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# Computation of Time Domain Scattering Parameters Through the Numerical Inversion of the Laplace Transform

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***Abstract*— Time-domain (TD) methods for the solution of Maxwell's equations are particularly appealing for their ability to provide the overall characteristics of an electrical system in a single simulation run. In many situations, such TD methods require computing the system's impulse response and using it in a convolution-based solver. In this work, we propose the evaluation of the scattering-parameters-type impulse response of partial element equivalent circuit (PEEC) models by firstly computing the scattering parameters pertaining to a unit-step excitation via the Numerical Inversion of Laplace Transform (NILT) technique, followed by recovering the corresponding impulse response. The accuracy and effectiveness of the advocated approach is validated by means of numerical experiments comparing its performance with that of more standard methods.**

***Keywords*— Numerical Inversion of Laplace Transform, Partial Element Equivalent Circuit method, scattering parameters**

## I. Introduction

Nowadays numerical electromagnetic/circuit simulations are increasingly used in virtual prototyping of electrical/electronic systems and devices and, thus, are widely used in the design process. Over the years, many different techniques have been developed in both the frequency and the TD. The most popular methods in the frequency domain are the Method of Moments (MoM) [1] and the Finite Element Method (FEM) [2]. They require the repeated computation of the solution at each frequency of interest. Time domain techniques include the Finite Difference Time Domain (FDTD) method ([3]), the TD Finite Element Method (TDFEM) [2] and the TD integral equation (TDIE) based methods [4]. They require the solution of a linear system over a properly sampled time window. In particular, all the TDIE methods based on a marching-on-in time (MOT) approach may exhibit stability issues which can be mitigated by adopting pertinent techniques [5].

Among the IE-based methods, the Partial Element Equivalent Circuit (PEEC) method [6] has proven to be well suited for the solution of EM problems especially for its capability to provide a circuit interpretation of the electric-field integral equation (EFIE) and the continuity equation. TD methods can, in principle, be used to achieve the impulse response of an electrical system that may further be employed in a higher-level computational framework performing time-convolution integrations. This approach has two main limitations: a) the input pulse cannot be ideal, it has to be approximated as a short finite pulse; b) the time domain technique may exhibit instabilities preventing to obtain any useful result.

In this work, we propose to compute the impulse response of PEEC models through the numerical inversion of Laplace transform (NILT) method, which has proven being able to guarantee the late-time stability [7]. Firstly, the scattering parameters step response of PEEC models is computed by applying the NILT method and then the scattering parameters impulse response is recovered by time differentiation. The numerical results confirm the accuracy of the proposed approach compared to more standard methods.

## II. The PEEC method

The Partial Elements Equivalent Circuit Method (PEEC) is an IE based method that permits a circuit representation of the electromagnetic (EM) phenomena that affect the electronic structures under examination. The foundations of the method are the EFIE and the continuity law for the electric current [6].

The method allows studying the behavior of complex structures in terms of standard circuit unknowns, namely node potentials and side electric currents.

First, the structure under investigation is discretized into a tessellation (mesh) consisting of a large number of elementary volumes and surfaces. The electric currents in the structure are assumed to flow through the elementary volumes, while the electric charges are assumed to exist over the elementary surfaces of the mesh.

Subsequently, the basic interactions among the currents flowing in volumes and among the charges on the surfaces have to be defined. The magnetic interaction among the currents are described by the the partial inductances $L_p$ [8], while the electric interactions among the charges are described by the coefficients of potential $P$ [9].

The partial elements matrices can be easily incorporated, in the Laplace-transform domain, in the Modified Nodal Analysis (MNA) circuit representation [10]

$$\begin{bmatrix} s\boldsymbol{P}^{-1} + \boldsymbol{Y}_{le}(s) & \boldsymbol{A} \\ -\boldsymbol{A}^T & \boldsymbol{Z}(s) + s\boldsymbol{L}_p \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{\Phi}(s) \\ \boldsymbol{I}(s) \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_s(s) \\ \boldsymbol{V}s(s) \end{bmatrix} \quad (1)$$

where $\boldsymbol{A}$ is the incidence matrix, $\boldsymbol{Z}(s)$ is the impedance matrix accounting for the impedance of conductors or dielectrics elementary volumes, $\boldsymbol{Y}_{le}$ is the lumped admittance matrix that contains all the lumped admittances connected to the nodes of the equivalent circuit.

