

Kavli Institute of Nanoscience
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Towards Fast Light at the Single Photon Level

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Abstract

We study the propagation of single photons through an atomic vapour cell where group velocity exceeds the velocity of light in vacuum. The mechanism behind this phenomenon is a result of interference of Fourier components due to the dispersive properties of the medium. By considering the distinction between group velocity and information velocity and its implications for special relativity, we show that fast single photons do not violate the principle of relativistic causality.

To produce fast single photons, we fine-tune the emission of a single GaAs quantum dot to the ^{87}Rb D_2 atomic transition, where a spectrally narrow region of anomalous dispersion enables a group velocity of $v_g = 1.21c$ at room temperature. It is shown that our current configuration does not allow the detection of fast single photons by means of a time resolved measurement of the exciton lifetime. Instead, we propose an alternative detection scheme based on single photon interference, where the peak of the fringe visibility is shifted forward in time by Δt = -0.4167τ_c (where τ_c denotes the coherence time of the photon).

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Chapter 1

An Introduction to Fast Single Photons

When Albert Einstein published his theory of special relativity in 1905, it completely changed our understanding of the nature of space and time. As presumably one of the most famous consequences of special relativity, nothing may travel faster than the speed of light in vacuum; a velocity so significant that physicist have exclusively assigned the letter c to it. For many decades this insight remained one of Einstein's biggest triumphs, establishing the dynamics of bodies moving at very high velocities. At closer inspection, however, there appears to be an inconsistency embedded in the theory of special relativity.

When Hendrik Lorentz evaluated the expression for the refractive index of atomic vapour in 1909 [1], it became clear that the speed of a pulse of light traveling through an atomic vapour can take on any value, and is not necessarily bounded by c. These findings initially questioned the validity of Einstein's special relativity, until the controversy was resolved by Sommerfeld and Brillouin in 1914 [2], who described the distinction between phase velocity and group velocity and its implications for special relativity. In their work, Sommerfeld and Brillouin explain that although the group velocity of a light pulse may become larger than c, the part of the pulse that carries the information cannot travel faster than c. This result suggests that Einstein was not wrong; instead his theory should be subjected to an alternative interpretation where it is merely information that cannot travel faster than c. It would take many more decades, however, until these theoretical predictions could be put to the test.

With the development of the laser in the late 1960's, the concept of fast light propagation received renewed interest. In 1966, Basov and Letokhov investigated the propagation of a highly intensive pulse through an optical amplifier [3]. Due to the nonlinear character of the amplifier, the front edge of the pulse depletes the atomic inversion density so that the trailing edge

propagates with much lower amplification. As a result, it appears at the output of the medium that the light pulse has travelled faster than c. After a more detailed mathematical treatment by Garrett and McCumber in 1970 [4], fast light propagation has been demonstrated in numerous experiments using a vast variety of optical techniques, ranging from gain [5] or attenuation [6, 7] assisted light propagation to single photon tunneling through a potential barrier [8].

While fast light propagation of laser pulses is completely described by semiclassical optics, there is a continued debate about fast light propagation of pulses containing one or few photons. It had in fact been argued by Aharonov, Reznik and Stern in 1998 [9] that the dominating influence of quantum noise prohibits the occurrence of fast light in an optical amplifier. In contrast, Milonni, Furuya and Chiao predicted in 2001 [10] that the peak probability of detecting a single photon can occur sooner than it would if the photon were traveling through vacuum. As no experimental data of fast single photons is available yet, the discussion remains largely unsettled.

The goal of this project is to demonstrate fast light at the single photon level. We aim at realizing this by coupling a single photon source to an atomic vapour cell. By demonstrating fast single photons, we verify the localization of information on a single photon; a relevant issue in quantum information processing. Morover, we contribute to a 100 year old discussion about the validity of Einstein's theory of special relativity.

In Chaper 2, a detailed treatment is presented of the relevant theoretical aspects of semiclassical fast light propagation. We specifically address the preservation of relativistic causality.
In Chapter 3, we introduce a quantum mechanical description of fast single photons and we give an overview of our experimental setup, where some important features are discussed in more detail.
In Chapter 4, various detection schemes for measuring fast single photons are discussed and some preliminary results are presented.
Finally, conclusions and recommendations are given in Chapter 5.

This research was performed at the optics lab in the Quantum Transport group of Delft University of Technology under supervision of Nika Akopian and Val Zwiller, to whom I owe my sincere acknowledgments. This master thesis is dedicated to Ilse.

Chapter 2

Theoretical Features of Fast Light

In this chapter we examine some of the theoretical features of fast light. We start by describing the different velocities associated with light propagation and show that fast light follows naturally from the wave description of light. Emphasis will be made specifically on the preservation of relativistic causality.

