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**DOI**

[10.1103/PhysRevB.100.064507](https://doi.org/10.1103/PhysRevB.100.064507)

**Publication date**

2019

**Document Version**

Final published version

**Published in**

Physical Review B

**Citation (APA)**

Vodolazov, D. Y., & Klapwijk, T. M. (2019). Photon-triggered instability in the flux flow regime of a strongly disordered superconducting strip. *Physical Review B*, *100*(6), Article 064507. <https://doi.org/10.1103/PhysRevB.100.064507>

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**Photon-triggered instability in the flux flow regime of a strongly disordered superconducting strip**D. Yu. Vodolazov <sup>\*</sup>*Institute for Physics of Microstructures, Russian Academy of Sciences 603950, Nizhny Novgorod, GSP-105, Russia  
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(Received 12 April 2019; published 9 August 2019)

We study, theoretically, the single-photon response of a strongly disordered thin superconducting strip in the flux flow state. We find that this resistive state, at a current  $I$  larger than the critical current  $I_c$ , jumps to the normal state by the absorption of a single optical photon. The absorbed photon creates a beltlike region with suppressed superconductivity and fast moving Josephson-like vortices across the strip. The formed Josephson-like link is not stable in such a superconductor and evolves into a normal domain which expands along the length of the superconducting strip, leading to the transition to the normal state.

DOI: [10.1103/PhysRevB.100.064507](https://doi.org/10.1103/PhysRevB.100.064507)**I. INTRODUCTION**

It is known that the absorption of a single photon of the optical or near-infrared wavelengths can switch a thin current-carrying superconducting strip from the superconducting to a resistive state [1,2]. The effect is based on the local heating of the superconductor, i.e., the creation of a hot spot with heated electrons and phonons, where the superconductivity is locally weakened. Such a local disturbance of the superconducting state distorts the current density distribution, and leads to the reduction of the Meissner state and to the nucleation of a vortex and antivortex inside the hot spot, depending on the distance from the edge. This nucleation occurs when the current in the strip exceeds a critical value, which is less than the critical current of the strip without the hot spot [3]. The motion of the vortex and/or antivortex due to the Lorentz force involves dissipation, leading to further heating of the superconductor and, eventually a switch to the normal state.

It is essential for the experimental observation of the photon event that the strip switches to the resistive state and does not spontaneously jump back. It implies that the current-voltage characteristics should be hysteretic. Photon detection is therefore only possible only at current biases larger than the so-called retrapping current  $I_r$ , where the superconducting strip goes back to the superconducting state, when the external pulselike perturbation is gone. Experimentally, the single-photon response is mainly observed in strongly disordered thin superconducting strips with a sheet resistance  $R_{\square}$  of  $\gtrsim 500$ – $1000 \Omega$ , a resistivity  $\rho_n \gtrsim 100$ – $200 \mu\Omega \text{ cm}$  and, hence, a rather small diffusion coefficient  $D \lesssim 0.5 \text{ cm}^2/\text{s}$  (NbN, NbTiN, MoSi, WSi, etc.). These material requirements are connected to the needed small electron-electron inelastic-scattering time  $\tau_{ee}$  and the small diffusion constant, which

provides a relatively compact hot spot with a high local temperature of the electrons  $T_e$ , which effects the superconducting state stronger than a large hot spot with a low  $T_e$  [4].

Here, we propose that single-photon response can occur also in a superconducting strip in which vortices are present induced by a perpendicular magnetic field. In such films, an applied current induces a resistive flux flow state, which transits into a more resistive state at a “quench” current  $I_q(H) > I_c(H)$  [5–16]. The critical current  $I_c(H)$  is magnetic field dependent, because it controls the density of vortices. Qualitatively, when the size of the hot spot, created by the photon, is of the order of the intervortex distance it strongly affects the vortex motion because the vortices are attracted to the region with suppressed superconductivity. Therefore, the local density of vortices increases, which may enhance the local Joule dissipation and deforms the vortex core. This process is connected to the finite relaxation time for the magnitude of the superconducting order parameter  $|\Delta|$  [16]. Both effects could trigger the transition of the superconducting strip to the more resistive, in particular the normal, state. Note that in contrast to a thin strip being in the, vortex-free, Meissner state we do not expect that the hot spot “creates” additional vortices. It just redistributes the existing ones in the superconductor.