Once the MNA equations are available, the unknown vectors of the node potentials and branch electric currents, $\boldsymbol{\Phi}(s)$, $\boldsymbol{I}(s)$ respectively, can be determined provided the voltage and current sources, $\boldsymbol{V}_s(s)$, $\boldsymbol{I}_s(s)$ respectively, are known.

The impulse response of the system described by (1) is typically obtained by solving it in the frequency domain ($s = j\omega$). To this aim, first the sources are set as Dirac pulses exciting one port at a time. Then (1) is solved over the frequency range of interest. Finally, the impulse response is recovered through the Inverse Fourier Transform (IFT). The main drawback of this approach is that, in order to build aberrations-free TD responses, the MNA system has to be evaluated over a large number of frequency samples, and this easily leads to excessively high computational costs, especially if the number of unknowns is high.

In this work, we propose to compute the impulse response of PEEC models described by (1) through the NILT [11]. The main advantage of this method is that, in contrast to the IFT based technique, the evaluation of each TD sample does not depend on the others. Consequently, an arbitrary number of TD samples can be evaluated without introducing aberrations [12].

## III. The Numerical Inversion of the Laplace Transform (NILT) method

The NILT is a powerful approach that permits to evaluate the approximate Laplace inverse transform of a complex-frequency domain function.

With the aim to briefly describe the method, it is convenient to express (1) in the form

$$\boldsymbol{M}(s)\boldsymbol{X}(s) = \boldsymbol{U}(s) \quad (2)$$

An approximation to $\boldsymbol{x}(t)$, representing the TD original of the unknown vector $\boldsymbol{X}(s)$, can be expressed as

$$\tilde{\boldsymbol{x}}(t) = -\frac{1}{t}\sum_{i=1}^{M/2} 2\text{Re}\{k_i \boldsymbol{X}\left(\frac{z_i}{t}\right)\} \quad (3)$$

where $z_i$ and $k_i$ denote the Padé poles and residues, which are known, while $M$ is the expansion order [12].

In contrast to the standard IFT-based approach it is apparent that the inverse represented through Eq. (3) requires, for a fixed instant $t$, to solve the system (2) just at a set of pre-specified $M/2$ points in the complex $s$-plane. Consequently, the computation pertaining to time $t$ does not depend on the previous evaluations, thereby avoiding potential error-accumulation-based late-time instabilities.

On the other hand, the drawback of the NILT method is the so-called late time inaccuracy of the solution. Indeed, the error introduced increases with $t$ as [13]

$$\boldsymbol{x}(t) - \tilde{\boldsymbol{x}}(t) = \Psi_{N,M} \frac{d^{N+M+1}}{dt^{N+M+1}}\boldsymbol{x}(t)\bigg|_{t=0} t^{N+M+1} + \mathcal{O}(t^{N+M+2}) \quad (4)$$

where,

$$\Psi_{N,M} = \frac{(-1)^M M!N!}{(M+N)! \cdot (M+N+1)!} \quad (5)$$

where usually: $N = M - 2$ to guarantee the stability of the method [13].

### *The resetting procedure*

Especially in applications where long and detailed transients have to be captured, the standard version of the NILT method can lead to unsatisfactory results in the most of the time window. In order to prevent the loss of accuracy expressed by (4), a time resetting procedure that supports the main body of the NILT scheme was introduced in [14] and explained in detail in [15]. Equations (1) can be reorganized, including the presence of the initial conditions, in a state form as

$$(\boldsymbol{G} + s\boldsymbol{C})\boldsymbol{X}(s) = \boldsymbol{B}(s) + \boldsymbol{C}\boldsymbol{x}(0) \quad (6)$$

where $\boldsymbol{C}$ and $\boldsymbol{G}$ are matrices that, respectively, represent the memory and the memoryless elements of the equivalent circuit, $\boldsymbol{B}(s)$ is a vector containing the independent sources and $\boldsymbol{x}(0)$ is the TD state vector at the initial time step. The standard NILT solution (3) can be viewed as an approximate TD solution of (6) considering the initial conditions $\boldsymbol{x}(0)$. With the aim to keep the error (4) constant, the key idea is to use the results $\tilde{\boldsymbol{x}}((q-1)h)$ obtained at the time $t = (q-1)h$ to move a step forward to $t = qh$, where $h$ is an appropriate time step and $q$ is an integer. In this way, for each time, we shift the time origin to $(q-1)h$ and, hence, (3) can be rewritten as:

$$\tilde{\boldsymbol{x}}(qh) = -\frac{1}{h}\sum_{i=1}^{M/2} 2\text{Re}\{k_i \boldsymbol{X}\left(\frac{z_i}{h}\right)\} \quad (7)$$

where $\boldsymbol{X}\left(\frac{z_i}{h}\right)$ is obtained by solving the system:

$$(\boldsymbol{G} + s\boldsymbol{C})\boldsymbol{X}(s) = \boldsymbol{B}(s) + \boldsymbol{C}\boldsymbol{x}((q-1)h) \quad (8)$$

with $s = z_i/h$, for $i = 1, \ldots, M/2$. Since the error now scales with $h$ and not with $t = qh$, this procedure enables to evaluate long TD responses using the NILT without any loss of accuracy.