2.1 The Velocities of Light

2.1.1 Phase Velocity and Group Velocity

Consider the propagation of a monochromatic plane wave through a medium. The complex electric field associated with this wave is given by

$$E(z,t) = E_0 e^{i(kz-\omega t)} + \text{c.c.} \tag{2.1}$$

where E_0 represents the amplitude, k is the wavevector, ω is the angular frequency, z is the position in space and t is the time. The phase velocity v_p is defined as the velocity at which points of constant phase travel through the medium. This motion is described by

$$\phi = kz - \omega t = 0, \tag{2.2}$$

where ϕ is the phase of the wave. If we write the wavevector as

$$k = \frac{n\omega}{c}, \tag{2.3}$$

where n is the phase refractive index of the medium, then substitution of equation 2.3 into equation 2.2 yields

$$z = \frac{\omega t}{k} = \frac{ct}{n} = v_p t, \tag{2.4}$$

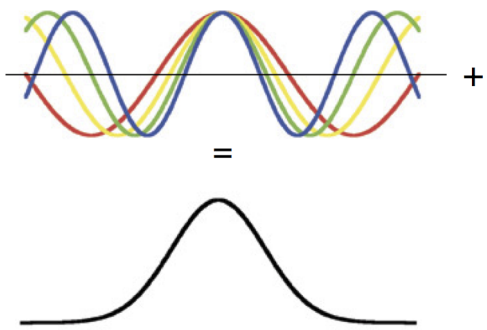


Figure 2.1: Schematic representation of the Fourier components of a light pulse.

where

$$v_p = \frac{c}{n} \tag{2.5}$$

is the phase velocity.

Let us now consider the more general case of propagation of a light pulse through a dispersive medium. Any pulse of light can be decomposed mathematically into a group of monochromatic waves with different angular frequencies, known as Fourier components. At the peak of the pulse, the various Fourier components interfere constructively, as illustrated in figure 2.1. If we require the pulse to travel through the medium without distortion, the constructive interference of Fourier components must occur at all values of the propagation distance z. This can be expressed mathematically by requiring that there is no change in the phase to the first order of ω, or

$$\frac{d\phi}{d\omega} = \frac{dn}{d\omega}\frac{\omega z}{c} + \frac{nz}{c} - t = 0. \tag{2.6}$$

Isolating z from this equation, we find $z = v_g t$, where

$$v_g = \frac{c}{n + \omega\, dn/d\omega} \tag{2.7}$$

is the group velocity. By construction, the group velocity represents the velocity at which the peak of the pulse travels through the medium. Additionally, the group velocity may be expressed as

$$v_g = \frac{c}{n_g}, \tag{2.8}$$

where

$$n_g = n + \omega \frac{dn}{d\omega} \tag{2.9}$$

denotes the group refractive index.

Equation 2.9 shows that the group refractive index is dependent on the dispersion $dn/d\omega$ of the medium. In vacuum, $dn/d\omega = 0$, so that the peak of the pulse travels exactly at c. In dispersive media, however, each Fourier component travels at a different velocity, resulting in an interference pattern where the peak of the pulse is shifted forward at a velocity that is not necessarily bounded by c. This suggests that one is given control of the group velocity of a light pulse by manipulation of the dispersive properties of the medium. Generally, one can discriminate between normal dispersive media where $dn/d\omega \gg 0$, so that the peak propagates slower than c, and anomalous dispersive media where $dn/d\omega \ll 0$, so that the peak propagates faster than c. Propagation of light at a group velocity higher than the speed of light in vacuum is commonly referred to as 'fast light'.

2.1.2 Front Velocity and Information Velocity

The notion of fast light naturally raises the question if such propagation would violate the laws of relativistic causality. Indeed, if it were possible for information to travel faster than c, then it would be possible to construct a causal loop in which an effect may precede its cause.

The first extensive work on this problem was done by Sommerfeld and Brillouin [12], who examined theoretically the propagation of a rectangularly shaped pulse through a fast light medium. They defined the front velocity as the velocity of propagation of the first non-zero value of the electric field and showed that, even though $v_g > c$, the front velocity can never exceed c. Moreover they showed that the initially rectangularly shaped pulse becomes significantly distorted throughout the propagation. This result can be understood physically by realizing that the atoms in the medium have a finite transition time so that they cannot respond instantaneously to the electric field associated with the sharp pulse front. Seemingly, these results do not apply to Gaussian shaped laser pulses and it is tempting to think of the peak of the pulse as carrying the information associated with the pulse. However, the information carried by the peak of the pulse can in principle be retrieved by preforming a Taylor expansion at the leading tail of the Gaussian, which arrives at a detector at an earlier time than the actual peak. Consequently, all information carried by the Gaussian pulse is localized at the first physically detectable point of the electromagnetic field, which is the analogue of Sommerfeld and Brillouin's pulse front. As a practical definition, we assign the pulse front to the first point where the value of the electric field exceeds the quantum noise level.

