



MASTER'S THESIS

## Dynamic Interaction of Subsea Pipeline Spans due to Vortex-Induced Vibrations

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## Abstract

When currents are present around a subsea pipeline, vortices are shed alternately from the upper and lower sides of the pipe. The vortex shedding can induce vibrations in areas where the pipe is suspended above the seabed. These vibrations are called Vortex-Induced Vibrations (VIV). VIV have been and still are a cause of concern for pipeline designers. If a pipeline vibrates during a large part of its design life, then damage due to fatigue is expected. It is therefore of great importance to understand how pipelines respond to the flow-induced vortices and how the vibrations can be mitigated.

Recent research has addressed the dynamic interaction of adjacent free spans. Dynamic interaction can occur when two or more parts of a pipeline that are suspended above the seabed are separated by a narrow support. In that case the vibration of a span can be different than when it is considered separately. This area is the focus of this research. The aim is to give more information on how the support properties effect the dynamic interaction of the free spans. The properties under investigation are: the support length and the type of soil model. For a multi-span, nonlinear soil behaviour such as pipe lift off could play a role in the vibration. The implication of this would be: change of dominating response frequency, different vibration shape of the pipe and impact loads on the pipe.

In this thesis a model is proposed that calculates VIV of a subsea pipeline; the Pipe-Wake model. This model allows for response of the pipeline in the time-domain, which makes it possible to add nonlinear soil effects. For clay soils a suction model is proposed that allows the pipe to break free from the seabed after the suction force reach a threshold value. For sand two models are possible: no suction or full suction. The former means that in upward motion no soil resistance is noted, while in the latter the soil resistance is equal in both directions.

The VIV are modelled using a wake-oscillator. A wake-oscillator is a phenomenological model that captures the behaviour of VIV accurately. The two main aspects of this are the lock-in region and the self-limiting vibration. The pipe itself is modelled as an Euler-Bernoulli beam using the Finite Element Method. A nonlinear static analysis is used to calculate the pipes initial position. For the dynamic calculation the stiffness of the beam is linearized around the static deflection. Modal analysis is used to calculate the response of the pipe. A soil model, which is dependent on soil type is then added to complete the Pipe-Wake model.

With this model the case study is replicated. The general conclusion of the research is that simplifying the support by either shortening it, or by assuming that soil behaves the same for both soil penetration and release, is an oversimplification in the case of a multi-span set-up. For high flow velocities this can lead to disconnection of the pipe from the support. The consequence of this is larger vibration amplitudes for lower flow speeds in relation to linear soil behaviour. Modelling the support shorter than it is in reality will lead to a drop in natural frequency and mode shape. It is therefore recommended to verify these conclusions experimentally and see if this would have implications for the design of multi-spanning subsea pipelines.

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# Nomenclature

[E]	Eigenmatrix	
[K]	Stiffness Matrix	
$[K^*]$	Modal Stifness Matrix	
[M]	Mass Matrix	
$[M^*]$	Modal Mass Matrix	
[T]	Secant Stiffness Matrix	
$\epsilon$	Tuning parameter [-]	
$\kappa$	Von Karmans constant [-]	
$\mu$	Friction coefficient [-]	
ν	Kinematic viscosity $[m^2 s^{-1}]$	
$\Omega_n$	Natural frequency matrix	
$\omega_n$	Natural frequency of the pipe $[rad s^{-1}]$	
$\omega_o$	Natural frequency of the oscillator $[rad s^{-1}]$	
$\phi$	Eigenvector	
$ ho_w$	Density of water $[kg m^{-3}]$	
ξ	Damping ratio matrix	
$\zeta$	Damping factor [-]	
A	Tuning parameter [-]	
$C_a$	Potential coefficient for added mass [-]	
$C_l$	Dynamic stiffness factor in lateral direction $[{\rm N/m}^{5/2}]$	
$C_v$	Dynamic stiffness factor in vertical direction $[\mathrm{N}/\mathrm{m}^{5/2}]$	
$C_y$	Coefficient of lateral force [-]	
$C_z$	Coefficient of vertical force [-]	
$C_{D0}$	Drag coefficient for stationary cylinder [-]	
$C_{L0}$	Lift coefficient for stationary cylinder [-]	
D	Pipe diameter [m]	
E	Young's Modulus [Pa]	
F	External Force [N]	

$f_n$	Natural frequency		
$f_s$	Shedding frequency $[rad s^{-1}]$		
Ι	Moment of Inertia $[m^4]$		
$k_c$	Cyclic loading factor for Fmax [-]		
$k_r$	Consolidation time factor for Fmax [-]		
$k_v$	Pull-out velocity factor for Fmax [-]		
$k_{DC}$	Cyclic loading factor for breakout distance [-]		
$k_{DT}$	Consolidation time factor for breakout distance [-]		
$k_{DV}$	Pull out velocity factor for breakout distance [-]		
L	Element length [m]		
m	Mass per unit length of the cylinder [kg]		
$m^*$	Mass ratio [-]		
$m^*\zeta$	Mass-damping		
$m_a$	Added mass per unit length of the cylinder [kg]		
N	Non-dimensional shape and depth factor [-]		
q	Wake variable [-]		
r	Modal displacement		
Re	Reynolds Number [-]		
$S_u$	Undrained shear strength [-]		
St	Strouhal number [-]		
T	Tension force [N]		
U	Relative velocity $[m s^{-1}]$		
u	Nodal displacement [m]		
$u^*$	Friction velocity $[m s^{-1}]$		
V	Undisturbed flow speed $[m s^{-1}]$		
W	Work done [N m]		
$z_0$	Seabed roughness length		
1N	One Node		
2N	Two Node		
DS	Distributed Springs		
g	Gravitational acceleration = $9.81 \text{ m s}^{-1}$		
IB	Initial Seabed		

## Chapter 1

## Introduction

## 1.1 Background

In the offshore industry pipelines are used to transport fluids and gas along the seabed. Pipelines can be located between two locations subsea, but are also used to transport the product to shore. Amazing water depths can nowadays be reached with depths ranging far over the 2000 m mark. The material used for the pipelines is mainly steel, sometimes covered by a coating of plastic or concrete. One of the failure mechanisms of steel is fatigue damage, which is why in pipeline design a lot of attention is paid to vibration of the pipeline in the long term.

The main cause of long term vibration is a phenomenon called Vortex-Induced Vibrations (VIV). VIV occurs when a free span, a part of the pipe that hangs free from contact with soil, is subjected to a current. An example of what a free span looks like is shown in Figure 1.1a. Due to the current vortices are formed in the wake of the pipeline, see Figure 1.1b. These vortices will cause the pipe to vibrate perpendicular to the flow. If the speed of the current is within a certain range, the lock-in region, these vibrations can become excessive and damage the free span in the long term. To analyse whether the free span is in danger of being damaged by VIV, use is made of the DNV (Det Norske Veritas) design code for free spanning pipelines (DNV-RP-F105).



(a) Single span configuration



(b) Vortex street

Figure 1.1: Span configuration and vortex street

When 2 free spans are located next to each other, another phenomenon is observed that has implications for fatigue life; dynamic interaction. Dynamic interaction means that the free spans have influence on each others amplitude of vibration. This can lead to an increase of the amplitude or even change the range of velocities for which a span is sensitive to VIV. In the calculation of fatigue life, the interaction of spans cannot be overlooked. In the DNV code, an addition is therefore made for cases where 2 spans are close together, or multi-spans as they are called.

The basis for the design procedure in the DNV code is based on experiments that were performed for the design of the Ormen Lange pipeline [Fyrileiv et al., 2006]. This is a

pipeline in which a lot of free spans were identified during design. A lot of different span configurations have been tested and from the results the model has been made as found in the DNV code. The model is a so-called response model: models that predict the maximum VIV amplitude based on basic structural and hydrodynamical parameters. These models do not give the user any insight into how certain parameters influence the dynamic behaviour of the VIV.

This thesis aims to give a better understanding of the phenomenon of dynamic interaction due to VIV. Modelling a pipeline on the seabed and analysing the effects of dynamic interaction due to VIV will be the main goal of the research.

## 1.2 Problem definition

The response model that is used to calculate the maximum amplitude of vibration due to VIV bases its results on only a couple of input parameters, of which natural frequency and current velocity are the most important. The model outputs the fatigue life. For investigation of dynamic interaction and the properties that influence it, a response model is unsuitable. A model that gives the user full freedom in inputs and outputs allows detailed investigation into the problem. Such a model is called a force-model.

One of the goals of this research is to come up with a force model. The model can then be used to give weight to existing pipeline designs. If any irregularities are noticed, the new model should be able to illustrate this. In addition to this, assumptions that are made in the response-model can be validated by the force-model. Finally the model can tell the user if dynamic interaction really is of any influence and whether it should be taken into account in the pipeline design.

## 1.3 Research objective

It is expected that the response model is not sufficient for investigating the effect of dynamic interaction. Therefore a force-model will be developed. Figure 1.2 illustrates a set-up where two free spans are separated by a support, such a set-up is also called a multi-span. For a similar set-up the following research question will be answered:

To what extent do the support properties influence the vibration of a free spanning pipeline?

To help answer the research question, a couple of subquestions have been written up as well. These question are divided into 2 categories: Pipe-Wake model and Case study.



Figure 1.2: A multi-span pipeline configuration

From here on in the pipeline model that will be created is called the Pipe-Wake model, to avoid confusion between other pipeline models and the model presented in this thesis.

#### Pipe-Wake model

- 1. What are the parameters that have effect on the pipeline response behaviour?
- 2. Which methods exist for modelling VIV behaviour?
- 3. How does the Pipe-Wake model relate to existing models?

#### Case study

- 1. What kind of methods are possible for modelling a support and how do they influence the amplitudes of vibration?
- 2. How does support length influence the amplitudes of vibration?
- 3. To what extent does soil type influence the vibration?
- 4. To what extent do the results relate to reality?

By answering these questions a valid conclusion can be drawn regarding the influence of the support on the vibration of pipeline spans.

### 1.4 Research methodology

To answer the questions, research has been performed that consists of a couple of stages. In each stage, a certain part of the research that needs to be performed is analysed. The stages are:

1. Literature study

The aim of the literature study is to provide solid background information on the problem. Knowing what already has been researched is a great help and saves a lot of time for your own research. Even though the first stage is a literature study, the study actually does not end. During the whole process literature is used for: validation, inspiration and information.

2. Modelling and programming

Building a model is a constant collaboration between modelling, designing how physical aspects can be interpreted in a computer, and programming; turning the equations into computer code.

3. Model Validation

Where possible the model should be validated with experimental results. Because the Pipe-Wake model as designed in this research has not been tested in a laboratory, literature and insight is used to provide validation of the model where appropriate.

4. Case study

To answer the research question an example case is used. This case is based on an existing pipeline design.

By performing the research in this manner, structure and validity is given to the thesis. These stages cover all the aspects that are needed to provide a realistic model and provide answers that are of use to INTECSEA.

## 1.5 Thesis outline

The thesis is built up of 6 chapters:

- *Chapter 1, Introduction*: the introduction gives background information on the subject and the research methods
- *Chapter 2, Theory*: this chapter provides theory on how vortices are created, what parameters are involved and the forces that the consequence of vortex creation
- *Chapter 3, VIV model*: the theory from chapter 2 is translated to a model using a wake-oscillator, a model that is discussed in this chapter
- Chapter 4, Pipe-Wake model: the model for the free spanning pipeline is presented and validated in this chapter
- Chapter 5, Time-domain analysis of a multi-span pipeline: the example case is investigated using the Pipe-Wake model and the results are presented in this chapter
- Chapter 6, Inspection of dynamic interaction in relation to support properties: a detailed look is taken at how nonlinear soil properties effect the vibration
- Chapter 6, Conclusion and recommendations: a conclusion is written for the entire research and a recommendation is done for the parts of the research that require more detailed investigation

## Chapter 2

## Theory

## 2.1 Introduction

Vortex-Induced Vibrations (VIV) can become a hazard when they act on an object. When vortices develop behind the cylinder they can cause the cylinder to vibrate at a certain frequency. Cylinders in both wind and water can feel the effects of these vortices. Examples of structures that are vulnerable to VIV are: long slender chimneys, pipelines and riser systems. It is predominantly fatigue damage that can occur due to these vibrations. If one imagines that a pipeline is designed for 50 years, these vortex-induced vibrations can occur for a long time during the pipelines lifespan. The aim of this chapter is to explain the basic theory behind VIV.

## 2.2 Vortex street

If a cylinder is located in a steady flow, vortices can be observed in the wake of the cylinder. When the flow hits the cylinder, the water particles are forced to pass around the cylinder, see Figure 2.1a. Because of the roughness of the cylinder, a boundary layer is formed where water particles close to the cylinder wall travel at a slower speed than particles further away, see Figure 2.1b. Near the widest point of the cylinder, point A and B in Figure 2.1c, the boundary layer separates from the cylinder and forms two shear flows that travel towards the wake of the cylinder as seen in Figure 2.1c. Because the water particles close to the cylinder as seen in Figure 2.1c. Because the water particles close to the cylinder travel at lower speeds than the particles further away, the shear flows tend to fold into each other alternately forming a steady vortex street as illustrated in Figure 2.1d [Blevins, 1990]. Such a vortex street is called the von Karman vortex street. An example of this is shown in Figure 2.2.



Figure 2.1: Stages of vortex generation

The fashion in which vortices are shed is dependent on the Reynolds number. The Reynolds number is a dimensionless quantity that gives indication of how turbulent the flow is. Low Reynolds numbers relate for more stable, laminar flows, while high Reynolds numbers indicate a turbulent flow.



Figure 2.2: von Karman vortex street

The equation for the Reynolds number is given by:

$$Re = \frac{VD}{\nu} \tag{2.1}$$

In which:

$$\begin{split} V &= \text{Flow velocity } [\text{m}\,\text{s}^{-1}] \\ D &= \text{Cylinder diameter } [\text{m}] \\ \nu &= \text{Kinematic viscosity } [\text{m}^2\,\text{s}^{-1}] \end{split}$$

Figure 2.3 shows how the vortex shedding is dependent on the Reynolds number. For a typical subsea pipeline the subcritical regime is of most interest  $(300 < Re < 3 \cdot 10^5)$ .

The frequency at which the vortices are shed is called the shedding frequency. This frequency is calculated using the *Strouhal relation*, which is shown in Equation (2.2).

$$f_s = \frac{V\,St}{D} \tag{2.2}$$

In which:

 $f_s =$  Shedding frequency [Hz] St = Strouhal number [-]

The new parameter that is introduced in Equation (2.2) is the Strouhal number. This number is an empirically derived value that is dependent on the Reynolds number. Figure 2.4 shows the relation between the Reynolds number and the Strouhal number. The range of Reynolds numbers that is applicable to subsea pipelines, the sub-critical regime, is 300 -  $3 \cdot 10^5$ , see Figure 2.3. From the figure it can be seen that the Strouhal number is approximately constant at St = 0.2 for this range of Reynolds numbers. A useful characteristic as will be seen later on.

### 2.3 Vortex induced forces on a cylinder

The alternate vortex shedding leads to pressure fluctuations around the cylinder. These fluctuations lead to an oscillating force. The characteristics of this force is dependent on whether the cylinder is fixed in position or is free to move. For this reason this section will discuss the vortex-induced forces for both a stationary cylinder as well as a moving cylinder.



Figure 2.3: Vortex shedding regimes, [Lienhard, 1966]



Figure 2.4: Strouhal number in relation to Reynolds number, [Lienhard, 1966]

#### 2.3.1 Stationary cylinder

The force on a 2D cylinder can always be split up into two directions: cross-flow, a force perpendicular to the flow direction, and in-line, a force in-line to the flow direction. Figure 2.5 illustrates how the forces in these directions behave during an oscillation. In this figure the cross-flow direction is  $C_L$ , the in-line direction is  $C_D$  and the total force is  $C_F$ . When the cross-flow force is at its maximum, the in-line force is as well. This situation is at t = 0.903 in the figure. At t = 0.968 the cross-flow force is 0, while the in-line force is also at its minimum. At t = 1.000 the cross-flow force is at its minimum and the in-line force is at its maximum again. An important implication of this is that the in-line force oscillates at *twice* the frequency of the cross-flow force.



Figure 2.5: Vortex induced force over half an oscillation period [Drescher, 1956]

For a stationary cylinder the cross-flow and in-line forces are also called the lift and drag forces respectively. These forces can be expressed in terms of dimensionless lift and drag coefficients. Equation (2.3) shows how this is done:

$$F_{D0} = \frac{1}{2} C_{D_0} \rho_w L D V^2$$
  

$$F_{L0} = \frac{1}{2} C_{L_0} \rho_w L D V^2$$
(2.3)

Where:

 $F_{D_0}$  = Stationary drag force [N]  $F_{L_0}$  = Stationary lift force [N]  $C_{D_0}$  = Stationary drag coefficient [-]  $C_{L_0}$  = Stationary lift coefficient [-]

The value of the stationary lift and drag coefficients are dependent on the Reynolds number. For the subcritical region  $(300 < Re < 3 \cdot 10^5)$  the value for  $C_D$  can be approximated at 1.1 [Blevins, 1990]. The value for  $C_L$  has a much bigger variation and lies between 0.2 and 0.6 [Blevins, 1990].

### 2.3.2 Moving cylinder

When the cylinder is allowed to move vertically, the lift and drag forces behave differently than for the stationary cylinder. This behaviour leads to the vibration of a cylinder due to VIV to be *self-limited*. What this means is that even when the shedding frequency is the same as the natural frequency of the cylinder, which would lead to resonance, the vibration amplitude is limited. An essential characteristic of VIV.

#### Lift force

As the cylinder oscillates due to the vortex induced forces the lift force influences the amplitude of vibration. The lift force can be split up into two parts, one in phase with the cylinder velocity and one in phase with the cylinder acceleration. To explain the implication of this first look at the lift coefficient in formula form:

$$C_{Lv} = \frac{F_{Lv}}{\frac{1}{2}\rho_w DLV^2}\sin(\Phi)$$
(2.4)

In this equation  $\Phi$  is the phase angle between the lift force and the cylinder velocity. This angle determines whether the amplitude of oscillation increases or decreases and thus governs the self-limiting behaviour. A lift force in phase with the velocity adds energy to the system while a lift force out of phase will damp the system. The same approach is valid for the force in phase with the acceleration.

#### Drag force

For a *stationary* cylinder, the drag force can be split up into both a mean drag force as well as an oscillating drag force. Figure 2.5 shows that although the force oscillates, there is always a mean component of the drag in line with the flow direction. The oscillating drag force is relatively small at around 10 % of the mean drag force, except at high frequencies. These frequencies are however rarely encountered, because very high flow speeds are required [Gopalkrishnan, 1993].

For an *oscillating* cylinder the big difference as opposed to a stationary cylinder lies in the mean drag force. For an oscillating cylinder a significant increase is noticed. This increase is related to vibration amplitude, and in some theories also to the vibration frequency. An example of an approach for the increase in mean drag force is e.g. [Blevins, 1990]:

$$\frac{C_D}{C_{D0}} = 1 + 2.1 \frac{y_{max}}{D} \tag{2.5}$$

### 2.4 VIV experiments

A couple of methods have been devised to analyse VIV, each with its own pros and cons. These are explained in this section. In all three methods the cylinder is assumed to be a rigid body.

#### 1. Forced vibration experiments

By forcing the cylinder to oscillate transverse to the flow, the user can determine the amplitude and frequency of vibration. By measuring the forces that the cylinder encounters, the lift and drag coefficients can be calculated. The advantage of this method is that lift and drag coefficients can be found for whatever combination between amplitude and frequency the user wants to investigate.

#### 2. 1 DOF free vibration experiments

In this case the cylinder is left to oscillate purely due to the flow, without external forcing. The degree of freedom is the cross-flow direction. The motion takes some time to reach its steady state vibration at which the cylinder oscillates freely. This type of experiment gives more insight into the physics of the vibration and the influence of flow speed on the actual behaviour of the cylinder.

#### 3. 2 DOF free vibration experiments

The 2 DOF experiment is a modification of the 1 DOF and allows the cylinder to move

in both cross-flow and in-line direction. This is the least used set-up, but it is the most realistic set-up. This experimental set-up gives insight to the in-line vibrations and the influence of the in-line on the cross-flow vibration. An effect of this was noted by Jauvtis and Williamson [2004]. They found that for low mass ratios the in-line motion starts to effect the transverse motion, which leads to an increase in vibration amplitude. Something that only could be found with 2 DOF experiments. The mass ratio is the ratio between cylinder mass and the mass that is brought in motion by the oscillation, also called the *added mass*.

## 2.5 Lock-in

Free vibration of an object occurs at its natural frequency. A logical consequence is that when the vortex shedding frequency is near the natural frequency, the vibration of the cylinder is at its maximum. Besides this another interesting phenomenon occurs when the shedding frequency approaches the natural frequency of the cylinder: when the shedding frequency is near the natural frequency, the Strouhal relation from Equation (2.2) is not valid anymore. Instead, the shedding frequency clamps onto the natural frequency for a range of flow velocities. This happens until a certain point is reached, where the shedding frequency releases and jumps back to the Strouhal relation. Figure 2.6 shows what this looks like graphically. The vertical axis is the ratio between the oscillation frequency and the natural frequency. The horizontal axis shows the flow velocity. The red line is the path the response frequency follows. The diagonal dashed line is the path of the Strouhal shedding frequency.

The velocity is expressed as a reduced velocity, which is a dimensionless parameter relating flow velocity to the natural frequency of the pipe:



$$V_r = \frac{V}{f_n D} \tag{2.6}$$

Figure 2.6: Idealised depiction of lock-in region

In the figure the range of velocities for correlation is between  $4 < V_r < 11$ , this is called the lock-in region. These boundaries aren't fixed, but these values are a good indication of the average lock-in region. The actual lock in region is dependent on the mass ratio of the pipe.

One can imagine that inside this range of velocities the amplitude of vibration reaches its largest value as it is oscillating near or at its natural frequency. One can even derive that the maximum amplitude is expected at  $V_r = 5$ , by equalling the natural frequency of the cylinder to the Strouhal frequency, using a Strouhal number of 0.2:

$$V_r = \frac{V}{f_n D} = \frac{D}{V \cdot St} \cdot \frac{V}{D} = \frac{1}{St} = 5$$
(2.7)

Another aspect of this phenomenon is that when the flow speed is in the lock-in region, the vortices start to correlate along the cylinder. This leads to a uniform direction of the force along the cylinder invoking an even larger amplitude of vibration.

### 2.6 Vibration amplitudes

The amplitudes of vibration in the lock-in region are not the same for each velocity. The amplitude curve splits into three distinct branches for low mass damping systems [Khalak and Williamson, 1999]. Mass-damping  $(m^*\zeta)$  is based on the pipeline properties. A pipeline in water is defined as a low mass damping system. A chimney that is surrounded by air is a high mass damping system, because air is much lighter than water. Figure 2.7 shows how the amplitudes vary over a range of reduced velocities ( $U^*$  in the figure). The switch between initial and upper branch is hysteric, while the switch between upper and lower branch is intermittent [Khalak and Williamson, 1997]. These switches are attributed to a change in phase between the exiting force and the motion of the cylinder [Govardhan and Williamson, 2000]. This phase difference also influences how cross-flow and in-line vibrations interact. To indicate this, Figure 2.8 shows the movement of cross sections over a range of reduced velocities. For the lock-in region the cross section moves in a figure 8 fashion, while outside of this region the figure 8 is less apparent.



Figure 2.7: Response amplitudes as function of  $V_r$  [Khalak and Williamson, 1999]



Figure 2.8: The relation between  $V_r$  and the movement of the cylinders cross-section in the x,y-plane

## 2.7 Conclusion

This chapter illustrates that VIV is a complex phenomenon. The vibration is dependent on a lot of different parameters, related to properties of both the pipe as well as to the flow. To simulate VIV, a model is needed that can capture the lock-in effect, the self-limiting behaviour and give correct amplitudes of vibration. The next chapter will demonstrate the 2D model that can do just this and is the basis for the Pipe-Wake model.

## Chapter 3

## VIV model

## 3.1 Introduction

To model the interaction between the pipeline and the vortices a couple of options are available. These can be classified in the following categories [Gabbai and Benaroya, 2004]:

• Semi-empirical methods

Phenomenological models that mimic the vortex shedding behavior. The models are based on experimental results, but give the freedom to be modified according to the users situation.

• Numerical methods

By using Computational Fluid Dynamics (CFD) programs, the flow and forces around a cylinder can be calculated numerically. These methods are however very time-consuming.

The model chosen in this research is a semi-empircal method called a wake-oscillator. The wake-oscillator is a second order differential equation that governs the oscillating behaviour of vortex shedding and relates the wake variable to the lift coefficient. By tuning some parameters in the model, the user can adjust the range of lock-in velocities and the amplitude of the lift coefficient [Facchinetti et al., 2003]. The biggest advantages of the wake-oscillater are that it allows for a lot of freedom to adjust it and it is much quicker in use than numerical methods. This chapter will explain how the wake-oscillator works for a 2D cylinder.

