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BETREFFENDE:

Local Structural Response.

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Contents.

Notation.

$[A]$	Matrix A.
$[a]$	Eigen vector a.
$D = \frac{Et^3}{12(1+\nu^2)}$	Plate rigidity per unit width.
E	Young's modulus.
K	Buckling factor.
N_x, N'_x	Load per unit width in x-direction.
N_y, N'_y	" " " " " y- " .
N_{xy}, N'_{xy}	Shear load per unit width.
t	Plate thickness.
α	Stress concentration factor in hatch corner.
γ, λ	Magnification factor.
ν	Poisson's ratio.
σ	Stress.
σ_{cr}	Critical buckling stress.

Other symbols used will be defined in the text.

LOCAL STRUCTURAL RESPONSE

Introduction.

This report has been prepared as a contribution to the report of committee 5 "Local Structural Response" of the I.S.S.C. 1973, Hamburg. The contents are limited to a general discussion on the purposes of local structural response analysis followed by a review of work carried out in France, Germany and Italy.

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1. Purposes of Local Structural Response Analysis.

It has been argued that local response is so much part of over all response that it does not merit separate attention. This might be true if the subject was wholly dependent on the boundary conditions "in situ". The following short discussion will show that this would be too limited a point of view.

Local response analysis can be defined as a detailed analysis of deformations and distributions of stress, strain and temperatures in a sub-structure subjected to a certain demand. Said analysis may be carried out theoretically and/or experimentally for one or more of the following reasons:

a) Improvement of either newly designed or existing structures,
b) Determination of the lowest limit load causing "damage",
c) " of post-damage behaviour,
d) " of the lowest limit load causing "collapse",
e) Detection of areas sensitive to low-cycle fatigue effects,
f) Diffusion of temperature differences and the ensuing stress and strain distributions.

The expressions "damage" and "collapse" may be associated with failure as defined in the report of committee 10, I.S.S.C. 1967, that is: for a (sub-) structure "damage" is tantamount to failure if "its original form has changed in a way which is detrimental to its future performance, even though there may be no immediate loss of function." The meaning of failure as "collapse" is clear for in this case the (sub-)structure "is damaged so badly that it can no longer fulfil its function."
No distinction has been made between elastic, elasto-plastic and plastic response since sub b, c and d cover the whole range* and provide a more meaningful sequence. Furthermore this division is better suited to provide information needed for the probabilistic design approaches. One of these has been discussed by Mansour and Faulkner /1/.

The above does not only apply to local response as part of overall response it is also valid in case of comparative studies. Some examples of these will be discussed in the review below. In the case of item a comparative studies are essential and the means to carry them out have become sufficiently sophisticated and accurate to cover very complex sub-structures. Another area for investigations concerns the influence of slight variations in boundary conditions on the magnitudes of the response parameters so as to provide more information on the accuracy of the calculations.

* If need be the items may be sub-divided, thereby reintroducing the concepts of elastic, elasto-plastic and plastic response. Alternatively linear and non-linear response may by used.

Information is also needed concerning the influence on response of combined loading. It can be expected for instance that there exists a value for the ratio of in-plane to normal loads beyond which the critical buckling load will be distinctly influenced.

It may be concluded from the foregoing that local response (and overall response too for that matter) is not limited to one clearly defined range of the material properties. For this reason a rearrangement of the relevant I.S.S.C. technical committees may have to be considered.

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2. Review of Work in France, Germany and Italy.

Efforts have been directed at solving stability problems and at the verification of the use of mathematical models in response analysis.
Buckling, especially under shearing loads, is requiring attention because of the relatively thin plates in very large vessels and because of the growing importance of local demand. The very size of ships has caused the local loads to grow to first order magnitudes. The resulting need for information on this kind of demand has already been stressed in the previous committee report /2/. Efforts to supply this information are being made in various countries.

a. Stability Problems.

