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# Reconciling transfer synchronization and service regularity: real-time control strategies using passenger data

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## ABSTRACT

Real-time holding control strategies are implemented, among other reasons, in order to protect transfers. In the context of high-frequency services, there is a need to reconcile between striving for single-line regularity and synchronizing inter-line arrivals. Their operationalization depends on the predictions regarding passenger flows across the network. We examine the influence of real-time passenger data on the performance of transfer synchronization control. To this end, we develop two real-time transfer synchronization controllers which make use of different passenger data sources. The controllers differ in their assumptions concerning capacity constraints as well as on-board crowding conditions. The results show that each transferring passenger saves on average 2–10 min thanks to the proposed strategy, while on-board passengers experience a delay of 1–2 min each in most cases. The highest time saving per transferring passenger is obtained when the demand level is low and the controller opts for synchronizing more frequently.

## Highlights

- Rule-based holding controller selects transfer synchronization or line regularity
- The impact of different passenger data on controller performance is investigated
- On-board crowding conditions are considered by the real-time controller
- On-board occupancy is the most valuable real-time passenger data source

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Public transport; control; transfer synchronization; short-term prediction; passenger flow data

## 1. Introduction

The public transport sector investigates ways to exploit the advancements made in information and communication technologies to improve the performance of the public transport system. One such way is the deployment of real-time control strategies that increase the system's capability to adapt to prevailing conditions. Among them, holding strategies

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are the most frequently studied control method. They are used to improve on-time performance, eliminate bunching and respond to unexpected demand (Ceder 2007). Moreover, they can be used to prevent passengers from missing their connection, thereby decreasing the transfer time between lines (Ibarra-Rojas et al. 2015). This, however, may have an adverse effect on on-board passengers and those waiting downstream.

Alongside travel time predictions, the availability of passenger flow data can play a role in the operationalization of these strategies. While the former has been extensively studied in the literature, the influence of the latter remains largely unknown. Most previous studies have used historic data on passenger flows in order to reach a control decision (see review in the next section). However, public transport systems are increasingly equipped with different types of passenger data that can be transmitted in real-time, such as the vehicle occupancy or the number and time of passenger arrivals at stops. Decisions concerning their deployment and availability should be based on scientific results. In the context of single-line operations, the study by Sánchez-Martínez, Koutsopoulos, and Wilson (2016) has investigated the attainable improvement when the number of passengers in vehicles and at stops is known to understand whether it is more important to have accurate estimates for the current or the future states. They found that more than half of the benefits stem from accurate estimations of the current state. Since they did not consider transferring passenger flows, the attainable control benefits from the different types of passenger data are yet to be determined in the context of transfer synchronization.

Additionally, the estimation of the level of service has so far been represented by the time that passengers have to wait until they can board and the time that through-going passengers need to be held at stops. Even though the waiting time of passengers at stops or inside vehicles is an important determinant of their satisfaction with the service, it does not fully capture their travelling experience. This can be complemented by the consideration of their perceived riding time when accounting for the on-board comfort.

The objective of this study is, thus, to develop a new controller that: (i) considers both transfer synchronization and service regularity; (ii) makes use of different real-time passenger data sources; (iii) is applicable in real-time and (iv) includes the on-board crowding component. The controller is designed for the context of transfers between services that are sufficiently frequent to regulate their services based on single-line headway and inter-line headway at transfer locations. Some principles of the proposed controller can be used with adjustments also in the context of low-frequency services where holding for schedule adherence is used to protect a transfer.

The main contributions are twofold: (i) comparing the impact of different passenger data sources and (ii) accounting for passenger on-board comfort in the controller.

The outline of this paper is as follows. A literature review of holding control strategies is provided in Section 2, while in Section 3 the methodology for the development of this controller is described. Section 4 applies the proposed controller to a case study in order to compare the attainable benefit from the different data sources. The main conclusions and recommendations of this study are discussed in Section 5.

## 2. Review of holding control strategies

In this section, the different implementation approaches of holding control strategies that have been adopted by other studies for controlling either a single line or transfer

synchronization are reviewed. These approaches can be distinguished depending on the holding criteria, the number and the location of the stops where holding can be implemented. Moreover, the types of data that are utilized are an important component in their application. Another relevant characteristic of real-time holding controllers is the way in which they model and include passenger flows. Each of these aspects is separately discussed in the following Sections 2.1–2.4.

### **2.1. Holding criteria**

Holding controllers can be classified into rule-based and optimization-based, as suggested by Zolfaghari, Azizi, and Jaber (2004). The former focuses, in the majority of the studies, on vehicle movements. The departure time or headway between consecutive vehicles is monitored and restored to a desired value – i.e. scheduled departure time or target headway – by the means of holding. Adhering to the timetable is an important objective for low-frequency services when passengers coordinate their arrival time with the expected vehicle arrival time or when transfer synchronization is an important consideration. In the context of high-frequency services, studies conventionally consider the headway between the present vehicle and its predecessor (Abkowitz and Lepofsky 1990; Barnett 1974; Daganzo 2009; Fu and Yang 2002; Turnquist and Blume 1980), while more recent studies look at the headway between the previous as well as the following vehicle (Cats et al. 2011, 2012; Cortés et al. 2010; Daganzo and Pilachowski 2011; Guevara and Donoso 2014; Xuan, Argote, and Daganzo 2011). Cats et al. (2011) combined the mean headway from the previous and the next bus with that of the planned headway from the previous bus in order to restrict the maximum allowable holding time and found it better than either of the two applied separately.

In the age that vehicle position data are available in real-time, holding for synchronization can be based on actual vehicle positions rather aiming to restore the transfer coordination planned for at the tactical planning phase. In case of multiple connecting lines, a rule-based controller needs to consider the arrival time of the connections. Dessouky et al. (2003) formulated a series of rules to hold a vehicle at a transfer stop so as to synchronize the transfers, i.e. the vehicle is dispatched after the transfers have been successfully completed. Some of the rules considered only the vehicle movements, either their scheduled or their forecasted arrival times, while others also took their consequences for passengers into account. The latter was performed, in the simplest case, by setting a minimum requirement of transferring passenger volume and, in the more complex ones, by selecting and applying the holding time that would inflict the minimum waiting time for passengers. Younan and Wilson (2010) estimated the net passenger time that can be saved while considering all impacted passengers and compared it to a minimum holding threshold assumed to be set by the transit agency. When the threshold was met, the vehicle was held. Another approach towards rule-based control with transfer coordination was proposed by Daganzo and Anderson (2016), who used the maximum holding time as a decision variable. They calculated the maximum permissible holding time of a controlled vehicle that arrives at a transfer point and then searched for connecting trips within this interval. Only if one or more such trips exist, is the controlled vehicle held and otherwise, it is dispatched once it completes stop operations.

Previous studies that applied mathematical programming and optimization considered the effect of control decisions on passengers. The objective function may consider only waiting passengers or incorporate also on-board passengers (Berrebi, Watkins, and Laval 2015; Delgado, Muñoz, and Giesen 2012; Delgado et al. 2009; Eberlein, Wilson, and Bernstein 2001; Sáez et al. 2012; Sánchez-Martínez, Koutsopoulos, and Wilson 2016) or even consider transferring passengers and their ability to successfully complete a direct transfer (Hadas and Ceder 2010; Hall, Dessouky, and Lu 2001; Manasra 2015; Yu et al. 2011). Zolfaghari, Azizi, and Jaber (2004) were the first to include in the objective function the extra waiting time of passengers who failed to board the first-arriving vehicle due to binding capacity constraints. Despite these additions, no holding controller has yet taken into account the riding discomfort caused by the on-board crowding conditions. The interrelation between service reliability and passenger congestion has been shown to carry significant consequences for passenger benefits in saturated networks (Cats, West, and Eliasson 2016).

## **2.2. Number and location of time point stops**

The subset of stops along a public transport route where the vehicle dispatch time is subject to regulation is called time point stops. In order to guide the timetable design and the installation of monitoring and control systems, the number and location of time point stops have been widely studied.

Sun and Hickman (2008) showed that it is beneficial in terms of headway regularization and cost reduction to have multiple time point stops. That is because, by introducing enough control points to restore the desired headway along the route, there is no need for large corrective actions, which inflict a higher passenger cost (Daganzo 2009). Similarly, Cats et al. (2012) found that, if each stop along the route is a control point, then the propagation of discrepancies is prevented by spreading the control over the entire route.

Despite these findings of improved performance when multiple control points are used, Cats, Rufi, and Koutsopoulos (2014) found that the performance is more sensitive to the location of the control points rather than their number. This could be attributed to service characteristics but also to the holding control strategy that was implemented. Higher service uncertainty could require more control points, while a strategy that considered the positions of all the vehicles, instead of just the previous and the following, could render the location of the control points less important. Regarding the location of the control points, there is a general agreement in the literature that the control points should precede a sequence of high-demand stops (Abkowitz and Engelstein 1984; Liu and Wirasinghe 2001; Turnquist and Blume 1980). Furth and Muller (2008) also concluded that the control points should be at stops with high boarding rates, preferably located at early stops along the route. Last but not least, stops with high through-passenger demand should be excluded, because of the negative effect of holding on on-board passengers (Hickman 2001).

## **2.3. Data utilization**

Prior to the rapid deployment of intelligent transport systems that can track the position of vehicles (automatic vehicle location systems, AVL) and the number of passengers (automatic passenger counting systems, APC), the application of real-time control strategies

**Table 1.** Real-time passenger data used in real-time holding controllers.

Author(s), year	Vehicle occupancy	Passenger arrivals	Passenger destination
Cortés et al. (2010)	x	x	x
Daganzo and Anderson (2016)	x		
Dessouky et al. (2003)	x		
Hickman (2001)	x	x	
Sáez et al. (2012)	x	x	x
Sánchez-Martínez, Koutsopoulos, and Wilson (2016)	x	X	x
Zhao, Bukkapatnam, and Dessouky (2003)	x		

required strategically located personnel to make the control decisions (Abkowitz and Lepofsky 1990). Most of those early studies assumed that the controller had little or no real-time information on the position of vehicles along the line and the holding strategy was applied at pre-specified control points on the basis of the timetable and possibly the distance between consecutive vehicles (Carrel et al. 2010). As noted by Bartholdi and Eisenstein (2012), the objective was generally to reduce the variation in the distribution of observed headways.