Consider the case where a Gaussian light pulse travels through a fast light medium with $n_g < 1$, so that the peak of the pulse travels faster than c, as shown in the light cone drawn in figure 2.2. The pulse front, which travels exactly at c, traces out the light cone and the remainder of the pulse is free to

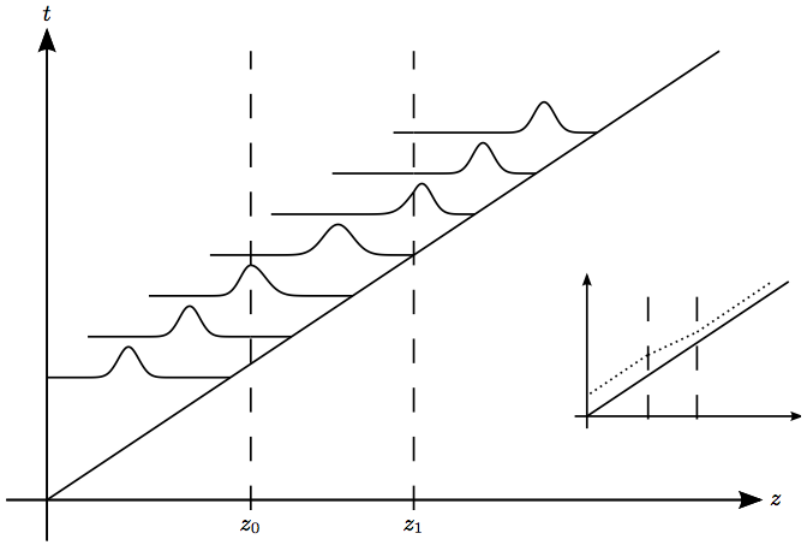


Figure 2.2: Forward light cone showing the propagation of a laser pulse in a fast light medium, positioned between coordinates z_0 and z_1. Inset: the trajectory of the peak of the pulse (dotted line) and of the pulse front (solid line) [11].

travel at any speed within the timeline region of the light cone. At the output of the fast light media, the peak is reconstructed from the initially leading tail of the pulse by means of extrapolation; it has no causal connection to the peak of the pulse before it had entered the fast light medium.

Brillouin's original concept of assigning the information to the pulse front can be generalized to any discontinuous point on a wavefront. In fact, it was proposed by Garrett et al. [4], that by encoding information on an optical pulse one necessarily creates a discontinuous point on its wavefront. Because a discontinuous point cannot be predicted by performing a Taylor expansion at the leading tail of the pulse, one genuinely retrieves new information by detecting the discontinuity. It was shown by Stenner et al. [13] that a discontinuous point on a wavefront always travels at a speed lower then c; in accord with the findings of Sommerfeld and Brillouin. Therefore, it can be concluded that any information encoded on a single photon cannot travel faster than c, hence relativistic causality is not violated.

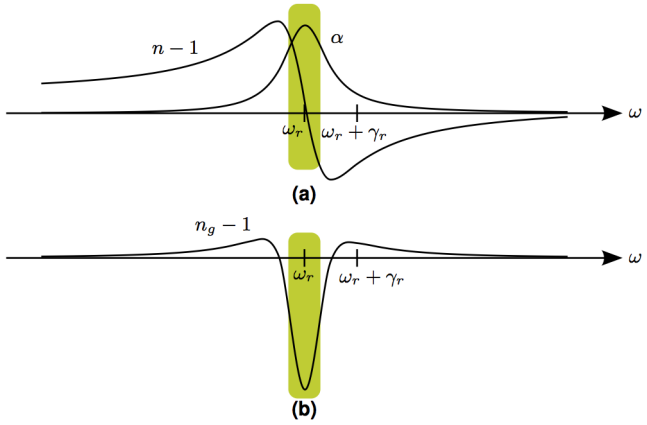


Figure 2.3: (a) Absorption coefficient and phase refractive index - 1 as a function of angular frequency. (b) Group refractive index - 1 as a function of angular frequency [11].

2.2 Fast Light in Atomic Vapour

We continue our discussion of fast light by considering pulse propagation through a dilute gas, such as an atomic vapour. We show that an atomic vapour exhibits a narrow spectral region of anomalous dispersion.

An atomic vapour may be represented by a collection of damped oscillating atoms where the electrical susceptibility $\chi(\omega)$ is given by [10]

$$\chi(\omega) \propto \frac{1}{\omega_r^2 - \omega^2 - 2i\omega\gamma_r}, \tag{2.10}$$

where ω_r is the central resonance frequency and γ_r is the damping coefficient. The phase refractive index $n(\omega)$ and the absorption coefficient $\alpha(\omega)$ of the medium are related to the electrical susceptibility by the Kramers-Kroning relations, which can be simplified to [11]

$$n(\omega) \approx 1 + \frac{1}{2}\,\mathrm{Re}\,\{\chi(\omega)\}, \tag{2.11}$$

$$\alpha(\omega) \approx \frac{\omega}{c}\,\mathrm{Im}\,\{\chi(\omega)\}. \tag{2.12}$$

Substituting equation 2.10 into equation 2.11 and equation 2.12, the phase refractive index and absorption coefficient can be evaluated as a function of the angular frequency of the incident field, as shown in figure 2.3. The absorption spectrum in figure 2.3(a) shows a Lorentzian absorption line centred around ω_r with linewidth γ_r. In the vicinity of this absorption peak, the value of $dn/d\omega$ is large and negative. The group refractive index is plotted in figure 2.3(b), where it can be seen that fast light is achieved when the

frequency of the incident field in tuned near the atomic absorption resonance. Fast light, however, necessarily occurs in regions of high absorption, which complicates performing a measurement.