Two methods for the calculation of the critical stress associated with the first mode of buckling have been developed for in-plane loaded plates without stiffening or with some form of stiffening.
Both methods are based on the classical expression given by Timoshenko /3/:

$$\gamma = \frac{D\iint\left\{\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2\,(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\,\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]\right\} dx dy}{-\iint\left[N'_x\left(\frac{\partial w}{\partial x}\right)^2 + N'_y\left(\frac{\partial w}{\partial y}\right)^2 + 2N'_{xy}\,\frac{\partial w}{\partial x}\,\frac{\partial w}{\partial y}\right] dx dy}$$

$$\left(\gamma = \frac{\text{strain energy in bending}}{\text{work done by in-plane forces/unit width of plate}}\right)$$

where:

$w\ (x, y)$ defines the deflected shape of the middle plane of the plate,
γ is the magnification factor required to bring one of the acting in-plane constant forces N'_x, N'_y, N'_{xy} to its critical value.

Castel and Finifter /4/ use a polynomial expression to approximate the function $w\ (x, y)$ and it consists of two components $w\ (x, y) = B\ (x, y) \times P_1\ (x, y)$. The boundary conditions are be defined by $B\ (x, y)$, and $P_1\ (x, y)$ is a complete third order polynomial dependent on ten constants $a_1, \ldots\ldots\ldots\ a_{10}$.

