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Fatigue predictions and scatter

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Abstract: Different aspects of fatigue design problems are indicated and uncertainties are listed. Scatter as observed in many laboratory studies is analyzed. It is argued that scatter of crack initiation and crack growth are different issues. Various sources of scatter are discussed and illustrative examples are presented. Comments are given on statistical distribution functions, scatter under Variable-Amplitude loading, and scatter in service. The discussion touches upon the meaning of experience on scatter of laboratory test series for practical problems.

List of symbols

CA	Constant-Amplitude
da/dN	crack growth rate
N	fatigue life
$N_{0.5}$	fatigue life until a crack of 0.5 mm
P	probability of failure
R	stress ratio (S_{max}/S_{min})
S_a, S_m, S_{max}, S_{min}	amplitude, mean, maximum and minimum of stress
VA	Variable-Amplitude
σ	standard deviation

INTRODUCTION

In practice fatigue failures occur in various parts of a structure. The consequences can be either unacceptable in view of possible fatalities (e.g. by explosions), or undesirable for reasons of economy or inconvenience. It is the task of the designer to develop structures which can be expected to be fatigue resistant. In other words, there is an issue of "designing against fatigue". Two different approaches with some variants can be indicated:

1. Design for an infinite life.
    1a: fatigue crack initiation should not occur.
    1b: existing cracks or material defects should not grow.
2. Design for a finite life, crack initiation and growth is accepted.
    2a: cyclic loads are constant.
    2b: cyclic loads have some variable character.

The first option is obviously a matter of threshold problems. For an "infinite" life, all load cycles in service should induce stress cycles below the fatigue limit, or ΔK values below some threshold level. Actually, if we say that we want an infinite life, the better statement is that we want no signs of fatigue in the useful life of the structure. This is not true for the second option. Let us consider some typical cases.

(i) Motor car engine: cracks should not occur, crack growth is of little interest. The number of cycles in service will be large.

(ii) Pressure vessels of welded steel. Initial flaws and defects in the welds are to be expected. Crack growth must be considered. The number of cycles in service will be limited, the variability of the load cycle may be small.

(iii) Aircraft wing structure. A finite life is acceptable, provided that the structure is damage tolerant. Both crack initiation and propagation are significant. The load spectrum represents a highly variable amplitude.

The last example is the most complex one. A survey of aspects involved is given in Fig.1. It clearly illustrates that predicted results depend heavily on a lot of influencing factors. Although the picture is based on aircraft problems, it can easily be translated to applications on various types of structures. Fig.1 will not be discussed in detail. *It is primarily presented to indicate that the output results (i.e. the predictions) are subject to various uncertainties of different origin. Part of the uncertainties is due to limited information (e.g. on the load spectra), to a limited accuracy (e.g. of the stress analysis) and to limited knowledge (e.g. the prediction methods). Another part of the uncertainties is due to statistical variations.*
Statistics will apply to:

- variations of the geometry of the structure (tolerances on dimensions)
- variations of the production quality (e.g. welding)
- variations of the material properties
- variations of the surface finish
- again variations of the load spectra (e.d. different utilizations of the structure).

Due to our limited knowledge about accurate predictions of crack initiation life and crack growth, fatigue predictions are often verified by tests. In the best case, such tests are realistic simulations of what happens to the structure in service. Such tests will bring us in a much better position to judge future fatigue problems, but a 1:1 relation between test result and service experience can not be expected. As a consequence, safety factors are adopted to cover possible uncertainties and variabilities. Because fatigue of structures can lead to hazardous situations, there are official safety regulations imposed by national or international agencies. Such requirements are supposed to cover "scatter", although it is not always clear which kind of scatter. The problem setting is rather complex.

The prime purpose of the present paper is to illustrate experience on scatter of fatigue properties with an attempt to trace the origins of scatter. Most of our experience on scatter is born in the laboratory. This experience should be evaluated to see the relevance for practical problems.

The paper starts with a brief summary of statistical descriptions of fatigue properties and possible goals to have such descriptions. Secondly, observations on scatter as observed in laboratory tests are presented. Some sources of scatter are indicated to explain the observations. It is followed by aspects of scatter under service conditions, and a discussion on some problems of dealing with scatter in practice. Some conclusions summarize the present analysis.

SCATTER OF FATIGUE PROPERTIES

The fatigue phenomenon has a long lasting reputation of considerable scatter of fatigue lives in nominally similar fatigue tests. It has drawn a lot of attention in numerous publications. In several studies the statistical aspects were the main subject. Large numbers of experiments were then carried out. As an example Tanaka et al. [1] carried out CA tests at three different amplitudes with 200 tests at each stress level. Several studies can be found in the literature, where large test series were carried out in order to reveal the statistical behaviour or fatigue properties (see e.g.[2]). In other experimental investigations the main topic was to determine fatigue properties of a material, e.g. an average S-N curve. In such test series the occurrence of much scatter is quite a nuisance, because large numbers of experiments are necessary to determine the properties with some statistical confidence. An illustrative example is presented in Fig.2a. Such a collection of data points is a nightmare for every experimentalist. The operator of the fatigue machine starts doubting about the quality of his experiments. It is even difficult to draw some average S-N curve through the data points. Usually, it is done by eye. Gatto [4] proposed the "rearrangement method" for this purpose. The data $(S_{a,i},N_i)$ are separated in one series of decreasing S_a-values ($S_{a,j}$), and another series of increasing N-values (N_j). The data are then paired and plotted as ($S_{a,j}$,N_j) values. The rearranged results of Fig.2a are shown in Fig.2b, where an average curve could indeed be drawn by eye. The curve is also presented in Fig.2a. It appears an elegant method to indicate the average trend, and Gatto has offered some statistical arguments to justify the procedure. Nevertheless, there are a few obvious and pertinent questions. The rearrangement method can introduce a bias at the beginning and the end of the S-N curve. There is another tricky consequence. If one of the test results should be expected to be completely out of order, its S_a and N value become normal results after the rearrangement treatment. It emphasizes that the fracture surfaces of all specimens should always be examined, because a very low N-value can be due to crack nucleation at some incidental surface damage.

Considering results as shown in Fig.2, the average S-N curve seems to be the most reliably defined information of a large and expensive test series. A more extensive statistical analysis can lead to a description in terms of probability density functions, see Fig.3:

- the distribution of the fatigue life at a constant stress level: $p(N_S)$
- the distribution of the fatigue strength at a constant life: $p(S_N)$

If these functions are available, predictions on confidence levels of statistical predictions can be made. If many results can be "pooled" to a single population, the statistical significance can be improved. However, it requires assumptions about the S-N relation and the statistical properties. Since this information can not be obtained in a rational way, the analysis acquires the status of an intelligent statistical guess.

An important question is: Why do we need statistical information about fatigue properties ? What are we going to do with it ? In general terms some goals are easily recognized:

(1) Comparison of fatigue properties of different design options.
(2) Allowable stress levels, required for design purposes.
(3) Allowable inspection periods if fatigue problems can arise.

The first case is related to questions whether material A is better than material B, or similar questions about surfaces treatments and other design and production variables. The question then is that A should be better than B, not only on the average, but in most statistically possible cases. Confidence levels for low probabilities of failure are then required. In the second and the third case, the problem is related to the prediction of the non-occurrence of unacceptable fatigue problems. In all three cases we are supposed to arrive at statistically justified conclusions for practical problems, with some knowledge of the variability of fatigue properties as observed in laboratory experiments. We will return to this issue later.

OBSERVATIONS ON SCATTER

Crack initiation life and the crack growth period

Most observations on scatter in the literature are made on fatigue lives until failure. However, the fatigue life can be divided in two periods:

- crack initiation life,
- crack growth period.

Scatter of the crack initiation life is not necessarily related to scatter of the crack growth period. The present understanding of fatigue implies that different sources of scatter can apply to the two periods. Crack initiation generally occurs at the material surface. It implies that initiation is easily affected by local conditions of the material surface. The surface roughness is specifically important, but the same is true for surface treatments (e.g. anodizing, nitriding, shot peening, etc.) and surface layers (e.g. clad layers on Al-alloys, decarburized layers of steel). These surface conditions have a negligible influence on crack growth through the bulk of the material. *Crack initiation is a surface phenomenon, whereas crack growth resistance is a matter of bulk properties.* A trivial example: incidental surface damage (e.g. a dent) can have a large effect on crack initiation, but not on propagation.

