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Towards spatially constrained gust models

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Abstract. With the trend of moving towards 10–20 MW turbines, rotor diameters are growing beyond the size of the largest turbulent structures in the atmospheric boundary layer. As a consequence, the fully uniform transients that are commonly used to predict extreme gust loads are losing their connection to reality and may lead to gross overdimensioning. More suiting would be to represent gusts by advecting air parcels and posing certain physical constraints on size and position. However, this would introduce several new degrees of freedom that significantly increase the computational burden of extreme load prediction. In an attempt to elaborate on the costs and benefits of such an approach, load calculations were done on the DTU 10 MW reference turbine where a single uniform gust shape was given various spatial dimensions with the transverse wavelength ranging up to twice the rotor diameter (357 m). The resulting loads displayed a very high spread, but remained well under the level of a uniform gust. Moving towards spatially constrained gust models would therefore yield far less conservative, though more realistic predictions at the cost of higher computation time.

1. Introduction

Over the past decades, there has been a strong trend towards larger rotor diameters, with 8 MW machines now entering production and 10–20 MW concepts lying on the drawing board. For rotors of such size, the weight requirements are very strict and there is little to no room for overdimensioning. It is therefore crucial to make a good prediction of the long-term loads.

Generating realistic responses makes it very important to correctly grasp the true nature of atmospheric gusts. Traditionally, gusts are modeled as perturbation velocities. These are scalar values that are added to or multiplied with the mean wind speed. Artificial gust models, like the Mexican hat wavelet that is now part of the IEC standards [1], are inherently one-dimensional and only model the longitudinal velocity component in time. Consequently, a gust is considered to be fully coherent over the rotor plane. This becomes less and less physical as the rotor diameter approaches the size of the largest turbulent structures.

The goal of this paper is to show what happens to the extreme response of the DTU 10 MW reference turbine when a gust shape is no longer uniform, but being constrained in space. The focus lies on a single longitudinal gust, which is then assigned various dimensions and positions in the frontal plane.

2. Method

2.1. A simplified gust model

The objective of this work is not so much to present a physically accurate gust model, but rather to show some of the consequences of limiting gusts in space. Therefore, a simple model is preferred that provides a clear definition of length scales. From the statistical treatment of gusts, it follows that a mean shape can be derived from the autocorrelation and cross-correlation functions of the turbulence spectrum [2]. The integral length scale can then be derived according to

$$L_0 = \frac{\lambda_0}{2} = \bar{u} \int_0^\infty r(\tau)\,\mathrm{d}\tau, \tag{1}$$

where λ is the eddy wavelength, $\bar{u}$, the mean wind speed, and $r(\tau)$ the normalized autocorrelation function. In a stationary frame of reference, the classical definition of a gust is a perturbation velocity, u', added on top of the mean wind speed, $\bar{u}$, according to the Reynolds decomposition:

$$u(t) = \bar{u} + u'(t). \tag{2}$$

In the present work, this perturbation velocity is modeled based on how the mean gust can be described by the autocorrelation function: $u'(t) = \hat{u}r(t)$. This leads to a relatively sharp waveform – a shape that has been shown to agree with the mean gust obtained from measurements [2]. An exponential decay function is taken with a velocity amplitude, $\hat{u}$, and (x_0, y_0, z_0) as the location of the gust center:

$$u'(x, y, z) = \hat{u}\mathrm{e}^{-2\sqrt{\left(\frac{x-x_0}{\lambda_x}\right)^2 + \left(\frac{y-y_0}{\lambda_y}\right)^2 + \left(\frac{z-z_0}{\lambda_z}\right)^2}}. \tag{3}$$

The longitudinal, lateral, and vertical dimensions of the gust are expressed here as the wavelengths λ_x, λ_y, and λ_z, respectively, because it offers the most intuitive way to interpret the area of effect. The longitudinal length scale of the gust can still be retrieved according to equation (1):

$$\frac{1}{\hat{u}} \int_0^\infty u'(x_0 + x, y_0, z_0)\,\mathrm{d}x = \frac{\lambda_x}{2} = L_x. \tag{4}$$

Moreover, the uniform gust is easily obtained by inserting $\lambda_y = \lambda_z = \infty$, which also makes it independent of y_0 and z_0.