Chapter 3

Fast Light at the Single Photon Level

In this chapter, we turn our attention from a laser pulse, consisting of many photons, to a pulse of light containing only one photon. In order to investigate the possibility of producing fast light at the single photon level, we need a fully quantum mechanical description of light. Since the position and momentum of a single photon are subjected to Heisenberg's uncertainty principle, it is only by subsequent measurement of many single photons that one can construct the probability of finding a photon at a certain position. In particular, while the peak of a laser pulse is associated with the highest value of the electric field, the peak of the statistical result of many subsequent measurements of single photons is associated with the highest probability of finding the photon. Hence, in the case of fast single photons, it is the peak probability of finding the photon that travels faster than the speed of light in vacuum.

First we present the key findings of a quantum mechanical model for fast light at the single photon level presented by Milonni et al. [10]. We then apply these results to our current experimental setup and describe some of its relevant features in more detail.

3.1 Quantum Mechanical Model for Fast Light

Consider a single photon source, such as a single two-level atom, emitting photons at transition frequency ω_0, upon resonant excitation by a laser pulse. The photons travel along a single direction z through a fast light medium towards an ideal broadband detector. We will assume that, initially, the atoms in the medium remain in their ground state and the field is in the vacuum state. We require furthermore that the excitation pulse duration is long compared with the radiative lifetime of the exciton, which allows for smooth excitation of the atom. It has been shown by Milonni et al. [10] that

under these conditions the probability of a single photon registering a count at the detector is proportional to

$$R(z,t) \propto e^{-\alpha z} P(t - z/v_g), \tag{3.1}$$

where $\alpha = 2\omega_0 n_I(\omega_0)/c$ is the absorption coefficient, where n_I is the imaginary part of the phase refractive index, and $P(t)$ is the probability at time t that the source atom is in the excited state. When $v_g > c$, equation 3.1 shows that the peak probability of detecting a single photon can occur earlier than than if it were traveling through vacuum, albeit that this probability is significantly reduced by absorption.

Milonni et al. also considered the case where the source atom is suddenly put in its exited state at $t = 0$, having previously been in its ground state. In this case, it can be shown that $P(z,t)$ vanishes when $t < z/c$, which according to equation 3.1 implies $R(z, t < z/c) = 0$. Thus, a suddenly exited atom cannot cause a photon to be counted before the time it takes a photon to travel from the atom to the detector in vacuum. This important result can be interpreted as the quantum analogue of the argument by Sommerfeld and Brillouin, who showed that a sharp classical wavefront cannot travel faster than c. Adapting their argument to the single photon level, we require that the peak probability of detecting the photon occurs some time after the earliest possible event of detecting the photon, as illustrated by the light cone in the inset of figure 2.2.

3.2 Experimental Setup

3.2.1 Single GaAs Quantum Dot

In our experiments, we employ an optical setup similar to the configuration considered by Milonni et al. Instead of a single two-level atom, however, we use a single GaAs quantum dot as a source of single photons. Essentially, a quantum dot is a semiconductor device where the exciton, a bound state of an electron and a hole, is confined in all three spatial dimensions, as illustrated in figure 3.1(a). As a result of the confinement, the energy levels are quantized and may be filled with two electrons or holes with opposite spin direction; a configuration remarkably analogous to real atoms. The use of a quantum dot offers several advantages compared to other sources of single photons, such as parametric down conversion schemes. Most importantly, a quantum dot produces single photons on demand, whereas the emission of photons by down conversion process is a purely random process.

Our quantum dots are fabricated by a multistep self-assembly process [15] where the width of the GaAs layer was optimized for photoluminescence around 780 nm. We excite our quantum dot by a Kerr-lens modelocked Ti:Shappire laser that produces highly intensive pulses (average power >

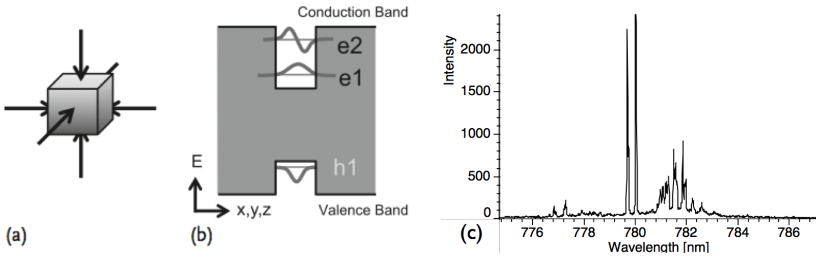


Figure 3.1: (a) Schematic of a quantum dot in which carriers are confined in all three spatial direction. (b) The confining potential, energy levels, and wavefunctions in a simple particle-in-a-box picture for one spatial direction [14]. (c) A typical photoluminescence spectrum.