In many fatigue tests on simple laboratory specimens the initiation period covers the larger part of the fatigue life. If a small crack becomes visible, specimen failure is imminent. The consequence should be, that most observations on scatter of fatigue life documented in the literature, reflect scatter on crack initiation. The conclusion will not be correct if significant cracking occurs early in the fatigue life.

A well-known trend on scatter in CA fatigue tests should be recalled. The scatter band is generally narrower at high stress amplitudes, and wider at low amplitudes. This trend is also visible in Fig.2a. Fractographic studies have indicated [5,6] that several crack nuclei can be observed in many cases if the stress level is high. If the stress level is low, only one crack nucleus is visible. An illustration for an Al-alloy specimen is presented in Fig.4. Perhaps it is not surprising that only one fatigue crack is found if S_a is decreasing to the fatigue limit S_f. The fatigue limit may be considered as a threshold stress level, i.e. the lowest stress amplitude, that can still successfully initiate a crack. That will occur at the "weakest" location of the specimen. Depending on the type of specimen, the weakest link may be expected to be sensitive to material surface condition variability. A wide scatter band at low stress levels seems to be a logical consequence. At high amplitudes crack initiation is no longer a threshold problem. It can occur relatively easy at several locations. The observations agree (at least qualitatively) with a weakest link conception of the material fatigue behaviour.

An interesting question is whether scatter of crack growth (depending on bulk properties) is smaller than scatter of the initiation period (depending on surface variability). A generally valid answer to this question can not be given. Fatigue crack growth is generally believed to show limited scatter only. This "low scatter" reputation is mainly based on the results of tests on Al-alloy sheet specimens with a central crack (through cracks). Numerous tests were carried out by the National Aerospace Laboratory NLR. As an example, in a comparative test series [8], each test was carried out in triplicate. This low number was considered to be sufficient to account for scatter in the comparisons. The ratio of the maximum and the minimum crack growth life ($a = 3$ mm to failure) of each three similar tests has recently been calculated. The average ratio of 35 groups of three results was 1.15, which is very low for similar fatigue test series. Virkler, Hillberry and Goel [9] carried out the well known test series of 68 similar tests. Scatter of the crack growth life was relatively small. During crack growth they found some more scatter of da/dN, which was supposed to be due to local material inhomogeneities. Nevertheless, it may be concluded that the average crack growth resistance will not significantly vary, if the crack front is moving through large numbers of grains. Scatter of fatigue crack growth of macro cracks should be expected to be low. An important conclusion then is that significant scatter of the total fatigue life must be mainly due to scatter of the crack initiation period. Illustrative results are presented in Fig.5 for hole notched specimens [10]. The initiation period was defined as the life ($N_{0.5}$) until $a = 0.5$ mm (still part through crack). The graph shows more scatter for the initiation period, while scatter increases at lower S_{max} values ($R = 0$). For the crack growth period, $N - N_{0.5}$, scatter is less, and moreover, it remains approximately constant for all stress levels. Another significant trend (see also Fig.5) was that some systematic correlation between $N_{0.5}$ and $N-N_{0.5}$ was not observed for three types of specimens. It confirms that scatter in the initiation period and the crack growth period depend on different conditions as pointed out before.

Scatter in different types of specimens

Three types of specimens will be considered here.

- unnotched specimens
- simple notched specimens (central hole, symmetric edge notches)
- joint specimens (welded, riveted, adhesive bonding)

Unnotched specimens are frequently used in order to determine basic fatigue properties of a material. Several decades ago, many rotating beam specimens ($S_m = 0$) were used for this purpose. Usually scatter was large. Fig.2a is an extreme illustration. It should then be questioned, whether this is a true picture of the fatigue properties of the material. Different sources of scatter must be recognized:

1. scatter related to specimen production,
2. scatter related to the fatigue test conditions (e.g. fatigue machine, clamping, operator),
3. scatter due to the material itself.

Such a list is almost embarrassing, because scatter due to sources in categories 1 and 2 are actually unacceptable. It has led to careful specimen production techniques, or to production procedures closely similar to standard practice in the workshops. The present closed-loop fatigue machines are definitely more reliable than the fatigue machines in the older days. However, clamping of specimens should always be a matter of concern. Nevertheless, the above considerations leave us with the unpleasant feeling that there is a problem: *scatter in our laboratory test series, is it representative for other conditions than the laboratory conditions ?* This question is rarely touched upon.

It is often thought that unnotched specimens are more sensitive to scatter than notched specimens. The surface area of unnotched specimens exposed to the nominal cyclic stress is usually larger than for notched specimens. From a statistical point of view, that does not necessarily imply more scatter. It depends on the character and the distribution of the fatigue prone locations in the material (weakest fatigue links). However, the production and handling of unnotched specimens, in view of the larger exposed surface area, can offer more problems to arrive at a uniform surface finish quality. More scatter in unnotched specimens may thus be expected, unfortunately due to sources of scatter in category 1.

In data compilations on scatter it is usual to present $\sigma_{\log N}$ (standard deviation of log N in a series of similar tests). Compilations by Stagg [5] and Mann [6] are instructive to show large ranges of $\sigma_{\log N}$ values, while some general trends were observed. However, systematic trends with respect to the type of specimen were not found. It is sometimes suggested that notched specimens are less sensitive to scatter, because of the small and well defined area of highly stressed material. This is not always confirmed by experimental results. Fig.6 is drawn from a recent brief analysis of older test series on unnotched specimens, side-edge notched specimens ($K_t = 2.85$) and riveted lap joints [11]. Values of $\sigma_{\log N} < 0.1$ are generally considered to indicate low scatter. The values in Fig.6 at the low endurance side confirm less scatter for the notched specimen and the lap joint. However, at $N \approx 10^6$ cycles the largest $\sigma_{\log N}$ value is found for the notched specimen.

The statistical distribution function

The distribution functions of fatigue properties have been an essential part of many statistical models. Several functions have been proposed in the literature, but two functions are frequently adopted, the (log)normal distribution and the Weibull distribution. The normal distribution is:

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}\left(\frac{v-\mu}{\sigma}\right)^2} dv \qquad (1)$$

with $x = \log N$, μ = mean value of x and σ the standard deviation.
The 3-parameter Weibull distribution function is written as:

$$P(x) = 1 - e^{-\left(\frac{x-x_0}{a}\right)^b} \qquad (2)$$

In this equation x_o is the location parameter (lower values of x are impossible, it is a lower limit), a is the scale parameter, and b is the shape parameter. Values of $b \approx 3.5$ give a shape which is somewhat similar to the normal distribution function, except for the low and high probability tails of the function. In Eq.(2) x can be taken as either the life N or as log N. For illustrative purposes, the results of two test series of 20 and 30 notched specimens are plotted in Fig.7. In Fig.7a and 7b the results are presented on normal probability paper. A normal distribution should then become a straight line, which is not a good approximation in either of the two series. The results of Fig.7b have been replotted in Fig.7c as a Weibull distribution. Minimizing the sum of squared deviations $(\Sigma(x(P_i)-x_i)^2$ with $P_i=(i-0.5)/n)$, $x_i=\log N$, n=number of specimens) has been adopted to obtain the values of a, b and x_o, which requires a numerical iteration calculation. The vertical scale in Fig.7c is adjusted in such a way that Eq.(2) becomes a straight line. Apparently, there is a very good agreement between the test results and the Weibull distribution function. However, it was tried to use the same procedure for the results of Fig.7a, but unfortunately, the sum of the squared deviations did not show a minimum. In other words, the 3-parameter Weibull distribution function could not be fitted to the data, which was not really a surprise. The data in Fig.7a do not suggest a lower minimum value, which is characteristic for the Weibull distribution. It is not clear why the same type of specimen tested at two different stress levels showed such essentially different trends with respect to scatter.

Of course, it can always be tried to fit some statistical distribution function to test results, but it can never be proven that the fatigue life does indeed obey a certain distribution function. For that reason, test results of larger test series are usually plotted on normal probability paper, which is more easily done than trying to fit a 3-parameter Weibull function.

