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Adaptive Neural Control for Pure Feedback Nonlinear Systems with Uncertain Actuator Nonlinearity

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Abstract. For the pure feedback systems with uncertain actuator nonlinearity and non-differentiable non-affine function, a novel adaptive neural control scheme is proposed. Firstly, the assumption that the non-affine function must be differentiable everywhere with respect to control input has been canceled; in addition, the proposed approach can not only be applicable to actuator input dead zone nonlinearity, but also to backlash nonlinearity without changing the controller. Secondly, the neural network (NN) is used to approximate unknown nonlinear functions of system generated in the process of control design and a nonlinear robust term is introduced to eliminate the actuator nonlinearity modeling error, the NN approximation error and the external disturbances. Semi-globally uniformly ultimately boundedness of all signals in the closed loop system is analytically proved by utilizing Lyapunov theory. Finally, the effectiveness of the designed method is demonstrated via two examples.

Keywords: Adaptive neural control · Robust control · Actuator input nonlinearity · Non-affine function

1 Introduction

As we all know, pure feedback systems have a more general form than strict feedback systems, and many industrial applications such as biochemical process, mechanical systems and dynamic model in pendulum control have the form of pure feed feedback systems. Many approaches have been investigated for this class of systems [1–4]. However, it is worth mentioning that, for non-affine nonlinear pure feedback systems, the main difficulty of controller design is that there is no affine control appearance of control input in systems; therefore, the approaches developed for affine nonlinear systems cannot be directly applied to control design for pure feedback systems in non-affine form. To overcome this difficulty, some remarkable methods have been presented for pure feedback systems such as in [5, 6]. Nevertheless, it is worth noting

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that the commonly used assumption in the above schemes is that the non-affine function of the pure feedback system is always assumed to be differentiable with respect to the control input or state variable. This is a restrictive condition for non-affine function due to the fact that the non-affine functions in some practical systems are always continuous but not differentiable.

Evident examples of such functions are non-smooth nonlinear characteristics such as dead-zone, backlash, and hysteresis, which extensively appear in mechanical connection, hydraulic servo valves, piezoelectric translators, and electric servomotors, and which may lead to instability of the closed-loop system if their effect is not taken into account properly. Although some constructive methods have been designed to eliminate the adverse influence in closed-loop systems such as in [7], it should be noted that the approach in [7] is only applicable to strict feedback nonlinear systems but is not be suitable for pure feedback systems. What's more, to the best of the authors' knowledge, the research for control design of pure feedback systems with uncertain actuator nonlinearity is an open problem, which motivates us to explore new methods to solve this problem.

Motivated by above discussion, this work proposes a novel adaptive neural control scheme for pure feedback systems with uncertain actuator input nonlinearity. The main contributions of this paper are as follows:

(1) The restrictive differentiability condition for non-affine function of pure feedback systems is removed and only a semi-bounded condition is required.
(2) Different from all the previous researches, in this paper, actuator input dead zone nonlinearity and backlash nonlinearity of pure feedback systems are both considered by modeling actuator nonlinearity appropriately when designing controller, which is a completely new work for pure feedback systems. The proposed method is not only applicable to the actuator dead zone nonlinearity, but also is suitable for the backlash nonlinearity, which has a more relaxed application scope than existing works such as in [1, 2].

2 Problem Statement and Preliminaries

Consider a class of pure feedback systems with actuator input nonlinearity as follows:

$$\begin{cases} \dot{x}_i = x_{i+1}, \quad i = 1, 2\ldots, n-1 \\ \dot{x}_n = f(\boldsymbol{x}, v(u(t))) + d(\boldsymbol{x}, t) \\ y = x_1 \end{cases} \tag{1}$$

where $\boldsymbol{x}(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in R^n$ and $y \in R$ denote the system states and output, respectively; $d(\boldsymbol{x}, t)$ represents systems external disturbance; $f(\boldsymbol{x}, v(u(t)))$ is an unknown function such that $f(\boldsymbol{x}, 0) = g(\boldsymbol{x})$; $u(t)$ and $v(u(t))$ are the actuator input and output, respectively. The actuator nonlinear model can be expressed as

$$v(u(t)) = k(u, t) \cdot u + \varepsilon_u \tag{2}$$

where $k(u,t)$ is an unknown positive constant and ε_u is a modeling error satisfying $|\varepsilon_u| \le \varepsilon_u^*$ with ε_u^* being an unknown positive constant.