Leveraging on the real-time availability of AVL data, many studies starting from Eberlein, Wilson, and Bernstein (2001), have introduced real-time vehicle information in the holding control problem. These studies have assumed accurate and real-time knowledge of the vehicle locations which facilitates headway regulation, especially in cases where the headway to the preceding vehicle is considered. Knowledge of the current location can also enhance the prediction of the arrival time of the next vehicle or connecting services which may be used as input to the controller. Yu and Yang (2009) developed a holding strategy, which holds vehicles arriving ahead of schedule, while ensuring their on-time performance at the next stop. This is done by forecasting the departure time of the controlled vehicle from the next stop using a model based on support vector machine.

Strategies that aim to minimize a passenger-related cost function require information about the current as well as the future passenger flows. This corresponds to vehicle loads and passenger arrival rates, as well as alighting and transferring fractions at each stop. With few exceptions, previous studies derived these rates from offline historic data. New technological developments have facilitated the provision of real-time passenger data, which, however, has only been used by a limited number of studies shown in Table 1. Three data types may be acquired in real-time: (i) vehicle occupancy, which represents the number of passengers that are on-board a specific vehicle and can be derived from the APC; (ii) passenger arrivals at a stop, which can be retrieved either by the number of ticket validations (hereafter referred to as tap-ins) if the validators are positioned at station gates or by sensors, which monitor the passenger movements around the stop and (iii) passenger destinations, in case that the fare collection system requires specifying in advance the alighting stop (e.g. long-distance train services).

#### **2.4. Passenger flow modelling**

Table 2 summarizes the approaches used for integrating passenger flows when making real-time holding control decisions. The first column provides a reference to the respective study. The second describes the objective, whether it is vehicle-based or passenger-based.

**Table 2.** Passenger modelling in real-time holding controllers.

Author(s), year	Objective	Vehicle capacity	Transferring passengers	Flow uncertainty	Time-dependent demand	Real-time passenger data	Solution method
Abkowitz and Lepofsky (1990)	Vehicle	–	–	–	–	–	Rules
Bartholdi and Eisenstein (2012)	Vehicle	–	–	–	–	–	Rules
Berrebi, Watkins, and Laval (2015)	Passenger	No	No	No	No	No	Optimization
Cats et al. (2011, 2012)	Vehicle	–	–	–	–	–	Rules
Cortés et al. (2010)	Passenger	No	No	Yes	Yes	Yes	Optimization
Daganzo (2009)	Vehicle	–	–	–	–	–	Rules
Daganzo and Pilachowski (2011)	Vehicle	–	–	–	–	–	Rules
Daganzo and Anderson (2016)	Passenger	No	Yes	Yes	No	Yes	Rules
Delgado, Muñoz, and Giesen (2012)	Passenger	Yes	No	No	No	No	Optimization
Dessouky et al. (2003)	Passenger	No	Yes	No	No	Yes	Rules
Eberlein, Wilson, and Bernstein (2001)	Passenger	No	No	No	No	No	Optimization
Fu and Yang (2002)	Vehicle	–	–	–	–	–	Rules
Guevara and Donoso (2014)	Vehicle	–	–	–	–	–	Rules
Hadas and Ceder (2010)	Passenger	No	Yes	No	No	No	Optimization
Hall, Dessouky, and Lu (2001)	Passenger	No	Yes	No	No	No	Optimization
Hickman (2001)	Passenger	No	No	Yes	No	Yes	Optimization
Li et al. (2011)	Passenger	No	No	No	No	No	Optimization
Liu et al. (2014)	Passenger	No	Yes	No	No	No	Optimization
Manasra (2015)	Passenger	Yes	Yes	No	No	No	Optimization
Nesheli and Ceder (2015)	Passenger	Yes	Yes	No	No	No	Optimization
Sáez et al. (2012)	Passenger	Yes	No	Yes	Yes	Yes	Optimization
Sánchez-Martínez, Koutsopoulos, and Wilson (2016)	Passenger	Yes	No	No	Yes	Yes	Optimization
Sun and Hickman (2008)	Passenger	No	No	No	No	No	Optimization
Xuan, Argote, and Daganzo (2011)	Vehicle	–	–	–	–	–	Rules
Younan and Wilson (2010)	Passenger	No	Yes	No	No	No	Rules
Yu and Yang (2009)	Passenger	Yes	No	No	No	No	Optimization
Yu et al. (2011)	Passenger	Yes	Yes	No	No	No	Optimization
Zhao, Bukkapatnam, and Dessouky (2003)	Passenger	No	No	Yes	Yes	Yes	Optimization
Zolfaghari, Azizi, and Jaber (2004)	Passenger	Yes	No	No	No	No	Optimization
This study	Passenger	Yes	Yes	No	No	Yes	Rules

When the holding strategy is aimed at regularizing the service, the focus is placed on the headways of consecutive vehicles (as a proxy for minimizing passenger waiting times and evening out passenger loads) and, therefore, the objective is considered vehicle-based. In the case of a passenger-based objective, the objective is explicitly formulated in terms of determining the holding time that is the most beneficial for the passengers. The reason for making this distinction is that in the cases of vehicle-based objectives, information concerning the passenger flows was not part of the controller, so the rest of the columns have not been filled in. It should, however, be noted, that in most of these studies the passengers were modelled in the simulation and the effect of the applied strategy on them was quantified using passenger-related performance measures at the evaluation phase.



The 'Vehicle capacity' column describes whether the controller considered vehicle capacity constraints (i.e. integrates predictions concerning the impacts of denied boarding). The 'Transferring passengers' column refers to the consideration of transferring flows between lines in the network in the objective function (i.e. considering interchanging passengers between two or more lines). The 'Flow uncertainty' column refers to whether the controller integrates information on probabilistic passenger flow scenarios (i.e. using a sample drawn from arrival and alighting distributions rather than using expected values). In 'Time-dependent demand', the consideration of time-dependent passenger arrival and alighting rates in the controller is indicated. Sánchez-Martínez, Koutsopoulos, and Wilson (2016) used time-varying mean arrival rates at the origin-destination-level and found that the inclusion of dynamics can improve the performance compared to static inputs only when the dynamics lead to significant overcrowding. The 'Real-time passenger data' column shows whether the controller utilizes passenger data that become available in real-time such as current on-board or waiting passenger flows (as opposed to historical passenger volume data).

Because there is a strong link between the solution approach and the objective, one more column has been added to demonstrate it. The solution methods have already been discussed in Section 2.1. This overview highlights that a rule-based approach is adopted for vehicle-based objectives. The only exceptions to this are the studies by Dessouky et al. (2003), Younan and Wilson (2010) and Daganzo and Anderson (2016), who used a rule-based approach to synchronize transfers, while looking at the effect on passengers. Rule-based approaches involve significantly lower computational requirements, a key factor in determining their real-time implementation prospects.

Based on this overview, we conclude that only very few studies integrate real-time data regarding the passenger flows into a real-time control decision. Furthermore, none of them considers both capacity constraints and transferring passengers (Table 2). Consequently, the benefits of their inclusion in the control decision remain unknown. To this end, the present study develops a holding controller that makes use of different types of real-time passenger data as described in the following section. Moreover, on-board crowding conditions will be considered for the first time when making a transfer synchronization decision.

### 3. Controller development and implementation

This section describes the development of the real-time transfer synchronization controller that makes use of passenger data. In order to evaluate the effect of including vehicle capacity constraints, two controllers have been developed. The first controller (*MinPassTime*) ignores the effects of vehicle capacity, while the second (*MinPassCongTime*) considers the limited residual capacity and distinguishes between standing and sitting passengers. This allows the consideration of passengers that may fail to board, as well as the level of comfort based on the on-board crowding conditions. In this section, the underlying assumptions (Section 3.1) and the formulation of the control decision rules (Section 3.2) are presented. The controllers' performance was evaluated using a simulation model. The interaction between the simulation and the controller is described (Section 3.3), followed by the definition of the performance measures used in the evaluation (Section 3.4).

### 3.1. Controller settings

#### 3.1.1. Assumptions

We propose controllers to synchronize in real-time transfers between two lines in a single interchange stop. The terms line and route are interchangeable in the following description. In developing the controllers, the following assumptions are made:

- AVL data are available for the entire fleet.
- Services have a sufficiently high frequency for passenger arrival at stops to be described as a random Poisson process.
- Passenger arrival, alighting and transferring rates per stop and route are known from historical data and are fixed over the analysis period.
- Real-time passenger data, i.e. vehicle occupancy (APC) and/or tap-ins (AFC), if applicable, are reliable.
- Passenger boarding process follows a FIFO (i.e. first-in-first-out) regime.
- Passengers prefer sitting over standing and those on-board have the priority over boarding passengers.
- Passengers left behind due to capacity constraints remain at the stop and wait for the next vehicle on the same line.
- Vehicle order is maintained.

Control decisions require making predictions on future system states. Each control decision involves predicting vehicle arrival times for vehicles succeeding the controlled vehicle and the corresponding passenger flows. It is assumed that the effects of a synchronization decision beyond the controlled vehicle and up to two succeeding can be considered negligible, an assumption backed by the findings of Eberlein, Wilson, and Bernstein (2001). The aforementioned assumptions are reasonable since their violation would indicate either a severe network disruption or a poor design of the network supply, both of which go beyond the scope of the devised controllers.

