Calculating shells through graphic statics

P2 Report

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Table of contents

Introduction

The main aim of this report is to present and explain the research proposal for graduating at Building Technology. The secondary aim is to give insight in the progress of the research done before the P2 evaluation.

In chapter 1, the background and the problem which are the starting points for this research are defined, the goal for the research is set and the research question is defined. The relevance of this research is explained and this chapter also covers the approach and the time planning for this research.

In chapter 2 the first results of the literature study are discussed. Not all of the research conducted will be included in this report, but the most important research for the direction in which the research will develop is covered. Note that the second chapter is a work in progress and will probably be included in a revised way in the report at the P3 or P4 evaluation. For this report, mainly the parts which are required to understand chapter 2 are included.

1. Research proposal

1.1 Background

Shell structures are usually calculated using the Finite Element Method (FEM). In this method, a structure is split up in small parts and for each part the forces working on it are calculated, including the forces working in between parts. Using this method, all the forces in the structure can be determined.

Using this method has two main disadvantages. The first of them being the efficiency of this method. These calculations can only be done for an almost complete structural design and are time-consuming. If the design turns out not to be strong enough or inefficient in material use, a new design has to be made, which can then be checked, after which the cycle probably needs to be repeated. The feedback on the structural performance is slow resulting in an inefficient design process.

The second, probably even more important disadvantage of FEM for shell structures is that it does not give any insight in the mechanics of shell structures in general. The calculations give numerical results, which can tell us something about a specific shell structure, and whether or not the structure will fail, but does not show the connection between the geometry and the structural performance, making it a kind of black-box method. The lack of knowledge on how these two are connected makes it hard to design a shell structure, it can only be done intuitively and by trial and error.

These problems can be solved if an alternative way of calculating shells is found. Some research is done already on this subject by several graduating students at the TU Delft, focusing on a way to calculate shell structures using graphic statics. The general aim of this research is to (i) find a graphical method to calculate these steps and to (ii) model these steps in a computational program linked to a 3D visualization, which results in a tool that can be used early in the design process and gives direct feedback on the structural performance of a shell. Some results of this research are discussed in chapter 2.

In the current situation, several steps are made in the direction of an expected solution, and in 1D (for beams and arches) a solution is found already (though not published yet). For more information on the research done on this subject, see paragraph 2.7.

1.2 Problem statement

The current method to calculate a shell does not give any insight in the relation between the geometry and the structural performance.

A solution for this problem is found for 1D structures (arches and beams) (see paragraph 2.7) but when translating this method from 1D to 2D, some problems emerge, one of them being the hoop forces which only occur in 2D. These hoop forces are forces which make equilibrium as can be seen in figure 9. These forces naturally don't occur in 1D situations. For more information on hoop forces see chapter 2.3.

Another problem with the current method of calculating a shell is that it is a time-consuming process because it only can be done on an almost finished structural design.

1.3 Objective

The main aim of this research is to find and describe a method to calculate shell structures in a quick and graphical way, so that the relation between the geometry and the structural performance is preserved.

This can probably be achieved by translating the equations form 1D to 2D and finding a way to determine the hoop forces in shells and to add these to the equations.

To be able to get direct feedback on changes in geometry, a computational algorithm will be designed and connected to a 3D visualization program.

A first direction to explore is how to add hoop forces to the current equilibrium equations.

1.4 Research question

The problem statement and objectives stated in previous paragraphs lead to the research question:

How can the structural performance of a shell structure be calculated in such a way that the relation between the geometry and the structural performance is shown?

To answer this questions, the subquestions that need to be answered are:

- *How can the graphical way to calculate an arch be made applicable to shell structures?*
- *How do hoop forces influence the structural performance of shell structures?*
- *How can this calculation method be translated into a computational algorithm?*

To get started on this subject, some research is done, starting with the question:

What methods can be used to calculate a shell structure?

1.5 Relevance

<u>Societal relevance</u>

This research aims to provide in a tool for designers which gives them earlier in the design process insight in the structural performance of a shell structure. This will lead to a less time-consuming design process, but also to a more direct feedback on the design changes. It will probably lead to more efficient structural design, in which less material can be used for a similar performance.