Our calculations, using a two-temperature model, combined with a modified time-dependent Ginzburg-Landau equation [4] confirms this idea. The single optical photon can trigger the transition from the flux flow state in a strongly disordered superconducting strip to the normal state. The effect exists not only in strips, whose width is about the size of the photon-induced hot spot, but also in much wider strips. In the latter case the photoresponse exists predominantly near the edge of the strip where vortices enter the sample. We find, numerically, that the photon induced hot spot perturbs the vortex motion and leads to the appearance of a beltlike region with locally suppressed superconductivity across the superconducting strip, analogous to a Josephson-like S-N-S

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junction (S is a superconductor and N a normal metal). Such a state is unstable in the studied superconductors and converts to an expanding normal domain, resulting in the transition of the strip to the normal state.

The paper is organized as follows. In Sec. II we present our model. In Sec. III we show the results of the numerical calculations of the single-photon response and in Sec. IV we discuss the obtained results.

## II. MODEL

The model we use was described in detail in Vodolazov [4]. It is based on the time-dependent Ginzburg-Landau equation to describe the dynamics of the complex superconducting order parameter  $\Delta$ . In its original version this equation is only valid close to  $T_c$  or for gapless superconductors [17]. In our case, we have to accommodate two aspects of the physical reality of the problem. First, we have to allow for an electron temperature  $T_e$ , different from the temperature of the phonon system  $T_p$ , with the unique property that  $T_e$  can both be lower and higher than  $T_p$ . Obviously,  $T_e$  is a spatially dependent quantity describing the electronic ‘‘hot spot,’’ which can be accompanied by an enhanced phonon temperature  $T_p$ . The temperature of the equilibrium environment is defined as  $T_{\text{env}}$ . Second, we have parts in the superconducting system in which  $\Delta < T_e$ , where the Ginzburg-Landau theory applies, and parts where  $\Delta > T_e$ , where we should use expressions from the Usadel theory. These parts are connected by current conservation for the supercurrent. Keeping in mind those two ingredients the studied equation is

$$\begin{aligned} & \frac{\pi \hbar}{8k_B T_c} \left( \frac{\partial}{\partial t} + \frac{2ie\varphi}{\hbar} \right) \Delta \\ &= \xi_{\text{mod}}^2 \left( \nabla - i \frac{2e}{\hbar c} \mathbf{A} \right)^2 \Delta + \left( 1 - \frac{T_e}{T_c} - \frac{|\Delta|^2}{\Delta_{\text{mod}}^2} \right) \Delta \\ &+ i \frac{(\text{div} \vec{j}_s^{Us} - \text{div} \vec{j}_s^{GL})}{|\Delta|^2} \frac{e \Delta \hbar D}{\sigma_n \sqrt{2} \sqrt{1 + T_e/T_c}}, \end{aligned} \quad (1)$$

where  $\xi_{\text{mod}}^2 = \pi \sqrt{2} \hbar D / (8k_B T_c \sqrt{1 + T_e/T_c})$ ,  $\Delta_{\text{mod}}^2 = [1.76k_B T_c \tanh(1.74 \sqrt{T_c/T_e - 1})]^2 / (1 - T_e/T_c)$ ,  $\mathbf{A}$  is the vector potential,  $\varphi$  is an electrostatic potential,  $D$  is the diffusion coefficient,  $\sigma_n = 2e^2 D N(0)$  is the normal-state conductivity, and  $N(0)$  is the one spin density of states at the Fermi level. The physically significant addition is  $\vec{j}_s^{Us}$  and  $\vec{j}_s^{GL}$ , the superconducting current densities in the Usadel and Ginzburg-Landau models (see Eqs. (33) and (34) in [4]). We do not use the extended time-dependent Ginzburg-Landau (TDGL) equation of Kramer and Watts-Tobin [18] because it is derived in the limit of  $\tau_{ep} \gg \tau_{ee}$  (with  $\tau_{ep}$  the electron-phonon inelastic-scattering time). It is valid only for relatively slow processes (on the time scale of  $\tau_{ep}$ ). In addition, it is rather difficult to incorporate Joule heating in this model in a consistent way. In Eq. (1) we introduce the phenomenological  $\xi_{\text{mod}}$  and  $\Delta_{\text{mod}}$ , in which the subscript ‘‘mod’’ means modified to take into account the temperature dependence of  $|\Delta|$  and for the depairing current  $I_{\text{dep}}$  at low temperatures, as discussed in Vodolazov [4].

