 2020). Some teachers are even hesitant to help children who work (slightly) differently from the existing methods, like creative or quick mathematicians, because they fear of not being able to (Keijzer, 2024).

Requirement: Exercise/tool should allow for collaboration between students

Wish: Exercise/tool should refer to real life situations and allow for a feeling of connection

Following this abstract and theoretical way of teaching, there are many children who do not enjoy maths, find it too abstract and difficult, or can even experience maths anxiety (it is expected that around 20% of the world population (similar in the Netherlands (Keijzer, 2024)) suffers from this (TED-Ed, 2017). Implementing design and inquiry based learning can help decrease the anxiety usually associated with mathematics and create a more enjoyable environment in class. Besides helping all students overcome their negative feelings towards maths, creative and interactive tools can stimulate students , including skilled mathematicians, in developing their spatial ability.

Requirement: Exercise/tool should allow for interactive learning (use of external tools)

Following this abstract and theoretical way of teaching, there are many children who do not enjoy maths, find it too abstract and difficult, or can even experience maths anxiety (it is expected that around 20% of the world population (similar in the Netherlands (Keijzer, 2024)) suffers from this (TED-Ed, 2017). Implementing design and inquiry based learning can help decrease the anxiety usually associated with mathematics and create a more enjoyable environment in class. Besides helping all students overcome their negative feelings towards maths, creative and interactive tools can stimulate students , including skilled mathematicians, in developing their spatial ability.

4.1 Interaction teachers and skilled mathematicians

4.1.1 Insufficient instructions and maths anxiety

As explained in the previous chapter, skilled mathematicians are often not treated in their best interest. An important reason for this is the lack of guidance that the teacher training college offers future teachers on how to support and challenge these students. There are multiple tips on supporting weaker students but for stronger students teachers are often instructed to hand out extra materials and let them work individually. The teacher training college also has interest in improving guidance for skilled mathematicians. However, this way of working is a general practice in the Netherlands (personal communication, 2025). Furthermore, teachers do currently not receive sufficient instructions on how to identify skilled mathematicians, let alone the three different types and the potential skilled mathematicians, as described previously by S. Sjoers. This can result in (potential) skilled mathematicians not even given extra practice materials, besides not being supported to reach their individual potentials. To guide teachers identifying good, quick an creative mathematicians, S. Sjoers created a tool (Sjoers, n.d.), see Appendix D.

A last reason for the possible difficult interaction between teachers and skilled mathematicians is that some teachers are also known to have maths anxiety, as R. Keijzer and a mathematics teachers at the teacher training college have experienced. For passing the secondary teaching education, all students must pass a mathematics test, which is not considered easy and for which some students have serious concerns or even experience anxiety. In addition, for the majority of future teachers, mathematics is not their favourite or strongest subject to learn or teach. Because there are multiple paths in the Netherlands to arrive at the teacher training college, some students who come there have a high school background of VMBO, at which they did not learn all mathematical topics needed to become a primary school teacher, they suddenly face these matters at the teacher training college and lack the sufficient experience and knowledge to build on and can have a hard time learning this.

4.1.2 Innovation is scary

Besides the possible difficult connection between teachers and skilled mathematicians, many schools and their teachers share a resistance to innovate in class. Innovation, however is necessary to keep challenging and guiding students to their best potential. As Barend Last mentioned; "You can only become an expert if you have experience, but experience only does not make you an expert" (2025). This means that a good teacher does not only have (sufficient) experience, but that they are also (constantly) open for innovation and improvement on their education style. The risk of being a teacher for a long time is to have once learned how to teach well and to continue working the same way as the years go by. However the education system, children's needs and the overall knowledge on subjects does change and therefore as a teacher you need to grow with these developments. Last also presented a list of requirements needed for teachers to implement innovation in their classes, mentioning five. The innovation needs to have a relative advantage to the current system, it needs to be test worthy, observable and include sufficient compatibility and complexity.

Wish: Exercise/tool should consist requirements of innovation, as presented by Barend Last

4.2 Methods do not allow for creativity and interaction

Besides the interaction of the teacher training college with teachers and teachers with students, other opportunities lay in the improvement of education methods. As mentioned before, these currently do not allow for the needed interaction between students and teachers, and students with each other. Furthermore, the current methods lack creative learning, which as mentioned above, can help children stimulate their spatial ability.

It does differ per school in the Netherlands how they support their students by achieving the needed learning objectives. However, most schools do not stimulate creative and interactive learning enough (personal communication, 2025). Barend Last explained that the need and demand to combine multiple teaching methods at the same time is not always received well by teachers. It is hard to ask them to change their ways of teaching to learn using new or different methods since it is not what they signed up for (2025). From the observations made at two different schools in Delft, the difference between two different teaching styles was noticeable. The first school nearly solely followed the method of the book while the second school used additional materials like objects in class and supporting materials the teacher finds online.

Two exercises that the teacher of the second school added to the method of the books were the usage of blocks to build towers viewed from different viewpoints and asking the students to collect objects in class that have the basic forms of cylinders, cubes and beams.

By considering adding creative and explorative interactions in the classroom, teachers can keep students intrigued. Former Swedish teacher Micke Hermansson created the teaching concept 'Grej of the day'. He gave his children a clue at the end of every day, made them think about it at home and explained the clue the next morning. Doing so, he experienced that his students were excited to come to school every morning and were much more involved during the day than before. The experience of Hermansson is in line with existing exercises for skilled mathematicians in the Netherlands. That is, children's interests in clues and puzzles. During the observation at the creatively oriented school in Delft, skilled mathematicians were asked which (type of) exercises they found most enjoyable. Most children excitedly showed the puzzle-, sudoku- and riddle exercises of the book. Examples of these exercises can be seen in appendix E.

4.2.1 Design- and inquiry based learning

The importance of adding creative and explorative exercises to the classroom is the base of design- and inquiry based learning. A way of teaching that is used and researched by Science Hub TU Delftand already used at some schools. Design and inquiry based learning explains the importance of creative and explorative learning, and learning through play. Design and inquiry based learning already has its adaptations at some high schools in the Netherlands and some even teach a course focusing on design and inquiry based learning: O&O (Onderzoekend en Ontwerpend Leren). "By using OOL the teacher offers children the opportunity to investigate, to think of a solution for the problem and to see if that solution works. O&O is a fitted approach for science and technology education." (Vanhooren & Mottart, n.d.). Furthermore, Science Hub developed a design cycle that presents the six stages of designing, alongside one for inquiry based learning, see figure 12. This circle can function as a guideline for exercises in class that make use of design and inquiry based learning.

Jonker and Wijers present the importance of design and inquiry based learning for mathematics education. They explain that while doing mathematics, students both develop concrete, mathematical skills and reflective skills (considering new situations) "If doing maths and reflecting go hand in hand, logically there is a need for an explorative attitude in class." (Jonker & Wijers, 2016). They continue by stating that it should be avoided to blindly execute tasks and follow mathematical rules. Since then, the opportunity to create sufficient connections between skills and implementations could be missed. Furthermore, Jonker and Wijers explain that the 'explorative attitude' in mathematics classes is also known as the 'mathematical attitude', which moreover means to be alert for the underlying mathematics of new situations and is needed for many every day situations (2016).

Requirement: Exercise/tool should allow for creative learning

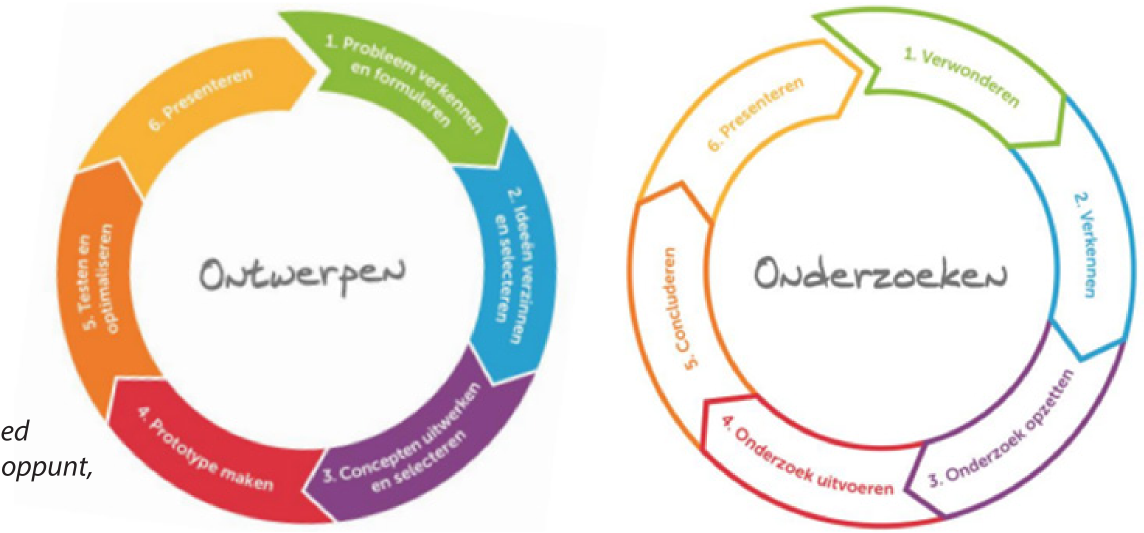


Figure 12: Design and inquiry based learning cycles (Wetenschapsknooppunt, 2018)

4.2.2 Methods are not challenging enough

The current reference levels in the Netherlands are not a perfect fit for most skilled mathematicians. 1S is the highest level given in primary school. However, this level is estimated to be structurally below the potential of 20% of all those who achieve it (Expertgroep Doorlopende Leerliinen, 2008). At some schools, students can practice with exercises that correspond to the final level of the secondary education program VMBO: 2F. Though, this level is still not challenging enough and does not support creative learning (Sjoers, 2018). This is why some researchers, like S. Sjoers, are discussing the possibility of adding an extra level to the mathematical education system. A study by Hawes, et al. suggests that there are many benefits to be had in offering young children a challenging yet engaging assortment of spatial activities." (2017).

Requirement: Exercise/tool should challenge the students further than the exercises in the current methods do

An example of such an additional level for the education system is level X, as tested by APS in 2009, an experimentational level of mathematical understanding. X stands for experimental, since that is what many high achieving students need instead of simply more exercises (Sjoers, S. & CPS Onderwijs- ontwikkeling en advies, 2018). Furthermore, a study by Hawes et al. tested the implementation of spatial visualisation exercises around spatial ability on children.

The children showed high engagement with these activities and the findings of the study suggest that "through participating in such activities, children demonstrated improvements in a number of important geometry standards outlined by NCTM (2000; 2006)". (Hawes, et al., 2007). During observations done at two schools in Delft, the interest of children in challenging exercises was also confirmed. Skilled mathematicians were asked on their experiences of doing harder exercises as addition to the normal classes and they said to enjoy these extra exercises since those of the normal classes do not challenge them enough. Besides, they enjoy to collaborate with other students to try and solve the harder exercises together. Two possibilities that S. Sjoers mentioned, to challenge skilled mathematicians, are by introducing future topics in earlier exercises or by combining different domains in in one exercise or in a group of exercises. Currently, most chapters in the current mathematics books tackle one domain at a time.

Wish: Exercise/tool should combine different domains or subjects

Wish: Exercise/tool can introduce future topics/exercises to students

Wish: Exercise/tool can be used for more than one exercise/ subject or as many subjects as possible

4.3 Design opportunities

Based on the research presented thus far, several shortcomings of the current curriculum for skilled mathematicians can be identified. These shortcomings hinder the effective stimulation of skilled mathematicians and can be reformulated into requirements or wishes, as they have been throughout the report thus far. These requirements and wishes will form the foundation of the ideation phase, ensuring that the final product will effectively stimulate skilled mathematicians.

Combining al these opportunities a list of requirements and wishes is made for the tool that is to be designed. Requirements are qualities that the future design has to meet and can be measured while wishes are qualities that are of less urgency but can serve as useful guidelines to design good solutions.

Requirements:

- Exercise/tool should allow for creative learning
- Exercise/tool should allow for interactive learning (use of external tools)
- Exercise/tool should allow for collaboration between students
- Exercise/tool should allow for reflection
- Exercise/tool should challenge the students further than the exercises in the current methods do
- Exercise/tool should be linguistically clear and not allow for misinterpretation
- Exercise/tool should be challenging and stimulating for quick and creative mathematicians, alongside that of the good ones
- Exercise/tool should be ergonomically designed for 3rd and 4th grade students
- Exercise/tool should provide opportunities for exercises that fit 3rd and 4th grade learning objectives of Dutch educational materials
- Exercise/tool should improve children's spatial ability and test different spatial skills

Wishes:

- Exercise/tool should refer to real life situations and allow for a feeling of connection
- Exercises should simplify real life situations as little as possible and stimulate students' knowledge
- Exercise/tool could allow for interaction between students of different levels
- Exercise/tool can be used for more than one exercise/subject or as many subjects as possible
- Exercise/tool can introduce future topics/exercises to students
- Exercise/tool should provide fitting guidance for potential skilled mathematicians
- Exercise/tool should combine different domains or subjects
- Exercise/tool should create an enjoyable interaction with mathematics
- Exercise/tool should inspire children to develop technical interest
- Exercise/tool should show the importance of spatial ability in real life situations
- Exercise/tool should consist requirements of innovation, as presented by Barend Last

5

Research through design

The list of requirements and wishes formed the starting point of the ideation phase. In this chapter that ideation phase is shown. It starts with five design directions, which after being compared based on the list of requirements and wishes were combined to one product idea. This idea was then embodied through prototypes and tested with children. Afterwards, the idea was improved and new iterations were prototyped and tested again.

5.1 Design directions

After conducting research, ideas were created and later clustered into possible design directions. These directions are options for the future design and can create a more specific scope on the existing design goal. This clustering process was done three times with a different division in directions each time, of which the results can be found in Appendix F. The chosen design directions differ in the type of challenge that the ideas will fulfil. The five directions are 'deepening', 'combining different domains', 'broadening/discovering', 'relating to reality' and 'personalising skills', all describing how the exercise fitted to its idea will challenge skilled mathematicians. The meanings behind the five design directions are described below, each accompanied by an example.

Direction 1: Deepening

The first way in which skilled mathematicians can be challenged is by allowing them to think about a topic they have learned in a new way. Doing so, they are asked to deepen their knowledge.
Example idea: using questions to inspire complex thinking (that use the topics of class but for a different context). These questions can also be repeated after new classes to make students realise that they can revise old situations with new knowledge to get a more profound outcome. Figure13 shows an example of such a question.

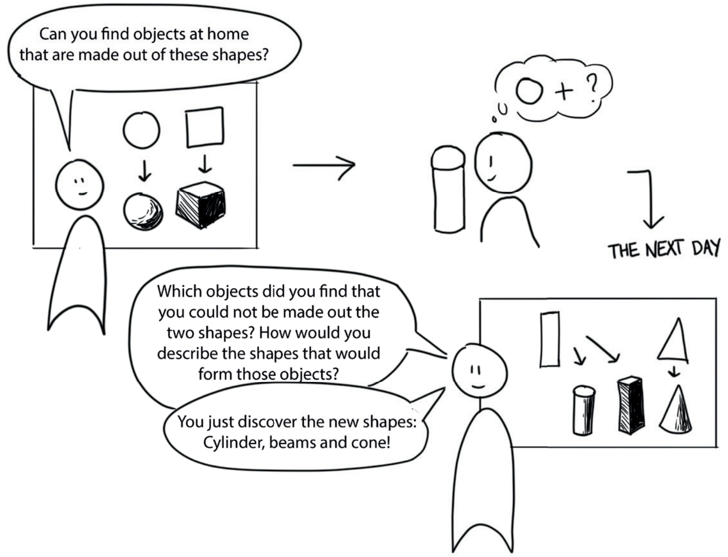


Figure 13: Idea presented for design direction 1

Direction 2: Broadening/discovering

Besides deepening, the students can also be challenged by discovering new applications of the topics they know. They can learn to answer current exercises by revising previously gathered knowledge or experiment with topics that are yet to be taught in a logical way.
Example idea : a riddle at the end of the day, that can be solved through a puzzle or exercise. The answer will be given the next day or in the next class. The riddle asks the students to think of something they learned 'long' ago or something they did not learn yet to make them broaden their knowledge.

Direction 3: Combining different domains

Thirdly, students can be challenged by combining the different domains of mathematics. Since currently, most exercises relate specifically to the domain that is handled that day or week. Combining different forms of mathematics forces students to make connections in order to understand the exercise.

Example idea: a flexible measuring tool consisting of different triangles that can be connected. This way they can function as a ruler for both straight as well as curved objects. They can be used for different domains because the triangles can function as 'cm', 'euro' or 'fraction' (if all triangles bend to the middle to form one whole shape). The triangles can be made of different sizes to increase possibilities.

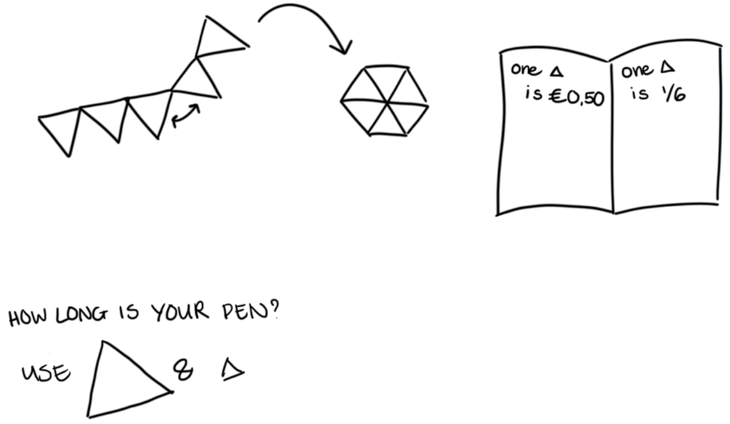


Figure 14: Idea presented for design direction 3

Direction 4: Relating to reality

Furthermore, an opportunity to improve the existing exercises for skilled mathematicians is by creating a relationship between mathematics and real life situations. There are currently exercises that do describe real life situations, however they often simplify reality or could present the importance of mathematical skills more. In that way, students will hopefully also feel the relevance of their spatial skills.

Example idea: exercise to design/build your own ‘dream’ room. This can also be used throughout different exercises and domains. For example, students are asked to start with building furniture, then measure them into the room, calculate their budget to buy different items, etc.

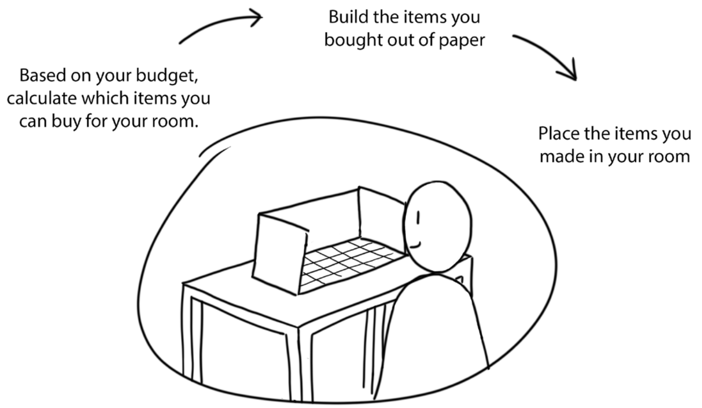


Figure 14: Idea presented for design direction 4

Direction 5: Personalising skills

Lastly, skilled mathematicians can be challenged by asking them to present their personal skills. By doing so, students can learn that even in the group of skilled mathematicians, there are differences in learning styles, thinking styles and in the type of exercises they are good at. This will hopefully stimulate them to deploy their personal skills and relate less to other students.

Example idea: characters that characterise different working fields. According to the exercises that students are good at, they can ‘unlock’ new skills that fit the different characters and thus relate their personal skills to different situations. Doing so, they can collect the skills of specific characters.

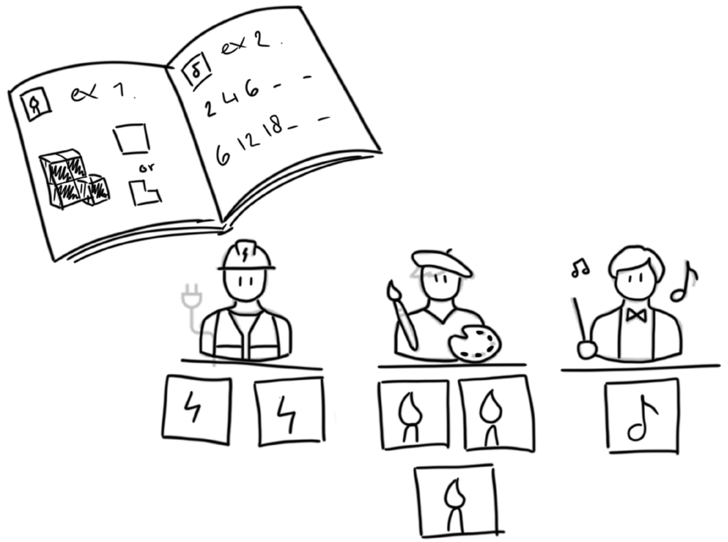


Figure 15: Idea presented for design direction 5

5.2 Comparing the design directions

In order to compare the five design directions, a Harris Profile is used, see table 3. The design directions are rated based on the wishes presented in chapter 5.1. Since every solution meets the requirements, these are excludes from the Harris profile. The wishes that are most important are placed higher up in the profile. A score is given to each direction on how well they score on each wish (ranging from ‘- -’, ‘-’, ‘+’ and ‘++’). In order to rank the design directions sufficiently, the example ideas are used as a guideline for that direction.

	Deepening questions				Broadening riddles				Combining domains				Relating to reality				Personalising skills			
	--	-	+	++	--	-	+	++	--	-	+	++	--	-	+	++	--	-	+	++
Exercise/tool refers to real life situations and allow for a feeling of connection			■	■			■			■					■	■			■	■
Exercise/tool creates an enjoyable interaction with mathematics			■				■	■			■				■			■		
Exercise/tool combines different domains			■				■				■	■			■	■			■	■
Exercise/tool provides fitting guidance for potential skilled mathematicians			■				■			■				■					■	■
Exercise/tool simplifies real life situations or skip students’ knowledge as little as possible			■				■			■					■				■	■
Exercise/tool allows for interaction between students of different levels			■			■					■			■				■		
Exercise/tool shows the importance of spatial ability in real life situations			■				■				■				■	■			■	■
Exercise/tool can be used for as many topics as possible			■				■				■	■			■				■	■
Exercise/tool can introduce future topics/exercises to students			■	■			■	■		■				■				■		
Exercise/tool inspires children to develop technical interest			■				■					■			■				■	■

Table 3: Harris Profile on presented wishes

Besides scoring well on the presented wishes above, the future design also needs to be in line with the wishes of the stakeholders of the project. Hence, a second Harris Profile is set up, see table 2. This is done for two of the three most important stakeholders: teachers and the designer, since those of children al already implemented in the first list of wishes. The light pink wishes are those of the teacher and the dark pink the ones of the designer. Again, the wishes are ranked from most to least important.

The directions 'combining domains' and 'personalising skills' and their example ideas score best overall, however all directions end up close together. These scores however do not yet include the weights of the wishes' importance. Taking those into account 'personalising skills' would score highest for the first set of wishes and 'combining domains' would score highest according to the stakeholder wishes.

	Deepening questions				Broadening riddles				Combining domains				Relating to reality				Personalising skills			
	--	-	+	++	--	-	+	++	--	-	+	++	--	-	+	++	--	-	+	++
The tool can be reused			■			■					■	■		■					■	
It takes little time to implement the tool in class			■	■			■	■			■			■					■	
It takes little time to understand how the tool should be used			■			■					■				■			■		
The tool is a physically embodied product		■				■					■	■			■			■		
The tool is visualised so that it is appealing and understandable by children		■				■					■	■			■	■			■	
The tool creates a fun/playful experience around mathematics		■					■				■				■			■		
It is easy to store the tool away after use			■	■			■	■			■			■					■	
The tool takes little storage space			■	■			■	■			■	■		■					■	■

Table 4: Harris Profile on stakeholder wishes

5.3 Towards final design

After comparing the Harris Profiles, It can be concluded that none of the design directions score significantly higher than others hence why none of them will be chosen explicitly according to the Profiles. However, an important wish of the designer was to create a physical product as final design. This is why the two ideas for which this wish was most feasible, were used as the base for future iterations. These were the ideas of a flexible measuring tool and that of designing a dream room. After considering their opportunities (Appendix I1), the idea with the best potential for a physical tool in the classroom, which was the starting point of this project, was the flexible measuring tool.

This idea was used as the base for the future design, however, it did not score excellent on each wish, as was concluded after filling in the Harris Profiles. Luckily, the other four directions score well on some of these wishes where the flexible measuring tool was lacking. Hence why some aspects of the other directions were used to improve this idea implemented in the final design. How each direction contributes to the final idea, can be seen in figure 16.