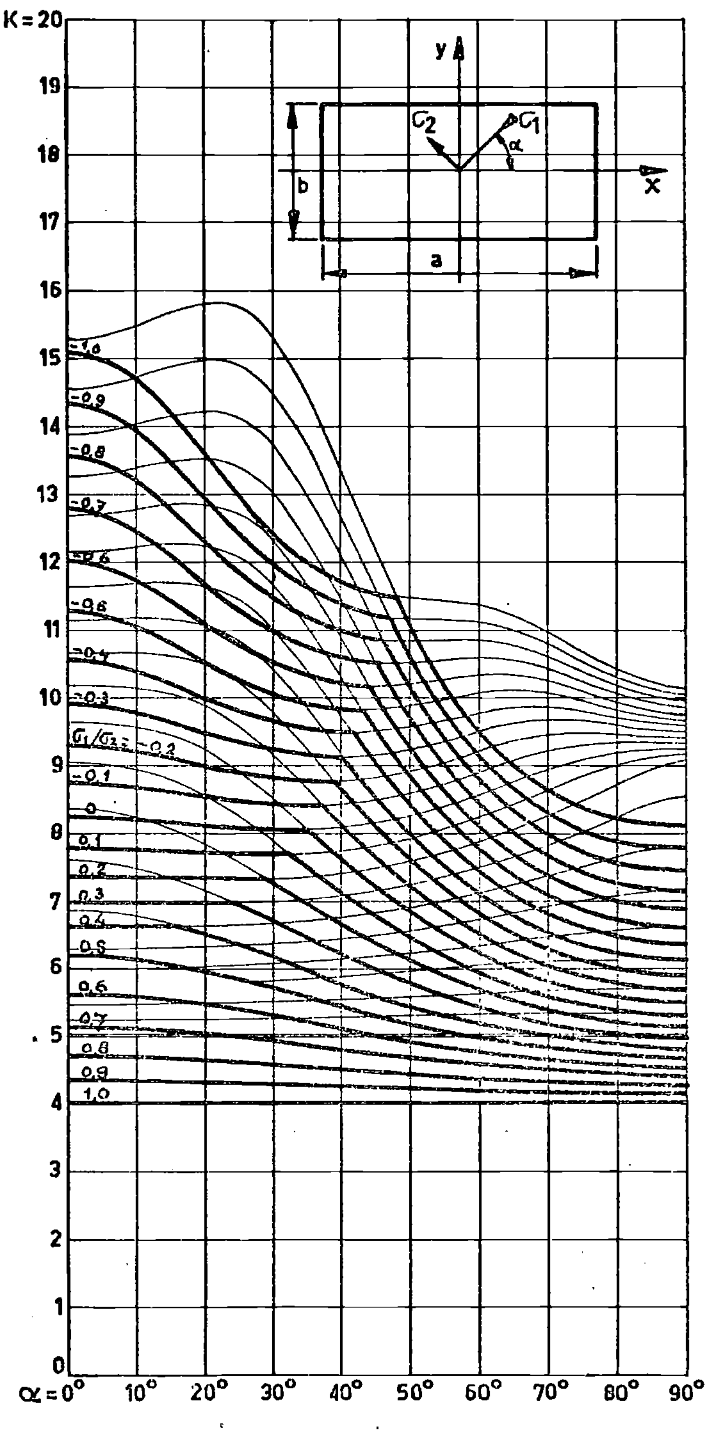


Figure 1. Rectangular plate, fixed on four sides.
a/b = 1,6, /4/.

Substitution of the selected function w (x, y) leads after integration to an expression for γ that is bi-linear in a_i. By virtue of matrix formulation and the condition that the required value of γ is an extreme, the expression finally appears as:

$$\left(\left[A^s\right] - \gamma^o \left[B^s\right] \right) \left[a^o\right] = 0,$$

that is: as an eigen-value problem and as such it is solved.
The critical stress is expressed as:

$$\sigma_{cr} = K \cdot \frac{\pi^2 D}{b^2 t}$$

where b is the short side of a rectangle or of the right angle side of a triangle.

Diagrams can be prepared where K-values are presented as functions of the angle between the compressive stress and X-axis for a number of stress (i.e. loading) ratios. (See figure 1).

The stress ratios range from -1 (pure shear) to +1 (uniform compression).

Castel and Finifter state that up to a slenderness ratio of 2 the error amounts to 5% at most, but it grows to 10 to 12% for a slenderness ratio of 3. For more slender plates the function P_1 (x, y) should be a higher order polynomial.

It is possible to apply this method also to a plate with a stiffener or to one where the unit width forces N'_x, N'_y, N'_{xy} vary linearly within the plate.
The authors have also studied other boundary conditions than simple support or complete fixity.

Broère /5/ has discussed a finite element approach to the stability problems of thin plates. The expression for the magnification factor γ (which is called λ in /5/) is directly written in matrix form and after various operations appears as the eigen-value problem:

$$\left(\left[K_F\right] + \lambda \left[K_G\right] \right) \left[\delta\right] = 0,$$

which is solved by iteration. The author has used two types of triangular elements, a constant stress and a pure bending element, the latter has been presented by Zienkiewicz /6/. This choice of elements was dictated by the available computer capacity, and the bending element though non-conforming gave good results. In fact the program has been tested and the error amounted to a maximum of 10% for the case of a square plate with a mesh having four triangular elements to a side. The error is on the safe side and is of less importance when the program is used to compare a range of structural solutions for one problem. During studies on buckling caused by pure shear

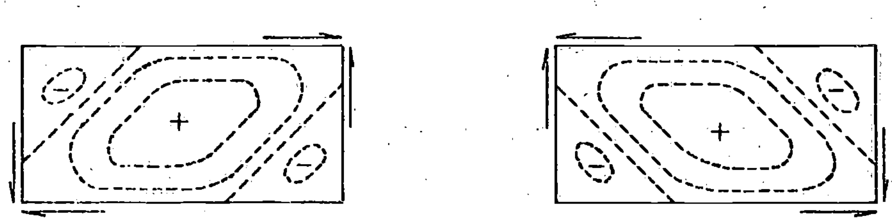


Figure 2. Deformations due to equal but opposite buckling loads, /5/.

equal but oppositely signed eigen-values were found, which is logical when considering that critical shear loads exist that have an opposite sign but an equal magnitude (figure 2).

The critical stress is again given by $\sigma_{cr} = K \cdot \frac{\pi^2 D}{b^2 t}$.

The program has been extensively used to investigate the influence on the factor K of openings in plates and of various kinds of stiffeners attached to one or more sides of the plates. The plates themselves were square, rectangular or they had the shape of an isosceles triangle with one angle of 90°.