In the time evolution of the photon-induced hot spot and the motion of the vortex the electrons and phonons are driven

out of equilibrium. In the case of dirty superconductors with small values of  $\tau_{ee}$  one can assume that electrons thermalize amongst themselves, leading to an effective electron temperature  $T_e$ . The phonon bath relaxes on a time scale larger than  $\tau_{ee}$ . It means that the energy distribution function can be described by the Fermi-Dirac expression with an effective local temperature  $T_e$ . In this limiting case, one can derive from the kinetic equations for the electron and phonon distribution functions a heat conductance equation for  $T_e$ , together with the energy balance equation for the phonon temperature  $T_p$ . The latter describes the transfer of energy from electrons to phonons (and vice versa) as discussed in Vodolazov [4]. The heat-transport equation for electrons is

$$\begin{aligned} & \frac{\partial}{\partial t} \left( \frac{\pi^2 k_B^2 N(0) T_e^2}{3} - \mathcal{E}_0 \mathcal{E}_s(T_e, |\Delta|) \right) \\ &= \nabla k_s \nabla T_e - \frac{96\zeta(5) N(0) k_B^2}{\tau_0} \frac{T_e^5 - T_p^5}{T_c^3} + \vec{j} \vec{E}, \end{aligned} \quad (2)$$

and for phonons

$$\frac{\partial T_p^4}{\partial t} = - \frac{T_p^4 - T^4}{\tau_{esc}} + \gamma \frac{24\zeta(5)}{\tau_0} \frac{15}{\pi^4} \frac{T_e^5 - T_p^5}{T_c}, \quad (3)$$

which we use as coupled equations to model the spatial and temporal evolution of  $T_e$  and  $T_{ph}$ . In Eq. (2), with the definition  $\mathcal{E}_0 = 4N(0)(k_B T_c)^2$ , the quantity  $\mathcal{E}_0 \mathcal{E}_s(T_e, |\Delta|)$  represents the change in the energy of electrons due to the transition to the superconducting state (see Eq. (26) in Vodolazov [4]). The heat conductivity in the superconducting state,  $k_s$ , is given by

$$k_s = k_n \left( 1 - \frac{6}{\pi^2 (k_B T_e)^3} \int_0^{|\Delta|} \frac{\epsilon^2 e^{\epsilon/k_B T_e} d\epsilon}{(e^{\epsilon/k_B T_e} + 1)^2} \right), \quad (4)$$

with  $k_n = 2D\pi^2 k_B^2 N(0) T_e / 3$  the heat conductivity in the normal state. The last term in Eq. (2) describes the Joule dissipation, with  $\vec{j}$  a current density and  $\vec{E}$  the electric field. The parameter is defined as  $\gamma = (8\pi^2/5)[C_e(T_c)/C_p(T_c)]$ , with  $C_e$  and  $C_p$  the electron and phonon heat capacities. The characteristic time  $\tau_0$  is in front of the electron-phonon and phonon-electron collision integrals in the kinetic equations (see Eqs. (3), (4), (6), and (7) in Ref. [4]) and controls both inelastic scattering times  $\tau_{ep}$  and the phonon-electron time  $\tau_{pe}$ . For example  $\tau_{ep}(T_c) = \tau_0/14\zeta(3) \simeq \tau_0/16.8$  as follows from the linearized kinetic equation for the electron distribution function in the limit of a small deviation from equilibrium at  $T = T_c$  for electrons at the Fermi level (see Refs. [19,20]). The time  $\tau_{esc}$  in Eq. (3) is the escape time of nonequilibrium phonons into the substrate.

The electrostatic potential  $\varphi$ , which enters Eq. (1), follows from the current continuity equation

$$\text{div}(\vec{j}_s^{Us} + \vec{j}_n) = 0, \quad (5)$$

where  $\vec{j}_n = -\sigma_n \nabla \varphi$  is the quasiparticle current density. As we mentioned above, the construction of our model assumes that at any moment of time the electrons are mutually in thermal equilibrium, due to the relatively short  $\tau_{ee}$ . Therefore our equations are valid for variations on time scales larger or comparable to  $\tau_{ee}$ , which is given by  $\hbar R_Q / [\ln(R_Q/2R_\square) k_B T R_\square]$  with  $R_Q$  the quantum unit of resistance defined as  $2\pi \hbar / e^2 \sim$

25.8k $\Omega$  [21]. This is about 13 ps for a metal with a sheet resistance of  $R_{\square} = 500\Omega$ , at a temperature  $T = 10$  K. This time is much shorter than  $\tau_{ep}(T_c) \simeq 55$  ps, estimated for the present NbN films from the theoretical estimate  $\tau_0 = 925$  ps made in a previous study by one of us [4].

Our model takes into account the nonequilibrium effect connected with Joule heating. However, it also includes the effect of the time variation of the local magnitude of the superconducting order parameter by a moving vortex, which leads to cooling of electrons in the moving vortex core [22] and large delay times of the restoration of  $|\Delta|$  [23,24]. It is known that this effect is most probably responsible for the jump in the current-voltage characteristics of the flux flow state near  $T_c$  [5–8,10]. In a previous publication by one of the authors [16], it is argued that the same effect may lead to a series of transitions in the moving vortex lattice. In our model the effect connected with the variation of  $|\Delta|$  in time and the associated electron cooling is present in the term  $\partial\mathcal{E}_s/\partial t$  of Eq. (2).

