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Nonlinear Iterative Learning Control: A Frequency-Domain Approach for Fast Convergence and High Accuracy ⋆

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Abstract:
Iterative learning control (ILC) involves a trade-off between perfect, fast attenuation of iteration-invariant disturbances and amplification of iteration-varying ones. The aim of this paper is to develop a nonlinear ILC framework that achieves fast convergence, robustness, and low converged error values in ILC. To this end, the method includes a deadzone nonlinearity in the learning update, which uses the difference in amplitude characteristics of repeating and varying disturbances to modify the learning gain for each error sample. A criterion for monotonic convergence of the nonlinear ILC algorithm is provided, which is used in combination with system measurements to select suitable design parameters. The proposed algorithm is validated using simulations, in which fast convergence to low error values is demonstrated.

Keywords: Iterative learning control, variable-gain control, nonlinear control, feedforward control, mechatronic systems

1. INTRODUCTION

Iterative learning control (ILC) is capable of attenuating repeating disturbances completely, yet it also amplifies iteration-varying disturbances. In ILC, the input that compensates the repeating disturbances is updated iteratively over a series of repeated experiments, resulting in high performance (Bristow et al., 2006). To achieve this, it is essential that the ILC algorithm meets criteria for (monotonic) convergence, taking into account robustness against model uncertainty (Oomen, 2020). In addition to these convergence requirements, the design of ILC controllers should be aimed at achieving small errors in a limited number of iterations.

The aim of achieving small errors in a limited number of iterations while meeting criteria for robust monotonic convergence leads to trade-offs and performance limitations in ILC. In particular, a limiting factor in the performance of ILC is that while iteration-invariant disturbances are compensated, iteration-varying disturbances such as noise are amplified up to a factor two (Butcher et al., 2008; Oomen and Rojas, 2017). In standard linear time-invariant (LTI) ILC approaches, including frequency-domain ILC (Blanken and Oomen, 2020) and lifted norm-optimal ILC (Gunnarsson and Norrlöf, 2001), limiting the amplification of iteration-varying disturbances typically results in both reduced attenuation of iteration-invariant disturbances (Bristow et al., 2006) and slower convergence (Butcher et al., 2008).

LTI ILC approaches offer limited possibilities to reduce the amplification of iteration-varying disturbances, because only frequency-domain characterizations are possible and therefore trade-offs cannot be avoided. In Butcher et al. (2008), multiple methods are proposed for frequency-domain ILC, including using a low-pass robustness filter or reducing the learning gain. While low-pass robustness filters are commonly used in ILC to achieve convergence (Bristow et al., 2006; Oomen, 2020) using these filters to attenuate noise may also lead to reduced attenuation of repeating disturbances and is therefore undesirable. A small learning gain achieves both a small error and minimal amplification of the iteration-varying disturbances, but at the cost of slow convergence. In norm-optimal ILC (Gunnarsson and Norrlöf, 2001), increasing the weights on the 2-norm of the input and the change in input has effects similar to respectively using a robustness filter and reducing a learning gain.

Instead of LTI ILC strategies, which are limited in the presence of iteration-varying disturbances, time-varying ILC strategies have been proposed. In Butcher et al. (2008), the learning gain is systematically reduced based on the iteration number, resulting in a time-varying system in the iteration domain. While this approach effectively limits the amplification of iteration-varying disturbances, convergence is slow. In Oomen and Rojas (2017), a sparse ILC algorithm is proposed that achieves time-varying ILC within one trial. To this end, the standard norm-optimal ILC criterion is extended by a convex relaxation of the ℓ_0-norm of the input signal that enforces sparsity. This approach reduces the amplification of iteration-varying disturbances significantly while retaining fast convergence properties. In contrast to norm-optimal ILC, the input update that minimizes the

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criterion in sparse ILC cannot be obtained in closed form, and instead an optimization problem is solved at each iteration. Because of the lack of closed-form expressions of the ILC controller, tuning for robustness is challenging in this approach.

Iteration-invariant and iteration-varying disturbances often have typical and different amplitude characteristics, which can be exploited in the design of ILC. The idea is to use nonlinear filters that essentially aim to filter out the low-amplitude iteration-varying disturbances. This idea relates to approaches that use nonlinear filters to create variable-gain feedback controllers, see, e.g., Heertjes and Steinbuch (2004) or the performance analysis in Pavlov et al. (2013). In Heertjes and Tso (2007), a related idea is proposed by adding a deadzone to the learning filter for lifted ILC without a robustness filter. In Heertjes et al. (2009) a low-pass filter is added to the learning filter, but as shown by Bristow et al. (2006) this implementation does not increase the robustness against model uncertainty. In addition, the Lyapunov-stability based analysis only provides a condition for convergence that cannot guarantee monotonic convergence, and therefore excessive learning transients may occur (Longman, 2000). The approach cannot be applied to frequency-domain ILC and it does not allow for the introduction of a robustness filter along the lines of Bristow et al. (2006).