Concerning the influence of openings in plates, figure 3

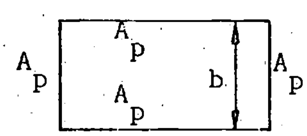
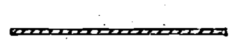
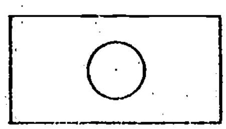
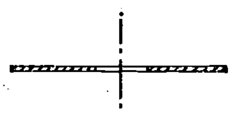
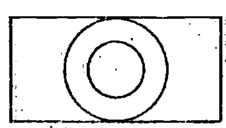
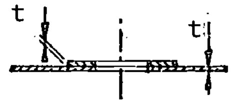
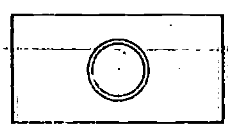
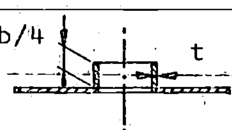
diameter of hole = $\frac{b}{2}$	slenderness ratio = 2	K 	K 
		2,23	6,21
		1,63	3,18
		5,27	12,72
		4,22	8,58

Figure 3. Table of results concerning a plate with a reinforced hole.
(A_p = simple support), /5/.

shows the results of calculations carried out in case of an oblique compression on the short side b of the rectangular plate and of pure shear. The marked increase of the K-values for circular doubling round the hole led to a search for the size of an equivalent doubling. It was found that a breadth of 0,077 b for the doubling plate gave a value for K equal to the one for an unperforated plate (figure 4).

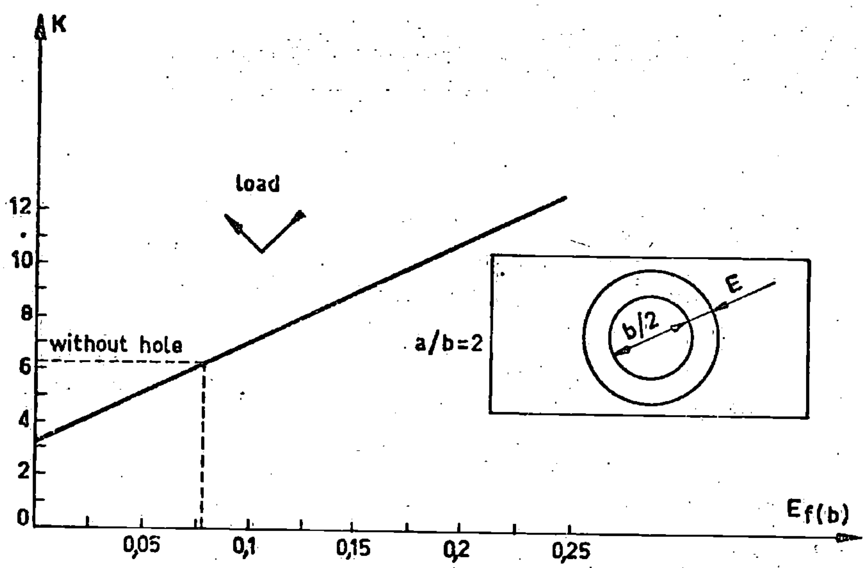


Figure 4. Diagram showing influence of size of doubling, /5/.

It should be realized however that the K-values were derived on the basis of the in-plane edge loads of the unperforated plate. This means that any stress concentrations around the hole (reinforced or not) have been ignored.

Both methods discussed above are concerned with elastic stability only and they give no indication of the ultimate collapse load, their value for parametric studies however can not be denied.

Weiss /7/ has addressed himself to the problem of post-buckling behaviour of elastically restrained rectangular plates.
The in-plane loads act in one direction only and initial unfairness has been included in the calculations. The differential equations, expressing non-linear behaviour have been solved by means of a sophisticated finite difference method. The results of the calculations are presented in a number of diagrams.

