THEORETICAL CONSIDERATIONS ON THE MOTION OF SALT AND FRESH WATER

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ABSTRACT

This paper gives a survey of the theoretical investigations in Holland on the motion of salt and fresh water in estuaries, locks, etc. The insight gained is set forth, and also questions yet unsolved are mentioned.

First the long wave phenomena in the interface of two sharply separated liquids are treated, and in connection herewith, the cases of critical flow. The saltwedge in a river without tides and the penetration of salt through locks are discussed.

Next, the stability of the interface is considered. The saltwedge with disturbed interface in a tidal river (corrupted saltwedge) and the desalting of a canal by a sluice with a screen are discussed.

Finally the mixing processes in a brackish water region are classified. The mixing in a corrupted tidal saltwedge is discussed and the intrusion of salt in an estuary. It appears that the main agent of mixing in estuaries is the exchange of water between the channel and storing basins, shoals, etc.

INTRODUCTION

Along the seaboard, salt and fresh water meet at many places: in inlets and estuaries, in navigation locks, sluices, etc. By its superior density the salt water tends to penetrate inland underneath the fresh water layers; this effect is accentuated where the depth of water is important. In many cases, the intrusion of salt water has detrimental consequences; for instance, by infiltration into the groundwater and increase of silting.

This calls for either preventing the penetration or flushing the salt water back to sea. Effective measures of this kind are only possible when the mechanics of the interactions between salt and fresh water layers are well understood.

In conjunction with measurements in Dutch estuaries and locks, the theoretical investigations set forth in this paper have been undertaken. As will be seen, they leave certain aspects still unsolved, although they have been useful for a rational analysis of empirical data. The theoretical considerations, and especially the gaps they have left, have made clear the need for systematic experiments which are being carried out.

As the problem occurs in different countries, and moreover, the treatment can also be applied to density currents of other media, it may be useful to submit our results, even in their present incomplete form, if only for the purpose of provoking comments and discussion.

MOTION OF SHARPLY SEPARATED SALT AND FRESH WATER

INTERNAL AND EXTERNAL WAVES - We consider a system of two homogeneous layers of liquids (salt and fresh water) separated by a sharp interface. The difference in densities is assumed to be small compared to the density itself. We put $\epsilon = (\rho_2 - \rho_1) : \rho_2 \approx (\rho_2 - \rho_1) : \rho_1$.

The equations of continuity and the dynamical equations of the two layers are (neglecting vertical accelerations)

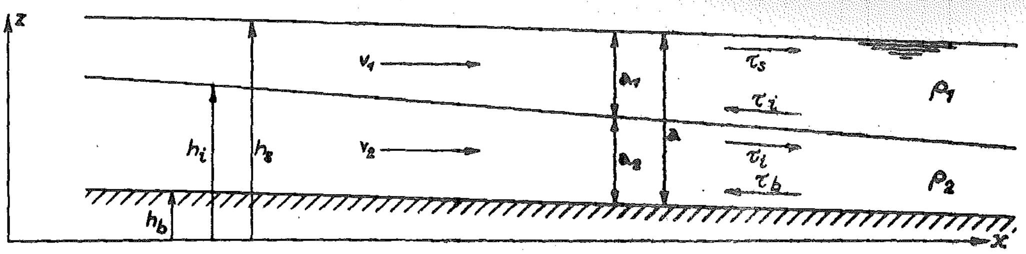


Fig. 1 - Definition sketch of two-layer system.

$$\frac{\partial a_1}{\partial t} + v_1 \frac{\partial a_1}{\partial x} + a_1 \frac{\partial v_1}{\partial x} = 0$$

$$\frac{\partial a_2}{\partial t} + v_2 \frac{\partial a_2}{\partial x} + a_2 \frac{\partial v_2}{\partial x} = 0$$

$$\frac{\partial v_1}{\partial t} + g \frac{\partial a_1}{\partial x} + g \frac{\partial a_2}{\partial x} + v_1 \frac{\partial v_1}{\partial x} + g(i_1 - i_b) = 0$$

$$\frac{\partial v_2}{\partial t} + (1 - \epsilon) g \frac{\partial a_1}{\partial x} + g \frac{\partial a_2}{\partial x} + v_2 \frac{\partial v_2}{\partial x} + g(i_2 - i_b) = 0$$

Here we have set

$$i_1 = \frac{\tau_i - \tau_s}{g \rho_1 a_1} \quad , \quad i_2 = \frac{\tau_b - \tau_i}{g \rho_2 a_2}$$

where τ_s, τ_i, and τ_b denote the respective shear stresses along the surface, the interface, and the bottom. Moreover, i_b denotes the bottom slope: $i_b = -dh_b/dx$.

Unless there is wind or an ice cover, we may put $\tau_s = 0$. Furthermore, we shall assume turbulent flow, and then put

$$\tau_i = \frac{\rho g |v_1 - v_2| (v_1 - v_2)}{4 C_i^2} \quad , \quad \tau_b = \frac{\rho g |v_2| v_2}{C_b^2}$$

where C_b is the coefficient of flow for the bottom, and C_i for the interface; in the above expression we have approximated ρ_1 and ρ_2 by $\rho = \frac{1}{2}(\rho_1 + \rho_2)$.