From Eqs. (2)–(4), one finds in the limit of small deviations from the equilibrium, and  $T_e$  and  $T_p$  just above  $T = T_c$  the commonly used set of linearized equations [25],

$$C_e(T_c) \frac{\partial T_e}{\partial t} = -C_e(T_c) \frac{(T_e - T_p)}{\tau_e(T_c)} + \vec{j} \vec{E}, \quad (6)$$

$$C_p(T_c) \frac{\partial T_p}{\partial t} = -C_p(T_c) \frac{T_p - T_{\text{env}}}{\tau_{\text{esc}}} + C_p(T_c) \frac{T_e - T_p}{\tau_p(T_c)}, \quad (7)$$

with  $\tau_e(T_c) \simeq \tau_0/76 \simeq \tau_{ep}(T_c)/4.5$  and  $\tau_p(T_c) = \tau_e(T_c)C_e(T_c)/C_p(T_c)$ . These are commonly used equations to describe the nonequilibrium response near the critical temperature of a superconductor, using the resistive transition of the superconductor as a thermometer. They assume a normal-state increase in electron temperature by an unspecified power input in which superconductivity does not play a role, although the transition to the superconducting state is, in practice, used as the thermometer. The relevance is that it allows a discussion of the relaxation times. From Eqs. (6) and (7) it follows that the relaxation time  $\tau$  for the electron temperature  $T_e$  can be expressed as (see Eq. (17) in [25])

$$\tau(T_c) \simeq \tau_e(T_c) + \tau_{\text{esc}}[1 + C_e(T_c)/C_p(T_c)] \quad (8)$$

In general it is determined not only by the electron-phonon scattering time but by the escape time for nonequilibrium phonons to the substrate. In addition it depends on the ratio of the heat capacities. The larger this time the stronger the influence of the nonequilibrium effects on the vortex dynamics at the same current. We find that a variation of  $\tau_0$ ,  $C_e/C_p$ , and  $\tau_{\text{esc}}$  leads to a change in the quench current  $I_q$ , but the photoresponse is present for all chosen parameters in the ranges  $\tau_0 = 100$ – $1000$  ps,  $\gamma = 10$ – $100$ ,  $\tau_{\text{esc}} = 0.005$ – $0.5\tau_0$ . Therefore, we present results only for  $\gamma = 10$ , i.e.,  $C_e(T_c)/C_p(T_c) \simeq 0.63$ ,  $\tau_{\text{esc}} = 0.05\tau_0$  and  $\tau_0 = 1000$  ps, which are close to the theoretical estimates for NbN [4].

In the numerical calculations we use the following parameters, typical for a NbN strip: sheet resistance  $R_{\square} = 500\Omega$ ,  $D = 0.5$  cm<sup>2</sup>/s, thickness  $d = 4$  nm,  $T_c = 10$  K. With these numbers the coherence length scale  $\xi_c = \sqrt{\hbar D/k_B T_c}$  is 6.2 nm and the time scale  $\tau_c$  is  $\hbar/k_B T_c = 0.76$  ps. The magnetic field is measured in units of  $H_0 = \Phi_0/2\pi\xi_c^2 \simeq 86$  kOe and the

