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Influence of electromagnetic fluctuations on electron cotunneling

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We analyze electron cotunneling in systems of small normal tunnel junctions embedded in a dissipative electromagnetic environment. As an example we consider two junctions in series with an Ohmic resistor R. We show that at low voltages the electromagnetic fluctuations can suppress the cotunneling current strongly $I \propto V^{3+2z}$, where $z = Re^2/h$. This effect can be used to improve the accuracy of prototypes of the single-electron dc current standard (the pump and the turnstile device).

I. INTRODUCTION

In view of the striking progress in experimental work on single-electron tunneling (SET) in ultrasmall tunnel junctions and the confirmation of the "orthodox theory" predictions[1] more elaborate descriptions of these phenomena gain importance.[2,3] In particular, the effect of macroscopic quantum tunneling of the charge or, simply, an electron cotunneling[4–10] was detected in several recent experiments.[11–13]

In a cotunneling event two (or, more generally, $n \geq 2$) electrons tunnel simultaneously and quantum coherently through two (n) junctions of some multijunction circuit. Such a transition occurs via $n-1$ intermediate virtual states with macroscopically different values of charges at the electrodes of the junctions involved. A single cotunneling event has a huge number $\sim E_c N(0)$ of realizations which lead to the microscopically distinguishable final states of the system [here $E_c = e^2/2C$ is the characteristic charging energy and $N(0)$ is the density of electron states in the junction electrodes at the Fermi level]. In each of these realizations the quantum interference occurs between $n!$ discrete trajectories of the transition, which correspond to the various sequences of *macroscopically* distinguishable virtual charging states. Therefore the macroscopic character of quantum mechanics clearly manifests itself in a cotunneling phenomenon.

The cotunneling process is the dominant channel for the electron transport in systems of tunnel junctions at low temperatures and small voltages, when the usual SET is blocked due to the Coulomb effects. For this reason it sets the ultimate limitations on the accuracy of SET devices.[14,15] This is particularly important for the turnstile device[16] and the pump,[17] which can be considered as prototypes of the first-principle dc current standard.

In Refs. 4–8, 14, and 15 cotunneling was analyzed using the simple model of the circuit electrodynamics, which takes into account only capacitances of the junctions and their electrodes. On the other hand, the consideration of the lowest-order tunneling processes in single junctions[18,19] and in multijunction circuits[20,21] has shown that in general the electromagnetic environment influences the electron tunneling.[22] The aim of this paper is to investigate the effect of the electromagnetic environment on the electron cotunneling in systems of normal junctions.

A similar problem was studied in Ref. 9. However, the analysis of the specific circuit of a "resistive current copier" in the limiting cases of small (in comparison with $R_K = h/e^2$) and infinitely large impedance $Z(\omega)$ of the environment did not reveal several crucial features of the cotunneling. For instance, the nonanalytic behavior of the transport characteristics (e.g., I-V dependence) at low voltages and temperatures has not been found. No results for moderate or large values of the impedance have been reported.

The paper is organized as follows. In Sec. II we describe the general formalism for the analysis of cotunneling in the presence of an arbitrary electromagnetic environment. This formalism is applied to a system of two junctions in series with an external Ohmic impedance R which models a dissipative environment (Sec. III). We find a suppression of the cotunneling rate at low voltages and temperatures. In particular, the zero-temperature I-V characteristic obeys a power law $I \propto V^{3+2z}$, $z = Re^2/h$ for an arbitrary R. We discuss the possibilities to observe this effect and to make use of it in order to improve the accuracy of SET devices (like pumps and turnstiles) without increasing the number of junctions (Sec. IV).

II. MODEL AND GENERAL FORMALISM

To describe the effect of cotunneling of two electrons in two junctions of some circuit, we consider the Hamiltonian

$$H = H_1 + H_2 + H_{\mathrm{em}} . \tag{1}$$

The terms $H_n = H_n^{(T)} + H_n^{(L)} + H_n^{(R)}$ correspond to the junctions $n = 1,2$ and H_{em} describes the electromagnetic environment. It is convenient to work in the interaction representation with respect to the sum of H_{em} and the Hamiltonians $H_n^{(L,R)}$ of the junction electrodes. The tunneling terms $H_n^{(T)}$ of the Hamiltonian (1) then take the form

$$H_{\rm int}=H^{(T)}_{{\rm int},1}+H^{(T)}_{{\rm int},2},\quad H^{(T)}_{{\rm int},n}=H_n^+ + H_n^- ,$$

$$H_n^+=\sum_{p,q} T^{(n)}_{p,q} c^\dagger_{n,p} c_{n,q} e^{i(\varepsilon_{n,p}-\varepsilon_{n,q})t} e^{i\phi_n(t)} , \tag{2}$$

$$H_n^- = (H_n^+)^+ ,$$

where $\varepsilon_{n,p(q)}$ and $c^\dagger_{n,p(q)}$ ($c_{n,p(q)}$) are the energies and creation (annihilation) operators of the electronic states in the junction electrodes.