<u>Scientific relevance</u>

Currently it is still unknown what the mechanics are behind shell structures. This research aims to give more insight in these mechanics.

1.6 Approach and methodology

Literature study

The whole research will be done within the field of structural mechanics. For this reason, the literature to be studied is mainly in the field of structural mechanics. To get the research started, a literature study on several subjects needs to be done. Part of these subjects are studied already. The following subjects will be studied:

- Complementary energy method (see chapter 2.1)
- Arches and thrust lines (see chapter 2.2)
- Graphic statics in arches (see chapter 2.3)
- Hoop forces (see chapter 2.4)
- Split in surfaces (see chapter 2.5)
- Curvature (see chapter 2.6)

Method development

From the literature study hypotheses will emerge. From these hypotheses a method to calculate shell structures will be developed.

Design computational algorithm

The found method will be translated into a computational algorithm. For this algorithm, the 3D program Rhino will be used, with the Grasshopper-plugin.

Validate method

The computational algorithm will be compared to FEM calculations for several case studies. Differences in results from these calculations will show whether or not the method is valid.

1.7 Time planning

P1 evaluation November 2013

- Orientation on graduation subject

P2 evaluation January 2014

- Research proposal
- Literature study

P3 evaluation April 2014

- Calculation method for shells
- Study on previous computational algorithms for shell structures
- First results of computational algorithm

P4 evaluation May 2014

- Computational algorithm
- Graduation report

P5 evaluation June 2014

- Revised report
- Final presentation

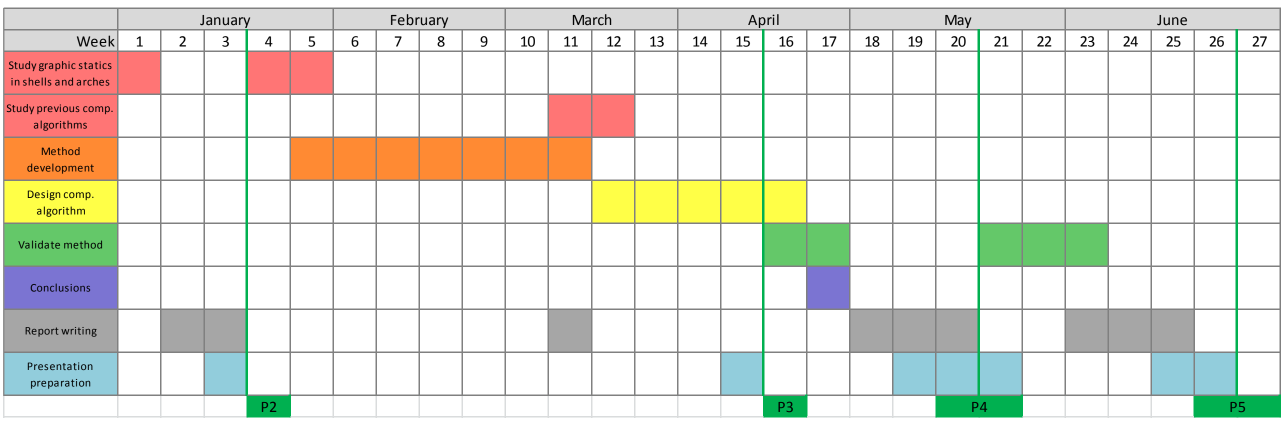

Figure 1 – Time planning

2. First results

2.1 Complementary energy method

[work in progress, more on this in the next report]

2.2 Arches and thrust lines

A thrust line is the line in which a 1-dimensional structure (arch or beam) with a given load (own weight for instance) would make equilibrium using compression only. This line can be found for masonry arches using the analogy of the chain models as shown in figure 2. The loads resulting from the own weight are discretized in a point load and projected on a chain (a). This chain line inverted in (a) shows the thrust line, the line along which all the masonry blocks make equilibrium through compression only.

In (b) all the forces working on a masonry block are drawn. To prove that these make equilibrium, a force polygon can be drawn, using the head-tail method. In this method all the forces working on the block are joined together, head to tail. If the forces make a closed polygon, it means that the sum of the vectors is equal to 0, proving that the block is in equilibrium. Since two of the three forces on block 6 have their reaction forces on other blocks (5 and 7 in this case), all the closed force polygons of the different blocks can be joined together, resulting in the force polygon in (d). This force polygon and the thrust line in (a) are each other's reciprocal figures, which means that if one of them is changed, the other one will change too.

