Total-Field Absorbing Boundary Conditions for the Time-Domain Electromagnetic Field Equations

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Abstract— **A method is described for generating absorbing boundary conditions (ABC's) that can be applied to total fields rather than the usual scattered fields. As compared with the traditional use of ABC's for total-field formulations, this method has the advantages that it does not require the introduction of a mathematical connection surface between the total-field region and the scattered-field region; the total field is computed in the entire domain of computation. The incident field is accounted for by augmenting the ABC used. The resulting code is much simpler than one using ABC's for scattered fields together with a connection surface and the numerical results are much more easily interpreted since they consist of total fields only.**

***Index Terms*—Absorbing boundary conditions.**

I. Introduction

Absorbing boundary conditions (ABC's) are used along (parts of) the outer boundaries of a finite-element or finite-difference mesh for absorbing fields scattered or radiated by obstacles or sources present inside the domain of computation. In case the configuration is illuminated from outside the domain of computation, the incident field can be taken into account by the introduction of a connection surface, located inside the domain of computation and surrounding the scattering obstacle [1]. Outside this connection surface, the scattered field is computed because ABC's can be applied along the outer boundary. Inside the connection surface, the total field is computed. Finally, the illumination is taken into account in the total-field/scattered-field formulation by applying continuity (connection) conditions. Unfortunately, the latter process requires a substantial amount of logic (see, for instance, [2]) and has the additional disadvantage that it produces a mixture of total and scattered fields on the mesh, which complicates the interpretation of the results. The present paper describes a method for circumventing these difficulties by accounting for the illumination in the ABC itself. The method is applicable to all linear ABC's that explicitly express the boundary value to be computed in terms of field values that are already known inside the domain of computation. To the author's knowledge this is valid for all conditions of this type that are presently available. The method cannot be used together with the perfectly matched layer (PML) method [3].

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II. Total-Field Absorbing Boundary Conditions

Assume we have an arbitrary ABC that can be written in operator form as

$$\mathcal{A}(\mathbf{E}^{\mathrm{sca}}, \mathbf{H}^{\mathrm{sca}}) = 0 \tag{1}$$

where $\mathbf{E}^{\mathrm{sca}}$ and $\mathbf{H}^{\mathrm{sca}}$ represent the scattered field. The operator $\mathcal{A}$ is assumed to be linear in terms of the field components and to express the boundary value to be computed in terms of field values that are already known inside the domain of computation. Using the linearity of $\mathcal{A}$, we can add the trivial identity

$$\mathcal{A}(\mathbf{E}^{\mathrm{inc}}, \mathbf{H}^{\mathrm{inc}}) = \mathcal{A}(\mathbf{E}^{\mathrm{inc}}, \mathbf{H}^{\mathrm{inc}}) \tag{2}$$

where $\mathbf{E}^{\mathrm{inc}}$ and $\mathbf{H}^{\mathrm{inc}}$ represent an arbitrary incident field having its sources outside the domain of computation, to (1) to obtain

$$\mathcal{A}(\mathbf{E}, \mathbf{H}) = \mathcal{A}(\mathbf{E}^{\mathrm{inc}}, \mathbf{H}^{\mathrm{inc}}) \tag{3}$$

with $\mathbf{E} = \mathbf{E}^{\mathrm{inc}} + \mathbf{E}^{\mathrm{sca}}$ and $\mathbf{H} = \mathbf{H}^{\mathrm{inc}} + \mathbf{H}^{\mathrm{sca}}$. Now (3) represents a total-field ABC for arbitrary incident fields and for all scattered-field ABC's satisfying the conditions mentioned above. Note that (3) is valid for any type of incident field, the only condition being that its sources are located outside the domain of computation.

Apart from the right-hand side and the fact that it refers to total fields rather than scattered fields, (3) is identical to (1). Consequently, the discretized versions of (3) and (1) have the same properties as regards accuracy and stability and any existing code for implementing scattered-field absorption can be used directly for implementing total-field absorption by merely adding to it the right-hand side of (3). Note that this right-hand side is, for a known incident field, known explicitly. The use of (3) does not require any additional logic such as the evaluation of connection conditions at a connection surface.