The influence of the electromagnetic environment is described by the terms $\exp[i\phi_n(t)]$, where the phases $\phi_n(t)=(e/\hbar)\int_{-\infty}^t V_n(\tau)d\tau$ are related to the voltages across the junctions.[18–22] The phases (and voltages) consist of the classical parts $\phi_n^{(\rm cl)}$ and fluctuations $\phi_n^{(\xi)}$. The latter should be treated as operators. The self ($m=n$) and mutual ($m\neq n$) correlation functions of $\phi_n^{(\xi)}$ can be computed from the fluctuation-dissipation theorem:

$$K_{m,n}(t)=\langle [\phi_m^{(\xi)}(t)-\phi_m^{(\xi)}(0)]\phi_n^{(\xi)}(0)\rangle_{\rm em}$$
$$=\frac{e^2}{\hbar}\int\frac{d\omega}{2\pi}\frac{ReZ_{m,n}(\omega)}{\omega}\left\{\coth\left[\frac{\hbar\omega}{2T}\right][\cos(\omega t)-1]-i\sin(\omega t)\right\}\quad \text{for } m(n)=1,2 . \tag{3}$$

In the last equation we have introduced an impedance $Z_{m,n}(\omega)$ which determines the linear response of the electromagnetic circuit $V_m(\omega)=Z_{m,n}(\omega)I_n(\omega)$ in the absence of electron tunneling. The average in Eq. (3) is taken over the equilibrium density matrix of the electromagnetic environment. In what follows we will use a bosonic model of the environment[18–22] characterized by the Hamiltonian $H_{\rm em}=\sum_\alpha \hbar\omega_\alpha b^\dagger_\alpha b_\alpha$. The fluctuational operators $\phi_n^{(\xi)}$ can be considered as a linear form of a large number of Bose operators $b^\dagger_\alpha$ and b_α corresponding to normal modes of the environment.

Provided that the tunnel conductances of the junctions $G_n^{(T)}$ are small, $G_n^{(T)} \ll \min[R_K^{-1}, ReZ^{-1}_{m,n}(\omega)]$ for $\hbar\omega \le eV_{m(n)}$, we can use perturbation theory in the tunneling Hamiltonian $H_{\rm int}$ (similarly to how it was done in Ref. 9). In the Coulomb blockade regime the usual second-order tunneling is exponentially weak, $I\sim \exp(-E_c/T)$ at low temperatures. The main contribution to the current arises in fourth order

$$I_n(t)=\frac{i}{\hbar^3}\int_{-\infty}^t dt_1\int_{-\infty}^{t_1} dt_2\int_{-\infty}^{t_2} dt_3 \langle\langle [[[\hat I_n(t),H_{\rm int}(t_1)]H_{\rm int}(t_2)]H_{\rm int}(t_3)]\rangle_{el}\rangle_{\rm em} . \tag{4}$$

Here $\hat I_n=(ie/\hbar)(H_n^+-H_n^-)$ is the current operator. The averages are taken over the equilibrium electron subsystem and electromagnetic environment. The operators $c^\dagger_{n,p(q)}$ ($c_{n,p(q)}$) and $\exp(\pm i\phi_n)$ related to these systems commute.

III. DOUBLE JUNCTION SYSTEM

We now apply the general formalism to a system of two identical junctions connected with an external circuit characterized by the impedance $Z(\omega)$ (Fig. 1). In what follows we assume that the capacitances C of the junctions are much larger than the gate capacitance C_g. The standard Coulomb blockade of SET occurs in the region

$$|V|<V_t=\frac{e}{2C}-\frac{|Q_0|}{C},\quad Q_0=C_gV_g-e\left[\frac{C_gV_g}{e}\right] \tag{5}$$

