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AN EXACT SOLUTION TO THE COMPRESSIBLE LAMINAR BOUNDARY-LAYER EQUATION FOR THE FLAT PLATE WITH CONSTANT HEAT FLUX

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SUMMARY

An exact solution is found to Chapman and Rubesin's transformed laminar boundary-layer equations, in the case of a flat plate with constant heat-flux. It is shown that the ratio of the heat-transfer coefficients for constant heat-flux and for constant temperature (i.e. isothermal case) is a constant independent of the Reynolds number, Mach number and Prandtl number. This property indicates that experimental results are obtained with a constant heat-flux technique which are simply related to results that could be obtained with the usual isothermal method.

The theory is experimentally checked by using an improved steady state technique derived from Seban's, which gives "uncorrected" data that are in agreement with the theory to within 10% at M = 2 and 20% at low speed.

INTRODUCTION

There exist two simple limiting cases in which the study of heat-conduction is eased : the isothermal case (i.e. constant wall temperature) and the case where heat is uniformly dissipated at the surface (i.e. constant heat-flux).

The former is very well known, specially in the case of a flat wall for which an exact solution of the boundary-layer equation is known (see for example ref.1). Moreover, most of the experiments are made with isothermal conditions.

The latter is not so well known and the purpose of the present study is to solve the laminar boundary-layer equations in the case of a flat plate with constant heat-transfer.

In particular, it is interesting as a first-step to relate the corresponding heat-transfer coefficient to the one obtained in the isothermal case, with a view to developing a steady-state technique of heat-transfer measurement which could be used to study more complicated types of flow.

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SOLUTION TO THE BOUNDARY-LAYER EQUATIONS

The problem consists in finding a solution to the boundary-layer equations for the supersonic flow over a flat surface on which heat is uniformly dissipated. One seeks the velocity and temperature profiles in the boundary-layer as well as the temperature distribution, heat-transfer and skin-friction coefficients at the wall.

The problem is treated in a way very similar to that used by Chapman and Rubesin (ref.1) who gave an exact solution for the flow over a "compressible" flat plate in the case of a polynomial wall temperature distribution and in particular for the isothermal case. The transformed boundary-layer equations given by Chapman and Rubesin are used here and solved for our particular boundary conditions.

The two-dimensional compressible boundary-layer equations are, in the case of zero pressure gradient :

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y} (\mu \frac{\partial u}{\partial y})$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

$$\rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial y} = \frac{1}{P_r} \frac{\partial}{\partial y} (\mu \frac{\partial T}{\partial y}) + \frac{\mu}{c_p} (\frac{\partial u}{\partial y})^2$$

By using von Mises' transformation, they are rewritten as :

$$\frac{\partial u}{\partial x} = \frac{\mu_\infty}{\rho_\infty} \frac{\partial}{\partial \psi} (Cu \frac{\partial u}{\partial \psi})$$

$$\frac{\partial T}{\partial x} = \frac{\mu_\infty}{P_r \rho_\infty} \frac{\partial}{\partial \psi} (Cu \frac{\partial T}{\partial \psi}) + \frac{\mu_\infty Cu}{c_p \rho_\infty} (\frac{\partial u}{\partial \psi})^2$$

where ψ is the stream function defined by

$$u = \frac{\rho_\infty}{\rho} \frac{\partial \psi}{\partial y} \; ; \quad v = - \frac{\rho_\infty}{\rho} \frac{\partial \psi}{\partial x} \qquad (1.1);(1.2)$$

and C is a coefficient introduced by :

$$\frac{\mu}{\mu_\infty} = C \frac{T}{T_\infty}$$

with (from Sutherland's formula) :

$$C = \sqrt{\frac{\overline{T}_w}{T_\infty}} \; \frac{T_\infty + S}{\overline{T}_w + S}$$

$\overline{T}_w$ is a mean wall temperature, S is a constant equal to 102.5 and the subscript infinity is related to free-stream conditions.

Finally, by introducing dimensionless quantities such as :

$$\psi^* = \frac{\psi}{\sqrt{\nu_\infty u_\infty L C}} \; ; \; u^* = \frac{u}{u_\infty} \; ; \; T^* = \frac{T}{T_\infty} \; ; \; x^* = \frac{x}{L}$$

(where L is the length of the flat plate), one gets the following system of equations :

$$\frac{\partial u^*}{\partial x^*} = \frac{\partial}{\partial \psi^*}\left(u^* \frac{\partial u^*}{\partial \psi^*}\right) \tag{2}$$

$$\frac{\partial T^*}{\partial x^*} = \frac{1}{P_r} \frac{\partial}{\partial \psi^*}\left(u^* \frac{\partial T^*}{\partial \psi^*}\right) + (\gamma-1)M_\infty^{\,2} u^* \left(\frac{\partial u^*}{\partial \psi^*}\right)^2 \tag{3}$$

By introducing a function $f(\eta)$ defined by

$$\psi = \sqrt{\nu_\infty u_\infty C x}\; f(\eta) \tag{4}$$

or

$$f(\eta) = \psi^*/\sqrt{x^*}$$

Chapman showed that the momentum equation (2) was satisfied by

$$u^* = \frac{1}{2} f'(\eta) = \frac{1}{2}\frac{df}{d\eta} \tag{5}$$

if f is the Blasius function - i.e. if f satisfies the following non linear differential equation

$$f f'' + f''' = 0$$

The solution (5) for the velocity profile differs from the incompressible one, only through the expression for η which remains to be determined.

By using (5) and by changing the coordinated from (x^*, ψ^*) into (x^*, η) Chapman finally wrote the energy equation (3) as :

$$\frac{\partial^2 T^*}{\partial \eta^2} + P_r f \frac{\partial T^*}{\partial \eta} - 2 P_r f' x^* \frac{\partial T^*}{\partial x^*} = -\frac{P_r}{4}(\gamma-1)M^2_\infty (f'')^2 \tag{6}$$

The solution to equation (6) must satisfy the following boundary conditions :

$$\psi^* = 0 \quad \text{or} \quad \eta = 0 \; ; \; q = -k_w\left(\frac{\partial T}{\partial y}\right)_w = \text{constant} \tag{7.1}$$

$$\psi^* = \infty \quad \text{or} \quad \eta = \infty \; ; \; T^* = 1 \tag{7.2}$$

which differs from Chapman's conditions

$$\psi^* = 0 \qquad T^*_w = T^*_{wa} + \sum_0^\infty a_n x^{*n} \tag{8.1}$$

$$\psi^* = \infty \qquad T^* = 1 \tag{8.2}$$

where T_{wa} is the adiabatic wall temperature and a_n are given constants.

A particular solution to the non-homogeneous equation (6) is obtained by considering the case where $q = 0$ (i.e. the adiabatic case). It is identical to the particular solution found by Chapman ($a_n = 0$), that is

$$T^*(\eta) = 1 + \frac{\gamma-1}{2} M^2_\infty r(\eta) \tag{9.1}$$

where the function $r(\eta)$ is given from figure 1. In particular, at the wall ($\eta = 0$)

$$T^*_{wa} = 1 + \frac{\gamma-1}{2} M^2_\infty r(o) \tag{9.2}$$

where $r(o)$ is the so-called recovery factor (equal to $\sqrt[2]{Pr}$).

At this point our solution starts to differ from Chapman's solution because the general solution of the homogeneous part of equation (6) must satisfy the condition (7.1) instead of (8.1).

SOLUTION TO THE HOMOGENEOUS EQUATION

a. Boundary conditions

The homogeneous part of equation (6) is :

$$\frac{\partial^2 T^*}{\partial \eta^2} + P_r f \frac{\partial T^*}{\partial \eta} - 2 P_r f' x^* \frac{\partial T^*}{\partial x^*} = 0 \qquad (10)$$

together with the following boundary conditions

$$\eta \to \infty \qquad T^* = 1 \qquad (7.2)$$

$$\eta \to 0 \qquad q = -k_w \left(\frac{\partial T}{\partial y}\right)_w = \text{constant} \qquad (7.1)$$

In order to rewrite the second condition in terms of the variables that are used in (10), we first try to relate η to y.

We introduce (4) in (1.1) :

$$u = \frac{\rho_\infty}{\rho} \sqrt{\nu_\infty u_\infty C x} \; \frac{\partial f}{\partial y}$$

or

$$u = \frac{\rho_\infty}{\rho} \sqrt{\nu_\infty u_\infty C x} \; \frac{df}{d\eta} \frac{\partial \eta}{\partial y}$$

But

$$u = \frac{1}{2} f' u_\infty \text{ and } f' = \frac{df}{d\eta}$$

therefore

$$\frac{1}{2} u_\infty f' = \frac{\rho_\infty}{\rho} \sqrt{\nu_\infty u_\infty C x} \; f' \frac{\partial \eta}{\partial y}$$

Because the pressure is constant in the whole flow field, the state equation gives

$$\frac{\rho_\infty}{\rho} = \frac{T}{T_\infty} = T^*$$

Thus

$$\frac{\partial \eta}{\partial y} = \frac{1}{2T^*} \sqrt{\frac{u_\infty}{\nu_\infty x C}} \qquad (11)$$

The condition (7.1) is then written as :

$$q = -k_w \left(\frac{\partial T}{\partial \eta}\right)_w \left(\frac{\partial \eta}{\partial y}\right)_w = -k_w \left(\frac{\partial T}{\partial \eta}\right)_w \sqrt{\frac{u_\infty}{\nu_\infty x C}} \; \frac{1}{2T_w^*}$$

As we assume constant values of P_r and c_p, we have

$$\frac{k}{\mu} = \text{constant} \quad \text{i.e.} \quad \frac{k_w}{\mu_w} = \frac{k_\infty}{\mu_\infty}$$