A. Total-Field Mur Conditions

For illustrating total-field ABC's we shall apply them to the conditions that have become known as the first- and second-order Mur conditions. We shall use Cartesian coordinates x_1, x_2, and x_3 and assume that the mesh is located in the region $0 \le x_1$. ∂_j will denote a partial differentiation in the x_j direction, ∂_t a partial differentiation with respect to the time coordinate t. Boundary conditions are given for E_3 on $x_1 = 0$ and all other conditions follow in a trivial manner. As regards the discretization, we shall assume a time increment Δ_t and lattice space increments Δ_1, Δ_2, and Δ_3.

1) First-Order Condition: The first-order condition uses the first-order approximation of the one-way wave equation that was obtained as [1]

$$(\partial_1 - c^{-1}\partial_t)E_3^{\mathrm{sca}}|_{x_1=0} = 0. \tag{4}$$

Using (3), the first-order total-field ABC follows as

$$(\partial_1 - c^{-1}\partial_t)E_3|_{x_1=0} = (\partial_1 - c^{-1}\partial_t)E_3^{\mathrm{inc}}|_{x_1=0} \tag{5}$$

where c denotes the speed of light. The finite-difference approximation of the first-order total-field ABC is finally obtained as

$$\begin{aligned} E_3^{\mathrm{N}+1}\left(0,\mathrm{J},\mathrm{K}+\frac{1}{2}\right) &= E_3^{\mathrm{N}}\left(1,\mathrm{J},\mathrm{K}+\frac{1}{2}\right) \\ &\quad + \frac{c\Delta_t - \Delta_1}{c\Delta_t + \Delta_1}\left\{E_3^{\mathrm{N}+1}\left(1,\mathrm{J},\mathrm{K}+\frac{1}{2}\right) - E_3^{\mathrm{N}}\left(0,\mathrm{J},\mathrm{K}+\frac{1}{2}\right)\right\} \\ &\quad + \frac{2\Delta_t\Delta_1}{c\Delta_t + \Delta_1}\{(\partial_1 - c^{-1}\partial_t)E_3^{\mathrm{inc}}\} \\ &\quad |_{x_1=\Delta_1/2;x_2=\mathrm{J}\Delta_2;x_3=(\mathrm{K}+\frac{1}{2})\Delta_3;t=(\mathrm{N}+\frac{1}{2})\Delta_t}. \end{aligned} \tag{6}$$

Note that the incident-field term is evaluated at the point in space–time around which the finite-difference approximation of the ABC for the total field is centered. In this way, we obtain the maximum accuracy possible.

2) Second-Order Condition: The second-order condition uses the second-order approximation of the one-way wave equation that was obtained as [1]

$$\left\{c^{-1}\partial_{1,t}^2 - c^{-2}\partial_t^2 + \frac{1}{2}(\partial_2^2 + \partial_3^2)\right\}E_3^{\mathrm{sca}}|_{x_1=0} = 0. \tag{7}$$

Using (3), the second-order total-field ABC follows as

$$\begin{aligned} &\left\{c^{-1}\partial_{1,t}^2 - c^{-2}\partial_t^2 + \frac{1}{2}(\partial_2^2 + \partial_3^2)\right\}E_3|_{x_1=0} \\ &\quad = \left\{c^{-1}\partial_{1,t}^2 - c^{-2}\partial_t^2 + \frac{1}{2}(\partial_2^2 + \partial_3^2)\right\}E_3^{\mathrm{inc}}|_{x_1=0}. \end{aligned} \tag{8}$$

The finite-difference approximation of the second-order total-field ABC is finally obtained as

$$\begin{aligned} E_3^{\mathrm{N}+1}\left(0,\mathrm{J},\mathrm{K}+\frac{1}{2}\right) &= -E_3^{\mathrm{N}-1}\left(1,\mathrm{J},\mathrm{K}+\frac{1}{2}\right) \\ &\quad + \frac{c\Delta_t - \Delta_1}{c\Delta_t + \Delta_1}\left\{E_3^{\mathrm{N}+1}\left(1,\mathrm{J},\mathrm{K}+\frac{1}{2}\right) + E_3^{\mathrm{N}-1}\left(0,\mathrm{J},\mathrm{K}+\frac{1}{2}\right)\right\} \\ &\quad + \frac{2\Delta_1}{c\Delta_t + \Delta_1}\left\{E_3^{\mathrm{N}}\left(0,\mathrm{J},\mathrm{K}+\frac{1}{2}\right) + E_3^{\mathrm{N}}\left(1,\mathrm{J},\mathrm{K}+\frac{1}{2}\right)\right\} \end{aligned}$$