Assumption 1: There exist unknown constants m_1 and m_2 such that

$$0 < m_1 \le k(u,t) \le m_2 \quad (3)$$

Assumption 2: There exists an unknown positive constant d^* such that $|d(t)| \le d^*$.

Assumption 3: Define $F(\boldsymbol{x}, v) = f(\boldsymbol{x}, v) - f(\boldsymbol{x}, 0)$, there always exists an unknown positive m_i $(i = 1, 2, 3, 4)$ making the following inequalities hold.

$$\begin{cases} m_1 v \le F(\boldsymbol{x}, v) \le m_2 v, & v \ge 0 \\ m_3 v \le F(\boldsymbol{x}, v) \le m_4 v, & v < 0 \end{cases} \quad (4)$$

Remark 1: It should be noted that the unknown function $f(\boldsymbol{x}, v(u(t)))$ is commonly assumed to satisfy $0 < g_1 \le \partial f(\boldsymbol{x}, v(u(t)))/\partial v \le g_2$ such as in [2, 8] with g_1 and g_2 being unknown positive constants, which is seen as the controllability condition of their systems. However, in this paper, Assumption 3 is utilized to guarantee the controllability of system (1), while the restrictive assumption that non-affine function must be differentiable has been removed. Moreover, we have also considered a class of uncertain actuator nonlinearity simultaneously when designing the control scheme.

According to Assumption 3, if $v < 0$, we have

$$F(\boldsymbol{x}, v) = [(1 - \theta_1(t))m_1 + m_2\theta_1(t)]v \quad (5)$$

where $\theta_1(t) \in [0, 1]$, if $v < 0$, we obtain

$$F(\boldsymbol{x}, v) = [(1 - \theta_2(t))m_3 + m_4\theta_2(t)]v \quad (6)$$

with $\theta_2(t) \in [0, 1]$.

Define

$$G(t) = \begin{cases} (1 - \theta_1(t))m_1 + m_2\theta_1(t), & v \ge 0 \\ (1 - \theta_2(t))m_3 + m_4\theta_2(t), & v < 0 \end{cases} \quad (7)$$

$$\text{Then, one has } 0 < \min_{i=1,2,3,4}\{m_i\} \le G(t) \le \max_{i=1,2,3,4}\{m_i\} = G_{\max} \quad (8)$$

Hence, the non-affine system (1) can be converted into the following affine system

$$\begin{cases} \dot{x}_i = x_{i+1},\ i = 1, 2..., n-1 \\ \dot{x}_n = g(\boldsymbol{x}) + k(u,t)G(t)u(t) + G(t)\varepsilon_u + d(\boldsymbol{x}, t) \\ y = x_1 \end{cases} \tag{9}$$

Radial basis function neural network can approximate any continuous nonlinear function $h(\boldsymbol{Z})$ with any precision. Namely

$$h(\boldsymbol{Z}) = \boldsymbol{W}^{*T}\boldsymbol{\psi}(\boldsymbol{Z}) + \varepsilon \tag{10}$$

where $\boldsymbol{Z} \in \boldsymbol{\Omega}_z \subset \boldsymbol{R}^n$ is a input vector; n is the input dimension of neural network; ε is the approximation error satisfying $|\varepsilon| \le \varepsilon^*$ with ε^* being an unknown positive constant; $\boldsymbol{\psi}(\boldsymbol{Z}) \in \boldsymbol{R}^l$ is commonly selected as Gaussian function, and $\boldsymbol{W}^* \in \boldsymbol{R}^l$ is the optimal weight vector defined by

$$\boldsymbol{W}^* = \arg\min_{\boldsymbol{W} \in \boldsymbol{R}^l} \left\{ \sup_{\boldsymbol{Z} \in \boldsymbol{\Omega}_Z} |h(\boldsymbol{Z}) - \boldsymbol{W}^T\boldsymbol{\psi}(\boldsymbol{Z})| \right\} \tag{11}$$

where $\boldsymbol{W}$ is a weight vector.