Therefore

$$\frac{k_w}{k_\infty} = \frac{\mu_w}{\mu_\infty} = C \frac{T_w}{T_\infty} = C\, T_w^*$$

or

$$\frac{k_w}{T_w^*} = C k_\infty$$

Thus

$$q = -T_\infty \left(\frac{\partial T^*}{\partial \eta}\right)_w \sqrt{\frac{u_\infty}{\nu_\infty x C}} \; \frac{C k_\infty}{2} = -\frac{k_\infty T_\infty}{2} \left(\frac{\partial T^*}{\partial \eta}\right)_w \sqrt{\frac{u_\infty C}{\nu_\infty x}}$$

The solution to the homogeneous equation (10) must then satisfy the condition :

$$\left(\frac{\partial T^*}{\partial \eta}\right)_w = -\frac{2q}{k_\infty T_\infty} \sqrt{\frac{\nu_\infty x L}{u_\infty C L}}$$

or, to simplify the writing

$$\left(\frac{\partial T^*}{\partial \eta}\right)_w = A \sqrt{x^*} \qquad (12)$$

with

$$A = -\frac{2q}{k_\infty T_\infty} \sqrt{\frac{\nu_\infty L}{u_\infty C}} \qquad (13)$$

b. <u>Solution to the equation</u>

Separation of the variables is obtained by writing

$$T^* = X(x^*)\ Z(\eta) \qquad (14)$$

Replacing in (10) gives

$$\frac{1}{\rho' Z}\ (Z'' + P_r \rho Z') = 2\ P_r x^* \frac{X'}{X} = K$$

The first member depends upon η only, while the second member depends upon x^* - Therefore, they should be both constant, say K.

Solving the second part of the equation first, we get

$$\frac{dX}{X} = \frac{K}{2P_r}\ \frac{dx^*}{x^*}$$

$$\ell n X = \frac{K}{2P_r}\ \ell n x^*$$

$$X = (x^*)^{K/2P_r}$$

Replacing in (14)

$$T^* = (x^*)^{K/2P_r}\ Z(\eta)$$

and differentiating with respect to η

$$\frac{\partial T^*}{\partial \eta} = (x\ )^{K/2P_r}\ Z'(\eta)$$

At the wall ($\eta = 0$)

$$\left(\frac{\partial T^*}{\partial \eta}\right)_w = (x^*)^{K/2P_r}\ Z'(o)$$

In order to satisfy the condition (12) we must have the following equality :

$$(x^*)^{K/2P_r}\ Z'(o) = A\sqrt{x^*}$$

which means that

$$K/2P_r = 1/2 \text{ or } K = P_r \text{ and } Z'(o) = A$$

Therefore the solution is

$$T^* = \sqrt{x^*}\ Z(\eta)$$

where $Z(\eta)$ satisfies the equation

$$Z'' + P_r f Z' = P_r f' Z \qquad (15)$$

with the boundary consition

$$Z'(o) = A$$

and $Z(\infty) = 0$; because from Chapman's analysis $r(\infty) = 0$.

Complete solution

Dividing equation (15) by $Z'(o)$ and intriducing a new function

$$W(\eta) = \frac{Z(\eta)}{Z'(o)}$$

one gets the following complete solution to the non-homogeneous equation (6).

$$T^*(x^*,\eta) = 1 + \frac{\gamma-1}{2} M^2_\infty r(\eta) - \frac{2q}{k_\infty T_\infty}\sqrt{\frac{\nu_\infty x}{u_\infty C}}\, W(\eta) \qquad (16)$$

where W(η) is a solution of

$$W'' + P_r f W' = P_r f' W \qquad (17)$$

with

$$W'(o) = 1 \qquad (18)$$

$$W(\infty) = 0 \qquad (19)$$

<u>Relationship for η</u>

By integration of (11) with respect to η at constant x, we get

$$\frac{y}{2}\sqrt{\frac{u_\infty}{\nu_\infty x C}} = \int_0^\eta T^* d\eta$$

and upon using the solution (16)

$$\frac{y}{2}\sqrt{\frac{u_\infty}{\nu_\infty x C}} = 1 + \frac{\gamma-1}{2} M^2_\infty \int_0^\eta r(\eta)d\eta + A\sqrt{x^*} \int_0^\eta W(\eta)d\eta$$

By introducing (see Chapman's paper or figure 1)

$$\overline{r}(\eta) = \int_0^\eta r(\eta)d\eta \qquad (20)$$

and $$\overline{W}(\eta) = \int_0^\eta w(\eta)d\eta \qquad (21)$$

We have

$$\frac{y}{2}\sqrt{\frac{u_\infty}{\nu_\infty x C}} = \eta + \frac{\gamma-1}{2} M^2_\infty \overline{r}(\eta) + A\sqrt{x^*}\, \overline{W}(\eta) \qquad (22)$$

where A is given by (13).

Heat transfer coefficient (h_q)

Let us denote by h_q and h_T the heat-transfer coefficients obtained for constant heat-flux and constant temperature respectively.

h is defined by convention as

$$h = \frac{q}{T_w - T_{wa}}$$

where q is the local heat-flux per unit surface and unit time, T_w the local wall temperature and T_{wa} the adiabatic wall temperature (q = 0). Then h_q corresponds to $q(x)$ = constant and h_T to $T_w(x)$ = constant.

By using the solution (16) written for $\eta = 0$ and the value of T_{wa} given by relationship (9.2) we get :

$$h_q = -\frac{k_\infty}{2} \sqrt{\frac{u_\infty C}{\nu_\infty x}} \; \frac{1}{W(o)} \tag{23}$$

From Chapman's theory, we get the value of h_T

$$h_T = -\frac{k_\infty}{2} \sqrt{\frac{u_\infty C}{\nu_\infty x}} \; Y'_o(o)$$

We can thus form the ratio of h_T to h_q, which is

$$G = \frac{h_T}{h_q} = Y'_o(o) \times W(o) = \text{CONSTANT} \tag{24}$$

It is therefore concluded that the ratio of the heat-transfer coefficients for constant temperature and for constant heat-flux is a constant, independent of the Mach number and Reynolds number. As shown later its numerical value is about 0.72, independently of the Prandtl number in the range 0.5 to 1.0.

Relationship (23) can be written in terms of the Nusselt number ($N_u = \frac{h_q x}{k_\infty}$)

It gives

$$\frac{N_u}{\sqrt{R_{ex}}} = \frac{\sqrt{C}}{2.44} \qquad (25)$$

with W(0) = -1.22 for P_r = 0.72. This ratio is independent of the streamwise coordinate x, and of the stagnation conditions. It varies with M_∞ through Chapman's constant C.

Boundary-layer thicknesses

By convention, y is equal to δ when u = 0.99 u_∞, that is for

$$f'(\eta) = 2\frac{u}{u_\infty} = 1.98$$

The tables of Blasius function then give : η = 2.5.

Therefore from (22) we get :

$$\frac{\delta}{2}\sqrt{\frac{u_\infty}{\nu_\infty x C}} = 2.5 + \frac{\gamma-1}{2} M_\infty^2 \,\overline{r}(2.5) + A\sqrt{x^*}\,\overline{W}(2.5)$$

By definition

$$\rho_\infty u_\infty \delta^* = \int_0^\delta (\rho_\infty u_\infty - \rho u)dy$$

or

$$\rho_\infty u_\infty \delta^* = \rho_\infty u_\infty \delta - \int_0^\delta \rho u dy$$

The integral of the secpnd member can be written as : (see 1.1)

$$\int_0^\delta \rho u dy = \int_0^\delta \rho_\infty \frac{\partial\psi}{\partial y} dy = \rho_\infty \Psi(\delta)$$

But from (4)

$$\Psi(\delta) = \sqrt{\nu_\infty u_\infty Cx}\, f(2.5) = 3.28 \quad \sqrt{\nu_\infty u_\infty Cx}$$

Therefore

$$\delta^* = \delta - \frac{3.28}{u_\infty} \quad \sqrt{\nu_\infty u_\infty Cx}$$

or

$$\frac{\delta^*}{2} \sqrt{\frac{u_\infty}{\nu_\infty xC}} = \frac{\delta}{2}\sqrt{\frac{u_\infty}{\nu_\infty xC}} - 1.64$$

Finally

$$\frac{\delta^*}{2} \sqrt{\frac{u_\infty}{\nu_\infty xC}} = 0.86 + \frac{\gamma-1}{2} M_\infty^2 \,\overline{r}(2.5) + A\sqrt{x^*}\, \overline{W}(2.5)$$

The momentum thickness is found to be, by a similar computation

$$\frac{\Theta}{2} \sqrt{\frac{u_\infty}{\nu_\infty xC}} = 1.64 - \frac{1}{4} \int_0^{2.5} f'^2 d\eta$$

independent of M_∞ and the temperature distribution, except through small changes in the value of coefficient C. Therefore, it is found that :

$$\Theta = \frac{0.664\ x}{\sqrt{R_{ex}}} \quad \sqrt{C}$$

The friction coefficient is equal to

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho_\infty u_\infty^2} = 2\frac{d\theta}{dx} = \frac{0.664\sqrt{C}}{\sqrt{R_{ex}}}$$

thus

$$\tau_w = \frac{0.332}{\sqrt{R_{ex}}} \sqrt{C} \quad \rho_\infty u_\infty^2$$

This shows that τ_w is inversely proportional to $\sqrt{x}$. As q is constant in the present case, the ratio q/τ_w is a functionof x, while that ratio is constant in the isothermal case (known as the Reynolds analogy).

