Mixing and Matching Building Blocks for Commodity Market Models: The Case of Phosphate Market Imbalances

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Abstract (90 words)

This paper advocates and demonstrates a novel approach of modelling: namely to use stock-flow mechanisms, decision-making mechanisms and market-clearing mechanisms to represent commodity markets. The focus is on the process of constructing a quantitative model through mixing and matching of building blocks that originate from different modelling methods: system dynamics, agent-based modelling and economic game theory. The new approach is illustrated using a case study on the phosphate market. Conclusions are drawn on how to take the utmost advantage of modern computational tools to flexibly model problems in commodity markets.

Keywords
Open modelling – Stock-flow – Decision-making – Market-clearing – Phosphate market

1. Introduction

Quantitative models are often used to investigate commodities markets. These models can provide insight into the dynamics and uncertainty that are inherent to these markets (Labys, 1999). Models also provide controllable environments in which future market developments can be explored and policy measures can be tested. Agent-based models are for example used to understand critical materials markets such as dysprosium and neodymium (Riddle et al., 2015). System dynamics models are for example used to scrutinise long term supply of rare earth elements (Kifle et al., 2012), and to investigate the copper market (Kwakkel et al., 2013; Sverdrup et al., 2014). Economic and game theoretic models have been used to investigate the gas market (Growitsch et al., 2014) and the petroleum market (Yang, 2013). These applications describe commodity markets from the perspective of a single modelling method.

Every modelling methodology has its focal problem characteristics. Agent-based methodologies are good at describing a real-world system on the level of intelligent decision-making agents that constitute the system of interest. Emergence of behaviour on the level of the whole system can then be described (Bonabeau, 2002). System dynamics is good at describing a real-world system on the aggregate level as being composed of stocks and flows. System behaviour is generated by accumulation, feedback loops and nonlinear relations (Sterman, 2000). Economic and game theoretic models are used to describe market fundamentals and strategic behaviour between competitors in commodity markets. Game theoretic models often describe oligopolistic markets (Shapiro, 1989). The different modelling methodologies are complementary in describing different elements of commodity markets.

Modelling methodologies can profitably be combined to describe different aspects of commodities markets within one model. Hybrid modelling is proposed as an important way of creating more accurate models (Lättilä et al., 2010). Different types of hybrid models can be build: interfaced, integrated and sequential (Swinerd & McNaught, 2012). Models of commodities markets can in the same way be built up of components that originate from different modelling methodologies. Mining company decision-making behaviour can in a natural fashion be described using agent-based modelling mechanisms. The dynamics of mine capacity over time can effectively be represented using stocks and flows originating from System Dynamics. Market fundamentals such as price formation can be modelled using economic fundamentals and game theory implementations. This work proposes a structured, open modelling process to combine model building blocks originating from totally different single modelling methodologies. An illustration and application of the proposed modelling process to the phosphate market is also given.

In section 2 the methodology of selecting model building blocks, quantitative model construction and quantitative model implementation in a programming environment is described. An elaborate illustration of the proposed modelling method on phosphate market imbalances is provided in section 3. Exemplary results are provided at the end of section 3 to demonstrate suitability and validity of the model results. Implications for quantitative modelling methodology are discussed in section 4. Conclusions on how open-minded quantitative modelling can serve to make the most of modern computational possibilities are provided in section 5.

2. Methodology

For this research, a quantitative model has been developed from scratch. The steps of this quantitative modelling process are shown in Figure 1. There are four modelling steps: conceptualisation, requirements analysis, model construction from building blocks and lastly model coding and model parameterisation. The conceptualisation provides a problem-specific description of the commodity market in question that serves as the basis for quantitative model construction. Via requirements analysis a list of model building blocks is derived from the essential conceptual market aspects. The building blocks found are typically not combined into one working model. Coding and parameterisation of a hybrid model also calls for special attention.

The stance of this work relative to modelling paradigms is non-conformist. It is neutral in the sense that no pre-analytical choices have been made on what modelling methods to apply. This has provided the liberty to analyse the problems of commodity markets without resorting to single methods and single thinking frames. It is hypothesised that such an open modelling process can strengthen the correspondence between the quantitative model and the real world commodity market it represents. Under circumstances the model more naturally fits the commodity value chain than do single method commodity market models. This section provides the background to the different steps of the open modelling approach that is advocated.