Next to the position of the gust, also the orientation of the rotor is of importance. Although this is included in the longitudinal position—a gust that arrives at a later time encounters a rotor with a different azimuth angle—it is more intuitive to rely on the rotational symmetry and introduce a phase lag, ϕ. Translating the above to a stationary cylindrical coordinate system with $x = \bar{u}t$, $x_0 = 0$, $y_0 = -r_0 \sin(\theta + \phi)$, and $z_0 = h + r_0 \cos(\theta + \phi)$, gives the alternate expression:

$$u'(t, y, z) = \hat{u}\mathrm{e}^{-2\sqrt{\left(\frac{\bar{u}t}{\lambda_x}\right)^2 + \left(\frac{y+r_0\sin(\theta+\phi)}{\lambda_y}\right)^2 + \left(\frac{z-h-r_0\cos(\theta+\phi)}{\lambda_z}\right)^2}}, \tag{5}$$

where h is the hub height, r_0 the radial position of the gust center, and θ the azimuthal position of the rotor. A sketch of the situation is shown in figure 1. The situation is further simplified through the following assumptions:

- No additional turbulence.
- No wind shear.
- No tower shadow.
- The gust is symmetrical around the x-axis ($\lambda_y = \lambda_z$).

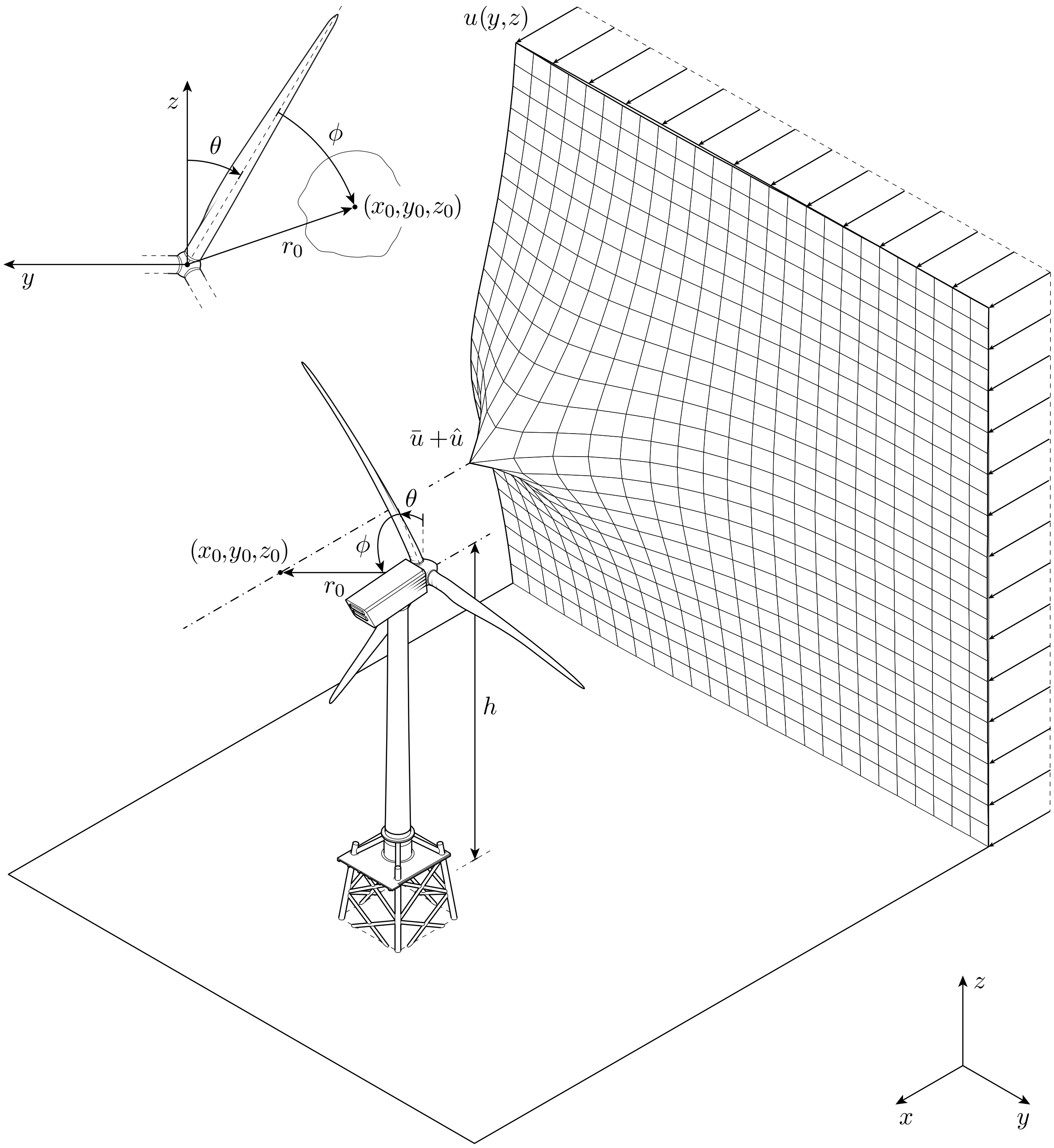

Figure 1. Sketch of the situation described by equation (5).

2.2. Turbine model
Load cases are evaluated in GH Bladed v4.4 (Sep 2013), using a model of the DTU 10 MW reference turbine of which the specifications are listed in table 1. The only deviation from the original setup is that the jacket foundation is assumed to be completely rigid, meaning that the tower is clamped at the interface height of $z = 26$ m. As can be expected, the first two tower mode frequencies are slightly higher than for the baseline case (0.32 Hz versus 0.30 Hz). Gusts are stored in separate wind files, containing a 49x49 grid with a width and height of 186 m.

Table 1. Specifications of the 10 MW reference turbine.

Wind regime	IEC class 1A
Rotor orientation	upwind
Number of blades	3
Control	variable speed collective pitch
Airfoil series	FFA-W3-xxx
Rated power	10 MW
Rated wind speed	11.4 m/s
Cut-in wind speed	4 m/s
Cut-out wind speed	25 m/s
Min. rotor speed	6.0 rpm