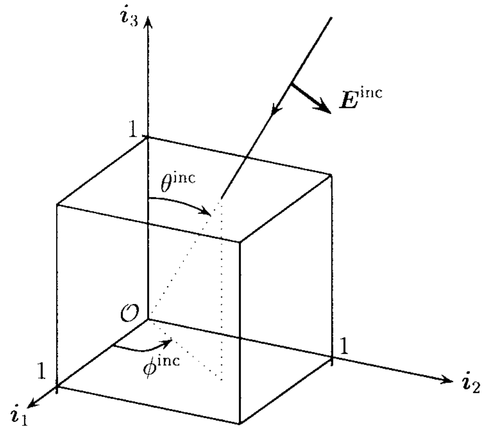

Fig. 1. The cubic domain of computation, the incident electric-field vector $\mathbf{E}^{\mathrm{inc}}$ is in the ϕ plane.

$$\begin{aligned} &+ \frac{\Delta_1(c\Delta_t)^2}{2\Delta_2^2(c\Delta_t + \Delta_1)}\left\{E_3^{\mathrm{N}}\left(0,\mathrm{J}+1,\mathrm{K}+\frac{1}{2}\right) - 2E_3^{\mathrm{N}}\left(0,\mathrm{J},\mathrm{K}+\frac{1}{2}\right) + E_3^{\mathrm{N}}\left(0,\mathrm{J}-1,\mathrm{K}+\frac{1}{2}\right)\right. \\ &\qquad \left. + E_3^{\mathrm{N}}\left(1,\mathrm{J}+1,\mathrm{K}+\frac{1}{2}\right) - 2E_3^{\mathrm{N}}\left(1,\mathrm{J},\mathrm{K}+\frac{1}{2}\right) + E_3^{\mathrm{N}}\left(1,\mathrm{J}-1,\mathrm{K}+\frac{1}{2}\right)\right\} \\ &+ \frac{\Delta_1(c\Delta_t)^2}{2\Delta_3^2(c\Delta_t + \Delta_1)}\left\{E_3^{\mathrm{N}}\left(0,\mathrm{J},\mathrm{K}+\frac{3}{2}\right) - 2E_3^{\mathrm{N}}\left(0,\mathrm{J},\mathrm{K}+\frac{1}{2}\right) + E_3^{\mathrm{N}}\left(0,\mathrm{J},\mathrm{K}-\frac{1}{2}\right)\right. \\ &\qquad \left. + E_3^{\mathrm{N}}\left(1,\mathrm{J},\mathrm{K}+\frac{3}{2}\right) - 2E_3^{\mathrm{N}}\left(1,\mathrm{J},\mathrm{K}+\frac{1}{2}\right) + E_3^{\mathrm{N}}\left(1,\mathrm{J},\mathrm{K}-\frac{1}{2}\right)\right\} \\ &+ \frac{2\Delta_t^2\Delta_1}{c\Delta_t + \Delta_1}\left[\left\{c\partial_{1,t}^2 - \partial_t^2 + \frac{c^2}{2}(\partial_2^2 + \partial_3^2)\right\}E_3^{\mathrm{inc}}\right] \\ &\quad ||_{x_1=\Delta_1/2;x_2=\mathrm{J}\Delta_2;x_3=(\mathrm{K}+\frac{1}{2})\Delta_3;t=\mathrm{N}\Delta_t}. \end{aligned} \tag{9}$$

Again, the incident-field term is evaluated at the point in space–time around which the finite-difference approximation of the ABC for the total field is centered.