## IV. TD scattering parameters computation

Let us consider a multi-port electrical system, described in terms of incident waves and reflected waves. The port waves can be then defined as

$$a_i(s) = V_i(s) + R_{0i}I_i(s) \quad (9a)$$

$$b_i(s) = V_i(s) - R_{0i}I_i(s) \quad (9b)$$

where $R_{0i}$ is the reference impedance at the $i$-th port. All the reflected waves can be related to the incident waves through the scattering parameters as:

$$\boldsymbol{b} = \boldsymbol{S}\boldsymbol{a} \tag{10}$$

where $\boldsymbol{b}$ is a vector that contains all the port reflected waves, $\boldsymbol{a}$ is a vector that contains the incident waves and $\boldsymbol{S}$ is the scattering matrix. The condition $a_k(s) = 0$ is easily obtained terminating the port $k$ with its reference impedance, setting $R_{0k} = R_k$. Hence, assuming a test configuration with $R_{0i} = R$ for all the ports, we can write:

$$S_{ij}(s) = \left. \frac{V_i(s) - RI_i(s)}{V_{sj}(s)} \right|_{a_k=0, k\neq j} \tag{11}$$

where $V_{sj}(s)$ is the port $j$ voltage generator, and $R$ is the termination resistance for the overall ports. Transforming (11) back to the TD, it is possible to write:

$$v_i(t) - Ri_i(t) = \int_0^t v_{sj}(t-\tau)s_{ij}(\tau)d\tau \tag{12}$$

If the voltage source $v_{sj}(t)$ is the Heaviside unit-step function $H(t)$, the previous equation becomes:

$$v_i(t) - Ri_i(t) = \int_0^t s_{ij}(\tau)d\tau \tag{13}$$

and finally:

$$s_{ij}(t) = \frac{d}{dt}[v_i(t) - Ri_i(t)] \tag{14}$$

where $v_i(t)$ and $i_i(t)$ are the port $i$ voltage and current when the step waveform is applied to the port $j$ and $R_{0k} = R \; \forall k$.

The main result of this procedure is that the scattering parameters can be obtained and saved as a one-time process, feeding the system with a canonical step voltage. When the the same system is excited by arbitrary port source waveforms and its ports are terminated by arbitrary loads, the knowledge of the TD scattering parameters can be exploited.

Firstly, assuming arbitrary terminations, the incident and reflected waves are related through the following condition that is the TD counterpart of (10).

$$\boldsymbol{b}(t) = \int_0^t \boldsymbol{s}(t-\tau)\boldsymbol{a}(\tau)d\tau \tag{15}$$

and $\boldsymbol{s}(t)$ is the TD version of the scattering matrix. Moreover, it is known that the port voltages and currents are related to the waves quantities by

$$\boldsymbol{v}(t) = \frac{1}{2}[\boldsymbol{a}(t) + \boldsymbol{b}(t)] \tag{16}$$

$$\boldsymbol{i}(t) = \frac{1}{2R_{0i}}[\boldsymbol{a}(t) - \boldsymbol{b}(t)] \tag{17}$$

Finally, the constitutive relations of the circuit connected to the ports must be enforced:

$$\boldsymbol{v}(t) = \boldsymbol{v}_s(t) - \boldsymbol{F}(\boldsymbol{i}(t)) \tag{18}$$

where $\boldsymbol{v}_s(t)$ is the vector containing the port voltage sources and $\boldsymbol{F}(\cdot)$ is a vector containing the models describing the passive behavior of the ports. Equations (15)-(18) represent a set of well posed equations for the electrical problem including the multiport system and arbitrary port circuits. To this aim, Eq. (15) can be approximated by efficient discrete convolution schemes.