Although significant steps have been taken towards ILC algorithms that achieve both fast convergence and limited amplification of iteration-varying disturbances, a closed-form approach that enables the use of robustness filters and for which conditions for monotonic convergence are available is still lacking. In this paper, a nonlinear ILC algorithm is developed that includes a deadzone in the learning filter for frequency-domain ILC. This results in an ILC algorithm with learning gains that vary over trials, as well as within a single trial. Explicit expressions of the ILC filters are available, and monotonic convergence criteria are provided. The proposed approach achieves fast convergence, robustness against model uncertainty, strong attenuation of repeating disturbances and limited amplification of iteration-varying disturbances. The contribution of this paper consists of the following elements.

- A nonlinear ILC algorithm containing a learning filter with deadzone is introduced, that achieves fast convergence, robustness, strong attenuation of repeating disturbances, and limited amplification of iteration-varying disturbances.
- A condition for the monotonic convergence of nonlinear frequency-domain ILC is provided, which is reminiscent of existing LTI ILC convergence conditions and which can be checked based on measured frequency response data.
- The approach is validated in simulations.

This paper is structured as follows. In Section 2, the problem is introduced. In Section 3, the potential for nonlinear ILC is illustrated. In Section 4 the nonlinear ILC algorithm is introduced and conditions for monotonic convergence are given. The approach is validated using simulations in Section 5. Conclusions are given in Section 6.

Notation: Throughout, $\|x\|_2 = \sqrt{\sum_{i=-\infty}^{\infty} |x_i|^2} < \infty$ denotes the ℓ_2-norm for $x \in \ell_2$. The $\mathcal{L}_\infty$-norm for discrete-time transfer functions is denoted by $\|G\|_{\mathcal{L}_\infty} = \sup_{\omega\in[0,2\pi)} |G(e^{i\omega})|$ for a rational transfer function $G \in \mathcal{RL}_\infty$. The power spectrum of signal x is defined as in (Ljung, 1999, Section 2.3) and is denoted by ϕ_x.

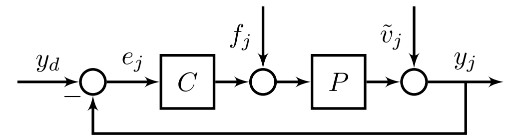

Fig. 1. Parallel ILC configuration.

2. PROBLEM FORMULATION

In iterative learning control (ILC) an input signal is updated iteratively in order to learn to attenuate repeating disturbances. In case of parallel ILC, the learned input signal is a feedforward signal, as illustrated in Fig. 1. For a SISO discrete-time linear time-invariant (LTI) system, this leads to a system of the form

$$e_j = r - Jf_j - v_j, \tag{1}$$

which is operating iteratively with error $e_j \in \ell_2$ in iteration $j \in \mathbb{Z}_{\geq 0}$, iteration-invariant disturbance $r \in \ell_2$, feedforward signal $f_j \in \ell_2$ and iteration-varying disturbance $v_j \in \ell_2$. In the parallel ILC configuration with reference y_d and measurement noise $\tilde{v}_j$, as shown in Fig. 1, $r = Sy_d$ with $S = (1 + PC)^{-1}$, $v_j = S\tilde{v}_j$ and $J = SP$.

While standard ILC can attenuate the iteration-invariant disturbance r completely, it may also amplify the iteration-varying disturbances v_j, such as noise, up to a factor two. This is further illustrated in Section 3. The amplification of iteration-varying disturbances can be reduced by reducing the learning gain, resulting in slow convergence.

The aim of this paper is to develop a frequency-domain iterative learning control (ILC) method that achieves both small converged errors and fast convergence. In general, for applications for which ILC is useful, the amplitude of iteration-invariant disturbances is higher than that of the iteration-varying disturbances. The main idea is to use this difference in amplitude characteristics to distinguish between iteration-invariant disturbances, which should be attenuated, and iteration-varying disturbances, which should not be amplified. In particular, a nonlinear deadzone is used to apply different learning gains based on the signal amplitude, leading to a learning gain that is time-varying within one iteration. In addition, as the error reduces over iterations, the learning gain is automatically decreased. Through this automated time-and iteration-varying learning gain, fast convergence to small errors is achieved.

3. POTENTIAL FOR NONLINEAR ILC

In this section the potential for nonlinear ILC is illustrated. First, frequency-domain ILC is introduced. Secondly, the trial-varying disturbances in standard ILC are analyzed, and thirdly the achievable performance of nonlinear ILC is estimated.