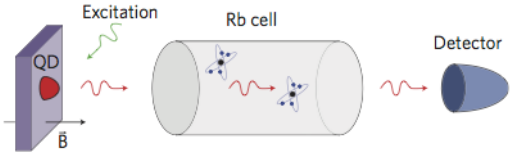


Figure 3.2: Schematic representation of the experimental setup [16].

800 mW) with a temporal width of 5 ps (FWHM) at a repetition rate of 80 MHz. The wavelength is tuned to $\lambda = 743$ nm with a spectral width in the order of 100 pm. A microscope objective (NA = 0.85) is used to focus the excitation pulse on quantum dot with a spot size of 1 mm. Depending on the density of quantum dots on our sample, we are able to address quantum dots individually. Recombination of a single electron-hole pair leads to the generation of a single photon with wavelength and linewidth set by the characteristics of the quantum dot. There are charge fluctuations in the vicinity of the quantum dot, however, causing inhomogeneous broadening of the linewidth. A typical measurement of photoluminescence spectrum is shown in figure 3.1(c) where two narrow Lorentzian emission lines can be seen.

As the quantized nature of the energy levels only becomes apparent at low temperatures (where k_bT is smaller than the quantum dot energy spacing), our measurements are performed in a helium bath cryostat at 4.2 K. An external magnetic field is applied across the sample by a 9 T superconducting magnet. This induces a Zeeman splitting of the emission line which allows for precise fine-tuning of the photoluminescence spectrum to the absorption peak of the atomic vapour. The photoluminescence signal is collected by the same objective and is lead to a Rb vapour cell. A characteristic sketch of the experimental setup is shown in figure 3.2.

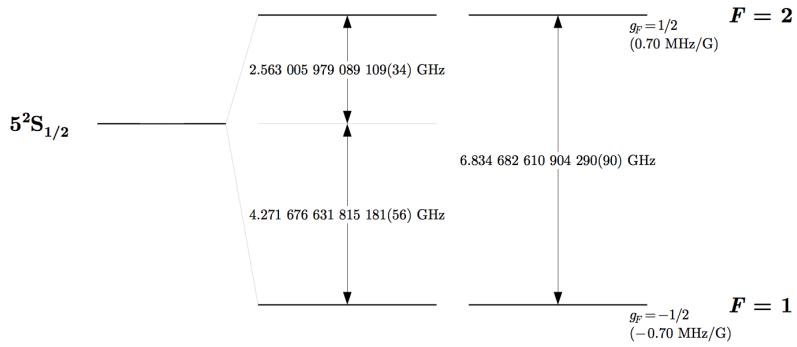


Figure 3.3: Rubidium 87 $5^2S_{1/2}$ (ground) level hyperfine structure [17].

3.2.2 ^{87}Rb Vapour Cell

The fast light medium in our setup is provided by a 75 mm long quartz cell of isotropically pure ^{87}Rb vapour; a highly reactive metal of the alkali group. The ground state of ^{87}Rb consists of a hyperfine splitting with two absorption peaks separated by 6.8 GHz, as shown in figure 3.3. By adjusting the strength of the magnetic field, the quantum dot emission can be tuned to one of the absorption peaks of the ^{87}Rb D_2 transition.

In the case of a double absorption resonance, the electrical susceptibility is described as a sum of the susceptibilities of the separate absorption peaks,

$$\chi(\omega) = A\left(\frac{g_1}{\omega_1^2 - \omega^2 - 2i\omega\gamma_r} + \frac{g_2}{\omega_2^2 - \omega^2 - 2i\omega\gamma_r}\right), \tag{3.2}$$

where g_1 and g_2 are the transition strengths, ω_1 and ω_2 are the transition frequencies and A is the total strength of the resonance, given by

$$A = \frac{N\mu^2}{\epsilon_0\hbar}, \tag{3.3}$$

where N denotes the atomic density and μ is the atomic dipole moment. The atomic density, and therefore the electrical susceptibility, depends exponentially on the temperature of the vapour cell [17].

Plugging in the relevant values for the ^{87}Rb D_2 transition and assuming room temperature in the vapour cell, the phase refractive index and absorption coefficient can be evaluated as a function of the frequency detuning, as we have done for a single absorption peak in section 2.2. They are plotted in the upper graph of figure 3.4, where it is clearly seen that there are regions of anomalous dispersion near the absorption peaks, while there is a large region of normal dispersion in the middle of the double resonance (which