So other thrust lines can be drawn along which this load will make equilibrium as well, simply by moving point O (called the polar coordinate) for instance closer to the own weight lines. If the point moves closer, the thrust line will become steeper (see figure 3). So for each load an infinite amount of thrust lines can be drawn. The correct thrust line is the one containing the least complementary

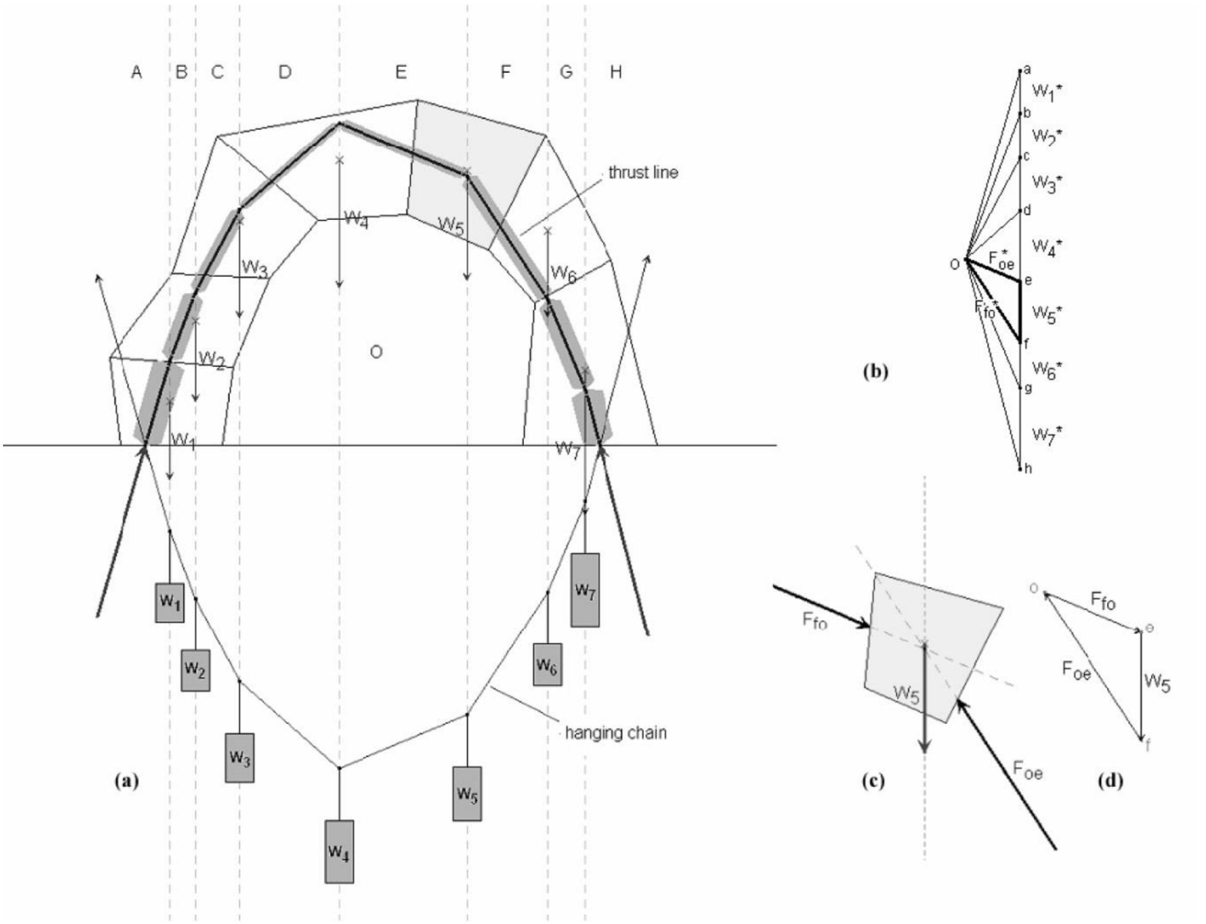

Figure 2 – The funicular line and the thrust line (Block, 2009)