By investigating the characteristics of the equations of the two-layer system, we find that four kinds of waves are possible, with the following velocities of propagation:

$$\begin{matrix} C_e^+ \\ C_e^- \end{matrix} = \frac{v_1 a_1 + v_2 a_2}{a} \pm \sqrt{ga}$$

$$\begin{matrix} C_i^+ \\ C_i^- \end{matrix} = \frac{v_1 a_2 + v_2 a_1}{a} \pm \sqrt{\epsilon g \frac{a_1 a_2}{a} - \frac{(v_2 - v_1)^2 a_1 a_2}{a^2}}$$

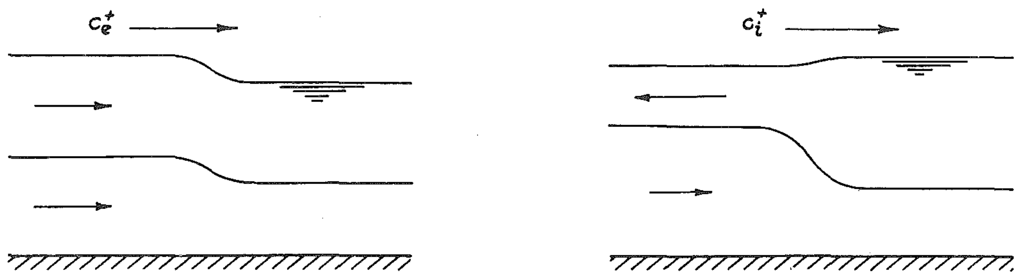
These formulas are approximative, based on the assumption $\epsilon \ll 1$.

The waves with velocities C_e^+ or C_e^- correspond to the long waves of an ordinary one-layer system; we shall denote them here as "external" waves. The waves with velocities C_i^+ or C_i^-, which usually are much less than C_e^+ and C_e^-, are called "internal" waves.

In an external wave (Fig. 2a) the velocities of the two layers are practically equal, and the displacements of the interface are practically equal to the vertical motion that would exist in a long wave in a one-layer system.

In an internal wave (Fig. 2b), the surface displacements are of the order ϵ less than the interface displacements. The two layers move in opposite directions with practically equal discharges.

An arbitrary disturbance of the equilibrium of the layers can generally be considered as the composition of two external waves in opposite directions, and two internal waves in opposite directions. If friction is negligible, the external and the internal waves are, to a large extent, mutually independent. Bottom friction, however, gives rise to mutual interference.



a. External wave. b. Internal wave.

Fig. 2

CASES OF CRITICAL FLOW - If one of the velocities of propagation becomes zero, the flow is called critical. When C_e^+ or C_e^- becomes zero, we obtain the ordinary critical flow, well known in the hydraulics of one-layer systems.

When C_i^+ or C_i^- becomes zero, we obtain internal critical flow. Here several cases may be distinguished.

When the lower layer is stagnant, the flow of the upper layer becomes critical when

$$|v_1| = \sqrt{\epsilon g a_1}$$

Similarly, in case of a stagnant upper layer, we have

$$|v_2| = \sqrt{\epsilon g a_2}$$

when the flow of the lower layer becomes critical.

Another typical case is the exchange flow, where both layers have equal discharges in opposite directions so that salt water and fresh water are exchanged in equal quantities. Consider, for example, the exchange between two basins connected by a relatively narrow pass (Fig. 3).

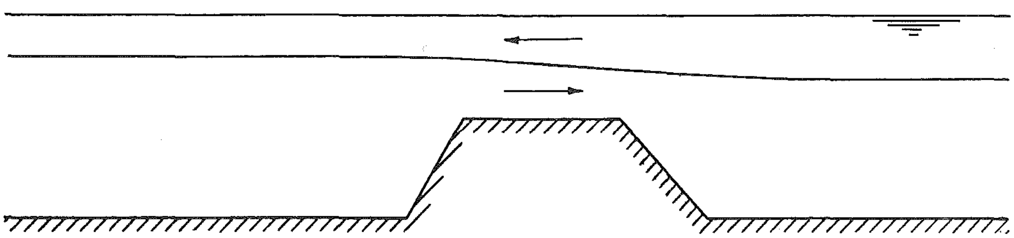


Fig. 3 - Exchange flow.

If the difference in level of the interface between the two basins (the internal head) is small, we have an exchange flow: the lower layer accelerates on the left side, passes into the right basin, and decelerates on the right; while the upper layer accelerates on the right, passes, and decelerates on the left. The decelerations are attended by losses, resulting in the difference of the internal head of the interface.

When the interface in the right basin is lowered, the exchange discharge increases until the velocity C_i^- of waves propagated to the left becomes zero. Then the flow in the pass becomes critical, insensitive to the right, and further lowering of the interface in the right basin is no longer influencing the flow in the pass.

When the interface in the left basin is raised, the exchange discharge increases likewise until the velocity of waves propagated to the right becomes zero. Then the flow in the pass becomes critical, insensitive to the left, and further raising of the interface in the left basin is no longer influencing the flow in the pass.

When the interface in the right basin is lowered below 3/8 a (a is the depth of the pass) and meanwhile when the interface in the left basin is raised above 5/8 a, both C_i^+ and C_i^-

become zero, and the flow is then double critical, insensitive in both directions (total deceleration losses have been assumed). In the pass, both layers then have the thickness 1/2 a, and the relative velocity of each layer with respect to the other then assumes the double critical value

$$v_{cc} = \sqrt{\epsilon g a}$$

INTERNAL JUMPS - Analogously to a surface jump (surge, bore), an internal jump (i.e., a sudden transition of the interface from one level to another) can propagate itself substantially without altering its form (Fig. 4).