#### 3.1.2. Features

The proposed controllers have the following features:

- Hybrid control functionality. Selects between holding for single-line regularity and transfer synchronization. This can be conceptualized as a bi-layer approach where passenger costs are minimized at the higher level and vehicle holding times are determined accordingly at the lower level.
- Passenger-oriented control. Opts for the holding option that results in the least total generalized passenger costs.
- The effect on different passenger groups is considered. Passenger costs consist of delays for those held on-board and waiting time for those transferring as well as for passengers waiting downstream.
- Capacity is considered in a refined version of the controller. It accounts for the additional waiting times caused by denied boarding at the transfer stop and downstream stops and the perceived in-vehicle time which depends on vehicle-specific on-board crowding.

- Involves passenger flow predictions using real-time data. Expected passenger flows are estimated based on a combination of historical data, real-time passenger data (if applicable) and predicted vehicle arrival times (and thus headways).

### 3.1.3. Notations

The following variables are used in the description of the control algorithms:

$m, M$	vehicle index, set
$s, S$	stop index, set
$\check{s}$	interchange stop
$q, Q$	passenger index, set
$i, j$	route indexes
$t$	time at which the control decision is taken
$t_{m,s}^a$	time at which vehicle $m$ arrives at stop $s$
$h^{\text{reg}}$	holding time in case of controlling for single-line regularity
$h^{\text{syn}}$	holding time in case of controlling for service synchronization
$\tau_{\check{s},j \rightarrow i}$	minimum time required for completing transferring process from route $j$ to route $i$
$\eta_m$	minimum headway requirement between vehicle $m$ and the successive vehicle
$\mu$	control decision number of stops horizon
$\beta$	perceived travel time weight
$\gamma$	on-board crowding multiplier
$C$	passenger cost component
$q_{m,s}$	passenger flow on vehicle $m$ at stop $s$
$\hat{q}_{m,s}$	predicted passenger flow on vehicle $m$ at stop $s$
$\lambda_{s,i}$	arrival rate at stop $s$ of passengers destined to board route $i$
$\alpha_{s,i}^{\text{alight}}$	probability that a passenger on route $i$ alights at stop $s$
$\alpha_{s,j \rightarrow i}^{\text{trans}}$	probability that a passenger alighting from route $j$ will transfer to route $i$ at stop $s$
$\varphi_m$	seat capacity of vehicle $m$
$\kappa_m$	total passenger capacity of vehicle $m$
$\hat{t}_q^{\text{a\_pass}}$	time at which passenger $q$ is recorded to arrive
$\omega$	real-time passenger data availability indicator

### 3.2. Control decision rules

The proposed controllers make use of different types of data about passengers when deciding whether to hold for transfer synchronization or not. Since the decision needs to be made fast, for the controller to be applicable in real-time, it is designed to be rule-based.

The controller is activated once a vehicle enters a transfer stop at time  $t$  and needs to decide what its dispatching time should be. In the following equations,  $m$  and  $s$  are used to denote vehicles and stops, respectively. The subscript  $i$  refers to the route of the controlled vehicle and  $j$  to the route of the connecting vehicle. The transfer stop where the two routes intersect is  $\check{s}$ . The local holding time optimization problem is solved by: first, determining alternative holding times based on either single-line regularity or inter-line synchronization (Equations (1)–(2)) and second, selecting the holding time which results in lower total generalized passenger costs (Equations (3)–(23)). In case of long headways, controlling

for regularity could be replaced with aiming for single-line punctuality by substituting Equation (2) with a schedule adherence control.

Two alternative holding times are considered: (a) the holding time required for synchronization with connecting lines,  $h^{\text{syn}}$  and (b) the holding time to regularize the controlled line,  $h^{\text{reg}}$ .

In order to synchronize transfers from the upcoming vehicle of the connecting line, the controlled vehicle  $m_i$  en-route  $i$  needs to be held until  $m_j$  en-route  $j$  arrives at time  $t_{m_j, \check{s}}^a$  and provide sufficient time for the transferring passengers to complete the transferring process,  $\tau_{\check{s}, j \rightarrow i}$ . Hence, the holding time required for synchronizing services is

$$h^{\text{syn}} = t_{m_j, \check{s}}^a - t + \tau_{\check{s}, j \rightarrow i}. \quad (1)$$

The holding time to restore service regularity is selected such that even headways between the preceding vehicle and the following one on a given line are kept, while respecting the minimum headway requirement,  $\eta_{m_{i-1}}$ . This rule-based strategy for equalizing headways was found effective in previous studies (Cats et al. 2011). The additional minimum headway requirement is introduced to accommodate rail-bound operational considerations. It gives the dispatch time of the controlled vehicle, which may be instructed to depart ahead of schedule, if the headway from the preceding vehicle exceeds the headway from the successive one. If, however, its arrival time is later than the computed departure time, then the holding time is set to zero:

$$h^{\text{reg}} = \max \left( \frac{t_{m_{i-1}, \check{s}}^a + t_{m_{i+1}, \check{s}}^a}{2}, t_{m_i, \check{s}}^a + \eta_{m_{i-1}} \right) - t. \quad (2)$$

The controller chooses between holding for synchronization between lines (Equation (1)) and holding for single-line regularity (Equation (2)) by assessing the expected implications on passengers' experience and selecting the one that is the least costly. The effect on passengers' experience is measured in terms of the total generalized passenger travel time within a certain horizon of downstream stops ( $\mu$ ). It consists of held and waiting passenger costs, and in the case, that vehicle capacity and crowding conditions are accounted for (i.e. *MinPassCongTime*), also denied boarding and riding passenger costs.

The calculation of this cost function requires estimating passengers in vehicles and at stops and predicting passenger boarding, alighting and transferring flows, as well as their arrival time at stops. In the absence of real-time data, passenger flows need to be predicted based on historical averages and passenger flow relations. The prediction needs to span from the beginning of the line in order to provide network-wide estimates. The availability of real-time passenger data shortens the horizon over which predictions need to be made thereby decreasing their uncertainty, albeit not eliminating the necessity to make predictions.

In order to study the impact of real-time passenger data on the controller performance, two real-time passenger data types are considered, namely vehicle occupancy and passenger tap-ins from ticket validation machines. The latter are assumed to be positioned at station gates and thus provide information on the arrival of passengers at stops.

Let us represent whether vehicle occupancy and tap-in AFC data are available using the dummy variables  $\omega^{\text{occ}}$  and  $\omega^{\text{tap}}$ , respectively. Three cases are distinguished: (1) no real-time passenger data ('None'),  $\omega^{\text{occ}} = \omega^{\text{tap}} = 0$ ; (2) real-time vehicle occupancy ('Occupancy'),

$\omega^{\text{occ}} = 1; \omega^{\text{tap}} = 0$ ; (3) real-time vehicle occupancy and passenger tap-ins ('+ Tap-ins'),  $\omega^{\text{occ}} = \omega^{\text{tap}} = 1$ . These cases allow testing the potential contribution of the availability of real-time passenger data for short-term passenger flow predictions in the context of real-time control strategies, an increasingly important modelling task.

### 3.2.1. Minpasstime controller

The passenger cost function consists of two elements: (a) *Held* – the delay caused to passengers held on-board the controlled vehicle at the transfer stop and (b) *Wait* – the total waiting time of passengers transferring at the transfer stop and of other passengers arriving there and up to  $\mu$  stops downstream. The latter entails the time between the arrival of passengers at the stop and the arrival of the first vehicle there:

$$C = \beta^{\text{held}} \cdot C^{\text{held}} + \beta^{\text{wait}} \cdot C^{\text{wait}}, \quad (3)$$

where the  $\beta$ s are the respective weights to account for the perceived travel time.

The total passenger held time when vehicle  $m_i$  on route  $i$  holds at transfer stop  $\check{s}$  is

$$C^{\text{held}} = q_{m_i, \check{s}}^{\text{onboard}} \cdot h, \quad (4)$$

where  $h$  is the holding time required for either regularity or synchronization (i.e.  $h^{\text{reg}}$  or  $h^{\text{syn}}$ ).  $q_{m_i, \check{s}}^{\text{onboard}}$  is the expected number of passengers on-board vehicle  $m$  upon departure from stop  $s$ :

$$q_{m,s}^{\text{onboard}} = [\hat{q}_{m,s-1}^{\text{onboard}} + q_{m,s}^{\text{wait}} - q_{m,s}^{\text{alight}}] \omega^{\text{occ}} + \left[ \sum_{k=1}^s (q_{m,k}^{\text{wait}} - q_{m,k}^{\text{alight}}) \right] (1 - \omega^{\text{occ}}), \quad (5)$$

where  $\hat{q}_{m,s-1}^{\text{onboard}}$  is the observed on-board occupancy upon departing the preceding stop.  $q_{m,s}^{\text{wait}}$  and  $q_{m,s}^{\text{alight}}$  are the expected number of passengers waiting at stop  $s$  for vehicle  $m$  and alighting from the same vehicle at this stop, respectively. These expected passenger flows are determined as follows:

$$q_{m,s}^{\text{wait}} = [\hat{q}_{m,s}^{\text{wait}} + \lambda_{s,i} \cdot (t_{m,s}^a - t)] \omega^{\text{tap}} + [\lambda_{s,i} \cdot (t_{m,s}^a - t_{m-1,s}^a)] (1 - \omega^{\text{tap}}), \quad (6)$$

$$q_{m,s}^{\text{alight}} = q_{m,s-1}^{\text{onboard}} \cdot \alpha_{s,i}^{\text{alight}}, \quad (7)$$

where  $\hat{q}_{m,s}^{\text{wait}}$  is the observed number of waiting passengers.  $\lambda_{s,i}$  is the arrival rate at stop  $s$  of passengers that want to board route  $i$ .  $\alpha_{s,i}^{\text{alight}} \in [0, 1]$  is the probability that a passenger en-route route  $i$  alights at stop  $s$  (i.e. the share of on-board passengers that alights).