3.1 Frequency-domain iterative learning control

When iterative learning control (ILC) is applied to system (1), input signal f_j is updated iteratively according to

$$f_{j+1} = Q(f_j + \alpha L e_j), \tag{2}$$

with learning filter L, which is chosen to approximate J^{-1}, robustness filter Q, which is typically a zero-phase low pass filter, and learning gain α which is typically chosen to be $\in (0, 1]$. In the case where $v_j = 0 \,\forall\, j$, (1) can be substituted in (2) to give the feedforward iteration

$$f_{j+1} = Q(1 - \alpha LJ)f_j - \alpha QLJr. \tag{3}$$

In frequency-domain ILC, it is assumed that the duration of an iteration is infinite and infinite-time signals are used. With this assumption, it is possible to design and analyze the ILC algorithm using transfer function expressions. Since the system J in transfer function form is invertible, the error iteration for a frequency-domain ILC update of the form (2) is given by

$$e_{j+1} = (1-Q)r + Q(1-\alpha JL)e_j. \tag{4}$$

The following lemma gives a the convergence condition for noise-free frequency domain ILC.

Lemma 1. The sequence of error iterates $\{e_j\}$ is monotonically convergent in the ℓ_2-norm $\|x\|_2 = \sqrt{\sum_{-\infty}^{\infty}|x_i|^2} < \infty$ if the mapping from e_j to e_{j+1} is a contraction, which is the case if

$$\|Q(1-\alpha LJ)\|_{\mathcal{L}_\infty} = \sup_{\omega\in[0,2\pi)} |Q(e^{i\omega})(1-\alpha L(e^{i\omega})J(e^{i\omega}))| < 1. \tag{5}$$

Essential in this condition is that it allows for non-causal filters Q and L (Oomen and Rojas, 2017, Theorem 2), and that it can be evaluated based on frequency-response measurements of $J(e^{i\omega})$ such that differences between the model and the measured system can be taken into account directly.

3.2 Amplification of iteration-varying disturbances in ILC

Consider the ILC system (1) with update (2). To analyze the propagation of the iteration-invariant and iteration-varying disturbances r and v_j over iterations, it is assumed that Q and L are chosen such that the the sequence of error iterates $\{e_j\}$ is convergent. In addition, assume that iteration-varying disturbance $v_j = Hn_j$, where n_j is i.i.d. zero-mean white noise and H is monic and bistable (Ljung, 1999). Then, for $f_0 = 0$, the error for iteration j is given by (Oomen and Rojas, 2017, Lemma 5)

$$e_j = \left(1 - J\frac{1-Q(1-\alpha LJ))^j}{1-Q(1-\alpha LJ)}Q\alpha L\right) r - v_j - J\sum_{n=0}^{j-1}\left(Q(1-\alpha LJ)^n Q\alpha L v_{j-n-1}\right). \tag{6}$$

For $j\to\infty$, the spectrum of the converged error is given by

$$\phi_{e_\infty} = \left|\frac{1-Q}{1-Q(1-\alpha LJ)}\right|^2 \phi_r + \left(1+\frac{|\alpha JQL|^2}{1-|Q(1-\alpha LJ)|^2}\right)\phi_v, \tag{7}$$

with ϕ_r and ϕ_v the spectra of the iteration-invariant disturbance r and the iteration-varying disturbance v, respectively. For the simple case with $Q = 1$ and $L = J^{-1}$, it is clear that the first term in (7) is equal to zero, such that the spectrum of the resulting converged error is given by

$$\phi_{e_\infty} = \left(1+\frac{\alpha^2}{2\alpha-\alpha^2}\right)\phi_v. \tag{8}$$

For learning gain $\alpha = 1$, this gives $\phi_{e_\infty} = 2\phi_v$, i.e., the iteration-varying part of the error is amplified by a factor 2 by ILC. For $\alpha \to 0$, this effect is mitigated and $\phi_{e_\infty} \to \phi_v$. This is the smallest achievable spectrum, since ILC cannot compensate for the unknown iteration-varying disturbance v_j in iteration j.