The following definition and lemma are instrumental to stability analysis.

Definition 1: A function $N(\cdot)$ is called a Nussbaum-type function if it has the following properties

$$\begin{aligned} \lim_{s\to\infty} \sup \tfrac{1}{s}\textstyle\int_0^s N(\zeta)d\zeta = +\infty, \\ \lim_{s\to\infty} \inf \tfrac{1}{s}\textstyle\int_0^s N(\zeta)d\zeta = -\infty \end{aligned} \tag{12}$$

Lemma 1 [9]: $V(\cdot)$ and $\zeta(\cdot)$ are smooth functions defined on $[0,\ t_f)$ with $V(t) \ge 0$, and $N(\zeta)$ is a Nussbaum-type function. If the following inequality holds

$$V(t) \le c_1 + e^{-c_2 t}\int_0^t [g(x(\tau))N(\zeta(\tau)) + 1]\dot{\zeta}e^{-c_2\tau}d\tau \tag{13}$$

where c_1 and c_2 are positive constants; $g(x(\tau))$ is a time-varying parameter which takes values in the intervals $I = [l^-, l^+]$ with $0 \notin I$, then, $V(t)$, $\zeta(t)$ and $\int_0^t N(\zeta(\tau))\dot{\zeta}d\tau$ are bounded in $[0, t_f)$.

We are now in the position to state the control objective.

Control objective: Design an adaptive neural network control law combined with the Nussbaum gain technology to make the system output y follow the desired trajectory y_d accurately. Assume that the desired reference trajectory y_d is bounded, namely, y_d, $y_d^{(1)}$, $y_d^{(2)}$, ...,$y_d^{(n)}$ are continuous and bounded, we define $\boldsymbol{x}_d = [y_d, \dot{y}_d, \ldots, y_d^{(n-1)}]^T$, $\boldsymbol{x}_d \in R^n$, where $\boldsymbol{e} = \boldsymbol{x} - \boldsymbol{x}_d$ is the tracking error.

For compactness, in the following let $|\bullet|$ denote the Euclidean norm of vector $\bullet$, $||\cdot||$ represents the 2-norm, $\hat{\bullet}$ is the estimate of $\bullet^*$ with $\tilde{\bullet} = \bullet^* - \hat{\bullet}$, and the Nussbaum function is chosen as $N(\zeta) = e^{\zeta^2}\cos(\pi\zeta/2)$ in this paper.

3 Controller Design and Stability Analysis

To begin with the design, we firstly define a filtered tracking. Using the idea of sliding mode control, the filtered tracking error is designed as follows

$$r = [\boldsymbol{\Lambda}^T \quad 1]\boldsymbol{e} \tag{14}$$

where $\boldsymbol{\Lambda} = [\lambda_{n-1}, \lambda_{n-2}, \ldots, \lambda_1]^T$ is a design vector with $s^{n-1} + \lambda_1 s^{n-2} + \cdots + \lambda_{n-1}$ a Hurwitz polynomial. The time derivative of the filtered tracking error is

$$\begin{aligned}\dot{r} &= g(\boldsymbol{x}) + k(u,t)G(t)u(t) + G(t)\varepsilon_u \\ &\quad + d(\boldsymbol{x},t) - y_d^{(n)} + [0 \quad \boldsymbol{\Lambda}^T]\boldsymbol{e}\end{aligned} \tag{15}$$

To consider the stability of (15), define a quadratic function as follows

$$V_r = \frac{1}{2}r^2 \tag{16}$$

The time derivative of V_r along (15) is

$$\begin{aligned}\dot{V}_r &= r[g(\mathbf{x}) + k(u,t)G(t)u(t) + [0 \quad \boldsymbol{\Lambda}^T]\boldsymbol{e} \\ &\quad + G(t)\varepsilon_u + d(\boldsymbol{x},t) - y_d^{(n)}]\end{aligned} \tag{17}$$