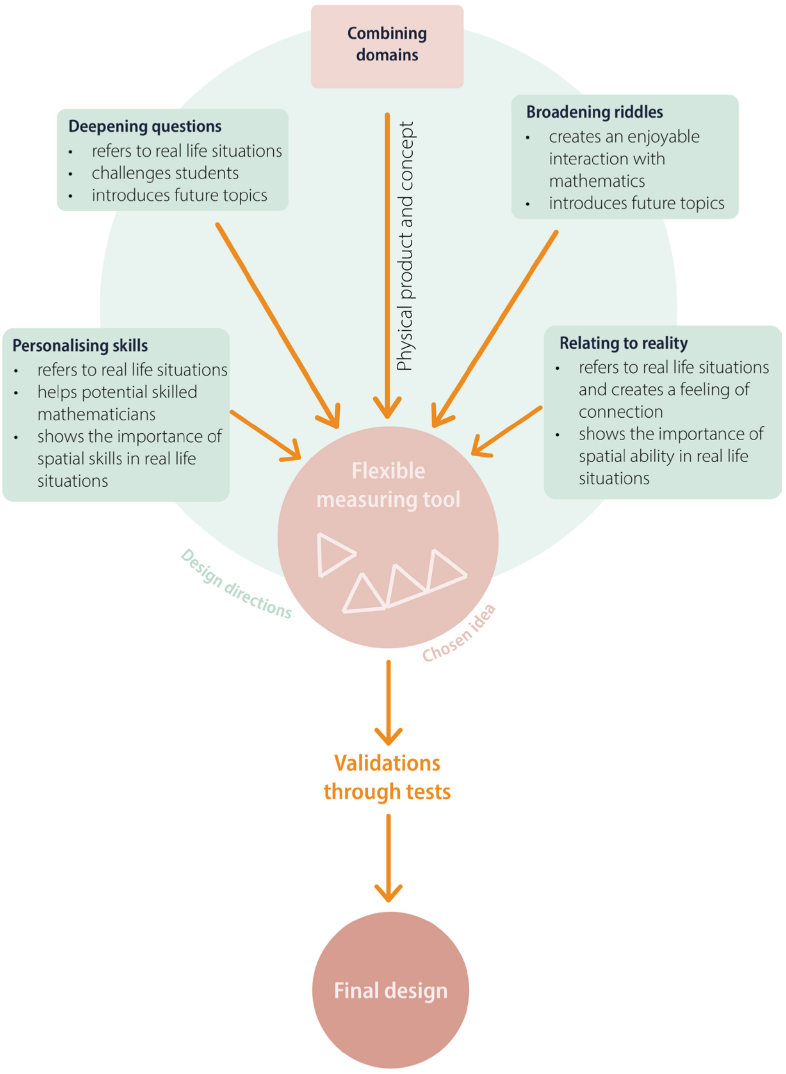


Figure 16: How each design direction contributes to the final design

Taking all these aspects, the final idea caught shape (Appendix I2) and first prototypes were made of cardboard, and later of wood, see figure 17 and Appendix I3.

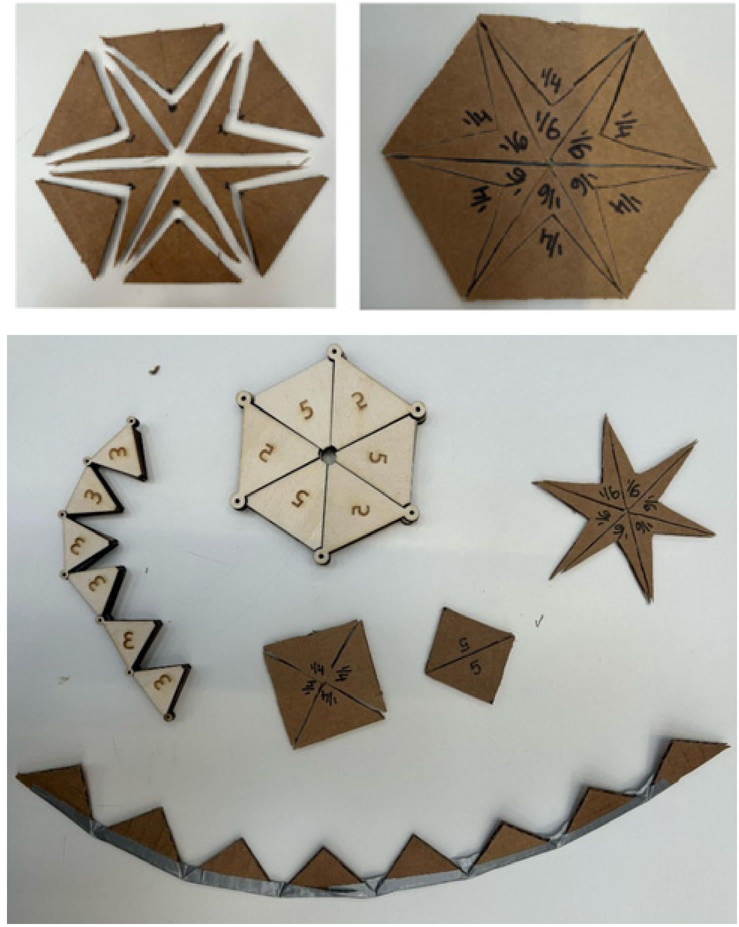


Figure 17: First rounds of prototypes

After these first prototypes, a new overview of improvements and opportunities of the idea was made, see Appendix I4. The exact improvements and how they each shape the final design will be shown and explained in Chapter 5. In figure 18 an overview of all the iterations in prototypes can be seen, where on the top the first cardboard prototypes are shown and on the bottom the final design in wood. Figure 19 shows some iterations of the map, part of the prototype that will be introduced in Chapter 6.

Figure 18: Prototype iterations on tiles

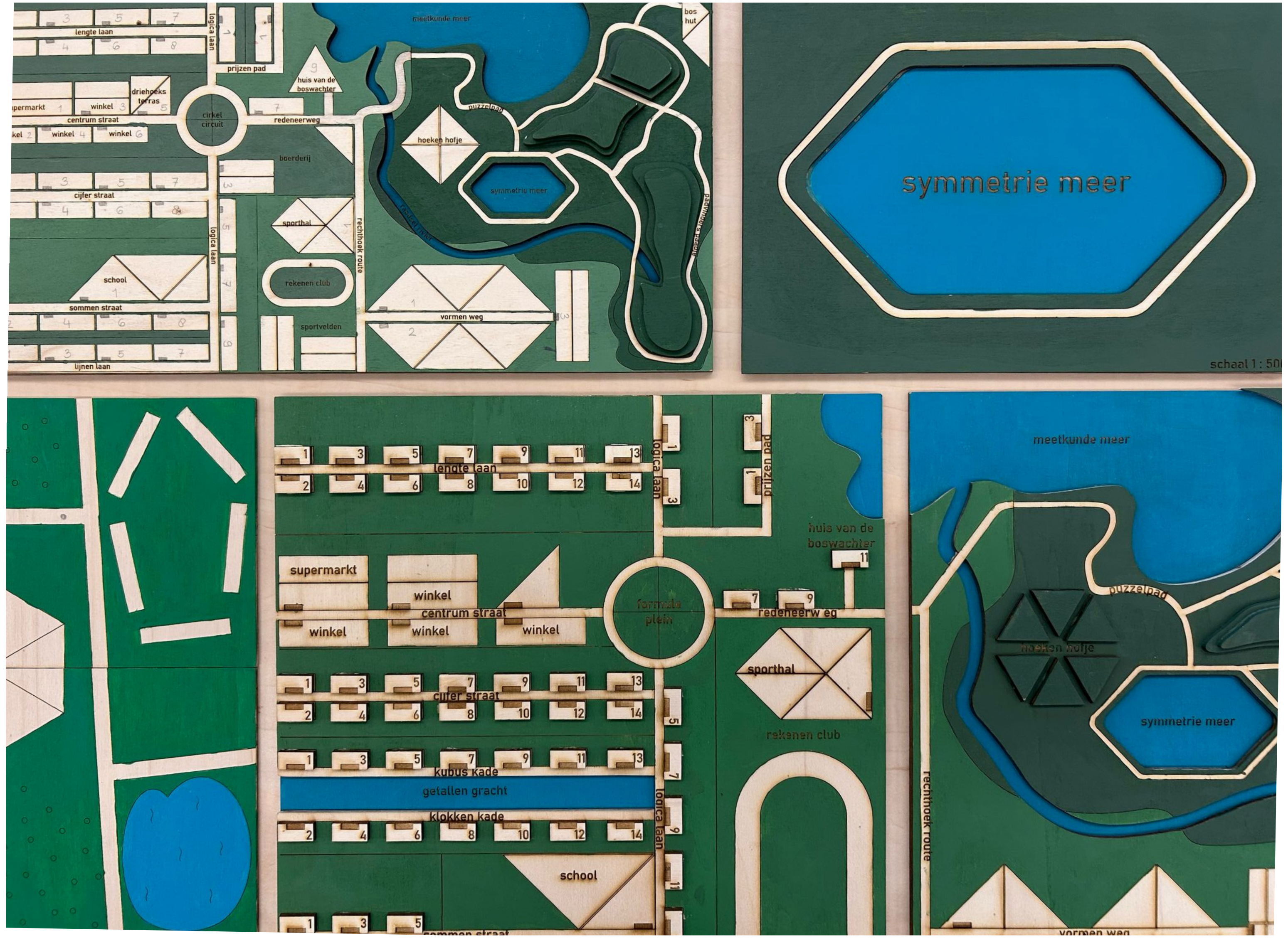


Figure 19: prototype iterations of the phsyical map (purpose is explained Chpater 6)

5.4 Testing with children

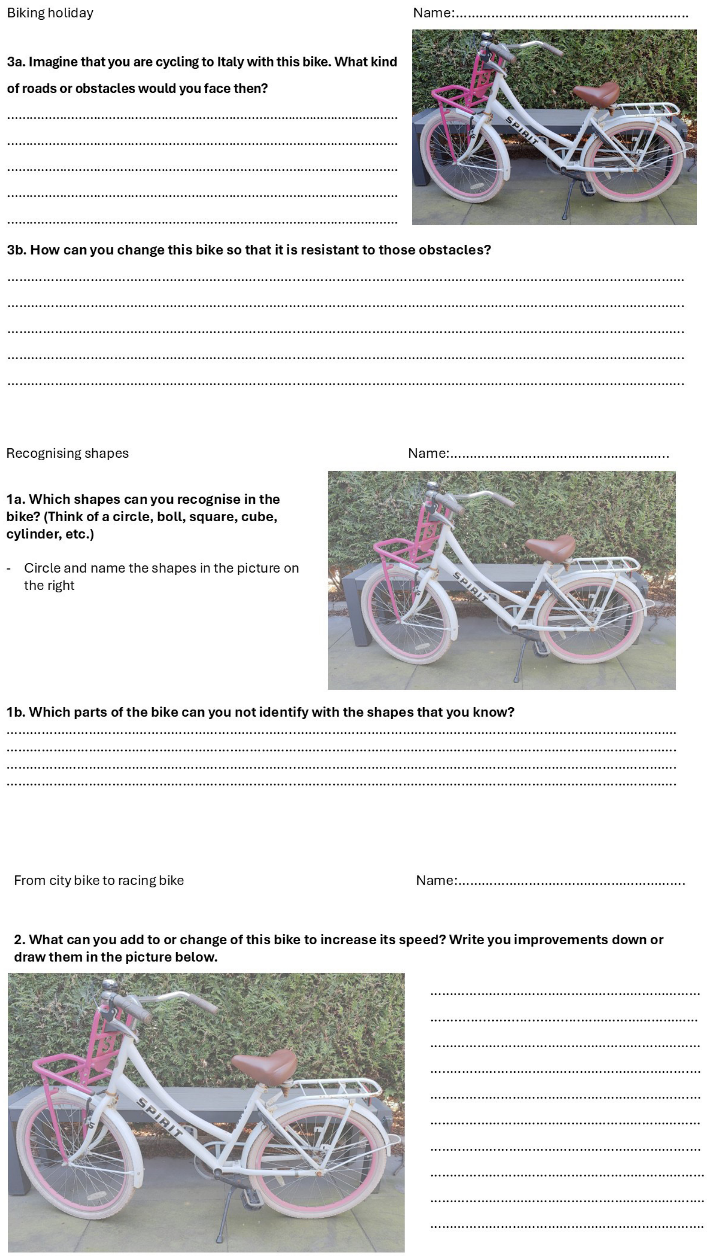
After the next iteration, the idea was tested with the target group. Two tests with children were set up to test different aspects of the idea. In order to conduct tests with children, ethical approval was obtained from the HREC of the TU Delft. Furthermore, an informed consent form was written and signed by the parents or caregivers of the children who participated in the tests, an empty version of this form is shown in Appendix G.

Test 1: Sponge Questions that spark creative thinking

Through research in this report was found that skilled mathematicians are in need of challenging exercises. One way of doing so is by providing questions that require them to think outside of the box, or combine previously gathered information with personally developed insights. For this reason I wanted to test if questions like this, related to spatial ability in some way, but different from those currently provided in most mathematics classes were indeed challenging and fun to answer for skilled mathematicians.

Besides being challenging enough for the skilled mathematicians , the questions also need to be doable by the rest of the class. This is why I came up with so called 'sponge questions'. These are open questions that students can answer differently regarding to their personal level of knowledge on the subject. This means that both skilled mathematicians and the rest of the class can answer questions related to mathematics without it being too hard or easy for either group.

I came up with three questions related to the subject of the current subject in class (repair related to bikes) that also require spatial ability. The three worksheets given to the children can be seen on the next page. The three questions in Dutch, the full explanation of the test and its results can be seen in Appendix H1. Some examples of what the children filled in can also be seen on the next page. In this class, 8 out of 20 children were skilled mathematicians.



Biking holiday Name:..

3a. Imagine that you are cycling to Italy with this bike. What kind of roads or obstacles would you face then?

....
....
....
....
....

3b. How can you change this bike so that it is resistant to those obstacles?

....
....
....
....
....

Recognising shapes Name:..

1a. Which shapes can you recognise in the bike? (Think of a circle, boll, square, cube, cylinder, etc.)

- Circle and name the shapes in the picture on the right

1b. Which parts of the bike can you not identify with the shapes that you know?

....
....
....
....

From city bike to racing bike Name:..

2. What can you add to or change of this bike to increase its speed? Write you improvements down or draw them in the picture below.

....
....
....
....
....
....
....
....
....
....

Figure 20: Sponge questions about bikes

Figure 21: Answers to exercise 1

Figure 22: Answers to exercise 2

3a. Stel je voor dat je met deze fiets naar Italië gaat fietsen.

Wat voor soort wegen en obstakels kom je dan tegen?

Krono Patje

schroef of spijker

dieren / insekte

3b. Hoe kan je de fiets zo aanpassen dat hij bestand is tegen die obstakels?

diken banden Ligt goet zadel Snelbinders

fiets pomp fiets tasen ~~tompeer~~ geld ~~fla~~ Plakertes

Figure 23: Answer to exercise 3

Some observations of the test were:

- Children filled the questions in both individually and collaboratively, however almost all wanted to fill in their own sheets.
- For exercise one, there were different approaches. Some drew the exact shape they saw, some circled every part of the bike in which they saw a shape and some drew in parts of the bike they recognised.
- All children seemed to see the photo as 2D and therefore only recognised or drew 2D shapes, instead of translating the bike and its shapes to 3D.
- For most questions, children drew on the photos, some also wrote down their answers but not everyone did so.
- For the second exercise, children asked questions to understand the boundaries of the exercise, like 'Can I also remove something?'. After hearing that there were no limits, they seemed to really think out of the box and drew everything they could think of. They also seemed to enjoy it more then.
- For the third question, the children really seemed to think out of the box, by writing and drawing many different types of obstacles and solutions.

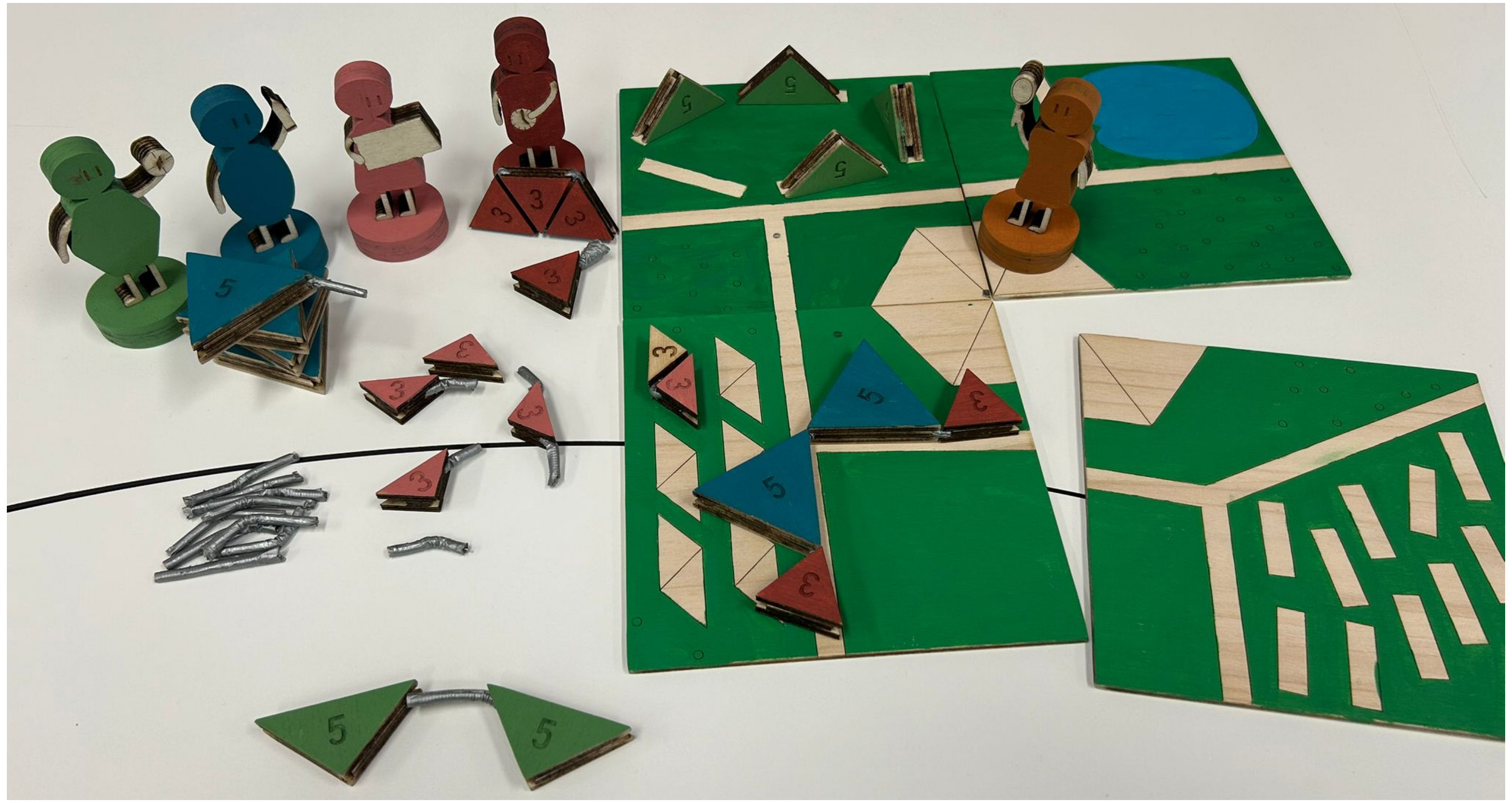
In conclusion, most children spend the full time (of 10 minutes) answering at least one of the three questions, most of them answered two or three. There was no noticeable difference between skilled mathematicians and the rest of the class on which questions they chose to answer or if they answered them by drawing, talking or writing. The only noticeable difference between skilled mathematicians and the other children is that skilled mathematicians found the questions related to 'puzzling', when they were asked to circle words related to the questions at the end of the test. 17 out of 20 children mentioned that they found the questions 'fun' (8 out of 8 skilled mathematicians did) and 0 out of 20 found them 'boring'. The rest of the words filled in by the entire class can also be read in Appendix H1.

Test 2: Testing the concept of the flexible measuring tool

For the second test I did, I wanted to test the concept of the designed idea thus far; the flexible measuring tool. The aspects I wanted to test were if using the measuring tool made sense for children to use instead of a normal ruler, if measuring with the tool was fun, if the tool was ergonomically well designed for children of 8-10 years old and if some questions I came up with had the right balance between being fun and challenging, while still fitting to the capabilities of 3rd and 4th grade students. All these aspect together could validate if the flexible measuring tool had potential for a product that can add value for skilled mathematicians.

During this test I brought my the prototype below, more photos are shown in Appendix I8. The full test set up can be found in Appendix H2. For the iteration I used for this test, I added a physical 'map', to accompany the tool and create possibilities for mathematical exercises that the tool can be used for. How this map develops into the final design can be read in Chapter 5.

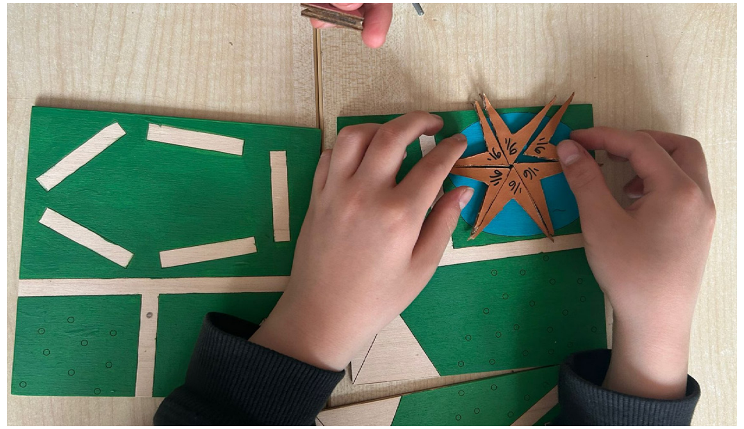
Figure 24: Prototype used in the concept test



the start of the test, I asked the children what they thought the tool could be used for. They did not recognise its measuring abilities, which led to the conclusion to write '3cm' or '5cm' at the tiles instead of just '3' or '5'. Later, I asked them to assemble the tiles together, measure a path on the map, the circumference of the lake on the map and to decide from which point on the map a specific photo was made. Finally, I asked the children's opinions on the idea, tool, map and exercises and asked them to draw possible additions for the physical map. These drawings and two children trying out to fit tiles on the map and are shown in figure 25 and 26.

Figure 25: Map ideas made by children

Figure 26: Children placing tiles on the map



The most mentionable conclusions and takeaways from this test were:

- The tiles are ergonomically fitted for 8-10 year old children, however, the connections need to be improved to not allow for deformation after many cycles of connecting tiles.
- The current exercises reached the right fit between challenging and doable, however more challenging exercises could be added.
- Adding a physical 'map' to play as an underground for exercises was a fun and valuable addition, allowing for collaboration and play. The map can be much bigger however and should include more aspects of a town.
- The flexible measuring tool is a fun addition for exercises made for skilled mathematicians, and with the correctly designed exercises around it could create fun, collaborative, challenging and playful learning opportunities.
- The children enjoyed drawing their own parts and exercises with solutions of a city so a drawing exercise will be included in the final design.
- The characters and physical map worked well to create a story/world around given exercises.

Figure 27: Overview of different design iterations

5.5 Conclusion

Through prototyping, ideating, testing and recalling research insights as well as the list of requirements and wishes, the flexible measuring tool was improved and the final design was created. This design will be shown and explained in Chapter 5. Overall, the two tests have validated the potential of the idea and thus this idea was kept and improved towards the final design. How specific prototype considerations, conclusions of the tests with children and specific requirements and wishes are used to improve the idea into its final design will also be elaborated on in this chapter.

6

The designed product: The Wednesday Afternoon Adventure Club toolkit

In this chapter, the designed product is presented, starting with an overview of the entire product and explanation of its advantages for skilled mathematicians and stimulations of spatial ability. Later, the different components of the product are explained in more detail. Throughout this chapter, each requirement or wish that the product meets (either partly or fully) is highlighted.