Max. rotor speed	9.6 rpm
Rotor diameter	178.3 m
Hub height	119.0 m
Hub overhang	7.1 m
Water depth	50 m
Interface height	26 m
Shaft tilt angle	5.0°
Rotor precone angle	-2.5°
Blade prebend	3.3 m

2.3. Parameters of interest

If the load of interest, M, is a function of multiple parameters ($\mathbf{k} = k_1, k_2, \ldots$), then its cumulative distribution function can be set up by using conditional probability:

$$F(M) = \int F(M|\mathbf{k}) f(\mathbf{k}) \, \mathrm{d}\mathbf{k}. \tag{6}$$

It is not hard to imagine that the above integral is very computationally expensive, if not impossible, to determine numerically for even a handful of parameters. This is the so-called 'curse of dimensionality'. In order to get around this, one should rather rely on stochastic simulation methods.

In this work, the scope lies on the maximum blade root flapwise moment, $\hat{M}_y$, which will be taken as the maximum response of the three blades:

$$\hat{M}_y = \max\left[M_{y1}(t), M_{y2}(t), M_{y3}(t)\right]. \tag{7}$$

The response of the turbine to equation (5) can be obtained by sampling the parameters according to the distribution function $f(\bar{u}, \hat{u}, y_0, z_0, \phi, \lambda_x, \lambda_y)$. The cumulative distribution can then be obtained by sorting a total of n responses and assigning the ith order statistic an estimated probability of

$$F_i = \frac{i}{n+1}. \tag{8}$$

Since the mean wind speed, amplitude, and longitudinal wavelength are shared with the uniform gust, the search is narrowed down by setting these to fixed values. The mean wind speed is taken as $\bar{u} = 8$ m/s with a gust amplitude of $\hat{u} = 4$ m/s. This is to stay away from the rated wind speed of the turbine, which should prevent triggering the pitch controller. Moreover, the longitudinal wavelength is taken as $\lambda_x = 16$ m to provide something similar to the IEC extreme operating gust (see figure 2). Any other combination of the remaining parameters (y_0, z_0, ϕ, and λ_y) would yield the exact same longitudinal gust shape through the gust's centerline and would be valid