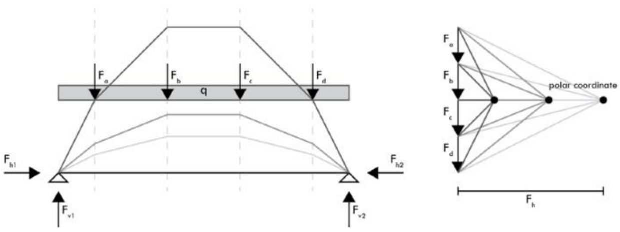

Figure 1 – Multiple thrust lines (Dool, 2012)

energy. For more information on this, see chapter xxx.

As long as the line lies within the masonry blocks, the blocks will make equilibrium through compression. Since these blocks can't handle tension or bending moments, the line has to lie within the structure, otherwise the structure will fail. However, when a structure can handle bending moments (reinforce concrete for instance), the line of thrust can lie outside the material. The force in the line of thrust can be translated onto the structure

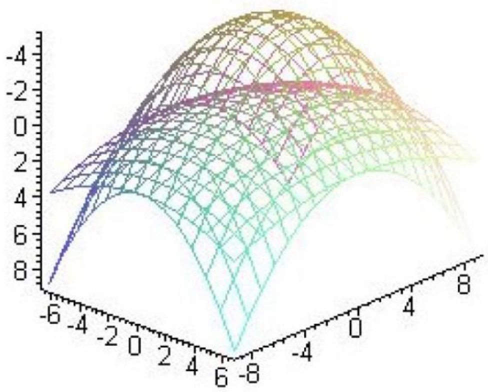

Figure 4 – Thrust surface (Tiggeler, 2009)

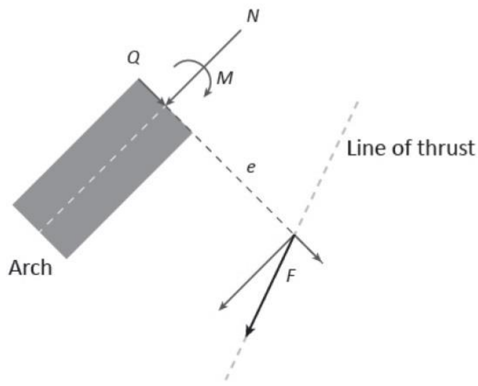

Figure 6 – Translating a force from the line of thrust to the structure (Dijk)

by adding a bending moment (see figure 6).

The thrust line has its 2D equivalent for plates and shells in the thrust network or thrust surface (figure 4). The thrust network also has a reciprocal figure as can be seen in figure 5. More on this thrust network and how to calculate it can be found in the graduation report of Tiggeler (2009) and the research of Block (2009).

2.3 Graphic statics in arches

For an arch which can handle bending moments, the correct thrust line is not just the line with the least complementary energy, since this line only deals with the complementary energy through normal forces. When the force is translated onto the structure like in figure 6, the complementary energy due to the bending moments can be calculated. In this case the correct thrust line

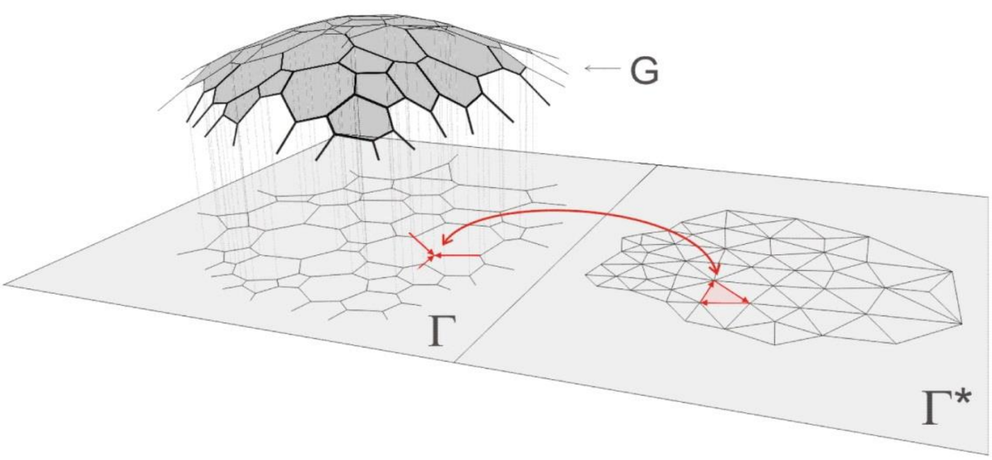

Figure 5 – Thrust surface and its reciprocal grid (Block, 2009)