The total passenger waiting time consists of the waiting times of the passengers (a) transferring at stop  $\check{s}$  from a route other than the one controlled and (b) at stops downstream of the transfer stop  $\check{s}$  that are within the controllers horizon  $\mu$ :

$$C^{\text{wait}} = C_{\check{s}}^{\text{wait}} + C_{>\check{s}}^{\text{wait}}. \quad (8)$$

The first component is relevant only in case transfer synchronization is not guaranteed:

$$C_{\check{s}}^{\text{wait}} = q_{m_j, \check{s} \rightarrow i}^{\text{trans}} \cdot (t_{m_i+1, \check{s}}^a - t_{m_j, \check{s}}^a) \cdot \delta, \quad (9)$$

where  $\delta$  is a dummy variable that takes the value of one if passenger costs are calculated for single-line regularity and zero if the calculation is made for transfer synchronization.

Equation (9) refers to the expected number of passengers alighting vehicle  $m$  at stop  $s$  and transferring from route  $j$  to route  $i$  which is defined as

$$q_{m,s,j \rightarrow i}^{\text{trans}} = q_{m,i,s}^{\text{onboard}} \cdot \alpha_{s,j}^{\text{alight}} \cdot \alpha_{s,j \rightarrow i}^{\text{trans}} \quad (10)$$

where  $\alpha_{s,j \rightarrow i}^{\text{trans}} \in [0, 1]$  is the transferring fraction out of the alighting passengers.

The second component in Equation (8) refers to waiting times at  $\mu$  stops downstream of  $\check{s}$ :

$$C_{>\check{s}}^{\text{wait}} = \sum_{s=\check{s}+1}^{\check{s}+\mu} \left[ \frac{(t_{m_i,s}^a - t_{m_i-1,s}^a)^2}{2} \cdot \lambda_{s,i} \right]. \quad (11)$$

It includes the headway in the numerator multiplied by itself because the expected number of waiting passengers is the product of the arrival rate and the elapsed time, and half of the latter corresponds to the average waiting time of those arriving during this interval.

The formulation in Equation (11) relies on historical arrival rates for estimating expected passenger flows. When tap-ins are available, the recorded arrival time  $t_q^{\text{a-pass}}$  of passenger  $q \in Q_{m_i,s}^{\text{wait}}$  of each detected passenger is known, along with the elapsed waiting time.  $Q_{m_i,s}^{\text{wait}}$  is the set of passengers that have arrived at stop  $s$  in anticipation of vehicle  $m_i$  serving route  $i$  at the time the control decision is made. Equation (11) is then replaced by the following term if  $\omega^{\text{tap}} = 1$ :

$$\begin{aligned} C_{>\check{s}}^{\text{wait}} = & \sum_{s=\check{s}+1}^{s_2} \left[ \sum_{q \in Q_{m_i,s}^{\text{wait}}} (t - \hat{t}_q^{\text{a-pass}}) + (t_{m_i,s}^a - t) \cdot |Q_{m_i,s}^{\text{wait}}| \right] + \sum_{s=\check{s}+1}^{s_2} \left[ \frac{(t_{m_i,s}^a - t)^2}{2} \cdot \lambda_{s,i} \right] \\ & + \left[ \sum_{s=s_2+1}^{\check{s}+\mu} \frac{(t_{m_i,s}^a - t_{m_i-1,s}^a)^2}{2} \cdot \lambda_{s,i} \right], \end{aligned} \quad (12)$$

where  $s_2$  is the last stop that the leading vehicle,  $m_i - 1$ , has departed from. In case,  $s_2 < \check{s} + \mu$ , i.e. the latest stop visited by the previous vehicle is upstream of the controller forward-looking horizon, then it is necessary to consider passenger arrivals between expected vehicle arrivals. The first term in Equation (12) sums, therefore, passenger waiting times insofar and expected remaining waiting times for all those passengers that have already arrived at downstream stops that vehicle  $m_i$  will be the next one to serve. The second term is the expected waiting time of these passengers that are expected to arrive in the relevant downstream stops by the time that vehicle  $m_i$  will arrive there. The last term refers to passengers arriving during the headway between vehicles downstream of stop  $s_2$  but still within the considered horizon (if applicable).

The *MinPassTime* controller is designed to resemble state-of-the-art rule-based transfer synchronization strategies that deploy a passenger-related objective (see Table 2; i.e. Daganzo and Anderson 2016; Dessouky et al. 2003) when assuming that vehicle occupancy data are available in real-time. In line with previous implementations, once a synchronization decision is made, the vehicle is held until the transfer takes place, thereby accounting for underestimations of the predicted arrival time. While previous implementations made a choice between holding for synchronization and no control, *MinPassTime* involves choosing between transfer protection and the even-headway strategy for single-line service regularity (Equation (2)).

### 3.2.2. *Minpasscongtime controller*

Accounting for vehicle capacity and crowding conditions, the total generalized passenger cost function considered by *MinPassCongTime* consists of two elements in addition to (1) *Held* and (2) *Wait* that were included in the previous controller:

$$C = \beta^{\text{held}} \cdot C^{\text{held}} + \beta^{\text{wait}} \cdot C^{\text{wait}} + \beta^{\text{denied}} \cdot C^{\text{denied}} + \beta^{\text{riding}} \cdot C^{\text{riding}}, \quad (13)$$

where the  $\beta$ s are the respective weights to account for the perceived travel time. The additional elements are: (c) *Denied* – the additional waiting at a stop due to denied boarding. This includes the time between the arrival of the first vehicle at the stop and the time at which the passenger can board a vehicle and (d) *Riding* – the perceived in-vehicle riding time which depends on the on-board crowding conditions.

Both held and riding times account for on-board comfort distinguishing between passengers that sit and stand:

$$C^{\text{held}} = (\gamma^{\text{sit}} \cdot q_{m_i, \check{s}}^{\text{sit}} + \gamma^{\text{stand}} \cdot q_{m_i, \check{s}}^{\text{stand}}) \cdot h, \quad (14)$$

$$C^{\text{riding}} = \left| \sum_{s=\check{s}+1}^{\check{s}+\mu} [(\gamma^{\text{sit}} \cdot q_{m_i, s}^{\text{sit}} + \gamma^{\text{stand}} \cdot q_{m_i, s}^{\text{stand}}) \cdot (t_{m_i, s}^a - t_{m_i, s-1}^a)] - \sum_{s=\check{s}+1}^{\check{s}+\mu} [(\gamma^{\text{sit}} \cdot q_{m+1, i, s}^{\text{sit}} + \gamma^{\text{stand}} \cdot q_{m+1, i, s}^{\text{stand}}) \cdot (t_{m+1, i, s}^a - t_{m+1, i, s-1}^a)] \right|, \quad (15)$$

where  $\gamma$ s are the on-board crowding multipliers. Note that these multipliers vary as a function of the number of passengers standing and sitting and can account for the non-linear effect of on-board crowding. The absolute value of the difference in perceived on-board time is taken in Equation (15) to penalize uneven passenger distributions among successive vehicles regardless of their order.

The expected number of passengers boarding vehicle  $m$  at stop  $s$  is

$$q_{m, s}^{\text{board}} = \min(q_{m, s}^{\text{wait}}, \kappa_m - q_{m, s-1}^{\text{onboard}} + q_{m, s}^{\text{alight}}), \quad (16)$$

where  $\kappa_m$  is the capacity of vehicle  $m$ . Out of the on-board passengers, the number of those sitting and standing can be determined as follows:

$$q_{m, s}^{\text{sit}} = \min(q_{m, s}^{\text{onboard}}, \varphi_m), \quad (17)$$

$$q_{m, s}^{\text{stand}} = \max(q_{m, s}^{\text{onboard}} - \varphi_m, 0), \quad (18)$$

where  $\varphi_m$  is the seat capacity of vehicle  $m$ .

The *MinPassCongTime* distinguishes between waiting time for the first vehicle (Equations (19)–(21)),  $C^{\text{wait}}$  and waiting time for successive vehicles in case of denied boarding (Equations (22)–(23)),  $C^{\text{denied}}$ . The former, waiting time for the first-arriving vehicle, consists of two components (similarly to Equation (8) for *MinPassTime*). The first component, the waiting time for transferring passengers in case of holding for single-line regularity,  $C_s^{\text{wait}}$ ,

remains the same as detailed in Equation (9). The second component, waiting times at stops downstream of the transfer stop,  $\check{s}$ , is

$$C_{>\check{s}}^{\text{wait}} = \sum_{s=\check{s}+1}^{\check{s}+\mu} \frac{(t_{m_i,s}^a - t_{m_i-1,s}^a)}{2} \cdot (q_{m_i,s}^{\text{wait}} - q_{m_i-1,s}^{\text{denied}}) + \sum_{s=\check{s}+1}^{\check{s}+\mu} \frac{(t_{m_i-1,s}^a - t_{m_i-2,s}^a)}{2} \cdot q_{m_i-1,s}^{\text{denied}} \quad (19)$$

Equation (19) consists of two terms: the first term refers to passengers for whom vehicle  $m_i$  is the first-arriving vehicle and the second term accounts for the initial waiting time for passengers that were denied from boarding vehicle  $m_i - 1$  and boarded vehicle  $m_i$  (assuming that denied boarding does not repeatedly occur on successive vehicles as mentioned in Section 3.1.1). Both terms calculate the initial waiting time based on the headway between the respective successive vehicles assuming a random passenger arrival process. The number of passengers that are denied from boarding vehicle  $m$  at stop  $s$  (i.e. passengers left behind) due to capacity constraints in Equation (19) is

$$q_{m,s}^{\text{denied}} = q_{m,s}^{\text{wait}} - q_{m,s}^{\text{board}} \quad (20)$$

The additional waiting time experienced by those passengers that are left behind is calculated in Equation (22).