While reducing α reduces the spectrum of the converged error, it also reduces the convergence speed significantly. In addition, reducing $\alpha \in (0,1]$ when $Q \neq 1$ may increase the contribution of the spectrum of the iteration-invariant disturbance to the error. To illustrate this, consider the case with $Q \neq 1$ and $L = J^{-1}$, such that

$$\phi_{e_\infty} = \left|\frac{1-Q}{1-Q(1-\alpha)}\right|^2 \phi_r + \left(1+\frac{|\alpha Q|^2}{1-|Q(1-\alpha)|^2}\right)\phi_v. \tag{9}$$

In this case reducing $\alpha \in (0,1]$ reduces the term $1-Q(1-\alpha)$ in the numerator of the term before ϕ_r, thus increasing the contribution of ϕ_r to ϕ_{e_∞}. It is therefore, in general, not desired to choose $\alpha \approx 0$. In Fig. 2 the effect of reducing α is illustrated using simulation results that are further elaborated upon in Section 5. It is shown that for high values of α, the convergence is fast but the converged error is relatively high. Reducing the learning gain reduces results in slow convergence, but the converged error is reduced significantly since iteration-varying disturbances are amplified less.

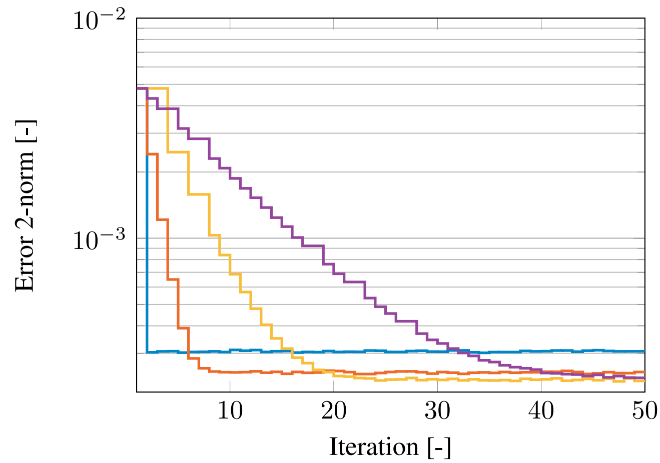

Fig. 2. Error 2-norm over iterations for $\alpha = 1$ (—), 0.5 (—), 0.2 (—) and 0.1 (—), averaged over 20 realizations. Small learning gains lead to lower converged errors at the cost of slower convergence.

3.3 Estimating iteration-varying disturbances

The achievable performance of ILC is determined by the size of the iteration-varying disturbances in a system, which can be estimated easily based on a series of standard-operation experiments. Consider a series of n_e experiments on the system (1) with $f_j = 0\,\forall\, j$. The output of each experiment is given by

$$e_j = r - v_j \tag{10}$$

with r the iteration-invariant part of the disturbances, and v_j a realization of the iteration-varying disturbances. An estimate $\hat{r}$ of the invariant part of the disturbances is given by the sample mean of the error signal over n_e experiments (Oomen, 2020):

$$\hat{r} = \frac{1}{n_e}\sum_{j=0}^{n_e-1} e_j = r - \frac{1}{n_e}\sum_{j=0}^{n_e-1} v_j. \tag{11}$$

Then, for each experiment e_j an estimate of the iteration-varying disturbances is given by

$$\hat{v}_j = \hat{r} - e_j. \tag{12}$$

This gives several realizations of the iteration-invariant part of the error. These realizations indicate the best performance that can be achieved through ILC, because ILC can never attenuate iteration-varying disturbances. In addition, they can be used to design the deadzone nonlinearity that is introduced in the next section, as is further illustrated in the simulations in Section 5.

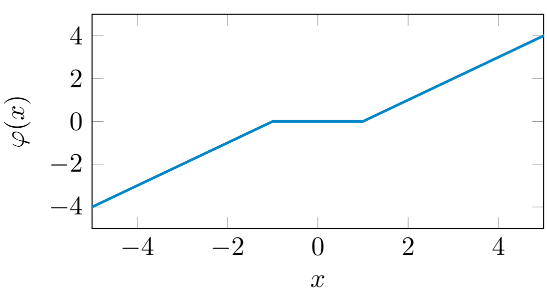

Fig. 3. Deadzone nonlinearity with $\gamma = 1,\ \delta = 1$.

4. NONLINEAR ILC

In the previous section the trade-off between limited amplification of iteration-varying disturbances and convergence speed in standard ILC was illustrated. In this section, the learning filter in ILC is extended by a deadzone, resulting in a nonlinear ILC algorithm that removes this trade-off. First, the algorithm is introduced and secondly a convergence criterion is developed.

4.1 Nonlinear ILC algorithm

To achieve both fast convergence and limited amplification of varying disturbances, a deadzone nonlinearity is included in the feedforward update. Through this filter, different learning gains are applied to the error signal based on amplitude characteristics. Small learning gains are applied to low-amplitude iteration-varying disturbances, resulting in limited amplification, and high learning gains are applied to high-amplitude iteration-invariant disturbances, resulting in fast attenuation.