## V. Numerical results

This section presents an application of the procedure outlined in the previous section. The structure under examination is a two ports microwave structure that can be analyzed through the PEEC method, in order to build an equivalent circuit. The NILT technique, applied to the PEEC model, is employed to compute the port step responses of the system and, hence, the TD scattering parameters. Finally, the effective port voltage responses, due to a trapezoidal input waveform, are computed.

### *Loaded microstrip*

The structure under examination is a 20 cm long stubs loaded microstrip, illustrated in Fig. 1. The copper structure, placed over the dielectric substrate with $\varepsilon_r = 4.4$ and thickness 1.6 mm, is composed by a microstrip which is periodically loaded by four microstrip stubs of length 38.5 mm, that are left open at the end of the dielectric. All the signal conductors have a width of 3 mm and a thickness of 35 $\mu$m. The distance between the microstrip line and the free edge of the dielectric is 8 mm. The device has two 50 Ω ports placed between the two ends of the main microstrip and the ground plane.

In order to obtain the TD scattering parameters, it is necessary to compute the TD port step responses. To this aim, the structure is excited with a step waveform $H(t)$ at one of the two ports and both the port step responses are then observed. The two step responses can be obtained by employing the NILT algorithm supported by the resetting procedure previously introduced. If a high number of samples (thousands) is desired, the NILT method can lead to very high computational efforts. In this case, Hermite interpolators can support the standard NILT procedure to obtain very dense detailed waveforms starting from a relatively small number of NILT computed samples (hundreds) [15]. The output port step voltage response is depicted in Fig. 2.

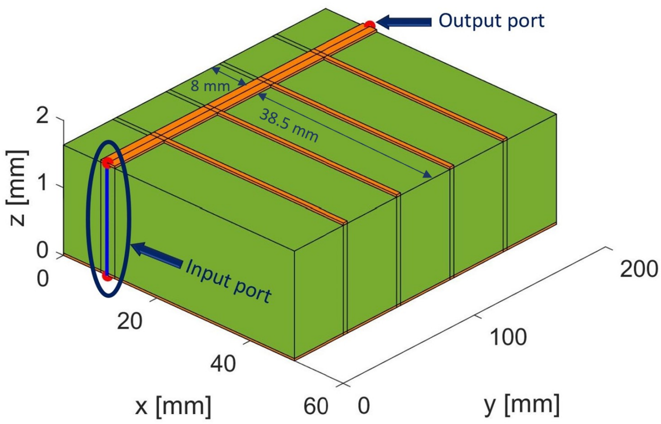


Fig. 1. Loaded microstrip geometry.

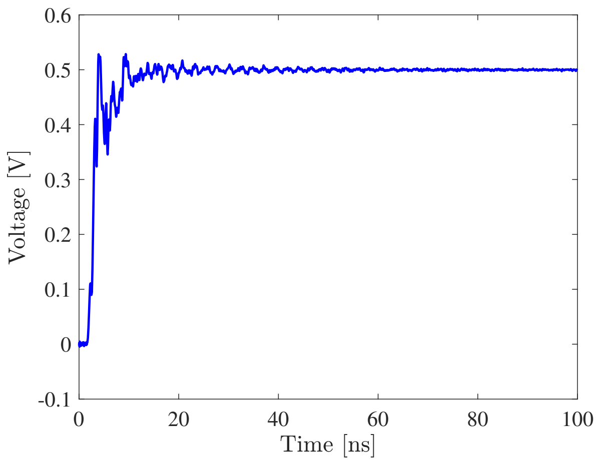


Fig. 2. Output port step response computed through the NILT method.

From the TD step responses it is possible to determine as a one time process the TD scattering parameters, which entirely characterize the structure. Once the scattering parameters of the structure are known and stored as library files, they can be employed to compute the voltage and current responses at each port when an arbitrary source waveform is attached to the source port. To illustrate this procedure, we consider a trapezoidal source voltage with a rising time 3 ns and width 8 ns. Then, the port voltages are obtained by solving equations (15)-(18) and using the pre-computed scattering parameters impulse response. The output port voltage is shown in Fig. 3. With the only purpose of validation, the results obtained via NILT are compared with those obtained with a conventional time-stepping solver [16], showing a very good agreement.