Solution to the W-equation

Values of $W(\eta)$, $W'(\eta)$ and $\overline{W}\ (\eta)$ were determined by a numerical integration of the W-equation (16).

This was done in two different ways which gave essentially the same results to within better than 1 percent. The first method consisted in a step-wise integration carried out from the outer-edge of the boundary-layer towards the flat wall. The initial values were given by an exact asymptotic solution to the W-equation. (Appendix A). The second method consisted in a step-wise integration carried out from the wall towards the outer-edge of the boundary-layer. The initial value of W was chosen arbitrarily, while W' was taken as unity and the computation was repeated until the correct asymptotic value of W was reached (i.e. $W(\infty) \doteq 0$).

The computations were made on an IBM 1620 computer either by a series expansion method or with the Kutta-Simpson technique.

The computation involves the use of the Blasius function and its derivatives. They are generally available in the form of tables with increments in η of 0.1 (ref.4). Therefore, they cannot be used for a numerical computation that includes smaller steps in η than 0.1. For that reason, the Blasius equation was reintegrated each time as indicated in Appendix B. Moreover, in these circumstances, there was no need of introducing Howarth's tables into the computer.

Practical details of the computation are given in Appendix C and the results are shown in table I (with four decimals only) or in figure 2. From these results, it is now possible to evaluate the ratio $G = h_T/h_q$. According to Chapman's theory, one has to within one percent accuracy :

$$Y_o'(o) = -\frac{f''(o)}{2} P_r^{1/3}$$

We thus get the following values of G.

P_r	0.50	0.72	1.00
G	0.730	0.726	0.723

It is concluded that G is constant to within one percent over a Prandtl number range of 0.5 to 1.0.

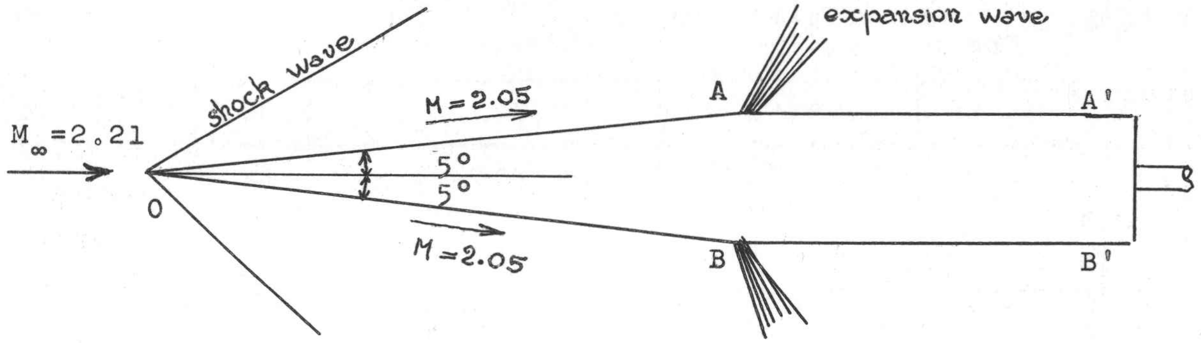
STEADY STATE TECHNIQUE FOR HEAT TRANSFER MEASUREMENTS

Principle of the technique

In the present technique, heat is uniformly dissipated at the surface of the model by Joule effect in a thin sheet of metal of constant thickness. The heat-flux per unit area and unit time (q) is determined from the measured voltage and current and the total area of the heating element. Temperatures are measuredd by thermocouples located at the model surface for power-off (T_{wa}) and power-on (T_w) conditions. The heat-transfer coefficient is then computed from the following relationship:

$$h = \frac{q}{T_w - T_{wa}}$$

The measurements were made at supersonic speeds on the symmetric wedge model shown in figure 3. It has a semi apex-angle of 5 degrees, thus giving a uniform flow at M = 2.05 along its upper and lower surfaces (OA and OB as shown in the sketch) when placed at zero incidence in a supersonic free-stream at M_∞ = 2.21.

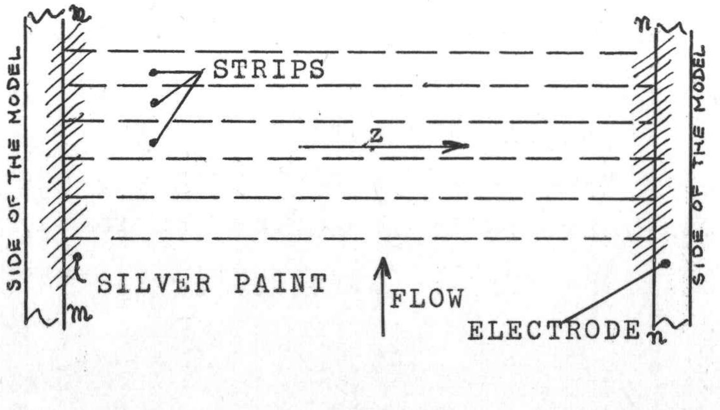


The model is made of araladite-type D - which has a low heat conductivity (k = 0.17 kcal/m hr °C, i.e. about 0.11 BTU/ft hr ° F). Heat is dissipated at the surface by Joule effect in a thin metallic layer that adheres to the model surface. This technique was previously used by Seban et al. (ref.3); their model was made of bakelite on which nichrome ribbons, 0.051 mm thick, were glued. In the present investigation, it was expected to improve the method by avoiding the difficulty of properly gluing a thin sheet of metal to the model surface and at the same time by further reducing the effective thickness of the metallic layer.

In the early part of the research, this was done by evaporating nickel under vacuum. The thickness of the nickel layer formed on the surface of the model was of the order of one micron. However, the strength of the coating depended very much on the smoothness of the surface, inasmuch as very tiny little scratches in the surface of the araldite produced small sparks when the voltage was applied, which destroyed the coating after a certain time.

A simpler method was then used which appears more successful and which is at the same time of great simplicity. It is based upon a standard method of silvering mirrors. Details of the technique are given in Appendix D. The adherence to the surface is extremely good. Indeed, it is necessary to use sand-paper to remove the silver layer.

The uniformity of thickness of the coating is determined by dividing the surface into a number of strips with a razor blade as shown in the sketch and then measuring the distribution of electric resistance of each strip along the z-axis. With proper care in preparing the surface to be silvered, it is possible to obtain a uniformity of thickness of the silver layer which is better than ten percent.



As suggested by McCroskey (ref.5), it is possible to improve this uniformity by rubbing the silvered surface with fine sand-paper in regions where the resistance is too small.

The mean thickness of the layer was determined on a typical sample by titration of the Silver Nitrate solution which was used for silvering the sample. The thickness was found to be of the order of 1 micron. It should be noted that a direct computation of the thickness based on the size of the surface, its total resistance and the resistivity of the bulk material gave a value which was quite evidently too small.

After checking the uniformity of its thickness the silver layer is electrically connected to the copper electrodes located at the sides of the model (see above sketch). Good contact is obtained between the layer and each electrode (along mm and nn respectively) by painting the surface locally with silver paint, as indicated in the sketch by a shaded area.

For the model shown in figure 3, the total resistance of the silver layers on the upper surface OA or on the lower surface OB is of the order of 1 ohm. In this case, one needs a power supply with low voltage and high current. Because of the high current, a sizeable voltage drop exists in the lines connecting the power supply to the electrodes and the voltage drop across the heating element must then be measured directly at the electrodes with auxiliary wires. In addition, the electrodes are running along the full length of the model to ensure a uniform dissipation of power on the whole surface of the model. Four independent power supplies were available during the tests in order to have independent control of the power dissipated on the four surfaces OA, OB, AA' and BB'.

There is a small temperature effect on the resistivity of the silver layer. This effect is expected to be the same for thin layers as for the bulk material; this was checked on a nickel layer in the early part of the investigation. The resistance increases approximately by one third of one percent, when the temperature increases by one degree. Therefore, if the wall temperature varies by $\pm$ 10° around a mean value, the resistance varies by $\pm$ 3% and q differs from a constant by the same amount. In the present investigation, the temperature changes are kept small because the araldite D cannot withstand high temperatures. In these circumstances, the effect of the temperature on the measurements is not large.

Seventeen flush fitting copper-constantan thermocouples are installed along the centre-line of surface OA to measure the wall temperatures. A few others are located on OB and also on the two parallel portions AA' and BB' of the model (see previous sketch) with a view to checking that symmetric conditions are obtained during the tests. The model was casted with araldite after correctly positioning the thermocouples in the mould. Details are given in Appendix E. Each thermocouple is connected to its individual reference thermocouple maintained at 0°C by melting ice in a thermos bottle. The reference thermocouple wires are not welded but merely twisted together and suspended in individual mercury reservoirs in the thermos bottle. Individual reservoirs are needed because the thermocouples, located at the surface of the model, are not insulated from the silver layer and therefore the thermocouple leads could locally by-pass the current from the silver coating. Rotary switches are used to measure the output voltages of the thermocouples, in turn, on a calibrated galvanometer.

Measurements were also made at low speeds. The model consists of a flat plate of araldite, silver plated on both surfaces in the same manner as the supersonic model. The plate is 285 mm wide and 550 mm long. It has a thickness of 10 mm and an elliptic nose.

Test conditions

The symmetric wedge model was tested in the TCEA continuous supersonic wind-tunnel S-1 (described in ref.6) at a free-stream Mach number of 2.21 and at stagnation pressures of 100 and 200 mm of mercury absolute. In these tests, the thickness of the silver layer was constant within 10 percent. Steady state conditions were achieved for both power-off and power-on conditions, after approximately one hour of running time. Adiabatic temperatures were of the order of 0°C and wall temperatures with power-on were limited to a maximum of about 40°C. The stagnation temperature in the tunnel was closed to ambient temperature and remained nearly constant after the tunnel had been running for more than one hour.