(we denote the nearest to x integer as $[x]$). The fourth-order tunneling of two electrons to (from) the central electrode of the system is also exponentially suppressed in this region and can be neglected. Moreover, we will not take into account the coherent electron propagation through both junctions which (for not extremely low voltages and temperatures) is weaker than the cotunneling by a factor $E_cN(0)$.[6] Taking averages over electronic states in Eq. (4) we obtain the cotunneling current at low voltages (5) and temperatures ($T\ll E_c$)

$$I=e(\gamma^{(+)}-\gamma^{(-)}) , \tag{6}$$

$$\gamma^{(+)}=2Re\int_{-\infty}^t dt_1\int_{-\infty}^t dt_2\int_{-\infty}^{t_2} dt_3\{ G_1(t_3-t)G_2(t_2-t_1)$$
$$\times\langle e^{i\phi_1(t_3)}e^{i\phi_2(t_2)}[e^{-i\phi_1(t)}e^{-i\phi_2(t_1)}-\Theta(t_1-t_2)e^{-i\phi_2(t_1)}e^{-i\phi_1(t)}]\rangle_{\rm em}$$
$$+G_1(t_2-t)G_2(t_3-t_1)$$
$$\times\langle e^{i\phi_2(t_3)}e^{i\phi_1(t_2)}[e^{-i\phi_1(t)}e^{-i\phi_2(t_1)}-\Theta(t_1-t_2)e^{-i\phi_2(t_1)}e^{-i\phi_1(t)}]\rangle_{\rm em}\} . \tag{7}$$

Here $\gamma^{(+)}$ is the forward tunneling rate. The backward tunneling rate $\gamma^{(-)}$ can be obtained from Eq. (7) by changing the signs in the exponents together with the sign of the whole expression. The functions G_n are determined as follows:

$$G_n(t)=\frac{1}{\hbar^2}\sum_{p,q}|T^{(n)}_{p,q}|^2\exp\left[\frac{i}{\hbar}(\varepsilon_{n,q}-\varepsilon_{n,p})t\right]f(\varepsilon_{n,q})[1-f(\varepsilon_{n,p})] , \tag{8}$$

where $f(\varepsilon)$ is the Fermi distribution. The Fourier transformed function

$$G_n(\omega)=\frac{\hbar G_n^{(T)}}{e^2}\frac{\omega}{1-\exp(-\hbar\omega/T)} \tag{9}$$

describes the electron tunneling rate in a single junction biased by a voltage $V=\hbar\omega/e$.

Keeping in mind that the operators $\phi_n^{(\xi)}$ are linear forms of Bose operators, we can compute[23] the averages of the exponents in Eq. (7). After Fourier transformation we obtain[24]

$$\gamma^{(+)}=2\int\frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi}\Gamma_1\left[\omega_1+\frac{eV_1}{\hbar}\right]ReD(\omega_1,\omega_2)\Gamma_2\left[\omega_2+\frac{eV_2}{\hbar}\right], \tag{10}$$

$$\Gamma_n(\omega)=\int dt e^{i\omega t}G_n(t)\exp[K_{n,n}(t)]\ , \tag{11}$$

$$\begin{aligned}D(\omega_1,\omega_2)=2\int_{-\infty}^{t}dt_1\int_{-\infty}^{t}dt_2\int_{-\infty}^{t_2}dt_3\{&e^{i\omega_1(t-t_3)}e^{i\omega_2(t_1-t_2)}e^{K(t_3-t_1)-K(t_3-t_2)+K(t_2-t)}[e^{-K(t-t_1)}-\Theta(t_1-t_2)e^{-K(t_1-t)}]\\&+e^{i\omega_1(t-t_2)}e^{i\omega_2(t_1-t_3)}e^{K(t_3-t)-K(t_3-t_2)+K(t_2-t_1)}\\&\times[e^{-K(t-t_1)}-\Theta(t_1-t_2)e^{-K(t_1-t)}]\}\ ,\end{aligned} \tag{12}$$

where

$$V_n=\frac{1}{2}\left[V+(-1)^n\frac{Q_0}{C}\right] \tag{13}$$

are the voltages across the junctions and $K(t)\equiv K_{1,2}(t)=K_{2,1}(t)$. The backward tunneling rate $\gamma^{(-)}$ can be obtained from Eq. (10) by changing the signs of voltages (13).

The functions $\Gamma_n(\omega=eV_n/\hbar)$ correspond to the rate of second-order tunneling through the nth junction embedded in an electromagnetic environment. To see this more clearly, we rewrite Eq. (11) as a convolution,[18]

$$\Gamma_n\left[\frac{eV_n}{\hbar}\right]=\int G_n\left[\frac{eV_n}{\hbar}-\Omega\right]P(\Omega)d\Omega\ , \tag{14}$$

of $G_n(\omega)$ [see text below Eq. (9)] and the conditional probability $P(\Omega)=(2\pi)^{-1}\int dt\exp[i\Omega t+K_{n,n}(t)]$ of electron tunneling with the transfer of energy $\hbar\Omega$ to the modes of electromagnetic environment [$\int P(\Omega)d\Omega=1$]. The propagator $D(\omega_1,\omega_2)$ describes the electromagnetic coupling of the junctions, which is responsible for the quantum correlations between two virtual tunneling events in the act of cotunneling.