6.1 Overview and advantages

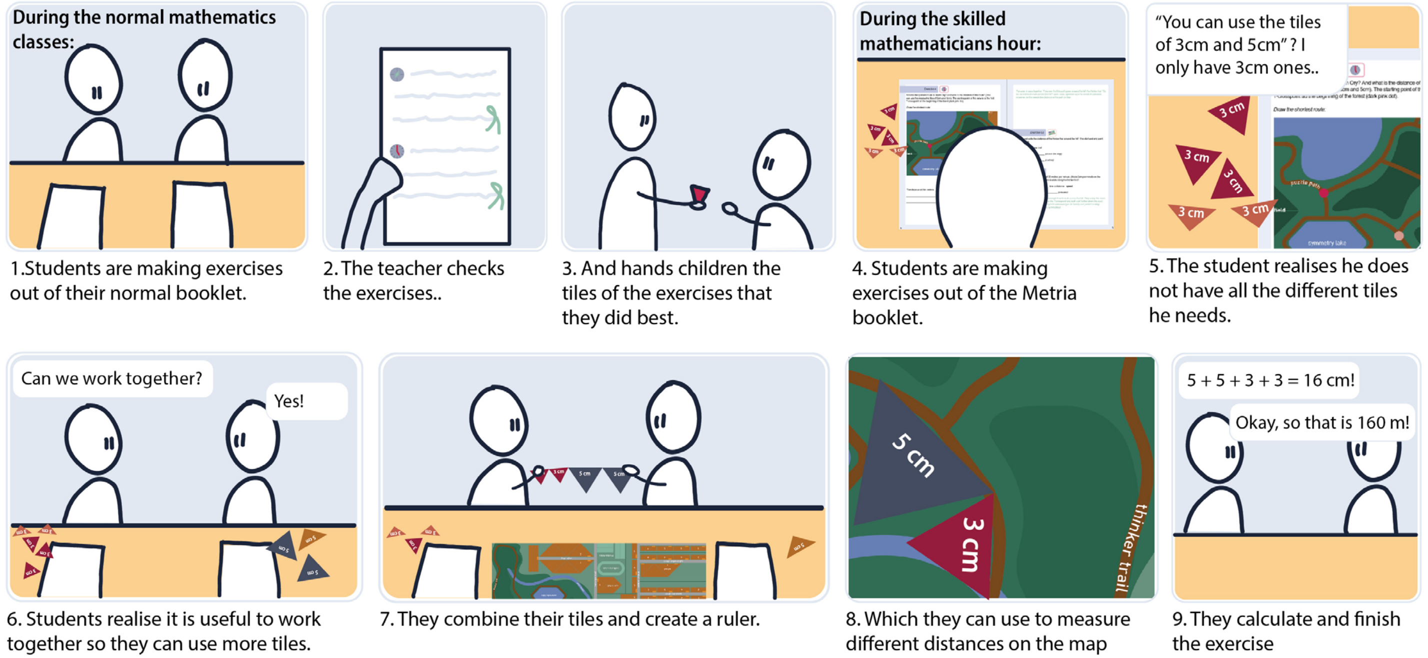
The Wednesday Afternoon Adventure Club is a toolkit that stimulates skilled mathematicians in developing their spatial skills in a physical, fun, interactive and challenging way. It consists of a booklet with exercises and additional map, characters and flexible measuring tool.

The measuring tool is made out of multiple triangular tiles. These tiles are collected by students after finishing different types of spatial ability exercises of the regular mathematics classes. Clicking multiple tiles together, these form a flexible ruler that children can measure straight and curved surfaces with, he tiles are either 3cm or 5cm long. The tiles can also be placed on the map separately to measure without connecting them.

Next to the measuring tool, a map is provided. The map forms the 'playground' for the students, providing various opportunities for exercises. For some exercises the measuring tool is needed while for others, the map alone forms as a guide to answer spatial questions. In Chapter 6.4, some specific exercises and questions around the tool and map will be shown and discussed.

Figure 28 shows the scenario of the Wednesday Adventure Club toolkit and figure 29 shows an overview of the product, a seperate wooden lake is provided for a specific exercise in the booklet, as can be seen later on.

Figure 28: Product scenario



1. Students are making exercises out of their normal booklet.
2. The teacher checks the exercises..
3. And hands children the tiles of the exercises that they did best.
4. Students are making exercises out of the Metria booklet.
5. The student realises he does not have all the different tiles he needs.
6. Students realise it is useful to work together so they can use more tiles.
7. They combine their tiles and create a ruler.
8. Which they can use to measure different distances on the map
9. They calculate and finish the exercise

Figure 29: Product overview of the Wednesday Afternoon Adventure Club toolkit

6.1.1 How the product benefits skilled mathematics

The toolkit has multiple advantages for skilled mathematicians. First of all, the product allows for a playful way to experiment with mathematics while still providing challenging exercises, because skilled mathematicians are in need of more challenge. Most exercises in mathematics books at school ask students to measure distances on a provided scale, mostly on straight surfaces. With this product, students are asked to measure both straight and curved surfaces as well as to add multiple measured surfaces together. Furthermore, the tool tests spatial ability since it differs from a normal ruler. In contrast to normal rulers, this tool does not have an indication of each centimetre, but only shows if an entire tile is 3 or 5 cm long. This challenges students because they need to indicate the exact measurement themselves. How the product challenges students in other aspects of spatial ability will be explained in the next paragraph.

Requirement: Exercise/tool should challenge the students further than the exercises in the current methods do

A second difference of this product compared to most mathematical exercises that test students' spatial ability is its transparency around learning spatial skills. In current classes, the word 'spatial ability' is barely mentioned and when making exercises of different spatial skills, students are not aware of which skill they are learning. Exercises are roughly divided in orientation, rotation, visualisation, spatial connections and symmetry, however these words are not mentioned, nor are their importances and applications explained. This product narrows the gap between spatial ability exercises and their relevance for everyday situations by visualising the spatial skills in the form of five characters. Giving these characters' characteristics and interests that fit the spatial skill they represent, students can gain understanding of their implications and meaning. How exactly the different skills are characterised is explained in Chapter 6.3.2.

Wish: Exercise/tool should show the importance of spatial ability in real life situations

In addition, the product creates another connection between mathematical exercises and real life situations through a physical map. This map forms the direct environment ('playground') of exercises, accompanied by a story that incorporates the characters mentioned above. The map illustrates an imaginative city. Through the additional story and the physical map, real life situations are resembled. The exercises that follow can then create an understanding of the importance of spatial skills in real life situations. Furthermore, the map and exercises stay as close to the real life situations they reassemble as possible because simplification of real life situations was found to interfere with skilled mathematicians' knowledge and thus confuse them while making such simplified exercises. Which means that the measurements and other values mentioned in the exercises represent realistic numbers. The visualisation of the map and its possibilities for different exercises are shown in the coming chapters.

Wish: Exercise/tool should refer to real life situations and allow for a feeling of connection

Another important need for skilled mathematicians is the possibility for interaction, collaboration and evaluation during and after doing an exercise. This product answers the need for interaction and collaboration by creating a playful and physical learning environment, encouraging students to collaboratively experiment with the provided map and tool. Because of the individual collecting system of the different tiles during class, students are encouraged to collaborate with each other in order to have access to as many, different, tiles as possible when they use them for measuring. Adding multiple tiles together creates a longer ruler, needed for longer distances and adding different tiles together creates a more specific ruler that creates a more precise tool to measure which needed for specific shapes or distances, see figure 30.

Requirement: Exercise/tool should allow for collaboration between students

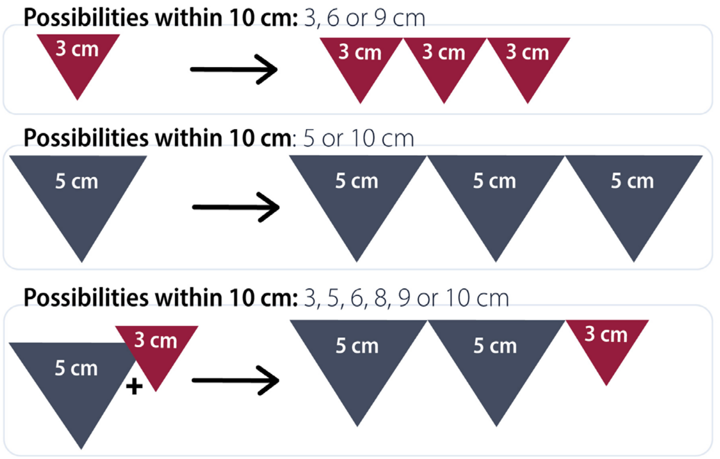


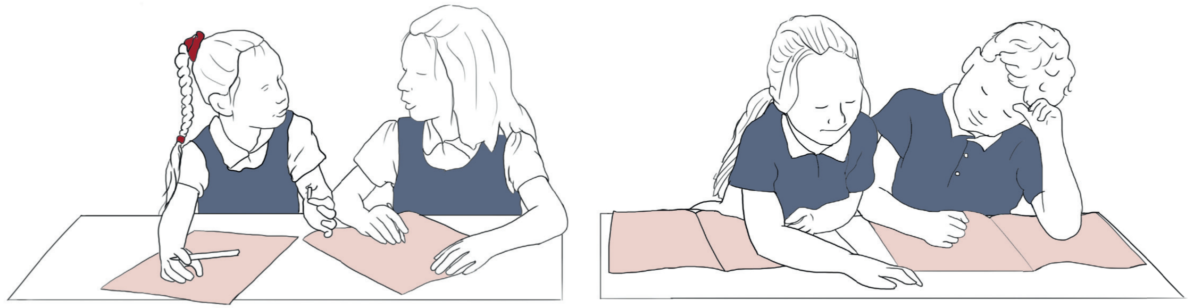
Figure 30: Measuring possibilities of 3cm and 5cm tiles

Why specific sizes and shapes are chosen for the tool and how these can be used for specific exercises is explained in Chapter 6.2.1. In addition, the collecting system asks students to reflect on their personal spatial skills since they receive tiles that belong to the exercises of the spatial skills that they perform best at. Visualising the different skills with different colours and reading about the personally traits that the different characters have, students can understand what it means to have specific spatial abilities. Furthermore, students are also stimulated to reflect on their answers because the exercises follow each other through a story, which requires students to look back on their previously given answers.

Requirement: Exercise/tool should allow for reflection

The last most important advantage of this product for skilled mathematicians, in comparison to existing materials, is that it is usable for different mathematical domains. Thus far, most exercises provided by educational methods focus on one out of four domains (numbers, proportions , measurement/geometry and connections). Even exercises specifically designed for better performing students lack in combining many different subjects and skills at once. In the booklet created around this product, different types of exercises follow after each other (see examples in Chapter 6.4). By implementing the different challenging aspects and the possibility for reflection and interaction, the tool fits to the most important needs for all skilled mathematicians, visualised below. How the toolkit fits to the need for instruction and guidance from teachers, is explained in Chapter 6.4.1.

Requirement: Exercise/tool should be challenging and stimulating for quick and creative mathematicians, alongside that of the good ones



Quick mathematician
- Focus on process
- Explanation on working strategies

Potential skilled mathematician
- Guidance
- Exercises that support personal development

Good mathematician
- Deepening exercises
- Extra challenges

Creative mathematician
- Logical real life exercises
- Guidance to understand the existing methods
- Challenges to use their gained knowledge

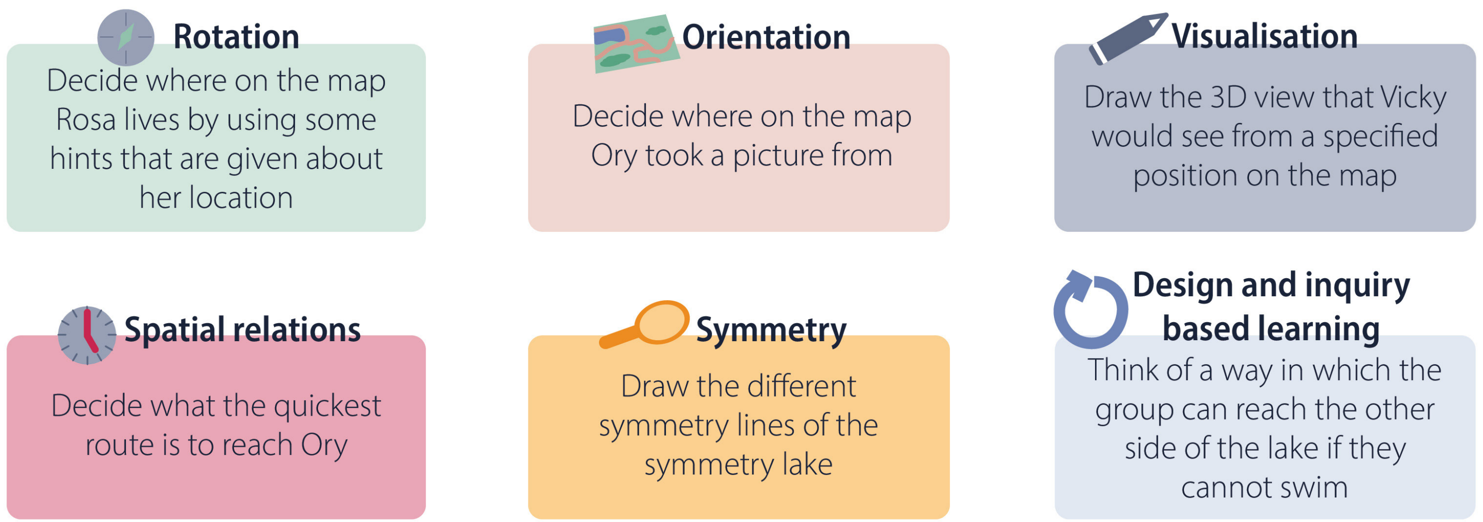
All
- Scheduled instruction moments
- Discussion and reflection moments
- Guidance from teacher
- Extra challnges
- Interaction with each other

Figure 31: Needs of different types of skilled mathematicians

6.1.2 How the product teaches spatial ability

As mentioned, the toolkit guides children in developing their spatial skill in different ways. The physical product and map are designed so, that there are multiple possibilities for exercises around different spatial skills. To test each of the five different skills, a fitted exercise is shown in figure 32. Furthermore the exercise booklet has an exercise that inspires design and inquiry based learning. Design and inquiry based learning itself includes all five spatial skills as well.

Requirement: Exercise/tool should improve children's spatial ability and test different spatial skills



Rotation
Decide where on the map Rosa lives by using some hints that are given about her location

Orientation
Decide where on the map Ory took a picture from

Visualisation
Draw the 3D view that Vicky would see from a specified position on the map

Spatial relations
Decide what the quickest route is to reach Ory

Symmetry
Draw the different symmetry lines of the symmetry lake

Design and inquiry based learning
Think of a way in which the group can reach the other side of the lake if they cannot swim

Figure 32: Example exercises in the booklet of different spatial skils

Because of the aspect of reality that the map adds to the product, students are confronted with realistic situations wherein spatial ability is needed. Besides, the characters with spatial ability related characteristics create a feeling of relatedness for the children and help them to recognise spatial characteristics within themselves. Both this environmental and human aspect can form understanding of spatial ability and hopefully increase the possibility of using spatial skills when the students face likewise situations in real life. Furthermore, the exercises provided by the booklet are logical results of the story around the map and characters and contribute to the understanding of the need for spatial skills.

In contrast to most spatial ability exercises in the existing curriculum, this product helps children to develop spatial skills in a physical way. In their mathematics books, they face exercises that are similar to the ones in the exercise book of this product, since the booklet should fit to and is inspired by existing educational methods of the 3rd and 4th grade. However, the difference with the new exercises is their physical component, namely the measuring tool and physical map. This 3D learning experience allows the students to 'move around' the situation presented in the exercise instead of only reading and seeing it as they would during the usual 2D learning experience. As mentioned earlier in the report, visualisation and experimentation are highly beneficial for the development of children's spatial ability.

Requirement: Exercise/tool should allow for interactive learning (use of external tools)

Requirement: Exercise/tool should allow for creative learning

6.2 Zooming in: The world

As mentioned before, the map illustrates an imaginative city and forms the playground for the booklets' exercises. In addition, the characters resemble the human aspects of their represented spatial skills.

6.2.1 Map

In the imaginative world, the five presented characters go on different adventures. A story with fitting exercises is made for each adventure. The map visualises the environments of the different adventures. For this graduation project, one story is written for which a booklet with fitting exercises is created. For future expansion, multiple stories with new exercises can be created. If needed, the map can than be expanded or new maps can be created if needed to support those stories.

To create more relatedness to spatial ability and mathematics in general, all names on the map have a mathematical aspect, such as symmetry street or angle alley, see figure 32. These names are mentioned in the story and exercises in the booklet, allowing students to follow the story on the map.

The map is made of two parts, which together has the measurement of 35 by 66 cm. The map is made out of multiple layers of plywood, these different layers add to the spatial reality of the map since it allows for height differences in the environment. On the map, different parts resemble the tiles of the measuring tool by sharing their measurements. This allows for children to place the tiles on the map in order to represent objects in 3D, such as houses. The map is made on a 1:1200 scale, mentioned in the booklet and engraved on the map. Because everything on the map fits the same scale and resembles measurements of real life objects, children face realistic numbers, measurements and situations while conducting the exercises.

Some elements on the map have the exact measurement of the measuring tiles, giving the possibility to build on the map with these tiles are letting the tiles resemble these elements. Examples are the school, stores, sports hall and fraction field bushes. The houses do not have measurements of the tiles since that would form unrealistic sizes. Distances that have been referenced to generate this map are: average width of a road and forest path, the width and depth of a canal, the depth of a lake, the height of a house and the distances of an athletics court. All these measurements are referenced for Dutch ones, since the story resembles an imaginative city in The Netherlands.

Wish: Exercises should simplify real life situations as little as possible and stimulate students' knowledge

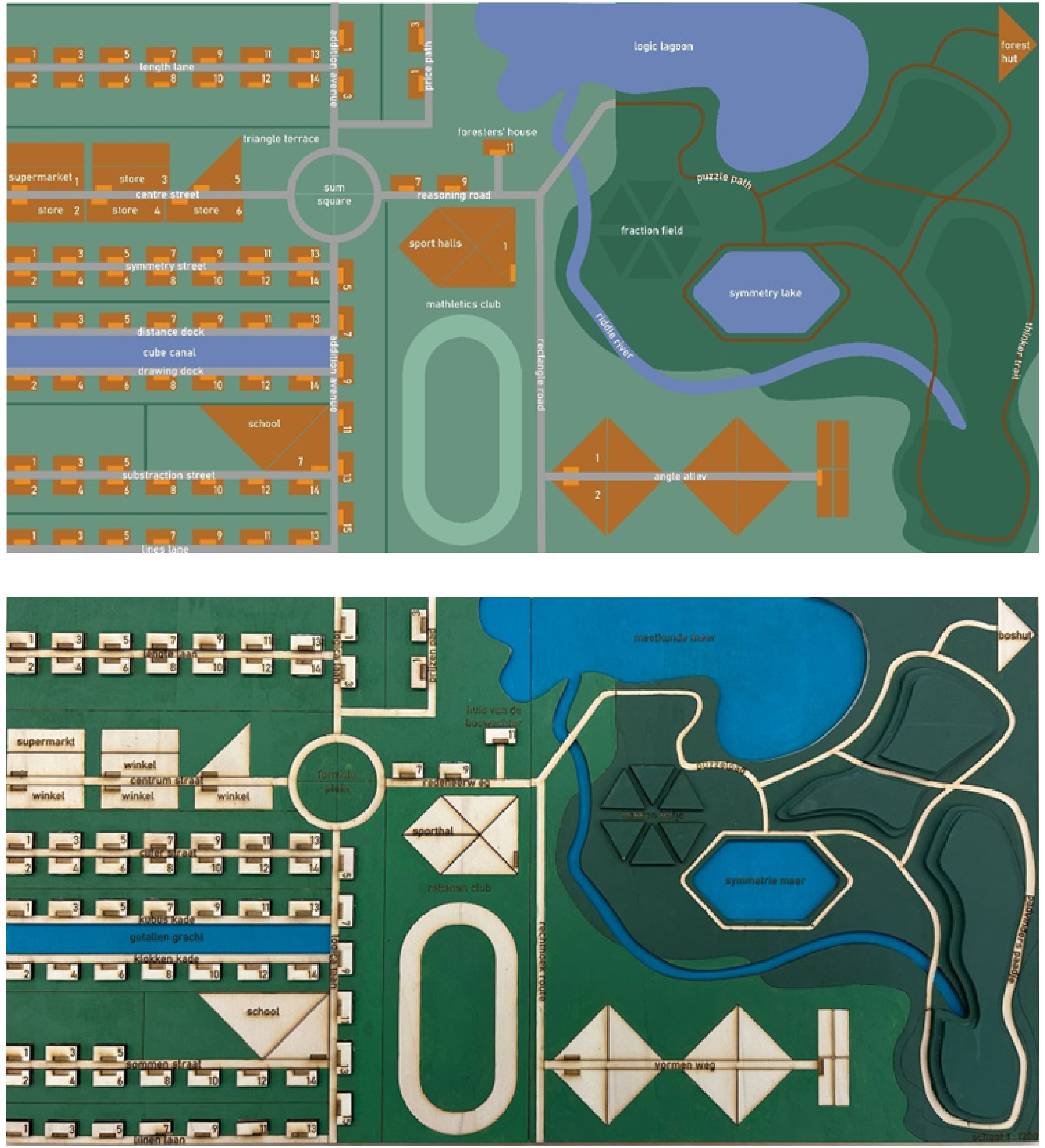


Figure 33: The map of the Wednesday Afternoon Adventure Club city

6.2.2 Characters

Next to the map, the presented characters also add to the concept of the 'world' and its story. As mentioned, there are five characters, each presenting a different spatial skill, see figure 34.

By embodying the different spatial skills that children face during mathematics exercises in class, the distinction between them is made clear. To show the different kinds of skills and introduce the characters, children will receive tiles associated with one of the five characters for the kind of spatial exercises they perform best at.

While making their usual mathematics exercises, students will learn which exercises belong to which spatial skill. Each day, students will collect one tile for the kind of exercise they finished best that day. If this was an orientation exercise, they will receive a tile from the orientation character, see figure 35. For one of the skills, the students will receive connection pieces instead of a tile, so each skill has their own reward. This could increase the need for collaboration since connection pieces are most necessary.

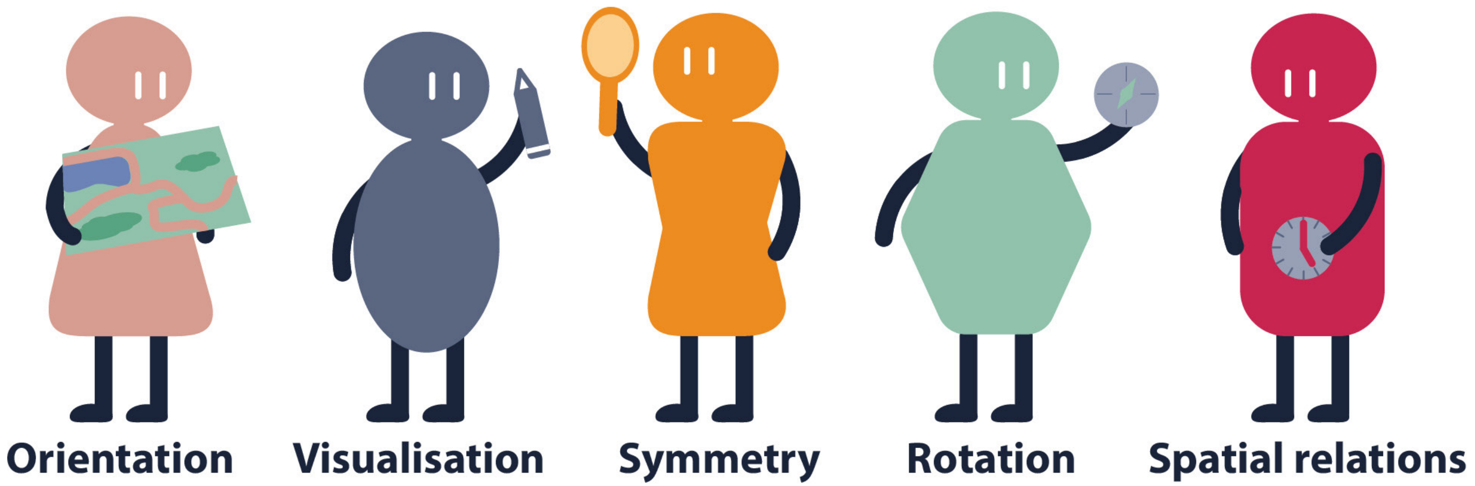


Figure 34: The five characters, each presenting a different spatial skill

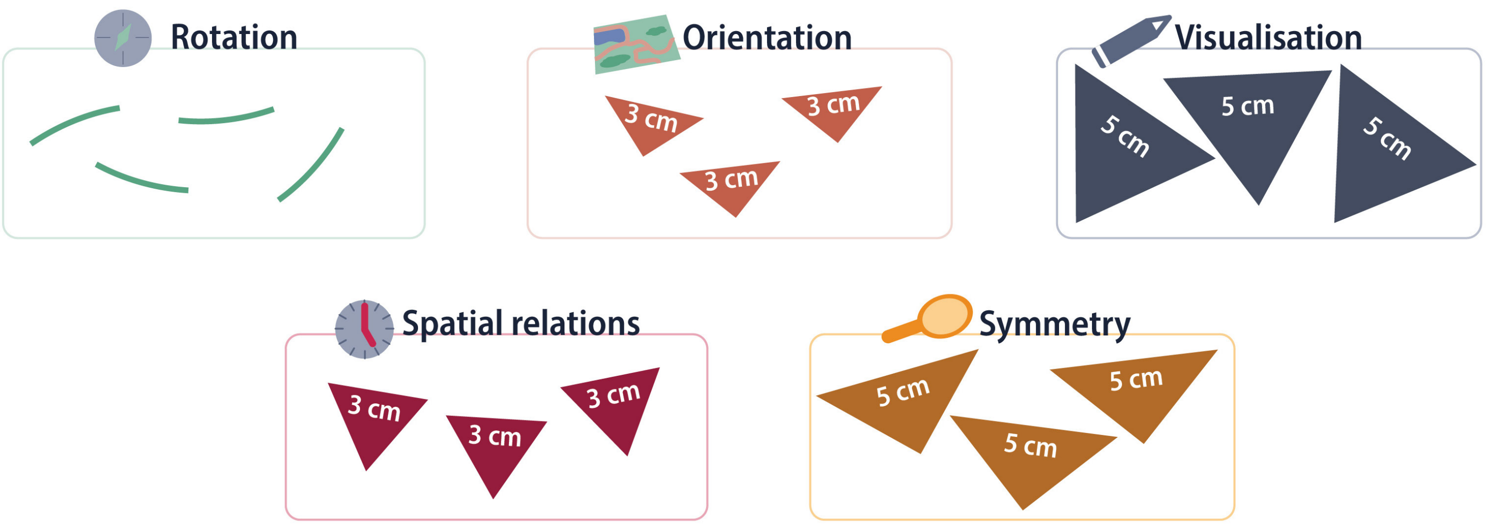
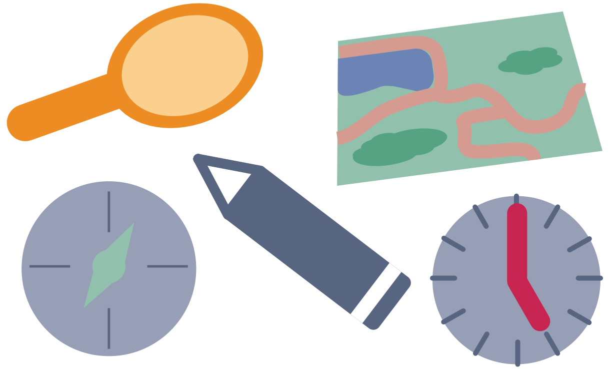
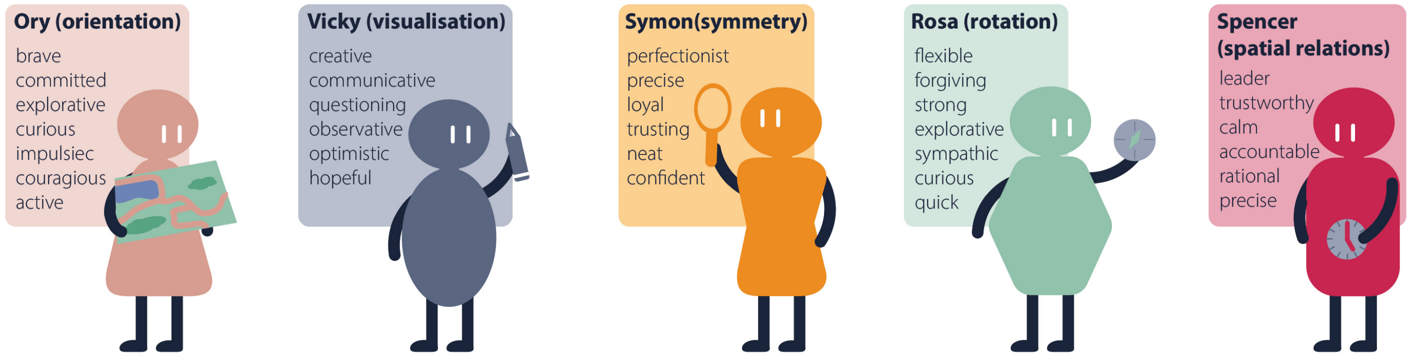


Figure 35: The different pieces that can be collected per spatial skill exercise

To increase the reality and relatedness with spatial skills for the students, the characters have been given specific characteristics based on the spatial skill they represent. These characteristics will hopefully spark recognition for the children and show them the human aspect of having sufficient spatial ability.