The flat plate model was tested in the TCEA low speed wind tunnel L-2 briefly described in ref.7.

Results and discussion

The experimental results are shown in figure 4 and compared with the theory. These results are not corrected for eventual heat-losses through the araldite, for non-uniformities in thickness of the silver layer or for temperature effect on the resistivity of the heating element.

Figure 4a gives the heat-transfer coefficient (h) as a function of the distance (x) from the leading-edge, for the supersonic test. Two different pressure levels were used in the tunnel (100 and 200 mm of mercury) and two different values of the power dissipated on the model surface (5 watts and 10 watts). The agreement between the experiment and the theory is better than ten percent over the full length of surface of the wedge. In these tests, the thickness of the silver layer was constant within approximately 10%. Also shown in the figure is the recovery factor which agrees with the theoretical value of $\sqrt{P_r}$ for laminar flow to within approximately 10%. As the recovery temperature was found to be very sensitive to the degree of humidity of the air in the tunnel, the tests were done at 0.1 gr. of water vapor per kg of air (10^{-4}). The influence of humidity has already been pointed out, in particular by Thomann (ref.8).

The same results are plotted in figure 4b which gives the ratio $Nu/\sqrt{R_{ex}}$ which is theoretically constant and equal to 0.39 for the present test conditions according to relationship (25).

The low speed data are shown in figure 4c which gives h vs x and in figure 4d, in which $Nu/\sqrt{R_{ex}}$ is plotted against x. The test was made with a free-stream velocity of 15 m/sec and a power dissipated on each surface equal to about 36 watts. The experimental points are off the theoretical curve by 20%. This was considered as satisfactory because the flow conditions are not ideal in the small scale wind tunnel L-2.

It is concluded that an excellent agreement exists between the "uncorrected" experimental data and the exact theory.

The possibility of correcting the experimental results will now be considered.

We first evaluate the heat-losses due to conduction through the araldite. Because the geometry of the model and the heating system are symmetric, there is no heat-flux across the plane of symmetry of the model. However, as the wall temperature increases from the leading-edge to the trailing-edge of the model, heat is conducted through the araldite in the stream-wise direction. This effect can be evaluated by computing the temperature distribution inside the wedge for given surface conditions. Under steady state conditions, the temperature must satisfy Laplace's equation which is written in polar coordinates as (see sketch) :

$$r\frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r} + \frac{1}{r}\frac{\partial^2 T}{\partial \theta^2} = 0 \qquad (26)$$

We assume that the wall temperature is given by the constant heat-flux theory, i.e.

$$T_w = T_{wa} - \frac{2q}{k_\infty}\sqrt{\frac{\nu_\infty}{u_\infty C}}\, W(o)\, \sqrt{r} \qquad (27)$$

for

$$\theta = \pm\, \alpha$$

It can be seen that

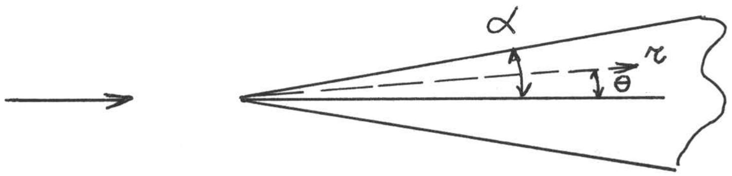
$$T = T_{wa} + C_2\, r^{1/2} \cos\left(\frac{\theta}{2} + \phi\right)$$

is a solution of (26), where C_2 and ϕ are constant. To satisfy the boundary conditions (27) we must have

$$\phi = 0 \; ; \; C_2 = - \frac{2q}{k_\infty} \sqrt{\frac{\nu_\infty}{u_\infty C}} \; W(o) \frac{1}{\cos\frac{\alpha}{2}}$$

Thus

$$T = T_{wa} - \frac{2q}{k_\infty} \sqrt{\frac{\nu_\infty}{u_\infty C}} \; W(o) \sqrt{r} \; \frac{\cos\frac{\theta}{2}}{\cos\frac{\alpha}{2}} \tag{28}$$



The heat flux through the surface of the model per unit time and unit area is then, from Fourier's equation :

$$q_a = -k_a \left(\frac{\partial T}{r\partial\theta}\right)_{\theta=\alpha} \tag{29}$$

where k_a is the coefficient of thermal conductivity of the araldite.
Using the solution (28), (29) is rewritten as :

$$q_a = -q \frac{k_a}{k_\infty} \frac{W(o)}{\sqrt{r}} \sqrt{\frac{\nu_\infty}{u_\infty C}} \; tg\frac{\alpha}{2}$$

The ratio of the heat-flux through the surface to the heat-flux dissipated by Joule effect at the surface is thus :

$$\frac{q_a}{q} = - \frac{k_a}{k_\infty} W(o) \sqrt{\frac{\nu_\infty}{u_\infty Cr}} \; tg\frac{\alpha}{2} \tag{30}$$

This ratio decreases as r increases and as the wedge angle decreases. For typical test conditions, (30) shows that q_a/q is smaller than 1%, when x is larger than about 5 mm. It is thus concluded that the heat-losses through the araldite have a negligible effect over most of the surface of the model. However, this computation is valid for an infinite wedge and the actual model has a finite length; it is thus possible that heat-exchanges exist between the model and the rear sting. This was checked by comparing the results of figure 4a with the results obtained without heating the rear surfaces of the model, AA' and BB' (see sketch on page 16). No difference was observed within the accuracy of measurements.

The effect of the temperature of the heating element on its resistance was experimentally checked by dissipating different amounts of power, keeping other conditions unchanged. As seen from figure 4a, no systematic difference was observed in the heat-transfer coefficients.

No attempt was made to correct the results for a non-uniform thickness of the silver layer. The actual distribution of the heat-flux can be determined by measuring the distribution of resistance of the heating element as indicated under "Principle of the technique". However, the width (or span) of the portion of the surface which affects the temperature along the centre line of the model, remains unknown.

The effect of the presence of thermocouples, below the silver-layer, on the power dissipated locally at the surface was checked by removing the silver layer just above the thermocouples. No difference was observed in the results.

For the sake of comparison, unsymmetric conditions were also tested, by keeping the lower surface of the model unheated. As expected, a large difference was observed in the results. Moreover, it was not possible to work out a simple method of correcting the data which was satisfactory. This shows the importance of symmetric conditions and the uncertainty of correcting the measurements for heat-losses, when necessary.

Conclusions

An exact solution was found to the compressible laminar boundary layer equations for the case of a flat plate with constant heat-flux. The problem was solved by using the transformed boundary-layer equation given by Chapman and Rubesin. The results showed, in particular, that there exists a constant ratio between the heat-transfer coefficients at constant heat-flux and at constant temperature.

A steady-state technique for heat-transfer measurements derived from Seban's, was developped which gave uncorrected data to within 10 % of the theory at high speed and 20% at low speed. It is based on a simple method of silvering the surface of a model made of araldite and instrumented with thermocouples.

This method seems particularly suitable and probably simpler than the isothermal method when used to study thin symmetrical two-dimensional wings. It is hoped that the relationship between the heat transfer coefficients still holds in the presence of a stream-wise pressure gradient.

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APPENDIX A.

Asymptotic solution to the W-Equation (17)

If $\eta \to \infty$, then the Blasius function and its first derivative can be expressed as

$$f = 2(\eta - 0.86038) \; ; \; f' = 2$$

So that (17) is rewritten as

$$W'' + 2 P_r (\eta - 0.86038) W' = 2 P_r W$$

Introducing a new independent variable z defined by

$$z = \sqrt{P_r}(\eta - 0.86038)$$

$$dz = \sqrt{P_r} \, d\eta$$

we have

$$\frac{dW}{d\eta} = \frac{dW}{dz}\frac{dz}{d\eta} = \sqrt{Pr}\frac{dW}{dz} \quad \text{and} \quad \frac{d^2W}{d\eta^2} = P_r \frac{d^2W}{dz^2}$$

Thus

$$P_r \frac{d^2W}{dz^2} + 2P_r z \frac{dW}{dz} - 2P_r W = 0$$

or, after simplification

$$\frac{d^2W}{dz^2} + 2z \frac{dW}{dz} - 2W = 0 \qquad \text{A.1}$$

One can find immediately a particular solution to the equation, i.e.:

$$W_o = z$$

and obtain the general solution, by writing

$$W = W_o t = zt$$

Where (t) can be determined by two successive quadratures as follows :

$$\frac{dW}{dz} = t + z\frac{dt}{dz} \quad ; \quad \frac{d^2W}{dz^2} = 2\frac{dt}{dz} + z\frac{d^2t}{dz^2}$$

Replacing in (A.1)

$$z\frac{d^2t}{dz^2} + 2\frac{dt}{dz}(1 + z^2) = 0$$

Defining

$$U = \frac{dt}{dz} \qquad \text{A.2}$$

we have

$$z\frac{dU}{dz} + 2U(1 + z^2) = 0$$

or

$$\frac{dU}{U} = -\frac{2}{z}(1 + z^2)\,dz$$

By integration

$$U = C_1 z^{-2} e^{-z^2}$$

where C_1 is a constant.