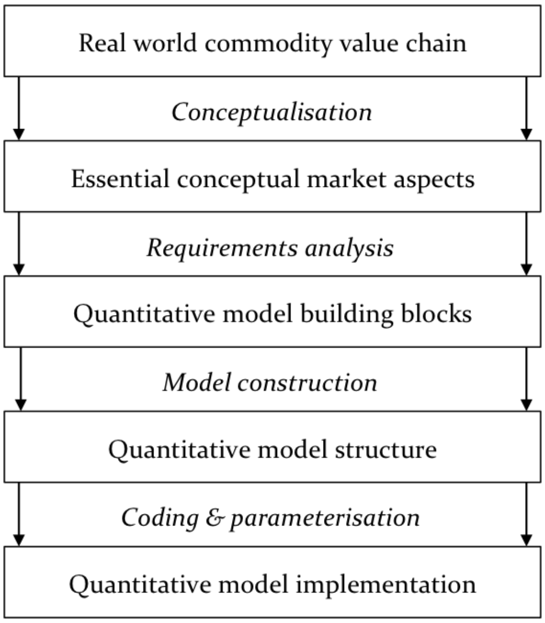

Figure 1 – Hybrid modelling steps

2.1 Conceptualisation and requirements analysis

The first step of the modelling process has been to translate the commodity value chain in question into a conceptual model. Such a conceptualisation belongs to standard practice for many fields of quantitative modelling. Conceptualisation is frequently both used to describe a system of interest as well as to select a problem of interest (cf. Labys, 1999; Sargent, 2013). A conceptual model can be a schematic representation as well as a verbal representation of the system of interest (Schlesinger, 1979). The conceptualisation phase of the proposed open modelling process can be performed in numerous ways.

Loosening the method of conceptualisation is a considerable change compared to single modelling methods. Most modelling methodologies have prescribed ways of conceptualising a system of interest. Within a single modelling method there are clear-cut restrictions on what concepts are allowed as part of a system conceptualisation. In developing System Dynamics models causal loop diagrams are often used to identify feedback loops in the system (Sterman, 2000). In developing agent-based models actors are identified that together compose the system of interest (van Dam et al., 2013). Most modellers do recognise that their preferred modelling methods are not suitable to model every problem within every commodity market. The conceptualisation of the proposed open modelling approach allows for the inclusion of concepts of different natures.

The second step of the process has been to systematically derive requirements that the quantitative model must satisfy. These requirements were mainly derived from the conceptualisation of the commodity value chain. The model is also assimilated to be easy to use to the analyst. Other requirements were that the model should reflect the dynamics and uncertainty that are inherent to commodities markets. The time scale used makes the problem of interest apparent and the model is able to show the effects of policy measures aimed at alleviating the policy problem.

To come to a set of quantitative model building blocks, the requirements were analysed. The line of reasoning that was followed is shown in Figure 2. Every requirement was first translated into a basic model aspect: the model should describe this or that concept from the conceptualisation embedded in the commodity market in this or that way. This results in a quantitative model characteristic. It is clear that this reasoning calls for difficult interpretations by the modelling analyst. There are always multiple ways to identify these model characteristics. Quantitative model building blocks then needed to be found that fulfilled the stated quantitative model characteristics.

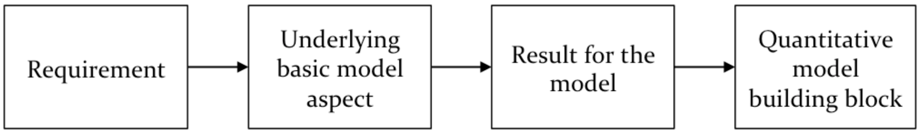

Figure 2 - Line of reasoning used to find quantitative model building blocks

2.2 Model construction

Quantitative model building blocks have been merged into an object-oriented simulation model (Joines & Roberts, 1999). The nature of the quantitative model building blocks is diverse. Yet, all model building blocks were described as objects or as interactions between objects. Actors active in the commodity value chain, both sellers and buyers, were interpreted as objects having attributes and exhibiting activities. Similarly a set of stocks and flows describing mine capacity over time was interpreted as an object. Game theoretic market equilibrium and price formation techniques were interpreted as interactions between the objects representing buyer groups and suppliers on a market. Dependent on the exact conceptualisation of the commodity market a novel interpretation of model building blocks was made.

The object orientation of the model has served to flexibly model commodities markets. It has allowed for the integration of all building blocks into one working model. This integration of building blocks can be done in numerous ways. It amounts to specifying relations between the objects representing model building blocks. Actor objects can for example be attributed a stock-flow structure. An actor object can use an equation-based reserve object holding mineable commodities to produce from. And an actor object can interact with buyers through a market mechanism.

The illustration of the modelling process applied to the phosphate market in section 3 exemplifies a number of object representations of commodity market concepts and their mutual relations that can be interpreted to hold in general. Numerous examples from modelling methods that are not taken up in the illustration can also be integrated into an object-oriented hybrid model: discrete event simulation, material flow analysis and other types of optimisation.

2.3 Model coding and parameterisation

To make the most of the combination of model building blocks from different modelling methodologies a flexible programming environment was used. Python was chosen to implement the model (Rossum, 1995). The language is often used for scientific computing purposes. In Python it is natural to build up a simulation model in an object-oriented manner (Goldwasser & Letscher, 2014). The programming language is fully open source and there are numerous packages that can be integrated to use all kinds of scientific computing applications. The language provides a suitable tool for quantitative model implementation using different building blocks.

The implementation and simulation of the hybrid model in a Python environment consisted of three main steps: (i.) declaration of objects with attributes and methods, (ii.) instantiation of objects and their mutual relations with parameters, (iii.) simulation as sequencing of interactions between objects. To execute these steps pure Python programming code was written. By varying parameters in the instantiation of objects and varying simulation characteristics it has been possible to adapt the model structure to different problem definitions. An operationalized illustration of the open modelling process as applied to the phosphate market is shown in the next section.