### 3.2 Van der Pol oscillator

This wake-oscillator model is based on the van der Pol oscillator. Van der Pol was an electrical engineer who developed an oscillator function to capture limit cycles in electrical circuits. Although he developed it for an entire different area of engineering, the characteristics of his limit cycles are similar to the vortex-induced vibrations that are modelled in this research. Both oscillations are self-limiting and both oscillations have a lock-in region. For this reason the van der Pol oscillator is a good basis for the VIV model. The wake oscillator used is a slightly modified van der Pol equation and reads as:

$$\ddot{q} + \epsilon \omega_o (q^2 - 1) \cdot \dot{q} + \omega_o^2 q = F(t) \tag{3.1}$$

Where:

$$q = \text{Wake variable [-]}$$
  

$$\epsilon = \text{Tuning parameter [-]}$$
  

$$\omega_o = \text{Natural frequency of the oscillator [rad s^{-1}]}$$
  

$$F(t) = \text{Forcing term}$$

The variable q is also called the wake-variable and is the variable that oscillates. The tuning parameter  $\epsilon$  and natural frequency  $\omega_o$  are chosen by the user. The parameter  $\epsilon$  influences the lock-in region and  $\omega_o$  describes the frequency of the oscillation in free vibration.

The self limiting behaviour of the oscillator is governed by the damping term:  $\epsilon \omega_o(q^2-1)\cdot \dot{q}$ . One can see that if q is small, the damping term is negative, leading to an increase in amplitude of q. However if q is large, the damping is positive and the amplitude decreases. For an unforced (F(t) = 0) oscillator the q-variable will find an equilibrium at q = 2.

To illustrate how the wake-oscillator works an example can be seen in Figure 3.1. This is a steady state vibration of the oscillator when unforced. To solve a second order differential equation two initial conditions are required, in this example the initial conditions are: q(0) = 0.01 and  $\dot{q}(0) = 0$ . The natural frequency is chosen at  $\omega_o = 2$  rad s<sup>-1</sup>. To show the influence of tuning parameter  $\epsilon$ , two values are used: 0.1 and 1.



Figure 3.1: The response for an unforced van der Pol oscillator

From these graphs a couple of conclusions can be drawn regarding the van der Pol oscillator:

- The wake-variable finds steady state at an amplitude of  $q = \pm 2$
- The larger  $\epsilon$ , the quicker steady state response is reached

Next step is to look at what happens when the wake-oscillator is coupled to a cylinder. The aim of this is to model VIV for a 2D case. The procedure is discussed in the following section.

### 3.3 Coupling the wake oscillator to a 2D cylinder

#### 3.3.1 Acceleration coupling

Relating Equation (3.1) to reality is done by choosing values for  $\epsilon$ ,  $\omega_o$  and the forcing term. This section will focus on the forcing term. The coupling of the wake-oscillator to the pipe is done through the forcing term. Facchinetti et al. [2003] found that the best results are found if the wake-oscillator is coupled to the acceleration of the cylinder in cross-flow direction. This is done using the following equation for the forcing term:

$$F(t) = \frac{A}{D} \cdot \frac{d^2 z}{dt^2}$$
(3.2)

Where:

A = Tuning parameter [-] D = Pipe diameter [m] z = Cross-flow displacement [m]

The tuning parameter A is a parameter chosen by the user. It influences the amplitude at which q oscillates. The acceleration coupling as in Equation (3.2) gives the best estimate in regard to the lock-in region and lift amplification of VIV [Facchinetti et al., 2003]. The full equation for the coupled wake-oscillator is:

$$\ddot{q} + \epsilon \omega_s (q^2 - 1) \cdot \dot{q} + \omega_s^2 q = \frac{A}{D} \cdot \frac{d^2 z}{dt^2}$$
(3.3)

In this equation the natural frequency of the oscillator is chosen to be the shedding frequency of the vortices. More information on the effect of this frequency and the tuning parameter A on the amplitude of vibration can be found in Appendix A.

#### 3.3.2 Equations of motion

With Equation (3.3) the manner in which vortex shedding takes place is described. What is left is coupling the equation of the wake oscillator to the equation of motion of a pipeline. Before looking at the coupling with a 3D pipeline, first the coupling with a 2D cylinder is described. Figure 3.2 shows the model of the cylinder that will be used for the coupling. It is a 2 degree of freedom system connected by a spring and dashpot in both directions. The equations of motion for the cylinder in Figure 3.2 are (assuming that the springs and dashpot coefficients are equal in y and z direction):

$$(m+m_a)\ddot{y} + c\dot{y} + ky = F_y(t)$$
  
(m+m\_a)\ddot{z} + c\dot{z} + kz = F\_z(t) (3.4)

Where:

m = Mass per unit length of the cylinder [kg]  $m_a = \text{Added mass per unit length of the cylinder [kg]}$   $c = \text{Dashpot value [kg s^{-1}]}$  $k = \text{Spring coefficient [N m^{-1}]}$ 



Figure 3.2: 2D wake-cylinder model

For convenience these equations are rewritten in terms of the natural frequency:

$$\ddot{y} + 2\zeta\omega_n \dot{y} + \omega_n^2 y = \frac{F_y(t)}{(m+m_a)}$$

$$\ddot{z} + 2\zeta\omega_n \dot{z} + \omega_n^2 z = \frac{F_z(t)}{(m+m_a)}$$
(3.5)

Where:

 $\zeta = \text{Damping ratio}$  $\omega_n = \text{Natural frequency of the pipe}$ 

Assuming that we have data on the pipe regarding the damping and spring coefficients, the remaining unknowns are the forces in y- and z-direction. Both y and z forces can be written in terms of dimensionless y and z coefficients. The force would then look as follows:

$$F_y = \frac{1}{2} \rho_w D L_{cyl} C_y V^2$$
  

$$F_z = \frac{1}{2} \rho_w D L_{cyl} C_z V^2$$
(3.6)

Where:

$$\rho_w = \text{Density of water } [\text{kg m}^{-3}]$$
  
 $L_{cyl} = \text{Cylinder length } [\text{m}]$ 
  
 $C_y = \text{Coefficient of lateral force } [-]$ 
  
 $C_z = \text{Coefficient of vertical force } [-]$ 

With the flow in positive y-direction the force in y-direction (in-line) will be governed by drag, while the force in z-direction (crossflow) is governed by lift. Both the drag and lift coefficients are available and are derived from experiments. To this extent it is necessary to rewrite the forces from Equation (3.6) in terms of lift and drag coefficients:  $C_D$  and  $C_L$ . Why  $C_D \neq C_y$  and  $C_L \neq C_z$  is demonstrated in Figure 3.3 (Note that in this figure the in-line direction is x and the crossflow direction is y). In this figure it can be seen that, because of the constant movement of the cylinder, the flow speed relative to the cylinder differs from the absolute flow speed of which the direction is unchanged over time. The lift and drag forces are always perpendicular and in-line to the *relative* flow speed.



Figure 3.3: Force decomposition [Ogink and Metrikine, 2010]

#### 3.3.3 Force decomposition

The first step in relating the relative motions to the absolute axis-system is defining the relative flow velocities. Assuming that the direction of the flow velocity V stays unchanged and in the direction illustrated in Figures 3.2 and 3.3, the relative velocities are:

$$U_{y} = V - \frac{dy}{dt}$$

$$U_{z} = -\frac{dz}{dt}$$

$$U = \sqrt{U_{y}^{2} + U_{z}^{2}}$$
(3.7)

The angle between the decomposed relative velocity  $(U_{x,y})$  and actual relative velocity U can be represented as:

$$\sin(\beta) = \frac{U_z}{U}$$

$$\cos(\beta) = \frac{U_y}{U}$$
(3.8)

Using these properties the following relation can be derived (see Figure 3.3 for clarification)

$$F_y = F_D \cos(\beta) - F_L \sin(\beta)$$
  

$$F_z = F_D \sin(\beta) + F_L \cos(\beta)$$
(3.9)

The Equations in (3.9) can be written in terms of coefficients, similar to (3.6):

$$\frac{1}{2}\rho_{w}DLC_{y}V^{2} = \frac{1}{2}\rho_{w}DLC_{D}U^{2}\cos(\beta) - \frac{1}{2}\rho_{w}DLC_{L}U^{2}\sin(\beta)$$

$$\frac{1}{2}\rho_{w}DLC_{z}V^{2} = \frac{1}{2}\rho_{w}DLC_{D}U^{2}\sin(\beta) + \frac{1}{2}\rho_{w}DLC_{L}U^{2}\cos(\beta)$$
(3.10)

Shuffling these equations around leads to the following relation:

$$C_{x} = (C_{D}\cos(\beta) - C_{L}\sin(\beta)) \cdot \frac{U^{2}}{V^{2}} = (C_{D}U_{x} - C_{L}U_{z}) \cdot \frac{U}{V^{2}}$$

$$C_{y} = (C_{D}\sin(\beta) + C_{L}\cos(\beta)) \cdot \frac{U^{2}}{V^{2}} = (C_{D}U_{y} + C_{L}U_{y}) \cdot \frac{U}{V^{2}}$$
(3.11)

Using this derivation for the force decomposition the equations of motion in Equation (3.5) can be written as follows:

$$\ddot{y} + 2\zeta\omega_n \dot{y} + \omega_n^2 y = \frac{\rho_w DL}{2 \cdot (m + m_a)} (C_D U_y - C_L U_z) U$$

$$\ddot{z} + 2\zeta\omega_n \dot{z} + \omega_n^2 z = \frac{\rho_w DL}{2 \cdot (m + m_a)} (C_D U_z + C_L U_y) U$$
(3.12)

This form of the equations of motion can be used to couple to the wake-oscillator. The only unknowns except for the input parameters are:  $C_D$  and  $C_L$ . The next section explains how these parameters are defined.

#### **3.3.4** Coupling equations of motion to the wake oscillator function

With Equations (3.3) and (3.12) the tools are at hand to describe a cylinder undergoing VIV. The only thing missing is the link between the equations. This link lies in a direct linear relation between q and  $C_L$  [Facchinetti et al., 2003], namely:

$$C_L = \frac{C_{L0}}{2}q\tag{3.13}$$

Where

 $C_{L0} =$  Lift coefficient from a stationary cylinder

The drag coefficient is also chosen to be the drag coefficient for a stationary cylinder,  $C_D = C_{D_0}$ . The values for these stationary coefficients can be determined experimentally.

In summary the set of equations that needs to be solved is a set of 3 second order differential equations. One equations for both the y and z direction, both coupled to the wake oscillator equation:

$$\ddot{y} + 2\zeta\omega_n \dot{y} + \omega_n^2 y = \frac{\rho_w DL}{2 \cdot (m + m_a)} (C_{D0}U_y - \frac{C_{L0}}{2}q \cdot U_z)U$$
$$\ddot{z} + 2\zeta\omega_n \dot{z} + \omega_n^2 z = \frac{\rho_w DL}{2 \cdot (m + m_a)} (C_{D0}U_z + \frac{C_{L0}}{2}q \cdot U_y)U$$
(3.14)
$$\ddot{q} + \epsilon\omega_s (q^2 - 1) \cdot \dot{q} + \omega_s^2 q = \frac{A}{D} \cdot \frac{d^2 z}{dt^2}$$

#### 3.3.5 Variable values for use of the model

Most of the parameters in Equation (3.14) are related to the set-up that needs to be investigated, e.g. mass, damping, flow speed. There are also a couple of variables however that are not related to this. Table 3.1 shows these variables and gives the values that are used in the rest of this research. The values are the same values found in experimental

Table 3.1: Constant model parameters

Variable	Value
$C_{D0}$	1.1856
$C_{L0}$	0.3842
St	0.1932

research done by Gopalkrishnan [1993] for a stationary cylinder. The paper by Ogink and Metrikine [2010] and Mina [2013] uses these values as well, because they give accurate results.

The remaining parameter that can not be chosen by the user is the added mass  $(m_a)$ . This is the mass of the water that is brought in motion by the oscillating cylinder. The expression for added mass per unit length is dependent on the area of the cross section of the pipe and the potential added mass coefficient:

$$m_a = C_a \frac{\pi D^2}{4} \rho_w \tag{3.15}$$

Where:

 $C_a$  = Potential coefficient for added mass = 1

Using the value of added mass, the mass-ratio can be calculated. This is done by dividing the cylinder mass by the added mass. The equation below demonstrates this:

$$m^* = \frac{m}{C_a \frac{\pi D^2}{A} \rho_w} = \frac{m}{m_a} \tag{3.16}$$

## **3.4** Determining A and $\epsilon$

The freedom of the wake oscillator lies in the tuning parameters A and  $\epsilon$ . To decide on what values to use for these parameters a comparison to experiments has to be done. A set of parameters needs to be found that approximates results of a real cylinder of which accurate measurements are known. For this report the results of experiments done on cylinders with variable mass and damping are used [Blevins and Coughran, 2009]. These experiments have been performed on cylinders with 2 degrees of freedom, a more realistic setting than the more commonly seen experiments with 1 degree of freedom.

In section 2.6 is described how there are roughly two distinct stages of vibration: the lower branch and the upper branch. An ideal set of parameters would describe both branches correctly and in addition to this they would capture the lock-in region accurately. Capturing both branches is not possible with a single set of parameters [Ogink and Metrikine, 2010]. For this reason a set of parameters is found that averages between the upper and lower branch in addition to capturing the lock-in region.

For very low mass ratios the behaviour of the vibration changes drastically. Therefor distinction is made between two regions, with the border at a mass ratio of 1.57. This value is chosen because in Figure 3.4a this is the mass ratio at which the difference is clear. This border is not fixed, and it is up to the user to judge which regime is present. For pipelines the mass ratio lies at around 2.

#### 3.4.1 Mass ratio > 1.57

The values used for the comparison are shown in Table 3.2. These parameters correspond to the parameters used in the experiments for the case of a mass ratio of 5.01. The reason the mass ratio in Table 3.2 is different, is because in the experiments a different expression is used for the mass ratio then is used in the model. The expression used in the experiments is  $m/(\rho_w D^2)$  instead of  $m/(\rho_w (\pi/4)D^2)$ .

Table 3.2: Model parameters for comparison with experiments

Parameter	Value	Unit
$\zeta$	0.0197	-
$\omega_n$	7.559	$\rm rads^{-1}$
$m^*$	5.3789	-
$ ho_{water}$	1000	${\rm kgm^{-3}}$

The best fit is found for the following tuning parameters: A = 10 and  $\epsilon = 0.07$ . For other mass ratios, different tuning parameters would have been obtained. The red line in Figure 3.4a shows graphically how the model compares with experiments. Figure 3.4b shows both the amplitude of vibration for different reduced velocities as well as the ratio between actual vibration frequency and natural frequency. The red line in the left graph corresponds to the blue line in the right graph.

**Note:** The vibration amplitudes are made dimensionless by dividing by the diameter of the pipe. From now on if spoken of amplitudes it will always be the dimensionless amplitude unless stated otherwise.





(a) Comparison of vibration amplitudes between model and experiments

(b)  $\frac{y}{D}$  and  $\frac{\omega_r}{\omega_n}$  in relation to reduced velocity

Figure 3.4: Model response for A = 10 and  $\epsilon = 0.07$ 

A couple of remarks have to be made regarding the results:

• The model locks-in at a reduced velocity that is slightly lower than is seen in the experiments. The only way to significantly change the start of the lock-in region in
the model is by altering the Strouhal number. The Strouhal number of 0.1932 is correct however, as can be seen in Figure 2.4 and will therefore not be changed.

• Each point in Figure 3.4b is modelled using initial conditions at 0, except for the wake variable for which the initial displacement was set to 0.01. The amplitude and frequency plotted in the graphs correspond to the steady state regime.

#### 3.4.2 Mass ratio < 1.57

The line corresponding to a mass ratio of 1.57 in Figure 3.4a (the black line with crosses) shows very different behaviour to the plots for higher mass ratios. Therefore, for low mass ratios, other parameters are needed. Because the amplitudes for this mass ratio tend to increase over the entire range of reduced velocities it is difficult to judge where the entrainment band ends. To determine the width of the lock-in region Equation (3.17) is used [Blevins and Coughran, 2009]. The same parameters as in Table 3.1 are used, except of course for the mass ratio, which is taken at 2.0185 in the model (and thus 1.57 in the Blevins and Coughran [2009] approach).

$$\frac{V_{r,max}}{V_{r,min}} = 0.25 \cdot \sqrt{\frac{m^*}{m^* - 1.4}} \tag{3.17}$$

The boundaries of  $V_r$  are taken where the amplitude reaches 0.4D. Using a mass ratio of 1.57 this leads to a lower boundary of  $V_r = 4.5$ , taken from the experimental results. Using Equation (3.17) the upper bound is found at 14.76D.

The best fit is found using A = 19 and  $\epsilon = 0.3$ . The blue line in Figure 3.5a shows the comparison between the model for low mass ratios and the experimental results. This line corresponds to the blue line in Figure 3.5b.





(a) Comparison of vibration amplitudes between model and experiments

(b)  $\frac{y}{D}$  and  $\frac{\omega_r}{\omega_n}$  in relation to reduced velocity

Figure 3.5: Model response for A = 19 and  $\epsilon = 0.3$ 

# 3.5 Conclusion

This chapter has shown how the wake-oscillator works and how it is related to the vortex shedding phenomenon for a 2D cylinder. The model manages to capture the lock-in region. Due to the way damping is incorporated it also governs the self-limiting behaviour of the vibration. By choosing the right tuning parameters the wake-oscillator can be tuned to amplitudes that are measured in experiments. In this case it is tuned to the crossflow amplitudes as measured in 2DOF experiments by Blevins and Coughran [2009]. The next step is to move from a 2D rigid cylinder to a 3D flexible pipeline structure. This is explained in the next chapter.

# Chapter 4

# Pipe-Wake model

### 4.1 Introduction

In Chapter 3 the coupling between a 2D cylinder and a wake oscillator is demonstrated. This chapter will describe how the model is expanded from 2D to a 3D pipeline model. There are a couple of commercial models available that can model a 3D pipeline that is excited by VIV. These models can be divided in roughly 4 categories: frequency domain vs. time-domain and linear vs. non-linear. The Pipe-Wake model that is described in this chapter is a non-linear, time-domain model. The model is programmed in MatLab, a numerical computing program often used in the engineering discipline.

This chapter will discuss how the Pipe-Wake model is built and show what the possibilities of the model are. In addition to this a comparison shall be made with existing pipeline/riser models.

# 4.2 Commercial pipeline models

There are quite some commercial VIV models available, each using its own method to calculate the responses. Where most models differ from the Pipe-Wake model, is that they focus on riser systems instead of pipelines. The implication of this is that soil is not taken into account. To include soil often some nifty programming is needed. To see where the Pipe-Wake model fits between its commercial counterparts, first the most used riser/pipeline models will be discussed:

• VIVANA

VIVANA is a method that solves the response for the frequency domain. For each mode that is excited a balance has to be found between excitation and damping along the length of the riser. Where the amplitude is low, near the support, the lift coefficient is positive (excitation zone) and where the amplitude is high the coefficient is negative (damping zone). An iteration procedure is used to find the balance between where along the length of the riser excitation is present and where along the riser it is damped. This is done for each frequency that participates in the vibration. When convergence is reached the steady state response is calculated by summation of all the participating frequencies [Koushan, 2009].

• SHEAR7

The response calculated with SHEAR7 is in the frequency domain as well. It is based on the same principle as VIVANA.

• Orcina wake oscillator

This is one of the only commercial software packages that uses a wake-oscillator similar to the one in this report.

The most popular VIV models use iteration to find the distribution of the lift and damping over the length of the riser. The wake-oscillator is rarely used, but holds the advantage that no iteration is needed to calculate the results for each participating frequency.

### 4.3 Pipe model

The fundamental part of the model is the pipe itself. The starting point of the Pipe-Wake model is a pipe without soil; a pure pipeline model. In the following sections the soil and gap influence will be added to create the complete Pipe-Wake model.

#### 4.3.1 Euler-Bernoulli beam

A pipe can be seen as a long slender beam influenced by lateral forces only. An Euler-Bernoulli beam is chosen as the model. An important assumption that is made with this model is that no deformation of the cross section takes place and that the cross-section is always normal to the axis of the beam. This is valid in cases where the beam is very long in relation to the cross-section, which is the case for a pipeline. The axis convention used for the Pipe-Wake model is shown in Figure 4.1. It is also assumed that the pipe is round, thus the structural properties are the same for both y- and z-direction. Equation (4.1) shows the analytical equations for a linear-elastic Euler-Bernoulli beam under these assumptions.



Figure 4.1: Axis system for Wake-Pipe model

$$EI\frac{\partial^4 y(x,t)}{\partial x^4} + c\frac{\partial y(x,t)}{\partial t} + \rho A\frac{\partial^2 y(x,t)}{\partial t^2} - T\frac{\partial y(x,t)}{\partial x^2} = F_y(x,t)$$

$$EI\frac{\partial^4 z(x,t)}{\partial x^4} + c\frac{\partial z(x,t)}{\partial t} + \rho A\frac{\partial^2 z(x,t)}{\partial t^2} - T\frac{\partial z(x,t)}{\partial x^2} = F_z(x,t)$$
(4.1)

Where:

A =Area of cross section of the pipe  $[m^2]$ 

E = Youngs modulus of steel [Pa]

I =Moment of inertia  $[m^4]$ 

T =Tension force [N]

#### 4.3.2 Finite Element Method

To model the properties of an Euler-Bernoulli beam, the finite element method (FEM) is applied. In the finite element method the beam is split up into small discrete elements, each having stiffness and mass properties, as if it were a small Euler-Bernoulli beam. These elements are connected through *nodes*. The external forces are applied to these nodes. At the extremities of the beam the movement is governed by the *boundary conditions*. The boundary conditions limit the degrees of freedom at these locations, e.g. vertical, lateral and/or horizontal movement or rotation of the pipe. The benefit of the finite element approach is that it is relatively easy to modify the model. Forces and moments can easily be changed and are simply applied to each node.

Figure 4.2 gives an impression of what a finite element beam looks like. It can be seen that each element has 12 degrees of freedom, 6 at each node of which 3 are displacements in x, y and z and 3 are rotations around the same axes. The boundary conditions in this



Figure 4.2: Finite Element Model used for the wake-pipe coupling

case are simple supports, where the right support can move in x-direction.

The finite element approach relates the element's stiffness and mass properties to each degree of freedom in which it can move. It does this using element stiffness and mass matrices, which can be found in Appendix B.1.1. Using this matrix approach Equation 4.1 can be rewritten as:

$$[M]\vec{\ddot{u}} + [K]\vec{u} = F(t) \tag{4.2}$$

Where :

 $\vec{u}$  = Vector of the nodal displacements in each degree of freedom per element [m]

[M] = Element mass matrix

[K] = Element stiffness matrix

By assembling the element stiffness and mass matrices, a global stiffness and mass matrix can be made. The result of this is an equation similar to Equation (4.2), but valid for the entire beam instead of for a single element. Appendix B.1.2 gives a more detailed explaination on the creation of the global matrices.

When the global matrices are known the boundary conditions can be implemented on the model. The boundary conditions are constraints on the movement of certain nodes of the model, most often in the extremeties. By removing each row and column in the global matrix related to the degree of freedom which is fixed the boundary conditions are implemented. Appendix B.1.3 provides more detail on this process.

#### 4.3.3 Static deflection

Using the finite element method as described above results in the mass and stiffness properties of a *straight* pipe. For small deformations around its equilibrium this approach is valid. When the beam is bend severely the stiffness properties of the beam change. For a pipeline this is can occur due to its own weight and the drag by the flow.

Implementing the effects of static deformation in the model requires a non-linear static analysis. The reason for this is that as the deflection gets larger, the pipe will develop internal forces resisting deformation which get larger when the deformation increases [Reddy, 2004]. The linear approach does not take this into account. In addition to this, the stiffness matrix after deformation is changed. This effects the natural frequencies of the pipe and can lead to a discrepancy between natural frequencies in z-direction as opposed to y-direction. Figure 4.3 illustrates a pipe that is deformed due to its own weight, the red line indicates how the span would look if the beam were rigid and the black line shows a sagged pipe.



Figure 4.3: Sag in a free span [Nielsen et al., 2002]

For the calculation of the static deflection the principle of virtual displacements is used. The principle states that if a body is in equilibrium, the total virtual work done by actual internal as well as external forces in moving through their respective virtual displacements is zero [Reddy, 2004]. The external forces in this case are the self weight, buoyancy and constant drag forces. The internal forces are taken from stress-strain relationships of the material.

The static forces in vertical direction are calculated by:

$$F_{z,s} = -F_{pipe} - F_{content} + F_{buoyancy}$$
  

$$F_{z,s} = -\rho_p A_p g - \rho_c A_i g + \rho_w A_o g$$
(4.3)

Where:

$$\begin{split} \rho_p &= \text{Density of the pipe} \\ A_p &= \text{Area of the pipe} \\ \rho_c &= \text{Density of the content} \\ A_i &= \text{Area of the inside of the pipe} \\ A_o &= \text{Total area of the pipe} \\ g &= \text{Gravitational acceleration} = 9.81 \text{m s}^{-1} \end{split}$$

The constant drag is calculated using the Morison equation:

$$F_D = \frac{1}{2}\rho_w DC_D V^2 \tag{4.4}$$

To find the new static position of the beam, iterations are necessary to converge to the final position. Use is made of the Newton-Raphson method. This procedure is as follows, with subscripts indicating the iteration number:

- 1. Define static force on each node in each DOF of the pipe:  $\vec{F}$
- 2. Define displacements of each node in each DOF of the pipe:  $\vec{U}$  (initial displacements are all 0)
- 3. Calculate  $[K(U_0)]$  (Stiffness matrix) and  $[T(U_0)]$  (Tangent stiffness matrix), using principle of virtual displacements. Appendix F elaborates on this procedure.

- 4. Calculate Residual force:  $\vec{R}_1 = F [K(U_0)]\vec{U}_0$
- 5. Calculate increment of displacements:  $\Delta U = [T(\vec{U}_0)]^{-1} \vec{R}_1$
- 6. Calculate new displacements:  $\vec{U}_1 = \vec{U}_0 + \Delta \vec{U}$
- 7. Calculate relative error:  $\frac{||R_1||}{||F||}$
- 8. Repeat step 3 7 until the error is lower than a predefined threshold,  $10^{-6}$  used in this case

This procedure has to be performed three times; for pre-tensioning, self-weight and drag. The split between the three cases to correctly incorporate the soil model. Figure 4.4 illustrates the procedure.