Therefore, from A.2

$$t = \int U dz = C_1 \int z^{-2} e^{-z^2} dz + C_2$$

where C_2 is another constant. Thus :

$$W = zt = C_1 z \int z^{-2} e^{-z^2} dz + C_2 z$$

In order to satisfy the boundary condition (19), i.e. $W(\infty)=0$, one must take $C_2 = 0$, because it is seen, for example, that by successive integrations by parts, that the integral is equal to

$$\frac{1}{z^2 e^{z^2}} \sum_{o}^{\infty} {}^{n} \frac{(-1)^{n+1} \, (2n+1)!}{2^{2n+1} \, n! \, z^{2n}}$$

which tends towards 0 as $z \to \infty$. Thus

$$W = C_1 z \int_z^\infty s^{-2} \, e^{-s^2} \, ds \qquad \text{A.3}$$

where C_1 is to be determined by boundary condition (18) i.e.

$$W'(o) = 1$$

The numerical integration is simplified by rewriting the relationship A.3 as follows :

$$\int_z^\infty s^{-2} e^{-s^2} ds = \left(-s^{-1} e^{-s^2}\right)_z^\infty - \int_z^\infty s^{-1} . 2s e^{-s^2} ds$$

$$= \frac{e^{-z^2}}{z} - 2\left[\int_o^\infty e^{-s^2} ds - \int_o^z e^{-s^2} ds\right]$$

Thus

$$\frac{W}{C_1} = e^{-z^2} - \sqrt{\pi} z \; (1 - \text{erf } z)$$

By differentiation, we get all the successive derivatives :

$$\frac{W'}{C_1} = - \sqrt{\pi}(1 - \text{erf } z) \; , \; \frac{W''}{C_1} = 2 \, e^{-z^2} , \text{ etc...}$$

APPENDIX B.

The Blasius function f satisfies the equation

$$ff'' + f''' = 0 \qquad B.1$$

with the boundary-conditions

$$f(o) = f'(o) = 0 \quad \text{and } f'(\infty) = 2$$

A stepwise numerical integration of equation (B.1) was done from the wall towards the outer-edge of the boundary-layer. The initial value of f"(o) was selected arbitrarily together with f(o) = f'(o) = 0 and the computation was repeated untill the correct asymptotic value f'(∞) = 2 was obtained. The integration was done by expanding f, f', f" into power series, such that :

$$f(\eta+d\eta) = f(\eta)+f'(\eta)d\eta + \dots + f^{V}(\eta)\frac{d\eta^5}{5!}$$

$$f'(\eta+d\eta) = f'(\eta)+f''(\eta)d\eta + \dots + f^{V}(\eta)\frac{d\eta^4}{4!}$$

$$f''(\eta+d\eta) = f''(\eta)+f'''(\eta)d\eta + \dots + f^{V}(\eta)\frac{d\eta^3}{3!}$$

The other derivatives were computed at (η+ dη) in order to satisfy the equation (B.1) and the other equations obtained by successive differentiations,i.e.

$$f'''(\eta+d\eta) = - f(\eta+d\eta).\ f''(\eta+d\eta)$$

$$f^{IV} = - f'f'' - ff'''$$

$$f^{V} = - f''^2 - 2f'f''' - ff^{IV}$$

With an interval of integration equal to $d\eta = 0.01$, it was possible to get values of f, f', f" which agreed with the values given by Howarth to within the fifth decimal.

When the integration was performed in the same manner, but from infinity towards the wall, it was found necessary to readjust twice the values of f, f', f" during the process in order to get Howarth's results. This is due to the fact that two initial values (for f and f") are now to be selected arbitrarily.

APPENDIX C.

Numerical integration of the W-equation (17)

1. Integration by series.

The integration was done by expanding W and W' in power series

$$W(\eta+d\eta) = W(\eta) + W'(\eta)d\eta + \ldots + W^{V}\frac{d\eta^5}{5!}$$

$$W'(\eta+d\eta) = W'(\eta) + W''(\eta)d\eta + \ldots + W^{V}\frac{d\eta^4}{4!}$$

and the other derivatives were computed at $\eta + d\eta$ in order to satisfy the W-equation and the other equations obtained by successive differentiation, i.e.

$$W''(\eta+d\eta) = P_r\left[f'(\eta+d\eta)W(\eta+d\eta)-f(\eta+d\eta)W'(\eta+d\eta)\right]$$

$$W''' = P_r\left[f''W-fW''\right], \text{ etc.}\ldots$$

The quantity $\overline{W} = \int_0^{\eta} W d\eta$ was computed by the following series.

$$\overline{W}(\eta+d\eta) = \overline{W}(\eta) + W(\eta)d\eta + \ldots + W^{V}\frac{d\pi^6}{6!}$$

2. Integration by the Kutta-Simpson rule.

The following system of differential equations is equivalent to the W-equation (17)

$$\frac{dW}{d\eta} = W_1$$

$$\frac{dW_1}{d\eta} = P_r(f'W - fW_1)$$

Therefore, from the Kutta-Simpson rule :

$$W(\eta+2d\eta) = W(\eta) + \frac{1}{3}(\Delta'+2\Delta''+2\Delta'''+\Delta^{iv})$$

$$W_1(\eta+2d\eta) = W_1(\eta) + \frac{1}{3}(\delta'+2\delta''+2\delta'''+\delta^{iv})$$

where

$$\Delta' = W_1(\eta)\, d\eta$$

$$\delta' = P_r\left[f'(\eta)W(\eta) - f(\eta)W_1(\eta)\right] d\eta$$

$$\Delta'' = \left[W_1(\eta) + \frac{1}{2}\delta'\right] d\eta$$

$$\delta'' = P_r\{f'(\eta+d\eta)\left[W(\eta)+\frac{1}{2}\Delta'\right] - f(\eta+d\eta)\left[W_1(\eta)+\frac{1}{2}\delta'\right]\} d\eta$$

$$\Delta''' = \left[W_1(\eta) + \frac{1}{2}\delta''\right] d\eta$$

$$\delta''' = P_r\{f'(\eta+d\eta)\left[W(\eta)+\frac{1}{2}\Delta''\right] - f(\eta+d\eta)\left[W_1(\eta)+\frac{1}{2}\delta''\right]\} d\eta$$

$$\Delta^{iv} = \left[W_1(\eta) + \delta'''\right] d\eta$$

$$\delta^{iv} = P_r\{f'(\eta+2d\eta)\left[W(\eta)+\Delta'''\right] - f(\eta+2d\eta)\left[W_1(\eta)+\delta'''\right]\} d\eta$$

The quantity $\overline{W}(\eta)$ was computed by Simpson's rule.

APPENDIX D

A mirror silvering method.

Chemicals involved.

1. Silver Nitrate - Dissolve 10 gr. of silver nitrate into 25 cc. of distilled water. By adding ammonia to the solution a precipitate or deposit starts to form and then disappears progressively. Ammonia is added until the deposit has almost vanished (it corresponds to approximately 9 cc. of ammonia at 25°C). Filter the solution and add distilled water to make 1 litre.

2. Tartaric acid - Prepare a solution of 5 gr. of tartaric acid for 100 cc. of distilled water.

3. Stannous chloride - (S_nCl_2-$2H_2O$)-Dissolve 2 gr. of stannous chloride in one litre of distilled water.

Procedure for silvering the models.

As indicated in appendix E, the model is cast in several steps. In most of the cases, it is thus impossible to obtain a uniform state of the surface. However, a uniform state is important in order to get a silver-layer of constant thickness. It is therefore useful to spray a thin layer of liquid araldite* with an air gun on the model surface. At the same time, by insulating the thermocouples from the silver layer, odd chemical reactions are avoided. The surface is then rubbed with sand-paper and very throughly degressed.

* Type EPOXYLUX 4720.

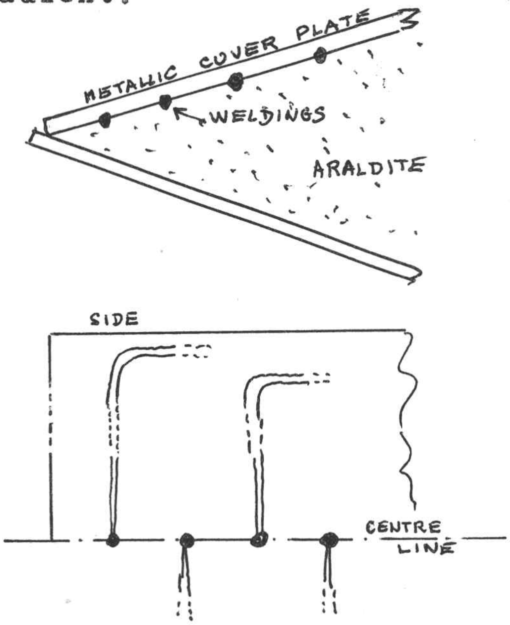
This being done, the model is immersed in solution (3) for a few minutes, then rinsed briefly in distilled water, and finally immersed in a mixture of 1 litre of solution (1) with 10 cc. of solution (2) (prepared just before use). One waits until the liquid gets slightly muddy. By that time, silver should have been deposited on the surface. It is preferable to work at a constant temperature of 25° C(the model must have a constant temperature).

It is generally easy, by watching the model surface, to predict wether a uniform thickness will be obtained or not. In case of an unsuccesful result, the deposit must be thoroughly removed with sand paper or with nitric acid and the complete operation repeated.

APPENDIX E.

Preparation of the araldite model

The wedge model was cast in a metallic mould, coated inside with vacuum grease, to facilitate withdrawal of the model from the mould. Each thermocouple is maintained in its correct position by inserting approximately half of the welding inside a small hole drilled in the cover plates of the mould. The wired are run spanwise, as indicated in the sketch, to minimize the heat-losses due to the streamwise temperature gradient.