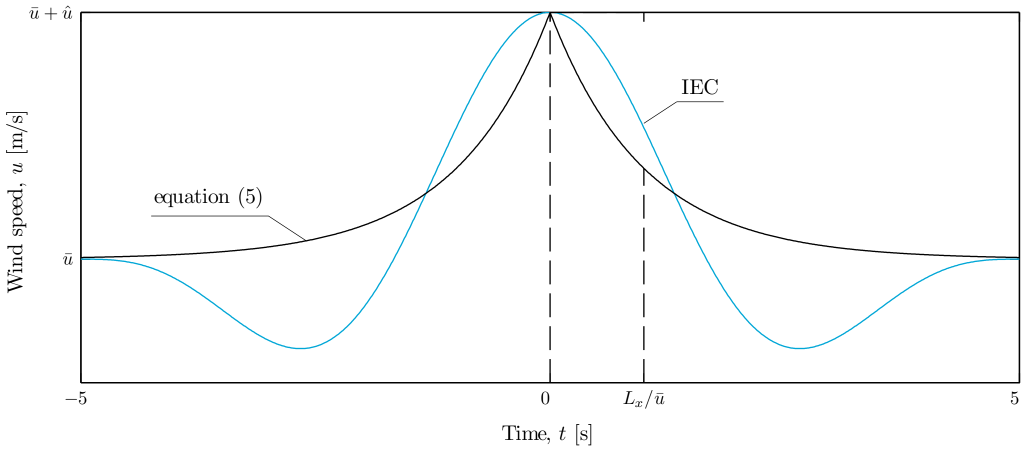

Figure 2. An arbitrary gust shape based on equation (5) with $\lambda_x = 2L_x = 16$ m. Included for comparison is the IEC extreme operating gust.

reproductions of a single-point measurement. These are left as variables and will be elaborated on in the next sections.

In a homogeneous wind field, the parameters should show independence of y_0 and z_0, which means that the probability density function can be rewritten to the product of the individual distributions:

$$f(y_0, z_0, \phi, \lambda_y|\bar{u}, \hat{u}, \lambda_x) = f(y_0)f(z_0)f(\phi)f(\lambda_y|\bar{u}, \hat{u}, \lambda_x). \tag{9}$$

The gusts' lateral and vertical positions are assumed to be uniformly distributed over the domain where $y_0 \in \{-D, D\}$ and $z_0 \in \{0, h + D\}$. The phase lag is uniformly distributed where $\phi \in \{-60°, 60°\}$. Because the function $f(\lambda_y|\bar{u}, \hat{u}, \lambda_x)$ is much harder to determine and would complicate the present discussion too much[1], the transverse wavelength is left variable and the results are binned separately for each λ_y.

3. Effects of gust three-dimensionality on wind turbine response

In cases where the lateral and vertical length scales are small compared to the rotor diameters, there is great variation in the response between different positions. This becomes clear when looking at figure 3, which shows the response to gusts of a variety of scales positioned at various heights. As can probably be expected, a larger area of effect causes a higher bending moment, but also smooths out the aforementioned variation. The maximum response tends to be found near the blade tips for very small gusts, and near $r = \frac{2}{3}R$ for large gusts. What is striking is the fact that even the largest gusts, having scales far beyond what one would expect in conjunction with $\lambda_x = 16$ m ($\approx 0.1D$), cause significantly lower loads than the uniform gust ($\lambda_y = \infty$).

Moreover, when the gust is relatively small, it is not hard to imagine that a gust can 'miss' the blades and trigger no response whatsoever. This is caused by what here is expressed as a gust's phase lag, of which its effects are illustrated by figure 4. For large positive ϕ (the gust is leading the blade) a gust can still generate a load when the blade slices through it. However, when ϕ is strong negative, a small gust can travel through the rotor unnoticed.