Figure 4.4: Three step static deformation procedure

For the pre-tension, the right support is free to move in x-direction. This results in slightly lengthening the pipe and changing the stiffness properties in x-direction.

After applying the tension, the right support is fixed in that position and the gravity force is added. This leads to a sag in the pipeline. In the presence of soil, this also leads to a deformation of the soil springs and thus the vertical soil forces.

In the last step, the constant drag is added. The soil forces from the previous calculation are used for the friction model, which will be discussed later in this chapter.

When convergence has been reached the vector  $\vec{U}$  represents the final displacements and [T] corresponds to the final stiffness matrix of the deformed system. The stiffness of the pipe is linearized around its static deflection. What this means is that the tangent stiffness [T], does not change during the dynamic analysis. Appendix E.2 shows the effect static deflection can have on a pipeline.

#### 4.3.4 Modal analysis

A structural model of the pipeline is now in place using finite elements. To solve the dynamic response of the beam use is made of *modal analysis*. The essence of modal

analysis is that the response of the beam is a summation of its eigenvectors, weighted by an unknown time-function [Spijkers et al., 2005]. In this section the concept of modal analysis is explained. Full mathematical background is found in Appendix C

The global mass and stiffness matrices are known from the finite element analysis and static deflection calculation. With this information it is possible to find the eigenvectors of the system. Using the equation of motion for free vibration the mode shapes and natural frequencies can be calculated, see Appendix C.2 for more detailed explaination on this.

Each eigenvector corresponds to a mode shape and to a natural frequency of the beam. Figure 4.5 illustrates the first 4 mode shapes for a simply supported Euler-Bernoulli beam. Note that for beams with equal stiffness properties in vertical and lateral direction, the mode shapes in both directions are the same as well as the natural frequencies in both directions. Note that Figure 4.5 only shows the modes for one of the two directions.



Figure 4.5: The first 4 mode shapes for a simply supported beam

A vibration of this beam will always be a superposition of a number of these modes, each with its on weighing factor. The equation of motion for such a system is:

$$[M^*]\vec{\dot{r}} + [C^*]\vec{\dot{r}} + [K^*]\vec{r} = F_{modal}$$
(4.5)

Where:

 $[M^*]$  = Modal mass matrix, see Appendix C.3 for explanation on modal matrices

 $[C^*] = Modal damping matrix$ 

 $[K^*] = Modal stiffness matrix$ 

 $\vec{r}$  = Weighing factor

It is this weighing factor that is the unknown which is calculated in the time-domain. By basically multiplying the weighing factor with its corresponding mode shape and summating the results gives the position of the beam in Cartesian coordinates. The more modes that are included in the calculation, the more accurate the response. Note however, that more modes also equals more computing time and they are not always necessary. The exact amount can be found by iteration. If adding more modes doesn't change the vibration, this means convergence has been reached.

# 4.3.5 System of differential equations

To obtain the vibration of the pipe the modal equation of motion, Equation (4.5) has to be coupled to the wake oscillator. In Section 3.3 is explained how the equations are coupled for a 2D system. For extension to a 3D pipe a wake-oscillator is coupled to each node of the finite element beam. If for instance a beam has been defined with 15 nodes, this means there are 15 wake-oscillator equations that have to be solved. The input for each wake-oscillator is the shedding frequency and the acceleration in z-direction for that particular node. For the rest the procedure is the same as in the 2D case.

The full set of differential equations now consists of the modal equations of motion for the beam, which size depends on the number of modes taken into account and a wakeoscillator for each node in the beam. Appendix C.4 goes into more detail of how these equations are set up. The differential equations can then be solved using MatLab's ODE45 solver. This solver is based on the 4th order Runge-Kutta method, a numerical method for solving differential equations.

All the tools are now at hand to calculate the vortex-induced vibrations on a pipeline. The model is still simple and doesn't take any external effects like soil into account. It is valid however for e.g. a riser or a pipe suspended between two points. That is why the model will be verified using experimental results for cases like these. This is done in the next section. Note that for both a riser and a pipeline the self-weight and buoyancy effects will still have to be added.

# 4.3.6 Model verification

The model has been verified by comparing its results to the results of experiments done on a riser. The goal of these experiments was to see how numerical models for VIV compared to reality. The experiments are therefore perfect for the validation of the pipe model.

## Riser vibration in flume

Experiments have been carried out at the Delta Flume in Delft on VIV of a hanging riser [Chaplin et al., 2005a]. Figure 4.6 shows the experimental set up and Table 4.1 gives the riser specifics. Four cases have been examined. The input variables for each case were incident velocity, applied only to the bottom 45% of the riser, and top tension, the tension applied to the top of the riser.

Figures 4.7, 4.8, 4.9 and 4.10 show comparison between measurements and model prediction. The top picture in each case displays the measured in-line and cross-flow vibration envelopes for the steady state response. A vibration envelope is a set of snapshots of the pipeline positions during an oscillation and indicates the maximum and minumum displacements of the pipe. The figure below, in colour, displays the model prediction of the envelope for the same input parameters. For more information on how these parameters are implemented in the model and for extra information on the results, see Appendix D.1.



Figure 4.6: Labratory set up of the riser [Chaplin et al., 2005b]

Table 4.1: Parameters for model validation	n
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Riser properties			Model properties		
Parameter	Value	Unit	Parameter	Value	Unit
Length	13.12	m	A	10	-
Diameter	0.028	m	$\epsilon$	0.07	-
EI	29.9	${ m N}{ m m}^2$	Nr. of modes	12	-
Damping ratio	0.0033	-	Nr. of elements	40	-
Mass ratio	3	-			

Case 1, V = 0.16  $\mathrm{m\,s^{-1}}$  and Top Tension = 405 N



Figure 4.7: Measured and predicted displacement for Case 1

Case 3,  $\mathrm{V}=0.31~\mathrm{m\,s^{-1}}$  and Top Tension = 457 N



Figure 4.8: Measured and predicted displacement for Case 3

Case 6, V = 0.6  $\,\mathrm{m\,s^{-1}}$  and Top Tension = 670 N



Figure 4.9: Measured and predicted displacement for Case 6

Case 9, V = 0.95 m  $\rm s^{-1}$  and Top Tension = 1002 N



Figure 4.10: Measured and predicted displacement for Case 9

#### Conclusion

The model shows very good behaviour for prediction of the cross-flow vibration. Both the amplitude of vibration as well as the mode shape correspond well to the measurements. For the in-line vibration the mode shape also corresponds well to measured mode shape. However, the equilibrium position in the in-line direction is not correct and is underestimated by approximately 50 % in the model. This can be attributed to the fact that the wake oscillator model is tuned to calculate cross-flow vibration. Because this is also the motion for which the dynamic interaction is most interesting, the underestimation of the in-line vibration is acceptable.

In comparison to other models the Pipe-model does not perform worse than other models of which the results are found in Appendix D.1.6. Most models tend to underestimate the in-line vibration, similar to the pipe model. The shape of the cross-flow vibration is also similar to most VIV models and is even superior to the numerical models (Norsk Hydro, USP, VIVIC and DeepFlow). In conclusion the Pipe model is a good model for estimating VIV response.

# 4.4 Soil model

Now that the pipe has been defined, the next step is to add soil to the equation. For a single free span soil will be present left and right from the span, see Figure 4.11. It is expected that at some distance from the middle of the span the soil effect will die out. This influences the choice at which distance the boundaries are chosen. The boundaries should be far enough to not influence the amplitude of vibration of the free span. Iteration is necessary to find this distance.



Figure 4.11: Illustration of freespan with soil on either side

In a dynamic system the soil will create a resistance force. The common method to transfer these forces to a beam is using springs as a soil model. The value of the spring force is dependent on the deformation of the springs. The further a spring is pressed down, the more resistance is created by the spring. The same approach generally holds for soil, the further one presses down in the soil, the more resistance is felt.

For the stiffness of the spring a difference is made between static and dynamic stiffness. The static stiffness is used for calculating the static deflection of the pipeline. These springs counteract the weight of the pipeline. After the pipe has reached its resting place, the dynamic springs are used to calculate oscillations around the equilibrium of the pipe. Reason for this is that before the pipe is installed, the soil is unconsolidated. This means the soil has not been compacted yet. Because of this the soil is relatively soft compared to the case when the soil is compacted.

While the static stiffness is not dependent on pipeline geometry, the dynamic stiffness is [DNV, 2006]. If one assumes non-stratified, homogeneous soil the DNV-RP-F105 code provides the Equations (4.6) and (4.7) for determining the dynamic spring stiffness in

vertical and lateral direction respectively. The values for both the static spring stiffness  $(K_{v,s})$  as well as the dynamic spring stiffness factors can be found in Table 4.2.

$$K_v = \frac{C_v}{1 - \nu} \left(\frac{2}{3}m^* + \frac{1}{3}\right)\sqrt{D} \quad [N\,\mathrm{m}^{-2}]$$
(4.6)

$$K_l = C_l(1+\nu) \left(\frac{2}{3}m^* + \frac{1}{3}\right) \sqrt{D} \quad [N \, \mathrm{m}^{-2}]$$
(4.7)

Where:

 $C_v = \text{Dynamic stiffness factor in vertical direction } [\text{N/m}^{5/2}]$ 

 $C_l = \text{Dynamic stiffness factor in lateral direction } [\text{N/m}^{5/2}]$ 

 $\nu =$ Poisson ratio, ratio between axial and transverse strains

= 0.5 for undrained soil

				Clay type	$\begin{pmatrix} C_V \\ (kN/m^{5/2}) \end{pmatrix}$	$C_L (kN/m^{5/2})$	$K_{V,S}$ $(kN/m/m)$
				Very soft	600	500	50-100
Sand type	$C_{V}$	C	Kus	Soft	1400	1200	160-260
Same OF C	$(kN/m^{5/2})$	$(kN/m^{5/2})$	(kN/m/m)	Firm	3000	2600	500-800
Loose	10500	9000	250	Stiff	4500	3900	1000-1600
Medium	14500	12500	530	Very stiff	11000	9500	2000-3000
Dense	21000	18000	1350	Hard	12000	10500	2600-4200
			_	-			

Table 4.2: Spring stiffness according to DNV-RP-F105

(b) Spring stiffness for clay

Another effect of the soil-pipe interaction is that energy is dissipated from the vibration by the soil. This is modelled using dashpots. Dashpots work slightly differently to springs and are not dependent on the displacement, but on the velocity in which the soil is pressed in. As long as the pipe is in contact with the soil, energy will be dissipated. The amount of energy is dependent on the value of the velocity. The DNV code does not give values for dashpots of different soil types. However, it specifies a way to relate the dashpot strength to the spring stiffness. This is done using the following equation:

$$c_i = 2\zeta \frac{k_i}{\omega} \tag{4.8}$$

Where:

 $c_i = \text{Dashpot at node i } [\text{Nsm}^{-1}]$ 

 $\zeta = \mbox{Damping ratio}$  at node i [-]

 $k_i = \text{Linear spring stiffness at node i } [\text{N}\,\text{m}^{-1}]$ 

 $\omega =$  Frequency of the mode considered [rad s<sup>-1</sup>]

There are still a couple of unknown variables in this equation, because except for the spring stiffness the other two parameters are unknown.

The damping ratio is a parameter that determines how quickly a vibration is damped out. For underdamped system it lies between 0 and 1, and for overdamped systems it is higher than 1. The damping ratio depends on the type of soil and on the shear strain. The shear strain is hard to calculate, as it is dependent on numerous parameters like: load history, soil structure, void ratio and many more. Measurements have been done on these parameters in relation to the damping ratio. Figure 4.12 shows this for both sand and

<sup>(</sup>a) Spring stiffness for sand



Figure 4.12: Damping ratios based on soil type and shear strain [Seed and Idriss, 1970]

clay [Seed and Idriss, 1970]. From these plots the average value for the factor in clayey soils is determined at 0.15 and in sandy soils at 0.08.

The manner in which the springs and dashpots are applied to the pipe can be varied and this section will elaborate on the various methods. To compare the soil models an example pipe is used with properties as in Table 4.3.

Parameter	Value	Unit	Parameter	Value	Unit
$L_{span}$	75	m	$\rho_p$	7850	${ m kg}{ m m}^{-3}$
D	0.5	m	$\rho_c$	800	${ m kg}{ m m}^{-3}$
$\mathbf{t}$	0.025	m	m*	2.14	-
$\mathbf{E}$	207	MPa	$K_{v,s}$	1300	${ m kg}{ m m}^{-1}$
$\zeta$	0.02	-	$C_l$	3900	$kN/m^{5/2}$
Т	50	kN	$C_v$	4500	$kN/m^{5/2}$

Table 4.3: Model properties

#### 4.4.1 Vertical soil models

Soil reaction forces can be split up into two direction, vertical and lateral. This section will discuss the methods available for vertical soil reaction.

#### **Static Analysis**

In vertical direction linear springs are used to model the static soil reaction. Linear springs are springs that relate the soil force linearly to the displacement. To incorporate this, the springs have to be added in the Newton-Raphson loop from section 4.3.3. Including soil stiffness requires adding a diagonal soil stiffness matrix to the global stiffness matrix in step 4 from Appendix B.1.2. For the rest all the steps in the Newton-Raphson stay the same. The soil stiffness matrix only affects the resistance in z-direction of each node and it can be found in Appendix E.3.1

#### Symmetric springs and dashpots

For this method the soil will provide the same resistance in both upward and downward motion of the pipe. Because the resistance is equal the springs are also said to be linearly dependent on displacement; *linear* soil behaviour. The spring stiffnesses used are the dynamic stiffnesses from Equations (4.7) and (4.6). Figure 4.13a illustrates what this setup looks like. Figure 4.13b gives the envelope of the cross-flow vibration for a pipe with the parameters from Table 4.3. From the figures it can be seen that the vibration at the shoulder is very small. Full results for the case of linear soil behaviour and the numerical implementation in the model are found in Appendix E.3.



(b) Envelope of the cross-flow response

Figure 4.13: Linear soil behaviour

#### Asymmetric springs and dashpots

One can imagine that symmetric springs as in the section above somehow simplify reality. Because the pipe is not buried, a strong case can be made for a situation where hardly any resistance would be felt if the pipe moved upwards. The only resistance is felt when the pipe is pushed into the soil. This assumption has also been investigated.

The soil springs for this model are asymmetric, or *nonlinear*. The springs have to decouple when the pipe moves above its initial position and behave as linear springs when below its initial position. For the dynamic response, the dynamic stiffnesses are used again. When the pipe looses contact with the soil, the dynamic springs do not apply any forde anymore. In this case there is still a constant force acting on the nodes, this force is equal to the static soil. When the nodes decouple, this force has to be counteracted by a force equal to the static force. The implementation of this force is illustrated in Figure 4.14a for nodes for which the static position is below the initial seabed (IB) and Figure 4.14c for nodes for which the nodes start above the seabed. The green line shows the soil force dependent on pipe position. The distinction between the two types of soil forces is made, because for nodes above the seabed, the springs only couple to the pipe when the pipe touches the bed. For nodes below the bed, the pipe starts in contact with the bed so the spring is coupled as soon as the pipe moves downwards. Figures 4.14b and 4.14d illustrate this difference. In these figures the springs are decoupled if the pipe above  $U_c$ .



Figure 4.14: Nonlinear soil stiffness

Figure 4.15b shows what the envelope of the cross-flow vibration looks like in that case. Appendix E.4 gives more detail on the results and numerical implementation of the non-linear springs.



Figure 4.15: Nonlinear soil behaviour

The amplitude of vibration for the case of nonlinear springs is approximately the same as for linear springs in the middle node. To that extent it does not matter which approach is chosen. There are however to significant differences to the previous case:

- The pipe moves upward at the shoulders. This is due to the fact that the pipe is not limited by soil in moving upwards. For a multispan this behaviour can pose a problem regarding the interaction. This upward motion could be present over the whole support and thus affect the vibration of its neighbouring span and vice-versa.
- The amount of participating modes is higher. In this case the soil is added as an external force, which means that the natural frequencies are calculated as if the free span ranged from boundary to boundary. In the case of linear springs, the soil is already incorporated into the natural frequencies, thus leading to a smaller effective span length and less frequencies. More frequencies equal more calculation time, so it is desirable to have the least participating modes as possible.

#### 4.4.2 Suction model

In the previous section it was allowed for nodes to lift up from the seabed. In pipelay projects it was found however that when a pipe was pulled up from the seabed, more force was needed than solely the weight of the pipe. Some kind of suction force seemed to be present. This force is especially apparent for clay soils. This is related to the structure of clay. Because clay particles are small and stacked tightly together, water has difficulty travelling through the soil. When pulling an object up from clay, the gap that is created between pipe and seabed can not easily be filled with water, which causes a suction force. For sand, the voids are bigger and it is easier to pull a pipe up from the soil because the gap between pipe and soil can be filled with water. In this chapter a suction model is proposed for clayey soils, based on experiments done in a laboratory. Besides adding a suction force to the model, it also proposes a method which is solved faster than the nonlinear soil model that is proposed in the section above.

#### Experiments on pull-out forces

The data for the suction model comes from experiments done on pipes that are pulled up from a trench in soft clay. These tests were performed for analysis of steel catenary risers, but the same principle applies for pipelines. During the pulling of the pipe the weight, pull-out time, consolidation time, pipe diameter, trench shape and uplift force were the variables. By changing these parameters and recording all the values a pattern is observed regarding the pull-out forces. The set-up is illustrated in Figure 4.16.



Figure 4.16: Test set up for suction test [Bridge, 2005]

Two different pipes were tested, both with a length over diameter ratio of 4, which ensures the end effects are negligible. The clay that is used is similar to clay in the deep waters of the Gulf of Mexico. The results of the tests give a force-displacement curve. What is noticed is that the shape of this curve is always similar, independent of pull-out force, velocity etc. Figure 4.17 shows an example of a force-displacement curve. There are 3 clear stages of the suction force. Firstly suction mobilisation, a linear dependence of force to the pull-out distance. Secondly the suction plateau, a constant suction force. Lastly the suction release, a negative linear dependence of the force related to breakout distance. When the breakout distance is reached, the suction force becomes 0.

The parameters that are needed to build the force-displacement curve are:  $\delta_B$  = breakout distance and  $F_{max}$  = Maximum uplift resistance force. Formulas to estimate these parameters have been made, based on the experimental results by Bridge [2005]

#### 4.4.3 Maximum uplift resistance force

The pipe-suction model proposed by Bridge [2005] is dependent on: pipe length, pipe diameter, undrained shear strength, hysterisis (cyclic loading), pull-out velocity, consol-



Figure 4.17: Force-displacement curve [Bridge, 2005]

idation time and load. Equation (4.9) calculates the maximum soil suction force. Each input parameter is discussed below.

$$F_{max} = k_c \cdot k_v \cdot k_t \cdot N \cdot L \cdot D \cdot S_u \tag{4.9}$$

Where:

 $k_c$  = Cyclic loading factor for Fmax  $k_v$  = Pull-out velocity factor for Fmax  $k_r$  = Consolidation time factor for Fmax N = Non-dimensional shape and depth factor D = Pipeline diameter  $S_u$  = Undrained shear strength

#### Cyclic loading factor: k<sub>c</sub>

The cyclic loading factor is taken from laboratory measurements and is set at 0.56. This has to do with the remoulding of the soil due to the repeated penetration.

#### Pull-out velocity factor: $k_v$

The equation for the pull-out velocity factor is:

$$k_v = k_f \left(\frac{V}{D}\right)^{n_F} \tag{4.10}$$

The factors  $k_f$  and  $n_f$  are empirically determined. For the clay types tested by Bridge [2005],  $k_f$  averages at about 1.08 and  $n_F$  at 0.2.

#### Consolidation time factor: k<sub>r</sub>

For cyclic loading the consolidation time factor is 1. This is because there is simply no time for the pipe to consolidate in the soil.

#### Non-dimensional shape and depth factor: N

The non-dimensional shape and depth factor is calculated according to a formula devised by Skempton. This formula is based on experimental and theoretical results [Bridge, 2005]:

$$N = min[5.14(1 + 0.23\sqrt{z/D}), 7.5]$$
(4.11)

Where:

z = Vertical penetration D = Pipe diameter

#### Undrained shear strength: $S_u$

This parameter is dependent on several factors: soil composition, void ratio, structure and loading history. It cannot be calculated but is a property of the soil in question and therefore a model input.

Solving Equation (4.9) gives the maximum soil suction force during pull-out. The other variable needed to calculate the force-displacement curve is the breakout distance, this is done next.

#### 4.4.4 Breakout distance

For the breakout distance an empirical equation has been set-up. The equation for the breakout distance is:

$$\delta_B = k_{DC} \cdot k_{DV} \cdot K_{DT} \cdot D \tag{4.12}$$

Where:

 $k_{DC}$  = Cyclic loading factor for breakout distance [-]  $k_{DV}$  = Pull out velocity factor for breakout distance [-]  $k_{DT}$  = Consolidation time factor for breakout distance [-]

#### $k_{DC}$ , cyclic loading factor

The cyclic loading factor is now set at 0.9 as taken from test data.

#### $k_{DV}$ , pull-out velocity factor

The equation for the pull-out velocity factor is:

$$k_{DV} = k_D \cdot V^{n_D} \tag{4.13}$$

The factors  $k_D$  and  $n_D$  are empirically determined. For the clay types tested by Bridge [2005],  $k_D$  averages at about 0.9 and  $n_D$  at 0.23.

#### $k_r$ , consolidation time factor

For cyclic loading the consolidation time factor is 1 for the same reason as stated above.

With the correct input parameters all the variables are known to calculate the soil suction force and the breakout distance of the pipe. Implementing the force-displacement curve in the model still requires some modification as is explained in the next section.

#### 4.4.5 Converting soil suction

Implementing the 3-stage suction force as described in Figure 4.17 leads to a very nonlinear system. MatLab needs a lot of time intervals to solve the system, which made the model inefficient. For this reason a solution was needed that would lead to faster solving of the equations. The conversion that is made is that from a nonlinear soil suction force to a linear soil suction force that cuts of at a certain distance above the static deflection. To decide what this distance is, use is made of the principle of work done. Work done is calculated by:

$$W = F \cdot s \tag{4.14}$$

Where:

$$W = \text{Work done [N m]}$$
$$F = \text{Force [N]}$$
$$s = \text{Distance [m]}$$

To illustrate how this can be applied to the case of soil suction, look at Figure 4.18. The left graph illustrates the force-displacement relation for a linear spring. The right graph is the soil suction graph from Figure 4.17. The area under the graph is the work done by the soil. The essence of this approach is that by comparing the work done by the linear spring to the potential work done by the soil suction in each node, the number of nodes that will disconnect during an upwards oscillation can be determined. The procedure is as follows:



Figure 4.18: Illustration of work done

- 1. Calculate steady state response for linear soil behaviour
- 2. Calculate maximum displacement  $(U_{max})$  and maximum velocity  $(V_{max})$  in each node
- 3. Using  $V_{max}$  calculate the  $F_{max}$  and  $\Delta_b$  and the work done needed for breakout.
- 4. Calculate the maximum work done in a soil spring, which is for  $z_s = U_{max}$ . The work done in that case is:  $1/2 \cdot K_z \cdot U_{max}^2$
- 5. Compare the work done by the spring to the work done needed for breakout. If the work done by the spring is *larger* than the work done by the possible suction, then it

is assumed the node will disconnect from the soil. This assumption is made because this means there is enough energy in the upward oscillation to totally overcome the suction force.

- 6. Calculate cut-off distance for disconnecting nodes. This is the distance for which the area under the graph for a linear spring is equal to the work done if suction were present. To do this the following equation has to be solved for  $z_s$ :  $1/2 \cdot K_z \cdot z_s^2 = W_{suction}$
- 7. Calculate new response with new suction nodes (nodes that may disconnect from soil after being deflected upwards a certain distance) together with fully connected nodes
- 8. Repeat steps until no new suction nodes appear from the analysis

To perform step 3, an assumption has to be made regarding the maximum upward velocity of the beam. This is that the velocity doesn't change over the distance over which the suction is active. Because this distance is very small relative to the total displacement (max of 10% of the maximum amplitude) this assumption is valid. Figure 4.19 illustrates this with time on the x-axis and displacement z and velocity  $\dot{z}$  on the y-axis.



Figure 4.19: Illustrative displacement and velocity plot of a node

In conclusion the soil suction model raises the threshold at which nodes would disconnect from the seabed. These *suction* nodes work the same way as the nonlinear nodes from the previous section. The difference now is that the threshold level at which the pipe disconnects is increased. In the previous section this threshold was expressed as  $\vec{U}_c$ . In this section the threshold is:  $\vec{U}_c + \vec{z}_s$  for nodes that disconnect. The nodes that do not disconnect, the linear soil spring approach is used.

#### 4.4.6 Results

Figure 4.20 shows the cross-flow vibration for the soil suction model. The undrained shear strength in this case is 1 kPa. The response is similar to the response for linear soil springs and the nodes at the shoulder do not disconnect like in the nonlinear springs case. To show how the 3 models compare, the pipe positions at the edge of the span are plot, see Figure 4.21. The red dotted line in this figure is the initial position of the pipe. The edge of the shoulder is at x = 50 m.

For linear soil springs the vibration above the shoulder is very small. For nonlinear soilsprings, nodes clearly disconnect at the shoulder and move upward freely. For the suction model the pipe positions over the shoulder is similar to the linear springs case. If one looks closely however, the nodes near the span do disconnect, with suction playing a part between 47 and 50 m.