3. Illustration

The novel approach to quantitative modelling is illustrated in a case study on the phosphate market. Section 3.1 introduces the problem of phosphate scarcity from the point of view of importing countries. From section 3.2 on, the hybrid modelling process is then started with a conceptualisation of the phosphate market value chain. Section 0 provides the requirements analysis that was performed to come to a set of quantitative model building blocks that would together describe the phosphate market. Section 3.4 to 3.5 discuss model construction, model coding and parameterisation and exemplary simulation results respectively.

3.1 Phosphate scarcity

Phosphorus is essential for all life on this planet. The majority of the demand for the element is due to its use as an artificially added macronutrient in the form of fertilizer (Cordell & White, 2014). It is needed to provide for high productivity yields in modern agriculture: a key part of the Green Revolution that took place in the 20th century. There is no substitute for phosphorus as a macronutrient, as it is essential for biological cell formation. Demand for fertilizer grows steadily with the growth of the world population (FAO et al., 2015). Yet, phosphate rock reserves are highly concentrated: only a few countries worldwide have considerable reserves of the commodity.

Phosphate rock is only found in a small number of countries: Morocco, China, Algeria, Syria and a few others have considerable reserves, in order of decreasing reserve size (USGS, 2016). Higher-grade reserves are emptying in the U.S. and in China, while Morocco has enough phosphate rock for a number of decades to come. It will thus even increase its market share in the coming century (Cooper et al., 2011). There are genuine geo-economical concerns about dependence from unstable countries for essential phosphate resources. Policy makers in the European Union and other importing regions should point their interest towards this problem (Rosemarin, 2004; van der Weijden et al., 2013). Exporting countries might very well be in the position to use their phosphate resources in a geo-strategic way through trade restrictions and strategic market behaviour.

3.2 Conceptualisation

The phosphate value chain is described by a set of ten market aspects: a verbal description serves as the system conceptualisation. These market aspects are phrased such that together they provide a full overview of the phosphate market that is suitable to build a quantitative model. The phosphate value chain is shown in Figure 3. The value chain is build up of a number of chemically engineered products that include standardised grade phosphate rock, phosphoric acid and finally a number of different phosphate fertilizer products including mono-ammonium and di-ammonium phosphate as well as fertilizers in which phosphate is mixed with nitrogen and potash (PotashCorp, 2014; Rutland & Polo, 2005). All these phosphate fertilizers are then traded in wholesale and retail markets all over the world.

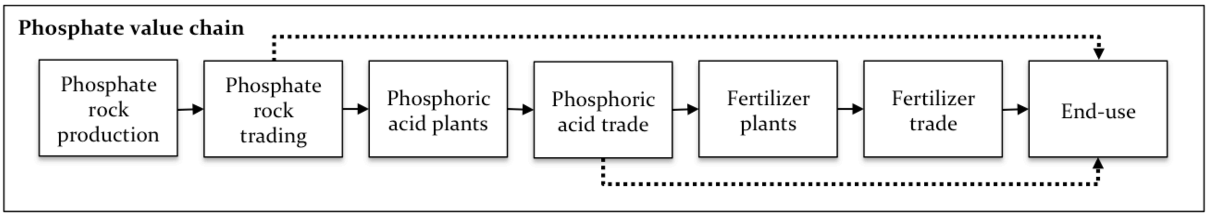

Figure 3 - A simplified view of the phosphate value chain (based on PotashCorp, 2014)

The full set of ten market aspects is shown in *Table 1*. They describe the different parts of the value chain through a number of observations that are sometimes characteristics of numerous other resources too. Exhaustibility of reserves, capacity and investment decisions are typically also seen in the markets of other resources. Other characteristics are highly specific to phosphate, these are: (i.) the very uneven distribution of resources over different countries, (ii.) large short-term inelasticity and the lack of a substitute in its agricultural use, (iii.) the huge implications that supply interruptions can trigger due to its connection to food security.

The uneven distribution of phosphate rock resources is seen from the low number of countries that have considerable reserves. The only country having large reserves of phosphate rock is Morocco (72% of the total according to USGS, 2016). Other countries having considerable reserves include Jordan, Algeria and Syria as well as the United States, Russia and China. Some of these countries however empty their reserves much faster than others – mainly United States and China have a large relative production. Exports of phosphate rock are even more concentrated, Morocco, a few other countries in the Middle East and Russia are the only other major exporters.

Demand for phosphate fertilizers comes almost fully from agricultural applications, 90% of phosphorus flows are used in agriculture (Heckenmüller et al., 2014). Phosphate is essential for cell growth and thus short-term price-elasticity of demand is very low (von Horn & Sartorius, 2009). The buffering capacity of the soils to which phosphate is regularly applied can however be used to delay the application of fertilizers for a few seasons strategically in the case of temporary high prices. There is thus a price plafond above which demand will temporarily retract. In the 2009 fertilizer price spike this plafond was reached at about 1200 $/ton of di-ammonium phosphate (Mew, 2016). In the long term the lack of a substitute again causes demand to be inelastic in size.