is found by looking for the line with the least

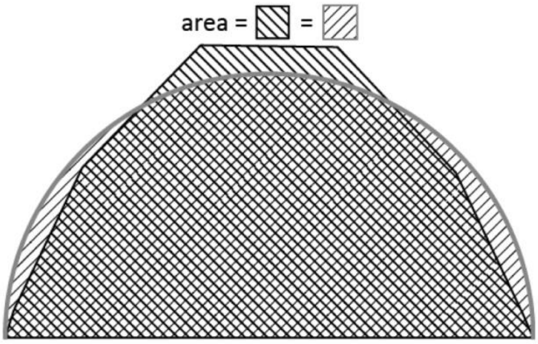

Figure 7 – The area under the structure is equal to the area under the line of thrust (Dijk)

total complementary energy (both due to bending moments and normal forces).

In the graduation report of Niels van Dijk, which is still in progress, a graphical way to calculate beams and arches is described. And a tool is made in Grasshopper to calculate arches in a quick way.

The first step is to draw an arch, which can be irregular shaped and with uneven supports.

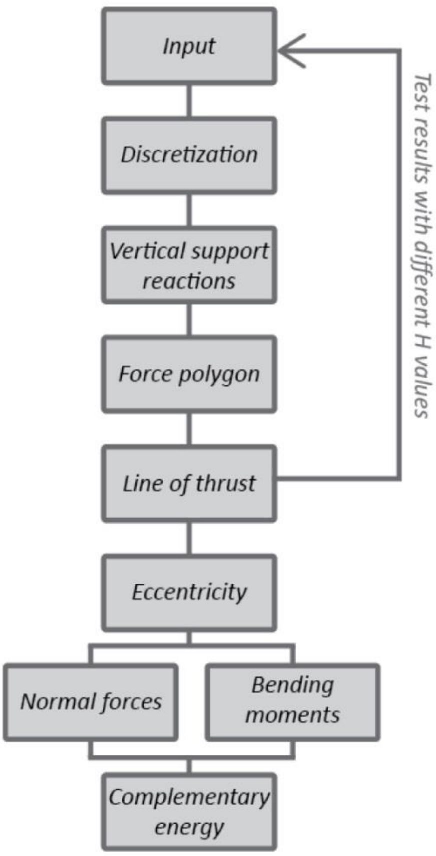

Figure 8 – The scheme to calculate a shell (Dijk)

The program discretizes both the projected load and the load due to own weight of the

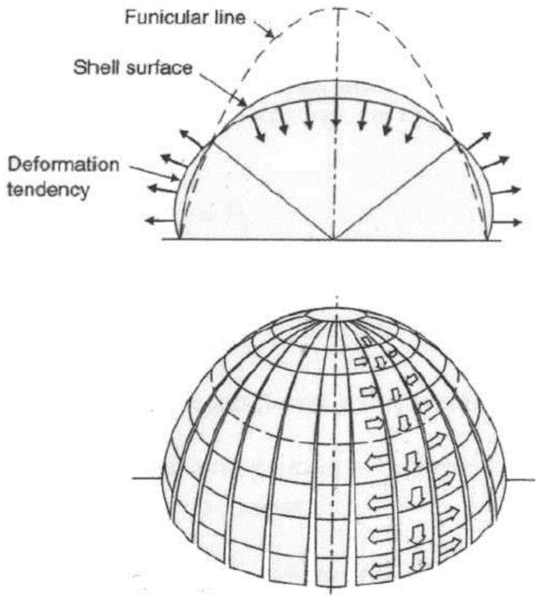

Figure 9 – Hoop forces (Schodek, 1998)

arch. This discretized load can lead to an infinite number of force polygons and their corresponding thrust lines as shown in par xxx. From a thrust line and the structure together, the complementary energy is calculated as shown in par. Xxx. With the height of the thrust line as a variable, the solution with the least complementary energy can be found, simply by changing the height until the lowest value is found. So, this is still not a direct method to calculate any arch, since there is still an iteration loop found.