Figure 4.20: Envelope of cross-flow response for suction model



Figure 4.21: Close up of pipe snapshots over the shoulder

The possible soil models for vertical soil resistance has been discussed in this section. Next is the soil resistance in lateral direction, for which different models are necessary.

#### 4.4.7 Lateral and axial soil resistance

There are basically two ways to model lateral soil and axial soil resistance: using springs and dashpots or using a Coulomb friction model. The latter is a model which allows an object to slide over a surface. For small displacements a linear-elastic approach using springs and dashpots is valid. For large displacements it is an oversimplification as the pipe will start to slide along the seabed. In the Pipe-Wake model a combination of the two is used. For the calculation of the static deflection a friction model is used, because the largest displacements in y-direction stem from the constant drag force. In the dynamic analysis the soil resistance is replaced by springs and dashpots, because the oscillating force is much smaller than the constant drag. This section will describe the approach used for both the static as well as the dynamic lateral soil resistance.

#### Static soil resistance approach

For the lateral direction a friction model is used. This friction model is based on the Coulomb model, which makes the resisting force as the object slides over a surface dependent on the direction of the movement and the weight of the object. The resisting force is only present if the external force is larger than a static friction force, Figure 4.22a shows the graph of how the friction force works. In terms of equation this looks like:

$$F_F = \begin{cases} -F_E, & \text{if } F_E < \mu_{stat} F_z \\ \mu_{dyn} F_z, & \text{otherwise} \end{cases}$$

Where:

 $F_F$  = Friction Force  $F_E$  = External Force  $F_z$  = Vertical Force  $\mu_{stat}$  = Static friction coefficient  $\mu_{dyn}$  = Dynamic friction coefficient



Figure 4.22: Friction models

This model, the stick-slip model, is nonlinear and because it is dependent on the direction the velocity of the pipe it is not possible to be implemented in the static calculation in its current form. To simplify the system first is looked at the flow velocity. In the static calculation it is assumed that there will be a constant drag present, which makes the deformation of the pipe dependant on the flow velocity as well as on the weight of the pipe as explained in section 4.3.3.

Because the oscillating forces aren't taken into account yet, the direction of the drag force is constant and in the direction of the flow. Knowing this allows for a simplification of the friction model. The new model that is introduced is taken from the DNV code for free spanning pipelines. The DNV code has based its model on experimental results, which are found in Verley and Lund [2005] and Verley and Sotberg [1994]. This model not only accounts for the friction, but also for the fact that if a pipe has sunk into the soil, it is harder to break out of its position, illustrated by Figure 4.23. The equations below describe the DNV model and Figure 4.22b illustrates the model.

$$F_F = \begin{cases} k_1 \cdot y, & \text{if } F_F < \mu F_z \\ \mu F_z + k_2 \cdot \left(y - \frac{\mu F_z}{k_1}\right), & \text{otherwise} \end{cases}$$

Where:

y = y-coordinate of pipe node

- $k_1 =$  Equivalent secant stiffness up to mobilisation of full friction
- $k_2 =$  Equivalent stiffness for deformations past mobilisation of full friction
- $\mu =$  Friction coefficient, 0.6 for sand and 0.2 for clay



Figure 4.23: Illustration of a pipeline in a trench [Bridge, 2005]

In this model no distinction is made between static and dynamic friction coefficient. The values for  $k_1$  are taken to be the same as the lateral dynamic stiffness. The values for  $k_2$  are dependent on soil type and pipe penetration. Table 4.4 shows the values for sand, dependent on dimensionless penetration. Equation 4.15 is valid for clay.

$$k_2 = 8.26 \cdot s_u \cdot \left(\frac{s_u}{D \cdot \gamma_{soil}}\right)^{-0.4} \cdot \frac{z}{D}^{1.3} \tag{4.15}$$

z/D	$k_2 \; [kN/m/m]$
0.00	0
0.25	19
0.35	28
0.50	44
1.00	105

Table 4.4: Values of  $k_2$  for sand

Where:

 $\gamma_{soil} = \text{submerged unit weight of soil } [\text{kg m}^{-3}]$ 

z =Penetration of the pipe [m]

 $k_2 =$  Equivalent stiffness for deformations past mobilisation of full friction [kN m<sup>-1</sup>]

 $\mu =$ Friction coefficient, 0.6 for sand and 0.2 for clay [-]

The Pipe-Wake model makes use of these equations to model the friction in both y- and x-direction. The big difference between the two directions is that for the axial direction the penetration plays no part and thus the term containing  $k_2$  is neglected. Also as the main deformation take place in the middle of the pipe, the axial friction force direction is different on both shoulders, see Figure 4.24.



Figure 4.24: Direction of axial friction force

For the model to work, first the vertical soil reaction forces are needed. That is why first the deflection due to self-weight is calculated before the constant drag is added. The soil reaction force in each node is equal to  $F_z$ . The friction model is then implemented between step 3 and 4 of the Newton-Raphson loop of section 4.3.3. Using the previous pipe position, the distance of each node in y-direction is known. With this both  $k_1 \cdot y \cdot L_{element}$ &  $\mu F_z$  can be calculated. If  $k_1 \cdot y \cdot L_{element} < \mu F_z$ , then a spring stiffness is added for that node equal to  $k_1$ . If however  $\mu F_z$  is smaller, then an external force is added in that node, equal to  $\mu F_z + k_2 \cdot \left(y - \frac{\mu F_z}{k_1}\right)$ . The same principle holds for the axial direction except the term containing  $k_2$  is neglected. This approach leads to quite some iterations of the Newton-Raphson loop. Especially when the flow speeds are high and the soil is softer, the friction model leads to a lot of iterations.

Figure 4.25 shows two examples of the static displacement in y-direction. The top graph is for a small flow velocity and thus low constant drag and the bottom graph is for a higher velocity. From the graphs can be concluded that for high velocities the pipe indeed slides over the seabed at the shoulders.

#### 4.4.8 Dynamic lateral soil reaction

For the dynamic soil reaction in y-direction a linear-elastic spring model is used. The values for the springs and dashpots are according to the DNV-RP-F105. In axial direction there



Figure 4.25: Static deflection for nonlinear friction model

is no soil resistance in the dynamic analysis. The reason for this is that the displacements in axial direction are negligible.

An important assumption that is done in the model is that the dynamic soil reactions are always present, also when the pipe loses contact with the soil. If it was allowed to loose contact with soil, the pipe is allowed to move freely in y-direction. When the pipe would return downward it will land in a different place then from where it took of. If one now looks at the elastic soil force expression:  $F_y = K_y \cdot y$ , it can be seen that the force is dependent on displacement from the static position. If the pipe lands far from this static position, the force pulling it back to the static position would be large and very sudden. This in turn leads to unrealistically big pipe accelerations in y-direction and as the yand z- directions are linked, it also leads to unrealistic z-displacements. To avoid this for lateral directions the pipe is always in contact with soil.

Figure 4.26 shows the in-line vibration for the example case with the suction model in place. The vibration at the shoulders is very small, this is because the oscillation in y-direction is relatively small as well.



Figure 4.26: In-line response for example case

# 4.5 Gap model

The last aspect to add to the model is the gap effect. Up to now the gap under the free span has been infinite. In reality the gap will not be infinite and the final pipeline model will look like Figure 4.27. This has a couple effects on the model. Firstly because of the presence of soil, there is a boundary layer above the seabed, which influences the flow speed near the seabed. Because of this a sheared current is present along the span, which affects correlation of vortex shedding along the pipe. Secondly the pipe can impact the seafloor if the gap is small enough.



Figure 4.27: Illustration of pipeline model

#### 4.5.1 Gap shape

The gap under a span is shaped by scour caused by flow along the seabed. The gap is dynamic and there is no exact description of what a gap looks like. The gap as shown in Figure 4.27 is only used as illustration, in the model a simplification of the gap has to be made. The approach taken in this research is that there are 2 parameters that effect the gap: slope angle and bottom depth,  $\alpha$  and  $H_{BG}$  respectively, see Figure 4.28. For sand, the slope angle can be taken as the angle of internal friction, for clay judgement of the user is needed for determining the angle.



Figure 4.28: Illustration of gap profile

#### 4.5.2 Flow speeds

The flow speed is dependent on height above the seabed. At the seabed the flow speed is 0 and can keep increasing for a couple of metres above the seabed [Soulsby, 1997]. The flow speed increases logarithmically according to the following relation, see Figure 4.29 for a illustration:

$$U(z) = \frac{u_*}{\kappa} \ln\left(\frac{z}{z_0}\right) \tag{4.16}$$

Where:

 $u_* =$  Friction velocity  $[m s^{-1}]$   $\kappa =$  Von Karmans constant = 0.40 z = Height above seabed [m] $z_0 =$  Seabed roughness length [m]

The friction velocity is calculated using the following equation:

$$u_* = \sqrt{C_{100} \cdot U_{100}^2} \tag{4.17}$$

Where:

 $C_{100} = \text{Drag coefficient dependent on seabed}$ 

 $U_{100} =$  Flow speed at 1 metre above the seabed

The flow speed at any given height can be calculated using Equation 4.16. Three inputs are needed for this:  $U_{100}$ ,  $C_{100}$  and  $z_0$ . The first parameter,  $U_{100}$ , is a user input that is dependent on the current conditions. The other two parameters are dependent on the seabed and can be taken from Table 4.5.

Table 4.5: Seabed parameters [Soulsby, 1997]

Table 7. Mean values of  $z_0$  and  $C_{100}$  for different bottom types

Bottom type	$z_0 (\mathrm{mm})$	C <sub>100</sub>
Mud	0.2	0.0022
Mud/sand	0.7	0.0030
Silt/sand	0.02	0.0016
Sand (unrippled)	0.4	0.0026
Sand (rippled)	6	0.0061
Sand/shell	0.3	0.0024
Sand/gravel	0.3	0.0024
Mud/sand/gravel	0.3	0.0024
Gravel	3	0.0047

Sources: see Soulsby (1983, Table 5.4).



Figure 4.29: Logarithmic current profile

In the model the input current speed is defined at  $H_B$  above the seabed, see Figure 4.28. Because the logarithmic input current speed is defined at 1m above the seabed, the current at  $H_B$  has to be related to a current speed at 1m above the seabed. The new current speed at 1m is calculated by Equation (4.16):

$$U_{100} = \frac{U_{H_B}}{\sqrt{C_{100}} \ln(\frac{H_B}{z_0})}$$
(4.18)

When  $U_{100}$  has been calculated, it can be used as the reference value to calculate the flow speeds along the entire span.

#### 4.5.3 Bottom impact

If the height of the gap is small, there is a chance that the pipe will impact the bottom and influence the steady state vibration. To take this effect into account the model has been modified such that when a node penetrates the bottom it will feel a soil resistance force. This force is only present when the bottom of the pipe is below the seabed. Suction and lateral soil force are neglected when the pipe impacts the soil.

# 4.6 Pipe-Wake model

The previous sections have described the various steps in setting up a realistic pipeline model, the Pipe-Wake model. Three versions have been presented, linear springs, nonlinear springs and a suction model. The version with the suction model is only valid for *clay soils*. For sand soils no suction data was available, so it is up to the user which soil model to use.

The Pipe-Wake model allows for various of pipe, soil and gap set-ups. It is also possible to have a sheared current acting on the model. Based on the inputs the model starts the procedure of calculating the response.

The outputs from the model are the modal displacements, modal velocities and the wakevariables in the time-domain. From these outputs a range of useful results can be given. For the example case that has repeatedly been used in this chapter, the most useful results are presented here using the suction model. In Appendix E.7 some examples is given for the Pipe-Wake model in combination with multispans.

#### 4.6.1 Nodal vibrations

The modal displacements can be transferred to nodal displacements, using the eigenmatrix. This process is explained in Appendix C. Figure 4.30 shows the resulting nodal displacements in cross-flow and in-line direction for the middle node in the freespan. This kind of figure shows whether steady state response is reached. The amplitude of vibration for each node can also be extracted from this plot.



Figure 4.30: Vibrations of the node halfway the free span

#### 4.6.2 Envelope of vibration

With all the nodal displacements at hand it is also possible to generate an envelope of the vibration. This means a snapshot of a pipeline position for each time-step. Figure 4.31 shows such an envelope for the steady state vibration of the pipe. In this case steady-state is defined as the last 25% of the results in the time-domain. Figure 4.30 confirms that indeed steady state is reached.



Figure 4.31: Envelope of vibration

#### 4.6.3 Lift and drag forces

In Chapter 3 is shown that the lift and drag forces are dependent on both the wakevariable q as well as on the velocity of the pipe in y- and z-directions. All these variables are calculated by the model. By inputting these results in the equations for the lift and drag forces, these forces can be reproduced and plotted. The envelope of these coefficients in the steady state regime is shown in Figure 4.32.



Figure 4.32: Envelope of lift and drag forces

#### 4.6.4 Frequency domain solution

From the time-domain results the frequency domain results can be calculated as well. This is done using a Fourier transform from a time-trace. In this case the time trace from the middle node, Figure 4.30, has been chosen for the transform. Again only the last 25% of the results are used to eliminate any transient effects.



Figure 4.33: Frequency domain

#### 4.6.5 Cross-section displacement

The last example of the output is a plot of the cross-section of the pipe. The plot in figure 4.34 is created by plotting the y-coordinate to the z-coordinate of the middle node over time. Because the steady state regime is used again, this time-trace stays the same for each oscillation.



Figure 4.34: Movement of the cross-section halfway the free span

# 4.7 Conclusion

A number of steps have been taken to arrive at the final Pipe-Wake model. These have been:

- 1. Coupling of the wake oscillator to a 2D cylinder model
- 2. Tuning the wake oscillator equation to 2DOF VIV experiments
- 3. A finite element model of an Euler-Bernoulli beam is created
- 4. The 2D cylinder equations are coupled to the FE model
- 5. The pipeline model is validated to experimental results
- 6. Three different soil models are created; linear, nonlinear and suction model
- 7. The bottom gap is added to the 3D pipeline model

The Pipe-Wake model offers the user a lot of freedom to play with the pipeline and environmental properties. It provides three soil models, which one is used is based on the soil type and user preferences. All three models show comparable behaviour in the span. For single spans, the vibration amplitudes in z- and y-direction are very similar for all three models, as well as the response frequencies, see Appendix E. The aspect that defines each model however is the behaviour of the soil at the shoulder, especially near the free span. For a single span this has little effect, but for a multispan case the soil behaviour in the support can be of influence. When the two spans in a multispan case are close together, the energy transfer between the spans travels over the support. In this case the choice of linear, nonlinear or suction model could change the way the spans vibrate.

Comparable results for a theoretical VIV model that incorporates any soil effects is not to be found. The same holds for any experimental results of the same case. This makes validating the Pipe-Wake a hard task. However both the riser/pipeline model and the soil suction model have been validated separately. Therefore it can be assumed that the Pipe-Wake model should give realistic results.

The next chapter will use the Pipe-Wake model to answer the research questions. A case-study will be used to illustrate effects of the support between two spans.

# Chapter 5

# Time-domain analysis of interacting free spans

# 5.1 Introduction

To investigate the effect of interacting free spans a test case is investigated. For the most relevant results a test case is chosen that reflects an existing set of free spans. The data for the pipeline and the free spans has been provided by INTECSEA and come from the *Liwan gas field* for which the design is done by Saipem.

The Liwan gas field is located in the South China Sea at 1500m waterdepth. Subsea wells are used which are connected to a central platform, which is built at approximately 200m waterdepth. The pipeline under investigation is a 79km long Mono-Ethylene Glycol (MEG) line running from the platform to the subsea distribution center. [Saipem et al., 2006]. MEG is used to reduce the freezing point in the pipeline and prevent blockage from hydrates.

# 5.2 Specifications of the free spans

The free spans are located at 200m waterdepth on a sandy seabed. There are two phases in the identification of free spans in a pipeline. Firstly the seabed is mapped by a seismic survey. Based on the results a pipe route is made and a FEM analysis is done of the pipe. During installation however there will always be discrepancies between the theoretical design of the pipe and the final position of the pipe. To investigate if no significant problems occur due to these small differences a complete survey is done of the pipeline after its been laid and the FEM model is updated according to the changes observed by the surveyor.

The free spans investigated in this thesis are interesting because in the preliminary design no problems were found regarding fatigue life of the spans. However, after the surveillance the support properties changed slightly and suddenly significant fatigue damage is predicted. Figure 5.1 shows the free spans before surveillance and Figure 5.2 shows the free spans after surveillance. Table 5.1 shows the environmental parameters and Table 5.2 the pipeline properties for the case study.

From the data provided it is unclear how the supports are modelled. In Figures 5.1 and 5.2 one can see distinct points where the soil makes connection to the pipe. For the case of one connection point or 'prop', Figure 5.2, the fatigue damage is much larger than for two props, Figure 5.1. The fatigue damage is indicated by the pink line in the graphs. A couple of remarks have to be made regarding the graphs:

• It is unclear what the criteria is for the props. It could be a small hill of sand or a rock lifting the pipe up. It could also be purely a visual feature, showing the corners of the hill. A third option is that Saipem used just two connection points to model the soil-pipe interaction.



Figure 5.1: Free spans before surveillance



Figure 5.2: Free spans after surveillance



Figure 5.3: First cross-flow vibration mode

- It could be the case that the pipe ended up landing on a rock, which means that in the model the support properties change from a distributed sand hill to a sharp rock.
- The mode shape for these spans in the case of one prop, Figure 5.3, suggests that a linear spring is used in the analysis, a coarse assumption especially if it were a rock.

Parameter	Value	Unit
$V_{max}$	1.6	${ m ms^{-1}}$
$C_v$	10500	$\mathrm{kN/m^{5/2}}$
$C_l$	9000	$\mathrm{kN/m}^{5/2}$
$K_{vs}$	250	$\mathrm{kN/m^{5/2}}$
$\zeta_{soil}$	0.08	-
$z_0$	0.4	$\rm mm$
$C_{100}$	0.0026	-

Table 5.1: Environmental parameters

Table 5.2: Pipeline parameters (As-Laid)

Parameter	Value	Unit	Parameter	Value	Unit
$L_{span1}$	28	m	Е	207	MPa
$H_{span1}$	0.86	m	$\zeta_m$	0.02	-
$L_{span2}$	26	m	Т	0	Ν
$H_{span2}$	0.84	m	$ ho_{pipe}$	7850	${ m kN}{ m m}^{-3}$
D	0.1683	m	$ ho_{content}$	0	${ m kN}{ m m}^{-3}$
$t_{wall}$	0.0127	m			

# 5.3 Research method

#### 5.3.1 Introduction

The free spans that are discussed in the above are investigated by the model. Because it is unsure how the support is modelled, a couple of cases will be run and investigated. These cases are dependent on: support length, soil properties and flow velocity.

The goal of the research is to answer the research question: To what extent do the support properties influence the vibration of a free spanning pipeline?. Three methods to describe the shoulder have been chosen to investigate its influence:

- 1. Distributed springs over the whole shoulder (DS)
- 2. Two contact nodes, one at either side of the shoulder (2N)
- 3. One contact node in the middle of the shoulder (1N)

The goal of this research is to find out which approach is the most realistic and what the consequences of each support type are on the dynamic interaction of free spanning pipelines.


Figure 5.4: Illustration of the set-up of the model

#### 5.3.2 Assumptions

A couple of assumptions have been made in order to perform this case study. These are listed below:

- 1. The soil response acts as linear springs. This assumption is based on the fact that the modal shape as seen in Figure 5.3 is made using some kind of linear spring.
- 2. The soil profile does not change in y-direction (in-line). The displacements in y-direction will be small, with a max of around  $4 \cdot D$ , which validates this assumption.
- 3. The initial position of the soil is horizontal and at the same level, except under the spans. Figure 5.4 shows what the multispan set-up looks like for the case study.

Based on these assumptions, the model can be used to calculate the response of the pipe.

#### 5.3.3 Cases

The cases that will be investigated using the model are described in this section. Note that for all the cases the behaviour is calculated for flow speeds ranging from  $0 \text{ m s}^{-1}$  up to  $2 \text{ m s}^{-1}$  using a dV of 0.05 m s<sup>-1</sup>. The maximum flow speed to occur once in 100 years for the free spans is 1.6 m s<sup>-1</sup> [Saipem et al., 2006].

#### Distributed springs (DS)

The distributed springs are located along the entire shoulder. From Figure 5.1 one can see that is around 3.5 m, but 3.6 m is chosen so it can neatly fit 18 elements. Linear springs are used to model the sand in the support, this is done because it is assumed that linear springs are used in the design of the pipe. Figure 5.5 illustrates this case.



Figure 5.5: Basic model for distributed springs

#### Two contact nodes case (2N)

The shoulder for this case is also 3.6 m. It is now modelled as two contact nodes, one at either side of the shoulder. This will leave a new free-span between the nodes, which will be very small. To prevent major static deformation in z-direction due to the reduced number of support nodes, the soil is modelled the same as in case 1 for the static analysis. In the dynamic analysis the soil is replaced by a spring at either side of the support. Figure 5.6 illustrates this case.



Figure 5.6: Basic model for 2 contact nodes

#### Single contact node (1N)

For the case of support reduction, one contact node is used to model the seabed. This node is chosen to be in the center of the support. For the sake of comparison, the span lengths have been kept the same as for the cases above. Figure 5.7 illustrates this case.



Figure 5.7: Basic model for 1 contact node

### 5.4 Results

#### 5.4.1 Relative Amplitudes

Firstly the relative amplitudes of the free spans in all cases will be discussed. The relative amplitude is the ratio between the maximum amplitude of the span in the multispan set-up and the amplitude of the middle node in the single-span set-up  $(A_{ms}/A_{ss})$ . The middle node has been chosen, because the shape of vibration of the single span is symmetric. For the multispan case the amplitudes along the freespan do not have to be symmetric, which is why the node with the maximum amplitude is used for comparison. Figure 5.8 shows the relative amplitudes for all three cases in all directions.

#### Remarks

The relative amplitudes of vibration give good indication if the multispan behaves very different from an isolated span, i.e. the amount of dynamic amplification due to its neighbouring span. Using Figure 5.8 the following remarks can be made regarding the vibration of the spans:

• The amplitudes for the 2N and DS case is approximately equal. This means the simplification of the support to the 2N case does not affect the amplitude of vibration.



Figure 5.8: Relative amplitudes of vibration

- In all three cases a peak is seen at around  $0.7 \text{ m s}^{-1}$ . It is clear that for the 1N case this peak is much larger than for the other cases.
- After the peak there is a difference between the 2N and 1N relative amplitudes, these differences get smaller as the flow speed increases.
- The relative amplitudes start to rise for higher flow speeds in all three cases. For very high flow speeds the rise is even (almost) vertical for span 1 in z-direction.

#### Conclusion

The first aspect that needs clarification is the peak at flow speeds of around  $0.7 \text{ m s}^{-1}$ . The peaks in the 2N and DS case can be put into perspective. Although the amplification is moderate with values between 2.5 and 3 for span 2, the absolute amplitudes are small and lie at 0.35 and 0.05 D for z- and y-direction respectively. Appendix G shows these absolute amplitudes. More interesting is the peak in the relative amplitudes for the 1N case, which in the largest case reaches 50. This amplification is related to the mode shape and natural frequency of the system.

Figure 5.9a shows the first two mode shapes for the DS case, which is very similar to the 2N case. Figure 5.9b shows the same for the 1N case. It is clear that there is a difference between the cases. The most apparent difference is in the first mode where the support clearly influences the shape of the mode for the 2N and DS case. In the second mode the mode shapes are similar for all cases, the only difference being that for the 1N the amplitudes of span 1 is slightly larger.

To say more about the influence of the mode shape on the vibration, the natural frequencies are also inspected. Because the static deflection is dependent on the flow speed,



Figure 5.9: First two mode shapes in z-direction for DS, 2N and 1N case for  $V=0.8 \text{ m s}^{-1}$ 

the natural frequency will also differ at each flow speed. For this reason the natural frequencies as listed in Table 5.3 are the frequencies for 4 different flow speeds.

Table 5.3: Natural frequencies of the 1st and 2nd mode for the 1N and 2N systems in rad  $\rm s^{-1}$ 

Mode	Direc-	$0.6 { m m s^{-1}}$		$0.9 { m m s}^{-1}$		$1.3 \text{ m} \text{s}^{-1}$		$2.0 { m m s^{-1}}$	
nr.	tion	1N	2N	1N	2N	1N	2N	1N	2N
1	IL	4.88	5.96	4.91	5.99	5.64	6.44	8.14	8.62
1	CF	4.97	6.41	4.50	6.43	5.71	6.80	8.18	8.79
2	IL	6.91	6.83	6.93	6.86	7.55	7.33	9.67	9.53
Δ	CF	7.49	7.36	7.53	7.40	8.17	7.88	10.15	9.83

From the table it can be seen that for the case of 1N the natural frequencies of the first mode are significantly lower than for the 2N set-up. The natural frequencies for the second mode are on the other hand quite similar in both cases. Because the natural frequencies of the first two modes for the 2N set up are closer together, multimodal vibration is clearly present. For the 1N set-up the multimodal vibration is less apparent. The consequence of this is lower relative amplitudes for the 1N case for speeds higher than  $0.9 \text{ m s}^{-1}$ .

To give more weight to this argument the vibration shape and corresponding frequency spectra are shown for three flow speeds for both the 2N and 1N case. Figure 5.10 shows the results for  $0.7 \text{ m s}^{-1}$ . Both vibrations show a different shape, while vibrating in their first mode. Because the 1N case is vibrating closer to its first natural frequency the vibration amplitude is larger. The peaks in the frequency spectra are sharp and are the same for both spans, which indicates that a single mode is activated in both cases.