We now evaluate the cotunneling rate (10) for the Ohmic external impedance $Z(\omega)=R$ at zero temperature. The real parts of impedances $Z_{m,n}(\omega)$ are

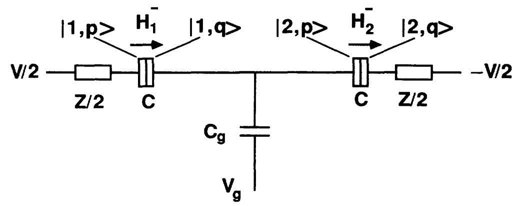

FIG. 1. Equivalent circuit of the system under consideration. Electronic states in the electrodes of the junctions are denoted by brackets.

$$ReZ_{m,n}(\omega)=\frac{R}{4}\frac{1}{1+(\omega/\Omega_z)^2}+(-1)^{m+n}\frac{\pi\delta(\omega)}{2C}\ ,$$

$$\Omega_z=\frac{2}{RC}\ . \tag{15}$$

The first term here describes fluctuations of the voltage across two junctions. To simplify calculations we approximate it by $(R/4)\exp(-|\omega|/\Omega_z)$, which is legitimate for low frequencies $\omega\ll\Omega_z$. The second term describes the voltage fluctuations at the central electrode. Its contribution $\exp(i\omega_c t)$ to $K_{m,n}(t)$ is related to the single-electron Coulomb energy $\hbar\omega_c=e^2/4C$.

Performing the integration in Eq. (3) we obtain

$$K_{m,n}(t)=(1+i\Omega_z t)^{-z/2}\exp\{(-1)^{m+n-1}i\omega_c t\}\ , \tag{16}$$

where $z=e^2R/h$ is the dimensionless external impedance. Substituting (16) in (11) we obtain

$$\Gamma_n(\omega)=\frac{\hbar G_n^{(T)}}{e^2}\Gamma^{-1}\left[2+\frac{z}{2}\right]\Omega_z\left[\frac{\omega-\omega_c}{\Omega_z}\right]^{1+z/2}\times\Theta(\omega-\omega_c)\ \text{ for } \omega-\omega_c\ll\Omega_z\ . \tag{17}$$

From Eqs. (12) and (16) we compute (see the Appendix) the function $D(\omega_1,\omega_2)$ with the result

$$ReD(\omega_1,\omega_2)=\pi\left[\frac{1}{\omega_1}+\frac{1}{\omega_2}\right]^2\delta(\tilde{\omega})\ \text{ for } z=0\ , \tag{18}$$

$$ReD(\omega_1,\omega_2)=\pi z\left[\left[\frac{1}{\omega_1^2}\right]^{1+z/2}+2\left[\frac{1}{\omega_1\omega_2}\right]^{1+z/2}+\left[\frac{1}{\omega_2^2}\right]^{1+z/2}\right]\tilde{\omega}^{z-1}\Theta(\tilde{\omega})\ \text{ for } z\ll 1\ , \tag{19}$$

$$ReD(\omega_1,\omega_2)=\frac{\pi}{\Gamma(z)}\frac{1}{\Omega_z}\left[\frac{1}{\omega_1}+\frac{1}{\omega_2}\right]^2\left[\frac{\tilde{\omega}}{\Omega_z}\right]^{z-1}\times\exp\left[-\frac{\tilde{\omega}}{\Omega_z}\right]\Theta(\tilde{\omega})\quad\text{for } z\gg 1\ . \tag{20}$$

Here $\tilde{\omega}\equiv 2\omega_c-\omega_1-\omega_2$ is assumed to be small, $\tilde{\omega}\ll\min(\omega_1,\omega_2)$.

The arguments ω_1,ω_2 of the functions $\Gamma_{1(2)}$ and D correspond to the energies of intermediate states, which arise after the first tunneling event in the cotunneling process. To see this, we note that in the Coulomb blockade regime (5) after such an event in the nth junction the Coulomb energy of the system increases by[25]

$$\hbar\Omega_n\equiv\hbar\omega_c-eV_n>0\ . \tag{21}$$

Simultaneously the electron energy changes by $\hbar\tilde{\omega}_n=\varepsilon_e^{(n)}+\varepsilon_h^{(n)}$, where $-\varepsilon_h^{(n)}$ and $\varepsilon_e^{(n)}$ are the electron energies (with respect to the corresponding Fermi levels) before and after the tunneling (Fig. 2). Therefore