Scatter under Variable-Amplitude loading

Quite a time ago we published results on scatter as observed in CA-tests and VA-tests on riveted lap joints [12]. Each type of test was carried out 10 times (CA-tests) or 7 times (VA-tests). Values of $\sigma_{\log N}$ were calculated. The VA-tests were a kind of program test with a periodic variation of the stress amplitude in accordance with a gust spectrum. Different sequences were applied and periodic overloads and underloads were also included. The values of $\sigma_{\log N}$ as a function of the fatigue life are shown in Fig.8. The open data points represent the CA-tests. There is an obvious increase of $\sigma_{\log N}$ for increasing N-values, as discussed before. A remarkable observation was made for the VA-tests. The $\sigma_{\log N}$ values all scattered around 0.1, without any noticeable dependence on the fatigue life. The average value for the VA-tests is of the same order of magnitude as $\sigma_{\log N}$ in the CA tests at the largest amplitude of the load spectrum. As pointed out in [12], it suggests that the highest stress level of the load spectrum is responsible for the amount of scatter. This observation seems to be logical. If scatter is mainly depending on scatter of the damage initiation, it might well be expected that the maximum stress cycle will have a predominant influence on the crack initiation process, and thus on its variability.

Similar observations on scatter in tests with random or programmed sequences were reported by others [6,13,14]. Mann [6] suggested that the maximum load of a spectrum will induce local plasticity, which will "smooth out" the influence of small inhomogeneities in the material, and cause load redistributions in the fatigue-critical regions of a built-up structure. It will result in a more uniform fatigue response. As pointed out by Jacoby and Nowack [15] the Miner rule overestimates the scatter under periodic VA-load histories. It apparently gives too much weight to the larger scatter of low amplitudes.

SCATTER IN SERVICE

Information about scatter of fatigue failures in service is extremely scarce. A fatigue accident can be due to some incidental case, which implies that it is not part of a regular population. Symptomatic fatigue failures do occur occasionally, but publicity is usually kept to a minimum. Everything is done to prevent similar failures in the future by some redesign action or by replacements. Usually, a good documentation of such failures is rarely available in the open literature. Exceptions are found in aeronautics, where reporting on failures is required by official regulations. Even then the information is not abundant.

Williams [16] reported on a catastrophic front spar failure of a Freighter 170 aircraft after 13000 landings in 1957. Other aircraft of the same type where then inspected. The results are shown in Fig.9. Apparently, the crack is a symptomatic one, but the crack growth rate is small.

In 1976 [17] a Hawker Siddely 748 crashed due to a 90 cm fatigue crack in the wing tension skin structure. Similar cracks were found at the same location in 18 aircraft of the same type, with a length varying from a few millimetres to 6 cm, but one crack of about 70 cm.

In 1977 [18] a Boeing 707 crashed after 16723 flights because it lost a horizontal tailplane due to a fatigue failure in a spar. In 38 old aircraft of the same type similar cracks were initiated, on the average still rather small with one exception.

The above examples show that systematic fatigue cracks have occurred in aircraft structures. Actually, it did occur in many more cases. Sometimes the fatigue cracks could be considered as nuisance cracks, which did not impair the aircraft safety, although the cracks are undesirable for economic reasons. In several other cases, local repairs of reinforcements were necessary, while in still other cases significant parts of the structure had to be replaced by redesigned components. Reskinning and wing replacements have occurred. The question in the context of the present paper is: how can predictions on such incidents and accidents benefit from our knowledge on scatter of fatigue properties. Even more, is it possible to use our statistical information in the design phase of a new aircraft structure, or to structures in general ?

DISCUSSION

In the previous chapter observations on scatter of fatigue properties were summarized, without attempting to be complete. Moreover, systematic trends, observed in some investigations, are sometimes contradicted in other ones. As a result, it is difficult to come to general statements. The crucial question to be considered now is: *What is the significance of experience on scatter in laboratory experiments for scatter of fatigue properties of structural elements in service ?* What are the similarities and the differences between test series in the laboratory and structures in service ? Which sources of scatter apply to structures in service ? The question almost suggests that there are tremendous differences, but let us look for the similarities first. In the introduction five possible categories of scatter have been mentioned (tolerances on dimensions, production quality, material, surface finish, and load spectra). It appears that the material and surface finish can be similar in specimens and in real structures. Specimens are made from the same material used for the production of structures. However, specimens for one investigation are usually made from a single batch of material, or even from a single sheet or bar. Batch to batch variations are not covered. The same applies to materials from different producers made according to the same material specification. At the NLR we procured 2024-T3 sheet material from different producers for comparative fatigue tests. Limited differences were found for the S-N curves of unnotched specimens, hole notched specimens and riveted lap joints, made from 2024-T3 of 5 different producers [19]. Material from one single producer was fairly systematically inferior to the other ones, even more in program fatigue tests (life ratios about 2). Systematic differences were found in fatigue crack propagation tests on sheets from 7 different producers [8]. The crack growth lives for the most superior material were about double the life for the most inferior material. Sheets from two different batches from the same producer were available for two manufacturers. Life ratios between the batches were about 1.5. The last difference appeared to be related to the ductility of the sheet, but the former differences were hard to correlate with any material characteristic.

Mann and Harris [20] reported on rotating beam fatigue specimens (unnotched and notched), produced from 2L.65 (similar to 2014) extruded bar material of 13 different batches. Scatter was very large and significant differences between the batches were found. It is quite well possible that extruded material is more sensitive to scatter.

A relevant observation was made in [21]. Full-scale fatigue tests were carried out on tension skins of a wing structure. Tests were carried out in duplicate. Each tension skin contained 40 finger tip configurations, which were fatigue critical. Scatter in one wing could thus be observed and compared to scatter in a similarly tested wing structure. An example of crack initiation lives (until a crack nucleus of 3 mm) is shown in Fig.10, plotted on log-normal probability paper. Depending on repairs the test could not be continued until cracks at all finger tips. However, there are sufficient results in Fig.10 to indicate fairly low $\sigma_{\log N}$ values. More important, it shows that the finger tip populations of the two wings are not identical. In other words, there is scatter in one structure with similar structural details, and there is another source of scatter between different structures. This aspect is rarely accounted for in statistical models.

In order to avoid scatter in laboratory tests, it is generally tried to prepare test specimens as carefully as possible with an excellent surface finish. Obviously, such a surface quality is not representative for the surface obtained in the production shop. The other sources of scatter mentioned before are also problematic. The laboratory LBF in Darmstadt fatigue tested 130 steering levers of cars under program loading [22]. Two levers were selected at random from the production each week. The standard deviation was $\sigma_{\log N} = 0.29$. Part of the scatter was due to dimensional variations. Accounting for these variations reduced the scatter to $\sigma_{\log N} = 0.24$. Unfortunately, the dimensional variations are a natural result of the production process.

Variations of the production quality of fatigue critical components are a matter of most serious concern in every production shop. However, close tolerances on production variables increase the production costs. They are adopted only if it has been shown to be necessary. In a recent investigation in cooperation with the Fokker Aircraft Industry, it was shown that the riveting force (squeezing) used in the automatic riveting machine has a large effect on the fatigue life, see Fig.11 [23]. As a consequence, variations of the riveting force can be a most significant source of scatter. W.Schütz [24] observed an average life ratio of about 3 between series of nominally identical lap joints, riveted by the same aircraft industry in different years. A similar observation was recently reported by Mayville and Sigelmann [25].

The last source of scatter listed before is the load spectrum. Actually, this source is not an inherent property of the structure and its material. However, it can not be ignored if considering scatter of fatigue life of structures in service.

It was questioned whether information about scatter obtained in laboratory test series can be relevant for scatter in service. In all honesty, it must be concluded that the relevance is rather limited. Several sources of scatter, that can affect the laboratory result, do not occur in service. Several other sources, that do occur in service, are not present in the laboratory. It implies that the designer has a problem if he wants to account for scatter in his fatigue life or crack growth predictions. Especially for fatigue life it is a difficult question. The value of laboratory experience is of a qualitative nature. It helps to recognize the character of sources of scatter, to understand whether they will affect crack nucleation or crack growth or both, and to know that scatter can depend on the type of fatigue loading. A profound knowledge of the fatigue process is essential for judging scatter problems in service.

In general, the designer will deal with scatter by adopting safety factors. There are two problems: (1) Is the safety factor applied to the stress level, or to the fatigue life, or to both ? (2) Which safety factors will be used ? We will not elaborate in detail on these questions here. There are no generally applicable answers, but some comments can be made. The answer to the first question depends on the conditions of the problem as briefly indicated in the introduction. Reference was made to a pressure vessel, loaded at an approximately Constant-Amplitude load to a limited number of cycles. The problem is to have a design point sufficiently remote from the S-N curve. It can be obtained by safety factors on stress or on life. The slope of the S-N curve should be considered, a conservative slope value should be adopted. Fully different conditions apply to a crankshaft of a combustion engine. It should not fail and even the most severe utilization will lead to very high numbers of load cycles. The fatigue limit is the relevant property. Safety factors should be applied to stress levels and not to life cycles. The second problem about the magnitude of safety factors is another delicate question. There is no rational answer, because we do not know the distribution functions. It is frequently assumed that the fatigue strength has a normal distribution. the problem then is, what is the standard deviation? Suggestion have been $\sigma_S = 0.05$ to 0.1, and for high level fatigue $\sigma_{\log N} = 0.1$ to 0.2. It should be recalled that safety factors for scatter are just one aspect of a wider problem setting of uncertainties, as discussed in the introduction. There are definitely many more uncertainties involved. It implies that "engineering judgement" of the designer, in addition to qualitative understanding, experience and consequences of fatigue failures, is an indispensable ingredient of the art of designing against fatigue !