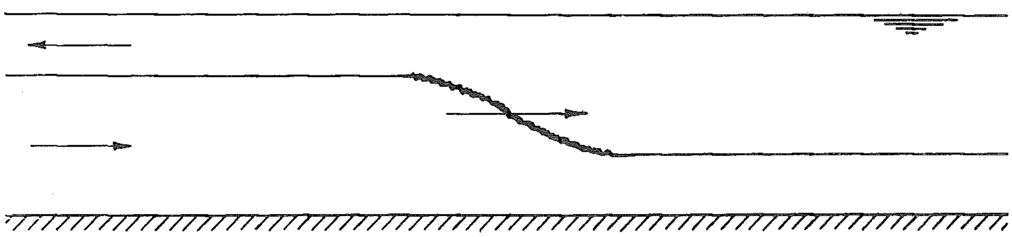


Fig. 4 - Internal jump.

The internal jump can be treated by considering it with respect to a coordinate system that moves with the same velocity as the jump. There must be satisfied conditions of continuity in both layers, and a condition of momentum in both layers together. One condition has yet to be added; for this we may take the energy equation in the relatively accelerated layer. In the relatively decelerated layer, the energy equation must yield a loss of energy. This last condition excludes definite types of jumps.

SALTWEDGE IN A RIVER WITHOUT TIDES - In rivers without an appreciable tidal motion, such as the Rhône (France) and the Mississippi (USA), the fresh water discharges over a practically stagnant body of salt water which has penetrated from the sea along the bottom (Fig. 5).

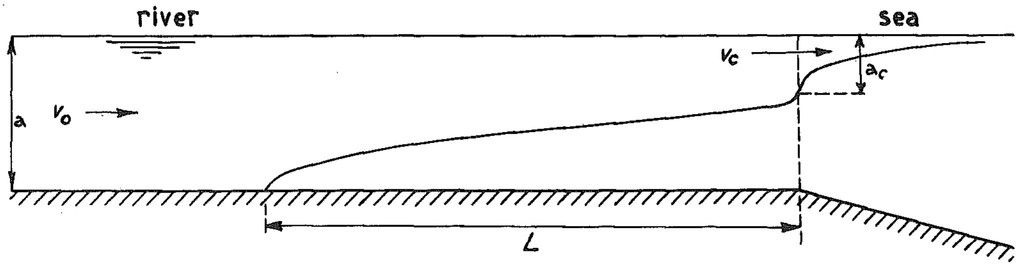


Fig. 5 - Saltwedge.

The flow of the upper layer is critical at the mouth. From the mouth upward the interface obeys the equation

$$\epsilon g \frac{da_1}{dx} + v_1 \frac{dv_1}{dx} + \frac{v_1^2 a}{4C_i^2\, a_1 (a-a_1)} = 0$$

obtained by combining the dynamical equations of the two layers; C_i is the coefficient of friction along the interface.

Integration yields

$$L = \frac{C_i^2}{g}\, a \left[\frac{1}{5} \frac{\epsilon g a}{v_0^2} - 2 + 3\sqrt[3]{\frac{v_0^2}{\epsilon g a}} - \frac{6}{5} \sqrt[3]{\frac{v_0^2}{\epsilon g a}}^{\,2} \right]$$

for the length of the wedge. When v_0 increases, the thickness of the upper layer at the mouth,

$$a_c = \frac{v_0^2}{\epsilon g}$$

increases. The friction at the interface and the slope increase likewise. Hence, the length of the wedge then decreases for a double reason.

When the velocity v_0 becomes greater than the double-critical velocity v_{cc}, the wedge is wholly expelled from the river.

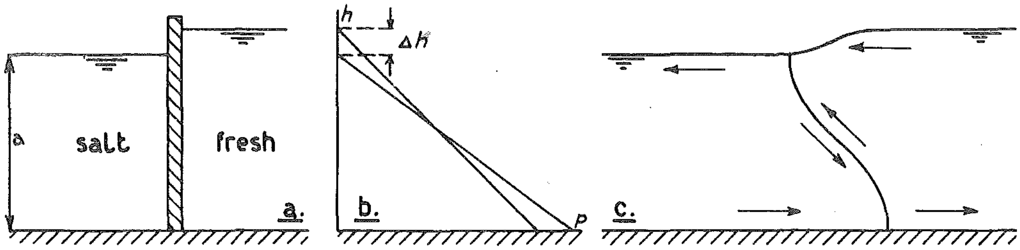
EXCHANGE OF SALT AND FRESH WATER IN A LOCK - We consider a gate of a lock, on one side of which there is fresh water, and on the other side salt water. When the gate is opened, the double-critical flow begins to establish itself. Hence, if friction losses are neglected, the exchange discharge is

$$\varphi = \frac{1}{4} a \ b \sqrt{\epsilon \ g \ a}$$

where a and b denote depth and width of the gate entrance.

This is valid, at least for the first stage of the exchange process, whether or not the lock chamber, the entrance, and the canal or harbor are of the same width and depth.

In case depth and width on both sides of the entrance are equal to those of the entrance itself, the following more detailed picture can be given:



a. Gate of a lock. b. Pressure distribution. c. Beginning exchange flow.

Fig. 6

In the normal procedure of locking, the lock chamber and the outside water have been in communication by means of culverts which have been closed after the pressures at the level of the culverts have reached equilibrium. The simplest case for the treatment of the flow pattern after opening the gate occurs when the culverts are supposed at mid-depth. Then the resulting pressure forces on both sides of the gate equal each other, whereas the level of the salt water is below that of the fresh water.