Interruptions in the supply of phosphate rock and phosphate fertilizers can cause severe difficulties due to the large productivity gains that are reached by applying fertilizer in the right rates. The combination of projected strongly growing populations and growing per capita demand for feedstock makes the need for a highly productive agriculture ever larger. Without fertilizer such a highly productive agriculture is fairly difficult to reach. Fertilizer demand is projected to grow strongly (FAO, 2015).

On the phosphate market there is perspective for export restrictive measures being implemented by exporters of phosphate rock and phosphate fertilizers (de Ridder et al., 2012). Because the phosphate market is a strongly oligopolistic market, these export restrictions can do a lot of harm to phosphate importers. Under an export tax imposed by a large exporting country, world rock and fertilizer prices would rise and trade from the exporting country in question would contract by much (Gandolfo, 2014; Latina et al., 2011; Piermartini, 2004). This has been seen in the case of China, which in 2009 strategically used an export tax of about 50 $/ton on phosphate rock to grow a fertilizer processing industry within China (Persona, 2014). Exporting country governments do not only let their power be felt on the market through the imposition of trade restrictions. Phosphate companies are frequently also in state ownership. There are possibly more market distortions. Lastly, instability of some key phosphate exporting countries such as Syria, Tunisia and Algeria shows that there is also a possibility of severe production restrictions.

Table 1 - Conceptualisation of the phosphate value chain through ten market aspects, this analysis has been based on approximately 30 references that are supplied in Keijser (2016)

Market aspect #1: Phosphate rock is an exhaustible commodity (*exhaustibility*) and its reserves are distributed highly unevenly across a small number of countries (*uneven distribution).* High prices trigger exploration for more reserves, becoming available with delay (*explorative delay*).
Market aspect #2: Phosphate rock mines operate at strongly differing cost rates (*differentiated mining costs*), furthermore they decide upon mining capacity recurrently – looking at market price and technology available – with new capacity only becoming available after a significant delay of about 5-10 years (*capacity decision and delay*).
Market aspect #3: Trade in phosphate rock is governed by bilateral contracts between miners and fertilizer companies (*bilateral contracts*), through regular negotiations these contracts are renewed (*contract negotiations*).
Market aspect #4: Phosphate rock mining companies increasingly also control phosphate rock processing to phosphoric acid and phosphate fertilizers (*vertical integration*).
Market aspect #5: The value chain of phosphate rock and phosphate fertilizer is somewhat technically complicated due to the large number of different fertilizer products (*numerous fertilizers),* as well as due to differences in grade ores and contaminations that need to be standardized to provide for trading (*beneficiation for trade*).
Market aspect #6: Trade in phosphate-containing fertilizers is governed by bilateral contracts between fertilizer companies and fertilizer importers (*bilateral contracts*). Trade takes place in a whole range of products including phosphoric acid, di-ammonium phosphate/mono-ammonium phosphate and nitrogen-phosphorus-potash-mixes via importers and subsequently wholesale and retail traders (*numerous fertilizers, numerous traders, numerous regional markets*).

Market aspect #7: End-use demand for phosphate rock and phosphate fertilizers is almost fully in the agricultural sector *(agricultural end-use)*, demand is price-inelastic in the short term, price elastic in the medium term due to soil buffering capacity and inelastic in size in the long term due to the lack of a substitute for phosphorus in growing feedstock, food security is thus at risk *(demand inelasticity and food security connection).*
Market aspect #8: Technological possibilities of recycling phosphorus flows are many and their application is recognized as an essential policy measure for sustainable phosphorus use (*potential of recycling*); their practical, cost-efficient application is however still lacking (*lack of practical application*).
Market aspect #9: Both the markets for phosphate rock and phosphate fertilizers have an oligopolistic structure *(oligopolistic structure),* oligopolistic behaviour is observed as well as an increasing influence of state ownership *(oligopolistic and state owner behaviour).*
Market aspect #10: The phosphate market is characterized by a number of export restrictions due to geopolitical events and state market measures *(presence of export restrictions)*; in the future export restrictive measures might very well be used by exporting countries for divergent reasons *(perspective of export restrictions);* the effect of these export restrictions are both felt on the domestic markets as well as on the global market *(effect of export restrictions).*

3.3 Requirements analysis

The commodity market conceptualisation is used to analyse the requirements that the quantitative model fulfils. Both the dynamical nature and the uncertainty inherent to the phosphate market are explicitly modelled. Strongly connected to the dynamical nature of the market is the time scale on which the model is simulated. A relevant time scale for long-term analysis of the phosphate market is 85 years. The explicit effects of trade and production restrictions can be investigated on a time scale of 15 years. The model has an empirical basis to provide applicable policy recommendations: it is initialized using empirical data on production, trade and consumption. The model is implemented in a flexible and modular programming environment and thus provides ease of use to the analyst.

The quantitative model describes the phosphate market: it represents mines and processing facilities as well as trade and demand for rock and fertilizer. It also describes the market's hypothesised oligopolistic structure and the resulting oligopolistic behaviour. On top of this, a suitable quantitative model is able to describe export and production restrictions and its effect on both exporting and importing countries. Both generic essential market aspects as well as those applied to oligopoly economics and restrictions have been described in the conceptualisation. To provide policy recommendations the effect of the importer policy measures of recycling and strategic buying is modelled.