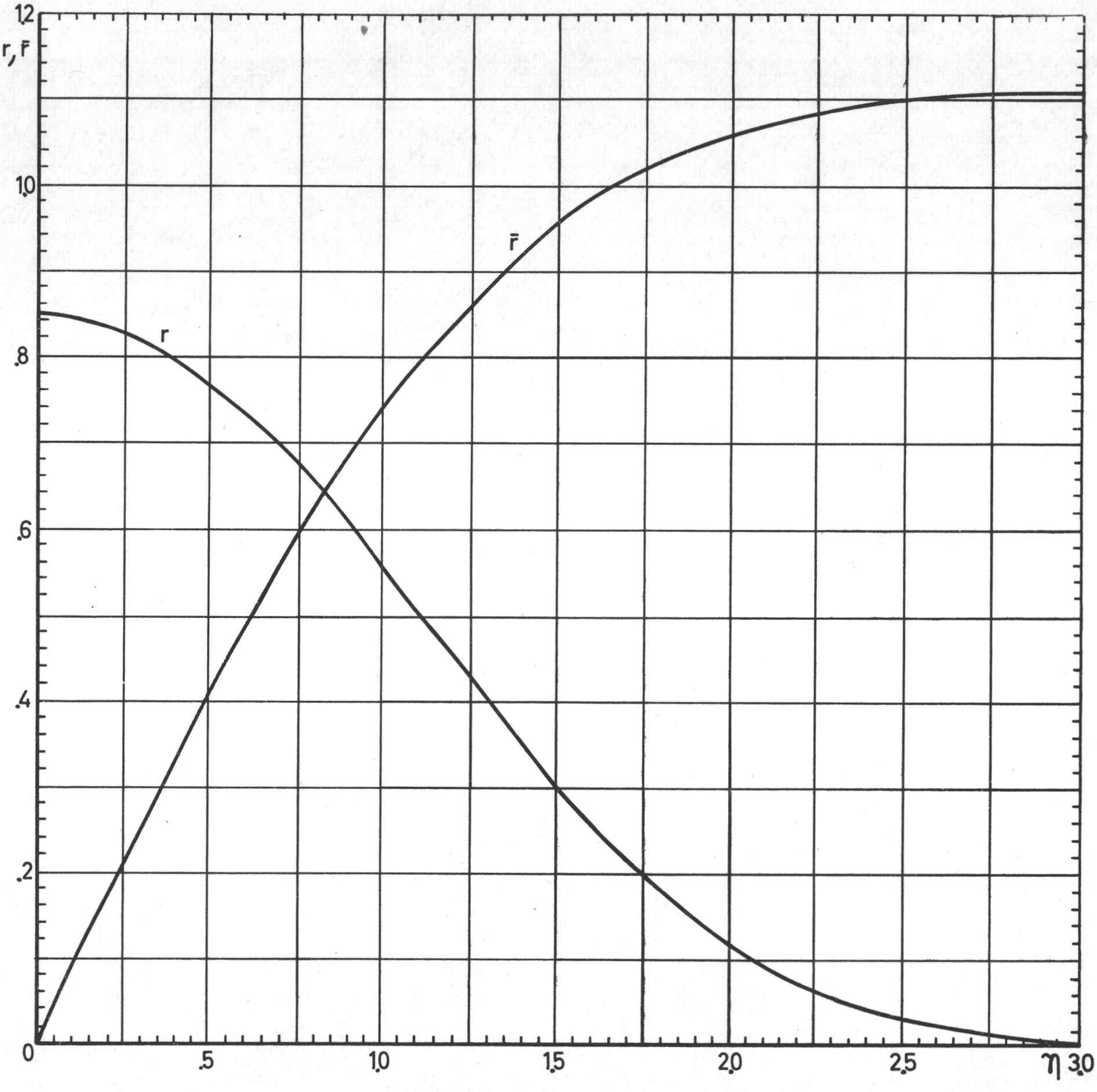
It was found necessary to cast the model in several steps in order to avoid excessive deformation of the araldite when removed from the mould. The model is then machined in order to correct possible defects and also to bring the thermocouple junctions flush with the surface.

The model was mounted in the wind-tunnel on a sting one end of which was inserted in the rear of the model during the casting process.

Table I

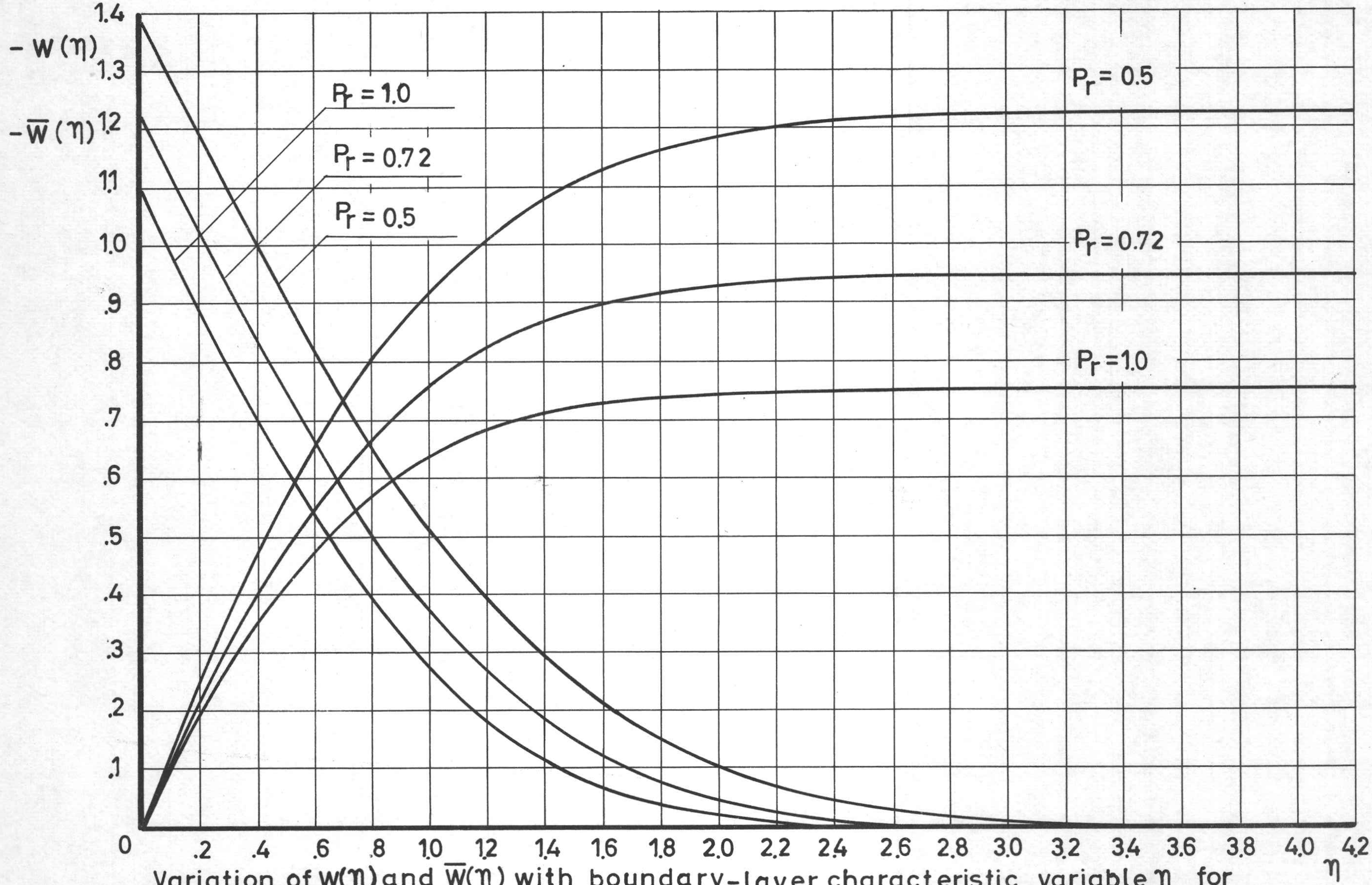
η	$P_r = 0.5$		$P_r = 0.72$		$P_r = 1.0$	
	$-W$	$-\overline{W}$	$-W$	$-\overline{W}$	$-W$	$-\overline{W}$
0	1.3850	0	1.2199	0	1.0894	0
0.1	1.2852	.1335	1.1201	.1170	.9896	.1039
0.2	1.1862	.2571	1.0214	.2241	.8912	,1980
0.3	1.0889	.3708	.9248	.3213	.7954	.2823
0.4	.9941	.4749	.8313	.4091	.7034	.3572
0.5	.9024	.5697	.7416	.4877	.6160	.4231
0.6	.8144	.6555	.6565	.5576	.5340	.4806
0.7	.7307	.7327	.5766	.6192	.4581	.5301
0.8	.6515	.8018	.5022	.6731	.3888	.5724
0.9	.5774	.8632	.4336	.7198	.3262	.6081
1.0	.5084	.9175	.3712	.7600	.2705	.6379
1.1	.4448	.9651	.3149	.7943	.2215	.6624
1.2	.3866	1.0066	.2647	.8232	.1792	.6824
1.3	.3338	1.0426	.2204	.8474	.1430	.6985
1.4	.2862	1.0735	.1817	.8674	.1127	.7112
1.5	.2437	1.1000	.1483	.8839	.0875	.7212
1.6	.2062	1.1225	.1198	.8973	.0670	.7289
1.7	.1731	1.1414	.0959	.9080	.0506	.7347
1.8	.1443	1.1572	.0759	.9166	.0376	.7391
1.9	.1194	1.1704	.0594	.9233	.0276	.7423
2.0	.0981	1.1812	.0460	.9286	.0199	.7447
2.1	.0800	1.1901	.0352	.9326	.0141	.7464
2.2	.0647	1.1973	.0266	.9357	.0098	.7476
2.3	.0520	1.2032	.0200	.9380	.0068	.7484
2.4	.0414	1.2078	.0148	.9397	.0046	.7489
2.5	.0327	1.2115	.0108	.9410	.0030	.7493
2.6	.0257	1.2144	.0078	.9419	.0020	.7497