[1] The distribution function $f(\lambda_y|\bar{u}, \hat{u}, \lambda_x)$ would describe the occurrence probability of a certain transverse wavelength, given a longitudinal wavelength. Although it is possible to say something about how the kinetic energy is approximately distributed as function of the wave vector, relationships between individual eddies are much harder to establish (a single eddy can trigger a whole range of wave numbers).

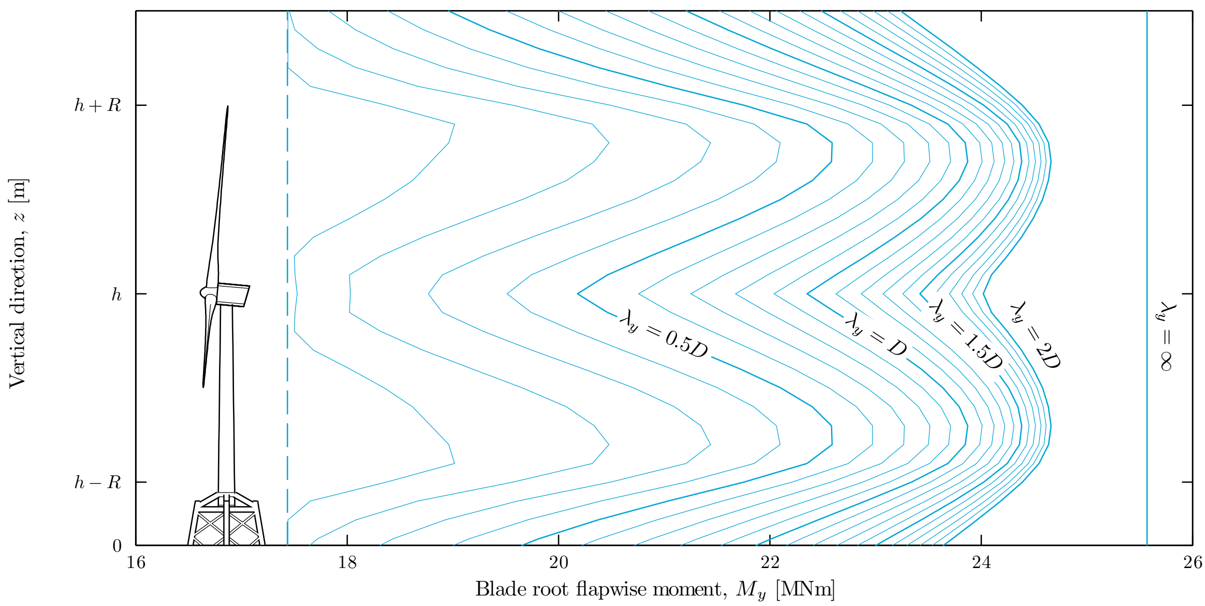

Figure 3. Effect of the gust position on the blade root flapwise moment for a several gusts with zero phase lag.