$$\hbar\omega_n=\hbar\Omega_n+\hbar\tilde{\omega}_n \tag{22}$$

are the energies of virtual states. The Θ function in Eq. (17) shows that at $T=0$ the electron energy can only increase,

$$\hbar\tilde{\omega}_n>0\ , \tag{23}$$

because of the Pauli principle. The total increase $\hbar\tilde{\omega}_1+\hbar\tilde{\omega}_2$ of electron energy cannot be larger than the difference eV of chemical potentials of the external electrodes (see Fig. 2),

$$\hbar\tilde{\omega}_1+\hbar\tilde{\omega}_2\le eV\ . \tag{24}$$

This condition clarifies the sense of Θ functions in the expressions (19) and (20) for D, provided that $\tilde{\omega}=eV/\hbar-\tilde{\omega}_1-\tilde{\omega}_2$ [see Eqs. (13), (21), and (22)]. The last two expressions determine actual domain of integration in Eq. (10).

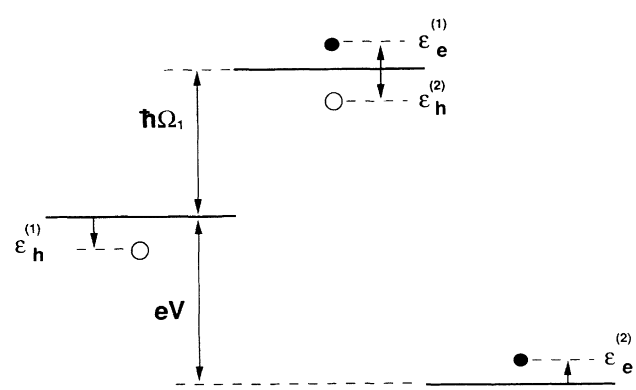

FIG. 2. Configuration of electrons (filled circles) and holes (empty circles) arising after a cotunneling event. Solid lines denote the positions of Fermi levels in electrodes of the junctions (Fermi level of the central electrode corresponds to the virtual state in an act of cotunneling, which arises after the tunneling of one electron through the left junction).

After an act of cotunneling an electron-hole pair with the energy $E_{e\text{-}h}=\varepsilon_e^{(1)}+\varepsilon_h^{(2)}$ is left at the central electrode (Fig. 2). Simultaneously the energy difference $\Delta E=eV-\varepsilon_h^{(1)}-\varepsilon_e^{(2)}$ arises between the initial and the final electron states in external electrodes. Purely capacitive electromagnetic environment ($z=0$) does not absorb energy, $\Delta E=E_{e\text{-}h}$, which is manifested by the δ function in Eq. (18). In this case the tunneling is elastic.[26]

In the presence of dissipation a part of the electron energy $\hbar\tilde{\omega}=\Delta E-E_{e\text{-}h}\ge 0$ can be transferred to the environment. For the Ohmic impedance Z the function D is proportional to $\tilde{\omega}^{z-1}\Theta(\tilde{\omega})$. Therefore for $z\ll 1$ the tunneling with low-energy transfer $\hbar\tilde{\omega}\ll eV$ dominates. Even for very small z the singularity in D is integrable, $\lim_{\varepsilon\to 0}\int_0^{\varepsilon}D d\tilde{\omega}\to 0$, which means that effectively only inelastic tunneling ($\hbar\tilde{\omega}>0$) occurs. [The alternative situation, where inelastic and elastic tunneling coexist, would be described by a function D, containing a term const$\delta(\tilde{\omega})$ together with some contribution at finite energies $\hbar\tilde{\omega}$.[27]] For larger impedances $z\sim 1$ the characteristic energy transfer to the environment is no longer small, $\hbar\tilde{\omega}\sim eV$. In particular, for $z\gg 1$ the processes with the energy transfers $\hbar\tilde{\omega}=2\hbar\tilde{\omega}_{1(2)}=eV/2$ dominate.