When tap-ins are available,  $\omega^{\text{tap}} = 1$ , the arrival time  $\hat{t}_q^{\text{a-pass}}$  of each detected passenger is known and Equation (19) is substituted with:

$$\begin{aligned} C_{>\check{s}}^{\text{wait}} = & \sum_{s=\check{s}+1}^{\check{s}+\mu} \sum_{q \in Q_{m_i-1,s}^{\text{denied}}} (t_{m_i-1,s}^a - \hat{t}_q^{\text{a-pass}}) + \sum_{s=\check{s}+1}^{s_z} \sum_{q \in (Q_{m_i,s}^{\text{wait}} \setminus Q_{m_i-1,s}^{\text{denied}})} (t_{m_i,s}^a - \hat{t}_q^{\text{a-pass}}) \\ & + \sum_{s=\check{s}+1}^{s_z} \left( \frac{(t_{m_i,s}^a - t)}{2} \cdot (q_{m_i,s}^{\text{wait}} - q_{m_i-1,s}^{\text{denied}}) \right) + \sum_{s=\check{s}+1}^{\check{s}+\mu} \frac{(t_{m_i,s}^a - t_{m_i-1,s}^a)}{2} \cdot (q_{m_i,s}^{\text{wait}} - q_{m_i-1,s}^{\text{denied}}). \end{aligned} \quad (21)$$

Equation (21) sums over the following terms for waiting times for passengers waiting downstream of the transfer stop, respectively: (a) passengers who arrived before vehicle  $m_i - 1$  and were denied boarding; (b) passengers who have already arrived at stops between  $\check{s}$  and  $s_z$ , the last stop visited by  $m_i - 1$ ; (c) passengers who are to arrive by the time that vehicle  $m_i$  will arrive at the same stops and (d) passengers arriving during the headway between vehicles downstream of stop  $s_z$  but still within the considered horizon (if applicable).

The total generalized passenger cost function, Equation (13), refers to the additional waiting time experienced by passengers that are denied boarding either at the transfer stop or at downstream stops within the control horizon:

$$\begin{aligned} C^{\text{denied}} = & \sum_{s=\check{s}+1}^{\check{s}+\mu} [(t_{m_i,s}^a - t_{m_i-1,s}^a) \cdot q_{m_i-1,s}^{\text{denied}}] + \sum_{s=\check{s}+1}^{\check{s}+\mu} [(t_{m_i+1,s}^a - t_{m_i,s}^a) \cdot q_{m_i,s}^{\text{denied}}] \\ & + q_{m_i,\check{s}j \rightarrow i}^{\text{trans\_denied}} \cdot (t_{m_i+1,\check{s}}^a - t_{m_i,\check{s}}^a) \cdot (1 - \delta) \\ & + \max(q_{m_i,\check{s}j \rightarrow i}^{\text{trans\_denied}} - \kappa_{m_i+1} + q_{m_i+1,\check{s}}^{\text{onboard}}, 0) \cdot \eta_{m_i+1} \cdot (1 - \delta) \\ & + q_{m_i+1,\check{s}j \rightarrow i}^{\text{trans\_denied}} \cdot \eta_{m_i+1} \cdot \delta. \end{aligned} \quad (22)$$



Equation (22) consists of the additional waiting times experienced due to denied boarding of passengers: (a) waiting downstream and denied by the previous vehicle; (b) waiting downstream and denied by this vehicle; (c) transferring that were left behind by the previous vehicle; (d) transferring that are left behind by this vehicle and will at least experience the minimum headway and (e) transferring that will be denied by the next vehicle, in case holding does not synchronize.  $q_{m_i, \check{s}}^{\text{denied}}$  is defined in Equation (20), while the number of passengers transferring from route  $j$  to route  $i$  who are left behind by vehicle  $m_i$  at stop  $\check{s}$  due to capacity constraints is

$$q_{m_i, \check{s} \rightarrow i}^{\text{trans\_denied}} = \max(q_{m_i, \check{s} \rightarrow i}^{\text{trans}} + q_{m_i, \check{s}}^{\text{onboard}} + \lambda_{\check{s}, i} \cdot h^{\text{syn}} \cdot (1 - \delta) - \kappa_{m_i}, 0). \quad (23)$$

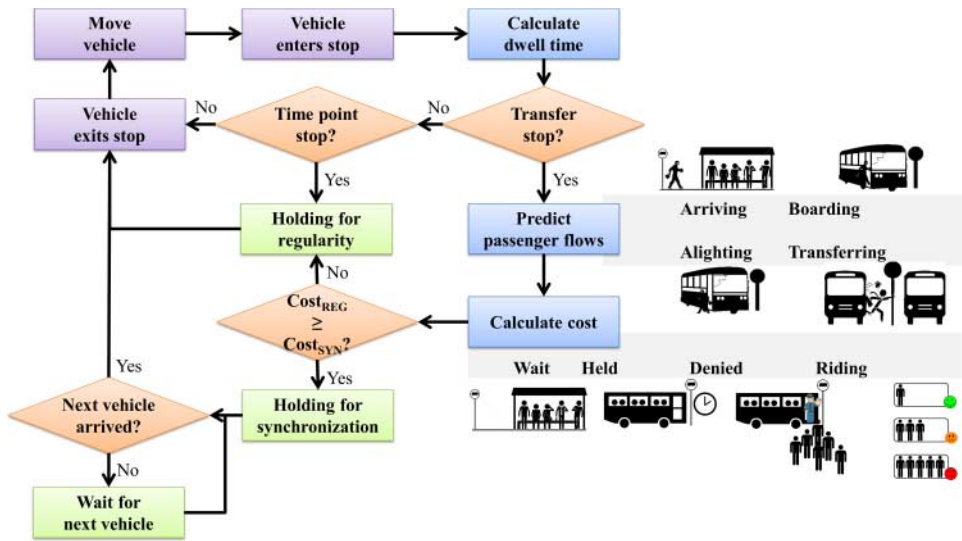
This definition takes into consideration the expected number of passengers that will arrive at the transfer stop during the time that the vehicle is held there for synchronization, if applicable (i.e.  $\delta = 0$ ).

### 3.3. Implementation

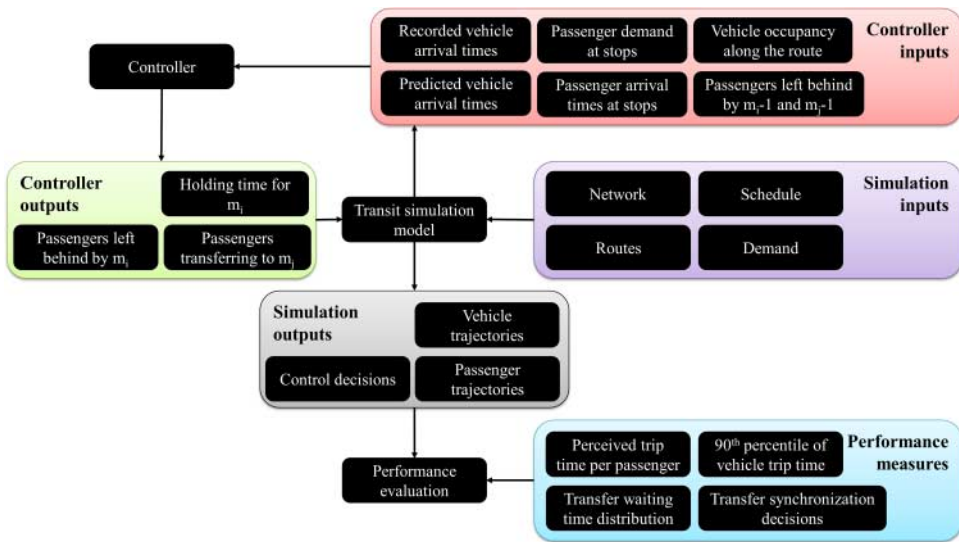
The aforementioned controllers were implemented in MATLAB. A public transport simulation model, BusMezzo, was used for mimicking real-world operations as a testbed for testing the performance of the controllers and their consequences under different scenarios. BusMezzo is a dynamic public transport operation and assignment model (Cats, West, and Eliasson 2016; Toledo et al. 2010), which simulates the progress of individual public transport vehicles and passengers using an agent-based approach. In this implementation, vehicle travel times between stops were simulated by sampling from distributions whose mean and standard deviation were specified, while the passenger arrivals at each stop follow the Poisson distribution. Vehicle capacity limitations are enforced in BusMezzo regardless of the control logic applied. Passengers that are left behind are retained in the flow of waiting passengers. Previous studies have demonstrated that BusMezzo can reproduce the bunching phenomenon (Cats et al. 2010) and represent dynamic congestion effects including variations in on-board crowding and denied boarding (Cats, West, and Eliasson 2016).

During a simulation run, each time a public transport vehicle enters a transfer stop, BusMezzo calls an instance of the control algorithm in MATLAB. Figure 1 shows the sequence of control decisions taken by the controller once a public transport vehicle enters a stop. If the stop is a transfer stop, then the passenger flows are predicted and the costs of the two candidate holding times, namely holding for single-line regularity and holding for transfer synchronization, are computed. In case, the controller decides to hold for transfer synchronization, the vehicle is instructed to wait until another vehicle (from either line) arrives at the stop in order to account for the uncertainty associated with the predicted arrival time of the connecting vehicle. At all other time point stops, only regularity control is applied. The maximum holding time is thus the longer of the headway to the successive vehicle on the same line and the connecting line. Finally, at all other stops, the vehicle is dispatched once the dwell time has been completed.

Figure 2 presents schematically the data exchange between the simulation model and the controller. The simulation model provides the controller with real-time vehicle and passenger data as well as predictions regarding vehicle arrival times at downstream stops



**Figure 1.** Sequence of control decisions taken by the controller.



**Figure 2.** Schematic relations between the controller and the simulation model and the resulting key performance indicators.

within a pre-determined horizon. These outputs are fed into the controller where the passenger data exchange depends on data availability. Following its execution, the controller informs the simulation model whether it should hold for regularity or synchronization and the respective expected vehicle dispatching time. The controller stores some of its estimates and retrieves them later if applicable to gain efficiency.