SUMMARIZING CONCLUSIONS

1. Scatter of fatigue life in laboratory test series mainly reflects scatter of the crack initiation life. In general, crack initiation is a surface phenomenon. Consequently, scatter of the initiation is highly depending on surface conditions. On the other hand, crack growth resistance is largely a bulk property of the material. It is hardly depending on material surface conditions, but rather on the homogeneity of the material. As a result, scatter of crack growth is usually smaller than scatter of the crack initiation life.
2. Two typical trends of scatter in fatigue tests are:

(i) $\sigma_{\log N}$ in CA-tests increases for lower stress levels, i.e. for higher endurances.

(ii) In VA-tests with some periodically recurring load spectrum, $\sigma_{\log N}$ is relatively small and of the same order of magnitude as $\sigma_{\log N}$ found in CA-tests at high stress levels. The maximum load level of a spectrum has some controlling influence on the fatigue mechanism, including scatter.

3. There are no indications that any statistical distribution function has a general applicability to the results of fatigue test series. For convenience, (log)normal probability paper is frequently used for presenting the results of large test series.
4. It should be recognized that the trends of scatter listed above, are all obtained in laboratory test series. It should also be realized that the results of laboratory test series are susceptible to different sources of scatter, mainly in three categories :(1) specimen production, (2) fatigue test conditions and finally (3) the material itself. Sources in the first and the second category are actually unacceptable. Their influence should be minimized by careful experimental techniques. Nevertheless, the statistical information of a large test series should primarily be associated with the conditions of that test series.
5. Several sources of scatter, that can affect the laboratory result, do not occur in service. Several other sources, that do occur in service, are not present in the laboratory.
6. The question whether safety factors for scatter should be applied to the stress level or the fatigue life, depends on the type of load spectrum.
7. In a fatigue design problem scatter must be considered. However, many more uncertainties are involved. "Engineering judgement", in addition to qualitative understanding and experience, is still an indispensable ingredient of the art of designing against fatigue.

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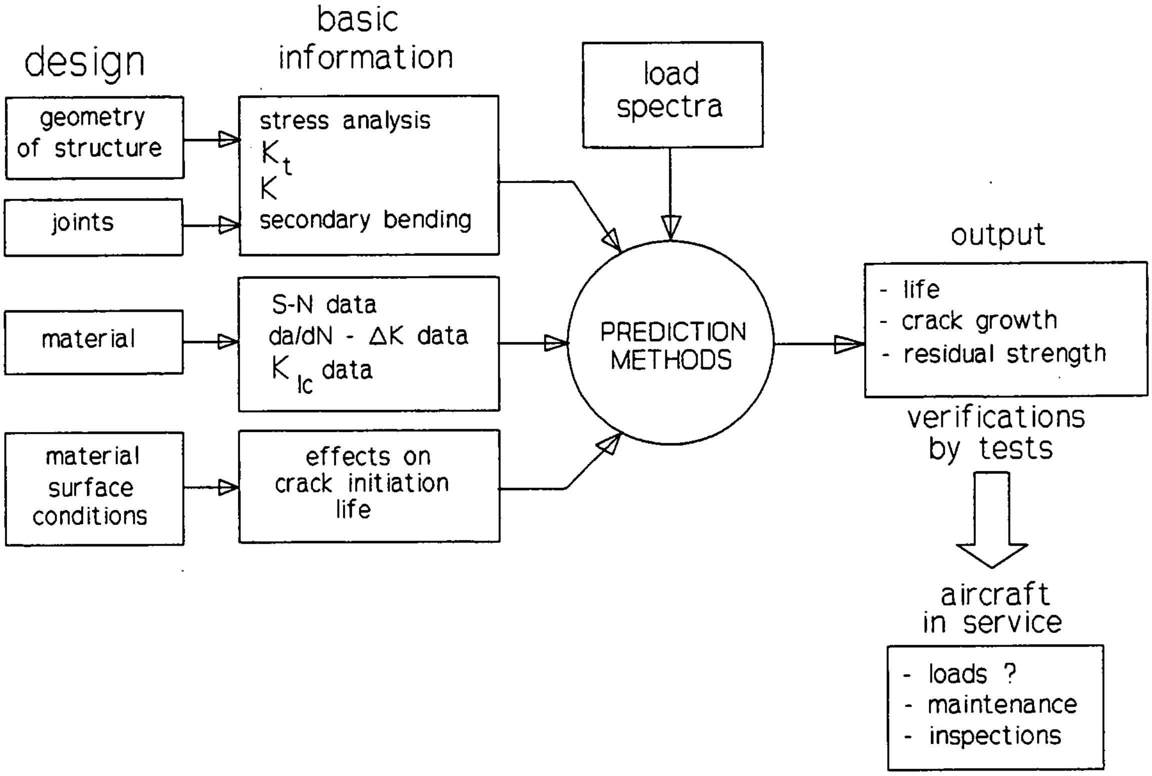

Fig.1 Survey of the various aspects of designing against fatigue in aeronautics.