The nonlinear frequency-domain ILC update for a system of the form (1) is given by the following expression.

$$f_{j+1} = Q(f_j + \alpha L e_j + L\varphi(e_j)), \tag{13}$$

with $\varphi(e_j)$ a deadzone nonlinearity, illustrated in Fig. 3, which is applied to each sample k of $e_j(k)$ according to

$$\varphi(e_j(k)) = \begin{cases} 0, & \text{if } |e_j(k)| \leq \delta \\ \left(\gamma - \frac{\gamma\delta}{|e_j(k)|}\right) e_j(k), & \text{if } |e_j(k)| > \delta. \end{cases} \tag{14}$$

Deadzone $\varphi(e_j)$ is a static nonlinearity that satisfies an incremental sector condition, i.e., for scalars e_1 and e_2, it holds that

$$0 \leq \frac{\varphi(e_1) - \varphi(e_2)}{e_1 - e_2} \leq \gamma. \tag{15}$$

This property enables the analysis of the convergence of the nonlinear ILC algorithm, as is shown in Section 4.

Note that any type of static nonlinearity that satisfies an incremental sector condition can be used in the feedforward update (13). In this paper, a deadzone is used because it can distinguish based on signal amplitudes to apply varying gains. However, for other purposes different nonlinearities could be considered.

4.2 Convergence of frequency-domain nonlinear ILC

Next, a condition for the monotonic convergence of frequency-domain ILC with a deadzone is developed. To analyze the convergence of algorithm (13), consider the situation with $v_j = 0 \forall j$. Then, the feedforward and error iterations that follow from the update law (13) are given by

$$f_{j+1} = \alpha Q L r + Q(1 - \alpha L J) f_j + Q L \varphi(r - J f_j), \tag{16}$$
$$e_{j+1} = (1 - Q) r + Q(1 - \alpha J L) e_j - Q J L \varphi(e_j). \tag{17}$$

The following theorem gives a condition for the monotonic convergence of the sequence of iterates $\{e_j\}$ in nonlinear ILC.

Theorem 2. The sequence of iterates $\{e_j\}$ for system (1) with feedforward update (13) converges monotonically in terms of the ℓ_2-norm to a fixed point e_∞ if

$$\left\| Q\left(1 - \alpha JL - \frac{\gamma}{2} JL\right)\right\|_{\mathcal{L}_\infty} + \frac{\gamma}{2}\|QJL\|_{\mathcal{L}_\infty} < 1, \tag{18}$$

with $\gamma, \alpha > 0$.

The following lemma is an auxiliary result used in the proof of Theorem 2.

Lemma 3. After a loop transformation, the feedforward update with $\varphi(e_j)$ satisfying sector condition (15) is equivalent to

$$f_{j+1} = Q\left(f_j + \left(\alpha + \frac{\gamma}{2}\right) L e_j + L\tilde{\varphi}(e_j)\right), \tag{19}$$
$$\tilde{\varphi}(e_j) = \varphi(e_j) - \frac{\gamma}{2} e_j, \tag{20}$$

with $\tilde{\varphi}(e_j)$ satisfying the symmetric sector condition:

$$-\frac{\gamma}{2} \leq \frac{\tilde{\varphi}(e_1) - \tilde{\varphi}(e_2)}{e_1 - e_2} \leq \frac{\gamma}{2}. \tag{21}$$

Proof. [Proof of Theorem 2] By the Banach fixed-point theorem, the sequence $\{e_j\}$ converges monotonically to a unique fixed point if the mapping $\mathcal{F}(e_j) = e_{j+1}$ is a contraction mapping (Kreyszig, 1978, Chapter 5). Mapping $\mathcal{F}$ is a contraction in the ℓ_2-norm if there exists a number $\rho < 1$ for which

$$\|\mathcal{F}(e_1) - \mathcal{F}(e_2)\|_2 \leq \rho \|e_1 - e_2\|_2. \tag{22}$$

Next,

$$\begin{aligned} &\|\mathcal{F}(e_1) - \mathcal{F}(e_2)\|_2 = \\ &\|Q(1 - \alpha JL)e_1 - JL\varphi(e_1) - Q(1 - \alpha JL)e_2 + JL\varphi(e_2)\|_2 \\ &= \|Q(1 - \alpha JL)(e_1 - e_2) - QJL(\varphi(e_1) - \varphi(e_2))\|_2. \end{aligned} \tag{23}$$