Now let the gate be opened suddenly. Then salt water begins to flow down and under the fresh water, while the fresh water begins to flow up and over the salt water (Fig. 6c).

Some time afterwards a motion as pictured in Fig. 7 has developed. The salt water intrudes under the fresh water by an internal jump I_1 traveling to the right with the rather small velocity $\frac{1}{2} \sqrt{\epsilon \ g \ a}$, and attended by a small surface depression $\frac{1}{2} \Delta h$. The fresh water dispels the salt water by an internal jump I_2 similar to I_1, but traveling to the left and attended by a small surface elevation.

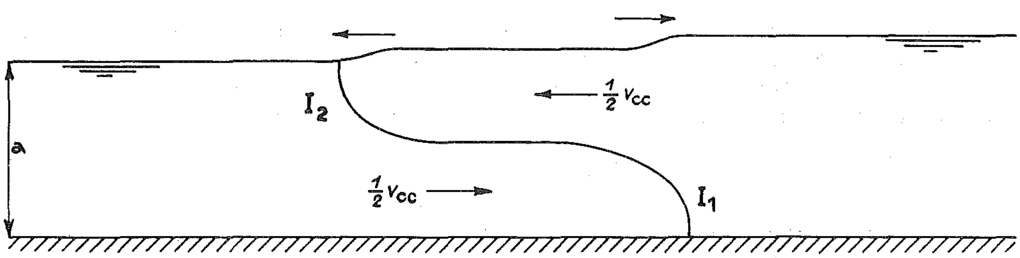


Fig. 7 - Developed exchange flow.

When the culverts are situated above or below mid-depth, two small surface waves, each with a maximum height of $\frac{1}{4}\Delta h$, travel, one to the right and the other to the left. If the culverts are at a high level, the surface wave to the right (that is toward the fresh water) is positive and the wave to the left is negative. If the culverts are at a low level, the reverse will be found.

These small surface waves have a much greater velocity than the internal waves, namely, $\sqrt{g\,a}$. They are of secondary importance and practically do not affect the behavior of the internal jumps at all.

It should be remarked that the flow pattern as described above has been derived with the friction and mixing left out of consideration. In fact, the exchange movement is attenuated by friction. Furthermore the interface is strongly disturbed by secondary waves.

THE INTERFACE AND ITS STABILITY

SHORT WAVES IN THE INTERFACE - We consider sine waves in the interface of which the wave length may be so short that the vertical accelerations are not negligible (short waves). If the amplitude is small, the wave travels undistorted with a velocity

$$\begin{matrix} C_i^+ \\ C_i^- \end{matrix} = \frac{v_1 \tanh \sigma a_2 + v_2 \tanh \sigma a_1}{\tanh \sigma a_2 + \tanh \sigma a_1}$$

$$\pm \sqrt{\frac{\epsilon g}{\sigma} \frac{\tanh \sigma a_1 \tanh \sigma a_2}{\tanh \sigma a_1 + \tanh \sigma a_2} - \frac{(v_2 - v_1)^2 \tanh \sigma a_1 \tanh \sigma a_2}{(\tanh \sigma a_1 + \tanh \sigma a_2)^2}}$$

This is accurate except for errors of the order ϵ to 1.

Viscosity has been left out of consideration. Furthermore σ denotes $\frac{2\pi}{\lambda}$.

When λ tends to infinity (and hence σ tends to 0) we arrive again at the formulas for long waves.

When λ is less than $\lambda_c = 2\pi : \sigma_c$, where σ_c is defined by

$$\frac{\tanh \sigma_c a_1 + \tanh \sigma_c a_2}{\sigma_c} = \frac{(v_2 - v_1)^2}{\epsilon g}$$

the velocities of propagation become complex imaginary. This must be interpreted in this way: the wave travels with the velocity

$$C_i = \frac{v_1 \tanh \sigma a_2 + v_2 \tanh \sigma a_1}{\tanh \sigma a_2 + \tanh \sigma a_1}$$

and that it is either attenuated or amplified in the course of time according to the factor exp $\pm \beta t$, where

$$\beta = \sqrt{\frac{(v_2 - v_1)^2 \tanh \sigma a_1 \tanh \sigma a_2}{(\tanh \sigma a_1 + \tanh \sigma a_2)^2} - \frac{\epsilon g}{\sigma} \frac{\tanh \sigma a_1 \tanh \sigma a_2}{\tanh \sigma a_1 + \tanh \sigma a_2}}$$

Thus waves shorter than those with length λ_c cannot be propagated without altering their amplitude.

When the relative velocity of the layers $v_2 - v_1$ increases, the critical wave length λ_c increases likewise. When $v_2 - v_1$ exceeds the double critical value v_{cc}, even the longest waves become of the type that cannot be propagated without either attenuation or amplification.

However, small the relative velocity $v_2 - v_1$, there is always a critical value λ_c of the wave length according to the above formulas. It should be noticed however, that all waves are subject to attenuation by viscosity, and it must be assumed that for very short waves the amplification deduced above is compensated or more than that by the viscous attenuation.

How great this lower critical velocity v_c^* is, is a question yet unsolved.

Hence, below some lower critical value of the relative velocity defined by viscosity, no growing waves in the interface are possible. Above that critical value, the range of wave lengths capable of amplification grows with increasing relative velocity. When the upper critical value v_{cc} is reached, this range has extended itself as far as to the longest waves.