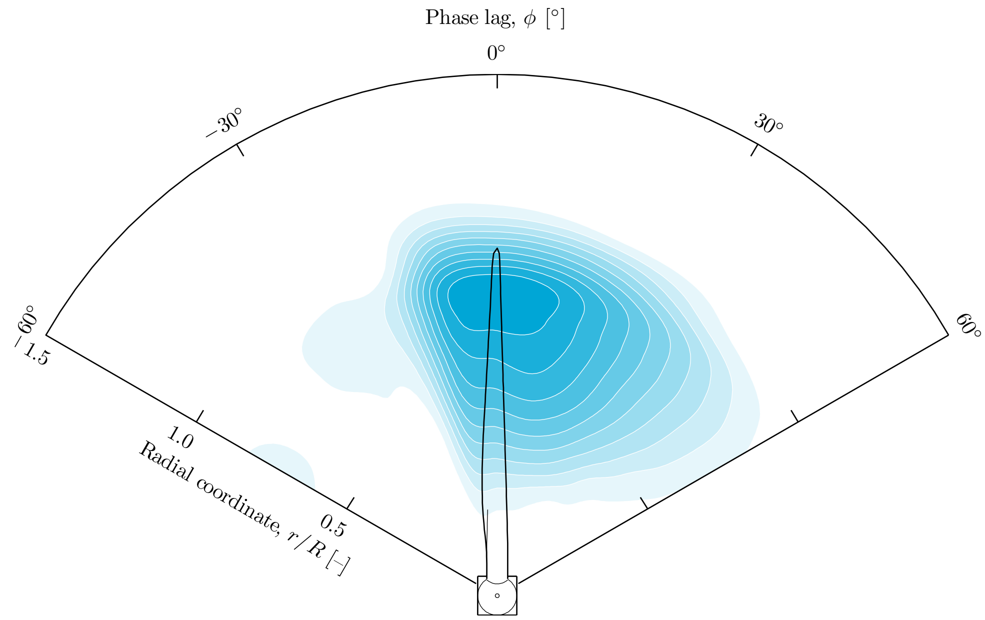

Figure 4. Effect of the phase lag on the amplitude of the blade root flapwise moment for a gust with $\lambda_x = \lambda_y = 16$ m. The dark colored area refers to the maximum load ($\hat{M}_y = 19.0$ MNm), while the uncolored area shows positions where no response is triggered ($\hat{M}_y = 17.4$ MNm).

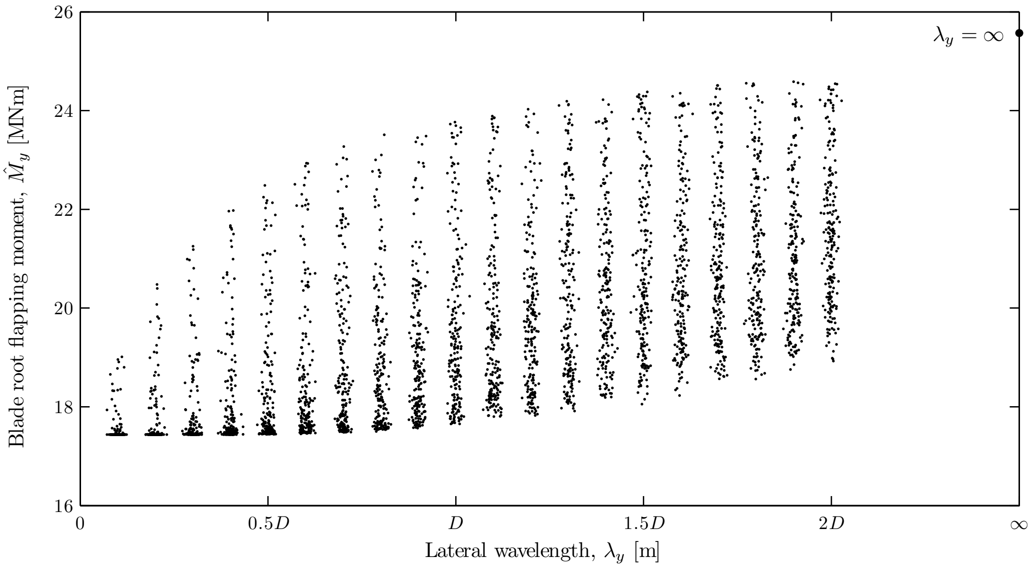

Figure 5. Monte Carlo simulation of $f(\hat{M}_y|\lambda_y)$ $n = 200$ load cases, showing the spread in M_y caused by various transverse wavelengths.

The combined effect of the parameters, y_0, z_0, ϕ, and λ_y can be visualized through a Monte Carlo simulation of the response to 200 gusts. The results, shown in figure 5, again emphasize that the spread increases at the medium scales where the area of effect is larger and the loads are higher. At the larger scales, the spread decreases due to the gusts starting to fill the domain. Ultimately, a fully uniform gust—this is denoted by $\lambda_y = \infty$—would always trigger the same response since it is independent of its position and the phase lag (see equation (5)). The corresponding cumulative distributions, shown in figure 6, indicate that even the high percentiles lie well below the response to the uniform gust. Sampling these load cases according to a certain distribution of length scales (where $\lambda_y < \infty$) will therefore inevitably lead to a lower N-year extreme load.