Quantitative model building blocks have been found that indeed have the needed function. An example of the reasoning used to find model building blocks is applied to the criterion of exhaustibility, uneven distribution and explorative delay – market aspect #1. The underlying basic model aspect is that a reserve object needs to be modelled including a reserve size and reserve increases through exploration. In conclusion, it is found that a model building block to be included is a dynamic reserve of phosphate rock. By continuing such a systematic investigation the model building blocks shown in Table 2 are found.

Table 2 - A list of phosphate market model building blocks

- ✓ A dynamic reserve and resource base of phosphate rock
- ✓ Decision-making mechanisms for mining companies on exploration and capacity
- ✓ The functional forms that provide the translation of production into phosphate rock, and P_2O_5-flows on the market (i.e. describe the technical value chain)
- ✓ Regional market-clearing mechanisms for both the phosphate rock and fertilizer markets (on which domestic and imported production compete) that take oligopolistic structure and possible resulting behaviour into account
- ✓ Decision-making mechanisms for market parties (possibly state-owned) to decide on when and to what extent they use their market power (including connections to the relevant market clearing mechanisms)
- ✓ Decision-making mechanisms for exporting countries governments to decide when they use what export restrictive measures (including connections to the relevant market clearing mechanisms)
- ✓ The market network structure to be able to describe transportation costs
- ✓ Functional forms to describe the demand of phosphate rock and fertilizer products on all regional markets
- ✓ Functional forms to describe the usage of recycling as well as its relative costs to employing phosphate rock for fertilizer products

3.4 Model construction and model coding

The quantitative model building blocks have been turned into a coherent quantitative model structure as shown in Figure 4. This model structure represents a strongly conceptual view of the global phosphate value chain consisting of rock production in mines and processing facilities producing P_2O_5-products, the aggregation of different phosphate fertilizers. The different objects of which the phosphate market model consists are shown in Figure 5. The model consists of:

- **Objects** representing reserves that are mined, companies mining phosphate rock and making decisions on capacity utilisation and investment in mining and processing capacity, and buyers that have certain aggregated characteristics on price and demand
- Objects representing **stock flow structures** representing two ageing chains – per company – for mining and processing capacity respectively, constructed, in operation and decommissioned due to end-of-lifetime unprofitability (Pruyt, 2010; Sterman, 2000)
- Interaction between objects in Cournot-Nash **market clearing mechanisms** representing non-cooperative oligopolistic market behaviour implemented using a quadratic optimisation solver for both the rock and the fertilizer market (grounded in economic theory, Kolstad & Burris, 1986; Kolstad & Mathiesen, 1991; Shapiro, 1989)
- A set of **generic control scripts** creating all objects and their relations, keeping time of the different parts of the model is implemented – stock flow structures run using a numerical integrator, while company decision-making behaviour and market clearing mechanisms are based on a quarterly schedule – and lastly sequencing object interactions, the sequencing of procedures is shown in Table 3.

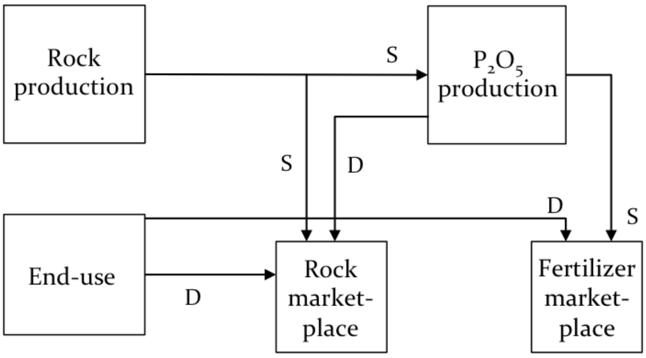

Figure 4 – An overview of the aggregate model structure, flows described include supply (S), demand (D), both an aggregated model and a disaggregated model are later used to describe the global phosphate market respectively the phosphate market of Africa, Europe, the Middle East and Russia

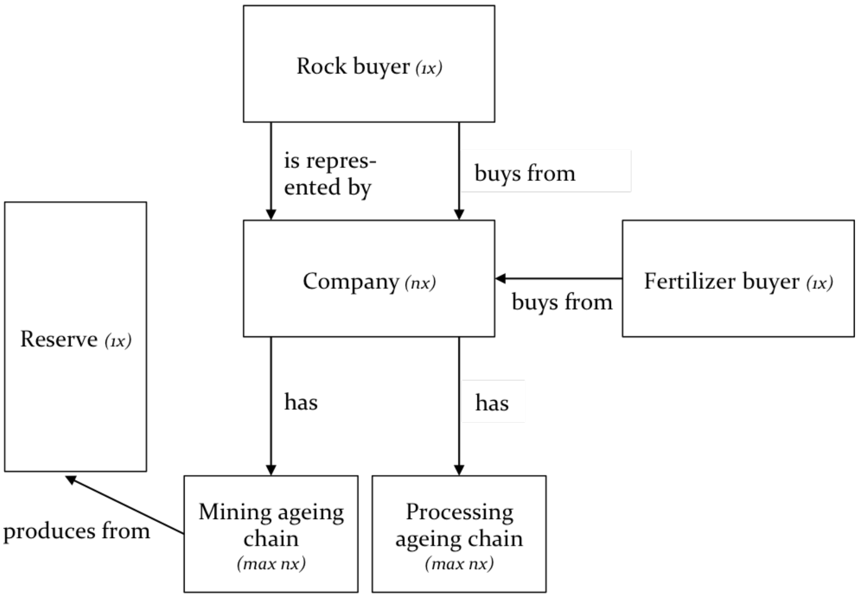

Figure 5 - Quantitative model object structure of the aggregated global phosphate market model