LAMINAR INTERLAYER - By molecular diffusion a usually very thin brackish interlayer between fresh and salt water will be established, constituting a gradual transition in salinity, density, and velocity between the two main layers.

Even if the motions of the fresh and the salt layers are turbulent, the interlayer may flow laminally, because of its greater stability, resulting from the density gradient. For, as a consequence of this, any disturbance of the laminar flow is counteracted, not only by viscous forces, but also by gravity forces. Hence, the stability of the interlayer depends on a Reynolds number and a Froude number,

$$R_i = \frac{\Delta v \Delta z}{\nu} \qquad F_i = \frac{\Delta v}{\sqrt{\frac{\Delta \rho}{\rho} g \Delta z}}$$

and must assume the form

$$\varphi \, (R_i \, , F_i \,) = 0$$

Here Δv is the difference between the velocity just above and below the laminar layer, $\Delta \rho$ is the difference in density, and Δz the thickness of the layer.

If we adopt Keulegan's proposition [2], we may introduce a new number

$$A = R_i \, F_i^{\,\alpha}$$

where $\alpha = 1.5$, and then

$$A = 77{,}000$$

is the condition of stability.

STABILITY OF THE INTERFACE AND INTERFACIAL FRICTION - When the Reynolds numbers of the upper and lower layers do not exceed their critical values, these layers move laminally, as well as the interlayer.

When the relative velocity is increased, the upper and lower layers become turbulent, but the interlayer remains laminar at first.

Now the interlayer as a whole (that is, practically the interface) is capable of wave motions. When the lower critical value $v_c{}^*$ of the relative velocity is exceeded, waves with wave lengths for which amplification is possible will develop from small random disturbances which are always present (an arbitrary disturbance is composed of an infinite spectre of sine waves; every wave length is represented, however small its amplitude). Those waves grow in amplitude. Then, however, the flow begins to distort them more and more, and finally the waves break down. Hence, the interface is more or less instable.

Assuming the critical velocity $v_c{}^*$ exceeds the velocity for which upper and lower layers become turbulent, it is then possible that the flow of the two main layers is turbulent, and that the interface is stable meanwhile. Then the interface may be considered as a smooth boundary for the turbulent flows above and below. We adopt the logarithmic velocity distribution both above and below the laminar layer:

$$v = v_m \pm \frac{v_f}{\kappa} \ln \frac{|z|}{z_0}$$

Here v_m is the velocity of the center of the laminar layer, $v_f = \sqrt{\tau : \rho}$ is the friction velocity, and $\kappa = 0.4$ represents von Karman's constant. We shall assume that $z_0 = \frac{1}{2} \Delta z : N$, where N is a constant to be defined experimentally, and Δz is defined by Keulegan's stability criterion for the interlayer. We can then deduce

$$C_i = \gamma \frac{\sqrt{g}}{\kappa} \ln \lambda \sqrt{g} \, \frac{B}{C_i}$$

for the interfacial friction. Here

$$B = R^{\beta} \quad F^{1-\beta}, \qquad \gamma = \frac{4 + 4\alpha}{4 + \alpha}, \qquad \lambda = \left(\frac{N}{e}\right)^{\frac{4 + \alpha}{4 + 4\alpha}} \; \frac{3\alpha}{2^{4 + 4\alpha}} \; \frac{1}{A^{2 + 2\alpha}},$$

where

$$R = \frac{(v_1 - v_2)a_i}{\nu}, \quad F = \frac{v_1 - v_2}{\sqrt{\epsilon g a_i}}, \quad \beta = \frac{2 + \alpha}{2 + 2\alpha}$$

and $a_i = \sqrt{a_1 a_2}$; for $\alpha = 1.5$ we have $\beta = 0.7$ and $\gamma = 1.8$. Assuming $N = 105$ as a laminar layer with half the thickness Δz along a rigid wall, we get $\lambda = 0.55$.

When the relative velocity increases and exceeds the critical value $v_c{}^*$ more and more, the interface becomes more instable and disrupted, and hence it must be even more considered as a rough boundary for the turbulent flows above and below. The interfacial coefficient of friction then assumes the form

$$C_i = \frac{\sqrt{g}}{\kappa} \ln 11 \frac{a_i}{k}$$

where k, the Nikuradse equivalent of the roughness of the interface, probably depends on F.

The laws of roughness and the possibility of a smooth interface still form questions unsolved.

After an amplified wave has broken down, salt volume elements which were left in the upper layer fall back in the lower layer; and similarly, fresh elements rise back from the lower to the upper layer. This is attended by a more or less intense mixing of salt and fresh water. When the relative velocity increases, more and more waves become instable, and mixing intensifies.

When the double critical value v_{cc} is exceeded, even the longest waves become unstable, and it must be assumed that a more or less distinct interface can then no longer exist (absolute instability).

CORRUPTED SALTWEDGE IN A TIDAL RIVER - Saltwedges with a sharp separation of salt and fresh water, found in several rivers without tides, are not found in tidal rivers such as the Dutch. Here we do find a wedge-like salt distribution with inclined isohalinic surfaces, but without the sharp separation of salt and fresh water. It seems that this corruption of the saltwedge is caused by the tidal motion. Perhaps the following consideration may lead to the explanation:

As stated previously, the saltwedge is expelled from the river when v_0 exceeds v_{cc}. This means that in a river without tides, the relative velocity along the interface never exceeds the value v_{cc}, so that the absolute instability is never attained in such a river.