From Assumption 2 and (8), we have

$$\begin{aligned}\dot{V}_r &\leq rg(\boldsymbol{x}) + rk(u,t)G(t)u(t) \\ &\quad + |r|(G_{\max}\varepsilon_u^* + d^*) + rY_d\end{aligned} \tag{18}$$

where $Y_d = -y_d^{(n)} + [0 \quad \boldsymbol{\Lambda}^T]\boldsymbol{e}$. Since $g(\boldsymbol{x})$ is an unknown continuous function, we apply a RBF neural network to approximate it on a compact set, namely

$$g(\boldsymbol{x}) = \boldsymbol{W}^{*T}\boldsymbol{\psi}(\boldsymbol{x}) + \varepsilon \tag{19}$$

where ε is the approximation error. It follows from (10) that there exists an unknown positive ε^* such that $|\varepsilon| \leq \varepsilon^*$ with ε^* being an unknown positive constant. Since the optimal weight vector $\boldsymbol{W}^*$ is unknown, we will use its estimate $\hat{\boldsymbol{W}}$ for the controller design. Substituting (19) into (18) yields

$$\begin{aligned}\dot{V}_r &\le r\boldsymbol{W}^{*T}\boldsymbol{\psi}(\boldsymbol{x}) + rk(u,t)G(t)u(t) \\ &\quad + |r|(G_{\max}\varepsilon_u^* + d^* + \varepsilon^*) + rY_d \\ &\le r\boldsymbol{W}^{*T}\boldsymbol{\psi}(\boldsymbol{x}) + |r|\delta^* + rk(u,t)G(t)u(t) + rY_d\end{aligned} \tag{20}$$

with $\delta^* = G_{\max}\varepsilon_u^* + d^* + \varepsilon^*$ being an unknown positive constant.

Consider the following Lyapunov function

$$V = V_r + \frac{1}{2}\tilde{\boldsymbol{W}}^T\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{W}} + \frac{1}{2\gamma}\tilde{\delta}^2 \tag{21}$$

where $\boldsymbol{\Gamma} = \boldsymbol{\Gamma}^{-1}$ denotes adaptive gain matrix; and γ is an adaptive gain coefficient; $\tilde{\boldsymbol{W}} = \boldsymbol{W}^* - \hat{\boldsymbol{W}}$ and $\tilde{\delta} = \delta^* - \hat{\delta}$ are parameter estimation errors.

It follows from (20) that the time derivative of V is

$$\begin{aligned}\dot{V} &\le r\boldsymbol{W}^{*T}\boldsymbol{\psi}(\boldsymbol{x}) + |r|\delta^* + rk(u,t)G(t)u(t) + rY_d \\ &\quad - \tilde{\boldsymbol{W}}^T\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{W}}} - \frac{1}{\gamma}\tilde{\delta}\dot{\hat{\delta}}\end{aligned} \tag{22}$$

Design the actual control law and adaptation laws as follows

$$u = N(\zeta)\left[\beta r + \hat{\boldsymbol{W}}^T\boldsymbol{\psi}(\boldsymbol{x}) + \hat{\delta}\cdot\tanh\left(\frac{r}{\tau}\right) + Y_d\right] \tag{23}$$

$$\dot{\zeta} = \beta r^2 + r\hat{\boldsymbol{W}}^T\boldsymbol{\psi}(\boldsymbol{x}) + \hat{\delta}\cdot r\tanh\left(\frac{r}{\tau}\right) + rY_d \tag{24}$$

$$\begin{cases}\dot{\hat{\boldsymbol{W}}} = \boldsymbol{\Gamma}(r\boldsymbol{\psi}(\boldsymbol{x}) - \sigma_1\hat{\boldsymbol{W}}) \\ \dot{\hat{\delta}} = \gamma\left(r\tanh\left(\frac{r}{\tau}\right) - \sigma_2\hat{\delta}\right)\end{cases} \tag{25}$$

where $\beta > 0$, $\tau > 0$, $\sigma_1 > 0$ and $\sigma_2 > 0$ are design parameters.

Then, the stability of the closed-loop system is analyzed as follows.