Using the loop transformation of Lemma 3,

$$\begin{aligned} \|\mathcal{F}(e_1) - \mathcal{F}(e_2)\|_2 &= \| \, Q\left(1 - \left(\alpha + \frac{\gamma}{2}\right) JL\right)(e_1 - e_2) \\ &\quad - QJL(\tilde{\varphi}(e_1) - \tilde{\varphi}(e_2))\|_2 \\ &\leq \left\| Q\left(1 - \left(\alpha + \frac{\gamma}{2}\right) JL\right)(e_1 - e_2)\right\|_2 \\ &\quad + \|QJL(\tilde{\varphi}(e_1) - \tilde{\varphi}(e_2))\|_2, \\ &\leq \left\| Q\left(1 - \left(\alpha + \frac{\gamma}{2}\right) JL\right)\right\|_{\mathcal{L}_\infty} \|e_1 - e_2\|_2 \\ &\quad + \|QJL\|_{\mathcal{L}_\infty} \|\tilde{\varphi}(e_1) - \tilde{\varphi}(e_2)\|_2, \end{aligned} \tag{24}$$

through application of the triangle inequality and multiplicative property for matrix norms, and (Zhou et al., 1996, Theorem 4.4). From (21), it follows that for each entry of e_1 and e_2,

$$|\tilde{\varphi}(e_1(k)) - \tilde{\varphi}(e_2(k))| \leq \frac{\gamma}{2}|e_1(k) - e_2(k)|. \tag{25}$$

Therefore, it also holds that

$$\|\tilde{\varphi}(e_1) - \tilde{\varphi}(e_2)\|_2 \leq \frac{\gamma}{2}\|e_1 - e_2\|_2. \tag{26}$$

Using this inequality,

$$\begin{aligned} \|\mathcal{F}(e_1) - \mathcal{F}(e_2)\|_2 &\leq \left\| Q\left(1 - \left(\alpha + \frac{\gamma}{2}\right) JL\right)\right\|_{\mathcal{L}_\infty} \|e_1 - e_2\|_2 \\ &\quad + \frac{\gamma}{2}\|QJL\|_{\mathcal{L}_\infty} \|e_1 - e_2\|_2 \end{aligned} \tag{27}$$

$$\leq \left(\left\| Q\left(1 - \left(\alpha + \frac{\gamma}{2}\right) JL\right)\right\|_{\mathcal{L}_\infty} + \frac{\gamma}{2}\|QJL\|_{\mathcal{L}_\infty}\right)\|e_1 - e_2\|_2$$

It follows that the mapping $\mathcal{F}$ is a contraction if

$$\left\| Q\left(1 - \left(\alpha + \frac{\gamma}{2}\right) JL\right)\right\|_{\mathcal{L}_\infty} + \frac{\gamma}{2}\|QJL\|_{\mathcal{L}_\infty} \leq \rho \tag{28}$$

with $\rho < 1$. □

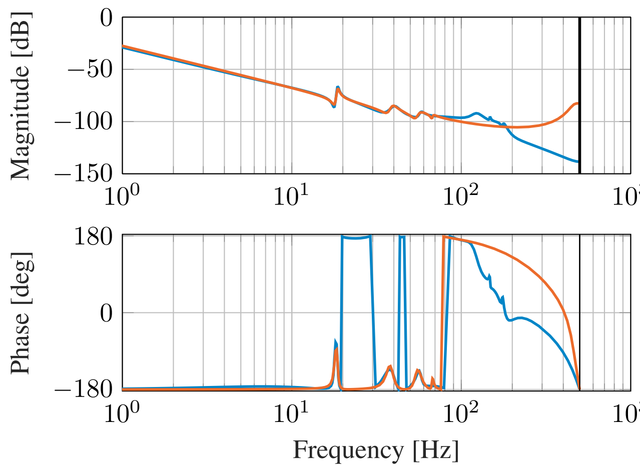

Fig. 4. Bode diagram of the system (—) and a low-order approximation (—).

Remark 4. The loop transformation to a symmetric sector condition in Lemma 3 is used to reduce the conservativeness of the convergence condition in Theorem 2, which is caused by the triangle inequality used to separate the norms in (24). If this transformation is omitted and the original non-symmetric sector condition is used instead in Equations 25 and 26, this results in the much more conservative convergence condition

$$\|Q(1-\alpha JL)\|_{\mathcal{L}_\infty} + \gamma \|QJL\|_{\mathcal{L}_\infty} \leq 1. \tag{29}$$

The convergence condition in Theorem 2 recovers several useful properties of the standard frequency-domain ILC convergence condition (Lemma 1). First, Theorem 2 uses $\mathcal{L}_\infty$-norms which can be evaluated easily using model knowledge or measured frequency-response data of the system. Second, the condition allows the use of a robustness filter Q, which can be designed intuitively because the norms can be visualized using Bode diagrams. Third, the $\mathcal{L}_\infty$-norms allow the use of non-causal filters, thus enabling the use of zero-phase filtering for Q, and a non-causal $L \approx J^{-1}$ that enables preview.