Figure 5.10: Frequency spectra and vibration for  $V = 0.7 \text{ m s}^{-1} \& \omega_s = 5.04 \text{ rad s}^{-1}$ 

Figure 5.11 shows the results for  $V = 1.1 \text{ m s}^{-1}$ . Both cases are now transitioning from the first to the second mode of vibration. An interesting observation is that while the 2N case shows a wide peak in the frequency spectrum, indicating that more modes are activated, the 1N case still shows a sharp peak. This leads to believe that although the shape of vibration is still changing, the frequency of vibration already made the switch to the second mode in the case of 1N. This validates the argument that multimodal vibration is less apparent for small supports.



Figure 5.11: Frequency spectra and vibration for  $V = 1.1 \text{ m s}^{-1} \& \omega_s = 7.93 \text{ rad s}^{-1}$ 



Figure 5.12: Frequency spectra and vibration for  $V = 2.0 \text{ m s}^{-1} \& \omega_s = 14.42 \text{ rad s}^{-1}$ 

The last case that is demonstrated is  $V = 2.0 \text{ m s}^{-1}$ , see Figure 5.12. The 2N case is oscillating still in a combination of its first two modes, as can be seen in both the vibration shape as well as in the frequency spectrum. Although the peaks are sharper than for the previous flow velocity, multimodal vibration is still occuring. If one inspects the frequency spectra closely it can be seen that the spans are oscillating at a slightly different frequency. The spans are oscillating out of sync.

The 1N case has made the total switch to the second mode of vibration. The vibration shape is the same as the mode shape of the second natural frequency. In addition to this, the peaks in the spectrum for span 1 and 2 are located at the same frequency.



Figure 5.13: Single span 28m

The final remark on this behaviour is that the second mode of vibration for the multispan is very different from the single span. Figure 5.13 shows how the first two mode shapes of a single span look. This difference is the reason why for higher flow speeds the relative amplitudes start to rise. At these speeds the single span is already making its change to the second mode. Span 1 actually reaches this second mode at  $1.9 \text{ ms}^{-1}$  if treated as an isolated span. As can be seen in the mode shape, the middle node of the span will hardly move when vibrating in the second mode. This is why there is a steep rise for these velocities. Figure 5.14 illustrates why the middle node of a span will hardly move in its second mode.



Figure 5.14: 2nd mode vibration of a single span

The next aspect that is investigated are the maximum soil reactions due to the vibration. These are discussed in the following section.

#### 5.4.2 Soil reaction

The soil reactions shown in this section are taken from the deformation of the dynamic soil springs in the support during the steady state vibration. The plots in Figure 5.15 show the displacements in both z- and y-direction around the static deflection. Note that for z-direction the dotted lines are the downward displacements, but they are shown as positive displacements. In y-direction the same principle is applied. For the 1N case however, the y-displacement never reached a negative value, thus its minimum displacements are shown to be 0. In Appendix G the same plots can be found, but scaled to pressures based on element length and pipe diameter.



Figure 5.15: Maximum soil reactions

#### Remarks

Again, the conclusions for the soil reactions are based on the remarks that can be made regarding the plots. These are:

- In z-direction the maximum displacements range from 4.5 8 mm and in y-direction from 3 5.2 mm.
- In z-direction the upward and downward displacements are approximately equal for each case, while in y-direction there is a difference between maximum and minimum displacements.

• The trend of the 2N and DS case is similar and increase fast before flattening out. The 1N case shows a constant rise for flow speeds larger than  $0.7 \text{ m s}^{-1}$ .

#### Conclusions

There is now a difference between the 2N and DS case, where for the amplitudes the reaction was the same. The trend in both cases is similar however, with lower displacements for the DS case. This is because the soil forces that are induced by the vibration are spread out over a longer area. The effect is illustrated in Figure 5.16, this figure shows snapshots during the steady state regime of the pipeline positions at the support. It is clear the vibration over the support is larger for the 2N case.



Figure 5.16: Snapshots of pipeline positions at the support

The trend of the soil reactions is similar to the trend of the absolute amplitudes in the multi-span case for both z- and y-direction. This leads to believe that the force on the soil is dependent on the amplitude of vibration.

For the 1N case the soil reaction in z-direction starts to increase when the vibration is moving to the second mode, so after the peak in the relative amplitudes. This is expected, because similar to the second mode of a single span, nodes in the middle hardly vibrate in the first mode. In y-direction the 1N case shows a different trend than in z-direction. The dip at a flow speed of  $0.9 \text{ m s}^{-1}$  is related to a dip in vibration amplitude.

The reason why for y-displacements there is a discrepancy between minimum and maximum displacement is because due to VIV there is a drag amplification, as explained in section 2.3.2. This means that the mean of the vibration moves. How far it moves depends on the type of support, flow speed and vibration amplitude.

#### 5.4.3 Critical support length

From the results it is apparent that for a small support, the natural frequency of the first mode of vibration is lower than for a longer support. The second natural frequency seems to be more similar for both cases. To give more insight, the relation between support length and *cross-flow* natural frequencies has been investigated. Figure 5.17 shows this relation for a support length ranging from 0.2 m - 3.6 m and a flow velocity of  $V_{flow} = 0.8 \text{ m s}^{-1}$ .

From this graph it can be seen that the first two natural frequencies show a very different relationship with the support length. The first natural frequency initially rises quickly and then seems to even out when the support gets larger than around 2 m. The second



Figure 5.17: Plot of cross-flow natural frequencies against support length

natural frequency however stays practically constant whatever support length is chosen. This behaviour can as well be related to the mode shapes of the pipe. Figure 5.18 shows four examples of the first mode shapes at different support lengths.

The choice of support lengths in Figure 5.18 is not done at random. From Figure 5.17 it seems that for a support length of about 2.2 m or larger, the first natural frequency stays approximately the same. If one looks at the mode shape of the first natural frequency it



Figure 5.18: Mode shapes for various support lengths

is also approximately the same for 2.2 m as for 3.6 m. The first mode shape for a support length of 1.4 m shows a more clear distinction from the support of 0.4 m as well as of 2.2 m, which would indicate that for this distance between the spans the mode shape and natural frequency is still heavily dependent on support length.

The plots of the second mode shapes can be found in Appendix G.2. The mode shapes for the second mode shows less distinct behaviour in relation to the support length. The difference is very small in relation to the first mode shape, which indicates why the second natural frequency is almost constant for each support length.

## 5.5 Conclusion

While simplifying the support from a set of distributed springs to 2-nodes does not severly effect the system, changing to 1 single node does significantly influence the system. The consequences of this alteration of the system are:

- 1. The first natural frequency of the system drops, both in in-line as well as crossflow direction.
- 2. The first mode shape changes, in both z- and y-directions.

The reason for both these consequences are linked to each other. The natural frequency is lower, because the first mode shape requires less effort to vibrate in the 1-node case. Picture it as follows, in the case of 1-node the support acts like the middle of a seesaw. Going up and down on a seesaw is not hard at all, see Figure 5.19a



Figure 5.19: Pipe deformations over the shoulder

Using 2-nodes or distributed springs however locks the system. The seesaw principle does not apply anymore as the beam is now forced to bend, whatever position the two spans are in (except if both spans are horizontal), see Figure 5.19b. This relates to a higher dynamic stiffness and thus a higher natural frequency. It can also be seen in the mode shape: whereas the beam is almost straight over the support in the 1-node case, for the 2-node and distributed springs case the beam is bend, see Figure 5.9a and 5.9b. This effect is present in both in-line and crossflow direction.

This drop in natural frequency when a single node case is used, is most likely to be the reason that the fatigue damage is very different between the 2-node and 1-node case as calculated by Saipem. The current with a return period of 100 years was 1.6 m s<sup>-1</sup>, but it is now apparent that a current of around 0.7 m s<sup>-1</sup> could theoretically do as much damage as a higher current speed, if the 1-node case is chosen. This speed is much lower and thus more likely to be present at the span. In all three cases the soil reactions have also been investigated. The aim was to see if the soil reactions are realistic. For the vertical direction the maximum displacements are 8mm, this is for the 2N system. Initially this doesn't seem to be very large. To relate this displacement to reality it is more useful to look at the strain this causes in the soil layer. The dynamic strength of soil is dependent on this strain. From the survey for pipeline the initial soil layer is estimated to be 0.3 m. The maximum strain is then 0.0266. Figure 5.20 shows a figure from the DNV-RP-F105, which relates dynamic soil strength to strain. The red line in this figure shows the reduction due to cyclic strain. The black lines are different plasticity indexes. The plasticity index for sand is 0.



Figure 5.20: Reduction of soil strength by cylic strain

The reduction indicates that even such a small displacement can impact the dynamic soil strength. For this reason the spring stiffness that is used could be an overestimation of the real soil strength. The other aspect that has not been taken into account yet is the fact that nodes could disconnect from the seabed. Because sand is a soil type that has relatively big particles, it is assumed that water travels easily through the soil. The consequence of this is a low cohesion of the soil, relative to most clay types. There is therefore all the more reason to believe that the pipe could lift up from the sand.

In horizontal direction it is harder to judge the soil displacements. This is because the process that leads to vertical soil stiffness is different to the process for horizontal soil stiffness. In horizontal direction it is assumed that the pipe entrenches itself due to the vibration or is already located in a trench. A concept for which a quantitative comparison is harder. With maximum displacement between 3 and 5 mm however, it can be concluded that the displacements are not unrealistically large or small.

The influence of the support length on the natural frequency has also been investigated. It is concluded that the support length has little influence on the second natural frequency and mode shape. For the first natural frequency this is not the case and the support has significant influence. In the multispan set-up under investigation, the critical support length is around 2.2m. For supports longer than this value, the frequency and mode shape for the first mode stay approximately the same.

This chapter has looked at how the shape of a support effects the vibration of interacting freespans and related this to an existing project. From the three possible support model

types, the 2N and DS case give similar results, with the 1N case giving distinctly different results. If the support were a hill with a sharp peak the 1N case would be valid. But because from the survey it was found that the hill was 2 metres wide, the 2N or DS model would give better results. From these two models, the distributed springs case gives the most realistic results as the forces due to the vibration are spread across the entire support.

To come to this conclusion an important assumption is made: the soil behaves linearly. Based on the maximum soil displacements a case can be made that possibly some nodes will disconnect from the seabed. To see how this effects the dynamic interaction a closer inspection on the effect of the support is necessary. Besides the cases described in this chapter, in the next chapter 2 new support types will be investigated: sand support with *nonlinear* springs, and a rock support with *nonlinear* springs. Another aspect that will be investigated is the distribution of soil forces and displacements in the shoulder. This chapter only looked at the maximum displacements of any node in the support, and it is interesting to see to what extent these maximum soil displacements are present along the shoulder.

## Chapter 6

# Inspection of dynamic interaction in relation to support properties

### 6.1 Introduction

To take a closer inspection on how the spans interact a couple of cases are investigated based on the pipeline of Chapter 5. Instead of looking at a range of flow velocities, only a couple of velocities will be investigated, so a more detailed analysis can be done of what is happening with the vibration of the free spans. The focus of this section is on the crossflow vibration, because this is the direction in which the support behaves nonlinear. The flow velocities that are investigated are:  $0.7 \text{ m s}^{-1}$  and  $1.5 \text{ m s}^{-1}$ , which correspond to shedding frequencies of:  $5.05 \text{ and } 10.8 \text{ rad s}^{-1}$  respectively. These velocities are chosen because  $0.7 \text{ m s}^{-1}$  hits the first mode for the 1N case and  $1.5 \text{ m s}^{-1}$  is hitting the second mode in all three cases from chapter 5.

The support types that will be tested are:

- 1. Distributed *linear* springs (sand)
- 2. Distributed *nonlinear* springs (sand)
- 3. Distributed *nonlinear* springs (rock)

Nonlinear springs are investigated, because there is a possibility that the pipe would disconnect over the shoulder. This has not been investigated in the previous chapter. For a comparison of nonlinear supports, also a situation is modelled where the support is made up of rock. The sand has the same properties as in the previous chapter, see Table 5.1. For rock the soil spring stiffnesses are 1000 times larger than for sand.

In this model only the support properties are changed. At the shoulders the springs are still assumed to be linear, to increase calculation time.

### 6.2 Flow Velocity = $0.7 \text{ m s}^{-1}$

For comparison of the different support situations the steady state regime is used for the pipe displacements. In the model this contains the last 25% of results in the time domain. To verify whether the steady state has been reached the vibration of a node is inspected. Figure 6.1 shows the nodal vibrations halfway span 1 and span 2 in crossflow direction. It can be seen that the vibration is steady over the last quarter of the time domain and therefore it is concluded that for all cases the steady state has been reached. This is also the case for the inline vibrations, as shown in Appendix H.2.

The static position of the pipe for this flow speed for both soil types is given in Figure 6.2.



(c) Nonlinear rock behaviour

Figure 6.1: Vibration of nodes halfway span 1 and span 2  $\,$ 



Figure 6.2: Static positions

#### 6.2.1 Envelope of vibration

To compare the cross-flow vibration for the different cases, the envelope of vibration is the best indicator. Figure 6.3 shows the snapshots of the pipeline position during the steady-state regime for all cases.

A very interesting thing is happening as we move from a linear soil to a nonlinear soil; the shape of the vibration is taking the shape of vibration for a 1N linear soil support, see Figure 5.10 in the previous chapter. For nonlinear rock this change in mode shape is more apparent than for nonlinear sand. The reason for this could be that because sand is softer than rock, the deceleration of the pipe on the shoulder is smaller, which allows more energy to be dissipated from the vibration. The hardness of the rock, leads to larger decelerations and thus the time in which the pipe has contact with the rock is smaller.



(a) Linear sandy soil behaviour



(b) Nonlinear sandy soil behaviour



(c) Nonlinear rock behaviour

Figure 6.3: Snapshots of pipeline positions in steady-state regime



Figure 6.4: Snapshots of pipeline positions in steady-state regime

Figure 6.4 shows snapshots of the pipeline vibration at the support in which the dotted red line is the static position of the pipe. It can be seen that especially at the edges of the support, the pipe in the nonlinear sandy soil penetrates the soil further than in the case of rock. Also one can see that for linear sandy soil the pipe does not rise from the static

	Lin. Sand		NonLin. Sand		NonLin. Rock		Lin. 1-Node	
Span nr.	1	2	1	2	1	2	1	2
Cross-flow	0.560	0.317	0.661	0.462	1.255	0.903	1.151	0.955
In-line	0.036	0.021	0.048	0.042	0.166	0.135	0.126	0.112

Table 6.1: Maximum amplitudes for each case A/D

position in the middle of the support, this prevents the vibration shape that is starting to show for nonlinear sand and is very apparent for nonlinear rock.

Table 6.1 shows the maximum amplitudes of vibration for all cases. The last 2 columns are the maximum amplitudes for the 1N linear soil support from the previous chapter. The amplitudes of the nonlinear rock case are very similar to these amplitudes. For this particular case the 1N approach would be valid for approximating the amplitudes of vibration if the pipe would be laid on rock. This correlation does not hold for higher flow velocities. Another factor that can contribute to this is that the spans are almost equal in length. For spans of very different lengths this relation could also not hold.

The nonlinear sand support also leads to larger amplitudes than the linear approach, however less than for the nonlinear rock. This increase is relatively small and there is less chance that the soil nonlinearity would severely affect fatigue life at this flow velocity.

#### 6.2.2 Response frequency

For the case of single action springs, looking at the natural frequencies does not give any insight into the vibration. The reason for this is the response shape of the pipe is now built up of multiple mode shapes of the pipe for the case that *no* support is present. To see at what frequencies the pipe responds, the frequency spectrum can be analysed. Figure 6.5 shows the spectra for left span. The peaks in the spectrum for the right span are at the same frequencies. As one can see a lot of frequencies participate in the vibration in the nonlinear soil cases. To come to a good result more mode shapes have to incorporated than for the linear soil behaviour case. In this analysis the first 40 modes are used to provide the results for the nonlinear sand and 50 modes are used for nonlinear rock. The number of participating modes has been determined by iteration.

#### Linear Sand

There is one big peak that can be identified in crossflow direction. This peak is located at  $6 \text{ rad s}^{-1}$ . In inline direction two peaks are identified, at 6 and 12 rad s<sup>-1</sup>. This frequency corresponds to the first natural frequency of the system as would be expected with a shedding frequency that is lower than  $6 \text{ rad s}^{-1}$ . In the previous chapter the first mode of linear springs has been discussed and it can be seen that the envelope of vibration corresponds with the first mode shape.

#### Nonlinear sand

A lot more peaks are present in the spectrum. This indicates the fact more modes participate in the vibration. These modes are necessary to calculate the exact displacements, because the support is not part of the mode shape anymore.



Figure 6.5: Frequency spectra

For the crossflow direction again the leftmost peak is most present, located at 5.5 rad s<sup>-1</sup>. In inline direction the two leftmost peaks also dominate, which are located at 5.5 and 11 rad s<sup>-1</sup> respectively. The response frequency of 5.5 rad s<sup>-1</sup> is close to the second natural frequency in crossflow direction, which is at 5 rad s<sup>-1</sup>. The modes are now taken over the entire freespan *excluding* the support. The second crossflow mode can be seen in Figure 6.6. In can be seen that the response for nonlinear sand approaching this vibration shape. Because this shape cannot exactly be taken by the pipe, higher modes are necessary to provide the accurate vibration shape.

#### Rock

For rock also 1 main peak can be identified in crossflow direction and it is located at  $4.5 \text{ rad s}^{-1}$ . The inline peaks lie at  $4.5 \text{ and } 9 \text{ rad s}^{-1}$ . The second natural frequency for the case of rock is at  $4.5 \text{ rad s}^{-1}$ . This indicates that for nonlinear rock the second mode of vibration is more dominating than for nonlinear sand, a fact that can also be seen in the envelopes of vibration. In addition to this the higher modes are less apparent than for nonlinear sand.



Figure 6.6: 2nd mode shape for nonlinear soil behaviour

#### 6.2.3 Soil reaction

To inspect the soil reaction, the maximum displacements and forces have been plotted over the length of the support, see Figure 6.7. This has been done for all three support cases.

In all three graphs the lateral displacements are similar, which is because of the assumption that the lateral soil reaction is always present, as explained in Section 4.4.8. For the lateral direction, clearly the displacements are larger at the sides of the shoulder than in the middle. Reason for this is that the pipeline is excited in the freespan and not over the support, so no external forces are present at the support. The lateral movements at the side come from the vibration in the freespan.

In z-direction however a difference is noted between the linear and nonlinear springs. For linear soil behaviour the soil reaction is similar to the reaction in y-direction, with larger excitation at the ends of the shoulder. The reason for this is also the same as for the y-direction.

For the nonlinear soil cases there won't be any tension in the soil, so no positive displacement is noticed. In the case of sand, the penetration is similar to the linear sand case, with the two peaks at the edges. The penetration is also similar with a maximum of 2 mm for nonlinear sand and 1.5 mm for linear sand. Both values are not unrealistically high or low.

For nonlinear rock two peaks are also registered, they are however less definite and not located at the very edge of the support. The penetrations are very small for rock, but because of the stiffness, the resulting pressures are extremely high, up to 8000 kPa. In reality these pressures will be even higher, because they are now calculated under the assumption that the pressure is evenly distributed over the diameter of the pipe. For rock a much smaller part of the pipe will be in contact and thus the pressure will be higher.



(a) Linear sandy soil behaviour



(b) Nonlinear sandy soil behaviour



(c) Nonlinear rock behaviour

Figure 6.7: Soil response for V=0.7  $\mathrm{m\,s^{-1}}$ 

#### Flow Velocity = $1.5 \text{ ms}^{-1}$ 6.3

20

40

t

60

Figure 6.8 shows the nodal vibrations halfway span 1 and span 2, to identify whether the steady state regime has been reached. It can be seen that the vibration is steady over the last quarter of the time domain and therefore it is concluded that for all cases the steady state has been reached. In the plots of Figure 6.8 can be seen that multi-modal vibration is present. The static position of the pipe for this flow speed for each case is given in Figure 6.9.



(c) Nonlinear rock behaviour

20

40

60

80

100

100

80

Figure 6.8: Vibration of nodes halfway span 1 and span 2 for  $V=1.5 \text{ m s}^{-1}$ 



Figure 6.9: Static positions

#### 6.3.1 Envelope of vibration

First is looked at a set of snapshots of the pipe position during the steady state vibration in cross-flow direction. Figure 6.10 shows the snapshots for all cases.



Figure 6.10: Snapshots of pipeline positions in steady-state regime for  $V=1.5 \text{ m s}^{-1}$ 

With the freedom to move upwards, the shape of the vibration changes drastically at this flow velocity. For nonlinear sand behaviour is very irregular, it seems to change its vibration shape during each oscillation. Although the amplitudes are lower than for linear soil, the response is not related at all, see Table 6.2 for the amplitudes of vibration. The pipe totally lifts up from the support at some point during the vibration, this will lead to impact loads upon return. This phenomenon is discussed later in this chapter.

Nonlinear rock has a totally different vibration shape as well. Three antinodes can be observed, one in each span and one at the support. The difference between the two nonlinear soil cases can again be related to the fact that for the hard rock, the pipe is less in contact than for sand. The pipe seems to want to have an antinode at the support and thus have a big vibration amplitude at the location of the support. The support

	Lin.	sand	NonLi	n. sand	NonLin. rock		
Span nr.	1	2	1	2	1	2	
Cross-flow	1.384	1.432	0.759	1.164	0.985	0.932	
In-line	0.275	0.284	0.124	0.149	0.2056	0.215	

Table 6.2: Maximum amplitudes for each case A/D

prevents the downward motion of the pipe. For sand the soil seems to dissipate a lot of energy during this impact, because the pipe doesn't vibrate far above the support. The stiffness of the rock however again leads to fast deceleration and fast acceleration at the support which sends the pipe upwards after impact. This phenomenon can be compared to dropping a marble on rock or sand. On rock it will bounce off, while in sand it will simply sink into the soil.

#### 6.3.2 Response frequency

The response is built up of multiple mode shapes. Figure 6.11 shows the frequency spectra for this vibration for span 1.



(c) Nonlinear rock behaviour

Figure 6.11: Frequency spectra

#### Linear sand

For linear sand there are actually two peaks close together in crossflow direction. These are located at 8.25 and 8.9 rad s<sup>-1</sup>. The fact that there are two peaks explains the beating behaviour that was noticed in Figure 6.8a. In inline direction the peaks are located at: 8.25, 8.9, 16.5 and 17 rad s<sup>-1</sup>. The reason why there are two peaks in crossflow direction is because the natural frequencies are close together in z-direction, at 8.05 and 8.5 rad s<sup>-1</sup>.

#### Nonlinear sand

The dominating frequency is located at  $8.5 \text{ rad s}^{-1}$  in crossflow direction. This frequency is located between the second and third natural frequency in crossflow direction. Because a lot of frequencies are present however the vibration shape consists of more modes than solely these two.

#### Nonlinear rock

There are two definite peaks for the case of nonlinear rock, located at 5 and 10 rad s<sup>-1</sup> for the crossflow vibration. The corresponding natural frequencies are located at 4.8 and 9.6 rad s<sup>-1</sup>, which are the second and third vibration modes in z-direction. Figure 6.12 shows the shape of these modes. From this figure can be seen why 3 antinodes are observed in the envelope of the vibration in Figure 6.10b and 6.10c. This is due to the strong presence of the third vibration mode. The shedding frequency for V=1.5 m s<sup>-1</sup> is 10.8 rad s<sup>-1</sup>, which is close to the frequency of the third mode shape. Because the downward motion is limited by the support, the vibration can't consist of purely this mode however, and the multimode vibration is thus observed.



Figure 6.12: Mode shapes nonlinear soil behaviour

#### 6.3.3 Soil reaction

Similarly to  $0.7 \text{ m s}^{-1}$ , the soil reactions have been determined for  $1.5 \text{ m s}^{-1}$ . They are shown in Figure 6.13.

For  $V=1.5 \text{ m s}^{-1}$  the lateral behaviour of the soil reaction forces is similar to  $V=0.7 \text{ m s}^{-1}$ . The reason for this is the same as well, and is due to the fact that the pipe is only excited at the free span.

The shape of the distributed displacements in z-direction for the case of linear soil behaviour is also similar to the previous flow speed. The size however, is different and is twice as large for  $V=1.5 \text{ m s}^{-1}$ .



(a) Linear sandy soil behaviour



(b) Nonlinear sandy soil behaviour



(c) Nonlinear rock behaviour

Figure 6.13: Soil response for V=1.5  $\rm m\,s^{-1}$ 

For nonlinear sandy soil the shape of the soil penetration is the same as for  $0.7 \text{ m s}^{-1}$ . The displacements are with a maximum of about 3.2 mm as opposed to 2 mm.

For nonlinear rock the shape of the maximum deformations looks more erratic as opposed to the previous flow velocity. The maximum displacement is more than twice the displacement of the previous case. A peak is also shown at both edges of the support.

#### 6.3.4 Accelerations

A new property that has been looked into are the accelerations of the pipe at the support. Because the pipe now lifts up from the support and hits it again upon downward motion, it is interesting to see whether high accelerations are felt. These accelerations are linked to the impact forces that are felt by the pipe. Newton's second law:  $F = m \cdot a$  relates acceleration to the force. If the acceleration is high, the force is large and there is a chance the material will deform plastically to account for this force. Compare it to a car crashing into a wall, the plastic deformation of the front of the car absorbs the force that is exerted by the fast deceleration of the car, the same could happen to a pipeline.