Substituting (23) into (22), we obtain

$$\begin{aligned}\dot{V} &\le r\boldsymbol{W}^{*T}\boldsymbol{\psi}(\boldsymbol{x}) + |r|\delta^* + k(u,t)G(t)N(\zeta)\dot{\zeta} \\ &\quad + rY_d - \tilde{\boldsymbol{W}}^T\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{W}}} - \frac{1}{\gamma}\tilde{\delta}\dot{\hat{\delta}}\end{aligned} \tag{26}$$

From (26) we can further have

$$\begin{aligned}\dot{V} &\le k(u,t)G(t)N(\zeta)\dot{\zeta} + \delta^*\left[|r| - r\tanh\left(\frac{r}{\tau}\right)\right] \\ &\quad - \tilde{\boldsymbol{W}}^T\boldsymbol{\Gamma}^{-1}\left[\dot{\hat{\boldsymbol{W}}} - r\boldsymbol{\Gamma}\boldsymbol{\psi}(\mathbf{x})\right] - \beta r^2 \\ &\quad - \frac{1}{\gamma}\tilde{\delta}\left[\dot{\hat{\delta}} - \gamma\cdot r\tanh\left(\frac{r}{\tau}\right)\right] + \dot{\zeta}\end{aligned} \tag{27}$$

and substituting (25) into (27), one has

$$\begin{aligned}\dot{V} \leq & -\beta r^2 + k(u,t)G(t)N(\zeta)\dot{\zeta} + \sigma_1 \tilde{\boldsymbol{W}}^{\mathbf{T}}\hat{\boldsymbol{W}} \\ & + \sigma_2\tilde{\delta}\hat{\delta} + \delta^*\left[|r| - r\tanh(\frac{r}{\tau})\right] + \dot{\zeta}\end{aligned} \tag{28}$$

Using the following inequality for any $\tau > 0$ and $r \in R$ [10].

$$|r| - r \cdot \tanh(r/\tau) \leq 0.2785\tau \tag{29}$$

Substituting (29) into (27), we obtain

$$\begin{aligned}\dot{V} \leq & -\beta r^2 + k(u,t)G(t)N(\zeta)\dot{\zeta} + \dot{\zeta} \\ & + \sigma_1 \tilde{\boldsymbol{W}}^{\boldsymbol{T}}\hat{\boldsymbol{W}} + \sigma_2\tilde{\delta}\hat{\delta} + 0.2785\tau\delta^*\end{aligned} \tag{30}$$

Utilizing the following inequalities

$$\sigma_1 \tilde{\boldsymbol{W}}^{\mathbf{T}}\hat{\boldsymbol{W}} \leq -\frac{\sigma_1}{2}\left\|\tilde{\boldsymbol{W}}\right\|^2 + \frac{\sigma_1}{2}\left\|\boldsymbol{W}^*\right\|^2$$

$$\sigma_2\tilde{\delta}\hat{\delta} \leq -\frac{\sigma_2}{2}\tilde{\delta}^2 + \frac{\sigma_2}{2}\delta^{*2}$$

Consequently, we can further have

$$\dot{V} \leq -\alpha_1 V + [k(u,t)G(t)N(\zeta) + 1]\dot{\zeta} + \alpha_0 \tag{31}$$

where $\alpha_1 = \min\left\{2\beta, \frac{\sigma_1}{\lambda_{\max}(\boldsymbol{\Gamma}^{-1})}, \sigma_2\gamma\right\}\alpha_0 = 0.2785\tau\delta^* + \frac{\sigma_1}{2}\left\|\boldsymbol{W}^*\right\|^2 + \frac{\sigma_2}{2}\delta^{*2}$.