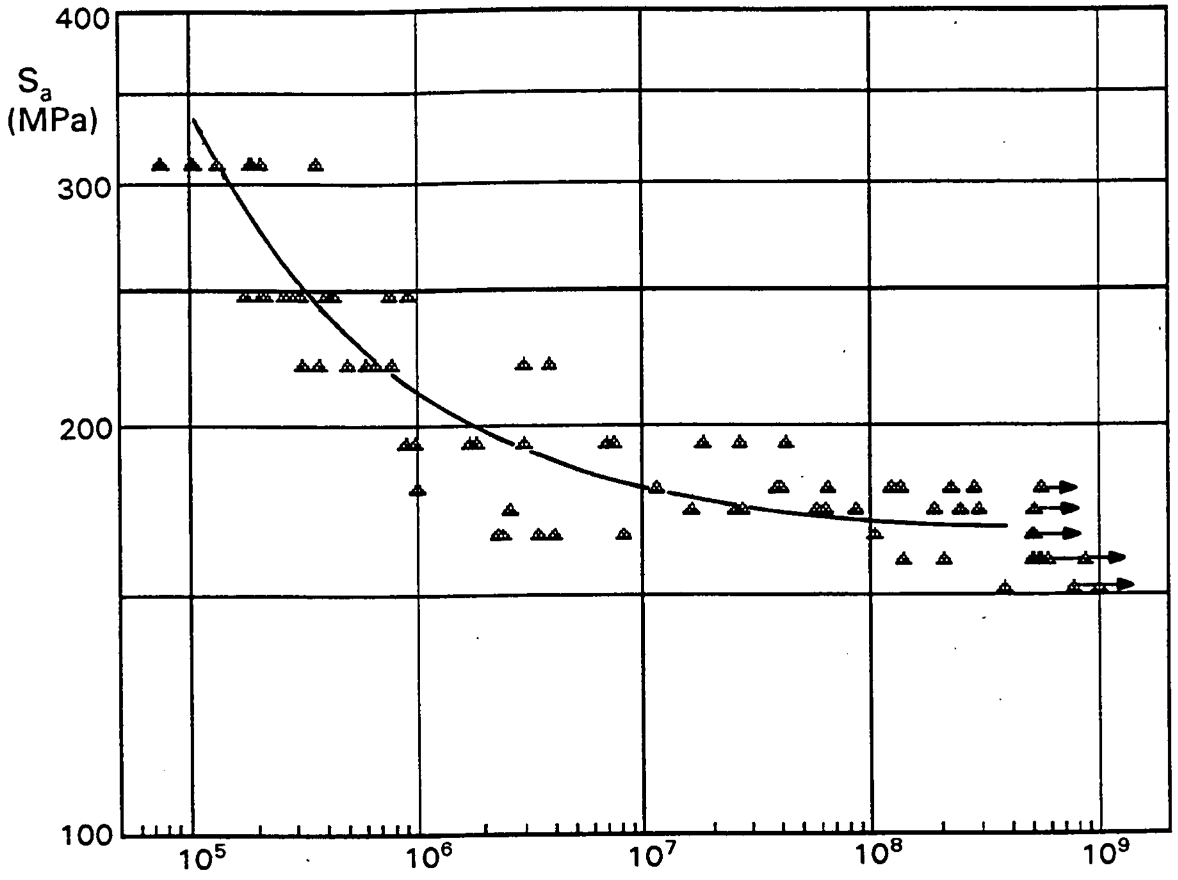

Fig.2a: Original data

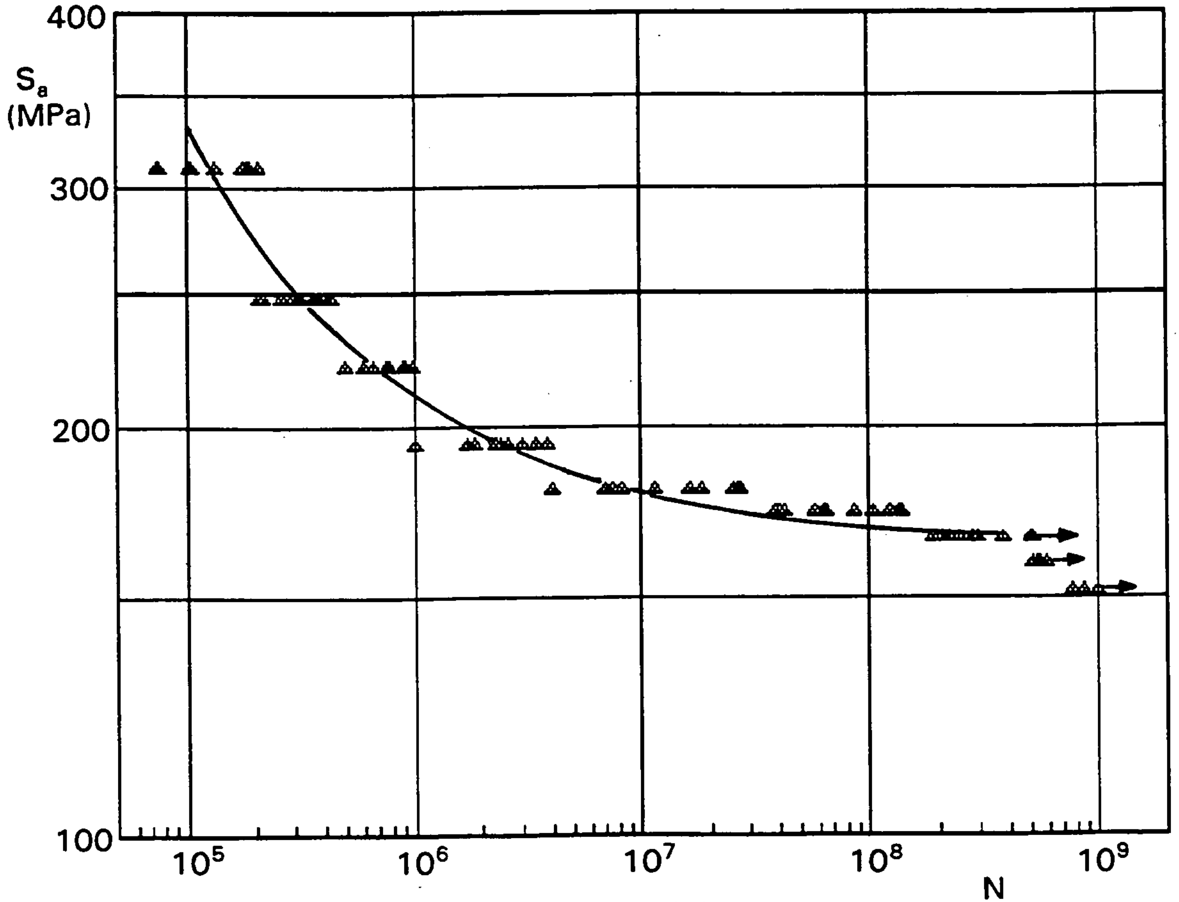

Fig.2b: Rearranged data

Fig.2 Results of rotating beam tests on unnotched 7075-T6 specimens [3].

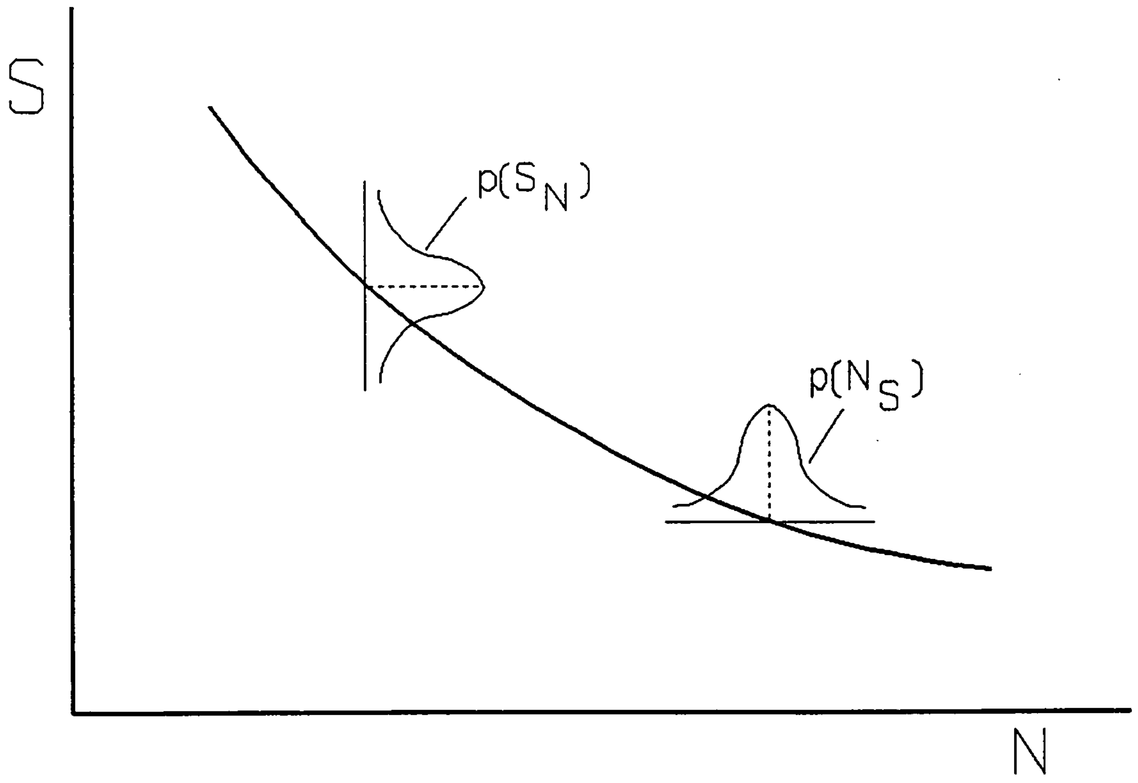

Fig.3 Distribution functions of fatigue life and fatigue strength.

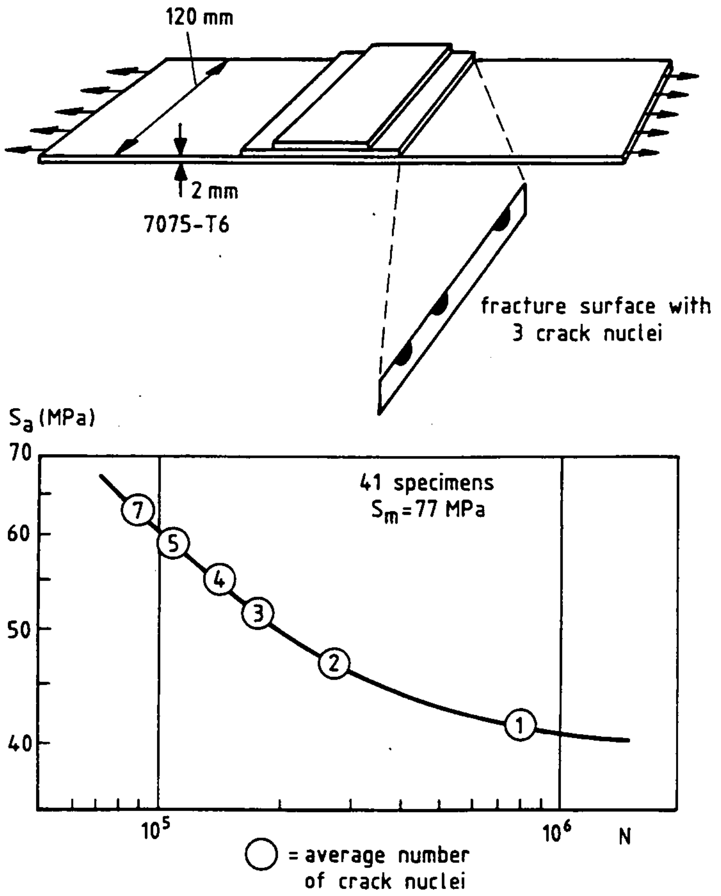

Fig.4 The influence of the stress level on the number of fatigue crack nuclei on the fatigue fracture surface [7].