The characteristics also allow for personal storylines within the exercises, read figure 36, Furthermore, the characters' names are based on their spatial skill and are each given an accessory that resembles their skill and personality. In the recommendations (Chapter 8.1) it is mentioned that real versions of the accesories of the characters can be included in the toolkit.





Ory (orientation)
brave
committed
explorative
curious
impulsiec
couragious
active

Vicky (visualisation)
creative
communicative
questioning
observative
optimistic
hopeful

Symon(symmetry)
perfectionist
precise
loyal
trusting
neat
confident

Rosa (rotation)
flexible
forgiving
strong
explorative
sympathic
curious
quick

Spencer (spatial relations)
leader
trustworthy
calm
accountable
rational
precise

- Always seeking for adventure
- Not afraid/takes other under his wings
- Can loose track of time
- Always wants to take pictures of his surroundings and the others
- Does not know what is manageable / aims for too much
- Can loose track of details

- Follows Spatial relations and trusts him everywhere
- Is scared of sudden changes
- Sees beauty in everything
- Eye for detail

- Follows Spatial relations and trusts him everywhere
- Is scared of sudden changes
- Sees beauty in everything
- Eye for detail

- Seeking for adventure, together with Ory
- Where orientation looses track, orientation finds the way back
- Always sees the positive side of the story
- Is therefore often forgiving and listens to the others
- Carrier of equipment on an adventure

- The planner and leader of the group
- Makes sure everyone is on time and prepared
- Is very thrustworthy and therefore the others listen to him
- Stays calm in stressful situations
- Finds it hard to think of innovative ideas in is not good at thinking outside of the box

Figure 36: The five characters with personal characteristics and storylines

Figure 37: Characters as included in the toolkit

6.3 Zooming in: The flexible measuring tool

Besides the map and characters, the toolkit includes a flexible measuring tool, consisting of different types of tiles that can be assembled together to form a ruler, both measuring straight and curved surfaces (figure 38 A). Throughout two validation tests (Chapter 7.1), the conclusion was drawn that it is not necessary to connect the tiles for every exercise, they can also be used separately and placed on the map next to each other.

Besides forming a ruler, the tiles can be used to count different fractions of a full circle (figure 38 B) or they play a role in other exercises of the booklet by resembling objects of the exercise (figure 38 C).

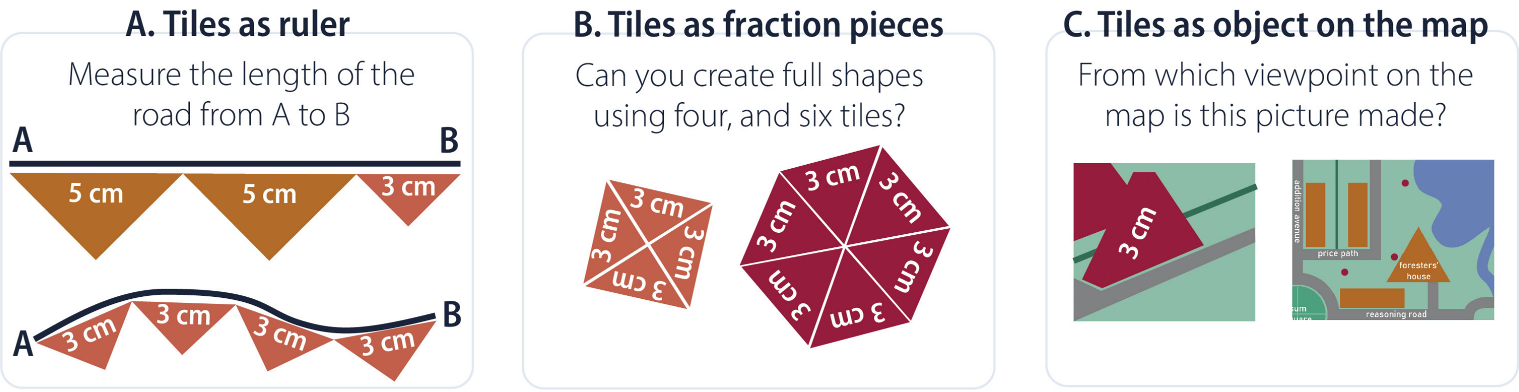

A. Tiles as ruler
Measure the length of the road from A to B
B. Tiles as fraction pieces
Can you create full shapes using four, and six tiles?
C. Tiles as object on the map
From which viewpoint on the map is this picture made?

Figure 38: Different applications of the tiles

There are four types of tiles, differing in length and shape. Through prototyping and creating assemblies of different sizes and shapes, the four types were chosen. First of all, the main shape of the tiles was chosen to be a triangle since this shape allows best for the applications of measuring straight and curved surfaces, see Appendix I5. The main measurements of the triangles (3 and 5 cm on the largest side) were chosen due to being the right combination between being ergonomically fitted for 8-10 year old children and having measurements that are useful to calculate with, Appendix I3.

The specific measurements of the triangles were chosen after creating different sized triangles and using those to create different fraction circles, Appendix I6. With the two shapes of tiles, different fractions circles can be made without having too many different tiles, figure 39. Larger circles than 6/6 can also be made, by creating more space between the tiles, in the same way as shown in 5/5 circle. As shown in the figure, for the 3/3 circle, two tiles together form 1/3.

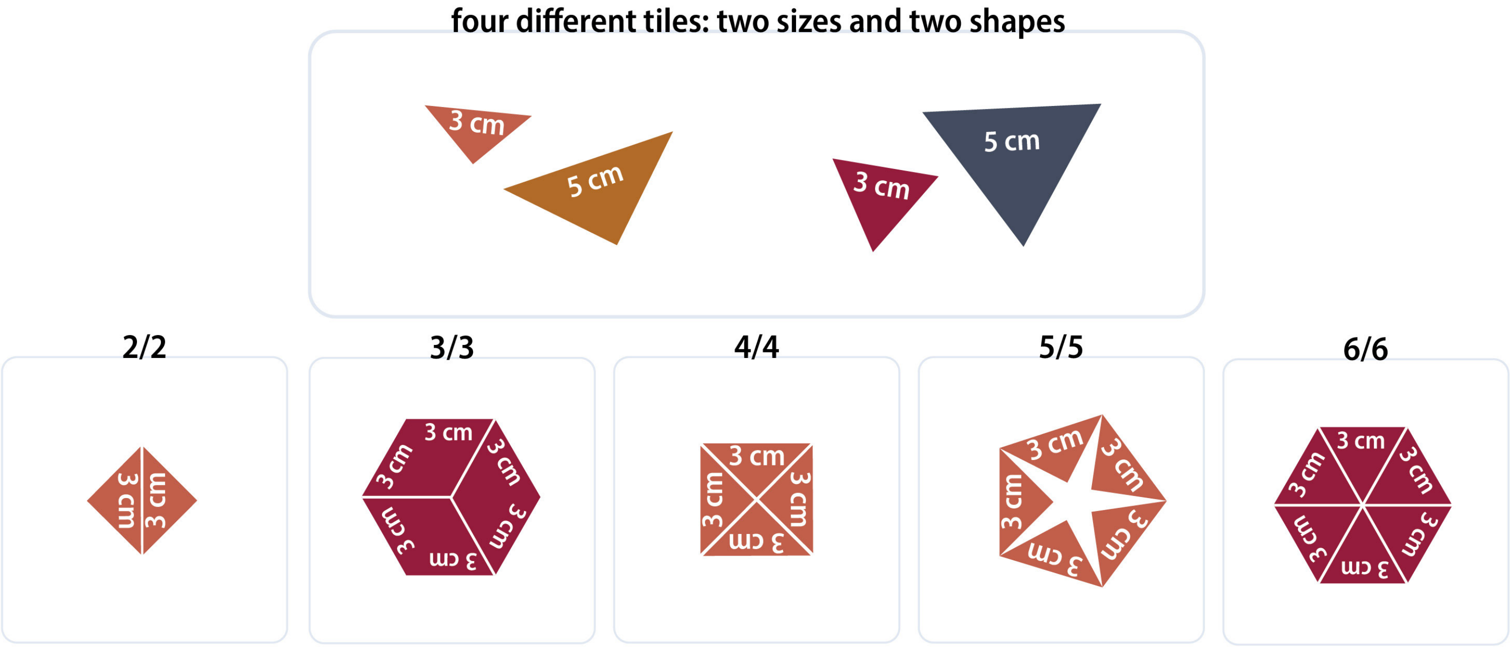

four different tiles: two sizes and two shapes

Figure 39: How the tiles can from different fraction circles

The different tiles are connected through each other by iron wires wrapped in plastic, see figure 40. This connection method was chosen after considering different options (Appendix I7) since it scores best on the most important requirements for assembling.

These requirements are:

- The tiles should function as separate pieces as well as a ruler when assembled together
- Assembling multiple tiles should be done quickly
- Assembling the tiles should be done by a 8-10 year old child and therefore not require too little pieces
- When multiple tiles are assembled, the largest side of the tiles should be completely straight, allowing for sufficient measuring
- When multiple tiles are assembled, the corners of the tiles should come together perfectly, both when aligned straight and when slightly bended, allowing for sufficient measurement

The tiles are laser cut out of wood. This method is chosen since it is a quick and simple way to create many tiles that are exactly the same. Besides, laser cutting allows for precise engraving of '3 cm' and '5 cm' on the tiles. One tile is made out of three different layers of 3,2 mm plywood. The middle layer has slots cut out of the wood that create a hole in the tiles allowing for the wires to fit in. Wood is a sustainable and durable material, that is easily accessible and cheap to laser cut. Besides, creating the tiles of wood creates a robust look and feel, resembling existing blocks that children play with. The wooden tiles are painted into different colours so that they fit the different spatial skills. The connection wires are existing wires that are used for gardening. That is why they are easily accessible and the plastic layer is already connected to the iron wires.

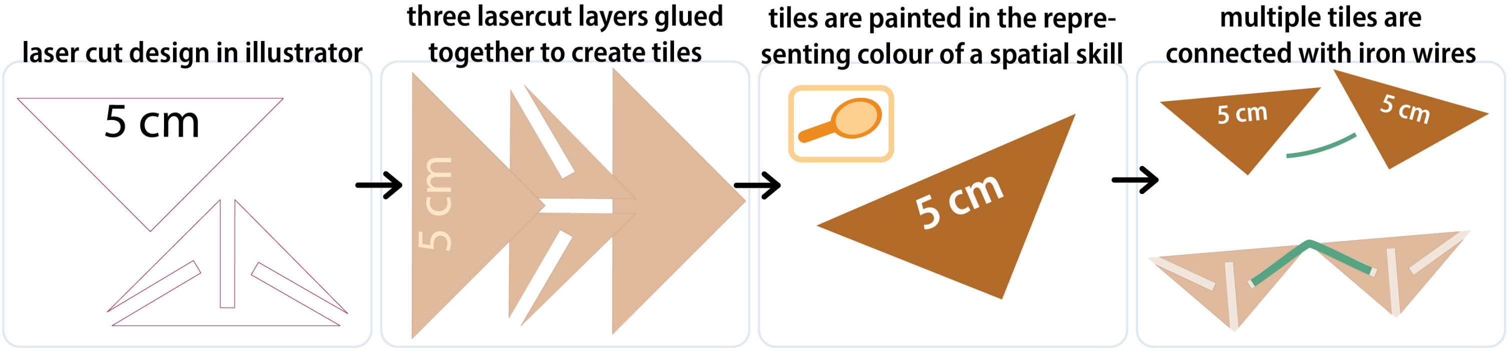


Figure 40: Laser cut and assembly process of the tiles

For the client, laser cutting the tiles in plywood is also a good fit since different schools that are interested in the product could then request the product to be laser cut. Science Hub TU Delft has access to plywood and laser cut machines and could possibly send the assembled product by post to different schools, since the tiles are 9,6 mm thick, which is thin enough for a postal package. Furthermore, Science Hub can collaborate with Science Centre TU Delft to laser cut and assemble the map and tiles. Exact arrangements on laser cutting, assembling and painting of the product need to be discussed further with the client.

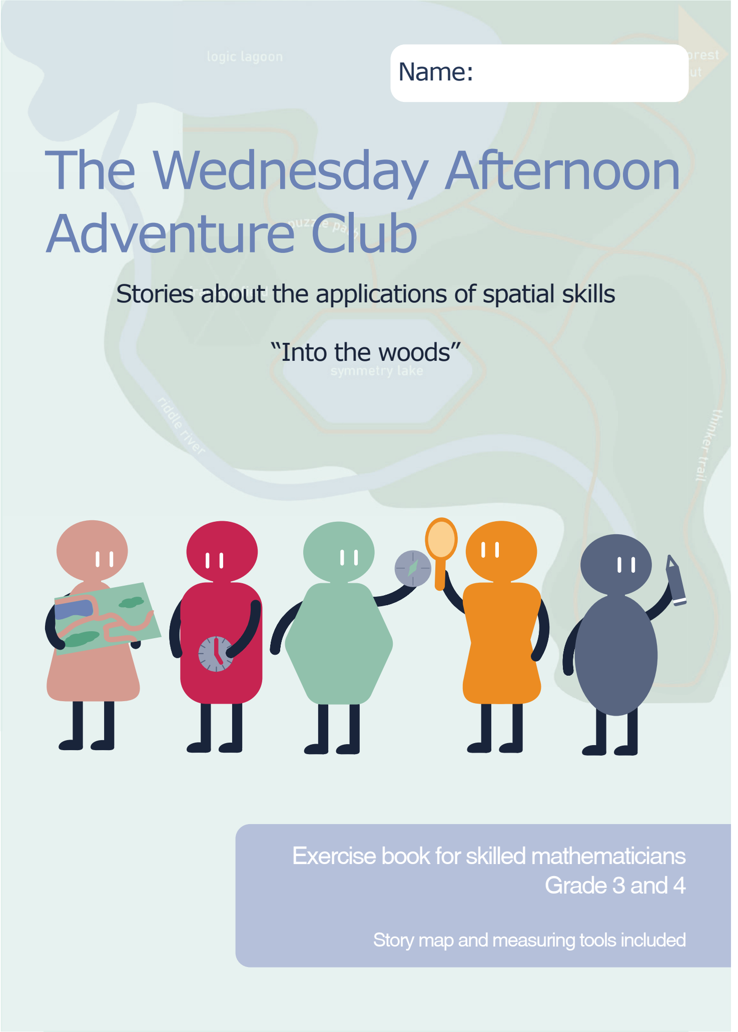
After meeting with experts to discuss this product, a comment that was made by two of them is that the measuring tool would not be necessary for the toolkit and that children could also use a normal ruler. However, during both validation tests done with children, it was observed that they did use the tiles in the right way and that it did add an extra challenge to count the measurements of each tile instead of reading the distance at once (as would have been the case with a ruler). Besides, using the tool adds an interactive element because the children can work together by all placing some tiles and holding them in the right spot for each other to add on to, as was also observed.

The tiles can also add a playful element to mathematics, since they can 'build' the ruler. One students said 'Can I please keep this piece, because it is nice to play with' when the tiles were taken away to create space for a design exercise during the test. Furthermore, for future expansion of the booklet, more exercises could be added for which students need the tiles to make certain buildings or objects on the map. Taking these meetings, observations and recommendation considerations in mind, the tiles are kept as part of the toolkit.

6.4 Zooming in: Exercise booklet

A booklet is created with exercises that combine the story of the characters, the map and the measuring tool, see figure 41. The exercises include the five different spatial skills, as well as design and inquiry based learning exercises, and are inspired by existing exercises of the 3rd and 4th grade. The exercises are based on the mathematics learning objectives of the 3rd and 4th grade, however they are more challenging then existing exercises since that was one of the most important requirements in order to create fitting material for skilled mathematicians. The name 'The Wednesday Afternoon Adventure Club' is chosen to relate the characters to primary school students. In The Netherlands, primary schools end before lunch on Wednesdays, allowing for an afternoon of adventures.

Requirement: Exercise/tool should provide opportunities for exercises that fit 3rd and 4th grade learning objectives of Dutch educational materials



Name:

The Wednesday Afternoon Adventure Club

Stories about the applications of spatial skills

"Into the woods"

Exercise book for skilled mathematicians
Grade 3 and 4

Story map and measuring tools included

Figure 41: Front page of the exercise booklet

The story written for this project is 'into the woods', where the characters travel from town to the woods and face different challenges on the way. In future design, multiple different stories can be written about the characters going on different adventures. Nine exercises are created for the booklet, tackling different spatial skills and different domains of mathematics. The booklet presents the story 'into the woods' by comic illustrations of the characters in the world that the map represents. During issues they face in the story, an exercise is presented in the booklet. With these exercises, children can feel like they are solving an issue that the characters are facing in the story. During some of the exercises, the measuring tool is required, together with the physical map. During other exercises, students face questions that require other types of spatial thinking. The language and questioning and answering formats have been tuned to existing exercises of the 3rd and 4th grade in The Netherlands. Furthermore, the questions have been written in such a way that misinterpretation is avoided, which is discussed with multiple experts.

Requirement: Exercise/tool should be linguistically clear and not allow for misinterpretation

On the first page of the booklet, the five characters are introduced and on the second page a legend of the map is shown and the story begins, The booklet consists of nine exercises shown on page 99-103. As mentioned before, the exercises are formed to test different spatial skills as well as other mathematical skills out of different domains. The last three exercises test visualisation skills and design and inquiry based learning. These three exercises take more time and can be done if the students have time left after finishing the other exercises. Some of the exercises recall topics that the students have learned before, besides the booklet allows for introduction of future topics.

Wish: Exercise/tool can be used for more than one exercise/subject or as many subjects as possible

Wish: Exercise/tool should combine different domains or subjects

Wish: Exercise/tool can introduce future topics/exercises to students

Figure 42 shows how the tiles are used in combination with the map and figure 43 shows a seperate map of the lake that is provided for children to use for exercise 9.