A progress report on experimental investigations concerning the elastic stability of welded plate beams has been presented by Tedeschi and Damilano /8/. The purpose of the tests was to check the validity of a simplified critical stress expression derived from a formula given by Bleich in his book "Buckling Strength of Metal Structures". The expression used has also been modified so that it can be applied both in case it is supposed that the cross-section of the beam does not change shape and in case of a non-rigid web being assumed. Furthermore the post-buckling behaviour of the web-flange combination was to be examined. The test pieces constituted a series of ten single web-beams with an attached strip of plating. Four beams had been tested up to the time when the paper was being prepared and the results are given in figure 5.

The values apply to the web-flange combination, the part of the critical load taken by the strip of plating being subtracted from the total. This had to be done partly theoretically because of insufficient instrumentation of the plate strip. These results seem to support the theoretical approach based on the assumption of a non-rigid web. The theoretical loads are consistent with the results obtained so far considering that model 1 suffered a (pre-

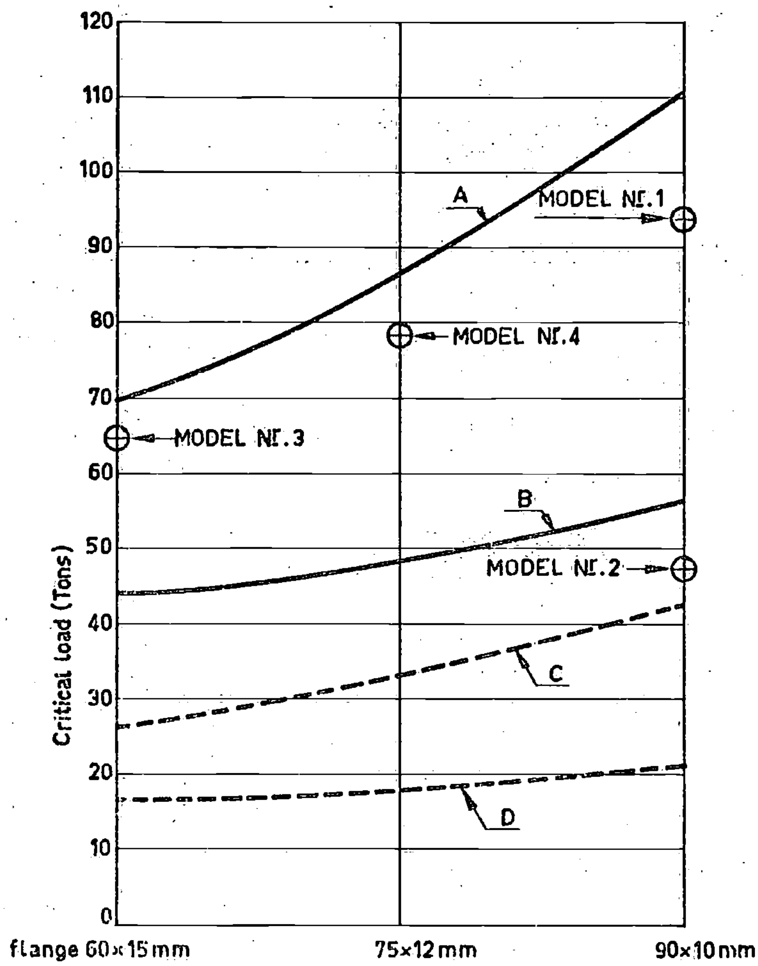


Figure 5. Comparison between theoretical and experimental critical loads.

Curve A = asymmetric flange } web elastically restrained
" B = symmetric flange
" C = asymmetric flange } rigid web - free to pivot, /8/.
" D = symmetric flange

mature) local collapse due to the manner of load application and that model 2 showed pronounced initial waviness at about mid-height of the web. A clearly defined post-buckling behaviour of the web-flange combination was not found.

b. Corners of large hatch openings.

The structural problems raised by the impetuous development of container vessels is reflected in the research carried out in Germany. Studies on local response are centred around the hatch corners and adjoining structures.