η	$P_r = 0.5$		$P_r = 0.72$		$P_r = 1.0$	
	$-W$	$-\overline{W}$	$-W$	$-\overline{W}$	$-W$	$-\overline{W}$
2.7	.0200	1.2167	.0056	.9426	.0013	.7498
2.8	.0154	1.2185	.0039	.9431	.0008	.7499
2.9	.0118	1.2198	.0028	.9434	.0005	.7499
3.0	.0090	1.2208	.0019	.9436	.0003	.7500
3.1	.0068	1.2216	.0013	.9438	.0002	.7500
3.2	.0050	1.2222	.0009	.9439	.0001	.7500
3.3	.0037	1.2226	.0006	.9440	.0001	.7500
3.4	.0027	1.2230	.0004	.9440	.0000	.7500
3.5	.0020	1.2232	.0002	.9441	.0000	.7500
3.6	.0014	1.2234	.0002	.9441	.0000	.7500
3.7	.0010	1.2235	.0001	.9441	.0000	.7500
3.8	.0007	1.2236	.0001	.9441	.0000	.7500
3.9	.0005	1.2237	.0000	.9441	.0000	.7500
4.0	.0004	1.2237	.0000	.9441	.0000	.7500
4.1	.0002	1.2237	.0000	.9441	.0000	.7500
4.2	.0002	1.2237	.0000	.9441	.0000	.7500
4.3	.0001	1.2238	.0000	.9441	.0000	.7500
4.4	.0001	1.2238	.0000	.9441	.0000	.7500
4.5	.0000	1.2238	.0000	.9441	.0000	.7500



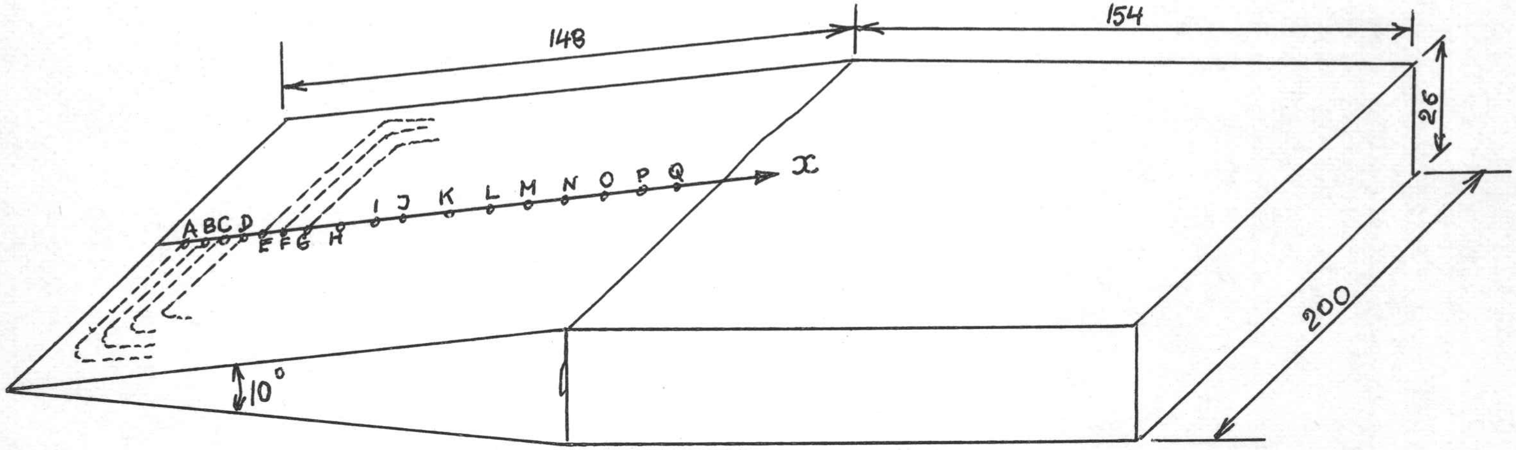
Variation of r and $\bar{r}$ with boundary-layer characteristic variable η, for P_r = 0,72

Figure 1



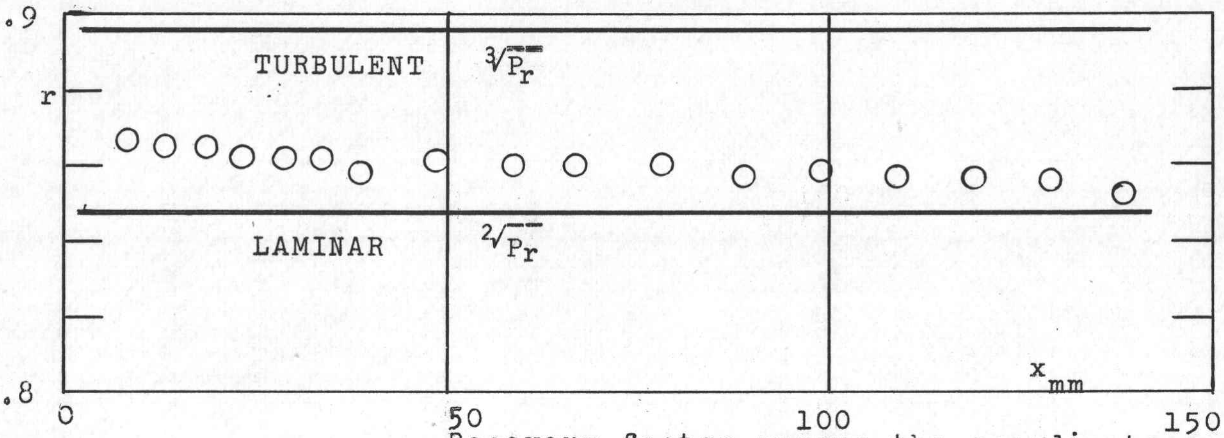
Variation of $W(\eta)$ and $\overline{W}(\eta)$ with boundary-layer characteristic variable η for P_r = 0.5 ; 0.72 and 1.0

Figure 2



Nr	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
x mm	8.5	13.5	19	23.5	29	34	39	49	59	67	78.5	89	99	109	118.5	129	139

Figure 3 - Wedge model for heat-transfer measurements.



Recovery factor versus the coordinate x

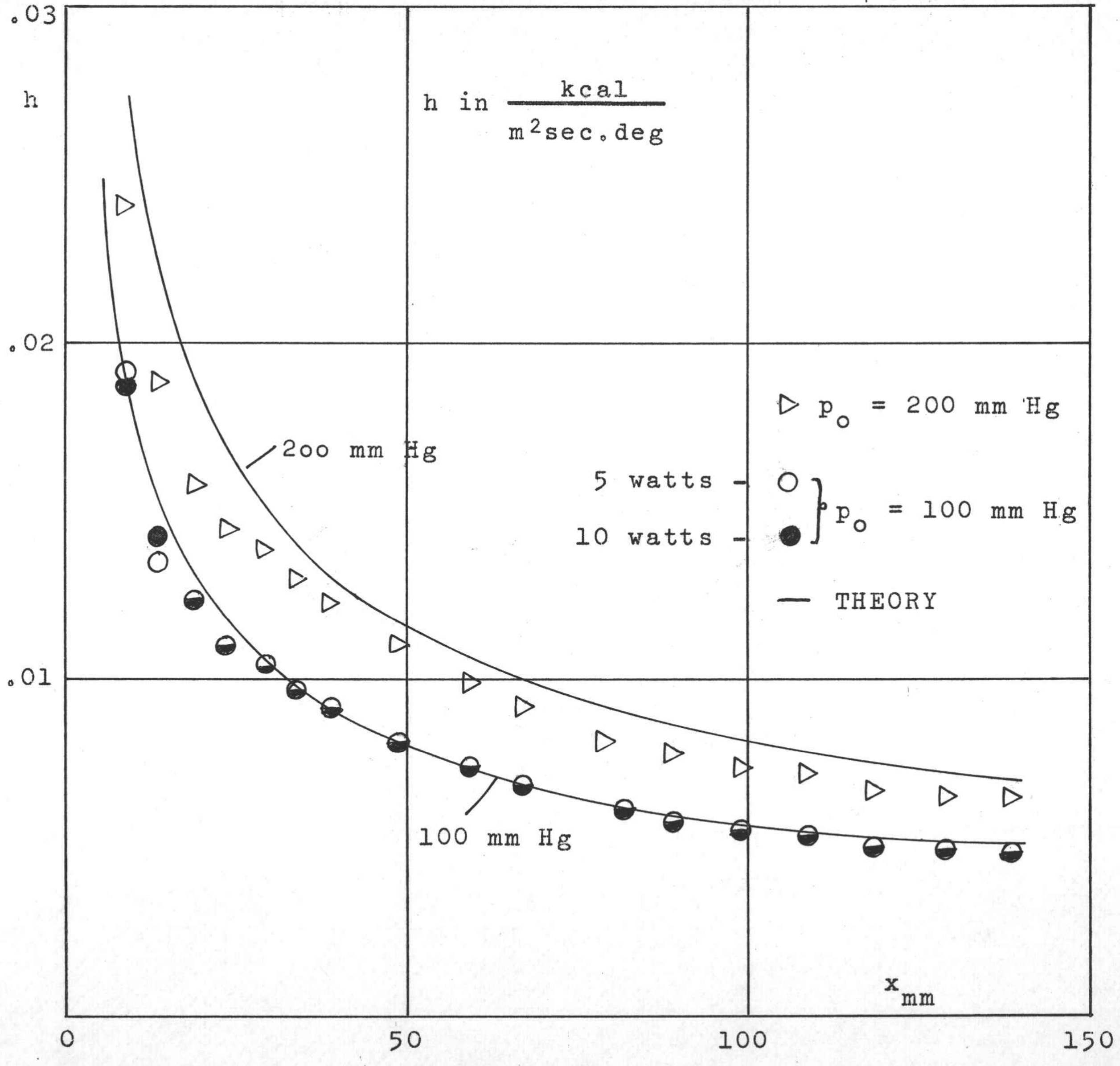


Figure 4a - Heat-transfer coefficient versus the coordinate x.