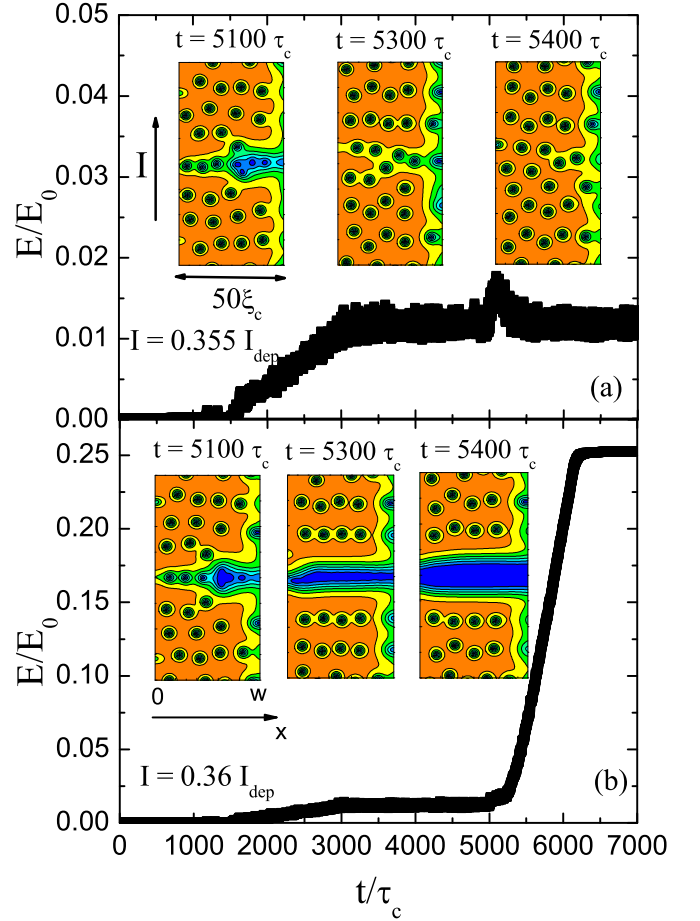


FIG. 1. Time-dependent electric field in superconducting strip. During time interval 0– $3000\tau_c$  the current linearly increases and at  $t = 5000\tau_c$  the instant local heating turns on. (a) The current is equal to  $0.355I_{\text{dep}} < I_q = 0.42I_{\text{dep}}$ . (b) The current is equal to  $0.36I_{\text{dep}}$ . In insets we show snapshots of  $|\Delta|$  at different times. Width of the strip  $w = 50\xi_c$ ,  $H = 0.05H_0$ ,  $T_{\text{env}} = 0.8T_c$ . Photon is absorbed on the distance  $12.5\xi_c = w/4$  from the edge of the strip where the vortices enter the strip (right edge).

electric field is in units of  $E_0 = k_B T_c/2|e|\xi_c$ . We use a finite length of  $L = 4w$ . The boundary conditions for  $\Delta$ ,  $T_e$  and the electrostatic potential  $\varphi$  in the  $x$  and  $y$  directions are chosen as described Ref. [4].

The absorption of the photon is modelled by instantaneous heating of both electrons and phonons by  $\delta T$  in an area of  $2\xi_c \times 2\xi_c$ , the initial hot spot, where  $\delta T$  can be related to the energy of the photon via energy conservation; see Eq. (23) in Ref. [4]. For a full comparison  $w^2 d$  should be replaced by  $6.25\xi_c^2 d$ , with the numerical constant, 6.25, arising from a choice of the grid and the step size  $\delta x = \delta y = 0.5\xi_c$ .

### III. PHOTON TRIGGERED TRANSITION

In the numerical calculations we start from a state with the current  $I = 0$  at  $t = 0$ . During the time interval from  $t = 0$  to  $3000\tau_c$  the current is linearly increased and at  $t = 5000\tau_c$  the local instantaneous heating, which models the absorption of the photon, is switched on. In Figs. 1(a) and 1(b) we show the evolution of the electric field with time,  $E(t)$ , in the

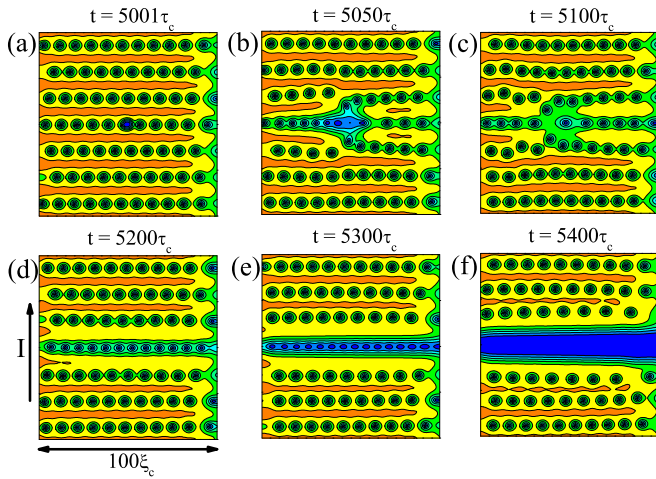


FIG. 2. Distribution of  $|\Delta|$  at different moments of time in the superconducting strip with width  $w = 100\xi_c$ . Current  $I = 0.4I_{\text{dep}} < I_q = 0.41I_{\text{dep}}$ ,  $H = 0.05H_0$ ,  $T_{\text{env}} = 0.8T_c$ . The photon is absorbed at  $t = 5000\tau_c$  in the center of the strip (a). The photon induced hot spot distorts the vortex motion (b),(c) and “creates” a Josephson-like S-N-S weak link (d),(e) which evolves into an expanding normal domain (f).

superconducting strip at slightly different values of the current in fractions of the depairing current. The magnetic field is fixed at  $H = 0.05H_0$ , the bath temperature at  $T_{\text{env}} = 0.8T_c$  and the width  $w$  of the strip is  $50\xi_c$ . For the chosen parameters the quench current, where the system jumps from the flux flow state to the normal state, is  $I_q = 0.42I_{\text{dep}}$ , while the critical current where the first voltage appears is  $I_c = 0.17I_{\text{dep}}$ . The center of the initial hot spot is located at a distance  $12.5\xi_c = w/4$  from the edge of the strip, where the vortices enter the superconductor in the flux flow regime.