Substituting the results (17), (19), and (20) into Eq. (10) we finally obtain the I-V characteristic

$$I=\frac{eR_K^2G_1^{(T)}G_2^{(T)}}{48\pi^3}\left[\left[\frac{1}{\Omega_1^2}\right]^{1+z/2}+2\left[\frac{1}{\Omega_1\Omega_2}\right]^{1+z/2}+\left[\frac{1}{\Omega_2^2}\right]^{1+z/2}\right]\left[\frac{eV}{\hbar\Omega_z}\right]^{z}\left[\frac{eV}{\hbar}\right]^{3+z}\quad\text{for } z\ll 1\ , \tag{25}$$

$$I=\frac{eR_K^2G_1^{(T)}G_2^{(T)}}{8\pi^3\Gamma(2z+4)}\left[\frac{1}{\Omega_1}+\frac{1}{\Omega_2}\right]^2\left[\frac{eV}{\hbar\Omega_z}\right]^{2z}\left[\frac{eV}{\hbar}\right]^{3}\quad\text{for } z\gg 1\ . \tag{26}$$

These results are valid for the voltages

$$V\ll\min(V_t,\hbar\Omega_z/e)\ , \tag{27}$$

small in comparison with the Coulomb blockade threshold V_t and the characteristic frequency Ω_z of the environment.

The I-V characteristic in the whole Coulomb blockade region (5) was computed numerically. To facilitate the calculations, we still use exponential approximation for $ReZ_{m,n}(\omega)$ [see text below Eq. (15)]. The results for several values of the impedance are shown in Fig. 3. One can see that increase of the impedance suppresses the cotunneling current at low voltages. The I-V curves plotted on a logarithmical scale [Fig. 3(b)] clearly display a power-law behavior $I\propto V^{3+2z}$ for all values of z considered. Approaching the edge of the Coulomb blockade ($V\to e/2C$) the current formally tends to infinity because of the energy denominators in Eqs. (19) and (20). This breakdown of perturbation theory can be overcome by taking into account the contribution of higher orders in the tunneling Hamiltonian. Such a procedure was car-

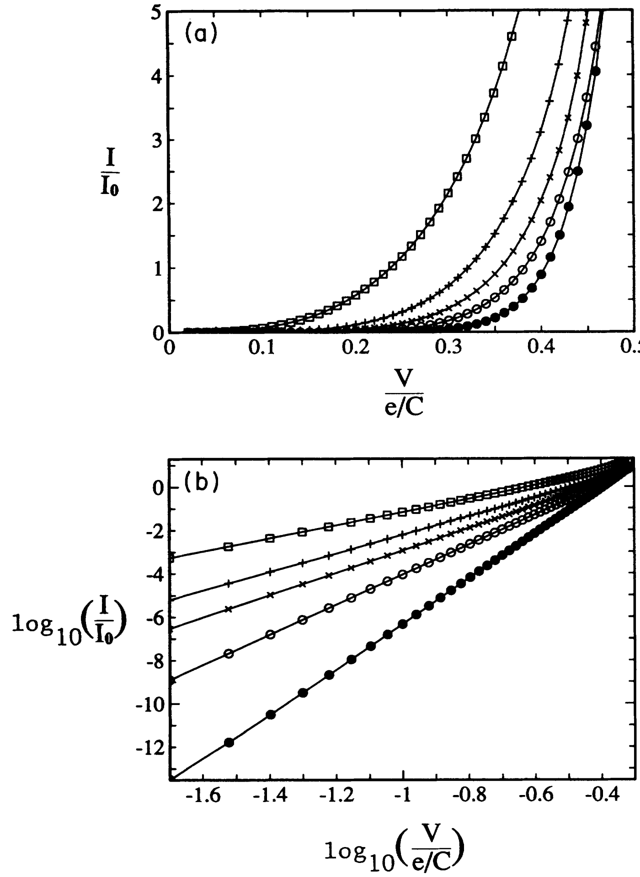

FIG. 3. *I-V* characteristics of double junction in the Coulomb blockade regime computed for $Q_0=0$ and $T=0$. The curves from top to bottom correspond to the impedances $z=Re^2/h=0,0.5,1,2,4$. The results are shown on linear (a) and logarithmic (b) scales. We denote $I_0=(e^3/\hbar C)\Pi_{n=1,2}(\hbar G_n^{(T)}/2\pi e^2)$.

ried out[8] so far only for the capacitive electromagnetic environment.

IV. DISCUSSION AND CONCLUSIONS

We discuss now the possibility to observe the effects of the dissipative environment in experiment. The main problem here is to place high Ohmic resistor with low stray capacitance in the vicinity of tunnel junctions. The best results, to our knowledge, were achieved by Kuzmin and coauthors[28] who fabricated thin resistive leads which have the stray resistance $R_l=3.4$ kΩ/μm, the stray capacitance $C_l=6\times10^{-15}$ F/μm and the length $l=28$ μm. A reasonable model of such an electromagnetic environment is an *RC* line.[28] An impedance of the line $Z/2$ is Ohmic $Z/2=lR_l$ at small frequencies $\omega\ll\Omega_{RC}\equiv1/l^2R_lC_l$ and decreases $ReZ(\omega)=R_K(\Omega_0/|\omega|)^{1/2}$, $\Omega_0\equiv8R_l/C_lR_K^2$ at higher frequencies $\omega\gg\Omega_{RC}$. The effect of dissipative environment is quite pronounced if its impedance is large $ReZ(\omega)>R_K$ for $\omega\sim eV/\hbar$. For the experimental parameters this is the case for low voltages $V<\hbar\Omega_0/e\approx0.45$ mV.