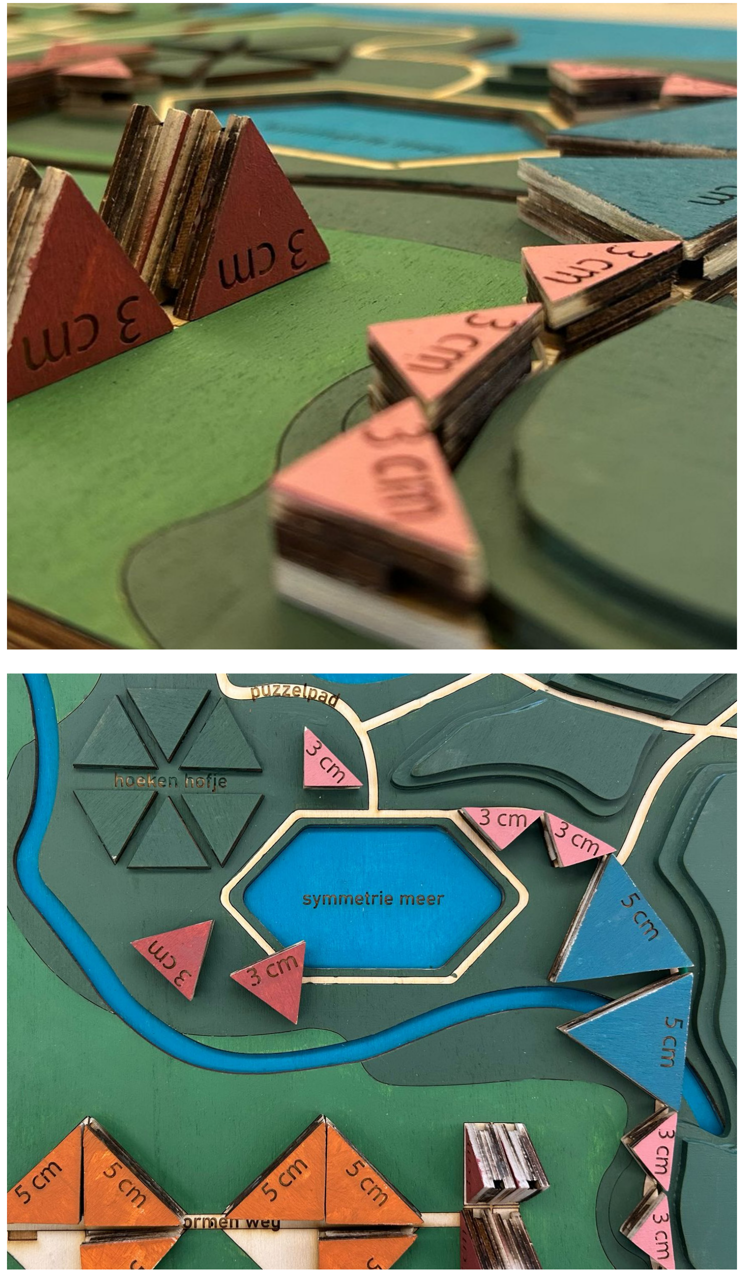
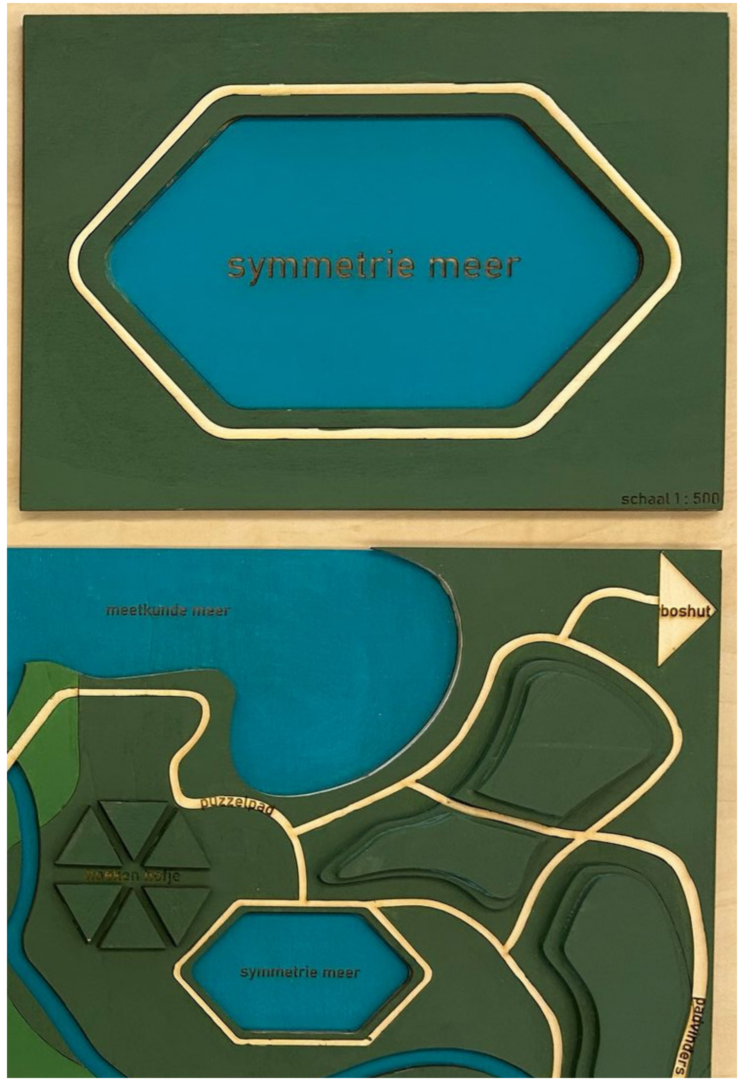
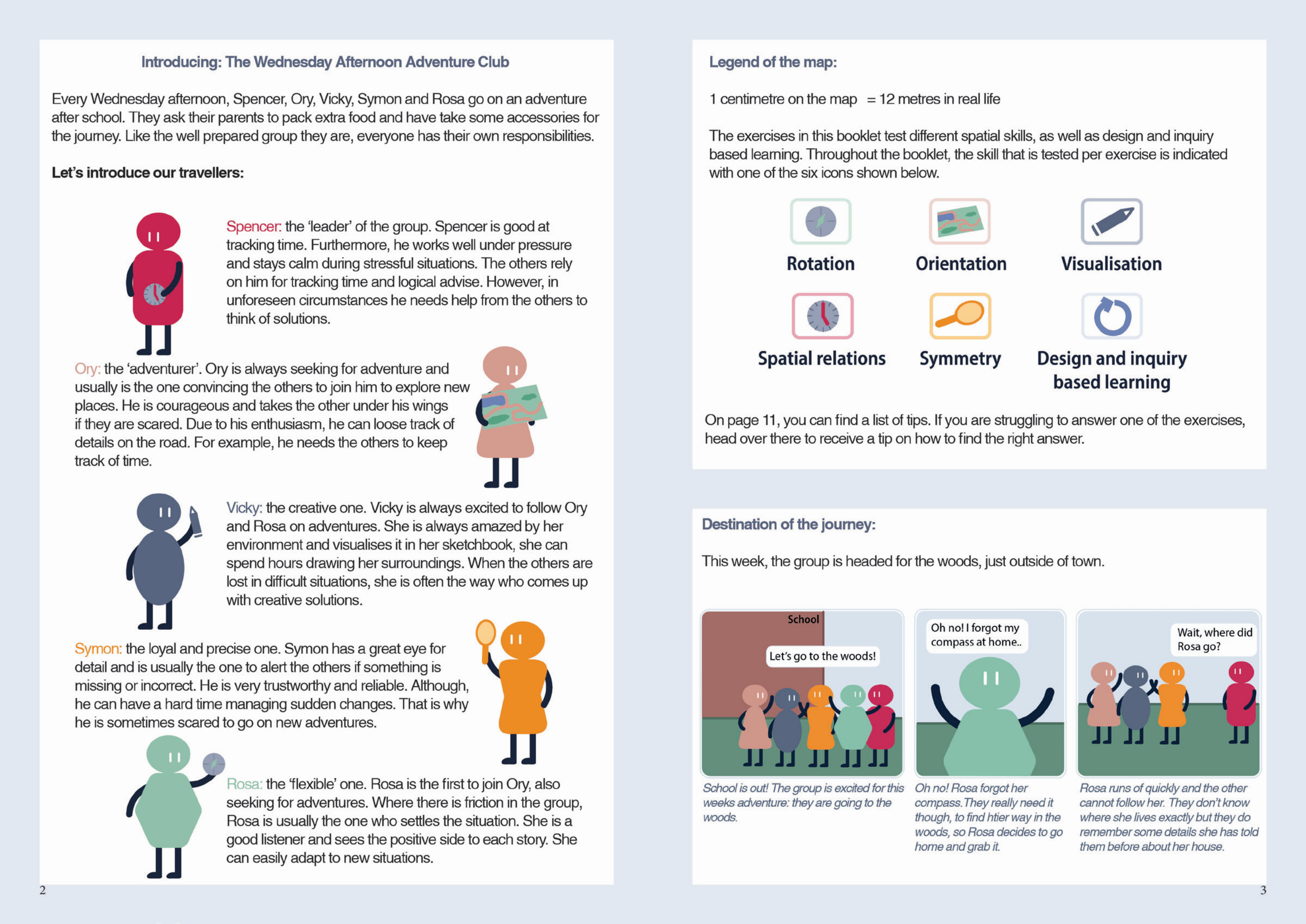


Figure 42: Tiles used in combination with the map

Figure 43: Upscaled version of the symmetry lake that can be used for exercise 9



The following pages show the exercise booklet.



Introducing: The Wednesday Afternoon Adventure Club

Every Wednesday afternoon, Spencer, Ory, Vicky, Symon and Rosa go on an adventure after school. They ask their parents to pack extra food and have take some accessories for the journey. Like the well prepared group they are, everyone has their own responsibilities.

Let's introduce our travellers:

Spencer: the 'leader' of the group. Spencer is good at tracking time. Furthermore, he works well under pressure and stays calm during stressful situations. The others rely on him for tracking time and logical advise. However, in unforeseen circumstances he needs help from the others to think of solutions.

Ory: the 'adventurer'. Ory is always seeking for adventure and usually is the one convincing the others to join him to explore new places. He is courageous and takes the other under his wings if they are scared. Due to his enthusiasm, he can loose track of details on the road. For example, he needs the others to keep track of time.

Vicky: the creative one. Vicky is always excited to follow Ory and Rosa on adventures. She is always amazed by her environment and visualises it in her sketchbook, she can spend hours drawing her surroundings. When the others are lost in difficult situations, she is often the way who comes up with creative solutions.

Symon: the loyal and precise one. Symon has a great eye for detail and is usually the one to alert the others if something is missing or incorrect. He is very trustworthy and reliable. Although, he can have a hard time managing sudden changes. That is why he is sometimes scared to go on new adventures.

Rosa: the 'flexible' one. Rosa is the first to join Ory, also seeking for adventures. Where there is friction in the group, Rosa is usually the one who settles the situation. She is a good listener and sees the positive side to each story. She can easily adapt to new situations.

2

Legend of the map:

1 centimetre on the map = 12 metres in real life

The exercises in this booklet test different spatial skills, as well as design and inquiry based learning. Throughout the booklet, the skill that is tested per exercise is indicated with one of the six icons shown below.

Rotation **Orientation** **Visualisation**

Spatial relations **Symmetry** **Design and inquiry based learning**

On page 11, you can find a list of tips. If you are struggling to answer one of the exercises, head over there to receive a tip on how to find the right answer.

Destination of the journey:

This week, the group is headed for the woods, just outside of town.

School is out! The group is excited for this weeks adventure: they are going to the woods.

Oh no! Rosa forgot her compass. They really need it though, to find htier way in the woods, so Rosa decides to go home and grab it.

Rosa runs of quickly and the other cannot follow her. They don't know where she lives exactly but they do remember some details she has told them before about her house.

3

Exercise 1

Can you find out where Rosa lives? These are the details the crew remembers:
- She always cycles home from school past the sum square.
- She has one next door neighbour on the left.
- She can see the lake from her front window and the house of the forester from her right side window.

Rosa's address: ______________________________

Everyone made it to Rosa's. She has already packed her compass and is waiting for the rest.

Now that everyone is complete and has their accesoires, it is time to go.

Exercise 2

Estimate the distance from Rosa's house to the border of the woods (where the road gets thinner) if you can only travel by road, and thus not through the grass. You can use the tiles of 3cm and 5cm. Remember: 1 centimetre on the map equals 12 metres in real life.

Distance from Rosa's house to the woods, traveling by road:

______________________ *cm on the map*

______________________ *metres*

Distance from Rosa's house to the woods, traveling through grass:

______________________ *cm on the map*

______________________ *metres*

We've made it into the woods! The road is already changing to be less straight and when we enter the woods it also narrows down a bit

While the others are still looking around, Symon realises that Ory is missing! He must have ran of when everyone was too distracted.

The group is walking around for a while when Spencer receives a message from Ory.

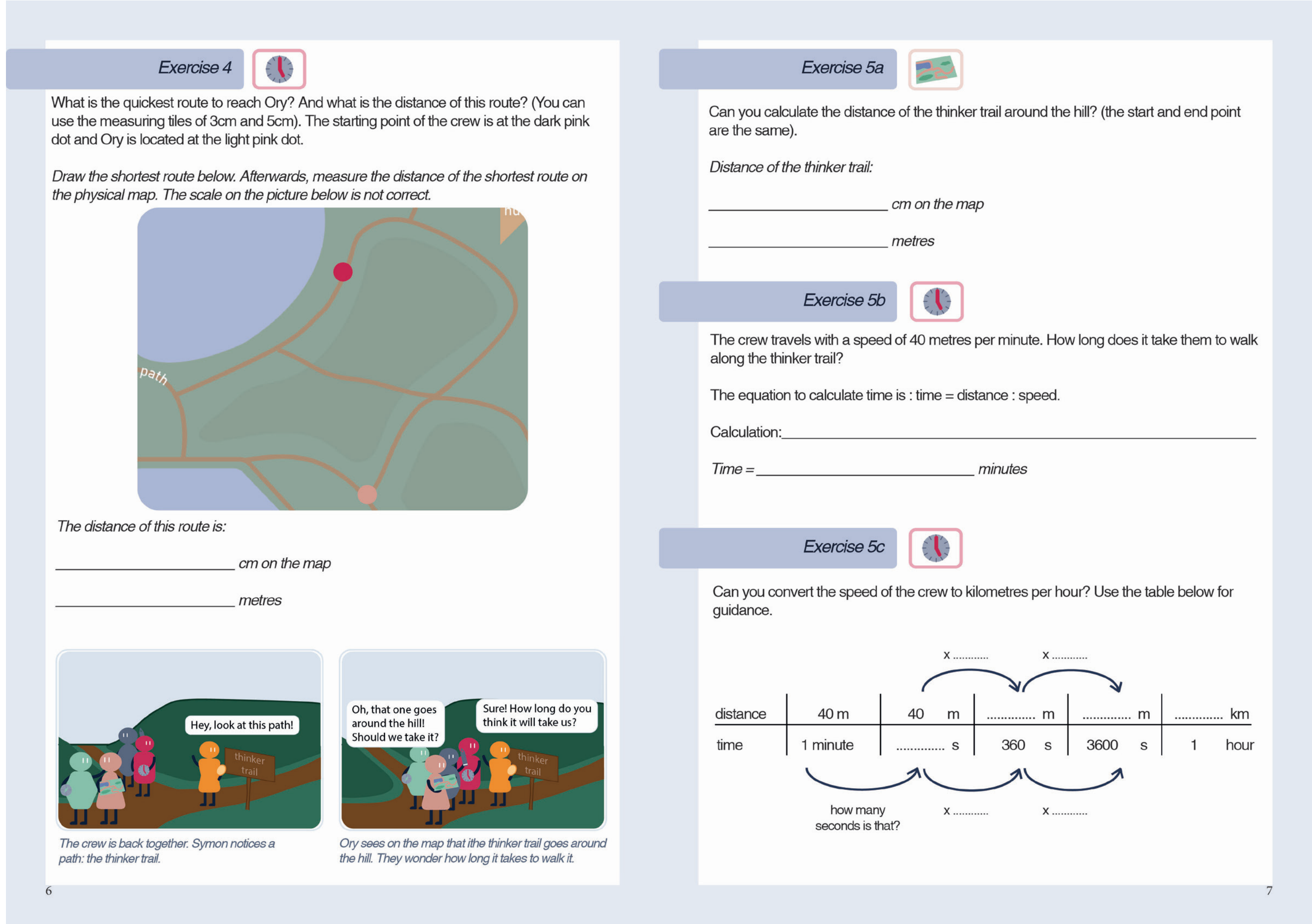
Attached to the message is a picture.

Exercise 3

Can you find out where Ory went? If you look at the picture attached to the message he sent to Spener, you can locate his exact location.

Ory's location:

☐ *A*
☐ *B*
☐ *C*
☐ *D*



Exercise 4

What is the quickest route to reach Ory? And what is the distance of this route? (You can use the measuring tiles of 3cm and 5cm). The starting point of the crew is at the dark pink dot and Ory is located at the light pink dot.

Draw the shortest route below. Afterwards, measure the distance of the shortest route on the physical map. The scale on the picture below is not correct.

The distance of this route is:

______________________ *cm on the map*

______________________ *metres*

The crew is back together. Symon notices a path: the thinker trail.

Ory sees on the map that ithe thinker trail goes around the hill. They wonder how long it takes to walk it.

Exercise 5a

Can you calculate the distance of the thinker trail around the hill? (the start and end point are the same).

Distance of the thinker trail:

______________________ *cm on the map*

______________________ *metres*

Exercise 5b

The crew travels with a speed of 40 metres per minute. How long does it take them to walk along the thinker trail?

The equation to calculate time is : time = distance : speed.

Calculation:______________________________

Time = ______________________ *minutes*

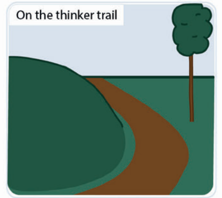
Exercise 5c

Can you convert the speed of the crew to kilometres per hour? Use the table below for guidance.

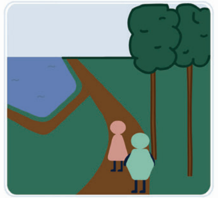
x .... x ....

distance	40 m	40 m	.... m	.... m	.... km
time	1 minute	.... s	360 s	3600 s	1 hour

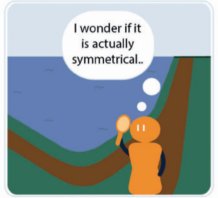
how many seconds is that? x .... x ....



The crew decides that they have enough time to walk across the hill. They enjoy the views along the way and make it back to the T-crosspoint.



When they walk a bit further down the road, they arrive at the symmetry lake.

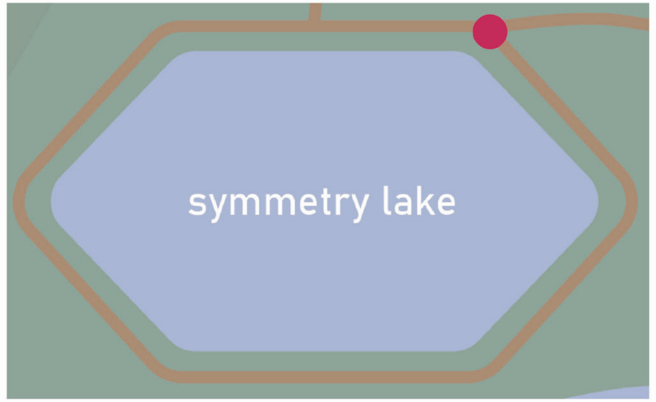


Symon acknowledges its beauty and perfect looking shape. He wonders If it is in fact, symmetrical.

Exercise 6

Symon was right, the lake is symmetrical. But how many symmetry lines does the lake have? Can you draw all the symmetry lines you see in the picture below?

Tip: A symmetry line is a line that divides a shape in two shapes that are exactly the same, but mirrored, like looking into a mirror.



I can count: ____________________ symmetry lines.

Vikcy is also admiring the lake, she wants to draw a picture to show her parents later.

Exercise 7

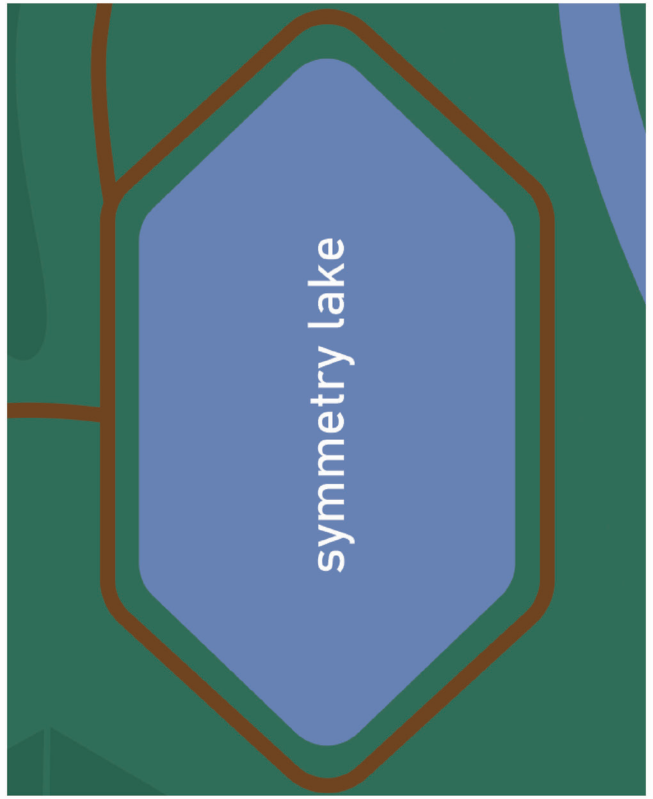
Can you draw a picture Vicky would draw for her parents? Imagine you are standing on the dark pink dot on the picture in the previous exercise. You can choose which point you are facing. What would you see? Draw it in the space below.

Exercise 8

The crew wants to cross the lake, however they did not take their swimming clothes and they don't want to make their normal clothes wet. Can you think of a way for the crew to cross the lake without getting wet? You can draw or write down your ideas below.

Exercise 9

Can you build the best idea you had for the previous exercise? Use the lake below as an underground and build something with which the group can cross it.



Hints

Exercise 1
Read the details one by one. If you keep in mind what the options are that are left after each detail, only one house remains.

Exercise 2
Before you start measuring, can you point out the route that you should take from Rosa's house. If you ca travel over grass, what is the route you can take?

Exercise 3
Look at the four options in the book and find out where these places are on the physical map. Which two options are not possible? And what is the difference between the remaining two?

Exercise 4
Which route do you think will be the shortest? How can you check if that one is the shortest?

Exercise 5a
Can you point out which route you will measure? Which tiles do you think would be best to use? Make sure that each tile starts at the end point of the last, so that you do not miss anything.

Exercise 5b
Write the formula of the question and fill in each number that you already know.

Exercise 5c
Look at the table and which units are written down. First write down each step that is taken to get from one number to the next and then fill in the table.

Exercise 6
You can use the mirror from the box. Place the mirror verticaly on the map and turn it around. When is the view in the mirror exactly the same as on the paper?

Exercise 7
Look at the comic story above exercise 6. On the third picture, you can see the lake in front of Symon. Use this picture as inspiration for your own drawing. Carefully watch the shape that the lake has on this picture. Also take a look at the physical map, if you stand on Rosa's location, what can you see in front of you?

Exercise 8
Think of objects that you know that can float. Can you think of something using these objects?

Exercise 9
Take a look at the materials in front of you. Can you recreate one of your ideas with these objects?

6.4.1 Guidance and collaboration

The exercises of the booklet are doable by students individually. However, it is an important requirement for skilled mathematicians to receive guidance and feedback from teachers. A previous version of the booklet therefore had exercises that required answers of previous ones, which students needed to check with their teacher before they could continue. However, after meeting with a teacher, previous teacher and PHD researcher on spatial ability, they all advised to remove this aspect since there is simply not the space and time for this. To keep the continuation of the story through the exercises, it is not required to have the correct answer to continue. When checking the booklets, a teacher will see if answers are wrong because the previous exercise was done incorrectly and will judge such an exercise on calculations more then answers. Because there is a strong need for guidance and reflection from skilled mathematicians and the teacher can not always be there while they are making the exercises, a hints page is added, as can be seen on the last page of the booklet. If a students struggles to answer an exercise, they can look at the last page of the booklet for a hint on how to tackle that exercise without giving the answer.

Besides the need for feedback being one of the requirements itself, this also creates an opportunity for potential skilled mathematicians to try exercises of the booklet. As previously mentioned, the toolkit stimulates collaboration between students because they will probably need tiles from peers to be able to use the flexible measuring tool. Because not every student has their own map, collaboration is also stimulated. The booklet is currently designed for 3rd and 4th grade students, who can do the exercises together. It could be investigated if the product can be extended for students of the rest of the class as well, allowing skilled mathematicians to collaborate with or design exercises for their peers.

Wish: Exercise/tool should provide fitting guidance for potential skilled mathematicians

Wish: Exercise/tool could allow for interaction between students of different levels

Throughout the booklet, each spatial skill is tested. Because the students are aware of their own best spatial skills, as well as those of each other, they know who to step to if they struggle to do one of the exercises. They know which spatial skill is tested per question because this is indicated by an icon presenting that skill, see figure 44. The design and inquiry based icon does not have a representative character in the story and toolkit. This is because design and inquiry based learning requires different spatial skills and does not differentiate enough compared to the five spatial skills mentioned. Therefore, a sixth character would have characteristics that are already mentioned in the other characters. Also, the story could be written in a way that the five characters are missing the exact skill of design and inquiry based learning and that they need the student making the exercises in the booklet to help them with the design exercises.

Figure 44: Icons of each spatial skill as presented in the booklet

For example, for exercise 9 the characters want to cross a lake but need help from the student to choose the best option to do so.

The toolkit will include an instruction manual for teachers so that they know exactly how to use it and what is needed from them while or after their students are making the exercises. This decreases the threshold that teachers have for using new materials in class.

Wish: Exercise/tool should consist requirements of innovation, as presented by Barend Last

6.5 Product details and future

This section will give an indication of the costs to create the Wednesday Adventure Club toolkit and explain how it could be packaged, shipped and reshipped to different schools. These factors have not been confirmed with manufacturing companies, however all of them have been discussed with the client and this indication is a plausible outcome for the feasibility of the product.

6.5.1 Costs and packaging

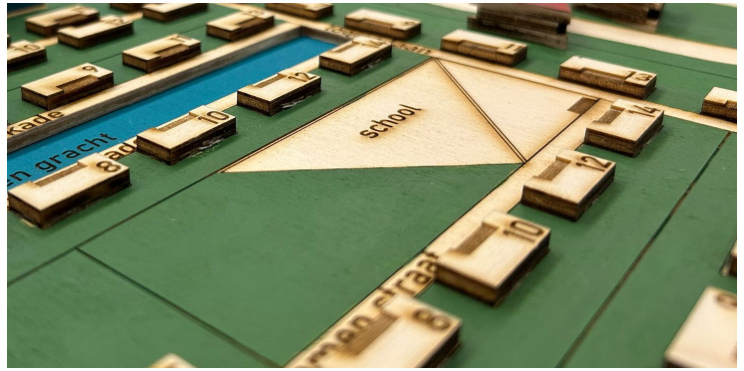
The exact costs of one toolkit will depend on some aspects that are not yet definite, however an indication can be made. Some specific considerations that have been taken to decrease costs are:

- Placing all materials to be laser cut on one wooden sheet.

Figure 45: Lasercut process

- Placing houses and buildings on the map directly towards the road to decrease painting time.

Figure 46: Building placement on previous and final map



- Laser cutting al houses in one sheet directly glued to the layer below to decrease gluing time.