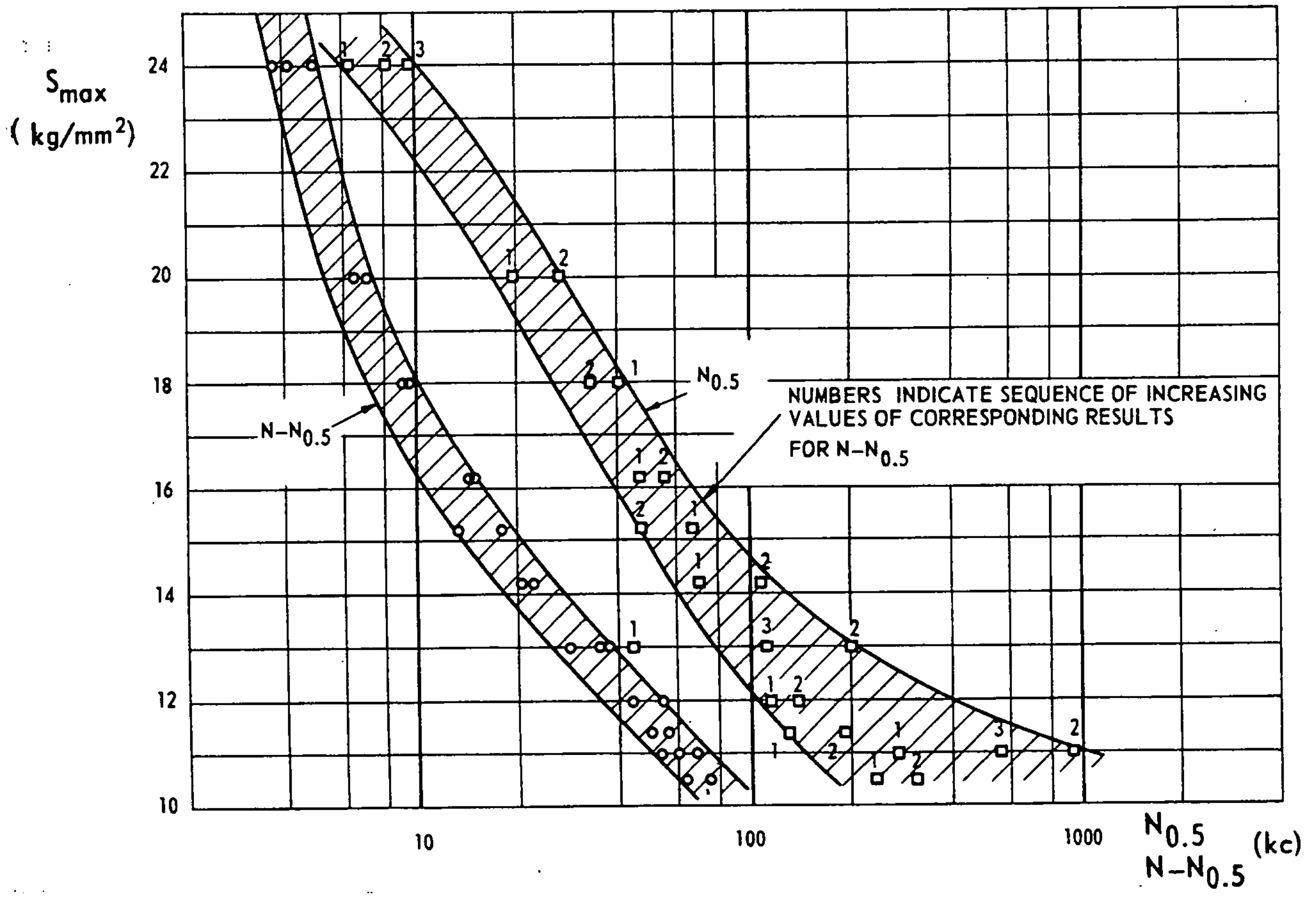

Fig.5 Scatter of fatigue crack initiation period (until a = 0.5 mm) and crack growth period. Results of 2024-T3 specimens with a central hole (d = 12.5 mm, K_t = 2.66) [10]. Less scatter at high S_{max}, no correlation between scatter of initiation life and crack growth period.

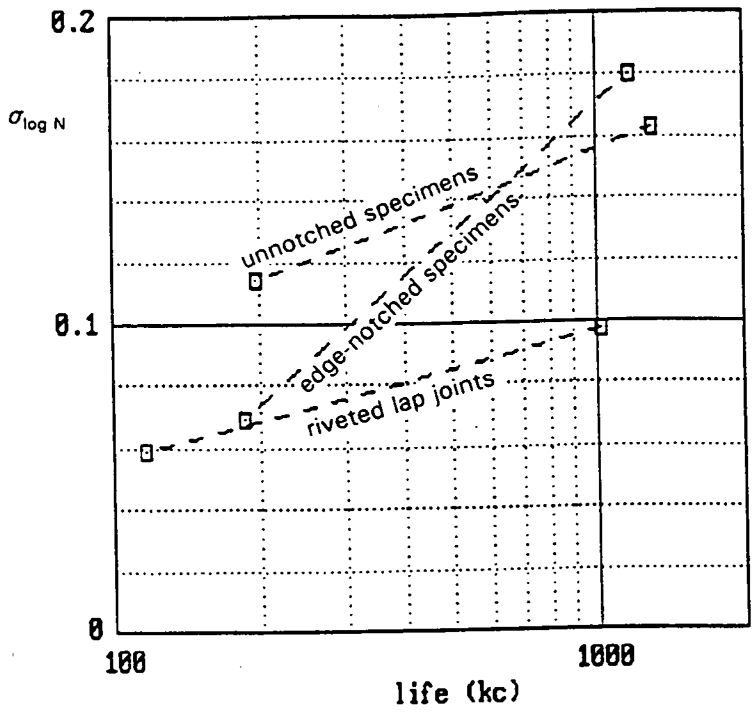

Fig.6 Scatter observed for three different types of specimens [11].