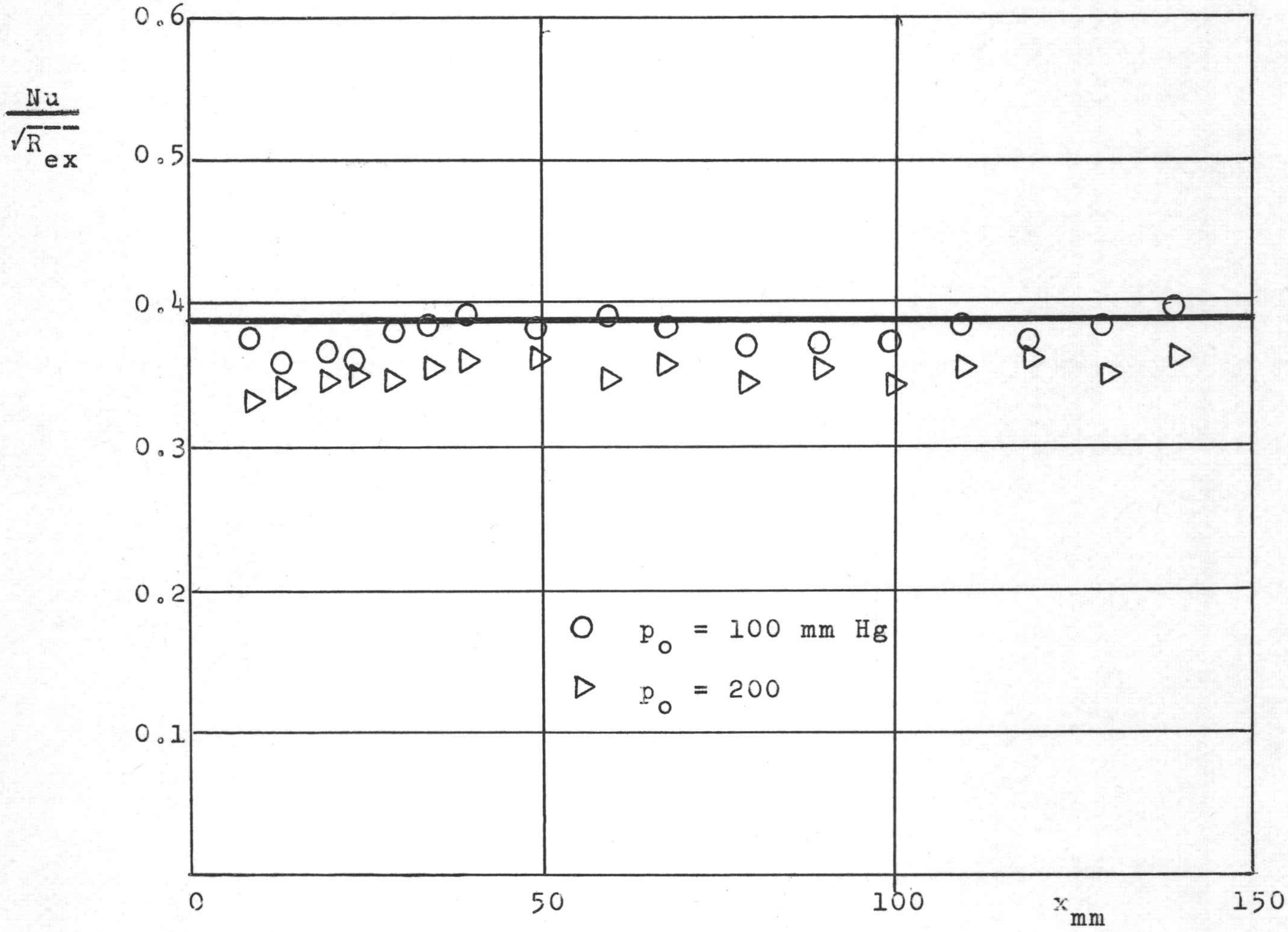


Figure 4b - Comparison between theory and experiment ; supersonic speed case.

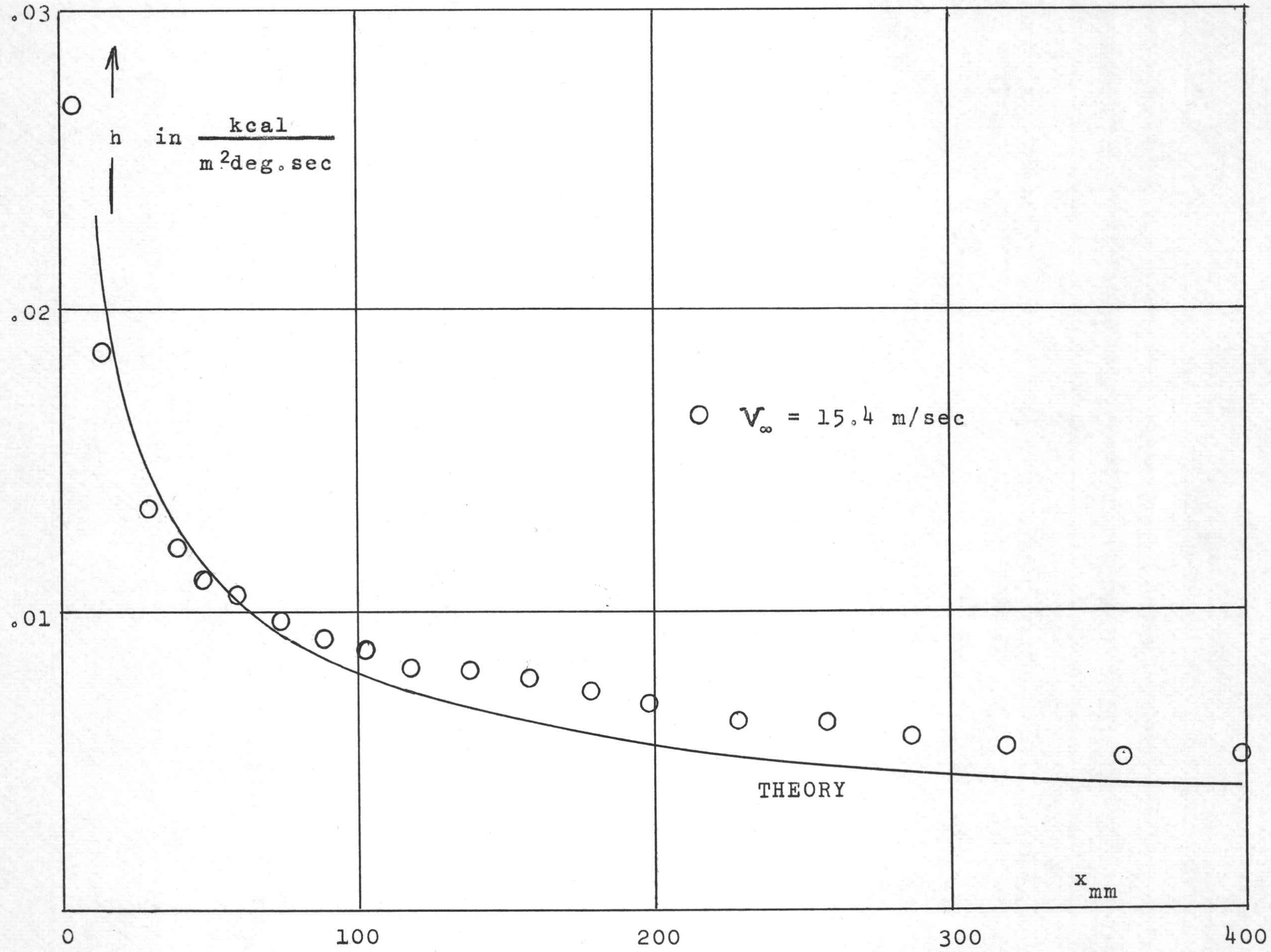


Figure 4c - Distribution of the heat-transfer coefficient at low speeds.

TCEA TN 11
Training Center for Experimental Aerodynamics

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II. TCEA TN 11

May 1963

AN EXACT SOLUTION TO THE COMPRESSIBLE LAMINAR BOUNDARY-LAYER EQUATIONS FOR THE FLAT PLATE WITH CONSTANT HEAT-FLUX.

An exact solution is found to Chapman and Rubesin's transformed laminar boundary-layer equations, in the case of a flat plate with constant heat-flux. It is shown that the ratio of the heat-transfer coefficients for constant heat-flux and for constant temperature (i.e.isothermal case) is a constant independent of the Reynolds number, Mach number and Prandtl number. This property indicates that experimental results are obtained with a constant heat-flux technique which are simply related to results that could be obtained with the usual isothermal method.

The theory is experimentally checked by using an improved steady state technique derived from Seban's, which gives "uncorrected" data that are in agreement with the theory to within 10 % at M = 2 and 20 % at low speed.

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