The maximum acceleration for the linear sandy soil is  $0.28 \text{ m s}^{-2}$ . The nonlinear soil types have far larger accelerations with 54 and 361 m s<sup>-2</sup> for sand and rock respectively, see Appendix H.1 for the plots of the decelerations. The forces induced by these accelerations could have an effect on the structural strength of the pipe in the long run and are not taken into account in the current response model by DNV.

## 6.4 Conclusion

The amplitude, soil reaction and shape of the vibration is heavily dependent on the way the support is modelled. Three methods have been investigated, linear sandy soil, nonlinear sandy soil and nonlinear rock. Each support type leads to a different vibration shape and to different soil reactions. Because rock is much harder than sand the reaction force was larger while the penetration was lower. For  $V=0.7 \text{ m s}^{-1}$  this lead to a very different shape for the rock support in relation to the linear sand support. The nonlinear sand support also was leaning towards the vibration shape of the rock support, however with smaller amplitudes. The reason for this is that more energy is dissipated due to the soft soil.

For  $V=1.5 \text{ m s}^{-1}$  there also was a difference between the three different support types. The rock again showed the most extreme difference between linear and nonlinear soil, with three antinodes in the vibration. This leads to the pipe completely losing connection with the support during half an oscillation period.

The nonlinear sand had a less extreme effect, but also lead to a short disconnection between pipe and support. The shape of vibration is made up of several modes and is much more irregular than the linear sand and nonlinear rock.

The three support cases have also been analysed for the accelerations that are felt during impact of the pipe on the support. This indicates how much force is felt by the pipe upon impact. For the nonlinear cases this was a lot higher than for the linear cases, which implicates that this could have an effect that is underestimated using the support model from the DNV code.

There are three conclusions that can be drawn in this chapter:

- 1. The vibration response for nonlinear sandy soil as shown in this chapter is plausible
- 2. The vibration response for nonlinear rocky soil as shown in this chapter is probable
- 3. The choice of soil model (linear or nonlinear) has influence on the dynamic interaction of freespans

The first conclusion is drawn based on the soil reaction forces for nonlinear sand. For both flow speeds the soil displacements for nonlinear sand is lower than for linear sand. This means that for nonlinear sand there is no reason to suggest that the soil would not behave according to the dynamic model. The only aspect that is questionable is whether to assumption of no suction is correct. The influence of suction would be small however in relation to clay. Another remark to be made is that, similar to the soil in the previous chapter, the dynamic stiffness would degrade due to the cyclic loading.

The second conclusion is drawn because for rock it is assumed that suction is not possible at all, because the pipe has not sunk into the ground at all. For this reason the predicted response is more certain than for sand, where the effect of suction is unknown.

The third conclusion is based on the envelope of cross-flow vibration. Nonlinear soil behaviour leads to a completely different response. It can lead a vibration which has 2 dominating frequencies. In addition to this the pipe can be subjected to impact loads by hitting the support.

In this particular case the 1N approach that was discussed in Chapter 5 is actually a good approximation for the soil reaction on rock for low flow speeds. For high flow the vibration is very different between the three cases and a 1N approximation makes no sense at all. In conclusion soil nonlinearity will effect the vibration.

## Chapter 7

# **Conclusions and Recommendations**

## 7.1 Conclusions

#### Contribution of the Pipe-Wake model

In this research a new model for modelling VIV reponse of a subsea pipeline on clayey soils has been proposed, called the Pipe-Wake model. The following conclusions can be drawn regarding the Pipe-Wake model:

- The Pipe-Wake model is an accurate VIV model. It captures the lock-in region and the cross-flow vibration very well. The in-line vibration is underestimated. This is because the model is tuned to the cross-flow vibrations.
- The Pipe-Wake model doesn't underperform in relation to commercial models. For the pipe-soil interaction no comparison is available. This conclusion is based on a pipeline model without soil interaction.
- The Pipe-Wake model is the first *time-domain* model to incorporate nonlinear soil reactions into the program. There is also no evidence that suggests that the soil suction model as implemented in the Pipe-Wake model has ever been applied in commercial models, time- or frequency domain alike. Because the soil suction model has been validated separately it is concluded that the Pipe-Wake model should give realistic results regarding pipe-soil interaction.
- Because the soil suction model relates the suction force to the undrained shear strength of clay, there is no suction model for sand. Only linear or nonlinear soil behaviour can be used to model sand. The model is therefore less accurate for sandy soils. However because sand has large voids and is loosely packed a nonlinear soil reaction without suction is assumed to be the most realistic sand model.

In conclusion the Pipe-Wake model is of added value to the models that are present today. It is an easy to use model that solves the response in the time-domain relatively quick. The suction model for clay is a novelty and can especially for multi-span cases shed new light on the interaction between free spans.

#### Effect of the support properties on dynamic interaction

An example case has been run for two interacting free spans. The aim of the case was to analyse how the support properties effected the dynamic interaction. Because the case study was on sandy soil, the suction model is not used, instead both linear and nonlinear springs are used. From this case study the following conclusions are drawn:

• Modelling a support that is not a sharp hill as a 1-node support is an unrealistic approach. Such a model severely changes the natural frequency and vibration behaviour in the system where linear springs are used.

- The vibration of the system is dependent on whether the soil is modelled with linear springs or nonlinear springs. Nonlinear springs leads to a different shape of vibration. This can also affect the flow velocities to which the span is sensitive. In the case-study, a support with nonlinear springs leads to higher amplitudes of vibration for lower flow velocities in relation to linear springs.
- The lateral and vertical soil displacements and pressures in the support are the highest at the edges and relatively low in the middle of the support. Because cohesion in sand is expected to be low, there is a good chance soil nodes will disconnect at the support. Therefore it is concluded that using nonlinear springs is a realistic approach.
- The lateral and vertical soil displacements are not unrealistically high or low and the maximum displacements lie between 5 and 8mm. The dynamic properties of soil however, are related to the strains in the soil layer and not the absolute displacements. In the case study the strains due to these displacements were sufficient to reduce the dynamic soil strength to 60% of its original strength. The spring stiffnesses used for the nodes at the edges of the support therefore are an overestimation of the true soil strength.
- When a support is modelled with nonlinear springs a steady state vibration can occur where the pipe lifts up from the support and impacts it when moving downwards. This leads to high decelerations, especially if the support is modelled as a rock. These decelerations induce impact loads which aren't taken into account in the current design codes.

To conclude, the approach that soil is modelled as linear springs is an oversimplification of the problem. Other vibration modes and response frequencies can occur if the pipe lets go of the soil. Optimally both linear and nonlinear soil springs should be tested. If nonlinear springs adversely affects the vibration then span intervention should be considered.

## 7.2 Recommendations

### Recommendations regarding the Pipe-Wake model development

The following recommendations are made to improve the Pipe-Wake model:

- To make the Pipe-Wake a more universal model the following aspects still need to be incorporated:
  - Degradation of dynamic soil stiffness due to cyclic loading
  - Allow for a change in cross-section along the length of the pipe

- Add the possiblities for altitude changes of the seabed so hills can be created in the seabed

- Improve the lateral dynamic soil stiffness method to allow the pipe to move while disconnected from the seabed

- Include a more accurate soil model for sandy seabeds.
- The model should be validated with experimental results. This means that model pipes on soil supports have to be exposed to VIV in a labratory environment. Both a clayey soil (or preferably different types of clayey soils) and a sandy soil should be used. By measuring the vibration and comparing them to pipelines which are

supported by springs it can be seen to what extent the soil suction effect comes into play.

- Another type of validation can be done by quantitatively comparing the Pipe-Wake to commercial models. In this research this comparison has only been done qualitatively.
- The in-line amplitude is currently underestimated. Finding a wake-oscillator set-up in which both cross-flow and in-line amplitudes are estimated accurately would be an improvement on the model.

### Recommendations regarding free span modelling

To improve the response calculation for free spans these recommendations are made:

- Model test a multispan set-up with nonlinear springs to validate the conclusion that nonlinear springs indeed severely influence the vibration due to VIV.
- If it is the case that asymmetric springs change the behaviour, then the response model as proposed by DNV-RP-F105 is not valid for all *multispan* cases. A force model, e.g. a simplified version of the Pipe-Wake model can then shed more light into the fatigue life of a set of free span.
- The effect of impact loads require further investigation.
- Do not simplify a support as a single node, except if it really is a sharp hill or rock. This will give inaccurate results regarding the natural frequencies and mode shapes.

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## Appendix A

# Forced van der Pol oscillator

To show the effect of the forcing term, a couple of combinations of different tuning parameters are shown in this section. For illustration a sinusoidal force is applied to Equation (3.1). The full equation for the van der Pol oscillator now reads:

$$\ddot{q} + \epsilon \omega_o (q^2 - 1) \cdot \dot{q} + \omega_o^2 q = A \cdot \sin(\omega_f t)$$
(A.1)

Where:

A =Tuning parameter

$$\omega_f =$$
 Forcing frequency

A couple of different cases are run to see the influence of the variables. The base case for each set of results is: A = 5,  $\epsilon = 0.2$ ,  $\omega_f = 2$  and  $\omega_o = 2$ . Figure A.1 shows the van der Pol for these parameters.



Figure A.1: Base case for forced van der Pol

The parameters for the different cases are:

- $A = 1, \epsilon = 0.2, \omega_f = 2$  and  $\omega_o = 2$
- $A = 10, \ \epsilon = 0.2, \ \omega_f = 2 \ \text{and} \ \omega_o = 2$
- $A = 5, \epsilon = 0.2, \omega_f = 1$  and  $\omega_o = 2$
- $A = 5, \epsilon = 0.2, \omega_f = 3$  and  $\omega_o = 2$

From figure A.2 two conclusion can be drawn:

- The bigger tuning parameter A, the larger the amplitude of q
- If the forcing frequency is close to the natural frequency the amplitude is larger than if further away.
- If the forcing frequency is far from the natural frequency the frequency of oscillation is different from the natural frequency.



Figure A.2: Forced van der Pol oscillator responses

## Appendix B

## **Finite Element Method**

This Appendix provides elaborations on the implementation of the finite element method for the pipeline model from Chapter 4.

Figure B.1 shows the pipeline model used for the pipe-wake coupling, with all the degrees of freedom of an element between nodes 5 and 6. In the real model however the axial degree of freedom (x) and the torsional degree of freedom ( $\Theta_x$ ) is not taken into account as they won't be influenced under the linear approach used here. Also the number of nodes chosen isn't the same as in the figure, but can be changed to whatever mesh size is needed for the analysis.



Figure B.1: Finite Element Model used for the wake-pipe coupling

## B.1 Mass, Stiffness and Tension matrices

#### B.1.1 Local matrices

To describe the behaviour of an element the mass and stiffness properties of the beam are needed. Structural damping won't be taken into account yet, this will be implemented in a later stage.

The mass and stiffness matrices relate the movement of an node in a particular degree of freedom to a reaction force in the element. Mass is related to the acceleration of the node, while stiffness is related to displacement, see Equation (3.4).

The equation of motion for the element from figure B.1 is:  $[M]\vec{u} + [K]\vec{u} = \vec{F}$ , where:

$$[M]\vec{\ddot{u}} = (m+m_a) \cdot \frac{L}{420} \cdot \begin{bmatrix} 156 & 0 & 0 & 22L & 54 & 0 & 0 & -13L \\ 0 & 156 & -22L & 0 & 0 & 54 & 13L & 0 \\ 0 & -22L & 4L^2 & 0 & 0 & -13L & -3L^2 & 0 \\ 22L & 0 & 0 & 4L^2 & 13L & 0 & 0 & -3L^2 \\ 54 & 0 & 0 & 13L & 156 & 0 & 0 & -22L \\ 0 & 54 & -13L & 0 & 0 & 156 & 22L & 0 \\ 0 & 13L & -3L & 0 & 0 & 22L & 4L^2 & 0 \\ -13L & 0 & 0 & -3L^2 & -22L & 0 & 0 & 4L^2 \end{bmatrix} \cdot \begin{bmatrix} \ddot{y}_5 \\ \ddot{z}_5 \\ \ddot{y}_6 \\ \ddot{z}_6 \\ \ddot{y}_6 \\ \ddot{\theta}_{z6} \end{bmatrix}$$

And:

$$[K]\vec{u} = \begin{bmatrix} \frac{12EI}{L^3} & 0 & 0 & \frac{6EI}{L^2} & \frac{-12EI}{L^3} & 0 & 0 & \frac{6EI}{L^2} \\ 0 & \frac{12EI}{L^3} & \frac{-6EI}{L^2} & 0 & 0 & \frac{-12EI}{L^3} & \frac{-6EI}{L^2} & 0 \\ 0 & \frac{-6EI}{L^2} & \frac{4EI}{L} & 0 & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 \\ \frac{6EI}{L^2} & 0 & 0 & \frac{4EI}{L} & \frac{-6EI}{L^2} & 0 & 0 & \frac{2EI}{L} \\ \frac{-12EI}{L^3} & 0 & 0 & \frac{-6EI}{L^2} & \frac{12EI}{L^3} & 0 & 0 & \frac{-6EI}{L^2} \\ 0 & \frac{-12EI}{L^3} & \frac{6EI}{L^2} & 0 & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 \\ 0 & \frac{-6EI}{L^2} & \frac{2EI}{L} & 0 & 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 \\ \frac{6EI}{L^2} & 0 & 0 & \frac{2EI}{L} & \frac{-6EI}{L^2} & 0 & 0 & \frac{4EI}{L} \end{bmatrix}.$$

In which:

L = Element length [m]

E = Young's Modulus of pipe [Pa]

I = Moment of inertia, equal in both x and y direction for circular cross section [m<sup>4</sup>]

The last matrix that will be introduced at this stage is the *Tension* matrix. This matrix is an addition to the stiffness matrix and takes the influence of an axial force into account. This is needed, because the axial force will have an influence on the natural frequencies of the system. **Note:** this matrix is only needed if the nonlinear static analysis is *not* used. The *Tension* matrix looks as follows:

$$[T] = \frac{T}{30L} \begin{bmatrix} 36 & 0 & 0 & 3L & -36 & 0 & 0 & 3L \\ 0 & 36 & -3L & 0 & 0 & -36 & -3L & 0 \\ 0 & -3L & 4L^2 & 0 & 0 & 3L & -L^2 & 0 \\ 3L & 0 & 0 & 4L^2 & -3L & 0 & 0 & -L^2 \\ -36 & 0 & 0 & -3L & 36 & 0 & 0 & -3L \\ 0 & -36 & 3L & 0 & 0 & 36 & 3L & 0 \\ 0 & -3L & -L^2 & 0 & 0 & 3L & 4L^2 & 0 \\ 3L & 0 & 0 & -L^2 & -3L & 0 & 0 & 4L^2 \end{bmatrix}$$

In which:

T = Tension force [N]

#### B.1.2 Global matrices

For the calculation of the motion of the beam the element matrices have to be combined to form a so-called *Global Mass and Stiffness Matrix*. This is done by adding the element matrices together. As can be seen in Figure B.1 two adjacent elements share a node. When adding the matrices together the properties allocated in each element matrix to a certain shared node shall also be added together. Figure B.2 illustrates this for a 2-element (3 nodes) stiffness matrix with 2 degrees of freedom per node. As one can see, the stiffness properties for the shared node overlap. The same principle applies to the mass and tension matrices.

$K_{11}^1$	$K_{12}^{1}$	$K_{13}^{1}$	$K_{14}^{1}$	0	0
$K_{21}^{1}$	$K_{22}^{1}$	$K_{23}^{1}$	$K_{24}^{1}$	0	0
$K_{31}^{1}$	$K_{32}^{1}$	$K_{33}^1 + K_{11}^2$	$K_{34}^1 + K_{12}^2$	$K_{13}^{2}$	$K_{14}^{2}$
$K_{41}^{1}$	$K_{42}^{1}$	$K_{43}^1 + K_{21}^2$	$K_{44}^1 + K_{22}^2$	$K_{23}^{2}$	$K_{24}^{2}$
0	0	$K_{31}^2$	$K_{32}^{2}$	$K_{33}^{2}$	$K_{34}^{2}$
0	0	$K_{41}^{2}$	$K_{42}^{2}$	$K_{43}^{2}$	$K_{44}^{2}$

Figure B.2: Assembly of a global stiffness matrix

The end result is a square matrix with the number of rows and columns equal to: Number of nodes multiplied by Degrees of freedom per node.

#### **B.1.3** Boundary conditions

The last step in the assembly of the Finite Element Model is imposing boundary conditions to the system. If this is not done the global matrices are singular and won't give results to any imposed force. Imposing boundary conditions is very simple, one has to simply remove all rows and columns that correspond to the degrees of freedom that limited by the boundaries. So e.g. for the beam in figure B.1 the left and right boundaries prevents vertical and lateral movement (the left boundary also prevents axial movement, but this degree of freedom is not yet taken into account). To this extent the rows and columns corresponding to this movement have to be removed from the global matrices, which are for the left boundary: 1 & 2 and for the right boundary: (nr. of nodes)  $\cdot 4 - 3 \&$  (nr. of nodes)  $\cdot 4 - 2$ . If the boundaries where also to be clamped, then the rows and columns corresponding to angular displacement would have to be removed (left: 3 & 4, right: (nr. of nodes)  $\cdot 4 - 1 \&$  (nr. of nodes)  $\cdot 4$ ).
# Appendix C

# Modal analysis

# C.1 Introduction

To calculate the dynamic behaviour of the use will be made of Modal analysis. The essence of modal analysis is that the response of the beam (or pipe) is a summation of its eigenvectors, weighed by an unknown time-function, called modal displacement. The equation for this is:

$$\vec{x}(t) = \sum_{i=1}^{n} \vec{\phi_i} \, r_i(t) = [E] \, \vec{r}(t) \tag{C.1}$$

Where:

 $\vec{x}(t)$  = Displacement vector  $\vec{\phi_i}$  = Eigenvector  $r_i(t)$  = Modal displacement [E] = Eigenmatrix  $\vec{r}(t)$  = Vector of all time functions

Using Modal analysis it is possible to uncouple the set of differential equations following from the finite element analysis (FEA). Solving the equations for r(t) gives the motion of the beam. This will be explained in more detail later.

# C.2 Mode shapes

Assuming that the global mass and stiffness matrices are known from the FEA, it is possible to find the eigenvectors of the system. Each eigenvector corresponds to a mode shape of the beam and to a natural frequency of the beam. Let's recall the equation of motion for the beam:

$$[M]\ddot{\vec{u}} + [K]\vec{u} = \vec{F} \tag{C.2}$$

The eigenmodes and natural frequencies are determined using the equation for free vibration, in which  $\vec{F} = 0$ . The global mass and stiffness matrices have to already be reduced by implementing the boundary conditions. After substituting  $\vec{u} = \vec{u} \sin(\omega_n t)$  The expression from which the eigenmodes and natural frequencies are determined reads as:

$$([K] - \omega_n^2[M]) \cdot \vec{\phi} = 0 \tag{C.3}$$

Figure C.1 illustrates the first 4 mode shapes for a simply supported square beam with properties as listed in Table C.1. Note that for beams with equal stiffness properties in vertical and lateral direction, the mode shapes in both directions are the same as well as

the natural frequencies in both directions. The figure only shows the modes for one of the two directions. Also the total number of modes of a beam is equal to the number of degrees of freedom for the beam.



Table C.1: Beam properties

Size

100

Unit

m

Property

Length

Figure C.1: The first 4 mode shapes for a simply supported beam

## C.3 Modal matrices

#### C.3.1 Mass and Stiffness matrices

With the eigenvectors so called orthogonality conditions can be defined in relation to the mass and stiffness matrices. These conditions are [Spijkers et al., 2005]:

$$\vec{\phi_r^T}[M]\vec{\phi_s} = 0 \qquad \text{for } r \neq s$$

$$\vec{\phi_r^T}[K]\vec{\phi_s} = 0 \qquad \text{for } r \neq s$$
(C.4)

Expanding this condition for all eigenmodes that are to be considered in the analysis, the *Modal mass*  $([M]^*)$  and *Modal stiffness matrix*  $([K]^*)$  can be defined. The beautiful property of the modal matrices is that they are diagonal, making the decoupling of equations possible.

$$[M^*] = [E]^T[M][E] [K^*] = [E]^T[K][E]$$
(C.5)

To find another useful property of the modal matrices use is made of equation (C.3), though slightly rewritten:

$$[K]\vec{\phi} = \omega_n^2[M] \cdot \vec{\phi} = 0 \tag{C.6}$$

By pre-multiplying this equation with an eigenvector, in a process same to the one described above one gets:

$$\vec{\phi_r}[K]\vec{\phi_s} = \omega_{n,r}^2 \cdot \vec{\phi_r}[M]\vec{\phi_s} = 0 \quad \to \quad [K^*] = \Omega_n^2[M^*] \tag{C.7}$$

In which,  $\Omega_n^2$  is a diagonal matrix with the natural frequency in each row corresponding to the modal mass in the same row of the modal mass matrix.

#### C.3.2 Equation of motion

Adding all the properties discussed above together a new equation of motion can be set up. This equation of motion is set up to acquire the modal displacement. When the equations are solved for  $r_i$  the *true* displacements can be found using equation (C.1). Note that a damping matrix is added at this stage, this will further be elaborated on in the next section.

$$\begin{split} &[M]\,\vec{r} + [C]\,\vec{r} + [K]\,\vec{r} = \vec{F} \\ &\to \quad \vec{r} + [M^*]^{-1}[C^*]\,\vec{r} + \Omega_n^2\,\vec{r} = [M^*]^{-1}\vec{F} \\ &\to \quad \vec{r} + [C^{**}]\,\vec{r} + \Omega_n^2\,\vec{r} = \vec{F^*} \end{split}$$
(C.8)

#### C.3.3 Damping matrix

For the damping matrix, it is assumed that the modal damping matrix is also orthogonal,  $[C^*] = [E]^T[C][E]$ . A good model to set up the damping matrix [C] that satisfies this condition isn't at hand. That's why when spoken of damping in relation to modal analysis, damping is expressed as a modal damping ratio. This ratio describes how much damping is present for the excitation of a certain mode. The value is chosen by the user based one either experimental data or his/her own insight. The method used in this research is related to the damping ratio of Equation (3.5). Furthermore in Equation (C.8) one can see that the modal damping matrix will be pre-multiplied by the inverse of the mass matrix. For this reason it is useful or relate the damping ratio to this new factor and it is done as follows:

$$[C^{**}] = 2\xi \,\Omega_n \tag{C.9}$$

Here  $\xi$  is a diagonal matrix of the damping ratios and  $\Omega_n$  is a diagonal matrix of the natural frequencies.

## C.4 System of differential equations

To calculate the vibration of the pipe, the system of Equations (3.14) has to be coupled to the finite element model. The positions at which the coupling takes place is at each node in the finite element model. Because modal analysis is used it is not possible to simply add Equation (3.14) to each node, as the displacements, velocities and accelerations in these equations are *true*, whereas *modal* displacements are the variables in the new equations of motion, (C.8). The following steps are taken for calculation of the true displacements:

#### 1. Method for solving the equations

The equations for both the modal displacements, as well as for the wake oscillator, Equation (3.3), are *second* order differential equations. Solving a set of *first* order differential equations is possible to do numerically, so it is necessary to re-write the second order differential equations. Below is illustrated how this is done for the wake oscillator, but the same principle applies to the modal y- and z-displacement.

Equation of motion:

$$\ddot{q} + \epsilon \omega_s (q^2 - 1) \cdot \dot{q} + \omega_s^2 q = \frac{A}{D} \cdot \frac{d^2 z}{dt^2}$$
(C.10)

Set:

$$p_1(t) = q(t)$$

$$p_2(t) = \dot{q}(t)$$
(C.11)

Create set of first order differential equations, filling (C.11) into (C.10):

$$\frac{dp_1}{dt} = p_2$$

$$\frac{dp_2}{dt} = \frac{A}{D} \cdot \frac{d^2z}{dt^2} - \epsilon \omega_s (p_1^2 - 1) \cdot p_2 - \omega_s^2 p_1$$
(C.12)

A method based on the 4th order Runge-Kutta is used to solve the differential equations.

#### 2. Unknowns

The differential equations shall be solved for modal displacements. So firstly the user needs to decide how many modes will participate in the vibration. For a simple beam a good rule of thumb is the mode number closest to the shedding frequency of the pipe, plus 2 or 3 extra modes. Reason for this is that the pipe will lock into the natural frequency of the mode closest to the shedding frequency.

For the sake of example, the beam of Figure C.1 is used. Say the velocity is such that the shedding frequency is 5 rad s<sup>-1</sup>, then the first 5 modes would be chosen to participate in the excitation. Note that for more complicated pipe geometry, this simple rule of thumb doesn't hold. In that case iterations will have to take place to find the minimum number of participating modes.

Sticking to the example above, 5 participating modes equals 5 unknowns for modal displacements in both y- and z-direction (so 10 unknowns in total for the displacements). Because the equations are of the second order other unknowns are the velocities, which also amount to 10 unknowns in total.

Besides the unknown modal displacements and velocities, the wake variable q has to be calculated. Because the wake oscillator function is also of the second order, see Equation

(C.10),  $\dot{q}$  also has to be taken as unknown. The number of unknowns related to the wake oscillator is not dependent on the number of participating modes, but on the number nodes of the beam.

#### 3. Coupling between true and modal displacements

After reading the above, it is clear there is a discrepancy between the way the displacements (and velocities) are calculated in relation to the wake variable. The displacements are in modal space, whereas the wake variable is coupled to the true displacements. Furthermore if one recalls Equation (3.14), the forcing term in the y- and z- equations of motion is also related to the *true* velocity, not the modal velocity.

It is necessary to realise that for switching between modal and true displacements, use can be made of Equation (C.1).