Fig. 7a

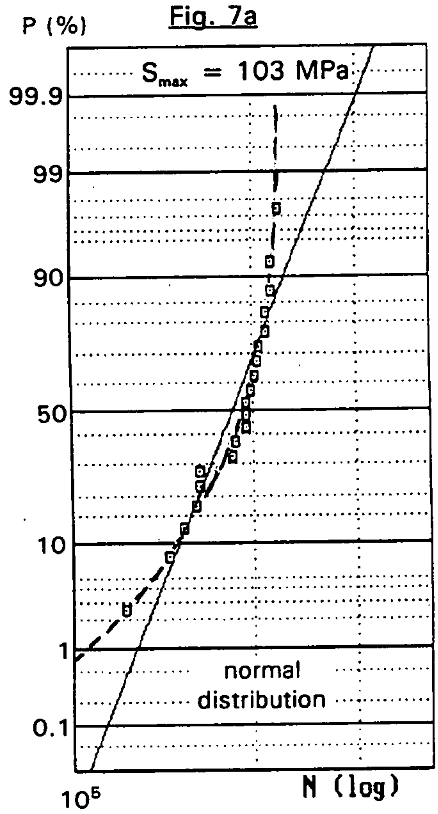

Fig. 7b

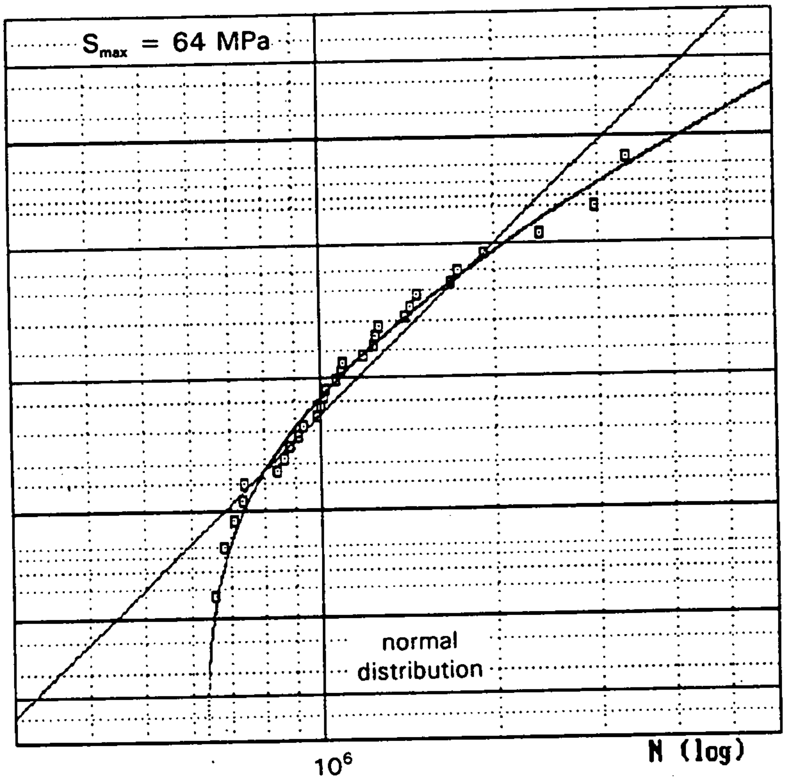

Fig. 7c

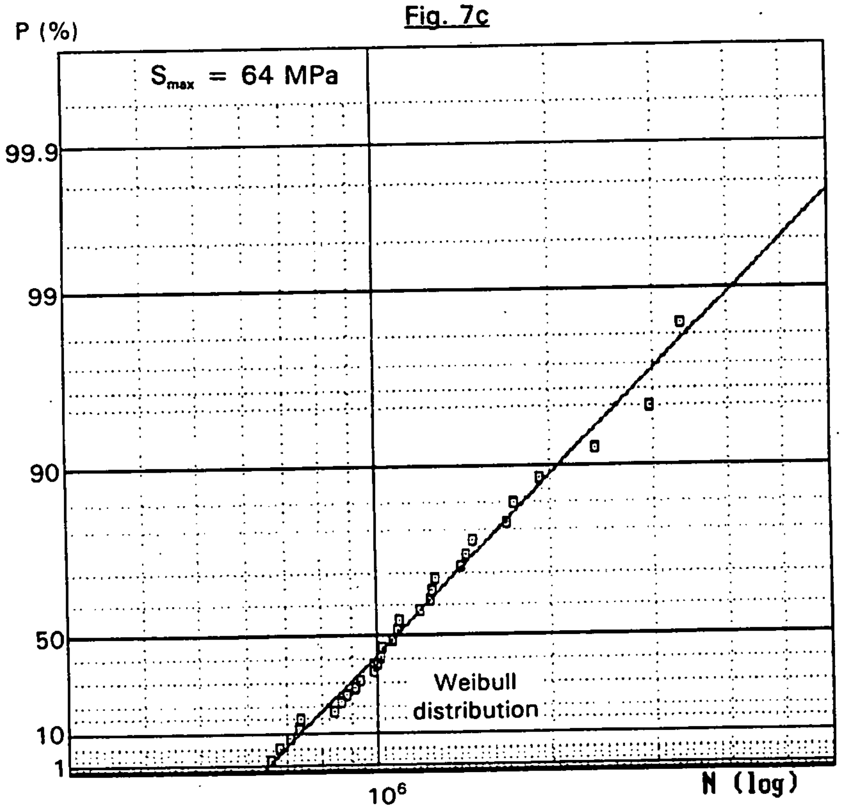

Fig.7 Statistical distributions of two test series on 2024-T3 notched specimens [11].

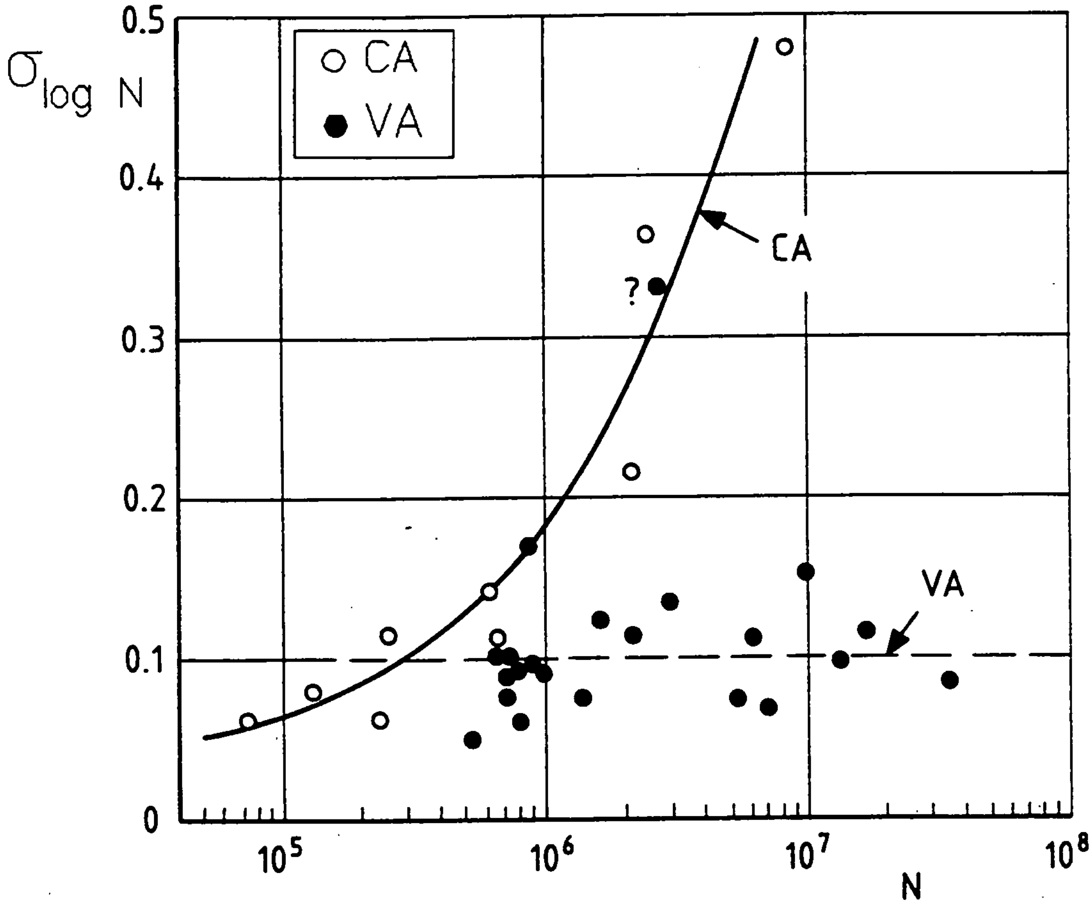

Fig.8 Scatter in fatigue tests on riveted lap joints under Constant-Amplitude loading and under Variable-Amplitude loading [12].