However, dealing with spatial gusts involves some difficulties concerning the definition of extremes. Uniform gusts would (theoretically) span the entire velocity field and therefore the amplitude measured at any fixed point would be a good representation of the 'severity' of the event. In reality, a single velocity amplitude can be linked to a variety of situations, ranging from rather meaningless fluctuations to large fronts. This means that it is not at all straightforward to connect the occurrence probability of a three-dimensional event to an N-year amplitude in a single point. This would require a different approach to distinguish damaging events from time series, for example based on the probability of entering a certain domain.

4. Uncertainty versus computational effort

The obvious downside to adding more parameters is the additional computation time it takes to arrive at the result. How this affects the prediction can be quantified by means of bootstrapping. For the sake of argument, say that a gust can take any transverse wavelength between $\lambda_y = 0$ and $\lambda_y = 2D$ with equal probability. The expected load is then the mean of a sample of n load cases. When repeated 100 times for a different random sample, it should give a fair estimate of the uncertainty that comes with evaluating more or less load cases. As can be expected, figure 7 shows that the error decreases consistently with the $1/\sqrt{n}$ rule associated with Monte Carlo methods – quadrupling the amount of load cases would halve the error. However, the

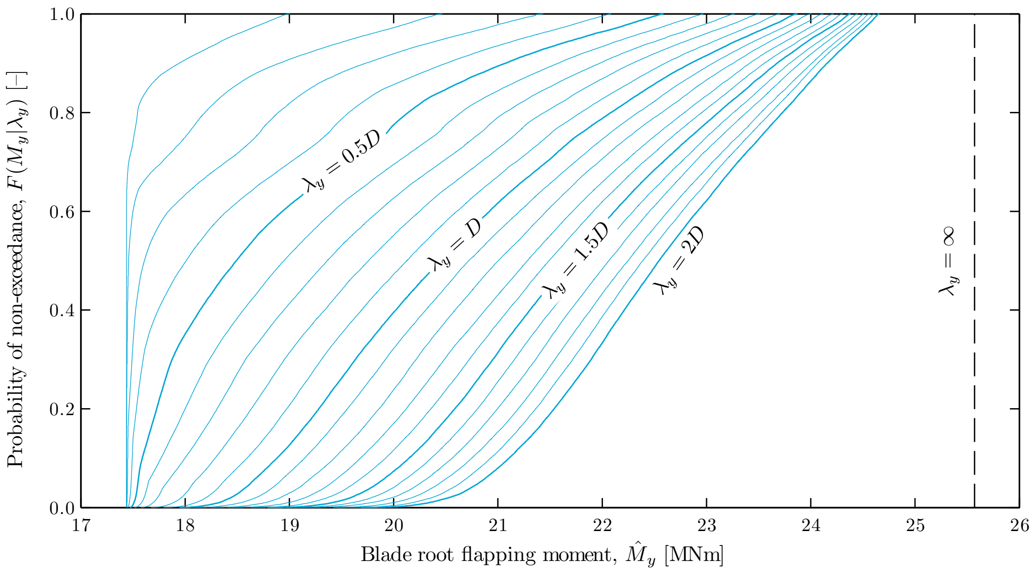

Figure 6. Cumulative distribution of the blade root bending moment for each transverse wavelength, matching the distribution of figure 5.

high spread indicates that adding spatial dimensions would require too much effort for even reasonable accuracy. In this particular case, making sure that 95% of the sample means would fall within ±1 MNm would already require 32 times as many computations as one would do for the uniform case.

In order to either reduce the amount of load cases (or the overall uncertainty), one can rely on importance sampling. Instead of choosing random combinations of parameters, the computation effort is best focused on the problem areas that contribute most to the integral in equation (6). A designer might already have an idea about the weak points on the rotor plane (e.g. see figure 3), certain damaging wavelengths, or situations that are generally difficult for the controller to handle. A strategy like this requires a method to generate a velocity field according to certain specifications. This is in line with the concept of constrained stochastic simulation (e.g. see [3]), which is an established tool that can be used to generate time series around some specific velocity amplitude. Future work will focus on extending this method to 3D.