Depending on the bias current the photon absorption leads either to a voltage pulse, small in amplitude and duration [see Fig. 1(a) at  $t \simeq 5000\tau_c$ ], or to a full switching of the strip to the normal state [see Fig. 1(b)]. The transition to the normal state occurs via the appearance of a region with suppressed superconductivity at the place where the photon is absorbed, followed by the creation of a Josephson-like S-N-S link, which evolves towards a growing normal domain. It can be seen from the insets of Fig. 1(b) and more clearly in Fig. 2, in which the results for a two times wider strip ( $w = 100\xi_c$ ) are presented. The photon induced hot spot attracts vortices and their density is locally enhanced [see Figs. 2(b) and 2(c)]. If the current is large enough a chain of fast moving vortices is formed due to the finite relaxation time of the amplitude of the order parameter  $|\Delta|$  [16]. Hence, a beltlike region with strongly suppressed superconductivity appears [see Figs. 2(d) and 2(e)]. This belt resembles a Josephson S-N-S-type weak link and the fast moving vortices resemble Josephson vortices. This intermediate state of an S-N-S link is unstable due to two reasons: the Joule heating and the instability of the N-S boundary at relatively large current. The latter mechanism comes from the short electric-field penetration depth  $L_Q$ , due to charge imbalance a quasiparticles with energy larger than  $\Delta$ , which is comparable to  $\xi(T)$  ( $L_Q$  can be found from the analysis of Eqs. (1) and (5) like in

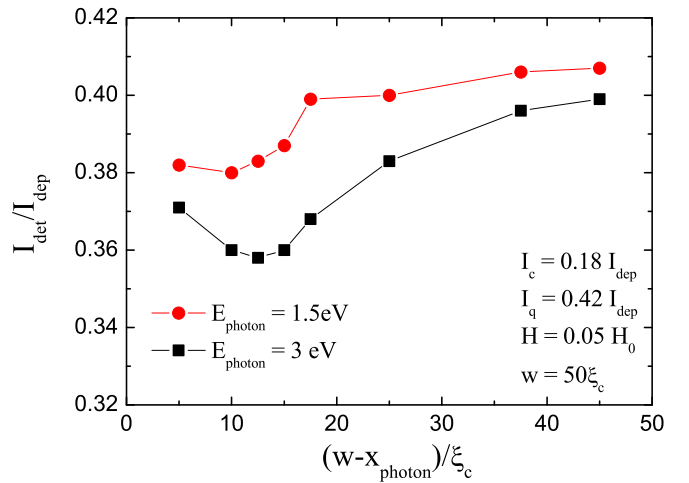


FIG. 3. Dependence of the detection current on the location, where the photon is absorbed for two different photon energies. Current direction as defined in Figs. 1 and 2.

the ordinary time-dependent Ginzburg-Landau equation [26]). When  $L_Q < \xi$  the N-S boundary moves in the direction of length of the superconductor even when the current density is smaller than the depairing current density [26] and one can neglect the Joule heating. This provides an additional channel to instability of the intermediate S-N-S link, important at  $T_{\text{env}}$  not far below  $T_c$ .

The value of the detection current  $I_{\text{det}}$ , above which the photon induced transition occurs, depends on the energy of the incoming photon  $E_{\text{photon}}$ . The larger the energy the larger the radius of the hot spot  $R_{\text{HS}}$ , which strongly effects the vortex motion. It also depends on the location of the hot spot (see Fig. 3). Near the edge, where vortices enter the strip, the local current density and local temperature are larger than further away from it due to the presence of the edge barrier for the vortex entry (see Fig. 4). This current density and the temperature gradient results in a gradual increase of  $I_{\text{det}}$ , when the location of the photon absorption moves away from that edge. We also find that the lowest value of  $I_{\text{det}}$  is reached when  $w - x_{\text{photon}}$  is  $\sim R_{\text{HS}}$ , with  $R_{\text{HS}}$  the radius of the hot spot, which resembles the result found for the case of the single-photon response in zero magnetic field [27]. We believe that our finding has the same origin and is connected to the change of the shape of the hot spot when  $w - x_{\text{photon}} < R_{\text{HS}}$ . In the limiting case  $x_{\text{photon}} = w$  the hot spot has the shape of a semicircle, which provides a different effect on the edge pinning and the distortion of the current flow.