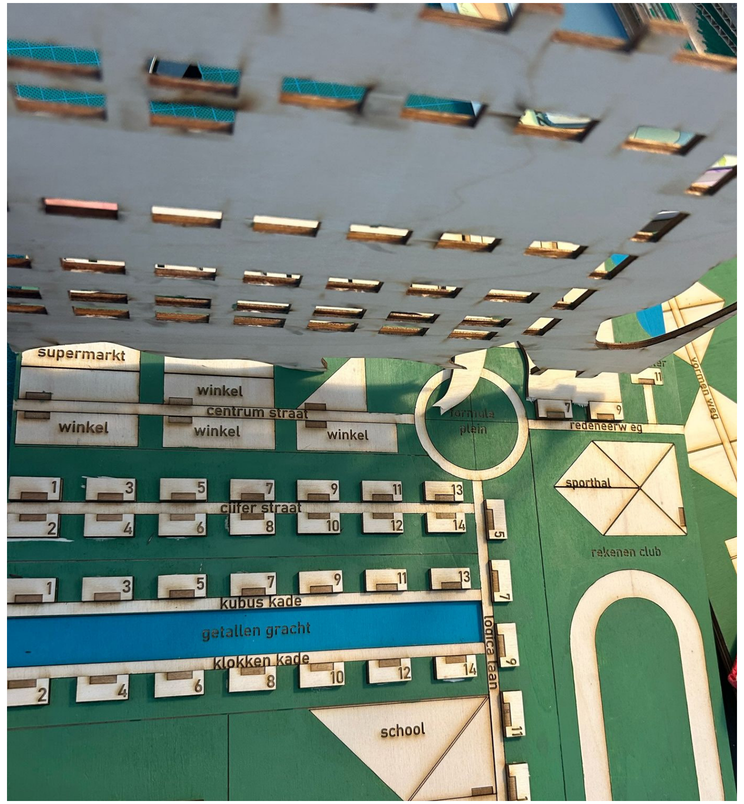


Figure 47: How rest wood is taken of after all houses are glued at once

Materials:

- Wood: 10 euros (an entire toolkit can be laser cut on one sheet of 1200 x 700 m plywood)
- Iron wires: 1 euro per toolkit
- Painting materials: 2 euros per toolkit
- Wood glue and sanding paper: free for TU Delft employees
- Exercise booklets: free for TU Delft employees

Production/labour:

- Laser cutting of the map and tiles (70 minutes): free for TU Delft employees
- Painting map and tiles (3 hours): 60 euros
- Gluing and sanding map and tiles (1,5 hours): 30 euros

Price of one toolkit: 103 euros

Some of these prices are exact and some are estimated. The labour costs are based on the average salary of TU Delft student asisitant. The cost for wood is based on the price of one sheet of plywood at TU Delft (Faciliteiten TU Delft, n.d.). Some matirial costs are free because these materials are available for employees of the TU Delft, including Sciencehub. Other materials, paint and wires, are estimated based on the price of full bottles of paint or packages of wires and divided by how many toolkits could be made with those. To decrease painting time, a future design could include engravings of spatial skills on the tiles instead of being painted in different colours and the map could be spray painted with overlays instead of painted by hand.

For an exact cost calculation, the materials, labour and production of the product should be evaluated and finalised.

The toolkit will most likely be shipped as a box or a little suitcase that can be shipped to schools to use for a short period of time. The toolkit could for example be used for a month and after usage send to the next school. All product specific materials (map, booklet, tiles) of the toolkit will be made at the TU Delft, either at Science Centre or at the workplace of the faculty of Applied Sciences. All components of the toolkit will then be packaged at the TU Delft and shipped to schools throughout the Netherlands.

A box will consist of the map, measuring tiles, characters and booklets of different stories. One booklet takes around an hour, which for most schools is the amount they spend on mathematics per day. This toolkit could be used one day of the week. Depending on the timeframe that a school would use the box, the amount of booklets could be decided. For example, four booklets are created to form a box that can be used for four weeks by a school.

Figure 48: All components of the toolkit

Most likely, ScienceHub will first send this toolkit to schools they are already in contact with, mostly in the area of Delft, Den Haag and Rotterdam. If it promises to be a success, they will consider shipping though more schools throughout the country.

The map is made out of two 3,3 mm layers of plywood. The tiles are made of three layers of 3,2mm plywood. It is not likely that these will break but there is a possibility since they will be used by many children and could be thrown on the floor, be stood on or stored inefficiently. Therefore it is possible that the wooden components will break. If this happens, the school can alarm Science Hub and ask for extra materials. Science Hub will then laser cut and assemble a new map or extra tiles and send these to the school. When a school has finished all booklets of the box, the box will be send back to Science Hub, they will revise if all components are still in tact and will fill it up with new booklets and send it to the next school.

6.5.2 Sustainability

During the design and finalising process, sustainability is taken into account, however there are still improvements that can be made or aspects that this project did not yet do sufficient research on. The most important considerations, either already thought of or written as recommendations, are discussed in this section.

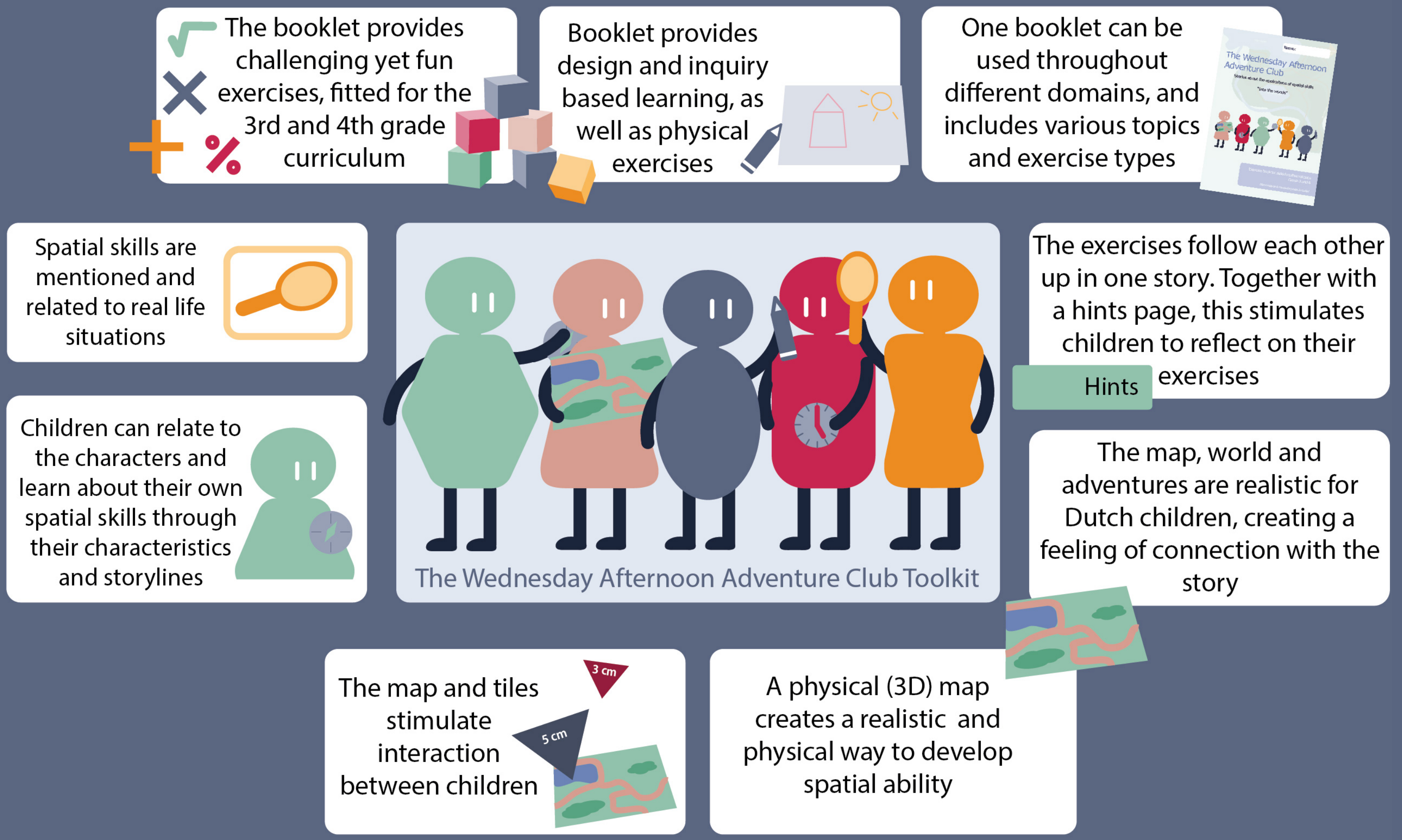
The first consideration made is about the material of the product. The product is mostly made out of printed paper and painted wood. Printed paper is well recyclable, if it is not finished of with a coating or consist of really saturated colours, which it is not. The booklets could be printed on slightly bigger paper or printed with a coated outside for better quality, however the recyclability would then decrease. The map is made out of plywood, glued and painted afterwards. Wood is well recyclable however adding glue and paint decreases this. For future improvements, the map could maybe be designed so that it does need to be glued, for example by designing a different laser cut document so that all height differences can be made out of one sheet of wood. Furthermore, the sheet is now painted with acrylic paint, which is quite sustainable since it is made out of water basis, however there are more sustainable options, like ecologic acrylic paint made of organic pigments.

Besides materials, the production method of the Wednesday Afternoon Adventure Club is fairly sustainable. The primary reason for this is because all materials are sourced and produced in the Netherlands and even more so, at Science Hub or Science Centre itself. This massively decreases the otherwise large impact that transport of materials or components would have on its sustainability. Furthermore, the sources and knowledge to create the toolkit are accessible to the employees of Science Hub, saving time compared to hiring or finding employees or companies.

An aspect of the product that gives it a sustainable advantage is that it will be transferred between different schools and classes and therefore will be reused many times. Most likely, the toolkit will stay in one class for one or two months and used every week. This gives the product a sustainable advantage over other educational objects at school that stay in one classroom all year and take up constant space, but are only used a few times a year (personal communication with teacher, 2025). Most likely, the map and tiles will stay in tact for a long time, however, if something breaks the school can contact ScienHub and even one tile can be remade without the necessity to remake the entire product. The booklets will be renewed for every class since students will write in them.

6.5 Conclusion

A summary of the strenghts of the Wednesday Afternoon Adventure Club toolkit is visualised below. The figure gives an overview of the most important aspects of the toolkit, namely how it answers to the most important requirements and wishes that were set up for this project.



The booklet provides challenging yet fun exercises, fitted for the 3rd and 4th grade curriculum

Booklet provides design and inquiry based learning, as well as physical exercises

One booklet can be used throughout different domains, and includes various topics and exercise types

Spatial skills are mentioned and related to real life situations

Children can relate to the characters and learn about their own spatial skills through their characteristics and storylines

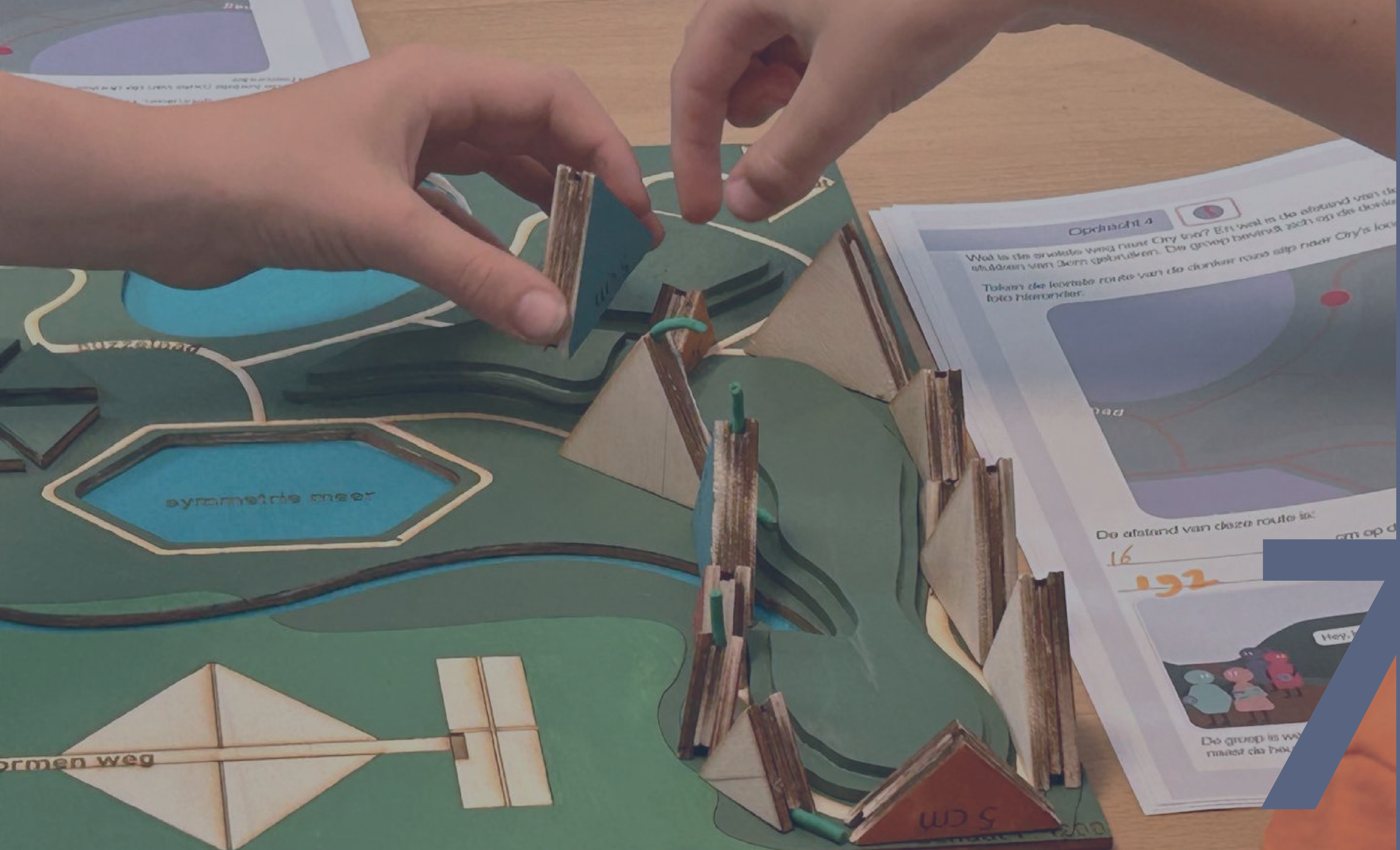
The exercises follow each other up in one story. Together with a hints page, this stimulates children to reflect on their exercises

The map, world and adventures are realistic for Dutch children, creating a feeling of connection with the story

The map and tiles stimulate interaction between children

A physical (3D) map creates a realistic and physical way to develop spatial ability

Figure 49: Strengths of the Wednesday Afternoon Adventure Club toolkit



7

Validation and revision

To validate the potential of the designed product, two tests were done with skilled mathematicians. Furthermore, three experts gave their opinion on the products' potential. These tests and expert meetings, as well as their outcomes are explained in this chapter.

After the value of the design was tested with children on the 16th of April, the product took its final shape. This form of the product was tested with the same group of skilled mathematicians on the 14th of May. Afterwards, one more iteration of the final design was made, based on this test and three meetings with experts. The version that followed is the last version of the product physically made for this project. This version was once again tested, on the 3rd of June, with a new class of skilled mathematicians. Some major insights that contributed to each design iteration are shown in the figure on the right. An elaborative explanation of the tests on the 14th of May and 3rd of June, as well as both expert meetings in between are mentioned in this chapter.

Both tests and the three expert meetings form the validation of the Wednesday Afternoon Adventure Club toolkit and indicate its potential, viability, desirability as well as improvements for a future design. These improvements, if not taken into the final design embodied for this project, will be written down as recommendations in this chapter. The Chapter ends with the limitations and an overall conclusion for this graduation project.

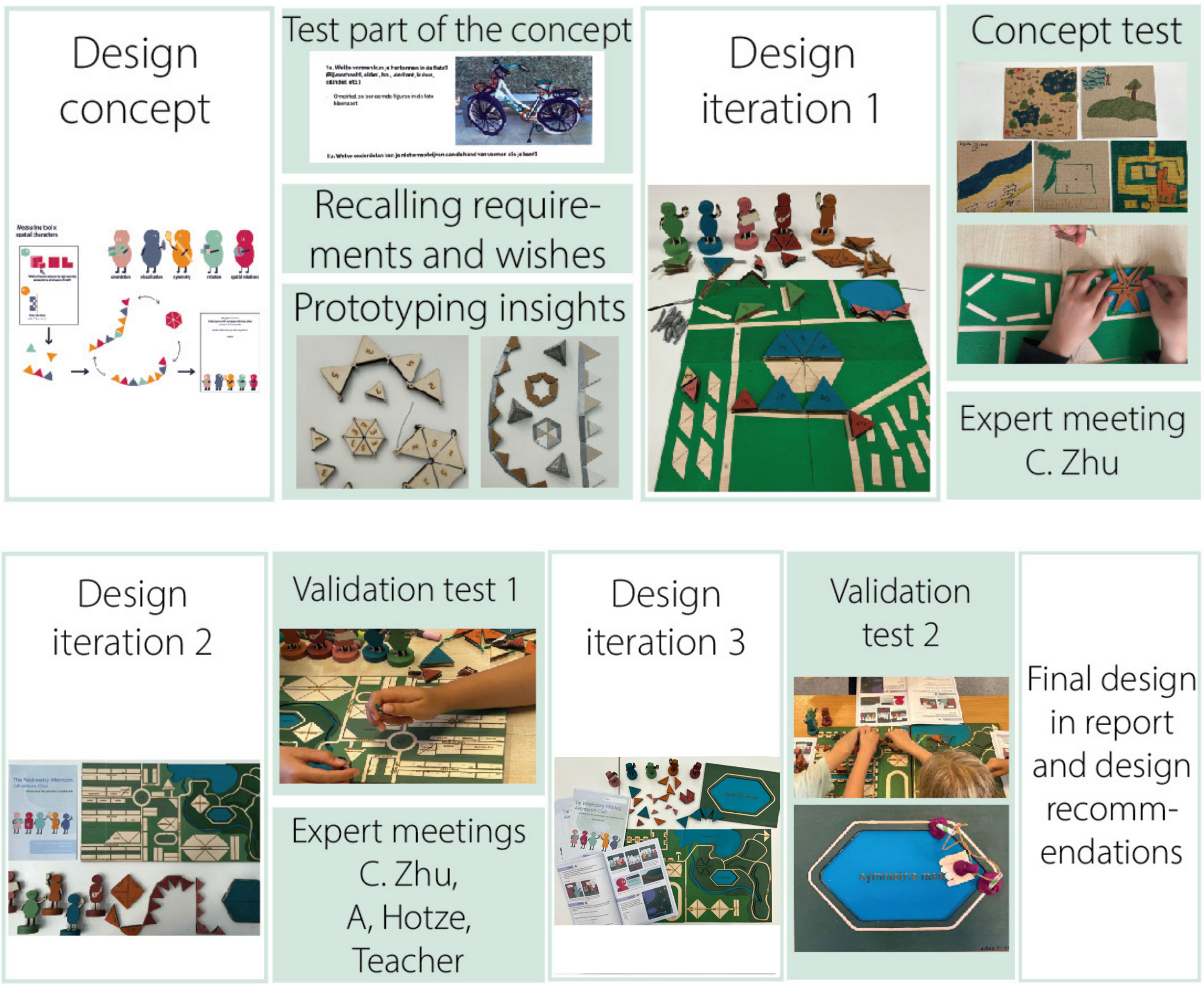


Figure 50: Overview of which tests, expert meetings and other validation moments influenced design changes

7.1 Validation through testing with children

7.1.1 First validation test

Aim of the test and test set up

On the 14th of May a test was conducted with children to test and validate the provisionally final design. The previous – concept – test helped to see if the concept had potential and if the measuring tool worked visually and ergonomically. For this test an exercise book was designed complete with different spatial exercises that stimulate children to use the measuring tool and a redesigned map.

The two most important aspects to test are if the product is fun and challenging for the children. This was tested by observing how the children were interacting with the product and what their reactions were during the test. Factors that will be observed like: facial expressions, body language, reactions and interaction with each other. At the end of the test the children were given an evaluation sheet in which they can write their experiences with the product, which can be seen in Appendix H3. booklet and map have been translated to Dutch for this test, since the language spoken at the school for the test is Dutch.

This test was done with nine skilled mathematicians of a combined 3rd and 4th grade class of a Waldorf school in Delft. The product was tested in three different groups of three children, 15 minutes per group. To be able to test all questions of the booklet, the questions were divided per group. The designer sat at the table together with all three children and the story and questions were read by the designer and the exercises were done mostly collaboratively between the children. The designer asked the children to think out loud in order to understand how they were answering the questions. After the three testing rounds, all children were given the evaluation sheet to fill in.

Figure 51: Test set up for first validation test

Observations and results

Overall, the children were intrigued by the story around the map and characters. They understood how the characters were moving along the map and could easily transfer the story they heard to the physical map. They were invested in the characters, asking 'Wait, who is Spencer, is this him?' and 'Rosa must be good at finding her way right, since she is holding a compass?' and taking characters in their hands when the story highlighted this character. Besides, they understood the physical features of the map. 'Is this a mountain?', 'No, that is a hill, it would have been way higher if it was a mountain'.

Regarding the tiles, some interesting observations were that the children often did not connect different tiles together. They used them separately by laying them next to each other. This did seem to spark interaction, since one student would hold a tile still, so that someone else could place the next tile exactly next to it. They did know that they could 'We can connect them, I think that is the purpose of it' but it did not seem necessary. When they did connect them, the connections worked well. Besides measuring with the longer sides of the tiles that had '3cm' or '5cm' on them, the children also measured with the other sides, assuming their lengths 'This is 1,5 cm'. Both when connecting them or not, the tiles were ergonomically well fitted for these children.

Exercise/tool should be ergonomically designed for 3rd and 4th grade students

While measuring paths on the map for the exercises, it seemed that some paths were too curved, making it hard to find exact answers to the questions. Because of this, some children had different results for the measuring exercises. Although, everyone's answers were close to the correct ones. Converting the measured values to the exact distances using the scale of 1:1000 was quite easy for all children.

Mentioned in the table:

*I explained that with 'product' I meant the tiles, map and exercise booklet all together
**rekentijgers = the booklet they use at this school to provide extra exercises for skilled mathematicians

Questions	Answers (filled in 8 times)
What did you think of doing maths with this product*?	- Fun (leuk) ; 8 times - Educational (leerzaam) ; 2 times - Challenge (uitdaging) ; 1 time
What grade would you give for doing maths with this product from 1-10?	Average: 8,6/10
How fun did you think it was to do exercises with this product, compared to the exercises of the normal maths classes (1-7)?	Average: 6,4/7
How fun did you think it was to do exercises with this product, compared to the exercises of rekentijgers (1-7)?**	Average: 5,7/7
How challenging did you think it was to do exercises with this product, compared to the exercises of the normal maths classes (1-7)?	Average: 5,3/7
How challenging did you think it was to do exercises with this product, compared to the exercises of rekentijgers (1-7)?	Average: 3,2/7
What would you think of using this product from now on during the 'rekentijgers' hour on Wednesday?	- Very fun (heel leuk) ; 5 times - Really good idea (heel goed idee) ; 2 times - Fun, but rekentijgers is also fun (Leuk, maar ik wil ook rekentijgers doen) ; 2 times - Please (heel graag) ; 1 time
Do you think this product fits to the rekentijgers exercises?	- No ; 4 times - Yes ; 2 times - A little ; 2 times
Why?	No: - this is bigger, - rekentijgers is harder, - you need a board and other things, - it is not as challenging as rekentijgers, - it is big Yes: - it is both extra work - it has a good challenge

Table 5: Evaluation sheet results for first validation test

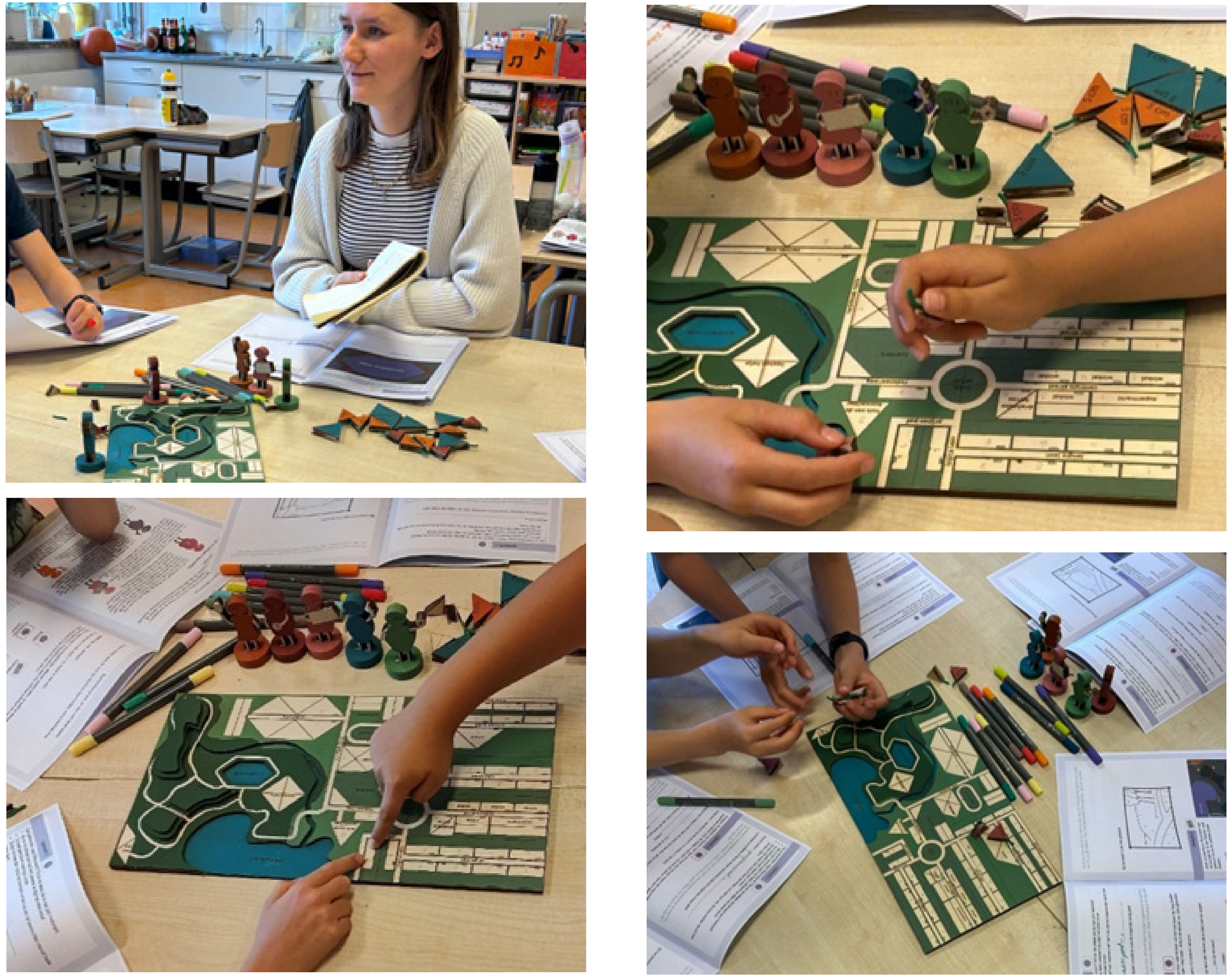


Figure 52: Photos of the first validation test

Conclusions of the test/design changes

A positive conclusion that can be drawn from this test is that the story around spatial characters with additional map is intriguing for the children and that all children experience the exercises as fun, both in relation to their normal mathematics exercises, as well as in comparison to the existing materials for skilled mathematicians. From the evaluation form, it can also be concluded that the children found this booklet more fun than their usual, and extra work, exercises.