However, even the best methods would not change the fact that extreme load predictions inevitably come with high uncertainty. Especially during the conceptual design phase, when a designer would rather minimize the computational effort, the N-year return level is meaningless without the corresponding confidence interval.

5. Conclusions

This work attempts to give a little more insight into what happens with the extreme gust loads when a single deterministic gust is no longer uniform, but limited in space. The situation was reduced to the most basic case: no wind shear, no turbulence, and no tower shadow.

The response of the DTU 10 MW turbine showed that the gust's size, or area of effect, has a large impact on the blade root bending moment. Larger velocity fronts naturally lead to higher loads, but are also less susceptible to variations in position or the rotor azimuth angle. Smaller gusts have a high probability of triggering no response at all, either by missing the rotor blades or landing close to the blade root. When interpreting a high velocity amplitude obtained from any single-point measurement, a designer has to keep in mind that there are numerous ways for

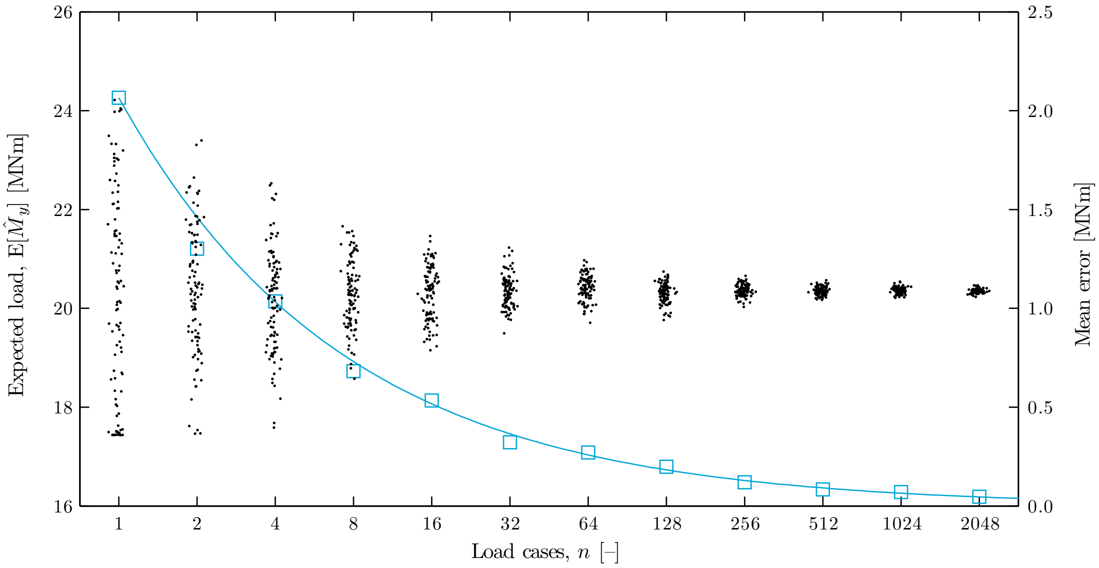

Figure 7. Effects of bootstrapping to simulate the uncertainty of the expected load, assuming that the gust's transverse wavelength is uniformly distributed in $\lambda_y \in \{0, 2D\}$. Added in the second axis is the error with respect to the true mean and the trend $1/\sqrt{n}$.

that gust to manifest itself in the transverse directions. What seems to be an extreme in a time series, might as well trigger no significant response at the turbine level. This is supported by the results of this exercise. Sampling from finite transverse length scales leads to a load distribution that lies below the level predicted by uniform gusts. The downside is that more parameters have to be considered, which inevitably leads to higher uncertainty.

The notion of a finite gust length scale is very relevant for future large wind turbines that may surpass the size of the largest turbulent structures in the atmospheric boundary layer. Still relying on current standards then may incorrectly cause the design to be driven by such extreme transients, leading to gross overdimensioning.

Acknowledgments
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