5. SIMULATION EXAMPLE

In this section the addition of a deadzone in frequency-domain ILC is validated and illustrated in simulations. First, a suitable choice of deadzone parameters is made. Then, simulation results are given and lastly it is illustrated that a less conservative convergence criterion could lead to even better performance.

The system used in simulation is the carriage of an industrial flatbed printer for which high- and low-order models are available. The high-order model represents the true system, which is approximated by the low-order model, resulting in a model mismatch at high frequencies, see Fig. 4. To ensure convergence in the linear case with $\alpha = 1$, the Q-filter is chosen as a first-order lowpass filter with a cutoff frequency of 100 Hz. The reference is shown in Fig. 5 and in the simulations Gaussian white noise with a variance of $0.005\,\mathrm{V}^2$ is added to the plant input, resulting in the mean and noise estimates shown in Fig. 7. For each ILC configuration, the results are averaged over 20 realizations.

5.1 Selection of the deadzone parameters

To implement the deadzone $\varphi(e_j)$ in (14), the parameters γ and δ should be chosen. The gain γ of the deadzone can be

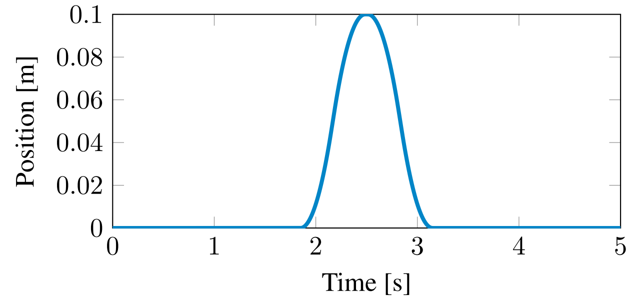

Fig. 5. Reference used in the simulations.

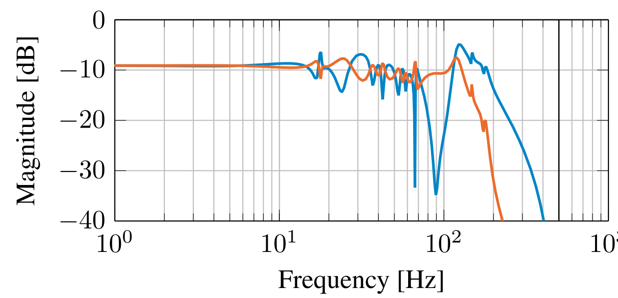

Fig. 6. Bode magnitude plots of $Q\left(1-\alpha JL-\frac{\gamma}{2}JL\right)$ with $\mathcal{L}_\infty = 0.5720$ (—) and $\frac{\gamma}{2}QJL$ with $\mathcal{L}_\infty = 0.4224$ (—). Since $0.5720 + 0.4224 < 1$, the system with $\alpha = 0.3$, $\gamma = 0.7$ meets the convergence condition of Theorem 2.

compared to the learning gain α in the linear case, in the sense that it influences the convergence speed and the amplification of iteration-varying disturbances. In contrast to the linear learning gain, nonlinear gain γ is only applied to errors outside of the deadzone. Since these errors are typically caused by iteration-invariant disturbances which should be attenuated fast, γ should be chosen as close to 1 as possible without compromising the convergence condition in Section 4. In addition, to limit the amplification of iteration-varying disturbances, the linear learning gain α should be chosen close to 0. In Fig. 6 examples of the Bode diagrams of the two terms in the convergence condition are shown for $\alpha = 0.3$, $\gamma = 0.7$, and it is shown that for these parameters, the convergence condition is met.

The deadzone width δ should be chosen such that learning is only applied to iteration-invariant disturbances based on their amplitude, i.e., it should be chosen such that $\varphi(v_j) \approx 0$. To determine δ, the amplitude of the iteration-varying disturbances is estimated using a series of $n_e = 20$ standard-operation experiments with $f_j = 0$ on the system, as explained in Section 3. This results in an estimate $\hat{r}$ of the iteration-invariant disturbances and 20 estimates $\hat{v}_j$ of different realizations of the iteration-varying part of the error, as shown in Fig. 7.

The estimates of v_j are used to determine a suitable value of δ that filters out the desired percentage of iteration-varying disturbances. Note that for values slightly larger than δ the gain is very small, because of the shape of the deadzone, see Fig. 3. Therefore, it is typically not necessary to choose δ such that it includes all iteration-varying disturbances. Based on Fig. 7, the deadzone width is chosen as $\delta = 5 \times 10^{-6}$ m.