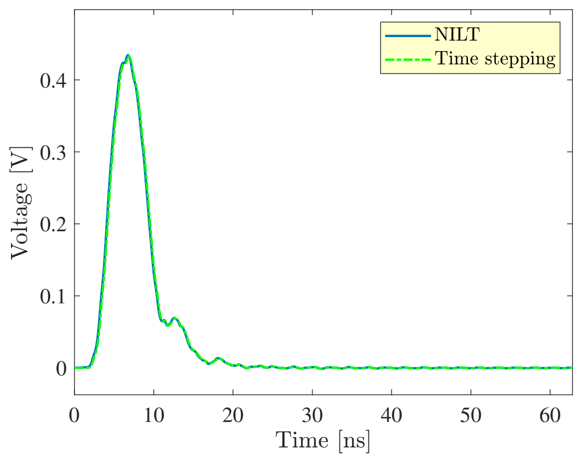


Fig. 3. Output port voltage response.

## VI. Conclusion

In this work, a novel technique is introduced which is able to extract the TD scattering parameters that characterizes a given electrical structure. The procedure is based on the NILT-based computation of the ports step responses from which the scattering parameters impulse response is recovered. The use of the NILT technique guarantees the stability of the response. Once the scattering parameters are computed, they can be used in a higher-level solver based on time-convolution integrals, which enables to incorporate any type of active/passive or/and linear/nonlinear circuit terminations. A microwave structure analysis, employing the PEEC method, is then presented, showing results that are in a satisfactory agreement with those obtained through a conventional time stepping technique.

## References


[1] R. F. Harrington, *Field Computation by Moment Methods*. New York: Macmillan, 1968.

[2] J. M. Jin, *The Finite Element Method in Electromagnetics*, 3rd ed. John Wiley and Sons, New York, 214.

[3] K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media," *IEEE Transactions on Antennas and Propagation*, vol. 14, no. 5, pp. 302–307, May 1966.

[4] R. Mittra, *Integral Equation Methods for Transient Scattering, in Transient Electromagnetic Fields*, L. B. Felsen, Ed. New York: Springer-Verlag, 1976.

[5] A. G. Tijhuis, *Electromagnetic inverse profiling: theory and numerical implementation*. Utrecht, NL: VSP, 1987.

[6] A. E. Ruehli, G. Antonini, and L. Jiang, *Circuit Oriented Electromagnetic Modeling Using the PEEC Techniques*. Wiley-IEEE Press, 2017.

[7] L. Lombardi, F. Loreto, F. Ferranti, A. Ruehli, M. S. Nakhla, Y. Tao, M. Parise, and G. Antonini, "Time-domain analysis of retarded partial element equivalent circuit models using numerical inversion of Laplace transform," *IEEE Transactions on Electromagnetic Compatibility*, vol. 63, no. 3, pp. 870–879, 2021.

[8] A. E. Ruehli, "Inductance calculations in a complex integrated circuit environment," *IBM Journal of Research and Development*, vol. 16, no. 5, pp. 470–481, Sep. 1972.

[9] A. E. Ruehli, P. A. Brennan, "Efficient capacitance calculations for three-dimensional multiconductor systems," *IEEE Transactions on Microwave Theory and Techniques*, vol. 21, no. 2, pp. 76–82, Feb. 1973.

[10] C. Ho, A. Ruehli, P. Brennan, "The modified nodal approach to network analysis," *IEEE Transactions on Circuits and Systems*, vol. 22, no. 6, pp. 504–509, Jun. 1975.

[11] M. Nakhla, K. Singhal, and J. Vlach, "Numerical inversion of the Laplace transform," *Electronics Letters*, vol. 9, no. 14, pp. 313–314, July 1973.

[12] J. Vlach and K. Singhal, *Computer Methods for Circuit Analysis and Design*, 2nd ed. Boston, MA, USA: Kluwer Academic, 2003.

[13] Y. Tao, E. Gad, and M. Nakhla, "Fast and stable time-domain simulation based on modified numerical inversion of the Laplace transform," *IEEE Transactions on Components, Packaging and Manufacturing Technology*, vol. 11, no. 5, pp. 848–858, 2021.

[14] L. Lu, M. Nakhla, and Q.-J. Zhang, "A resetting algorithm for transient analysis of coupled transmission line circuits," *IEEE Transactions on Microwave Theory and Techniques*, vol. 42, no. 3, pp. 494–500, 1994.

[15] E. Gad, Y. Tao, and M. Nakhla, "Fast and stable circuit simulation via interpolation-supported numerical inversion of the Laplace transform," *IEEE Transactions on Components, Packaging and Manufacturing Technology*, pp. 1–1, 2021.

[16] G. Antonini and D. Romano, "Adaptive-cross-approximation-based acceleration of transient analysis of quasi-static partial element equivalent circuits," *IET Microwaves, Antennas Propagation*, vol. 9, no. 7, pp. 700–709, 2015.