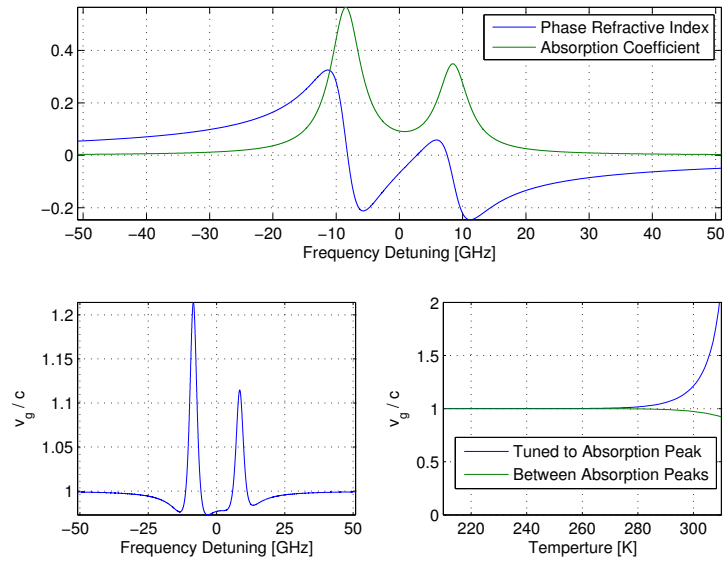


Figure 3.4: Upper graph: phase refractive index - 1 (blue) and absorption coefficient (green) as a function of frequency detuning at room temperature. Lower left graph: the ratio v_g/c as a function of frequency detuning at room temperature. Lower right graph: v_g/c as a function of temperature with the wavelength tuned to the absorption peak (blue) and tuned in between absorption peaks (green).

can be used to obtain slow light). The ratio v_g/c as a function of frequency detuning is plotted in the lower left graph, where it is shown that the group velocity exceeds the speed of light in vacuum within the regions of anomalous dispersion. The maximum value of $v_g = 1.214c$ corresponds to a temporal advancement of Δt = -41.67 ps while the minimum value of $v_g = 0.9730c$ corresponds to a temporal delay of Δt = 7.732 ps. The lower right graph shows the ratio v_g/c as a function of temperature, where the blue graph indicates tuning to the absorption peak (causing fast light) and the green graph indicates tuning in between absorption peaks (causing slow light).

Chapter 4

Detection Methods

In the previous chapter, it was shown that fast single photons can be produced by tuning the emission of a single quantum dot to the D_2 transition of a ^{87}Rb vapour cell. In this chapter we present and discuss two methods for measuring fast single photons. In particular, we take the implications of the preservation of relativistic causality, described in section 2.1, into consideration.

4.1 Lifetime Measurement

After excitation of the quantum dot, the exciton state is subjected to an exponential decay. This decay is caused by decoherence processes of the quantum states and by spontaneous emission to the ground state [14]. The lifetime of the exciton is mainly determined by the quality of the coupling of the quantum dot to the microcavity; in the case of our sample the lifetime is approximately 300 ps.

When we consider the limiting case of excitation by a Dirac Delta pulse, the highest probability of photon emission occurs instantaneously with the event of excitation. This means that the first detectable point of the wavefront coincides with the peak probability of detecting the photon. Following the analysis of Milonni et al, relativistic causality dictates that there can be no fast light advancement in this situation. Hence, to obtain fast single photons, a time difference is required between the arrival of the pulse front and the arrival of the peak of the pulse. In the more realistic case of excitation by a finite excitation pulse, this requirement is in principle always satisfied. The delay of the peak of the pulse with respect to the pulse front is given by the temporal width of the excitation pulse and it can be calculated by means of convolution, as illustrated in figure 4.1.

In our current configuration, the width of the excitation pulse is 5 ps, corresponding to a temporal advancement of 7.5 ps. The photoluminescence signal of the quantum dot is dispersed by a blazed grating and the spectrum is imaged on a CCD camera to measure the spectrum with a spectral resolution

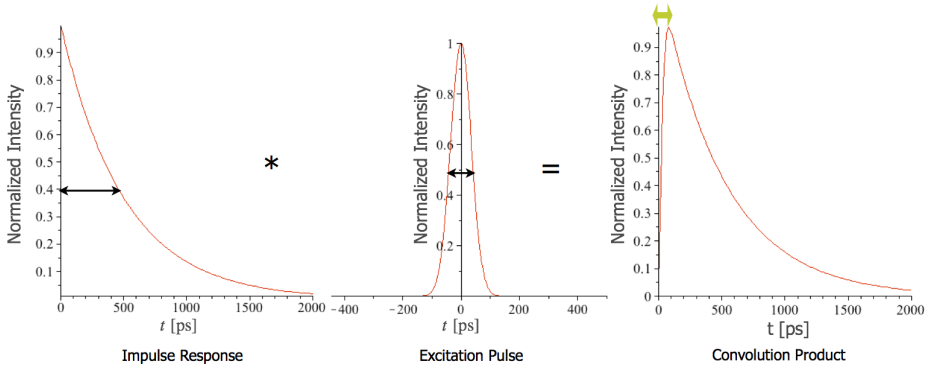


Figure 4.1: The left graph shows the lifetime of the exciton in the limiting case of excitation by a Dirac Delta pulse, the middle graph shows the temporal width of a finite excitation pulse and the right graph shows the convolution product. The arrows represent the lifetime of the exciton, the temporal width of the excitation pulse and the delay of the peak of the pulse with respect to the pulse front, respectively.

of 30 μeV or a streak camera to measure the lifetime with a time resolution of 15 ps. Thus, the time resolution of the streak camera is currently insufficient to measure the advancement of the peak of the pulse.