For the voltages $eV>\hbar\Omega_{RC}$ our results are valid qualitatively (because a non-Ohmic dispersion of the impedance is essential). For smaller voltages $eV<\hbar\Omega_{RC}$ only low-frequency Ohmic parts of the impedance contribute. Therefore we expect quantitative agreement, and, in particular, the power-law behavior (25) and (26) of the *I-V* characteristic.

The effect of cotunneling current suppression by the dissipative environment can be used to improve the accuracy of SET devices,[16,17] which transfer electrons one by one. The simplest way to do this is to feed the devices by means of high Ohmic resistors situated close to the junctions (as shown in Fig. 1 for the double junction). The suppression of cotunneling should be especially effective in the pumplike devices, where an adiabatic transfer of electrons occurs. A typical Coulomb energy change $eV_{\text{eff}}\simeq(e/C)[(NfC/G^{(T)})^{1/2}]$ for a SET event in a pump with N junctions decreases with decrease of the operating frequency f.[14,15] The value $eV_{\text{eff}}/\hbar$ determines a characteristic frequency at which an environment contributes. The less this frequency is, the larger is an impedance of the environment (*RC* line) and the more effective is the cotunneling suppression. For typical experimental parameters $C=10^{-15}$ F and $f=0.05G^{(T)}/NC$ we obtain $eV_{\text{eff}}/\hbar=5.4\times10^{10}$ s^{-1}. An impedance of the environment at this frequency corresponds to $z\simeq3.5$. This estimate shows that suppression of cotunneling current, characterized by the increase $2z\simeq7$ of the power of voltage in Eq. (26), can be quite effective.

The effect of inelastic cotunneling considered in this work for tunnel junctions can also occur in semiconductor nanostructures containing quantum dots, where a conventional cotunneling was recently observed.[13] In this case our consideration should be generalized to include possible effects of the discreteness of energy levels and of the finite traversal time. We stress that a bosonic model of electromagnetic environment used here is rather general: it can describe the influence of various types of electronic system excitations (e.g., plasmons,[29] or electron-hole pairs[30]) on the tunneling.

In summary, we have calculated in the lowest nonvanishing order of perturbation theory the cotunneling current through two tunnel junctions coupled to a general electromagnetic environment. For a dissipative Ohmic environment a cotunneling event is shown to be accompanied by a finite energy transfer $\hbar\bar{\omega}$ to the environment, i.e., cotunneling is inelastic. For the low impedance $ReZ(\omega)\ll h/e^2$ of environment the small energy transfers dominate $\hbar\bar{\omega}\ll eV$ whereas for $ReZ(\omega)\gg h/e^2$ we obtain $\hbar\bar{\omega}\approx eV/2$. In the last case a drastic suppression of cotunneling rate occurs at low voltages. This effect can be used to improve the accuracy of the devices which pass electrons one by one.

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APPENDIX

In this appendix we present the calculation of the function $D(\omega_1,\omega_2)$ (12) for high and low impedances of the environment in the limit $T=0$. The rates $\Gamma_n(\omega_n+eV_n/\hbar)$ (17) are nonzero for

$$\omega_n > \omega_c - \frac{eV_n}{\hbar} > 0 \tag{A1}$$

[the right inequality follows from Eq. (5)]. Substituting the correlator (16) in Eq. (12) and performing integration in respect to t_1 and t_3 along imaginary axis (by means of substitution $t-t_1 \to -i\tau_1$, $t_2-t_3 \to i\tau_3$, $t-t_2 \to t_2$ in the first and third terms and $t-t_1 \to i\tau_1$, $t_2-t_3 \to i\tau_3$, $t_1-t_2 \to t_2$ in the other two terms) we obtain

$$D(\omega_1,\omega_2)=\int_0^\infty d\tau_1 dt_2 d\tau_3 e^{-(i\tilde{\omega}+\epsilon)t_2}\left[[e^{-\omega_2\tau_1-\omega_1\tau_3}+e^{-\omega_1\tau_1-\omega_2\tau_3}]\left[\frac{(1+\Omega_z\tau_1)(1+\Omega_z\tau_3)}{(1-i\Omega_z t_2)(1+\Omega_z\tau_1+\Omega_z\tau_3-i\Omega_z t_2)}\right]^{z/2} + [e^{-\omega_1(\tau_1+\tau_3)}+e^{-\omega_2(\tau_1+\tau_3)}]\left[\frac{(1+\Omega_z\tau_1)(1+\Omega_z\tau_3)}{(1+\Omega_z\tau_1-i\Omega_z t_2)(1+\Omega_z\tau_3-i\Omega_z t_2)}\right]^{z/2}\right], \tag{A2}$$