III. An Example

The simplest way to demonstrate the validity and usefulness of total-field ABC's is to compute the total field in a homogeneous domain without any obstacles in it. (The demonstration can, of course, also be given with an obstacle present, but that would only obscure the point we want to make.) When using a total-field ABC, any incident field will be coupled into the domain of computation at the side(s) of the domain of computation through which the field enters the domain of computation. After traversing the domain of computation, the field will leave it again through the side(s) opposite to the side(s) through which it entered. For simplicity, we have cho-

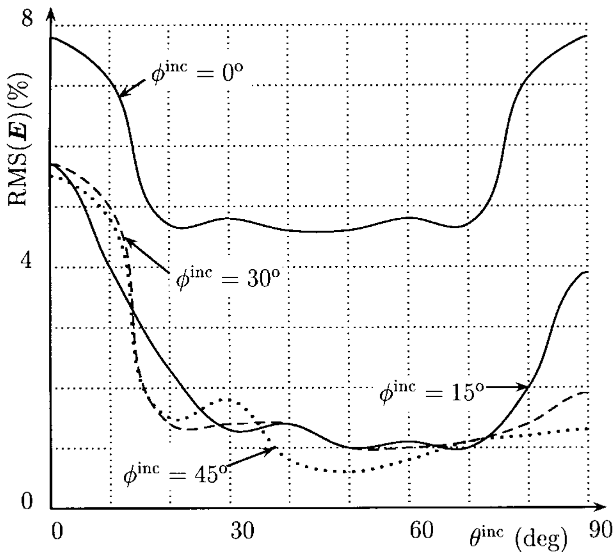

Fig. 2. The rms error in percent as a function of $\theta^{\rm inc}$ and $\phi^{\rm inc}$.

sen a cubic domain of computation with dimensions of 1*1*1 m (see Fig. 1). The incident field is taken as a monochomatic plane wave incident from the direction $(\theta^{\rm inc}, \phi^{\rm inc})$ and having a frequency of 600 MHz. We have used a $33*33*33$ mesh. Second-order conditions were used at all faces of the cube and first-order conditions along its edges.

Numerical results depicting the root mean square (rms) error in the total field, which, for this example, should be indentical to the incident field as a function of the angles of incidence; the polarization of the incident field are depicted in Fig. 2. From these numerical results we observe that an excellent accuracy is obtained for almost all directions of incidence and all orientations of the incident field vector $\mathbf{E}^{\rm inc}$. Some loss of accuracy is observed when the direction of propagation is (almost) parallel to a plane in the outer boundary of the domain of computation, in particular, when the incident field vector is also (almost) parallel to such a plane.

In the example given, we have assumed an incident plane wave, which implies a homogeneous embedding. The method can be applied straightforwardly to inhomogeneous embeddings such as, for instance, configurations bounded by conducting half-spaces where the incident field cannot be a plane wave. In those cases, the evaluation of the right-hand side in (3) becomes more complicated, a difficulty that would also have been encountered when evaluating the equations along connection surfaces.

IV. Conclusion

For total-field finite-difference time-domain computations, total-field ABC's are an attractive alternative to the use of scattered-field ABC's and connection surfaces. The main advantages of total-field ABC's as compared with scattered-field ABC's are: 1) their simpler logic; 2) their lower computational costs; and 3) the fact that the total field is directly available in the entire domain of computation. Existing codes using a total/scattered-field formulation and a scattered-field ABC can be easily converted to a total-field formulation with a total-field ABC by adding the known right-hand side terms of the total-field ABC to the existing ABC and removing all parts of the code related to the connection conditions. Carrying out such a conversion yields a considerable simplification of the code together with a substantial improvement of its efficiency and user friendliness.

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In 1963, he was employed as a teacher at the Polytechnic School in Utrecht. From 1968 to 1970 he was employed as a Research Student of the Laboratory of Electromagnetic Research, Faculty of Electrical Engineering, Delft University of Technology. Since 1970 he has been a member of the Scientic Staff of this laboratory. He is currently an Associate Professor there. During the academic year 1979–1980 he held a 12-month Fellowship in the European Science Exchange Program at the Department of Electrical and Electronics Engineering, University of Nottingham, U.K. His main research interests are the development of finite-difference and finite-element methods for computing electromagnetic fields in complicated configurations containing inhomogeneous media.