The inhomogeneous distribution of the current density and the temperature makes it impossible to obtain a single-photon response in wide strips with  $w \gg R_{\text{HS}}$ , when the photon is absorbed far from the edge where the new vortices enter the sample. The reason is that the photon-detecting instability occurs first near that edge. In such wide strips only the region with a width of about  $2R_{\text{HS}}$  near the edge is photosensitive, where the current density is maximal.

We find that with increasing magnetic field the detection current  $I_{\text{det}}$  approaches  $I_q$ , which we explain by the decrease of the quench current with  $H$  and hence a smaller influence of the nonequilibrium effects originating from Joule heating

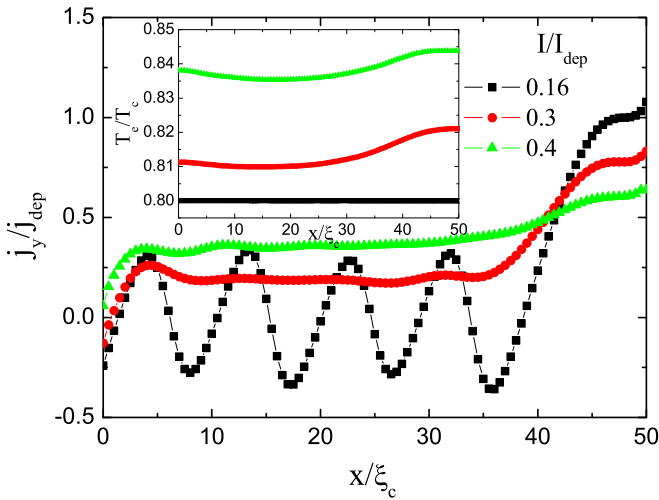


FIG. 4. Distribution of the current density and electron temperature (see inset) across the superconducting strip being in the superconducting and flux flow states. Temperature and current density are averaged on large time interval ( $500\tau_c$ ) and on distance  $50\xi_c$  along the strip ( $y$  direction). Width of the strip  $w = 50\xi_c$ ,  $T_{\text{env}} = 0.8T_c$ ,  $H = 0.05H_0$ .

and the time variation of  $|\Delta|$ . An additional reason might be connected to the larger vortex density at large  $H$ . Indeed, due to the vortex-vortex repulsion one obtains a smaller increase in vortex density inside the hot spot region, which leads to a relatively smaller impact of the photon induced hot spot on the vortex motion.

We also find that a photon triggered transition occurs both at lower ( $T_{\text{env}} = 0.5T_c$ ) and higher ( $T_{\text{env}} = 0.9T_c$ ) temperatures. In both cases it was connected to the chain of events from fast vortices to the Josephson-like S-N-S weak link, and from there to the growing normal domain.

#### IV. DISCUSSION

This numerical analysis of the single-photon response of superconductors in the flux flow state resembles, in many respects, the photon response of a superconducting strip in the vortex-free Meissner state. In both cases the photon can trigger the transition to the normal state if the current in the strip exceeds some critical value  $I_{\text{det}}$ . The value of the detection current depends on the point where the photon is absorbed and the minimal value of  $I_{\text{det}}$  is reached near the edge of the strip (compare Figs. 4 and 9 in Zotova *et al.* [27] with Fig. 3). A main difference is the initial stage of the photon induced instability. In the case of the vortex free Meissner state, the absorbed photon allows for the nucleation of vortices inside the strip and their motion leads to appearance of a Josephson-like S-N-S link (see Fig. 3(a) in Ref. [3]). In the case of a strip in the flux flow state the hot spot “redistributes” the already present vortices in the superconductor, leading eventually to the appearance of a Josephson-like S-N-S link. The destruction of superconductivity is in both cases similar, consisting of the expansion of the normal domain.

The theoretically found nonuniform current and temperature distribution in the resistive, flux flow state affects not only the position dependent  $I_{\text{det}}$ , but also, in the absence of a photon

flux, the transition of the superconducting strip to the normal state. We find that at  $I > I_q$  the transition to the normal state starts near the edge of the strip, where the current density and the temperature are maximal. Near that edge the region with locally suppressed superconductivity and fast moving vortices is formed. It spreads to the opposite edge, resembling the response of a moving vortex array due to the absorbed photon. When the Josephson-like S-N-S junction across the strip is formed, such a state becomes unstable and the superconductor switches to the normal state. This result demonstrates that in superconductors with a pronounced edge barrier for vortex entry, a nonuniform current density and temperature distribution need to be taken into account in the analysis of vortex-motion instability. In particular, for disordered superconductors like NbN or TiN [8,12,13] where the present model should be applicable.