### 3.4. Performance evaluation

At the end of a BusMezzo simulation run, the model generates output files that summarize vehicle and passenger metrics. Since the proposed controllers are designed to improve passengers' experience (i.e. minimizing perceived passenger travel time), the following passenger-oriented key performance indicators were selected. Next to these, an additional metric is included to assess the impact on vehicle operations, an important practical consideration:

- Passenger perceived travel time components for four passenger groups defined in relation to the transfer stop: (i) boarding and alighting upstream ('Upstream'); (ii) boarding and alighting downstream ('Downstream'); (iii) transferring ('Transferring') and (iv) boarding upstream and alighting downstream of the transfer stop ('Traversing'). The travel time calculation considers all passengers served within the simulation period from their origin to their destination and all the components encountered along their trip.
- Transfer waiting time distribution, defined as the difference between the arrival times at the transfer stop of the two vehicles passengers used to perform their trip.
- The share of the control decisions that opted for transfer synchronization.
- The 90th percentile of the vehicle trip time per line which is commonly used for determining the fleet size.

## 4. Application

The controllers proposed in the previous section were applied to a case study of two tram lines in The Hague, the Netherlands, which were selected for demonstration purposes. The performance of the alternative controllers was evaluated using the simulation model. We first describe the case study (Section 4.1) and then turn into analysis and discussion of the simulation results (Section 4.2).

### 4.1. Case study description

The case study consists of tram lines 3 and 17 which are depicted in Figure 3 and their key characteristics are summarized in Table 3. Line 3 is part of the RandstadRail network which includes high-capacity lines connecting the urban agglomeration in the South-Holland province. Its long route extends from the westernmost neighborhood, through the city center and the central train station into neighboring suburbs to the east of The Hague where the line has a fully segregated right of way. Line 17 is an L-shaped urban tram with shorter distances between stops and served by smaller vehicles. It connects the seaside to the southernmost neighborhoods through the city center and a major train station.

The controller aims at synchronizing transfers at the common stop (marked by a black star in Figure 3). The operations of the case study lines are simulated for the eastbound and southbound directions, respectively. The line configuration and link travel times are based on publicly available data provided by the public transport operator. The planned headway for each of the lines is 10 min between 7 am and 6 pm on weekdays. The line operation is simulated for two hours within this period and, therefore, the minimum headway requirement was set accordingly ( $\eta_{m-1,m} = \eta_{m+1,m+2} = 10$  min). In order to ensure that the



**Figure 3.** Routes of the two simulated tram lines in The Hague.

**Table 3.** Summary of key characteristics of the case study lines.

Characteristic	Line 3	Line 17
Route length (km)	33	17
Number of stops	37	35
Seat capacity	84	76
Total vehicle capacity	214	188
Planned headway (min)	10	10
Number of boarding passengers per hour	4654	3054

performance of the real-time controller is assessed rather than the tactical timetable planning, dispatching times were shifted so that the two lines are scheduled to synchronize at the transfer stop.

Since the case study lines are relatively long, two additional time point stops, which serve as transport hubs, were added and are marked by the stars in Figure 3. The horizon length,  $\mu$ , corresponds to the number of stops between the transfer stop and the next downstream time point stop. Adopting the common industry standard, vehicle arrival time predictions were based on the assumption that current delays persist at downstream stops. Hence, arrival times at downstream stops are estimated by shifting the scheduled arrival time according to the currently observed delay as described in the study of vehicle arrival predictions in Cats and Loutos (2016) and Fadaei, Cats, and Bhaskar (2017).

The weights in the generalized travel time function in Equations (3) and (13) were specified based on values estimated and reported in the literature of value of times and passenger route choice (Cats, Rufi, and Koutsopoulos 2014, 2016; Wardman 2004):  $\{\beta^{\text{held}}, \beta^{\text{wait}}, \beta^{\text{denied}}, \beta^{\text{riding}}\} = \{1.5, 2, 7, 1\}$ . Moreover, the crowding multipliers in Equations (14) and (15),  $\gamma^{\text{sit}}$  and  $\gamma^{\text{stand}}$  were applied to account for the perceived in-vehicle time as function of the vehicle load factor, i.e. the ratio between vehicle occupancy and seat capacity, based on a meta-analysis performed by Wardman and Whelan (2011).

Furthermore, because the transfer stop location is modelled to be on the same physical point, the walking time required for the transferring passengers to transfer between the two lines is considered negligible (i.e.  $\tau_{s,j \rightarrow i}^w = 0$ ) in order to improve the interpretability of the results.

Each controller was simulated with the three passenger data levels and three demand levels. The three passenger data levels – without real-time data, with real-time APC data, and with real-time APC and AFC data as detailed in Section 3.2 – were simulated. Note that APC is not available in real-time in the case study area at the moment and AFC devices are located on-board next to vehicle doors. In the simulation model, we treated real-time observations of passenger flows depending on the assumed real-time availability. The simulation experiments allow, therefore, to estimate the potential benefits of real-time data streaming and moving the fare validators to station gates for improved real-time control strategies.

Three demand level scenarios were designed to reflect a low, a medium and a high average load in order to test the performance of the controllers under different service utilization levels. The medium load is considered the base case scenario, while the other two were constructed by applying a uniform  $\pm 25\%$  change in passenger volumes. In addition to the *MinPassTime* and *MinPassCongTime* controllers, a benchmark case, in which the transfer stop was used as an ordinary time point stop where regularity control is implemented (Equation (2)), was simulated for each demand level to assess the added-value of real-time transfer synchronization. This scenario design results with a total of 21 ( $= 2*3*3 + 1*3$ ) cases. The reference scenario is, thus, also subject to control, albeit where each line is controlled independently without seeking to attain transfer protection in real-time.

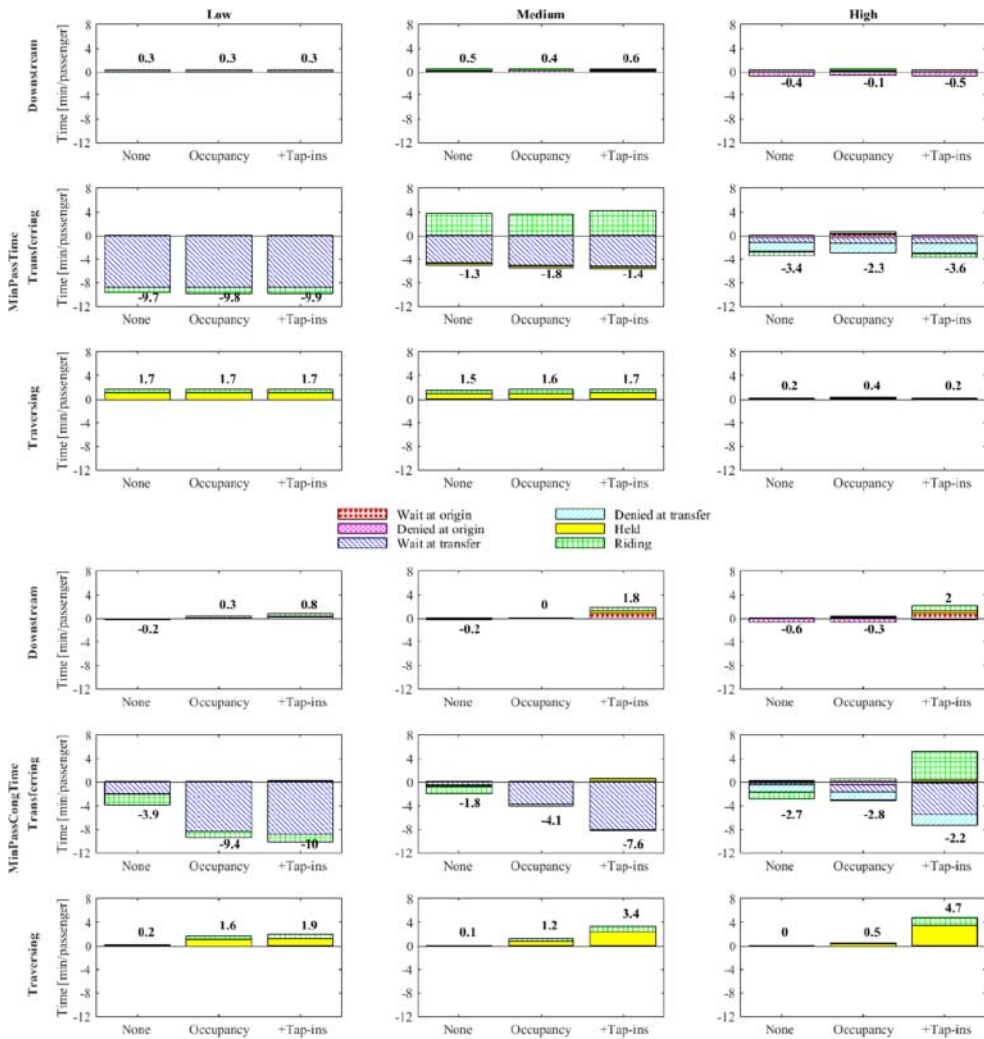
The time required for processing each control decision amounts to 0.05 s regardless of the passenger data level and the demand, thereby rendering it applicability in real-time. In order to attain statistically robust results from the stochastic simulation model, 30 runs were found necessary to attain a 95% confidence level.

## 4.2. Analysis and results

The results of each scenario (a combination of controller, data availability and demand level) were analyzed per passenger group by taking the average over 30 simulation runs.