#### 4. System to be solved

Using the information above, it is possible to set up a routine that has to be solved for each time-step in the integration. It looks as follows.

#### **Define Initial conditions**

Initial conditions for the modal displacements, velocities and wake variable and deritave of the wake variable are shown in Table C.2

Table C.2: Initial conditions

Parameter	Initial condition
$\vec{r}$	0
$ec{r}$	0
$ec{q}$	0.01
$ec{q}$	0

After this has been defined the integration loop is started. Without any soil effects included, the loop consists of 5 steps, each described below:

#### 1. Real velocities of each node

$$\vec{\dot{y}} = [E_y]\vec{\dot{r}}$$

$$\vec{\dot{z}} = [E_z]\vec{\dot{r}}$$
(C.13)

Where:

$[E_y] =$ Eigenmatrix containing all participatin	ıg
eigenvectors related to the y-direction	

 $[E_z] =$ Eigenmatrix containing all participating eigenvectors related to the z-direction

#### 2. Real relative velocities of each node

$$\vec{U}_z = -\vec{z}$$
  

$$\vec{U}_y = \vec{V}_y - \vec{y}$$
  

$$\vec{U} = \sqrt{U_y^2 + U_z^2}$$
  
(C.14)

Where:

 $V_y =$  Vector of the flow velocities in y-direction at each node

#### 3. Nodal Forcing terms

$$\vec{F_y} = \frac{1}{2} \rho_w DL_{element} \cdot (C_{D0} \vec{U_y} - \frac{C_{L0}}{2} \vec{q} \cdot \vec{U_z}) \vec{U} \vec{F_z} = \frac{1}{2} \rho_w DL_{element} \cdot (C_{D0} \vec{U_z} + \frac{C_{L0}}{2} \vec{q} \cdot \vec{U_y}) \vec{U}$$
(C.15)

#### 4. Modal Forcing term

$$\vec{F_r} = [E_y]\vec{F_y} + [E_z]\vec{F_z}$$
 (C.16)

#### 4. Second derivatives

For the *modal* accelerations, based on the equation of motion (C.8):

$$\vec{\ddot{r}} = \vec{F_r^*} - [C^{**}] \, \vec{\dot{r}} - \Omega_n^2 \, \vec{r} \tag{C.17}$$

For the *nodal* accelerations in y-direction:

$$\vec{\ddot{z}} = [E_z]\,\vec{\ddot{r}} \tag{C.18}$$

For the wake variable:

$$\vec{\ddot{q}} = \frac{A}{D} \cdot \ddot{z} - \epsilon \vec{\omega_s} (q^2 - 1) \cdot \dot{q} - \vec{\omega_s}^2 q \tag{C.19}$$

#### 5. System of first order differential equations

Using the method described in Equations (C.10) and (C.11), the following set of first order differential equations is set up:

$$\frac{d\vec{p_1}}{dt} = \vec{p_2}$$

$$\frac{d\vec{p_2}}{dt} = \vec{F_r} - [C^{**}]\vec{p_2} - \Omega_n^2 \vec{p_1}$$
(C.20)
$$\frac{d\vec{p_3}}{dt} = \vec{p_4}$$

$$\frac{d\vec{p_4}}{dt} = \frac{A}{D} \cdot ([E_y]\frac{d\vec{p_2}}{dt}) - \epsilon \vec{\omega_s} (\vec{p_3}^2 - 1) \cdot \vec{p_4} - \vec{\omega_s}^2 p_3$$

One can solve this system for  $\vec{p_1}$ ,  $\vec{p_2}$ ,  $\vec{p_3}$  and  $\vec{p_4}$ . Displacement is the main focus of this research, which is found in  $\vec{p_1}$ . Multiplying this vector in each time step by  $[E_y]$  or  $[E_z]$  results in the true displacements in y- and z-direction respectively.

## Conclusion

All the tools are now at hand to calculate the vortex-induced vibrations on a pipeline. It is valid however for e.g. a riser or a pipe suspended between to points. For soil extra steps are included in the loop, which are explained in Appendix E.

# Appendix D

# Model verification

# D.1 Riser experiments

The aim of this appendix is to provide more background information on the riser experiments performed in the Delta Flume in Delft. For clarification the riser specifics and model parameters are repeated in tables D.1 and D.2.

Parameter	Value	Unit
Length	13.12	m
Diameter	0.028	m
EI	29.9	${ m N}{ m m}^2$
Damping ratio	0.0033	-
Mass ratio	3	-

Table D.1: Riser specifics [Chaplin et al., 2005b]

<b>T</b> 11 <b>D</b> 0	7 1 1		C	•	•
Table D 2	Model	parameters	tor	riser	comparison
10010 1.1.	model	parameters	101	TTOOL	comparison

Parameter	Value	Unit
A	10	-
$\epsilon$	0.07	-
Nr. of modes	12	-
Nr. of elements	40	-

The following cases have been examined, with the cases corresponding to the cases in Chaplin et al. [2005a]:

- Comparison of natural frequencies
- Case 1,  $V = 0.16 \text{ m s}^{-1}$  and Top Tension = 405 N
- Case 3,  $V = 0.31 \text{ m s}^{-1}$  and Top Tension = 457 N
- Case 6,  $V = 0.60 \text{ m s}^{-1}$  and Top Tension = 670 N
- Case 9,  $V = 0.95 \text{ m s}^{-1}$  and Top Tension = 1002 N

**Note that:** The weight of the pipe has to be included for the comparison. This is done by deducting the self weight from the top tension at each node. This means that the further down the riser you look, the smaller the tension in the pipe becomes:  $T = T_{top} - \rho_{pipe} \cdot A \cdot L_{above}$ , where  $L_{above}$  is the length of the pipe above the node considered.

#### D.1.1 Comparison of natural frequencies

For the riser the first 7 natural frequencies were measured in still water for different top tensions. To make sure the riser in the laboratory experiments is the same as the one in the model these natural frequencies have to correspond. Table D.3 shows how the frequencies compare, with the measured natural frequencies taken from Chaplin et al. [2005b]. Also the flow velocity needed to have a shedding frequency equal to the measured natural frequency is shown, using Equation (2.2). This indicates which mode is expected to be dominant for a certain flow velocity.

Mode nr	Measured	Calculated	Flow Velocity		
mode III.	nat. freq. [Hz]	nat. freq. [Hz]	$[\rm ms^{-1}]$		
1	0.633	0.646	0.092		
2	1.329	1.296	0.193		
3	1.954	1.955	0.283		
4	2.625	2.626	0.380		
5	3.312	3.313	0.480		
6	4.028	4.020	0.584		
7	4.753	4.750	0.689		

Table D.3: Natural frequencies for riser with a top tension of 813 kN

The natural frequencies calculated are close to the frequencies that are measured. Only the first and second mode differ slightly. The fact that they don't correspond exactly can be due to the following reasons:

- The riser is assumed to be clamped at the top and the bottom, thus preventing rotation at these points. The actual supports aren't known and could therefore differ.
- The calculated mass ratio or displaced mass could not be exactly 3 in the model. Actually an added mass coefficient of  $C_a = 1.2$  is used instead of 1. With this factor the natural frequencies can be tuned slightly, as was necessary in this case.

With the natural frequencies determined and verified the model can be trusted to show realistic behaviour in relation to riser specifics. Now what is left is to test whether the wakeoscillator coupled to the pipe shows accurate behaviour, corresponding to the experimental results.

# D.1.2 Case 1, $V = 0.16 \text{ m s}^{-1}$ and Top Tension = 405 N



Figure D.1: Measured displacement for Case 1

Comparing the graphs in figures D.1, D.2 and D.3 give the following results:



Figure D.2: Calculated cross-flow and in-line response for Case 1



Figure D.3: Node displacement,  $C_L$  and  $C_D$  and Frequency spectrum for Case 1

- The cross-flow natural frequency that would be expected to be excited is the second, see Table D.3. Both the measurements and the model correspond to this.
- Both in-line and cross-flow mode shapes are the same in both measurements and model predictions
- Maximum cross-flow responds is measured to be  $\pm 0.8\,z/D$  and is calculated to be  $\pm 0.8\,z/D$
- Maximum in-line response is measured to be  $\pm 1.4 \, y/D$  and is calculated to be  $\pm 0.6 \, y/D$

# D.1.3 Case 3, $V = 0.31 \text{ m s}^{-1}$ and Top Tension = 457 N

Comparing the graphs in figures D.4, D.5 and D.6 give the following results:

• The cross-flow natural frequency that would be expected to be excited is the third or fourth, see Table D.3. Both the measurements and the model correspond to this.





Figure D.5: Calculated cross-flow and in-line response for Case 3



Figure D.6: Node displacement,  $C_L$  and  $C_D$  and Frequency spectrum for Case 3

- Both in-line and cross-flow mode shapes are the same in both measurements and model predictions
- Maximum cross-flow responds is measured to be  $\pm 0.9\,z/D$  and is calculated to be  $\pm 0.9\,z/D$
- Maximum in-line response is measured to be  $\pm 3.8 \, y/D$  and is calculated to be  $\pm 1.9 \, y/D$

# D.1.4 Case 6, $V = 0.6 \text{ m s}^{-1}$ and Top Tension = 670 N



Figure D.7: Measured displacement for Case 6



Figure D.8: Calculated cross-flow and in-line response for Case 6



Figure D.9: Node displacement,  $C_L$  and  $C_D$  and Frequency spectrum for Case 6

Comparing the graphs in figures D.7, D.8 and D.9 give the following results:

• The cross-flow natural frequency that would be expected to be excited is the sixth, see Table D.3. Both the measurements and the model correspond to this.

- Both in-line and cross-flow mode shapes are the same in both measurements and model predictions
- Maximum cross-flow responds is measured to be  $\pm 0.8\, y/D$  and is calculated to be  $\pm 0.8\, y/D$
- Maximum in-line response is measured to be  $\pm 8.5 z/D$  and is calculated to be  $\pm 4.5 z/D$

D.1.5 Case 9,  $V = 0.95 \text{ m s}^{-1}$  and Top Tension = 1002 N



Figure D.10: Measured displacement for Case 9



Figure D.11: Calculated cross-flow and in-line response for Case 9



Figure D.12: Node displacement,  $C_L$  and  $C_D$  and Frequency spectrum for Case 9

Comparing the graphs in figures D.10, D.11 and D.12 give the following results:

- The mode shape from the measurement isn't very clear, but if one look carefully 8 anti-nodes can be recognised, the same as the model predicts.
- Maximum cross-flow responds is measured to be  $\pm 1.0\,{}^{x/D}$  m and is calculated to be  $\pm 0.75\,{}^{y/D}$
- Maximum in-line response is measured to be  $\pm 13.0\,{}^{x/D}$  m and is calculated to be  $\pm 7.0\,{}^{y/D}$

#### D.1.6 Other VIV-models

Figure D.13 shows the results of the same VIV problem as calculated by other models.



Figure D.13: Comparison of different VIV models

# Appendix E

# Pipe-Wake model

# E.1 Introduction

To give more information on the influence of the various aspects of the model this appendix will demonstrate how the model is built up. Assuming one knows data about the seabed profile and pipeline specifics, then a perfect model would consist of the following steps:

- 1. Calculate static deflection of the pipe using static springs in vertical direction and a friction model in lateral directions
- 2. Define modeshapes and natural frequencies and decide how many modes will participate in the vibration
- 3. Create nonlinear soil spring model. Because the pipe will lift of the soil during some part of the vibration single contact springs are needed, which can couple and decouple from the model based on the pipeline position.
- 4. Model dynamic soil resistance in axial and lateral direction
- 5. Define the specifics of the gap under the free-span and adjust flow speed for each position along the pipe dependent on depth of the gap under each point
- 6. Calculate steady state vibration of the pipeline

To show the effects described above on the model, each step will be implemented separately. For comparison a predefined pipeline span is defined. Table E.1 shows the specifics of the pipeline and the seabed for the comparison and Figure E.1 illustrates the final set up. By inspecting the vibration after each step in the model it is clear what its effect is on the behaviour of the pipe. The pipeline under investigation is based on an existing pipeline to give realistic results. The L/D ratio is taken as 150. The reason for this is that for such a ratio the sag effect starts to come into effect [Soreide et al., 2001].

# E.2 Effect of static deflection

This section describes how a pipe without static deflection behaves under VIV. The aim of this Appendix section is give background information on the effect of static deflection.

### E.2.1 Linear free hanging pipe

Figure E.2 illustrates the free span for this step. Because it is a linear model, the mass does not effect the stiffness properties of the pipe and is applied as a distributed load instead of implemented as static deformation.

Parameter	Value	Unit
$L_{span}$	75	m
D	0.5	m
$\mathbf{t}$	0.025	m
${ m E}$	207	MPa
$\zeta$	0.02	-
Т	50	kN
$ ho_p$	7850	${ m kg}{ m m}^{-3}$
$ ho_c$	800	${ m kg}{ m m}^{-3}$
$m^*$	2.14	-
$C_a$	1	-

Table E.1: Pipeline specifics for model comparisons



Figure E.1: Illustration of the pipeline under investigation



Figure E.2: Model used for a geometrically linear free hanging pipeline

#### Results

For the checking of the results, the response in the first mode will be analysed. The first 5 natural frequencies in both cross-flow and in-line are taken into account for the analysis.

Also 150 elements are used, with a element length of 0.5 meter. Table E.2 shows the first 5 natural frequencies. Since the first natural frequency is  $2.4 \text{ rad s}^{-1}$ , a shedding frequency of the same value must be used. A flow speed of  $1.0 \text{ m s}^{-1}$  is used which leads to a shedding frequency of  $2.4 \text{ rad s}^{-1}$ . This should lead to good lock-in behaviour. All the results for the displacements, frequencies and coefficients will be taken for the steady state regime. This is taken to be the last 50 seconds of the time-domain solution. The top plots in Figure E.3 illustrate that this indeed the steady state regime.

Table E.2: Natural frequencies for the linear free hanging pipe

Mode	$\omega_{CF}  [\mathrm{rad}\mathrm{s}^{-1}]$	$\omega_{IL}  [\mathrm{rad}\mathrm{s}^{-1}]$
1	2.405	2.405
2	6.577	6.577
3	12.85	12.85
4	21.21	21.21
5	31.65	31.65



Figure E.3: Displacements and coefficients for a linear free hanging pipe



Figure E.4: Response frequencies at x = 37.5 for linear free hanging pipe

#### Natural frequencies

See table E.2:

• The natural frequencies are the same in CF and IL direction. This is logical as the properties of the beam is symmetric in both directions.

#### **Cross-flow vibration**

See figure E.3:

- Maximum amplitude  $A_z/D = 1.18$
- Mean deflection = -1.6 z/D

#### In-line vibration

See figure E.3:

- Maximum amplitude  $A_y/D = 0.095$
- Mean deflection =  $0.514 \, y/D$

#### **Response frequency**

See figure E.4:

- The crossflow response frequency at the peak in the spectrum is 2.4 rad s<sup>-1</sup>. This is expected as its located between the first natural frequency and the shedding frequency
- The inline response frequency is 4.85 rad s<sup>-1</sup>. This is also expected as its double the cross flow response frequency as explained in section 2.3

#### E.2.2 Nonlinear free hanging pipeline

To compare the geometrically linear pipe with a pipe under static deflection, this section gives the results for the same pipeline as above with the static analysis included. The static deflection is included in the same way as explained in Section 4.3.3. The model for this step is illustrated in Figure E.5.



Figure E.5: Model of the pipe for analysis including static deflection

#### Numerical implementation

The procedure for the calculation of displacements in the time-domain from Chapter C.4 has to be altered slightly. Because of the static deflection a degree of freedom is added to the system; the x-direction. This has a couple of effects on the model.

Firstly there is no need for a tension matrix as was used in the case of no static deflection. The alternative that is easier and better to use is adding a nodal tension force on the right most node (at the support). During the static analysis the tension will effect the deformation and thus the stiffness matrix.

Another implication is that the mass matrix has to be expanded. Whereas the stiffness matrix is equal to the secant stiffness matrix following from the virtual displacements procedure, the mass matrix has to be redefined. Two degrees of freedom have to be added, the x-direction in both nodes of the element. The matrix will take the following form:

$[M]\vec{\ddot{u}} = (m+m_a) \cdot$	$\frac{L}{420} \times$
-------------------------------------	------------------------

140	0	0	0	0	70	0	0	0	0	$\ddot{x}_1$
0	156	0	0	22L	0	54	0	0	-13L	$\ddot{y}_1$
0	0	156	-22L	0	0	0	54	13L	0	$\ddot{z}_1$
0	0	-22L	$4L^2$	0	0	0	-13L	$-3L^2$	0	$\ddot{\theta}_{y1}$
0	22L	0	0	$4L^2$	0	13L	0	0	$-3L^2$	$\ddot{\theta}_{z1}$
70	0	0	0	0	140	0	0	0	0	$\ddot{x}_2$
0	54	0	0	13L	0	156	0	0	-22L	$\ddot{y}_2$
0	0	54	-13L	0	0	0	156	22L	0	$\ddot{z}_2$
0	0	13L	-3L	0	0	0	22L	$4L^2$	0	$\ddot{\theta}_{y2}$
0	-13L	0	0	$-3L^2$	0	-22L	0	0	$4L^2$	$\ddot{\theta}_{z2}$

**Note:** For the solution procedure one has to realise that now also natural frequencies in the x-direction will be calculated, and thus also eigenmodes. This means more participating modes have to be taken into account. In addition to this, the displacements calculated in the system of differential equations will be relative to the new static position. When plotting the results, the static deformation has to be added to the calculated displacements.

#### Results

For the analysis of the nonlinear beam, a different flow speed is chosen than for the linear analysis. Reason for this is because the natural frequencies of the system are different now. A flow speed of  $1.6 \text{ m s}^{-1}$  is used, because it hits the first natural frequency in vertical direction. The same number of elements is used as for the previous analysis; 150 elements. To check whether the number of elements is sufficient for the calculation of the static deflection more elements have also been used, this did not lead to a significant difference for the static deflection, so 150 elements is enough.



Figure E.6: Static deflection for the nonlinear free hanging pipe

Table E.3: Natural frequencies for the linear free hanging pipe



Figure E.7: Displacements and coefficients for a nonlinear free hanging pipe

#### Static deflection

See figure E.6:

- The static deflection for the z-direction is -0.99 z/D
- The static deflection for the y-direction is 0.36 y/D

### Natural frequencies

See Table E.3:



Figure E.8: Response frequencies at x = 37.5 for nonlinear free hanging pipe

- Firstly the modes of vibration are linked in this depiction of the natural frequencies, this is because MatLab scales the eigenmodes so the modal mass matrix is and identity matrix. Because of this, e.g. the first cross-flow mode also leads to excitation in in-line direction and vice-versa. The natural frequencies in table E.3 are sorted such that the direction of the mode is the dominating mode for that particular frequency.
- The natural frequencies for the first 2 modes are significantly higher for the geometrically nonlinear beam than for the geometrically linear beam.

#### **Cross-flow vibration**

See Figure E.7:

- Maximum amplitude  $A_z/D = 1.07$
- Mean deflection = -0.913 z/D, slightly higher than the static deflection
- The CF displacements are similar for the nonlinear case in relation to the linear case, however the lock-in region is at a different shedding frequency

#### In-line vibration

See Figure E.7:

- Maximum amplitude  $A_y/D = 0.2714$
- Mean deflection =  $0.755 \, y/D$
- The in-line displacements are higher for the nonlinear beam in relation to the linear beam, this is due to cross-flow induced vibration, as will be illustrated in the section on response frequencies.

#### **Response frequency**

See Figure E.8:

- The cross-flow vibration is multimodal, but is dominated by the first mode at  $3.8 \text{ rad s}^{-1}$ .
- The in-line response frequency shows 2 distinct peaks, at 3.85 and 7.7 rad s<sup>-1</sup>. Instead of only vibrating at double the response frequency of the first mode, it is also vibrating at the same frequency of the first CF mode



Figure E.9: Cross-sectional displacements in the steady state regime

#### Conclusion

Changing from a geometrically linear model to a geometrically nonlinear model clearly has influence on the response of the system. The first natural frequencies are higher, which shifts the lock-in region of the system. Another implication is that the first inline natural frequency is very different from the first crossflow natural frequency, this difference is present in all symmetrical modes, but gets smaller as the natural frequency increases. While the amplitude of the cross-flow vibration doesn't change much, the inline behaviour does change both in amplitude and response frequency. This has to do with the phasing of the lift and drag coefficients. Figure E.9 shows the crosssection displacement halfway down the pipeline. Where the linear model shows a clear figure 8 shape, the nonlinear model does not.

Note: The nonlinear effect dies away as the pipe gets lighter and/or the span gets shorter.

#### **E.3** Linear soil behaviour

This section provides background information on the implications of linear soil behaviour, i.e. symmetric springs and dashpots, on the model.

#### E.3.1 Numerical implementation

#### Static analysis

A distinction has to be made between the dynamic and static analysis, because different principles apply. For the axial and lateral directions a friction model is applied, while for the vertical direction a spring model is applied. These models are explained in sections 4.4.1 and 4.4.7.

#### Element stiffness matrix

The global soil matrix is built up of soil element matrices, assembled in the same way as in Figure B.2. The only difference is that the soil element matrix only has to be added in the global matrix for nodes that are in contact with soil. The soil element matrix for the static calculation looks as follows:

$$[K_{z,stat}]\vec{u} =$$

[*TZ* 

#### Lateral Directions

In lateral direction the procedure is used as described in Section 4.4.7.

#### E.3.2 Dynamic Analysis

To include the soil stiffness in the model the stiffness matrix from the static analysis has to be expanded with a soil stiffness matrix. This matrix has the following form:

0	0	0	0	0	0	0	0	0	0
0	$K_L$	0	0	0	0	0	0	0	0
0	0	$K_V$	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	$K_L$	0	0	0
0	0	0	0	0	0	0	$K_V$	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

In addition to the spring stiffness, there is also soil damping present. The soil damping is added as external forces on the pipe. The damping force is equal to the value of the dashpot multiplied by the velocity of the pipe. This is done both in z- and y-direction. Because every time-step the velocity of the pipe changes, the force due to the dashpots is also update each time-step. Off course the dashpots are only present where the pipe is in contact with the soil, so the forces due to dashpots is always zero for nodes in the free span.

#### E.3.3 Results

For the soil a total pipe length of 175m is chosen with element lengths of 0.5m. The span length is still 75m. The flow speed is still kept at 1.6 m s<sup>-1</sup> as was the same in the nonlinear beam without soil. The soil type that will be used for the following analysis is *stiff clay*. This results in a dynamic spring coefficient of 4500 kN/m<sup>5/2</sup> and 3900 kN/m<sup>5/2</sup> for vertical and lateral directions respectively. The static spring stiffness is taken to be 1300 kN m<sup>-1</sup> and the friction coefficient is  $\mu = 0.2$ .

Table	E.4:	Natural	frequencies	for th	e pipe	on	linear	soi	1
-------	------	---------	-------------	--------	--------	----	--------	-----	---

Mode	$\omega_{CF}  [\mathrm{rad}\mathrm{s}^{-1}]$	$\omega_{IL}  [\mathrm{rad}\mathrm{s}^{-1}]$
1	3.423	2.581
2	6.422	6.323
3	11.88	11.66
4	18.97	18.63
5	27.23	27.79

#### Static deflection

See Figure E.10:



Figure E.10: Static deflection for the pipe on linear soil



Figure E.11: Displacements and coefficients for a pipe on linear soil



Figure E.12: Response frequencies at x = 100 for pipe on linear soil

- The static deflection for the z-direction is -1.54  $z/{\scriptscriptstyle D}$
- The static deflection for the y-direction is 0.79 y/D

• The static deflection especially in z-direction is larger than for previous cases. This is because at the shoulders near the free span the pipe also sinks into the soil. Herefore the span seems to be a bit larger giving it more room to sag in the middle.

#### Natural frequencies

See Table E.4:

• The natural frequencies are lower than in the previous cases. This is because the effective span length has increased as opposed to the situation without soil (section E.2). The effective span length is the distance between the two points on the pipe where the vertical displacement doesn't change. Previously this was at the boundaries, now however because of the presence of soil this point is somewhere between the boundary and the free span.

#### Cross-flow vibration

See Figure E.11:

- Maximum amplitude  $A_z/D = 1.14$
- Mean deflection = -1.43 z/D, slightly higher than the static deflection
- The CF displacements is slightly larger with soil present in relation to the span only. This is due to the fact that the effective length of the free span is larger with soil present. The natural frequency is however lower as well, which would indicate that the a lower flow speed is needed to hit the first mode completely. If the flow speed were lower then the amplitude of vibration would be even larger.

#### In-line vibration

See Figure E.11:

- Maximum amplitude  $A_y/D = 0.2722$
- Mean deflection = 1.29 y/D
- The IL displacements are slightly larger with soil present. The mean deflection is larger because of the drag amplification.

#### **Response frequency**

See Figure E.12:

- The crossflow response frequency has the peak in the spectrum at  $3.5 \text{ rad s}^{-1}$
- The in-line response frequency has peaks at 3.5 and 7.0 rad  $\mathrm{s}^{-1}$
- The first frequency lies between the shedding frequency  $(3.8 \text{ rad s}^{-1})$  and the first natural frequency in crossflow direction. The second inline frequency is double the first frequency.

# E.4 Nonlinear soil behaviour

This section provides results and numerical implementation of nonlinear soil behaviour as discussed in Chapter 4.4.1. The pipeline model for this section is shown in figure E.13.