A straightforward solution to this problem would be to increase the temporal width of our excitation pulse, thereby downscaling the required time resolution of the detection system. This can be achieved in a vast variety of optical schemes, two of which will be briefly discuss here.

Dispersive and Distortive Properties of Optical Waveguides Propagation of a light pulse through an optical waveguide is known to cause temporal broadening of the pulse due to chromatic dispersion and, in the case of multimode waveguides, modal distortion. Chromatic dispersion has the undesirable feature of destroying the spectral symmetry of the laser pulse. Modal distortion, on the other hand, preserves spectral symmetry at the cost of having to use a large core diameter waveguide, thereby disabling sharp focussing of the laser pulse on the quantum dot sample. For these reasons, we found the use of dispersive and distortive features in optical waveguides to be disadvantageous for our purpose.

Electro-Optical Modulation An an alternative solution, we have tried to modulate the output of a continuous wave laser source by an fiber-integrated Electro-Optical Modulator (EOM) device. An EOM essentially makes use of the Pockels-effect in a $LiNbO_3$ crystal, where an external electric field induces a change in refractive index. This effect creates a phase shift, which can be used in a Mach-Zehnder configuration to obtain amplitude modulation by

means of interference. When driving the potential by an ultra-fast function generator, pulses with a tunable temporal width of at least 200 ps can be obtained. Because the optical input of an ultra-fast EOM device is limited to relatively low powers, the modulated pulses should be amplified by an external optical amplifier. Amplification is preferably performed by an Erbium-Doped Fiber Amplifier at telecom wavelengths, after which the desired wavelength of 780 nm can be achieved by means of parametric down conversion.

4.2 Single Photon Interference

From the findings in the previous section, it can be concluded that a lifetime measurement cannot easily be made with our current configuration. For this reason, we consider an alternative detection scheme that does not involve time resolved measurement of single photons. This scheme is based on single photon interference; an extraordinary demonstration of the quantized nature of light.

Consider the Mach-Zehnder interferometer illustrated in figure 4.2. A single photon emitted by the quantum dot is incident on a 50/50 beamsplitter, where the wave function of the photon splits into an entangled state. The photon remains in a superposition of propagation through both arms, until it is recombined by a second 50/50 beamsplitter. By adjusting one of the two arm lengths, the wave function can be set to overlap at the second beamsplitter so that interference fringes are created. A single photon detector is placed after the beamsplitter to measure the interference fringes. By scanning the Optical Path length Difference (OPD) and measuring the corresponding fringe visibility, one essentially performs a measurement of the coherence length of the photon [18].

When the ^{87}Rb vapour cell is placed in one of the arms of the interferometer, an additional phase shift is induced. In order to compensate for this phase shift, one needs to readjust the optical path length difference. Since this readjustment is proportional to the induced phase shift, it enables one to determine the magnitude of the phase shift and hence the group velocity in the vapour cell. A similar setup based on two photon interference has previously been used to measure faster than c single photon tunneling times through a potential barrier [8].

Because this scheme does not involve time resolved detection of a single photon, one does not directly suffer from the implications of relativistic causality. Clearly, this configuration does not allow faster than c propagation of information; it is merely a method to determine the group velocity in the vapour cell. Because this scheme relies on interference of frequency components, a limiting factor is imposed by the coherence length of the photon [19]. Indeed, the generation of fast single photons requires a narrow linewidth,

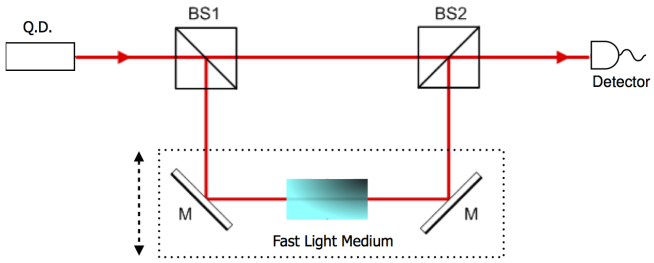


Figure 4.2: A Mach-Zehnder type interferometer used for interferometric measurement of fast single photons.

corresponding to a long coherence length [20]. Based on measurements on similar quantum dots [18], we assume that the coherence time of our single photons is approximately $\tau_c = 100$ ps, corresponding to a coherence length of $L_c = 30$ mm.