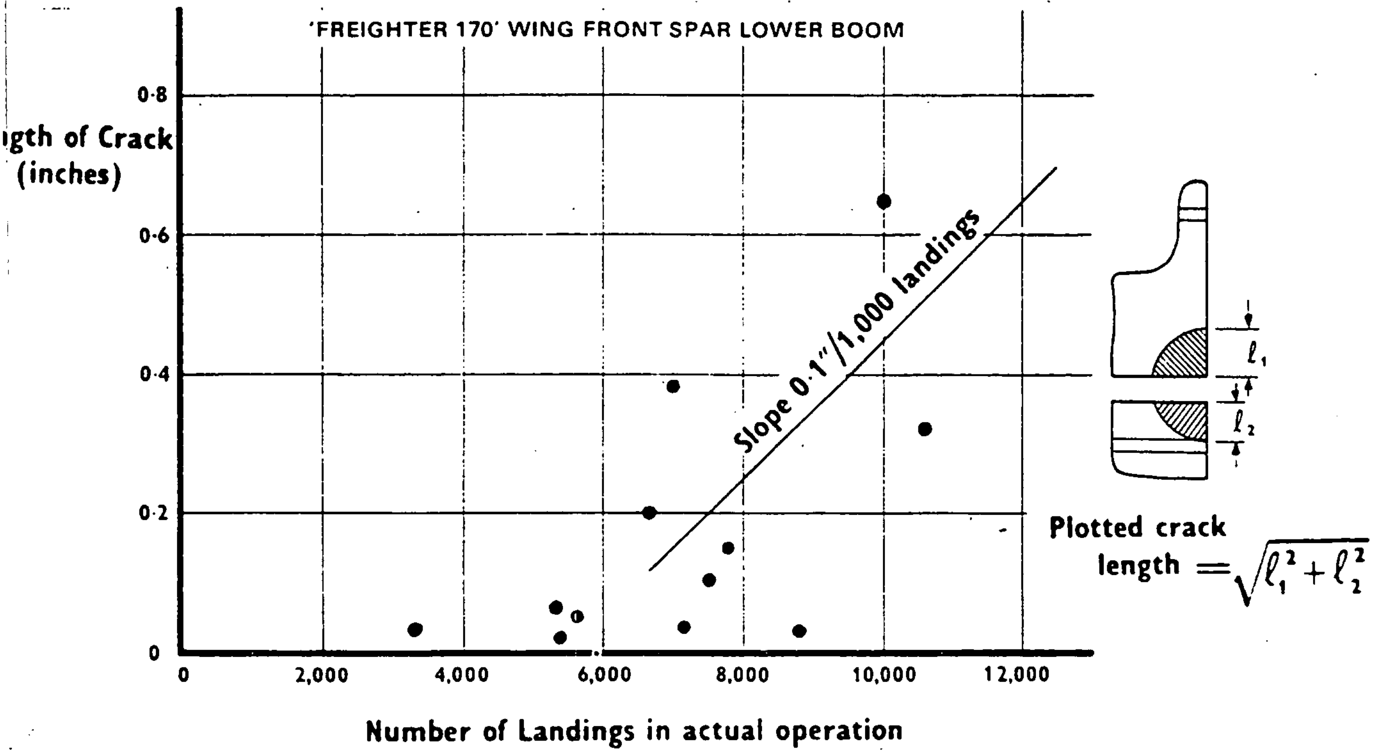

Fig.9 Cracks in a wing spar found after an accident [16].

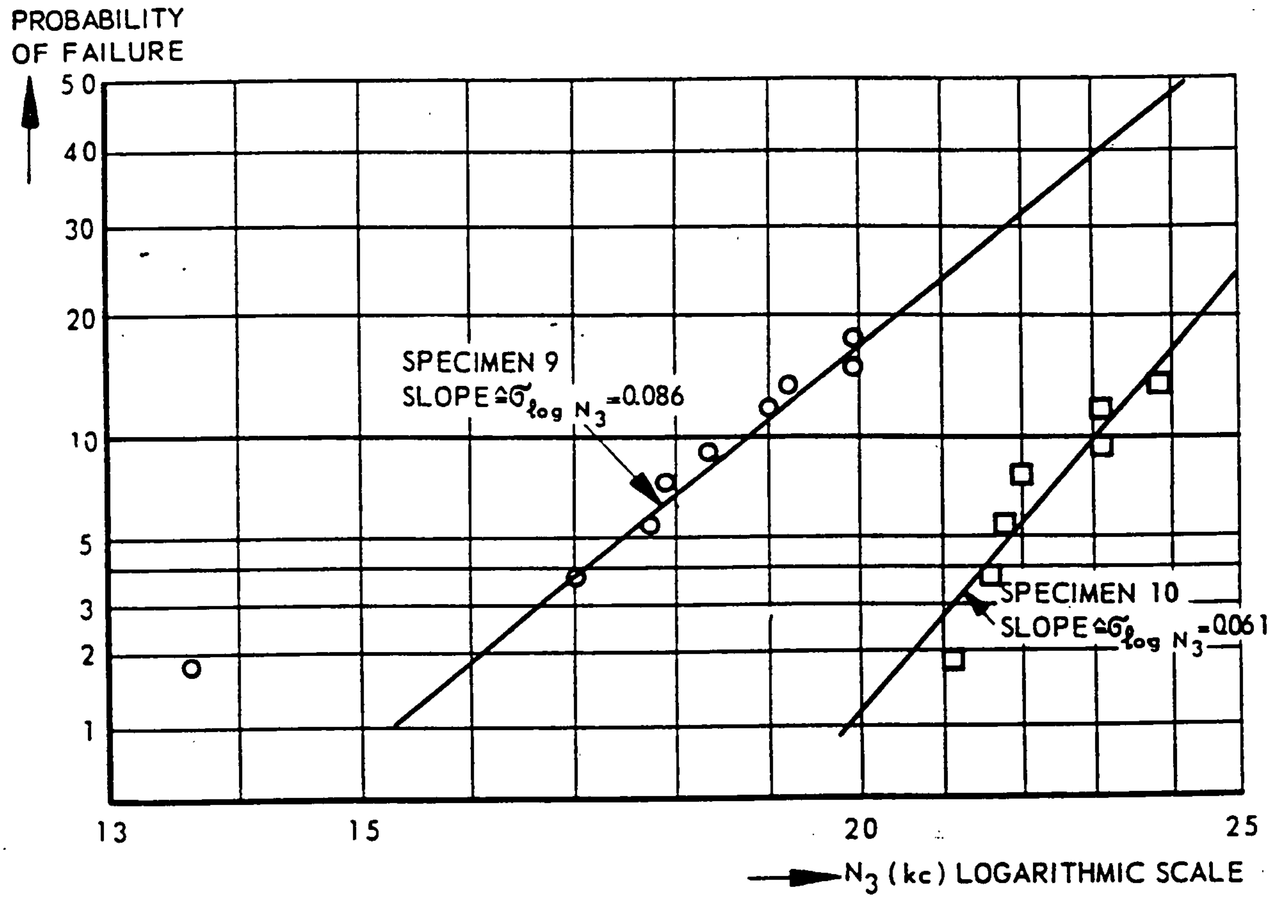

Fig.10 The occurrence of crack nuclei at "finger tips" in similar fatigue tests on two full-scale aircraft wing tension skin structures [21]. Scatter in each structure, and scatter between similar structures.

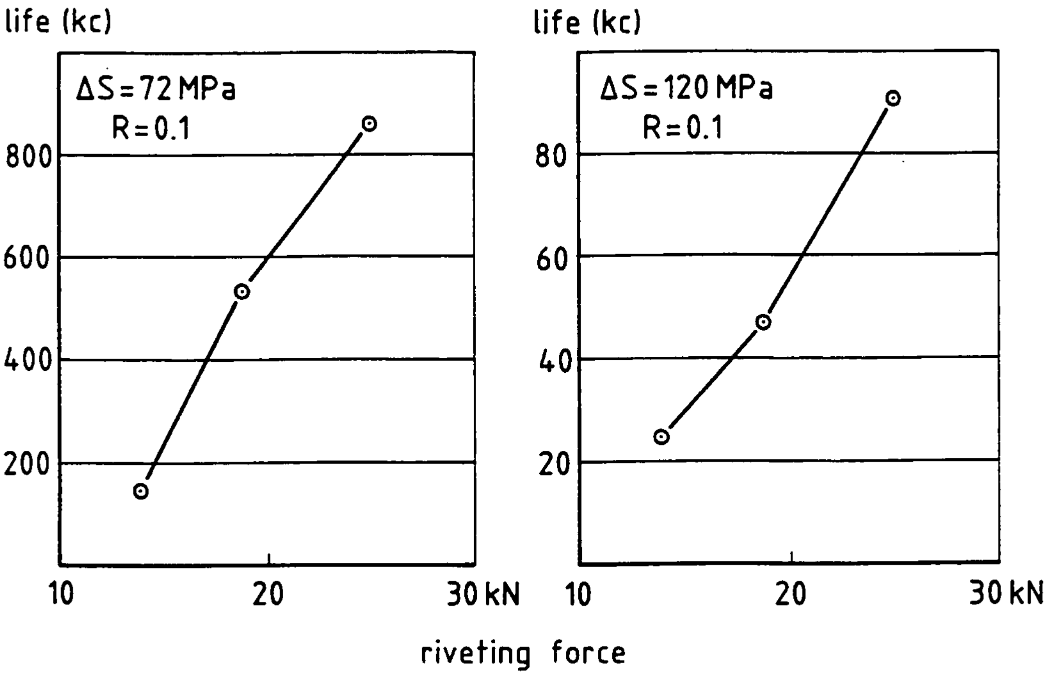

Fig.11 Effect of the riveting force on fatigue life of a riveted lap joint [23]. Production quality as a source of scatter.