Table 3 - Sequencing of model procedures in the phosphate market model

1. Run the stock-flow structures of mines and processing facilities for a quarter
2. Calculate resulting production of rock and P_2O_5
3. Store the produced rock and P_2O_5
4. Add amounts of rock produced from the reserves to its cumulative output
5. Calculate each company's extraction and processing costs
6. Clear the market for P_2O_5 (find imperfectly competitive equilibrium)
7. Update capacity utilization and investment and divestment in P_2O_5-producing facilities
8. Determine demand for rock that will be used next quarter
9. Clear the market for rock (find imperfectly competitive equilibrium)
10. Update capacity utilization and investment and divestment in mines
11. Update fertilizer demand for next quarter

For practical and computational reasons it has not been possible to fulfil the requirements for a quantitative phosphate market model fully. Firstly, some different aggregations are used. Fertilizer products have been aggregated into P_2O_5-containing products, and multiple smaller traders into one conceptual buyer. Every component has been implemented in such a way that the model is implementable within a reasonable amount of time and effort – thus fulfilling the condition of ease of use for the analyst. All this makes sure that the model is computationally tractable.

The model structure contains a dynamic reserve of phosphate rock, although its size is not explicitly calculated. Exploration is not decided upon, mining companies can simply mine at a certain extraction cost – that increases when cumulative output from the reserve increases. Mine capacity decisions are made, effectuating with a delay. Rock flows are functionally translated into P_2O_5-flows. Market price always reflects the market power of suppliers – they do not separately decide upon this. Ownership of companies varies, some companies are of an integrated nature and others are specialised. Vertical integration dynamics over time can however not easily be simulated in the current implementation. The total market share of each type of company however does differ due to diverging investment decisions. Both rock and fertilizer demand is represented in demand curves: these curves are thus a conceptual substitute for seller-buyer negotiations. The dynamics of negotiations and inflexibility of contracts cannot be fully taken into account.

To create all objects and their relations and to govern all object interactions a considerable number of Python packages have been used. These include: Scipy, Numpy and Pandas for data structure, calculation and numerical integration purposes (Numpy Reference, n.d.; Pandas, 2016; SciPy Reference, 2016), a Python implementation of stock-flow structures (Herman, 2014), a symbolic mathematics package for the Cournot-Nash formulation, SymPy (Sympy Development Team, 2016), a convex optimisation package for implementing and solving the Cournot-Nash equilibrium finding problem, Cvxopt (Andersen et al., 2015), a design of experiments package for generating Latin Hypercube Sampling parameters used in running the model, pyDOE (Lee, 2014), and lastly the Python-integrated multiprocessing package for running the model in a parallel core setting speeding up runtime by roughly a factor twelve on the computer setup used.

3.5 Model parameterisation

The quantitative phosphate market model can be used to experimentally simulate both the development of the global phosphate market throughout the rest of the 21st century as well as the implications of export and production restrictions for phosphate importers. The use and effectiveness of importer policy measures can also be investigated. For these purposes, two different model instantiations are used.

The first instantiation describes the global phosphate market in an aggregated way. There is one regional market and there are between five and seven companies active in the phosphate value chain. These companies are of three different types: mining companies, processing companies and integrated companies having both mines and processing facilities. Model parameters include: global size of rock production of about 220 million tonnes of rock yearly

(USGS, 2016) and phosphate fertilizer demand of about 45-50 million tonnes (FAO, 2015). This model instantiation is used to get to know endogenous phosphate market development in the long term to 2100. Determinants of phosphate scarcity can be explored.

The second instantiation describes a collection of regional phosphate markets in a disaggregated way. There are three regional markets in the model – South America & Africa, Europe and lastly the Middle East, Russia and India – on each of these markets there are two or three companies active. Model parameters are set such that the size of fertilizer demand is approached in each of the regional markets, in total 50 % of global phosphate demand is represented in this model instantiation. To reach approximate market balance rock production is scaled so that demand fits supply roughly in the phosphate rock market. Processing facilities as well as mines are divided over the regions of Africa, Europe and Middle East/Russia in the same proportion as empirically observed. This model instantiation is used to investigate the implications of export and production restrictions on rock and fertilizer prices on the different regional markets as well as on trade dynamics and regional phosphate scarcity.

3.6 Exemplary model results

This section provides insight into the types of experiments that can be run with the quantitative model developed. This is done by showing the results of a set of experiments of 2000 runs of the model that was performed to investigate long-term plausible global phosphate market development.