In a tidal river, the saltwedge is carried to and fro by the tidal flow. This, of course, produces internal waves, and we may consider whether this can be the cause for absolute instability. Now in absence of friction, it is not easily explainable how internal waves would cause absolute instability. The absolute instability may be explained, however, by the bottom friction; during the ebb, the water is pushed seaward, but the friction at the bottom impedes the seaward motion of the lower layer. True, the motion of the upper layer is also impeded by the friction along the interface, but this is a secondary effect, and it seems intelligible that the relative velocity may be so much increased that the double critical value v_{cc} is exceeded.

When the saltwedge is corrupted during the ebb, a gradual transition between salt and fresh water is established, which is maintained throughout the whole tidal period.

EXPELLING SALT FROM INLAND WATER BY A SLUICE WITH A SCREEN - In certain cases of salt water intrusion into an inland body of water, only limited quantities of fresh water are available for flushing the saline water back to sea. This problem occurs, for instance, at the locks of IJmuiden (giving access to the Noordzeecanal and to Amsterdam). Adjacent to the locks, a capacious sluice serves to evacuate superfluous water from the canal and the surrounding areas. When the rainfall is high, and as a consequence the quantities to be discharged are abundant, the salt water that has intruded through the locks and assembled near the bottom of the canal can be expelled readily. In dry periods, however, the small amount of fresh water available for flushing has to be utilized in the most effective manner.

For this purpose it is considered to build, at a certain distance inland from the sluice, a screen according to Fig. 8, the lower edge of which is situated below the interface separating

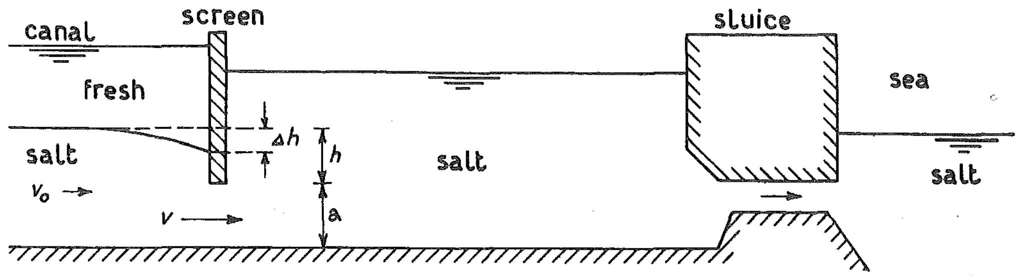


Fig. 8 - Sluice with screen for desalting.

the fresh layer from the salt water layer. The flow from the canal to the sluice will then have to pass through the low slit of fairly great length below the screen. The object of this arrangement is to confine the flow to the deep saline layer while the fresh water on top remains stagnant.

Near the screen the interface in the canal is depressed as a compensation for the increase of velocity head. In order to prevent the discharge of fresh water, which would detract from the effectiveness of the screen, the depression must not exceed a critical value defined substantially by the ratio $h : a$ [6].

It is also inefficient when the lower layer transports fresh water as a result of mixing at the interface. This can be sufficiently avoided by keeping the velocities near the slit well below the double-critical value v_{cc}. This question still has to be further investigated quantitatively.

MOTION OF SALT AND FRESH WATER WITH INTERMIXING

CLASSIFICATION OF MIXING MECHANISMS - Brackish water is formed by mixing salt water and fresh. This mixing may be brought about by quite different mechanisms.

According to scale and regularity, the motion of fluid particles may be distinguished into the following:

(1) Molecular movement.
(2) Turbulence.
(3) Average flow.

All three kinds of motion may provoke mixing.

The thermal movement of the molecules and ions results in a diffusion of salt from places with great concentration to places with less concentration of salt. The scale of this mixing mechanism is of the order of 10^{-11}m. The diffusivity (coefficient of molecular diffusion) of salt in water is of the order of 10^{-8} sq m per sec, which is very small, even if compared to the kinematic viscosity (10^{-6} sq m per sec), which is caused likewise by the molecular movement. Hence, molecular diffusion is seldom of direct influence.

The irregular motion which we call turbulence brings elements of water from more saline layers into fresher layers, and vice versa. Thus, the surface along which the saltier and the fresher water are in contact with each other is increased considerably, which strongly promotes the formation of brackish water by molecular diffusion.

If the average flow follows such a pattern that quantities of water with different salinities, which were first separated by a great distance, are brought in close proximity, the intermixing by turbulence is made possible. The formation of brackish water, ultimately, is again due to molecular diffusion.

So, turbulence and average flow do not strictly form brackish water. The role of a larger scale mixing mechanism consists in the activation of the smaller scale mechanism, and in each mixing process, the final act is played by molecular diffusion.

TURBULENCE IN A FLOW WITH A VERTICAL DENSITY GRADIENT - As discussed previously, the salt-wedge in a tidal river may be corrupted so that a brackish region is formed with a gradual transition between rather saline water at the bottom, and rather fresh water at the surface.

The turbulence in a flow with such a vertical density gradient undergoes the influence of that gradient.

If there is little or no density gradient, the development of turbulence is defined by the equilibrium of the work done by driving forces, and the dissipation by turbulent diffusion of momentum, and finally by viscosity.

If there is a density gradient, a new form of dissipation is introduced. When, for example, a volume element of denser water from a lower layer is raised to a higher layer, and in the lower layer replaced by an element of less dense water from the higher layer, both elements acquire an amount of potential energy. If then the elements dissolve in the new environment by molecular diffusion, the potential energy acquired dissipates. Hence, in a flow with a density gradient, the equilibrium between energy supply and dissipation is attained by less developed turbulence than in a flow without a density gradient. This means that the density gradient attenuates friction as well as mixing.