Figure E.13: Model for the pipe with single action soil behaviour

#### E.4.1 Numerical implementation

To implement the nonlinear soil model, the relative position of the pipe has to be checked every time step. The soil force is dependent on two things: the z-position and the initial pipe position, see Figures 4.14. Firstly a selection vector that separates the nodes according to their initial position. This is done as follows:

$$SN_1 = \begin{cases} 1, & \text{if } z_0 < 0. \\ 0, & \text{otherwise.} \end{cases}$$
(E.3)

$$SN_2 = \begin{cases} 1, & \text{if } z_0 > 0. \\ 0, & \text{otherwise.} \end{cases}$$
(E.4)

Where:

 $SN_1$  = Selection vector for nodes that start below the seabed

 $SN_2$  = Selection vector for nodes that start above the seabed

 $z_0 =$ Initial position of the pipe [m]

Note: All nodes that are in the freespan and thus not in touch with soil are also made 0.

The next vector that is needed is  $U_c$ , which indicates the threshold at which the springs connect/disconnect. This threshold is dependent on the static soil force for nodes which static position is below seabed and on the initial position of nodes above the seabed. It is calculated as follows:

$$U_{c} = \frac{K_{v,s} \cdot \vec{z_{0}}}{K_{dyn}} \cdot S\vec{N}_{1} + \vec{z_{0}} \cdot S\vec{N}_{2}$$
(E.5)

The last parameter that is needed before the system of differential equation can be solved is the static soil force needed to compensate for gravity when the springs disconnect. This is done as follows:

$$F_{z,s} = \vec{z_0} \cdot K_{v,s} \cdot L_{element} \tag{E.6}$$

These vectors are transferred to the differential equation from section C.4. In this loop for each time step the following selection vectors are made. These vectors are dependent on the z-position of the pipe in each time step and decide whether the springs are (de)coupled:

$$z_{c,1} = \begin{cases} 1, & \text{if } z - U_c < 0. \\ 0, & \text{otherwise.} \end{cases}$$
(E.7)

$$z_{c,2} = \begin{cases} 1, & \text{if } z - U_c > 0. \\ 0, & \text{otherwise.} \end{cases}$$
(E.8)

The calculation of the soil spring forces is then done as in Equation E.9 (the first equation). The soil damping forces are solved in the second equation in E.9 and are dependent on velocity. These forces are only active when the pipe is in contact with the bottom. The equations are solved for each time step, because  $\vec{z}_{c,1,2}$  is updated each time step. After this using the procedure from equation (C.16) onwards is used to add the soil forces in the differential equation.

$$\vec{F}_{s,z,k} = \vec{z}_{c,1} \cdot \vec{z} \cdot K_{dyn} \cdot \vec{SN}_1 + \vec{z}_{c,1} \cdot (\vec{z} + \vec{U}_c) \cdot K_{dyn} \cdot \vec{SN}_2 + \vec{z}_{c,2} \cdot \vec{F}_{z,s} \vec{F}_{s,z,c} = \vec{z}_{c,1} \cdot \vec{z} \cdot C_{s,z} \cdot (\vec{SN}_1 + \vec{SN}_2)$$
(E.9)

Where:

 $\vec{F}_{s,z,k} =$  Nodal soil force due vertical soil stiffness  $\vec{F}_{s,z,k} =$  Nodal soil force due vertical soil damping  $\vec{K}_{s,z} =$  Soil stiffness coefficient in each node  $\vec{C}_{s,z} =$  Soil damping coefficient in each node

In y-direction the springs and dashpots are always connected, so there is no need for the selection vectors. The values for the stiffness and damping terms are different however. The values can be found in section E.3.

#### E.4.2 Results

The total length of the pipe used is 175 meters in this case, with an element length of 0.5 meter and a span length of 75m. The soil parameters are the same as for the linear soil behaviour. An important thing to note is that the soil springs now also act as an external force. Because of this, the mode shapes don't include the presence of the soil. This also leads to a dramatic reduction in the values of the natural frequencies, which has two implications: the natural frequencies calculated don't give correspond to the actual mode of vibration of the span and much more modes have to be included in the analysis. In this case 30 modes participate in the vibration. This value is not based on anything physical, but is large enough to give a good description of the vibration.



Figure E.14: Static deflection for the pipe with asymmetric soil behaviour



Figure E.15: Displacements for a pipe with asymmetric soil behaviour



Figure E.16: Response frequencies at x = 87.5 for pipe with asymmetric soil behaviour

#### Static deflection

See Figure E.14:

- The static deflection for the z-direction is  $-1.54 \ z/D$
- The static deflection for the y-direction is 0.79 y/D
- The approach for the static deflection is exactly the same as for the case of linear soil behaviour

#### Natural frequencies

No conclusion can be taken from the calculated natural frequencies. The reason for this is that the span for which the frequencies are calculated is now 175m long. It is expected that the apparent natural frequencies of the 75m free span is very similar to the ones calculated in table E.4. Most likely they will be slightly lower, because the effective span length is longer due to the upwards motion of the pipeline. But again, the frequencies of the span itself cannot be calculated analytically.

#### **Cross-flow vibration**

See Figure E.15:

- Maximum amplitude  $A_z/D = 1.13$
- Mean deflection = -1.43 z/D, slightly higher than the static deflection
- The CF displacements in the free span are similar to the previous case. At the shoulders however the pipe clearly lifts off the soil, which is different from the previous case

#### In-line vibration

See Figure E.18:

- Maximum amplitude  $A_y/D = 0.304$
- Mean deflection = 1.26 y/D
- The IL displacements are slightly lower than previously.

#### **Response frequency**

See Figure E.16:

- The crossflow response frequency has the peak in the spectrum at 3.5 rad  $\rm s^{-1}$
- The in-line response frequency has peaks at 3.5 and 7.0 rad  $s^{-1}$
- The frequencies are the same as for the case of linear soil springs.

## E.5 Suction model

This section provides background information for the soil suction model. The procedure for the suction model is described in Chapter 4.4.2.

#### E.5.1 Numerical implementation

The essence of the suction model is that it compares symmetric springs to the suction model. If the work done by symmetric springs is larger than by the suction model, then the nodes would disconnect at a new threshold level:  $z_s$ . This requires some manipulation of the differential equation. For each loop of the model, the nodes that stay connected are called: *soil nodes* and nodes that disconnect are *suction nodes*. Because the soil nodes do not disconnect, they are included in the calculation of the natural frequencies. The consequence of this is that they don't need to be included as external soil forces.

For the suction nodes this is not the case. The calculation of the forces in these nodes is very similar to the case of nonlinear springs in the previous section. The only difference is made is to the selection vectors:  $z_{c,1}$  and  $z_{c,2}$ . In the previous case, e.g.  $z_{c,1}$  looked like:

$$z_{c,1} = \begin{cases} 1, & \text{if } z - U_c < 0. \\ 0, & \text{otherwise.} \end{cases}$$
(E.10)

For suction nodes the threshold level for (dis)connection is made at  $z_s$ . To include this the new selection vector is:

$$z_{c,1} = \begin{cases} 1, & \text{if } z - U_c - z_s < 0. \\ 0, & \text{otherwise.} \end{cases}$$
(E.11)

The same principle applies for  $z_{c,2}$ .

#### E.5.2 Results

The total pipe length for this run was 175m, with elements of 0.5m long and the freespan at 75m long. The flow speed is kept at 1.6 m s<sup>-1</sup>. For the soil an extra parameter is needed, the undrained shear strength  $(s_u)$ . To keep it simple a value is chosen, similar to what is used in experiments done on a steel catenary riser in a harbour;  $s_u = 1$  kPa [Bridge, 2005]. The rest of the values are the same as for the linear soil behaviour case. To come to a final result 4 iterations of the loop were needed. After the last loop the first 6 nodes on either side of the span are modelled as suction nodes. This is equal to 3 metres. The results shown in this section are the results for the last iteration.



Figure E.17: Static deflection for the pipe on linear soil

Mode	$\  \omega_{CF}  [\mathrm{rad}\mathrm{s}^{-1}]$	$\omega_{IL}  [\mathrm{rad}\mathrm{s}^{-1}]$
1	3.15	2.33
2	5.72	5.64
3	10.6	10.3
4	16.8	16.5

24.1

24.5

4 5

Table E.5: Natural frequencies for the pipe with suction model



Figure E.18: Displacements and coefficients for a pipe with suction model



Figure E.19: Response frequencies at x = 87.5 for pipe with suction model

#### Static deflection

See figure E.17:

- The static deflection for the z-direction is  $-1.43 \ z/D$
- The static deflection for the y-direction is 0.79 y/D

#### Natural frequencies

See Table E.5:

• The natural frequencies are a little lower than for the case of linear soil behaviour. This is because in the last iteration the effective freespan is longer, due to the presence of the suction nodes right next to the span. This length increase is 3 metres at either side. The forces in the suction nodes are applied as external forces and are therefore not present in the calculation of the natural frequencies.

#### **Cross-flow vibration**

See Figure E.18:

- Maximum amplitude  $A_z/D = 1.12$
- Mean deflection = -1.43 z/D, slightly higher than the static deflection
- The CF displacements are similar to both linear and nonlinear springs

#### In-line vibration

See Figure E.18:

- Maximum amplitude  $A_y/D = 0.284$
- Mean deflection = 1.26 y/D
- The CF displacements are similar to both linear and nonlinear springs

#### **Response frequency**

See Figure E.19:

- There are more frequencies present in the vibration than in the previous cases. There are however still definite dominating modes.
- The dominating crossflow response frequency has the peak in the spectrum at 3.5  $\rm rad\,s^{-1}$
- The dominating in-line response frequency has peaks at 3.5 and 7.0 rad s<sup>-1</sup>
- The dominating frequencies are the same as for the case of linear and nonlinear soil springs.

### E.6 Gap influence

#### E.6.1 Numerical implementation of bottom impact

To include the impact of the pipe on the bottom a new calculation step is added in the differential equation. At each timestep the distance between the pipe and the seabed is calculated for each node:

$$\vec{z}_{bi} = \vec{z} - \vec{H}_G \tag{E.12}$$

Where:

 $H_G = \text{Height of initial gap } [m]$ 

 $\vec{z}$  = Vertical position of each node [m]

 $\vec{z_{bi}}$  = Vertical position of each node relative to the bottom gap [m]

If  $z_{bi}$  is negative, that means that the pipe is penetrating the seabed. If positive, it is not, therefore at each time step each positive value is set equal to 0. All negative values are equal to the displacement relative to the seabed at that time step. To get the impact force, the next step is to multiply  $z_{bi}$  by the spring stiffness of soil and by the flow vector  $(\vec{FV})$ . The flow vector is a vector of zeros and ones, where each node where node that is not in the free span is equal to 0 and each node that is present in the freespan is equal to 1. This last step is needed, because otherwise there would be bottom impact forces at nodes which already have soil forces acting on them.

$$\vec{F}_{k,bi} = \vec{z}_{bi} \cdot K_Z \cdot \vec{FV} \tag{E.13}$$

The same principle applies for damping, except that it uses the velocities instead of the displacements. Because of this the vector that decides which nodes have damping and which don't has to be altered a little. Instead of making all positive values 0 and leave all negative values as the displacements, the negative values are made equal to 1. This vector of zeros and ones is called:  $dz_{bi}$ . The damping force is thus:

$$\vec{F}_{c,bi} = \vec{dz}_{bi} \cdot \vec{z} \cdot C_Z \cdot \vec{FV} \tag{E.14}$$

#### E.6.2 Results

The model for this section is the same as the previous version of the model, except for the gap effect. Two gaps heights have been chosen, one for no bottom interaction and one for bottom interaction. The heights for the two cases are 6D and 2D respectively. The slope angle of the gap is 25 degrees. For the case without bottom interaction 15 participating modes where used, by iteration this has been verified to be enough. For bottom interaction 50 modes were used, to account for deformation of the pipe when it hits the bottom, see Figure E.20.



Figure E.20: Possible occurrence of pipe deformation by impact

#### E.6.3 No bottom impact

As there is no bottom impact, the only change with the previous case is the flow velocity. All discrepancies with the previous case are linked to this. Figure E.21 shows the flow velocities for this case. The reason that flow velocities in the middle of the span are lower than at the edges is because that part is closer to the seabed.



Figure E.21: Flow velocities



Figure E.22: Static deflection for the pipe with a gap of 6D



Figure E.23: Displacements and coefficients for a pipe with a gap of 6D



Figure E.24: Response frequencies at x = 137.5 for a pipe with a gap of 6D

#### Static deflection

See Figure E.22 for the results of the static deflection calculation:

• The static deflection for the z-direction is -1.704 z/D
- The static deflection for the y-direction is 0.887 y/D
- The static deflection is the same as for the previous case

### **Cross-flow vibration**

See Figure E.23:

- Maximum amplitude  $A_z/D = 1.001$
- Mean deflection = -1.62 z/D, slightly higher than the static deflection
- The crossflow vibrations slightly different than for the previous case. This has to do with the change in flow velocities along the pipe.

### In-line vibration

See Figure E.23:

- Maximum amplitude  $A_y/D = 0.199$
- Mean deflection = 1.14 y/D
- Slight discrepancy with the previous case, this also is down to the change in flow velocity

#### **Response frequency**

See Figure E.24:

• There are multiple frequencies present in the vibration. Besides the influence of nonlinear soil, the variation of shedding frequencies along the pipe have influence on this.

### E.6.4 Bottom impact

In this case both the change in flow velocity as well as the bottom impact influence the vibration. Because the middle of the pipe is close to the seabed the change of flow velocities along the pipe show more variation than in the previous case, see Figure E.25.

#### Static deflection

See Figure E.26 for the results of the static deflection calculation:

- The static deflection for the z-direction is -1.704 z/D
- The static deflection for the y-direction is 0.49 y/D
- The static deflection is lower in y-direction, because in the middle of the span the flow velocities are lower.



Figure E.25: Flow velocities for a gap of 2D



Figure E.26: Static deflection for the pipe with a gap of 2D



Figure E.27: Displacements and coefficients for a pipe with a gap of 2D

### **Cross-flow vibration**

See Figure E.27 for the displacements:



Figure E.28: Response frequencies at x = 137.5 for pipe with a gap of 2D

- Maximum amplitude  $A_z/D = 0.384$
- Mean deflection = -1.61 z/D, slightly higher than the static deflection
- The bottom clearly influences the vibration. There is a restriction in downwards motion, while in upwards motion the pipe can go freely. The amplitude is smaller due to this and also due to the discrepancy in shedding frequencies.

#### In-line vibration

See Figure E.27:

- Maximum amplitude  $A_y/D = 0.052$
- Mean deflection = 0.59 y/D
- Smaller vibration than before, again due to the flow velocities.

#### **Response frequency**

See Figure E.28:

• There are multiple frequencies present in the vibration, more than in the previous cases. This is because the pipe is being deformed when it impacts the bottom.

### E.7 Pipe-Wake multispan results

This section gives some results for the Pipe-Wake model. This is done for a multi-span case. The properties of the pipeline are shown in Table E.6. The soil chosen is : *very stiff clay.* The aim of this section is to give an impression of the Pipe-Wake model. A total of 65 cases are shown, these are depedent on support length and flow speed. Only cross-flow vibration is shown, because for cross-flow the suction effect is most apparent.

- Span 1 =75m, Span 2 =25m, Support length = 5m,  $V_{flow} = 1 \text{ m s}^{-1}$
- Span 1 =75m, Span 2 =25m, Support length = 1m,  $V_{flow} = 1 \text{ m s}^{-1}$

Parameter	Value	Unit
D	0.5	m
$\mathbf{t}$	0.025	m
Ε	207	MPa
$\zeta$	0.02	-
Т	50	kN
$ ho_p$	7850	${\rm kg}{\rm m}^{-3}$
$ ho_c$	0	${\rm kg}{\rm m}^{-3}$

Table E.6: Pipeline specifics for model comparisons

- Span 1 =75m, Span 2 =25m, Support length = 1m,  $V_{flow} = 1 \text{ m s}^{-1}$ , flow only hits the *left* span
- Span 1 =75m, Span 2 =25m, Support length = 1m,  $V_{flow} = 1 \text{ m s}^{-1}$ , flow only hits the *right* span
- Span 1 =75m, Span 2 =75m, Support length = 1m,  $V_{flow} = 1 \text{ m s}^{-1}$
- Span 1 =75m, Span 2 =75m, Support length = 1m,  $V_{flow} = 1 \text{ m s}^{-1}$ , flow only hits the *left* span

Case 1, Span 1 =75m, Span 2 =25m, Support length = 5m,  $V_{flow} = 1 \text{ m s}^{-1}$ 



Figure E.29: Cross-flow response envelope and response of middle node for Case 1



Figure E.30: Close-up on support for Case 1

```
Case 2, Span 1 =75m, Span 2 =25m, Support length = 1m, V_{flow} = 1 \text{ m s}^{-1}
```



Figure E.31: Cross-flow response envelope and response of middle node for Case 2

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Figure E.32: Cross-flow response envelope and response of middle node for Case 2





Figure E.33: Cross-flow response envelope and response of middle node for Case 3

Case 4, Span 1 =75m, Span 2 =25m, Support length = 1m,  $V_{flow} = 1 \text{ ms}^{-1}$ , flow only hits the right span



Figure E.34: Cross-flow response envelope and response of middle node for Case 4

Case 5, Span 1 =75m, Span 2 =75m, Support length = 1m,  $V_{flow} = 1 \text{ m s}^{-1}$ 



Figure E.35: Cross-flow response envelope and response of middle node for Case 5





Figure E.36: Cross-flow response envelope and response of middle node for Case 6

### E.7.1 Remarks

Regarding the plots above a couple of remarks can be made:

- Because for the case of the different length spans the left span dominates the vibration it does not matter whether both spans or only the left span is hit with flow. If the right span is hit in this case however, the whole pipe hardly vibrates. This is because the natural frequency for the right span would be much higher than for the left.
- Decreasing the support leads to more interaction of the two spans in the case of different span lengths.
- If both spans are of equal length, whether the flow hits 1 or both spans has slight effect on the vibration. The effect is far less than for spans of different lengths.

### Appendix F

## Virtual displacements

In this section the method of virtual displacements for a finite element beam is shortly explained. The explaination is written for a 2D beam. The approach is the same for a 3D beam however. Figure F.1 shows reference system used in this chapter.



Figure F.1: Axes system

The principle of virtual displacements states that: the external virtual work due to any admissible virtual displaced state is equal to the internal virtual work due to the same virtual displaced state. In equation this would look lide:

$$W_{int} = W_{ext} \tag{F.1}$$

For a beam element the internal virtual work consists of a part due to bending and a part due to extension. If L = element length, the equation for internal virtual work reads as follows:

$$W_{int} = \int_0^L EI\kappa \,\delta\kappa \,\delta x + \int_0^L EA\epsilon \,\delta\epsilon \,\delta x \tag{F.2}$$

Where:

 $\begin{aligned} \kappa &= \text{Curvature} \\ \epsilon &= \text{Strain} \end{aligned}$ 

Assuming that EI and EA are constant over the element length, the curvature and strain are the variables that differ over x. For a 2D beam these equations are shown below in both global coordinates and nodal coordinates:

$$\kappa = \frac{x'z'' - z'x''}{(x'^2 + z'^2)^{3/2}} = \frac{(1+u')w'' - w'u''}{((1+u')^2 + w'^2)^{3/2}}$$
(F.3)

$$\epsilon = \sqrt{x^{\prime 2} + z^{\prime 2}} - 1 = \sqrt{(1 + u^{\prime})^2 + w^{\prime 2}} - 1$$
 (F.4)

The *cubic Hermite spline* is used to interpolate the vertical displacements of a point in the beam and linear interpolation is used for horizontal displacements. In matrix form this looks like:

$$\begin{bmatrix} u(x)\\ u'(x)\\ u'(x)\\ w(x)\\ w'(x)\\ w'(x)\\ w'(x)\\ w''(x)\\ w''(x)\\ w''(x)\\ \end{bmatrix} = \begin{bmatrix} \frac{L-x}{L} & 0 & 0 & \frac{1}{L} & 0 & 0 \\ \frac{-1}{L} & 0 & 0 & \frac{1}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 - \frac{3}{L^2}x^2 - \frac{2}{L^3}x^3 & x - \frac{2}{L}x^2 + \frac{1}{L^2}x^3 & 0 & \frac{3}{L^2}x^2 + \frac{2}{L^3}x^3 & \frac{2}{L}x^2 + \frac{1}{L^2}x^3 \\ 0 & -\frac{6}{L^2}x - \frac{6}{L^3}x^2 & 1 - \frac{4}{L}x + \frac{3}{L^2}x^2 & 0 & \frac{6}{L^2}x + \frac{6}{L^3}x^2 & \frac{4}{L}x + \frac{3}{L^2}x^2 \\ 0 & -\frac{6}{L^2} - \frac{12}{L^3}x & 1 - \frac{4}{L} + \frac{6}{L^2}x & 0 & \frac{6}{L^2} + \frac{12}{L^3}x & \frac{4}{L} + \frac{6}{L^2}x \end{bmatrix} \begin{bmatrix} u_1 \\ w_1 \\ w_1 \\ w_1 \\ w_2 \\ w_2 \\ w_2 \end{bmatrix}$$
(F.5)

Using these expressions,  $\kappa$ ,  $\delta\kappa$ ,  $\epsilon$  and  $\delta\epsilon$  can be defined in each point along the element. By rewriting the Equation (F.2) in terms of U and  $\delta U$ , in which U is a vector of the nodal displacements and  $\delta \vec{U}$  are the virtual displacements, the following expression for virtual work is created:

$$W_{int} = EI \int_0^L \delta \vec{U}^T [\delta \kappa(U)]^T [\kappa(U)] \vec{U} dx + EA \int_0^L \delta \vec{U}^T [\delta \epsilon(U)]^T [\epsilon(U)] \vec{U} dx$$
  
$$= \delta \vec{U}^T \left( EI \int_0^L [\delta \kappa(U)]^T [\kappa(U)] dx + EA \int_0^L [\delta \epsilon(U)]^T [\epsilon(U)] dx \right) \vec{U}$$
(F.6)  
$$= \delta \vec{U}^T [K(U)] \vec{U}$$

Where:

[K(U)] = Stiffness matrix dependent on nodal coordinates (U)

If this expression is filled into Equation (F.1) and with the external work also written down in terms of virtual displacements, the equation for the balance of internal and external work is:

$$\delta \vec{U}^T[K(U)] \vec{U} = F_{ext} \,\delta \vec{U} \to [K(U)] \,\vec{U} = F_{ext} \tag{F.7}$$

A  $\vec{U}$  has to be found that satisfies this equation. But because small perturbations of the external force influence the internal force this has to be calculated using iterations. If a small perturbation is applied, Equation (F.7) becomes:

$$[K(U_0 + \Delta U)](\vec{U}_0 + \Delta \vec{U}) = F_{ext} + \Delta F$$
(F.8)

This equation is expanded using a first order Taylor series expansion:

$$[K(U_0)]\vec{U_0} + \frac{\delta[K(U_0)]\vec{U_0}}{\delta\vec{U}}\Delta\vec{U} = F_{ext} + \Delta F$$
(F.9)

Where:

$$\frac{\delta[K(U_0)]\vec{U_0}}{\delta\vec{U}} = \text{Tangent stiffness matrix } [T(U_0)]$$

This expression is used in the Newton-Raphson loop to calculate the final position of the beam. Given that the initial position is horizontal and initial force  $F_{ext} = 0$ , then  $\Delta F$  is the external force that is applied. The stiffness matrix  $(K[U_0])$  and tangent stiffness matrix  $(T[U_0])$  are only dependent on the position of the beam, *before* the load is applied. The following equations are then solved in this order to calculate the next position of the beam:

1. 
$$\vec{R}_1 = \Delta F - [K(U_0)]\vec{U}_0$$
  
2.  $\Delta U = [T(\vec{U}_0)]^{-1}\vec{R}_1$  (F.10)  
3.  $\vec{U}_1 = \vec{U}_0 + \Delta \vec{U}$ 

When the difference between to iterations is below a certain threshold, then the solution is said to have converged and the final pipe position is known. The tangent stiffness matrix for that pipe position is the matrix for displacements in relation to the final pipe position. Therefore it is the stiffness used for the calculation of the dynamic displacements.

## Appendix G

# Time-domain analysis results

### G.1 Amplitudes and soil pressures for different support types

### 1. Results for case of distributed springs



Figure G.1: Relative amplitudes of vibration for distributed springs



Figure G.2: Amplitudes of vibration for each span in case of distributed springs



Figure G.3: Soil pressures for DS

### 2. Results for case of a 2-node support



Figure G.4: Relative amplitudes of vibration for 2 nodes



Figure G.5: Amplitudes of vibration for each span in case of 2 nodes



Figure G.6: Soil pressures for 2N

### 3. Results for case of a 1-node support



Figure G.7: Relative amplitudes of vibration for 1 contact node



Figure G.8: Amplitudes of vibration for each span for 1 contact node



Figure G.9: Soil pressures for 1N

### G.2 Mode shapes for different support lengths



Figure G.10: 2nd mode shapes for various support lengths

## Appendix H

# Dynamic interaction results

H.1 Accelerations for  $V = 1.5 \text{ ms}^{-1}$ 



Figure H.1: Accelerations for V=1.5  ${\rm m\,s^{-1}}$ 

### H.2 Nodal vibrations for V = 0.7 and 1.5 ms<sup>-1</sup>



(c) Nonlinear rock behaviour

Figure H.2: Vibration of nodes halfway span 1 and span 2 for  $V = 0.7 \text{ m s}^{-1}$ 



(a) Linear sandy soil behaviour



(b) Nonlinear sandy soil behaviour



Figure H.3: Vibration of nodes halfway span 1 and span 2 for  $V=1.5 \text{ m s}^{-1}$