The discovery that the area under the correct line of thrust equals the area under the structure (figure 7) leads to a shorter iteration loop (figure 8). This research is still in progress, so more on this subject in a later report.

2.4 Hoop forces

As discussed in paragraph 2.6, if the thrust line of an arch lies outside the material, a bending moment is needed to make equilibrium inside the structure. If a thrust surface lies outside a shell structure, equilibrium is not only made by a bending moment, hoop forces also

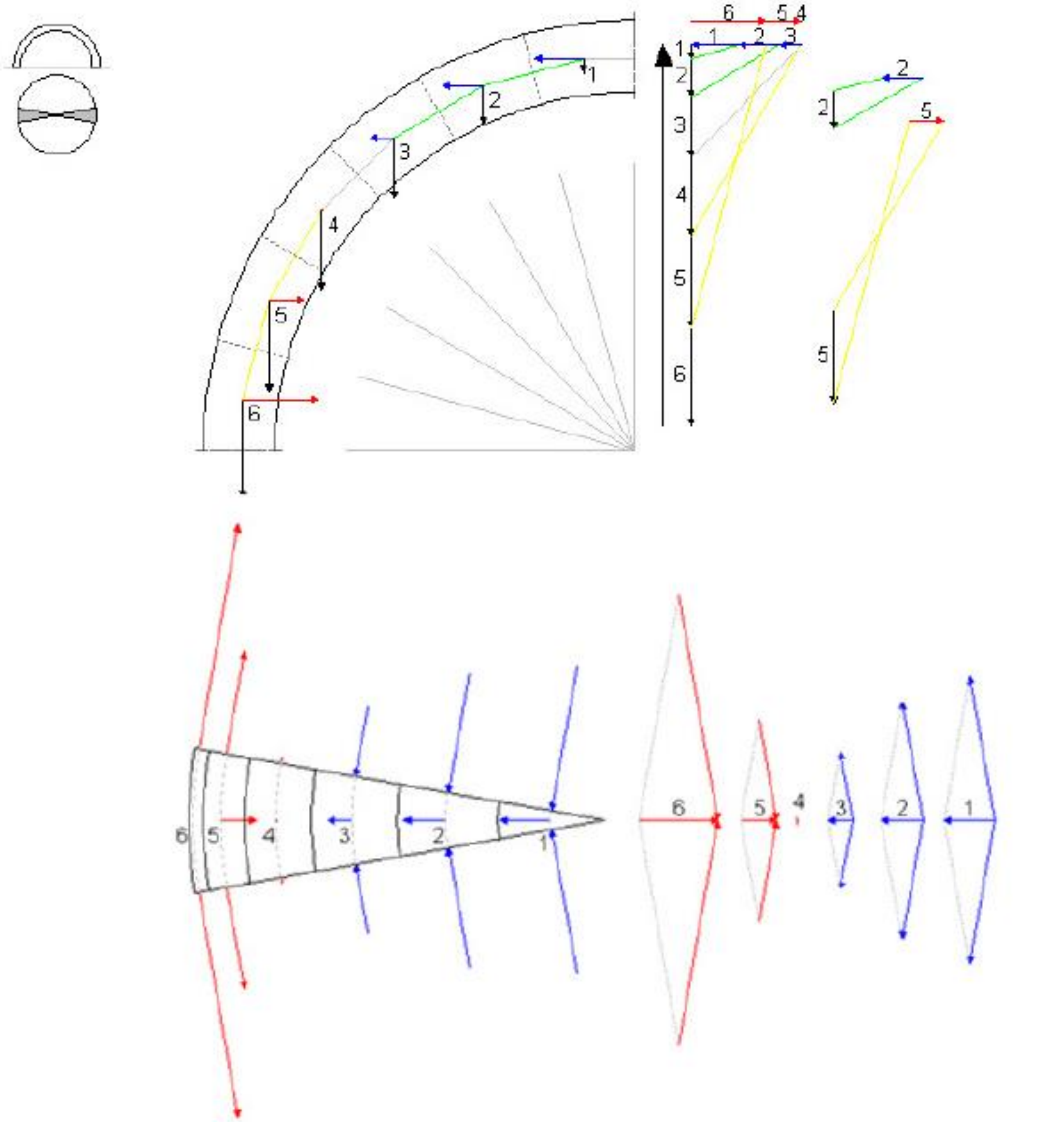

Figure 10 – Equilibrium through hoop forces

contribute to making equilibrium. In figure 9 (a), a segment of a spherical masonry shell is shown. As can be seen, the shell has a deformation tendency to the outside at the lower part, and to the inside at the upper part. The hoop forces contributing to make equilibrium (b) are in compression for the upper part and in compression in the lower part. These hoop forces only occur in 2D situations, and of course can't occur in arches. In figure 10 is shown how the hoop forces make equilibrium for each block of a segment of a shell. The upper gravitational forces are smaller because the segments of the shell are smaller. The shell can only handle normal forces, so there needs to be a resultant force through the normal line of the shell (the green line). The blue force in combination with the gravitational force result in a force along the normal line of the shell. In the top view is shown how this blue force relates to the hoop forces to make equilibrium.

2.5 Split in surfaces

To be able to calculate a shell, C.R. Calladine (1977) describes an element of a shallow shell which is conceptually split into two elements, one element which can only stretch and one of which can only bend. The stretching element is similar to for instance a bar network with hinges only, it can only transport normal forces. For the bending element there is no well-known equivalent, but it can be visualized as the beam shown in figure 11 (d).

In figure 11 this conceptual split is visualized. Surface (a) is the actual shell surface with all stress resultants. Figure 11 (b) shows the stretching surface (S-surface), with all the stress resultants working in plane. The external load is represented by a pressure p. Figure 11 (c) shows the bending surface (B-surface) with the bending and twisting stress