When introducing Richardson's parameter

$$\tau = \frac{g \frac{d\rho}{dz}}{\rho \left(\frac{dv}{dz}\right)^2}$$

we have [3]

$$D_m = \ell^2 \left|\frac{dv}{dz}\right| F(\tau)$$

for the vertical diffusion of momentum, and

$$D_{sv} = \ell^2 \left|\frac{dv}{dz}\right| f(\tau)$$

for the vertical diffusion of salinity. Here we have

$$\ell = \kappa \left|\frac{dv}{dz}\right| : \left|\frac{d^2 v}{dz^2}\right|$$

according to Von Karman, while F and f denote functions that might be obtained from experiment, and about which the following may be remarked in general:

For $\tau = 0$ (no density gradient) we have $F = 1$ in order to obtain the equation of Prandtl-Von Karman. Experiment indicates that $f > F$ for small values of τ. This can theoretically be understood by considering the influence of the pressure-forces on the motion of the elements [3].

The condition that the supply of energy to a volume element by the turbulent stress must exceed the dissipation by mixing, yields that

$$f \tau < F$$

so that for greater values of τ we have $f < F$.

Assuming the validity of the above theory also for a density gradient resulting from suspended material (at least approximately), application of the theory to some data published by Ismail [4] suggests that F and f decline rapidly with increasing values of τ (for $\tau = 1$, F and f are of a much lower order than 1).

Both when treating the subject by the concepts of turbulence [3] and when treating it by the concepts of internal waves [1], there appears to be evidence that in a flow with a density gradient, motions with a definite period,

$$T = 2\pi : \sqrt{g \frac{d\rho}{dz} : \rho}$$

will be dominating in the spectre of turbulence.

A better understanding of the phenomena is hoped to be gained from measurements of velocity and salinity fluctuations in the Dutch tidal waters, which are being prepared.

INTRUSION OF SALT IN AN ESTUARY - In an estuary with a fresh water discharge, the salt may penetrate from the sea against the expelling action of the flow of fresh water, firstly in the form of a saltwedge along the bottom. Inward of this saltwedge, and in the whole estuary if the wedge is so corrupt that the vertical gradient is nearly absent, the salt penetrates by horizontal diffusion. Introducing a coefficient of horizontal diffusion D, we may put

$$v_0 \sigma = D \frac{d\sigma}{dx}$$

where v_0 is the velocity obtained by dividing the fresh water discharge by the cross sectional area.

The diffusivity of salt in water is of the order of magnitude of 10^{-2} sq m per sec. This is much less than the values of the diffusion coefficient that follow from the average salinity distribution along an estuary.

Turbulence is a diffusion mechanism of irregular motions on different scales. The prevailing motions have a scale of the order of one-tenth of the depth. In the Dutch estuaries this scale is about 1 m.

In order to discuss the turbulent diffusion, we assume a logarithmic vertical velocity distribution and a small vertical density gradient. Then the coefficient of the vertical diffusion of momentum varies with the height z above the bottom by

$$D_{mv} = \frac{\kappa \sqrt{g}}{C} \bar{v} \, z(1 - \frac{z}{a})$$

where $\bar{v}$ is the instantaneous velocity, averaged along the vertical. Putting λD_{mv} for the coefficient of horizontal diffusion of salinity, the average value of this coefficient along the vertical is

$$D_{sh} = \frac{2}{3} \lambda \frac{\kappa \sqrt{g}}{C} \bar{v} \, a$$

In the Dutch estuaries this coefficient of diffusion may be estimated at about 0.1 to 1 sq m per sec, which is still much less than the values of D following from the average salinity distribution.

The main flow may form more than one mechanism provoking diffusion. One such mechanism is the following:

The upper layers in a flow usually have greater velocities than the layers near the bottom. This means that the paths of tidal excursion of the upper layers are greater than those of the lower layers. Hence, the tides do not only bring about an oscillating movement of the whole body of water, but also an oscillatory shifting of the upper layers relative to the lower layers.

An element of water from an upper layer may therefore descend to a lower layer by vertical turbulent diffusion, stay there some time, and then return to the upper layer. During this time the upper layer has progressed a certain distance relative to the lower layer. As a result of this, the element considered does not return to its initial environment.

It has thus been transported along the upper layer by the intermediary of the lower layer. In the same way a transport along the lower layer by the intermediary of the upper layer is possible. This way of transport of water has the effect of a diffusion.

The scale of this diffusion mechanism is the average distance that elements of water travel in relation to their original layer before they return. This distance, which depends on the vertical turbulent diffusion, is of the order of 10 to 10^2 m, and hence but a fraction of the amplitude of the shifting motion (10^3 m).

The diffusion coefficient of the shifting-layer mechanism is about 10 to 20 times that of the horizontal turbulent diffusion D_{sh}, i.e., of the order of 1 to 10 sq m per sec in Dutch estuaries. This is still insufficient to explain the actual penetration of salt.