where $\tilde{\omega}\equiv 2\omega_c-\omega_1-\omega_2$ and $\epsilon \to +0$. Poles for all terms in formula (A2) lie on the negative part of the imaginary axis in the complex plain of t_2. This means that if $\tilde{\omega}$ were negative we could rotate the axis of integration over t_2 in the positive direction by $\pi/2$, $t_2 \to i\tau_2$, which in turn would produce purely imaginary D. Thus, in order to have nonzero real part of D, its arguments should satisfy the condition

$$0<\tilde{\omega}<eV/\hbar \tag{A3}$$

[the last inequality comes from Eq. (A1)]. For zero impedance from Eq. (A2) follows the result (18).

Below we concentrate on the small voltages $V \ll V_t$ [see Eq. (5)] when

$$\omega_{1(2)} \sim \omega_c(1\pm 2Q_0/e), \quad \tilde{\omega} \ll \min(\omega_1,\omega_2) . \tag{A4}$$

For low impedance $z \ll 1$ (i.e., $\Omega_z \gg \omega_c$) exponential functions under the integral (A2) determine the area from which the main contribution to D is coming. For example, for the first term, $\tau_1 \sim 1/\omega_2$, $\tau_3 \sim 1/\omega_1$, $t_2 \sim 1/\tilde{\omega}$. This, together with (A4) gives $\Omega_z\tau_1 \gg 1$, $\Omega_z\tau_3 \gg 1$, $t_2 \gg \max(\tau_1,\tau_3)$. Therefore we can approximate this term by the integral

$$i^z \int_0^\infty d\tau_1 dt_2 d\tau_3 e^{-(i\tilde{\omega}+\epsilon)t_2} e^{-\omega_2\tau_1-\omega_1\tau_3}(\tau_1\tau_3)^{z/2} t_2^{-z} ,$$

which can be easily computed. Using the same line of reasoning for the other three terms we obtain

$$D(\omega_1,\omega_2)=i\Gamma(1-z)\Gamma^2(1+z/2) \times\left[\left[\frac{1}{\omega_1^2}\right]^{1+z/2}+2\left[\frac{1}{\omega_1\omega_2}\right]^{1+z/2}+\left[\frac{1}{\omega_2^2}\right]^{1+z/2}\right]\left[\frac{1}{-\tilde{\omega}+i\varepsilon}\right]^{1-z} . \tag{A5}$$

Taking the real part and omitting the terms of order of z^n, $n \ge 2$ we arrive at the formula (19).

In the opposite limit $z \gg 1$ (to be more precise $z^{-1} \ll 1-2|Q_0|/e$) the main contribution to the first term in Eq. (A2) comes from $\tau_1 \sim 1/\omega_2 \ll 1/\Omega_z$, $\tau_3 \sim 1/\omega_1 \ll 1/\Omega_z$, $t_2 \gg \max(\tau_1,\tau_3)$. Thus we can approximate it by

$$\int_0^\infty d\tau_1 dt_2 d\tau_3 e^{-(i\tilde{\omega}+\epsilon)t_2} e^{-\omega_2\tau_1-\omega_1\tau_3}(1-i\Omega_z t_2)^{-z} .$$

Considering the other terms in the same way, we obtain Eq. (20).

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[23]The formulas $e^A e^B = e^{A+B} e^{[A,B]/2}$ and $\langle e^A \rangle = \exp\langle A^2/2 \rangle$ are handy at this step.

[24]The similar expression for $D(\omega_1,\omega_2)$ obtained in Ref. 9 contains extra terms, which in our case are exponentially small [see text below Eq. (5)].

[25]In Refs. 4 and 6 the energies $\hbar\Omega_n$ are denoted as E_n $(n=1,2)$. It follows from Eq. (17) that the inequality (21) indeed defines the region where SET is blocked, $\Gamma(eV_n/\hbar)=0$.

[26]Note that the sense of "elastic tunneling" in this work differs from the one in Ref. 6.

[27]We conjecture, it could be the case for non-Ohmic low frequency dispersion of the impedance, e.g., for $ReZ(\omega)\propto\omega^\alpha$, $\alpha>0$.

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