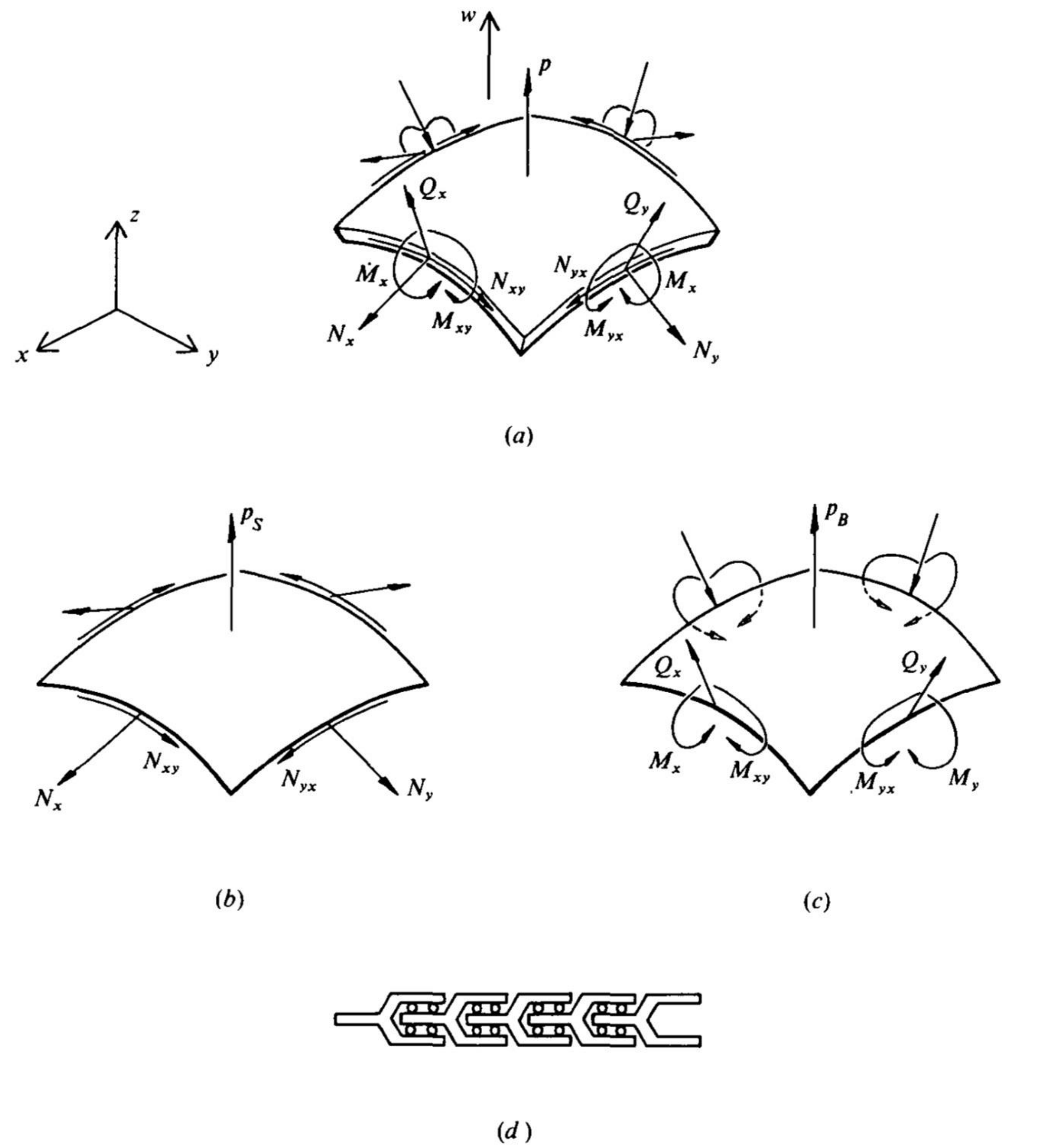

Figure 11 – The conceptual split in surfaces (Calladine, 1977)

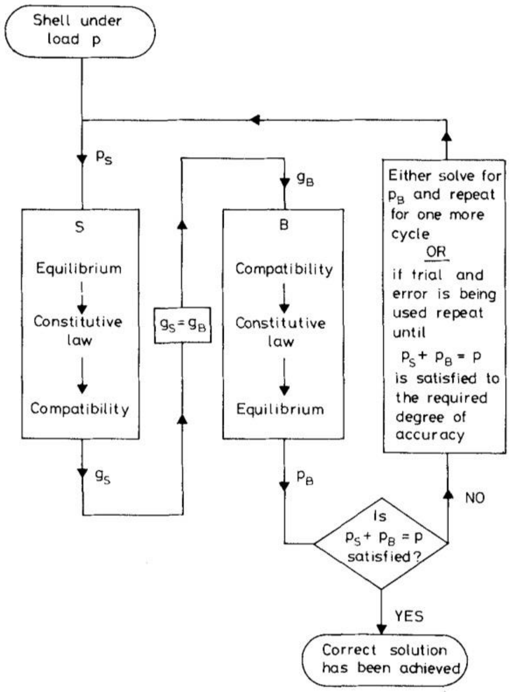

Figure 12 – The scheme to calculate shells using the split in surfaces (Pavlovic, 1984)

resultants and the shear stress resultants working out of plane.

These two surfaces are separated so the equilibrium equations can be written out separately. To relate these two surfaces together, it is stated that the sum of the pressure of each surface should equal the pressure in non-split surface. So:

$p = p_S + p_B$

The two surfaces are also related through the equation

$g_S = g_B$

which means that the geometry of the S-surface should always be equal to the geometry of the B-surface, so the deformations of the two surfaces should be the same.

Pavlovic (1984) made a scheme to use this theory to solve an element of a shell (figure 12). In this scheme, a value for p_S is chosen and with this value, the deformation is calculated. The new geometry g_S is set equal to g_B and from g_B the value for p_B is calculated. If the resulting p_B and the chosen p_S together are equal to p, the solution is reached. If not, the value for p_S should be changed and the cycle repeated until the correct solution is found. So the solution for each element in this case should be found through trial and error.

So the ratio between p_B and p_S is unknown. Once the ratio between these is known for each part of the structure, it is possible to make a relation between the geometry and the structural mechanics of a shell.

2.6 Curvature

[work in progress, more on this in the next report]

2.7 Rain flow analysis

[work in progress, more on this in the next report]

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