Lehmann and Niessen /9/ reported on experiments with a plexiglass model representing the transverse box-girder and connected longitudinal parts of a third generation container ship. Stress levels were studied separately for simulated longitudinal bending and torsion. In both cases attention was directed at stress levels in the entire region and it was found that those in area adjacent to the corners were high. Various shapes of rounded corners were investigated in connection with the maximum stress value occurring along them and some stress concentration factors are given. The use of these factors is made somewhat uncertain because no dimensions are given for the corner shapes. Calculations based on a two dimensional F.E.M. model gave results that were in agreement with the "average" value of the measured parameters. Notwithstanding this result the use of two dimensional models for the analysis of local response should not be encouraged since the aim of such an analysis is not the magnitude of "averaged" response parameters. The most significant conclusion of Lehmann and Niessen is that the connection between the transverse box girder and the longitudinal structure can be improved by the introduction of local horizontal shearplates. (See figure 6).

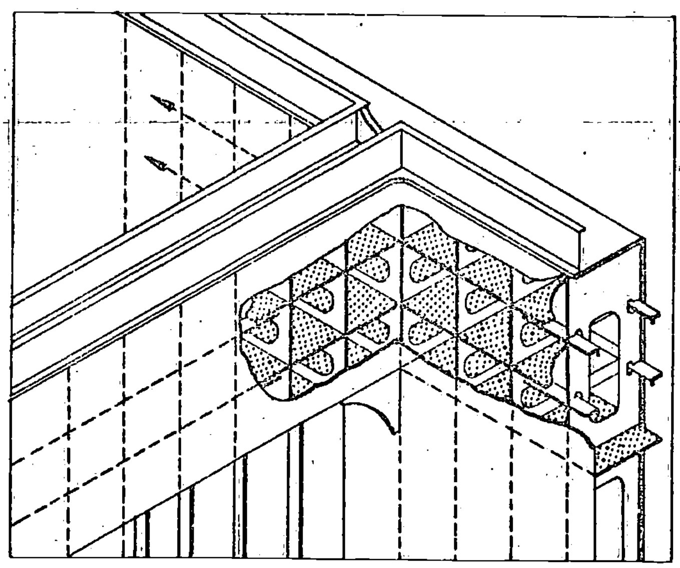


Figure 6. Position of horizontal shear plates, /9/.

Their effect would be an improved distribution of the support reactions from the point of view of transverse strength. Also an improved fixation of the transverse box girder would be obtained resulting in lower shear stress values under torsional loads in the deck.

Alte, Behr and Oei /10/ studied the response of a hatch corner structure adjacent to the engine room in the weatherdeck of a container vessel, (figure 7).

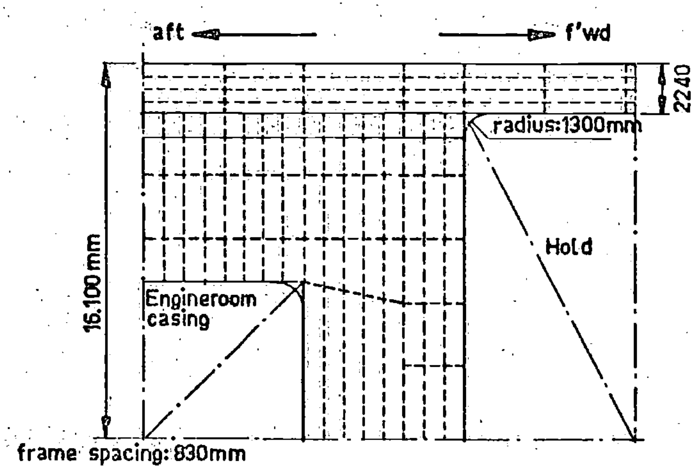


Figure 7. Maindeck in engineroom area, /10/.

This was done with the aid of a F.E.M. model, details of which are not given except that ASKA has been used. The structural variations consisted of:

a) cornerdeck area reinforced by insertion of a plate with increased thickness and a large area;
b) as a) but with reduced area;
c) no reinforcement.