Wish: Exercise/tool should create an enjoyable interaction with mathematics

However, the most important conclusion for a next iteration is that most exercises are relatively simple for the children. This is why the exercises will be increased in difficulty for the final design. All kinds of exercises will be kept in the next version, however the measuring exercises will increase in difficulty. The orientation, symmetry, drawing and design questions will stay the same since these seemed to be challenging enough.

Some specific design changes that will be made (or used as recommendations), for the map or exercise booklet, are:

- The map should be larger, mostly in order to make the roads longer and to increase the distances. Then, the 5cm tiles are of more value next to the 3cm ones.
- The map can consist of two separate parts so it is not too big to store or transfer.
- The roads should have less sharp turns in order to create more precise measurements.
- The scale of the map should change because multiplying by 10 is too easy.
- The pictures in some exercises should be of low opacity in order to make drawing on them easier.
- Doors and house numbers should be laser cut on the map.
- Think of a way/instruction to increase stimulation of connecting multiple tiles instead of using them as separate tiles. Or change the design to separate blocks and skip the 'connection ruler?'
- Idea: add an exercise that requires children to build something on the map (with the tiles).
- Either give measurements to each side of the triangle or give instructions to only use one side of the tiles.

7.1.2 Validation test after final prototype

Aim of the test and test set up

On the 3th of June a final test was done to test the last physical iteration with children, however most aspects of the product and test are the same. Thus, his test will mainly provide extra data and insights for the aspects that were tested before. Therefore, still, the two most important aspects I want to test are if the product is fun and challenging for the children. I will test this by observing how the children are interacting with the product and what their reactions are during the test. I will observe factors like: facial expressions, body language, reactions and interaction with each other. At the end of the test I will give the children an evaluation sheet in which they can write their experiences with the product.

The product that was used for this test was an improved version compared to the one that was tested with last time. These improvements can be read in the conclusion from last test.

This test could therefore give a validation if those design changes are improvements compared to the last design and test. The most important changes compared to the last design are that the map is much bigger, that some exercises are increased in difficulty (whereunder increasement of the scale of the map) and that the written story in the book is replaced by comic illustrations. Besides increasing some exercises in difficulty, a design exercise was added for which students have to physically design and make something. Furthermore, during this test the exercises and story were not read out loud by the designer but children were asked to read the comic story and questions themselves. This was done to see if the students would follow and understand the story and exercises themselves or if it was perhaps still too much or unclearly written text. In addition to creating a better understanding of the story, the designer introduced the characters at the start of the test, unlike done for the previous test.

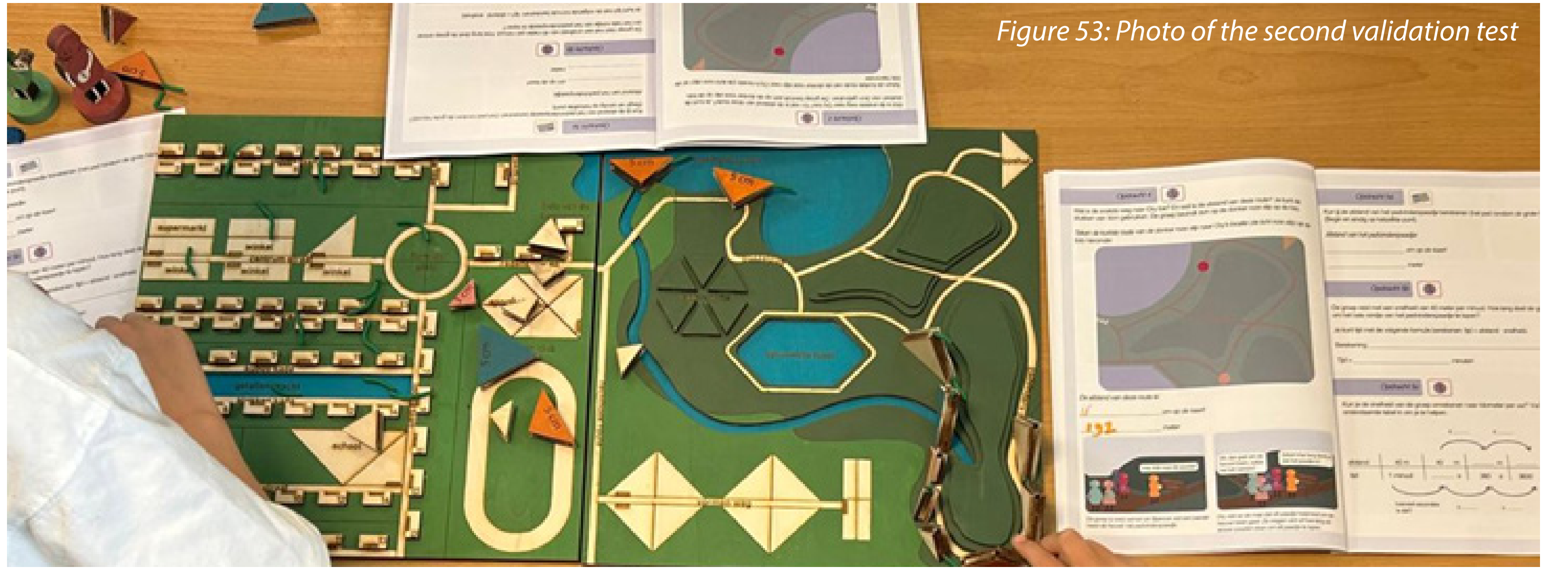


Figure 53: Photo of the second validation test

Figure 54: Set up of the second validation test

Observations and results

Like during the last test, the children were intrigued by the story, map and characters. 'Ah can we take home of them?'. What worked even better this time is that the characters were introduced at the start of the test and that they were visualised throughout the story in the comic visuals. Besides, during some exercises, the characters were picked up and placed on the map according to the story. A student picked up Ory and placed it outside of the map 'Ah, Ory walked away'.

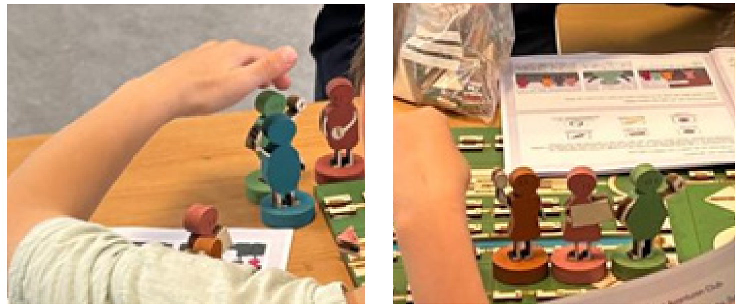


Figure 55: Students playing with the characters

Furthermore, the students were asked to read the comics and exercises themselves, which worked well because they did read everything and knew (better than the students during the previous test) were to write answers and how to answer the exercises. However, it did result in some students working quicker than others, which made it more difficult to manage the flow of the test.

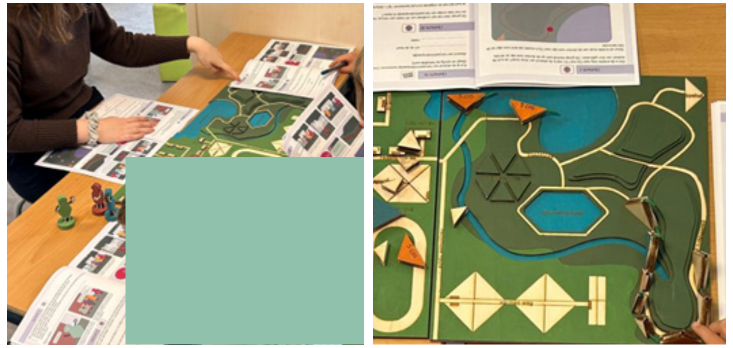


Figure 56: Students reading booklets and placing tiles on the map

Increasing the map in size also had a positive impact. This allowed for students to measure with 3cm and 5cm tiles, instead of only with 3cm ones (which was the case during the previous test). Increasing the map in size and decreasing the amount of curves on the paths on the map both ensured that it was easier to measure distances of the paths on the map because the blocks fitted better.

For this test, some exercises were changed to increase their difficulty levels. One change that was made to increase the difficulty of measuring exercises was increasing the scale of the map from 1cm=10m to 1cm=12m. This did make the exercises more challenging since for each measurement, the students had to calculate the real life distance. They did succeed each time to get the correct answer in the end, however sometimes they did have to help each other or recalculate. Question 5c was added in comparison to the previous test, where students had to fill in a table to reformulate a calculated speed from m/min to km/hour. One student said, while reading this exercise, 'No I don't want to do that one', because it seemed like too much work. His peer did try it out and realised it was actually very doable if he followed all the steps of the table, after which the first student also filled it in.

The drawing and design exercise were perceived as the most challenging ones during the previous test, which is why they stayed the same for this test. The drawing exercise was still the only exercise that none of the students really succeeded in and the design challenge also sparked much discussion and thinking. During this test, the symmetry exercise was also perceived as difficult since the students did not realise that the lake is only symmetrical in two axes. Overall, it could be concluded that the exercises are a bit more challenging than they were during the previous test and that some of them reach the right level of challenge where students try them out but do still get them wrong without help from a teacher, however, there is still plenty of room for even more challenging exercises.

Since the previous test, the measuring tiles had not been changed since there was no clear conclusion if the connection system should be left out or not. During this test, the tiles were connected more than during the previous one. Which is interesting because these children had not seen the product before (in comparison to the previous class) and did not know that the tiles could be connected. They figured it out themselves by seeing them laid out with iron wires and they started connecting them themselves. Both groups did so, however one asked 'do we have to connect them', to which the designer said they did not if they found it easier to use them separately, to which the student in return said that he found it easier to use them separately. Some students also just liked to play with the tiles and wires, (re)connecting them, it seemed to add an enjoyable play-experience. When the designer removed the map and tiles to create space for the arts and crafts materials: 'Can I keep this wire, I like to fidget with it'.

Figure 57: Students assembling tiles and placing them on the map

Both groups that conducted measuring exercises placed many tiles on the map at once (some connected, some separately) and only calculated the distance after revising how many tiles were used and adding these altogether 'Okay so we had five times 5cm and two times 3cm, that's 31 cm!'.

The students all thought of different ideas for the characters to cross the lake for exercise 8. They discussed it together and reread the question to make sure that the options they though of still followed the rules of the questions. 'It says that their clothes cannot get wet, so if they swim without their clothes it is still okay', or 'it does not say you cannot walk around the lake right?'. Afterwards, the students were presented with arts and crafts materials to make on of their ideas from the previous exercise physically. When the materials were laid out their faces seemed to light up and they immediately stood up and started grabbing different materials. They also all started thinking out loud as to what they would make. The design that they build are shown below. From left to right: a drone to fly over the lake, a floater that moves by pulling wires, a boat and a human catapult.

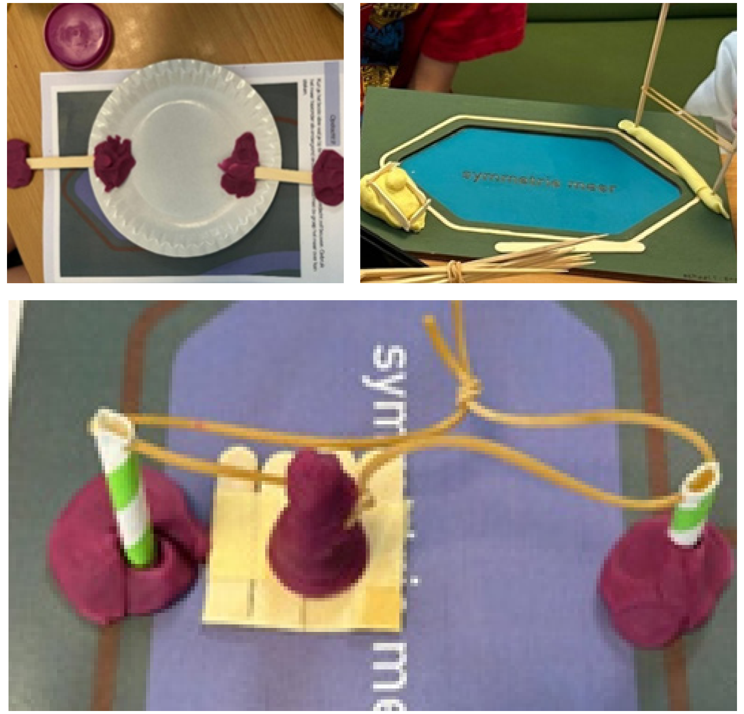


Figure 58: Solutions to cross the lake, made by students

Questions	Answers (filled in 6 times)
What did you think of doing maths with this product*?	- Fun (leuk) ; 6 + because you could build something (1 time)
What grade would you give for doing maths with this product from 1-10?	Average: 9/10
How fun did you think it was to do exercises with this product, compared to the exercises of the normal maths classes (1-7)?	Average: 6,2/7
How fun did you think it was to do exercises with this product, compared to the exercises of rekentijgers (1-7)?**	Average: 3,8/7
How challenging did you think it was to do exercises with this product, compared to the exercises of the normal maths classes (1-7)?	Average: 6,2/7
How challenging did you think it was to do exercises with this product, compared to the exercises of rekentijgers (1-7)?	Average: 3,8/7
What would you think of using this product from now on during the 'rekentijgers' hour on Wednesday?	- More fun (leuker) ; 3 times - (Really) fun ((super) leuk) ; 3 times
Do you think this product fits to the rekentijgers exercises?	- No ; 2 times - Yes ; 3 times - Both ; 2 times
Why?	No: - Because rekentijgers is equations (sommen) Yes: - 'Just because' - Because there is a lot of challenge in this Both: - Because this is more arts and crafts

Table 6: Evaluation sheet results for second validation test

Conclusions of the test/design changes

Overall, the test showed that the design changes done after the previous test had a positive result, namely, increasing the size and scale of the map, adding a comic story and design exercise and increasing some exercises in difficulty. The biggest improvement, still, is to increase the difficulty of exercises even more. However, not for all, since some exercise all already challenging enough as they are. Furthermore, some exercises could still be added, like another orientation exercise or an exercise for which students have to build something physically on the map with the tiles.

An element of the product for which there is still not yet clear conclusion if it should be changed is the connection element of the tiles. During last test, none of the students connected the tiles however during this one some of them did. The connection pieces do not seem necessary, however the students during this test did seem to enjoy putting different tiles together, since it adds a 'building element'. It could also be beneficial if a real building exercise is added.

The changes mentioned above will form recommendations for a possible future design, however some specific changes that can still be done during this project for the booklet, to increase students' experience while making the exercises are:

- Add a line for the measuring exercises to write the calculation of cm to metres.
- The picture of exercise 4 should have the exact same dimensions as the physical map, or should clearly be smaller.
- Add one already drawn element to exercise 7 to indicate the perspective of the lake.
- Add hints for each exercise that students can look for if the do not understand an exercise immediately but cannot ask their teacher.

7.1.3 Results of the two tests combined

The results of the evaluation forms of the classes of both tests are combined in the table below. Besides, the difference is visualised between the answers of the students of the 3th and 4th grade.

As far as differences between the two classes the most mentionable differences were, firstly that the first class scored the exercises a 5,7/7 for 'fun compared to rekentijgers' and the second class 3,8/7. Second, the first class scored the exercises 5,3/7 for 'challenging compared to normal class exercises' and the second class 6,2/7. Since the test was only done with 14 students in total, these numbers should not be taken as quantitative data however these differences could be because the second class has done other exercises in class, as well as in rekentijgers in the past few weeks. Besides, all scores are hard to compare since not all students did every exercise of the booklet. A possible conclusion could be that the booklet overall did increase in level of challenge for the second group and thus why it scored higher on this aspect.

The results have also been divided in the scores given by students of the 3rd grade compared to those in the 4th grade. These scores can be read in the table below. The two mentionable differences here are that 3rd grade students rate the exercises higher on 'fun compared to normal class exercises' as well as 'fun compared to rekentijgerss'. Again, it is hard to draw conclusions from this but an explanation could be that the 3rd grade students are slightly more into playing with characters and blocks still than that 4th graders are, hence why it is more fun for them. If this product was to be tested with 5th and 6th graders as well and the fun-score kept decreasing, that could be an indication that the play-element of the product is more fitting to 3rd grade students. This would have to be researched further and with mentionable more students.

Questions	Answers total (14 students)	3rd grade (7 students)	4th grade (7 students)
What did you think of doing maths with this product*?	Fun; 13 times Educational; 2 times Challenge ; 1 time	Fun; 7 times Educational; 1 time	Fun; 7 times Educational;1 time Challenge ; 1 time
What grade would you give for doing maths with this product from 1-10?	8,8/10	8,6/10	8,9/10
How fun did you think it was to do exercises with this product, compared to the exercises of the normal maths classes (1-7)?	6,3/7	**6,7/7**	**5,9/7**
How fun did you think it was to do exercises with this product, compared to the exercises of rekentijgers (1-7)?**	4,9/7	**5,4/7**	**4,4/7**
How challenging did you think it was to do exercises with this product, compared to the exercises of the normal maths classes (1-7)?	5,7/7	5,7/7	5,6/7
How challenging did you think it was to do exercises with this product, compared to the exercises of rekentijgers (1-7)?	3,5/7	3,3/7	3,6/7

Table 7: Evaluation sheet answers combined, 3rd vs 4th grade

7.2 Validation through Experts

As mentioned, in the period of creating the final two versions of the product, three experts have taken a look at the Wednesday Afternoon Adventure Club toolkit. These meetings are summarised in this section by highlighting their overall opinion of the product as well as listing improvements that they mentioned. After this report will be submitted, a fourth expert (lecturer from the Thomas More Hogeschool) will look at the product, unfortunately this meeting will therefore not be included in this report, but a conclusion will be shared at the Graduation presentation.

7.2.1 Teacher & education materials designer

On the 15th of May I met up with the teacher of the class I tested with the day before (validation test 1) to get her feedback on the product and its potential. The most important improvements that she suggested are listed below. Some of them are in line with what I had already concluded, which strengthens their importance.

- The map should be larger. You can create a folding line or two parts so that it still fits in the classroom and protects the inside of the map.
- You can use loose pieces of iron wires to measure the bended roads more easily.
- You can indicate which exercises are least and most difficult by adding one, two, or three stars. If you do so, it is okay to have 'really difficult' exercises since the children will feel less disappointed knowing that they weren't able to complete those compared to the easier ones.
- Indicating a distance between 1-5 is not extremely challenging for some of these children
- It is good to keep the speed calculation exercise; you can increase difficulty by letting them recalculate from seconds to minutes, or minutes to hours. Include a table which they can use to make these calculations.
- Add lines on which children can write their calculations; helps them to reflect and helps teachers to check answers.
- Adding a scale to the map is very relevant for these ages. You can indeed increase difficulty by changing 1 cm = 10 m to 1cm = 12 m.
- You can create more exercises around measuring and calculating with scales.
- Add short explanations to exercises that use specific terms like 'symmetry lines'. These children just revised it but for some children it might have been some time

Overall she was excited about the product and said that the children could definitely use it in class. She liked the physical aspect and mentioned how well children react on learning with physical objects and materials. This is something that she has previously experienced in her own class and learned throughout her position as an education material designer. The most practical remark that she had was if there were more booklets and exercises, since this booklet would be done in an hour. I explained that this booklet is one story I created but that the idea is to have multiple stories, in which she saw more potential. To invest in this product, she wanted at least a few booklets so that it was worth investing in.

Furthermore, she highlighted that the level of mathematical understanding in this class (and school) is not average because many parents in this neighbourhood have studied at the TU Delft. So questions that might not be extremely challenging for these children, could very well be way more challenging for children in other classes, schools, or cities.

7.2.2 Spatial ability researcher (C. Zhu)

The earlier mentioned PHD researcher C. Zhu, revised two different iterations of the exercise booklet. She mostly gave practical tips on improvement in language, questioning (to make sure it is exactly clear what should be done to answer questions correctly) and the visual style of the booklet and pictures that support the exercises. Some specific improvements or tips that she gave (of which most were implemented in the final design and some used as recommendations) were:

Implemented in the final design:

- Decrease the introduction text for the characters a little.
- Ensure that it is clear where the front side (and door) of a house is.
- You could add a difficulty indication by rating an exercise with 1, 2 or 3 stars but it could also have a demotivating effect if students are unable to solve a 1 or 2 star exercise.
- For the drawing exercise; let children use their own viewpoint. In that way, they can look at the physical map and choose a direction to look towards.
- For the design exercise you want to make, laser cut the lake in a bigger scale so students can use this to build something on top of.
- Add the 'spatial relations' icon to the table calculation exercise (ex. 5c) because filling in such a table is part of that skill.

Used as recommendations:

- Add an exercise for which children have to use spatial rotation skills (like folding and unfolding pieces of paper).

Overall, she was enthusiastic about the booklet and additional map and she said that is was a good way to test different spatial skills and that the physical map was definitely part of this. She also thought that the story around the spatial characters was a fun and helpful addition for the booklet. Furthermore, she highlighted that is beneficial to add the icons of different spatial skill throughout the booklet, and mention them at the start because sometimes teachers are not aware of the different spatial skills that are tested during mathematics exercises.

7.2.3 Previous teacher, lectorate in science and technology education (A. Hotze)

A. Hotze revised the product on the 27th of June. She has experience in science and engineering education as well as in conducting research on skilled mathematicians. Hotze said that the booklet is very well designed and that the questions are well thought of and realised for the short amount of time and little experience in education of the designer. Besides, she said that the map is a good addition to the exercises and that it would definitely help with physical activity and development of spatial skills. Furthermore, she mentioned that 3rd and 4th grade students are interested in physical learning and learning though games. Adding a gaming element to the map/toolkit could therefore be an option. Lastly, she advised to keep in mind that if an exercise has multiple layers (a, b, c) children sometimes loose interest for the last one since they expect those to be more difficult than the first one. Adding a page for hints could help as a little push to keep trying for that last part of the exercise.

Some specific improvements or tips that she gave (of which most were implemented in the final design and some used as recommendations) were:

Implemented in the final design:

- Add a page with hints for the children to consult if they struggle to answer an exercise and cannot ask their teacher.
- In agreement with C. Zhu: do not add a star indication for difficulty of the exercises: it will demotivate children.
- (after suggesting it:) You could indeed improve the spatial element of the map by creating more 3D aspects, next to the hills and lakes.
- Mention the scale of the map on the map itself instead of (or besides) in the booklet, children of this age sometimes already look for a scale indication since they have knowledge on maps and scales.
- If you are adding a building exercise, for high buildings for example, you could create a light and shadow exercise for this.

Used as recommendations:

- You could add an exercise that asks students to fold or unfold one of the building on the map in paper, for example the school.
- The tiles would work better for the 1st and 2nd grade, for the 3rd and 4th grade a real ruler would maybe work better.