5.2 Comparison between linear and nonlinear ILC

Two combinations of the linear learning gain α and the nonlinear gain γ are applied in simulation. For linear ILC, smaller

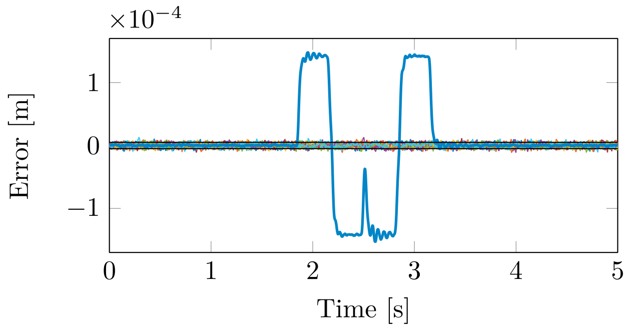

Fig. 7. Mean $\hat{r}$ of the error signal over 20 iterations (—) and the noise estimates $\hat{v}_j$ for $j = 1, 2, ..., 20$. The black lines indicate the interval $[-5\times10^{-6}, 5\times10^{-6}]$, which contains most of the noise.

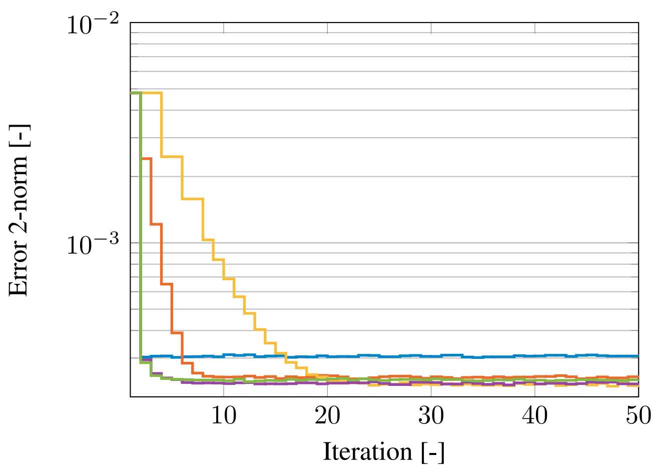

Fig. 8. Error 2-norm for linear ILC with $\alpha = 1$ (—), 0.5 (—) and 0.2 (—) and nonlinear ILC with $\alpha = 0.1$, $\gamma = 0.9$ and $\delta = 5\times10^{-6}$ (—) and $\alpha = 0.3$, $\gamma = 0.7$ and $\delta = 5\times10^{-6}$ (—). Nonlinear ILC removes the trade-off between convergence speed and converged error.

values of $\alpha \in (0, 1]$ result in slower convergence and lower errors, see Section 2 and Fig. 2. In Fig. 8 it is shown that nonlinear ILC removes this trade-off between convergence speed and converged error value. For $\alpha = 0.1$ and $\gamma = 0.9$, nonlinear ILC matches the convergence speed of linear ILC with $\alpha = 1$ for the first iteration. After that, it converges in six iterations to the error value that linear ILC only achieves after 25 iterations with $\alpha = 0.2$. Thus, adding a deadzone to frequency-domain ILC results in both fast convergence and small errors due to limited amplification of the iteration-varying disturbances.

5.3 *Conservativeness of the convergence criterion*

The convergence criterion of Theorem 2 is a sufficient condition for monotonic convergence, and in some cases it is conservative. This is shown by comparison of the results for two different configurations for nonlinear ILC. For $\alpha = 0.3$, $\gamma = 0.7$ the convergence criterion is met, as shown in Fig. 6. However, in Fig. 8 it is shown that for $\alpha = 0.1$, $\gamma = 0.9$ a comparable convergence speed leads to an even smaller converged error. For this second configuration, $\|Q\left(1 - \alpha JL - \frac{\gamma}{2}JL\right)\|_{\mathcal{L}_\infty} = 0.5651$ and $\|\frac{\gamma}{2}QJL\|_{\mathcal{L}_\infty} = 0.5431$, such that the convergence criterion in Theorem 2 is not met. The illustrated conservativeness of the convergence criterion requires further research.

6. CONCLUSIONS

In this paper, a nonlinear frequency-domain ILC algorithm is developed that achieves both fast convergence and a small converged error in the presence of iteration-varying disturbances. The approach removes the traditional trade-off between convergence speed and amplification of iteration-varying disturbances in ILC through a deadzone nonlinearity, which applies various learning gains to different elements of the error signal depending on their magnitude. A condition for monotonic convergence of the frequency-domain algorithm is given that enables the use of robustness filters and that can be evaluated using measured frequency-response data of the system. The proposed algorithm is validated using simulations, in which fast convergence to small errors is demonstrated. Future research will be aimed at reducing the conservativeness of the convergence criterion, extension of the approach to time-domain ILC with robustness filters, and experimental implementation.

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