### 4.2.1. Perceived travel time

Figure 4 shows the perceived trip time per passenger and passenger group compared to the value obtained in the respective benchmark case. Hence, the graph shows the impact of reconciling transfer synchronization with service regularity as compared to the case where only individual line regularity is sought. Negative values correspond to time savings, i.e. travel time reductions attained by using the controller. The results are presented in order of increasing demand level scenario from left to right, while the results of controller *MinPassTime* and *MinPassCongTime* are displayed in the top and the bottom part of the figure, respectively. The rows in each part correspond to a passenger group. The results for the upstream group are not shown, since these passengers are not affected by the control strategy that is applied at the transfer stop. The bars within each plot refer to the three passenger data levels and display the change in each of the time components discussed in Equations (3) and (13). This allows analyzing the impact of the proposed controller on each user group. Initial and excessive waiting time due to denied boarding are computed and shown separately for the first boarding location and the transfer location for transferring passengers. All travel time components are evaluated for all passengers served within the simulation time from their origin to their destination.



**Figure 4.** Generalized travel time changes per passenger for each passenger group (rows), for each controller (boxes), demand level (columns) and passenger real-time data (bars).

When comparing the performance of the controllers, the results show that each transferring passenger saves on average 2–10 min thanks to the proposed strategy, while prolonging traversing passengers’ trip by 1–2 min each in most cases. The highest time savings for transferring passengers are obtained when the demand level is low and the controller opts for synchronizing more frequently. The effect on downstream passengers varies depending on the demand scenario, the controller used and the passenger data level. The first simpler controller, *MinPassTime*, with passenger occupancy data are equivalent to strategies previously used in the literature (i.e. Daganzo and Anderson 2016; Dessouky et al. 2003). As can be seen in Figure 4, the *MinPassTime* with real-time occupancy data results in –1.8 min for transferring passengers at the cost of +1.6 min for traversing passengers and +0.4 for downstream passengers in the Medium demand level scenario. This can be used as a benchmark when assessing the performance of the *MinPassCongTime* controller for the

same scenario (Occupancy, Medium):  $-4.1$  min for transferring passengers while reducing the extra cost of traversing passengers to  $+1.2$  min for traversing passengers and no impact for those passengers waiting further downstream. Hence, the crowding and capacity sensitive controller, *MinPassCongTime*, attains significantly greater time savings for transferring passengers while reducing the delay caused for traversing passengers.

The passenger data level was found influential only when applying *MinPassCongTime*. The incorporation of passenger congestion effects – on-board discomfort and denied boarding – leads to decisions that try to evenly distribute the demand over the vehicles and hence the quality of vehicle load estimates becomes crucial. The availability of real-time data greatly influences these estimates and consequently leads to different control decisions. In this case study, transferring passengers benefit most from the availability of tap-ins under the low and medium demand scenarios. As can be seen in Figure 4, when occupancy and tap-in data become available in real-time, the travel time savings of transferring passengers in the presence of Medium demand level and the advanced controller, *MinPassCongTime*, significantly increase to  $7.6$  min albeit with an increase of  $3.4$  min for traversing passengers and  $1.8$  min for downstream passengers. Given the adverse effect on downstream and traversing passengers, the results obtained in the case on-board occupancy data are available are likely to be better for the system as a whole under most passenger group composition circumstances.

Next, the impact of the demand level was investigated. When the demand is high, the knowledge of tap-ins in real-time leads to greater waiting time savings for transferring passengers, but increases their riding time, thereby compromising the total effect. This increase is due to the myopic view of the network conditions inherent to a limited prediction horizon. This is confirmed in a simulation scenario where all downstream stops are included in the horizon in the case of *MinPassCongTime* and when tap-ins are available and the demand level is high, the results are similar to those obtained under the corresponding scenario with the short horizon and *MinPassTime*. Besides the limited horizon, another reason for the underperformance of *MinPassCongTime* when tap-ins are available is that the service reliability deteriorates as the load rises, which renders the vehicle arrival time predictions less reliable. Given the controllers' reliance on these predictions, the expected time required for synchronization is underestimated when taking the control decision, leading to a longer than expected holding time for traversing passengers and longer waiting time for passengers further downstream.

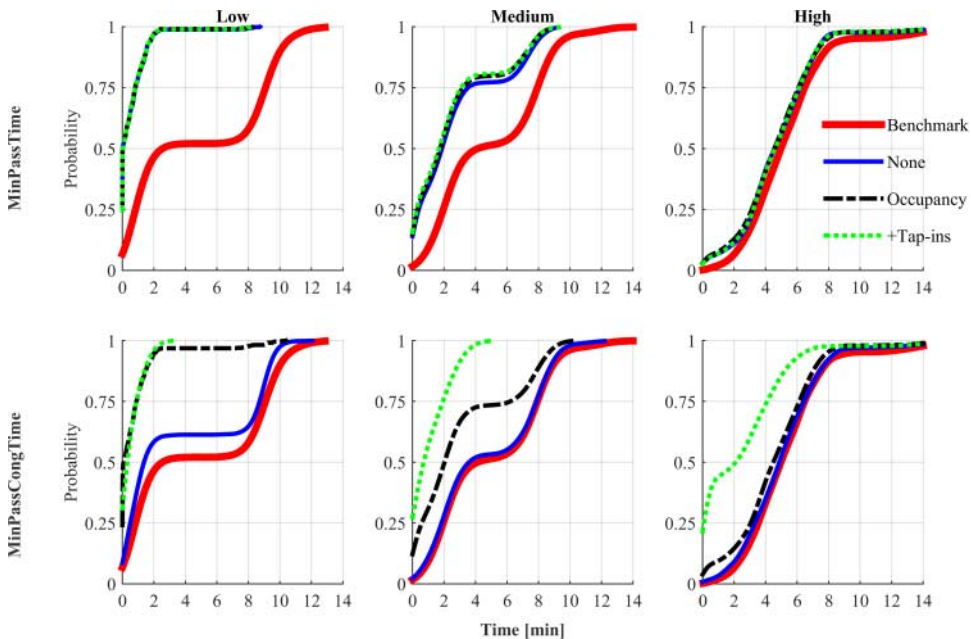
Based on the above-mentioned results concerning the impact of real-time passenger data, it can be stated that the availability of tap-ins does not offer any added value to the proposed controller – when taking its consequences for all passenger groups into consideration – when the vehicle occupancy is known in real-time. *MinPassCongTime* achieves better results, compared to *MinPassTime*, in case that the capacity constraints can become binding and vehicle occupancy is available. Consequently, it can be concluded that vehicle occupancy is the most valuable real-time data source.

The proposed real-time transfer synchronization controllers are designed to forecast and assess the implications of a transfer protection decision on different passenger groups – downstream, transferring and traversing. It is, therefore, expected that the share of these passenger groups will greatly influence the outcomes of the control strategies in their attempt to maximize the travel time savings for those transferring and minimize the delays it induces for those waiting further downstream or held on-board the vehicle. The sensitivity

of the controllers with respect to different transfer flow levels was, thus, tested. In addition to the base case share of 6% of all passengers travelling in the case study network, scenarios with shares of 3% and 9% were investigated. The results indicate that with an increasing share of transferring passengers, the overall pattern is that the controller seeks to further reduce the travel time of transferring passengers at the cost of traversing and downstream passengers. As can be expected, the controller opts more frequently for transfer protection with an increasing number of passengers benefiting from it and accepting longer delays for other passengers in the system as long as their joint generalized marginal costs do not exceed the expected time savings for those transferring.

#### 4.2.2. Transfer waiting time distribution

The time savings for transferring passengers are further investigated by examining the cumulative distribution of the transfer waiting time, depicted in Figure 5 per demand level, i.e. Low, Medium and High, and controller, i.e. *MinPassTime* (top) and *MinPassCongTime* (bottom). The respective results of the benchmark scenario when only controlling for the regularity of each line are also displayed in all cases for comparison. The shape of these curves represents the headway between successive arrivals of vehicles from different lines at the transfer stop. When the demand is low, there is a step in the Benchmark case between 2.5 and 8 min at 50% probability. This clearly distinguishes two classes of transferring passengers, those who either transfer directly or alight the vehicle controlled for synchronization and need to wait for the connecting one to arrive, and those who have just missed their connection and have to wait a full headway. This step fades as the demand rises, indicating the randomness in the vehicle arrivals and, thereby, the unreliability of the service.



**Figure 5.** Cumulative distribution of transfer waiting time per demand level (columns) and controller (rows).

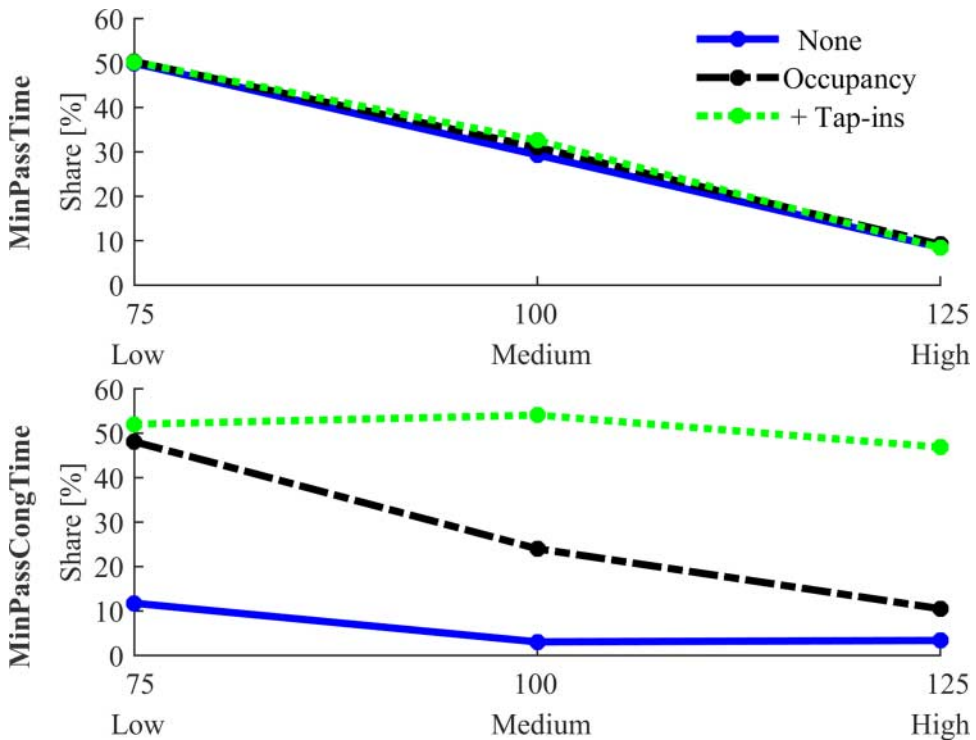


In the controlled cases, these curves shift upward due to the synchronization of transfers. Passengers from the second group experience a direct transfer, i.e. have a negligible waiting time, and depending on the share of synchronization, the corresponding part of this group is served, changing the shape of the curve accordingly.