7.3 Revision of requirements and wishes

Most requirements and wishes have been implemented throughout the design phase. Some were implemented immediately in the design, like 'exercise/tool should allow for interactive learning' or 'exercise/tool should combine different domains or subjects'. While others needed to be tested with the target group, like 'exercise/tool should be ergonomically designed for 3rd and 4th grade students' or 'exercise/tool should allow for collaboration between students'. In the figure below, an overview is made of how well each requirement or wish is implemented in the final design. As can be seen, all requirements and wishes have at least been partly implemented in the design however some could be implemented more. Besides, not all of them could be validated through the tests done for this project and need to be tested through a long term research, like 'exercise/tool should improve children's spatial abilities'.

Requirements	**Score**
Exercise/tool should allow for creative learning	🟩
Exercise/tool should allow for interactive learning (use of external tools)	🟩
Exercise/tool should allow for collaboration between students	▨
Exercise/tool should allow for reflection	▨
Exercise/tool should challenge students further than the exercises in the current methods do	▨
Exercise/tool should be linguistically clear and not allow for misinterpretation	🟩
Exercise/tool should be challenging and stimulating for quick, creative and good mathematicians	🟩
Exercise/tool should be ergonomically designed for 3rd and 4th grade students	🟩
Exercise/tool should provide opportunities for exercises that fit 3rd and 4th grade learning objectives of Dutch educational materials	🟩
Exercise/tool should improve children's spatial ability and test different spatial skills	🟩 🟥

Wishes	**Score**
Exercise/tool should refer to real life situations and allow for a feeling of connection	🟩
Exercise/tool should simplify real life situations as little as possible and stimulate students' knowledge	🟩
Exercise/tool could allow for interaction between students of different levels	▨
Exercise/tool can be used for more than one exercise/subject or as many subjects as possible	🟩
Exercise/tool can introduce future topics/exercises to students	🟩
Exercise/tool should provide fitting guidance for potential skilled mathematicians	▨
Exercise/tool should combine different domains or subjects	🟩
Exercise/tool should create an enjoyable interaction with mathematics	🟩 🟥
Exercise/tool should inspire children to develop technical interest	▨ 🟥
Exercise/tool should show the importance of spatial ability in real life situations	🟩
Exercise/tool should consist requirements of innovation, as presented by Barend Last	▨

🟩 Fully implemented ▨ Partly implemented ⬜ Needs improvement 🟥 Needs more testing to validate

Table 8: How well the requirements and wishes are implemented in the final design

7.4 Revision on stakeholder wishes

Beside the list of requirements and wishes for the product to meet, there were some more specific needs of some of the stakeholders for the project. As can be seen on the visual below, as shown in Chapter 2 of this report, the three most important stakeholders were children, teachers and the designer. Since the wishes of the children are implemented in the previously shown list of requirements and wishes, this figure shows those of teachers and the designer. The first five are those of teachers and the last three those of the designer. There are three wishes that have been taken into consideration in the design of the toolkit, however are not fully met. 'it takes little time to understand how the tool should be used' and 'it takes little time to implement the tool in class' require an extra test in order to know how long implementation and understand will take for the teacher. Furthermore, the tool does take some storage space in class, however, the map is designed so that it can be stored in two pieces instead of one big one.

The wishes and needs of the stakeholders in the second and third ring of the stakeholder map are impacted or influenced by the wishes of the three inner circle stakeholders. However, during this project many meetings were held with the client (Wetenschapsknooppunt) to ensure that their wishes were still involved and met through the development of the product. Besides, meetings were held with experts, including employees of the PABO and SLO to ensure that the development of the product would also fit those stakeholders' wishes. The stakeholders of the third circle were not contacted directly during this project, however through research and talking to the stakeholders closer in the circle, their wishes were discussed and considered within this project. Mentionable is that teachers operate on the needs of school boards, who again operate on those of the government, so if needs of teachers are discussed, those of the government are implemented.

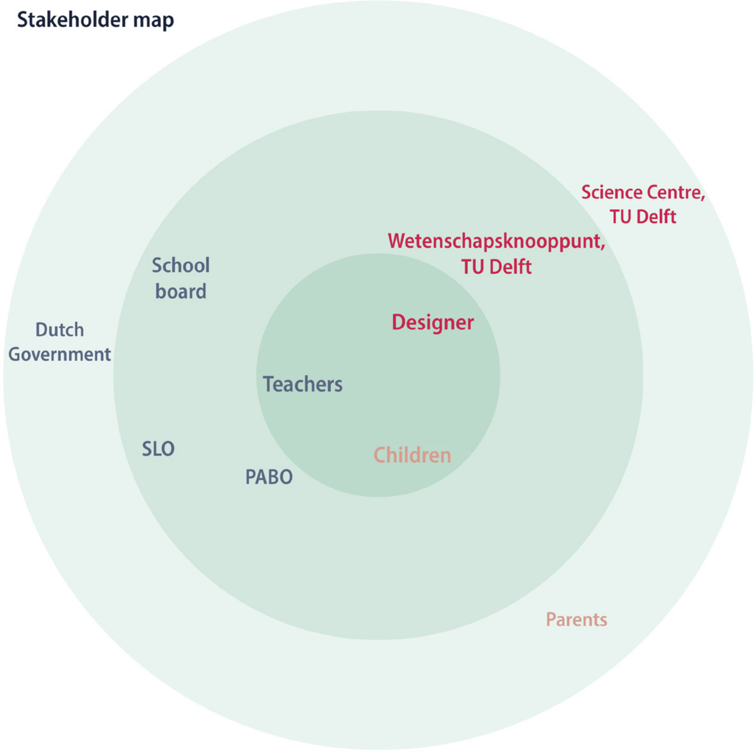


Figure 59: Stakeholder map

Wishes	**Score**
The tool can be reused	🟩
It takes little time to implement the tool in class	▨ 🟥
It takes little time to understand how the tool should be used	▨ 🟥
It is easy to store the tool away after use	🟩
The tool takes little storage space	▨
The tool is a physically embodied product	🟩
The tool is visualised so that it is appealing and understandable by children	🟩
The tool creates a fun/playful experience around mathematics	🟩

🟩 Fully implemented
▨ Partly implemented
□ Needs improvement
🟥 Needs more testing to validate

Table 9: How well stakeholder wishes are implemented in the final design

Naam:

De Woensdag Middag Avonturen Club

Verhalen over de toepassingen van ruimtelijk inzicht

"Naar het bos"

Opdracht 1

Kan jij bedenken waar Rosa woont? Dit zijn de details die de rest zich herinnert:

- Als ze van school naar huis fietst, passeert ze altijd het cirkel circuit
- Ze heeft één buur links van haar wonen
- Ze kan het meer zien vanuit haar voorraam en het huis van de boswachter vanuit haar rechter raam.

Rosa's adres: ____________________

Iedereen is aangekomen bij Rosa. Ze heeft haar kompas al gepakt en wacht op de rest.

Nu de groep compleet is, is het tijd om te gaan. Maar hoe lang is de weg naar het bos eigenlijk?

Opdracht 2

Bereken de afstand van Rosa's huis naar de rand van het bos (waar de weg smaller wordt) als je alleen over de wegen mag lopen. Je kunt de stenen van 3cm en 5cm gebruiken.

Ze hebben het bos bereikt. De weg is smaller en kronkeliger geworden.

Na een tijdje realiseert Symon zi dat Ory er niet meer is. Hi weggelopen toen ieder was met om zich hee

De groep loopt een tijdje rond als Spencer een bericht krijgt van Ory.

In het bericht is ook een foto g

Opdracht 3

8

Discussion and conclusion

This chapter revises the outcome of the project and discusses recommendations for future designs, limitations of the research and design . Furthermore it includes a conclusion in which each research and design question is answered and ends with a personal reflection on the project.

8.1 Recommendations

As could be seen on the previous page, all requirements and wishes have at least been partly implemented in the final design. Given the time frame of this project, it was not possible to implement or test every wish or requirements perfectly. However, that means that there are still enough opportunities to improve the product to meet all requirements and wishes perfectly. In order for a possible future design to perfectly meet each requirement or wish, or simply improve the product, a list of recommendations is written. These can be concrete recommendations to lift the viability of the product, make the product fitting for a larger audience or aspects that require further research.

Firstly, concrete ways to increase the value and viability of the product are:

- Creating extra types of exercises in the booklet, for example:

1. An exercise that asks students to build something physically on the map.
2. A viewpoint exercise (like exercise 4) where the viewer stands on a high building on the map, looking down on the map.
3. An exercise that asks students to fold a piece of paper into the correct 3D shape of one of the buildings on the map.
4. An exercise that asks students to create different fraction amounts with the tiles.
5. An exercise for which students need a physical mirror.
6. An exercise for which students need a physical compass.
7. An exercise for which students need a physical clock.

- Adding multiple booklets with different stories, next to the one created for this project 'to the woods'. Examples are 'to the beach', 'to the zoo', 'adventure at the sports club' or 'adventure in town'.
- For each booklet, specific exercises should be created, that tackle different spatial skills as well as encourage design and inquiry based learning.
- For some stories, an additional map could be created, including a beach or zoo for example. Otherwise, the current map needs to be adapted to fit a more elaborate town. This can also be done by creating different modules of a map that all fit to the main part of town.
- Add physical versions of the accessories of all characters to the toolkit: a compass, clock and mirror. These accessories could then be used for some exercises as well.

Furthermore, these are recommendations that could improve the product, but would have to be tested or researched to decide if implementation is beneficial:

- The collecting system around the tiles needs to be tested on the long term. For example, if it would create too much hassle in class, if it is doable for a teacher to give children a specific tile per class, where these tiles would be stored and if it adds to the collaborative element to let children have different tiles.
- The exact embodiment of the tiles, as well as its ideal assembly system needs to be studied. Perhaps, adding magnets to assemble the tiles is the best option.
- There could be more variations of the tiles, made of different sizes or shapes.
- Since the client mentioned that they would ideally create an educational tool that lasts through all grades of primary and high school, the options for this toolkit to fit to different ages should be studied and created. Ideas for this are;

1. Kindergarten: Introduce the map and characters to students,
2. 1st/2nd grade: Create exercises that require using the map but are fitting to the 1st and 2nd grade
3. 5th/6th grade: Create exercises that require children to build objects in 3D using the tiles. For this the assembly options of the tiles needs to be studied further in order to create both 2D rulers as well as 3D objects with them.

- It could be studied how this toolkit can be used by all children of the class, instead of only by skilled mathematicians. Perhaps this could be done by creating different booklets, or by dividing questions.
- It could be interesting to add a gaming element to some exercises of the toolkit. The map could function as a gaming board and children can choose their own character, with its own spatial skills. They could race against each other to whom is the first to answer different exercises and is the first to 'travel across the entire map'.

8.2 Limitations

During this project, considerations were made in order to create a full understanding of the market and to design a product that is as fitted for its target group as possible. However, given the short timeframe and limited resources, there are still many aspects that should be considered while looking at the outcome of this research, design and test results.

There are a few limitations regarding the children who participated in the tests. Firstly, the observations and testing of this project was done at three different schools. One observation was done at one school who follows a strictly way of teaching, while all other observations were done at a Waldorf school. The testing was done at two different classes of that Waldorf school. Mentionable as well is that both Waldorf locations are in the same neighbourhood in Delft and both classes are known to have many children with technically very well educated parents, which can lead to their children having a fair advantage in mathematics understanding. To get a complete understanding of 3rd and 4th grade students' experiences during mathematics classes, observations would have to be done at more and different types of schools, as well as schools at more locations in the Netherlands. Furthermore, out of the 14 children that participated in the tests, 11 were boys and only 3 were girls. For a future test, it would be interesting to see more girls participating in the tests as well. Lastly, all three product tests were done with already labelled skilled mathematicians and not with potentially skilled mathematicians. The latter should be included in future tests to ensure that their needs are also met with this product. For an even more complete image regarding if the needs of the entire target group are met, It would be interesting to identify good, quick and creative mathematicians and see if there are different results for each group.

Besides the participants, there were some organisational limitations during the tests as well. First of all, since there were too many exercises in the booklets to test with every participant. This means that not only was every exercise tested about three to six times on average, but also that not all participants did all exercises. This is important to keep in mind, especially when looking at the results for how challenging and fun the exercises were because that could have had a different result If all participants would have done all exercises. Furthermore, there are elements of the product that were not tested during this project because of the given time frame, but would need to be tested to ensure that the product fully works. The first is that the collecting system of the tiles was not tested, for future research this could be done by following a class for an entire months and giving the students one tile per day during normal mathematics classes . After, it would be interesting to see if all children having different tiles when they start with the toolkit, improves the level of interaction between students. If it does not, the collecting system of the tiles could be excluded.

Lastly, this test could not capture any long term results of using the toolkit. Most importantly, it would be interesting to see if using this toolkit would actually improve children's spatial ability. Also interesting to test if using this toolkit would create a more positive attitude towards mathematics for children and if possible to test, If it would actually increase their interest to choose technical career paths later in life. The latter was the hardest wish to test during this project.

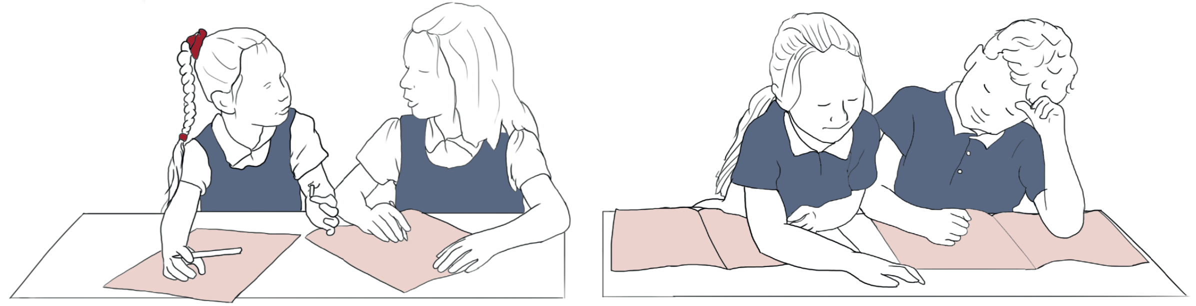
Wish: Exercise/tool should inspire children to develop technical interest

8.3 Conclusion

This thesis report started by introducing the environment of Dutch primary school mathematics, followed by four research, and one design question. Those were investigated and answered throughout the report. Beneath is a summary per question.

RQ1: Which characteristics in children lead to them not achieving their full potential in mathematics because of the current education methods?

This report chose to focus on the scope of skilled mathematicians, since these children do not receive sufficient guidance in the Dutch education system to pursue their full potential, which on its turn could decrease enjoyment and interest in technical fields for these children. Four types of (potential) skilled mathematicians were identified by S. Sjoers: good mathematicians, quick mathematicians, creative mathematicians and potential skilled mathematicians.



Quick mathematician

- Focus on process
- Explanation on working strategies

Potential skilled mathematician

- Guidance
- Exercises that support personal development

Good mathematician

- Deepening exercises
- Extra challenges

Creative mathematician

- Logical real life exercises
- Guidance to understand the existing methods
- Challenges to use their gained knowledge

All

- Scheduled instruction moments
- Discussion and reflection moments
- Guidance from teacher
- Extra challnges
- Interaction with each other

Figure 60: Needs of different types of skilled mathematicians

RQ2: Which characteristics of the current mathematics classes and methods lead to children not achieving their full potential and how could these be improved?

The four identified types of skilled mathematicians have shared and personal needs for them to achieve their full potential in mathematics classes. These needs are what is currently lacking in the Dutch education system to sufficiently guide these children.

RQ3: How can a new tool help to stimulate children to developing spatial skills in a creative and interactive way?

A tool designed for an improved experience and guidance for the four types of children should keep all their needs in mind in order to be an improvement compared to the currently available education methods. By designing a tool that considers all of them, these children can achieve their full potential in mathematics classes.

RQ4: What is needed for a good interaction between children and teachers in class in order to let the children achieve their full mathematical potential?

Besides the already mentioned needs for instruction, discussion, reflection and interaction/guidance between children and children with their teacher, there are other factors needed for teachers to allow their students to achieve their full potential:

- Teacher training college has way less instructions on how to help stronger students compared to weaker ones
- Most teachers are not aware of the differences between different skilled mathematicians and their personal needs or do not know how to identify them
- Teachers can experience maths anxiety, leading to fear on how to answer and help very skilled students
- Some teachers reach the teacher training college on other routes then the most common one, meaning that they did not learn every subject that they have to teach their future class
- There is a natural threshold to implement innovation in class, leading to schools and teachers not often implementing new tools

All these factors need to be considered when designing a tool to help skilled mathematicians.

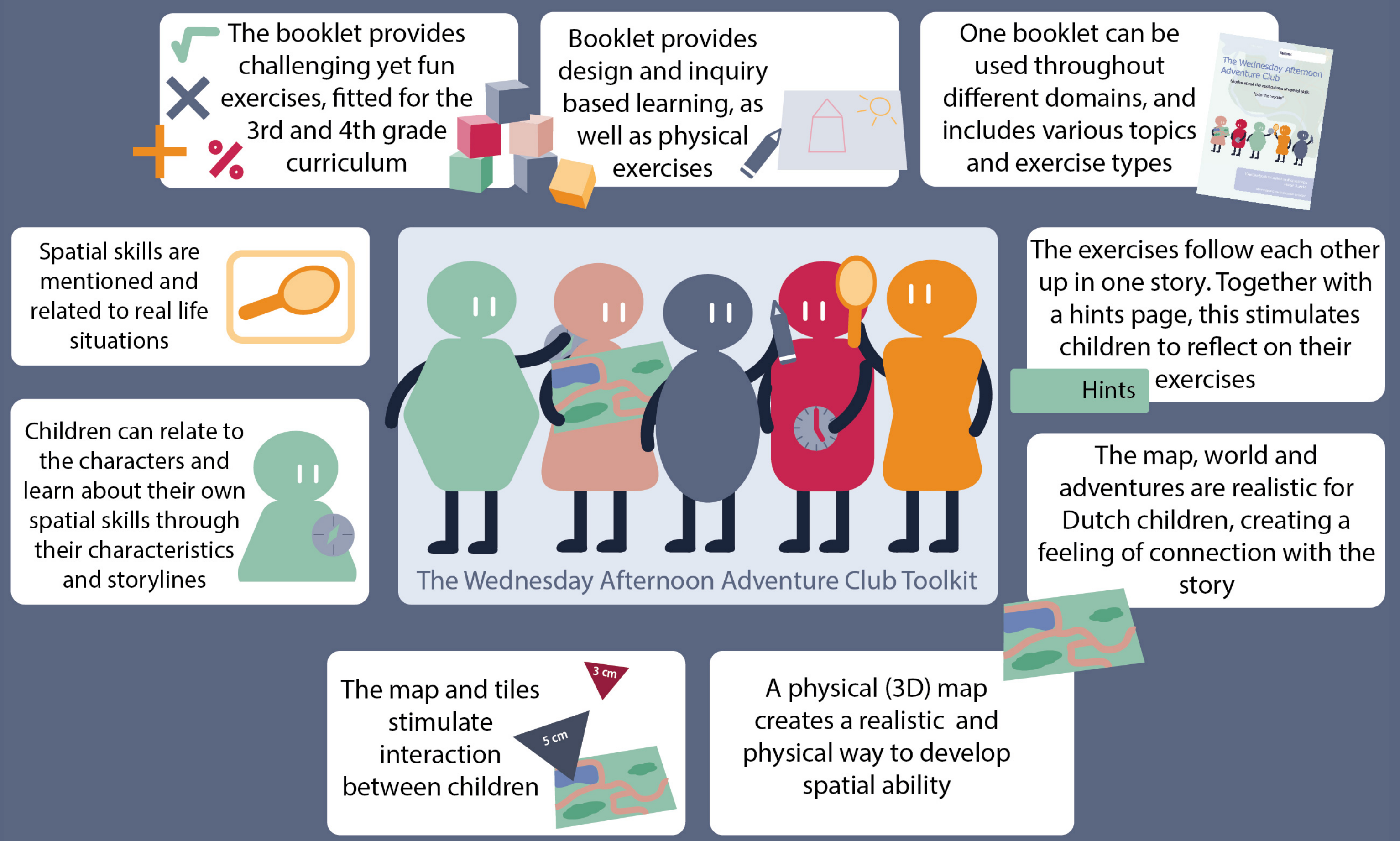
DQ: How would a new tool inspire innovation in the current mathematics classes?

The Wednesday Afternoon Adventure toolkit was designed to guide skilled mathematicians to develop their spatial ability in a physical and playful way. this toolkit inspires innovation in the classroom in a few ways. First of all, it will be offered for schools to use for a scheduled amount of time. Teachers will know that one booklet can be used in one hour (the average mathematics time per day). They can easily provide one booklet for one day of the week for a few weeks. Furthermore, the toolkit is a complete package and has an instruction manual for teachers. If used well, its physical and design and inquiry based learning aspect make the tool innovative compared to most existing education methods.

Design goal:

Design a tool that makes use of design and inquiry based learning in mathematics classes to improve the development of children's spatial ability for skilled mathematicians at primary school in grades 3 and 4 (groep 5 en 6) in the Netherlands.

How the designed product answers the design goal, is visualised by highlighting its unique selling points.



The booklet provides challenging yet fun exercises, fitted for the 3rd and 4th grade curriculum

Booklet provides design and inquiry based learning, as well as physical exercises

One booklet can be used throughout different domains, and includes various topics and exercise types

Spatial skills are mentioned and related to real life situations

The Wednesday Afternoon Adventure Club Toolkit

The exercises follow each other up in one story. Together with a hints page, this stimulates children to reflect on their exercises

Children can relate to the characters and learn about their own spatial skills through their characteristics and storylines

The map, world and adventures are realistic for Dutch children, creating a feeling of connection with the story

The map and tiles stimulate interaction between children

A physical (3D) map creates a realistic and physical way to develop spatial ability

Figure 61: Strength of the Wednesday Afternoon Adventure Club toolkit

8.4 Personal reflection

I started this project with some personal goals and aspirations. To summarise, these were: learn to implement children's needs and wishes in a design, design a physical product that adds value to an existing system, visualise the product to be understandable and attractive for children and learn about the difficulties that children have around mathematics classes (Appendix A: project brief).

I had designed for children before during my studies however never had it been so interactive and possible to visit them often, due to covid or simply not having the time or connections. During this project I really wanted to push myself to visit the target group often and not only 'check off' if my product worked, but really listen to the children and learn about their experiences and thoughts they have. I wanted to experience if designing for children is something I want to pursue after graduating. Education is a field I had no experience in outside of, of course, my own time at school, and tutoring high school students. When I started this project I thought my scope was quite small: spatial ability, 3rd and 4th grade, making maths more fun and later on: the narrowing down to skilled mathematicians. But I must say it was quite overwhelming at times to see and read how much research is already done in even in this 'small' scope. That is why I by no means learned everything there was to learn and had to choose which articles, books, instances and issues to focus on.

This focus required me to plan and analyse well. What exactly is my focus and what do I want to 'solve' with a product. It is a common setback in design processes to feel like you are not adding anything with your design and I definitely had those moments in this project, too. Although, the scope of skilled mathematicians is what helped me to believe that I was adding something. To learn about this group of children and their needs and wishes really inspired me to design something especially for them. To see the pride and happy feelings on their faces when I explained that I designed something for them, and had taken their opinions and experiences in mind while doing so, was really sweet to see.

I think that the biggest lesson for me during this project was to believe in what sufficient research and analysis can add to a product and to listen to my own ideas and opinions. I found it hard to believe during the research phase that I would draw new conclusions when I realised how much research was already done and how many product concerning a fun mathematics experience are already out there. However, I think that the focus of skilled mathematicians helped me to stay focused. I ended the research phase with requirements and wishes that were both clear, logical and inspiring and keeping those in mind throughout the rest of the project I kept reminding myself that the product would add something for this group of children. I'm glad that I could visit the schools often and test multiple design iterations. Both the tests with children and meetings with experts added many improvements to the design and I'm glad with the outcome. Of course, there are many things that can be reconsidered and I could continue for a while to improve this product (read limitations and recommendations).

I started this report with the following visual, showing the importance of spatial ability in every day situations and I would like to end the report by revising my own spatial ability throughout this project.
Some that I would like to highlight are:

- Analysing data: I had to choose which paths to follow or not within the research and sufficiently summarise the needs and requirements for a product that would best add value for the target group.
- Construction play/playing games: by designing exercises for children I had to keep in mind how they could be fun, what play element I could add and what the rules would be to allow for a challenging, yet still solvable exercise.
- Mathematics: of course, my own mathematics was also put to the test during this project. First of, I helped skilled mathematicians with their exercises during classes and understanding a (for me new) exercise on the spot an helping these intelligent children was a challenge. Furthermore, I understand better then ever how making your own exercises is the ultimate test for understanding a topic, because it required many, many calculations before adding an exercise to the booklet that will be solved in 30 seconds.
- Visualisation and orientation: these were mostly used throughout the visual aspects of the project, for example how the research data could be summarised in an appealing and clear visual and how the booklet would guide children and connect to them.
- Cooking/playing sports: during the deadline weeks of this project it was easy to eliminate everything outside of studying but I tried to manage my time as sufficient as possible and plan for relaxing moments as well.

Figure 62: How spatial ability was used in this project

(Expertis Onderwijsadviseurs, 2025)

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