The development of the global phosphate market over time can be scrutinised by looking at five variables over time. These are: rock price, fertilizer price, mining capacity, processing capacity and lastly ratio of demand and supply of phosphate. Together these variables show the extent to which the phosphate market is in balance over time – the results are shown in Figure 6. From a further analysis of the model results it can be concluded that extraction cost acts as a rising price floor in the market, marginal cost overcharge is about 100-200 $/ton. It is not plausible that phosphate reserves will empty before the end of the 21st century. The model results show that a cumulative output of 9 to 35 billion tonnes of phosphate rock between 2015 and 2100 is plausible, compared to currently projected reserves of over 70 billion tonnes.

The most important determinant of the balance between demand and supply is the sufficiency of investment in mine capacity and fertilizer production capacity. If companies only slightly overreact to phosphate scarcity then mine capacity and fertilizer production capacity can be doubled over the 21st century. In this case, phosphate scarcity is fended off. By dissecting the decision-making process of companies active in the phosphate value chain, attention was drawn to the fact that companies first need to see phosphate scarcity through rising demand over time after which investment can take place. Close attention should thus be paid to the influence of the investment decision-making process of companies on market balance over time. In this way it has been demonstrated that the model can provide insights by combining model building blocks from seemingly different modelling methodologies.

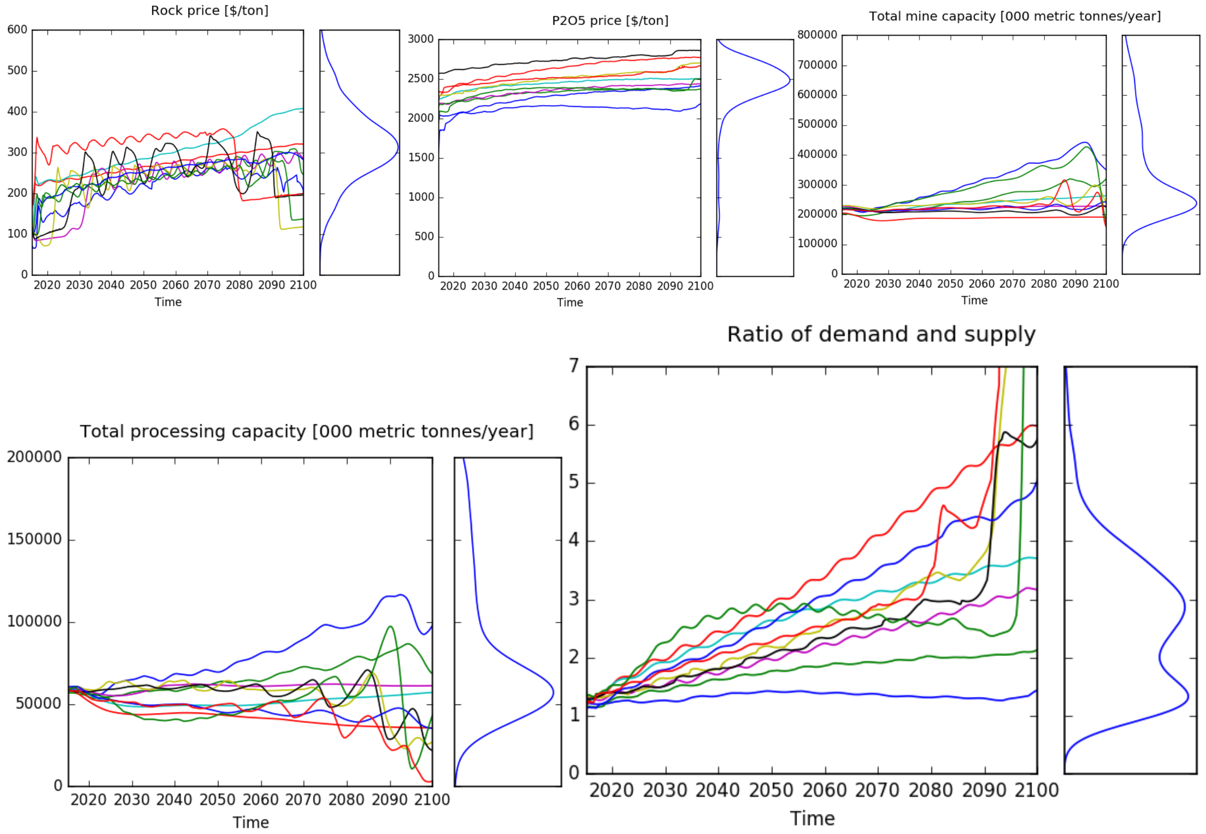

Figure 6 – Five key global phosphate market variables, from left to right and from top to bottom: Rock price [$/ton], Phosphate price [$/ton], Total mine capacity [10^3 tonnes/year], Total processing capacity [10^3 tonnes/year], Ratio of demand and supply of phosphate [dimensionless]. Ten representative dynamic scenarios from 2000 runs are shown. Kernel density estimates show distribution of end values.