Rounding of the hatch corner was by a quarter circle (r = 1300 mm full scale) in all cases while in some cases the effect of rounding by a parabola was also studied. Loading conditions were longitudinal bending and torsion, for which stress distributions were determined. Results show that the circular rounding of the hatch corner gives generally the lowest stress concentrations. This finding is consistent with the results obtained by Lehmann and Niessen /9/ who found that the radius suggested by DnV gives good results too. Alte c.s. in /10/ present four differently defined stress concentration factors. (See figure 8):

$$\alpha_i = \frac{\sigma_{max}}{\sigma_i} .$$

The most logical factor seems to be α_4 which is also used by Lehmann c.s. /9/.

The main conclusion arrived at by Alte c.s. is that where the corner rounding must be limited a circular shape combined with a reinforced corner plate is the best solution to the problem of keeping the stress peaks in this location (hatch opening adjoining engine room) as low as possible.
Care should be taken to make the transition from the curved corner inset to longitudinal and transverse coamings as smooth as possible, f.i. by grinding.

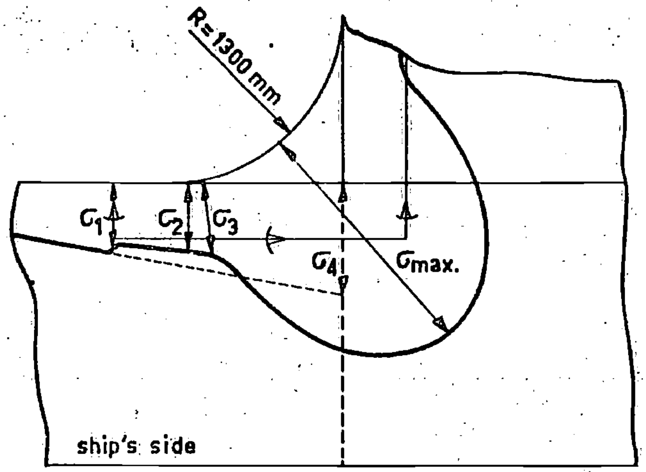


Figure 8. Stresses used to define stress concentration factors, /10/.

This with a view to reducing the sensitivity to low-cycle fatigue effects.

Alte et al. /11/ have discussed the torsional rigidity of large container vessels and they pay considerable attention to local response in way of the hatch corners. This was done on the one hand on a large plexiglass model of the entire vessel and on the other hand on a steel model of the connection between the transverse box-girder and the longitudinal structure scale 1:5. The steel model was tested with three different values of the corner radius and it contained the horizontal shearplates recommended by Lehmann and Niessen /9/.
The experimental values also served to check the output of a 3-dimensional F.E.M. analysis. This paper by Alte et al. contains so much useful information that only an extensive discussion would do it justice. This is outside the scope of this review however and the reader is advised to study the valuable paper in its entirety.

c. Stiffened Panels.

Amati and Damilano /12/ discuss the results of a series of experiments carried out on a full-scale model of a 2.70 × 2.51 m helicopter pad aboard an Italian naval vessel.
Loading consisted of a pair of concentrated loads simulating the helicopter and a uniformly distributed load. For the concentrated loads four different positions were selected. Instrumentation with strain gauges (fil. length 20 mm) was extensive but on one side of the plating only. Stiffeners (forming a square meshed grillage) were instrumented at mid-span. Theoretical calculations were carried out with the STRESS-program, the whole structure being considered.
The authors claim good agreement between measured and calculated stress values, though the diagrams show that especially for peak stress values there is a large discrepancy between theory and experiment.

3. Final Remarks.

The foregoing review has been based on directly available material, therefore it will not cover all the work on local response analysis carried out in the countries considered.

If any recommendations were to be made they would be the following:

a) When carrying out experimental analysis of local response the instrumentation should be such that the components of the total stress can be clearly distinguished. This means strain gauges on both sides of the plating and a sufficient number of them on stiffeners.

b) When using F.E.M. for the theoretical calculations 2-dimensional models of 3-dimensional structures should be avoided.
In case this proves to be impossible, proof is needed that the model is sufficiently accurate for the purpose. This inclusion of proving the model would greatly facilitate the study of the relevant papers.

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A.T.M.A. = Association Technique Maritime et Aéronautique.