Another key performance indicator of a transfer synchronization method is the share of direct transfers. The probability to attain a direct transfer is 25% when the demand is low and it decreases as the demand rises, except for the case when *MinPassCongTime* has knowledge of the tap-ins and synchronization is favored. Moreover, the maximum waiting time that transferring passengers may experience increases (up to 24 and 40 min for the medium and high load, respectively), representing cases of denied boarding at the transfer stop. The difference between the passenger data levels is again visible for *MinPassCongTime*, where no real-time data lead to results close to the benchmark, knowledge of the occupancy results in a similar performance to *MinPassTime* and the availability of tap-ins causes all vehicles to synchronize.

#### 4.2.3. Share of synchronization control decisions

All vehicles arriving at the transfer stop, from both lines, are subject to a control decision. The share of control decisions that opt for transfer synchronization is shown in Figure 6. Since both lines have the same planned headway and the timetable is planned for transfer coordination, a slight deviation from planning will result with vehicles arriving in pairs and all transfers can be synchronized when the first-arriving vehicle waits for the upcoming one,



**Figure 6.** Share of synchronization control decisions per demand scenario (base case level = 100) and controller.

leading to 50% of the vehicles holding. While a higher share of synchronization decisions can happen locally, a global synchronization rate considerably higher than 50% reflects a significantly unstable system (i.e. unreliable service or predictions). Noticeably, as the demand increases, fewer vehicles are synchronized by *MinPassTime*, regardless of the passenger data level. In contrast, *MinPassCongTime* follows this pattern only in the case that vehicle occupancy data are available. In the case of no real-time passenger data, it rarely synchronizes irrespective of the demand level, while the acquisition of tap-ins leads to the synchronization of all transfers. Interestingly, in the case of the long horizon in the high demand controlled by *MinPassCongTime*, the availability of tap-ins does not result in 50% synchronization decisions but merely 7.5%. This demonstrates that the highest share of synchronization decisions does not necessarily lead to the greatest time savings for transferring passengers.

#### 4.2.4. Vehicle trip time

In addition to the impact on passengers, the controllers were also evaluated with respect to their influence on vehicle travel times. The last performance measure relates to the 90th percentile of the vehicle trip time, which remains unchanged for line 3 but increases for line 17. The explanation for this outcome is that the arrival time of line 17 at the transfer stop is more reliable due to the fact that it has fewer stops upstream of the transfer stop. Therefore, line 17 often arrives with a shorter interval between vehicles of different lines at the transfer stop and is thus more likely to be held for synchronization. The increase in the 90th percentile of the vehicle trip distribution is in the order of 1–2 min with an average holding time of 0.5–1 min, except for in the cases that tap-ins are available for medium and high demand. In these cases, the increased share of transfer synchronization decisions by *MinPassCongTime* along with the decreased service reliability cause substantially longer holding times ( $\sim 5$  min) resulting in up to a 7-minute increase in the 90th percentile of the vehicle trip time for line 17.

## 5. Conclusion

### 5.1. Key findings and implications

Two real-time holding controllers for transfer synchronization of public transport vehicles were developed and the benefits of using different types of real-time passenger data were investigated. The first controller, *MinPassTime*, neglects vehicle capacity limitations, while the second, *MinPassCongTime*, explicitly considered capacity constraints as well as on-board crowding conditions. Both controllers apply a rule-based strategy that aims at selecting the holding time that yields lower total generalized passenger travel times. Regarding the data types, three cases were distinguished: (1) no real-time passenger data; (2) real-time vehicle occupancy; and (3) real-time vehicle occupancy and passenger tap-ins. In all cases, it was assumed that data concerning historical passenger demand is available. The proposed controllers incorporate for the first-time passenger tap-in as a real-time source of information and on-board crowding and capacity into a rule-based transfer synchronization strategy. The performance of these controllers was assessed by simulating public transport operations for a case study network. A benchmark case was also implemented, where only regularity control was applied at the transfer stop. Each of these cases was simulated for three demand levels.

The performance of the controllers was evaluated for synchronizing transfers between two trams lines using a simulation model that mimics tram operations. Our findings indicate that there is value in considering holding for synchronizing transfers when the scheduled arrival times are synchronized at the transfer stop, since transferring passengers can save up to a full headway. Each transferring passenger saves on average 2–10 min in the case study network thanks to the proposed strategy, while on-board passengers experience a delay of 1–2 min each in most cases. The highest benefit was achieved when the demand level was low and more vehicles could be synchronized. In case of high-demand levels, the prevalence of synchronization decisions did not yield higher savings for transferring passengers, because it decreased their riding comfort due to overcrowded vehicles. Compared with the *MinPassTime* controller, the crowding and capacity sensitive controller, *MinPassCongTime*, yielded significantly greater time savings for transferring passengers while reducing the delay caused for traversing passengers. *The former further increase if tap-in data are available in real-time, albeit at the cost of increasing passenger travel costs for other passenger groups.*

The control algorithm involves making predictions about future system states. Consequently, the performance of the controller depends on the quality of the predictions made and the latter depends in turn on the data available in real-time. The comparison of the results for the different passenger data types reveals that the vehicle occupancy is the most valuable source of information and it is unsurprisingly best exploited by the controller which considers on-board crowding and denied boarding. Information from the AFC devices was found beneficial for transferring passengers when making transfer synchronization decisions but induces greater delays for downstream and traversing passengers. In the case that data from AFC devices is available, it was assumed to be indicative of passengers' arrival time. While this is the case for some high level of service bus and light rail systems, it may not be applicable in other situations. Notwithstanding, the real-time availability of data from on-board AFC devices can still be valuable in predicting downstream passenger flows and improved upon forecasts based on historical values. Regarding the selection of the horizon length, a long horizon was found appropriate when the demand level is high, because it prevents a limited view of the anticipated network saturation levels. The determination of the optimal horizon length is a topic for future research.

The performance of the proposed controllers suggests that even when timetables are designed to allow for coordinated transfers, public transport operators can significantly improve transfer synchronization by adopting real-time transfer holding strategies. In the case study application, the likelihood of a synchronized transfer increased from virtually non-existing to 25% under certain circumstances. The potential impacts of the proposed controller on vehicle scheduling can be further analyzed in future research by considering a longer horizon at the control algorithm, including consequences for trip chaining.

The benefits of adopting the proposed control strategy depend on the passenger demand distribution, and the conditions under which this policy is advantageous, need to be further investigated to support the provision of more explicit guidelines for the deployment of the control strategy. A sensitivity analysis demonstrates that the controllers respond to changes in the demand profile by assessing the consequences for the total generalized passenger travel time while taking the composition of passenger groups into consideration. It is expected that an increasing share of transferring passengers, especially in relation to traversing passengers will yield the greatest benefits due to an increasing

likelihood of opting for transfer synchronization as well as an increased in the number of positively affected passengers. Hence, the performance reported in this study is expected to be an underestimation given the low share of transferring passengers in the case study. The proposed controller is expected to be most beneficial in hubs where there is a high passenger turnover. The promising results can facilitate the implementation of a real case study that will allow testing the extent to which the envisaged controller can be realized and highlight relevant practical considerations.

## **5.2. Limitations and future research**

The findings of this study pave the way towards future research to extend the control and prediction rules. The negligible computation time calls for testing the scalability and real-time applicability of the controller for real-world networks. The local rule-based approach taken in this study can potentially be incorporated into technological developments of transit management software. The passenger flow prediction scheme could be enhanced by incorporating the uncertainty in the vehicle arrival time predictions, as well as the deterioration in prediction reliability over time and space. Last but not least, the controller should allow the re-evaluation of a synchronization decision in case the originally expected holding time is found to be insufficient or alternatively estimate a threshold value which defines the upper limit for favoring a transfer synchronization decision. The added-value of the real-time availability of different passenger data sources should then be estimated again to verify the generalizability of the conclusions made in this study. Future implementations may consider dependencies among passenger alighting and transferring probabilities based on an empirical analysis of passenger demand patterns.

Further research is needed to determine the range of passenger demand compositions that will yield a net positive effect in terms of total generalized travel costs. The scalability of the approach proposed in this study and its performance in more complex settings where optimization methods might be required should be investigated. Furthermore, more advanced controllers should be developed to deal with various network configurations, such as the coordination of (a) more than two lines intersecting at one stop; (b) multiple transfer stops along one line and (c) a common corridor where transfers are not restricted to specific transfer location. In the latter case, trade-offs exist not only between synchronization for transferring passengers and single-line regularity but also the regularity of the common corridor section for those who travel along the corridor. The results of this study demonstrate that even when timetables aim at coordination, real-time transfer protection measures can be beneficial. We expect that increasing inter-line headways will increase the benefits of controlling but only until a certain point after which holding for synchronization will become too costly and the controller will opt for single-line regularity. While transfer coordination at the timetable planning phase is expected to impact the likelihood of transfer synchronization and consequently the extent to which real-time control is needed, further research is needed to determine the exact relation.

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