4. Discussion

The phosphate market model constructed as part of this modelling research has clearly shown that quantitative modelling of commodity markets through mixing and matching model building blocks can strengthen commodity market understanding. The quantitative model has been of an integrated hybrid design – company agents were represented having a rich internal structure of stock-flow structures (one of the types named by Swinerd & McNaught, 2012). Going further, agents were aggregated into sets of active market players on two types of dynamic markets – the phosphate rock and phosphate fertilizer market. The markets were cleared every quarter to determine rock and fertilizer prices.

This quantitative model design has provided for an open mind-set in modelling of commodity markets. More open even than by just building an agent-based model and a system dynamics model of a commodity market next to each other. The quantitative model of the given illustration described the phosphate market in a natural way. Instead of prescribing conceptual units that describe only parts of commodity markets such as feedback loops or agents, the model described multiple objects having a direct conceptual counterpart in the real-world market. This open-minded type of modelling complements single-methodology studies of commodity markets. As has been demonstrated in the case study of phosphate, mixing and matching model building blocks can gather meaningful insights on commodity markets.

The programming environment in which the quantitative model has been implemented – pure Python with a number of further packages – has provided a large degree of liberty in mixing and matching model building blocks. There are a number of reasons for this. Firstly, the programming environment is numerically fast. Lastly, there is no stringent limit on the applications and packages that can be implemented in an open-source language. The programming environment is also relatively easy to use for modellers experienced in thinking in object-based terms. The object-oriented structure of the model also allows for rapid creation of multiple versions of the same quantitative model describing different parts of the same commodity market. Modular design of quantitative models minimises the investment of labour needed to create models describing different commodity markets.

Three key difficulties remain in the implementation of quantitative models through mixing and matching of building blocks. Firstly, there is no easy step-by-step process for modelling by combining model components from different paradigms. The lack of formalisation of this step-by-step process can cause a delay in the typical length of a modelling project. Translation from the standard modelling cycle, consisting of conceptual analysis, formal analysis, verification, validation, conclusions, should be logically continued when mixing and matching different model building blocks. This sometimes calls for improvisation in assimilating models. There are no clear-cut choices to model commodity market aspects in a quantitative model assimilated from scratch. Building block choices must be justified by elaborate reasoning. The work presented has shown an outline for the translation of the standard modelling cycle to design models from scratch by combining building blocks.

Quantitative model building blocks can only be coded in formalised ways: the computer needs to be told exactly what it needs to do. The quantitative model implementation has shown a number of possible formalisations of model building blocks as programming objects. The proposed way of working allows for model formalisation. The correspondence between conceptual model and formalised programming model should still be proven by extensive verification and validation. Existing validation methods can be combined to provide the same level of trust in the model results. There are also time synchronization problems between different model parts. Every modelling method has a different way of coping with time. Some methodologies implement continuous conceptions of time others use discrete conceptions. A last difficulty in model formalisation is the lack of visualisation of model structure and model behaviour. Off-the-shelf modelling applications have visualisation features of both structure and behaviour available. When building a quantitative model in a generic programming environment visualisation features are not always available. This makes it important to conscientiously think about how to present the model and its results.

Lastly, the breadth of modelling methodologies that can be applied within such a flexible environment can well transcend the boundaries of the skill set of the analyst. The analyst ideally has experience building different types of quantitative models. Programming skills are also needed to be able to assimilate the quantitative model from scratch. Ideally, the analyst has a large number of model building blocks available to choose from. If a modeller with limited experience within a single methodology would build a quantitative model according to the same modelling process, a model comprised of only similar building blocks might be the result. The proposed way of working is highly dependent upon earlier experience with building quantitative market models.

5. Conclusion

In conclusion, there is room for improvement in modelling the multi-disciplinary policy problems of commodity markets. In some cases it is still far better – and foremost more practical – to use a mono-method software package to model these markets. In other cases quantitative models can improve on reflecting the diverse nature of the problems of commodity markets. This paper has demonstrated the merit of the proposed approach in a case study on the phosphate market. Mixing and matching model building blocks originating from system dynamics models, agent-based models and game theory models produces a quantitative model describing the market in a natural way. The quantitative model that has been build has a highly object-oriented structure and – allowing for parameter changes and a few structural changes – it is interpretable as describing other commodity markets. In this way, the next case study of mixing modelling methods to describe commodity markets might take up a quantitative investigation of the potash market. For, the potash market and the phosphate market are highly comparable. By building more models for more case studies, the effort inherent to mixing and matching model blocks can be decreased over time.

Some of the problems of commodity markets do not stay within the boundaries of well-defined modelling methodologies. Instead, they cross inter-methodology lines. Commodity markets consists of both feedback mechanisms and decisions made by intelligent actors. They consist of both accumulations in dynamic stocks and of man-made institutionalised structures that we call markets. The current, modern possibilities of general-purpose programming languages such as Python, can strengthen our computational models. Some multi-disciplinary modellers then need to invest in strengthening the methods of quantitative modelling that are available as well as invest in strengthening the availability of visualisation of programming simulation models. Intricate and highly complex policy problems as the problem of phosphate scarcity can be analysed in new and eye-opening ways.

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