The first-order correlation function for a wave packet with a Lorentzian linewidth centered around angular frequency ω is given by [20]

$$g^{(1)}(\tau) = e^{-i\omega\tau} e^{-\frac{|\tau|}{\tau_c}}, \tag{4.1}$$

where τ is the time delay and τ_c is the coherence time. The visibility of the interference fringes is defined as

$$\text{Visibility} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = |g^{(1)}(\tau)|, \tag{4.2}$$

where $I_{\max}$ and $I_{\min}$ denote the intensities recorded at the fringe maxima and minima, respectively. Plugging in the results from section 3.2.2 into equation 4.1 and assuming a photon flux of $3.3 \cdot 10^9$ under continuous wave excitation, the observed interference fringes can be calculated. In the upper graph of figure 4.3, the photon flux is plotted as a function of OPD / L_c and the corresponding visibility is shown in the lower graph of figure 4.3. The green graph indicates frequency tuning to the absorption peak, the magenta graph indicates frequency tuning in between absorption peaks and the blue graph indicates frequency tuning far-off resonance, where the time delay is zero. For clarity, the green graph has been multiplied by 10^4. As can be seen from inspection of this figure, the shift of the peak of the fringe visibility fits well within the coherence length.

When the temperature in the vapour cell is increased, the group velocity is increased accordingly. However, it is dictated by the Kramers-Kronig relations that an increase of group velocity is accompanied by an increase of the absorption coefficient. Hence, the photon flux of the advanced (or delayed) wave packet will be reduced compared to the photon flux corresponding

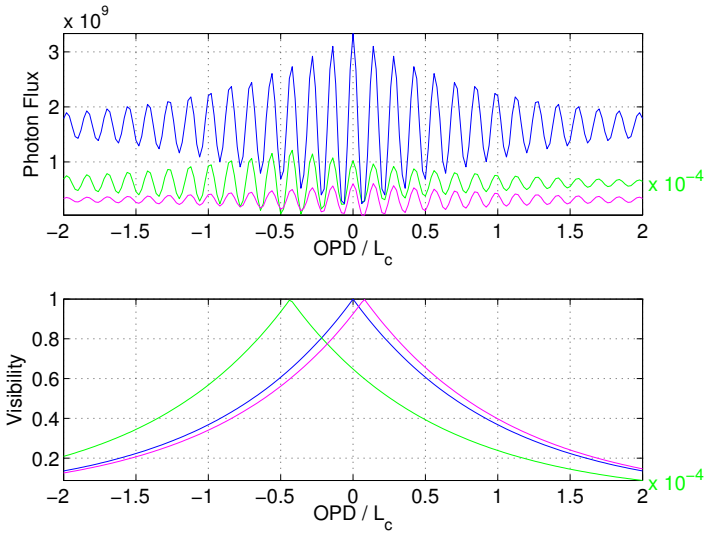


Figure 4.3: Upper graph: photon flux as a function of the ration OPD/L_c. Lower graph: visibility as a function of the ratio OPD/L_c. The green graph indicates tuning to an absorption peak, the magenta graph indicates tuning in between absorption peaks and the blue graph indicates far-off resonance tuning. The green graph has been multiplied by 10^4.

to far-off resonance frequency tuning. This is clearly shown in figure 4.3, where the photon flux of the advanced wave packet is reduced by 4 orders of magnitude. The lowest measurable photon flux is set by the level of quantum noise; when the magnitude of the photon flux drops below the quantum noise level no interference measurement can be performed.

To summarize, we propose a measurement of fast single photons based on single photon interference. By gradually changing the optical path difference between the arms of a Mach-Zehnder interferometer, a measurable change in fringe visibility is created. Inserting the ^{87}Rb vapour cell into one of the two arms, a phase shift is induced which produces a spatial shift of the peak of the fringe visibility. By adjusting the temperature in the vapour cell while repeating the procedure, one can experimentally reproduce the theoretical predictions given in figure 3.4, albeit that the magnitude of the photon flux is significantly reduced by absorption.

Chapter 5

Conclusions and Recommendations

From the results reported in the previous chapters, several conclusions may be drawn.

By making a distinction between the group velocity (the velocity at which the peak of the pulse travels) and the information velocity (the velocity at which the information carried by the pulse travels), we have shown that the occurrence of fast single photons does not contradict relativistic causality. In particular, the pulse front coincides with the lightlike surface of the light cone, and the peak of the pulse may travel at any speed in the region bounded by this surface. Accordingly, a temporal separation between the arrival of the pulse front and the arrival of the peak of the pulse is required to produce fast single photons.

There are several solutions available to modify our current setup to match the above requirement in a time resolved measurement of the exciton lifetime. These involve modulating the amplitude of a continuous wave laser by an EOM device and amplifying the modulated output, preferably at telecom wavelengths, in order to excite the quantum dot more smoothly. Another solution is to obtain a new Ti:Sapphire laser that features tunability of the pulse duration in the sub-nanosecond regime or to use a detection system with sub-picosecond time resolution, both of which are commercially available.

Additionally, we have proposed an alternative detection scheme based on single photon interference, which allows one to measure the phase shift induced by insertion of the ^{87}Rb vapour cell in a Mach-Zehnder interferometer. This setup requires a relatively simple optical arrangement and does not involve the purchase of expensive optical systems. For this reason, it is the most recommendable procedure for the detection of fast single photons.

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