In the present calculations we neglect the pinning of the vortices. We expect that our results will also be valid in the regime where the depinning current density  $j_p \ll j_s$ , where  $j_s$  is the edge current density at which vortices can enter the strip [28]. The latter value is equal to the depairing current density for strips, without edge defects. The regime with  $j_p \ll j_s$  is experimentally possible as can be inferred from the experimental dependence of  $I_c(H)$  for NbN, TaN, Nb [29], and MoGe strips [28]. These results demonstrate a linear decay of  $I_c$  at low magnetic fields and for  $I_c(0)$  about half of the depairing current, both of which are fingerprints of an edge barrier controlled  $I_c$ , for samples with a uniformity. In such samples, a region with a gradient of  $j \sim j_s > j_p$  should exist near the edge of the strip, where vortices enter the superconductor and where the temperature should be locally larger. Therefore, we believe that vortex pinning does not have a strong effect on the photon induced transition to the normal state if it occurs at  $j \lesssim j_q = I_q/wd \gg j_p$ .

The present results can be used to conclude that practical single-photon detectors can only be made by using relatively narrow strips with a width of a few  $R_{\text{HS}}$ , when the whole cross section of the detector is photosensitive for  $I < I_q$ . The radius of the hot spot can be estimated from the energy conservation law and the assumption that it has the largest effect on the superconducting properties when the electron temperature is equal to  $T_c$  [4]. For photons with energy 3 eV and typical parameters of a NbN strip: thickness  $d = 4$  nm, one spin density of states on Fermi level  $N(0) = 25.5 \text{ eV}^{-1} \text{ nm}^{-3}$ , critical temperature  $T_c = 10$  K,  $\gamma = 10$ ,  $\xi_c = 6.2$  nm, and  $T_{\text{env}} = 0.8T_c$ , one obtains a radius of the hot spot  $R_{\text{HS}} \sim 56 \text{ nm} \sim 9\xi_c$  which is about of  $w/6$  when  $w = 50\xi_c \simeq 310$  nm. In strips with  $w \gg R_{\text{HS}}$  only relatively narrow part ( $\sim 2R_{\text{HS}}/w$ ) is sensitive to the absorption of the single photon [assuming that one is not interested in small amplitude and short duration voltage response at  $I < I_{\text{det}}$ ; see Fig. 1(a)]. In this respect, the presented results are not very promising from an application point of view. This result is in contrast to the single-photon response in micron-wide bridges in the vortex-free Meissner state, biased at currents close to the depairing current [30].

The experimental confirmation of a photon-triggered transition in the flux flow state could prove indirectly the importance of the nonuniform current and temperature distribution in the resistive state. It would allow the extraction of the size

of the photon induced hot spot. Indeed, the illumination of the superconducting strip, in the resistive state at  $I \lesssim I_q(H)$ , with a relatively low photon flux should trigger, according to our calculations, the transition to the normal state. From the known photon flux, photon absorption coefficient, and the number of photons one can estimate which part of the strip is photosensitive and compare it with theoretical expectations. We believe that the photon induced transition should be observable, when the intervortex distance is about  $2R_{HS}$ . In this case, the photon can effectively change the structure of moving vortex matter. Using the above estimate for  $R_{HS}$  one finds that the photon-triggered transition could be observable with a thin NbN strip at  $H \gtrsim \Phi_0/(2R_{HS})^2 \gtrsim 1.8$  kOe and  $E_{\text{photon}} = 3$  eV.

We do not expect that the photon-triggered transition is possible in thick strips, due to the small change of  $T_e$ , nor in relatively pure thin superconductors with a large diffusion

coefficient. In this case, due to the large size of the hot spot [4] and the correspondingly smaller variation of  $T_e$ . In such systems, as well as in thin disordered superconductors, one may use a scanning tunnel microscope (STM) and heat locally the sample using the STM tip, as done in the recent work by Ge *et al.* [31]. The local heating may trigger a transition to the normal, or more resistive, state. The result should depend on the position of the tip (far from or close to the edge of the strip) and, of course, the applied tunnel current.

#### ACKNOWLEDGMENTS

The authors acknowledge the support from the Russian Science Foundation through Grant No. 18-72-10027 (D.Yu.V.) in the part concerned with the optical response of superconductors and Grant No. 17-72-30036 (T.M.K.) in the part concerned with superconducting detectors.

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