Another mechanism of diffusion is formed by ports or similar storing basins along the estuary (Fig. 9). This can be set forth as follows:

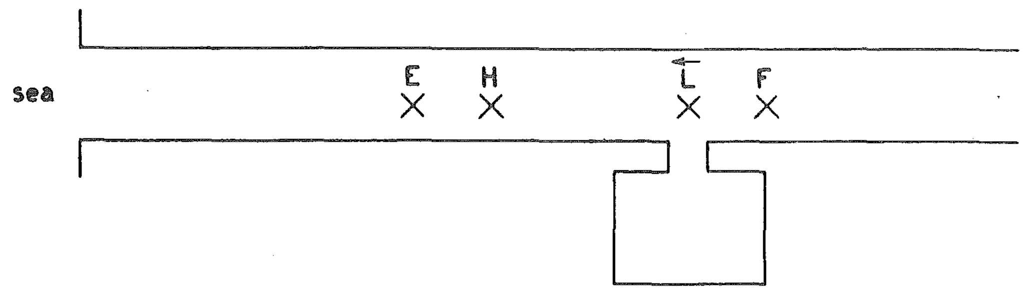


Fig. 9 - Effect of storing basin.

Let L be the element of water in the channel (that is, just before the entrance of the basin) when it is low tide. Suppose the ebb is then still flowing. Let F be the element that reaches the basin when the tide turns on the flood, H the element arriving at high tide, and E the element arriving when the tide turns on the ebb. Hence during the rising tide, first the elements between L and F, and then those between F and H pass by the entrance of the basin. During the falling tide, first the elements between H and E pass by, and then those between E and L. This means that the basin accumulates water of the elements H to F during the rising tide, and that this water is returned to the elements between E and L during the falling tide, thus provoking mixing.

The effect of the basin is schematically equivalent to an exchange, each tidal period a volume V equal to the tidal volume of the basin between two places on a distance $\frac{\pi}{2} \hat{x} \cos \varphi$, the one upstream and the other downstream from the basin; here $2\hat{x}$ denotes the tidal excursion path, and φ the angular phase-lag of the vertical with respect to the horizontal tide.

Shoals in an estuary have a similar effect as a storing basin. Let us schematize the estuary as a combination of a channel (width b_1, depth a) that transmits the flow, and shoals that store water without transmitting the flow (Fig. 10). Let b_2 denote the average width of the shoals, so that $2b_2\hat{h}$ represents the tidal volume stored on the shoals per unit length, if $2\hat{h}$ denotes the tidal range.

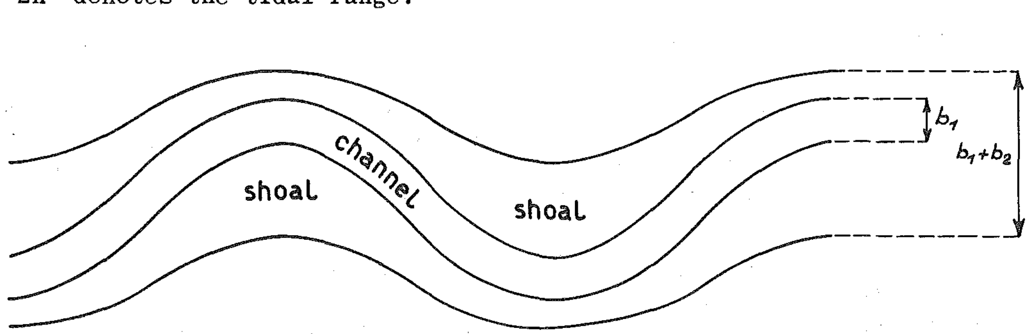


Fig. 10 - Effect of shoals.

The effect of the shoals is a diffusion along the channel, of which the diffusion coefficient can be estimated at

$$D = \frac{\pi}{4} \frac{b_2 \hat{h}}{b_1 a} \hat{v} \hat{x} \cos^2 \varphi$$

where $\hat{v}$ is the amplitude of the velocities of tidal flow.

The above formula for D may be considered as a refinement of the mixing-length theory of tidal flushing [5].

This formula yields values of 10^2-10^3 sq m per sec for the coefficient of diffusion in Dutch estuaries and this corresponds well to the values deduced from the salinity distribution along the estuaries.

We may therefore infer that the storing-basin mechanism is the primary promoter of salt intrusion. This mechanism has the largest scale of the mechanisms considered, viz, a scale of the order of the tidal excursion path (10^4 m).

The above conclusion does not imply that the other diffusion mechanisms are irrevelant. Turbulence, for example, continues the work of the storing-basin mechanism, by a more detailed mixing of the quantities of water discharged by the basins, with the water in the channel passing by. Molecular diffusion finishes the work by making homogeneous brackish water.

BIBLIOGRAPHY

[1] Groen, P. "Contribution to the Theory of Internal Waves." Meded. Verh. KNMI de Bilt, Series B, Vol. 2, No. 11, 1948.

[2] Keulegan, G. H. "Wave Motion." Chapter XI of Engineering Hydraulics, edited by Hunter Rouse, 1949.

[3] Schönfeld, J. C. "De turbulentie in de zout- en zoetwaterbeweging." (with English abstract) (not published), 1950.

[4] Ismail, H. M. "Turbulent Transfer Mechanism and Suspended Sediment in Closed Channels." Proceedings of ASCE, Vol. 77, Separate 55, 1951.

[5] Arons, A. B. and Stommel, H. "A Mixing-length Theory of Tidal Flushing." Transactions of American Geophysics Union, Vol. 32, pp. 419-421, 1951.

[6] Veen, J. V., Dronkers, J. J., Notenboom, W., and Schönfeld, J. C. "Penetration of Salt Water in Tidal Rivers and Their Tributaries in Maritime Canals, and in Ports." Comm 3, Section 2, International Navigation Congress, Rome, 1953.