Multiply (31) by $e^{a_1 t}$ and integrate (31) over $[0,t]$, we have

$$\begin{aligned}V(t) \leq & \frac{\alpha_0}{\alpha_1} + (V(0) - \frac{\alpha_0}{\alpha_1})e^{-\alpha_1 t} + \\ & e^{-\alpha_1 t}\int_0^t [k(u,t)G(\tau)N(\zeta) + 1]\dot{\zeta}e^{\alpha_1\tau}d\tau \\ & \leq \frac{\alpha_0}{\alpha_1} + V(0) \\ & + \int_0^t [k(u,t)G(\tau)N(\zeta) + 1]\dot{\zeta}e^{-\alpha_1(t-\tau)}d\tau\end{aligned} \tag{32}$$

In view of Lemma (1), we know that $V(t)$, $\zeta(t)$ and $\int_0^t k(u,t)G(\tau)N(\zeta)\dot{\zeta}d\tau$ are bounded on $[0,t_f)$. Therefore, let

$$\int_0^t |k(u,t)G(\tau)N(\zeta) + 1|\dot{\zeta}e^{-\alpha_1(t-\tau)}d\tau \leq Q \tag{33}$$

From (32) and (33), one has

$$\lim_{t\to\infty} V(t) \le \frac{a_0}{a_1} + V(0) + Q \tag{34}$$

Since $V(t)$ is bounded, we derive that all signals of closed-loop system are SGUUB.

Then, define

$$C_0 = \sqrt{2(a_0/a_1 + V(0) + Q)} \tag{35}$$

According to (21) and (35), we obtain

$$r \le \sqrt{2V(t)} \le C_0 \tag{36}$$

Therefore, by invoking the definition of filter tracking error r, we know the tracking error $\boldsymbol{e}$ is bounded, moreover, when $r \to 0$, tracking error $\boldsymbol{e} \to 0$. Consequently, the system output y can track the desired trajectory y_d accurately.

4 Simulation Results

Example 1: Consider a class of pure feedback nonlinear systems with actuator input dead zone nonlinearity as follows

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -p_1 x_1 - p x_2 - x_1^3 + q\cos(wt) \\ \qquad\qquad + h(\boldsymbol{x}, v(u(t))) + d(\boldsymbol{x}, t) \\ y = x_1 \end{cases}$$

where $h(\boldsymbol{x}, v(u(t))) = (1 + 0.1\cos x_1)(1 + 0.2\cos v(u(t))) \times v(u(t))$ is a non-affine term and $d(\boldsymbol{x}, t) = 0.1(x_1^2 + x_2^2)\cdot \sin t$ denotes external disturbance; $p_1 = -0.2$; $q = 5 + 0.1\cos(t)$; $p = 0.2 + 0.2\cos(5x_1)$ and $w = 0.5 + 0.1\sin(t)$ are uncertain parameters, initial conditions $x_1(0) = 0.5$, $x_2(0) = 0$ and we assume the desired trajectory $y_d = 0.5 \times (\sin t + \sin(0.5t))$.

The model of actuator dead zone nonlinearity is as follows

$$v(u(t)) = \begin{cases} (1 + 0.3\sin(u))(u - 0.5) & 0.5 \le u \\ 0 & -0.3 \le u \le 0.5 \\ (0.8 + 0.2\cos(u))(u + 0.3) & u \le -0.3 \end{cases}$$

In the simulation, the controller $u(t)$ is designed as (23), the parameter adaptation laws are chosen as (24) and (25), respectively. The Gaussian function is selected as the basis function of RBF neural network as follows

$$\psi(Z) = \mathrm{e}^{-(\mathbf{Z}-\mu_i)^T(\mathbf{Z}-\mu_i)/\upsilon_i^2}, i = 1, 2, \ldots, l$$

The parameters of RBF neural network are set as: $\hat{\boldsymbol{W}}^T \boldsymbol{\psi}(\boldsymbol{x})$ contains $l = 27$ nodes, the center $\mu_i (i = 1, 2, \ldots, 27)$ is evenly distributed $[-10, 10] \times [-10, 10]$, width $\upsilon_i = 2 (i = 1, 2, \ldots, 27)$. The initial values of neural networks weights $\hat{\boldsymbol{W}}(0)$ are set to zero and the initial conditions of parameters estimations are set as $\hat{\delta}(0) = 0$ and $\zeta(0) = 1$. The remaining parameters are selected as: $\boldsymbol{\Gamma} = diag[0.5]$, $\lambda_1 = 1.5$, $\sigma_1 = \sigma_2 = 0.5$, $\gamma = 1.5$, $\tau = 0.2$, $\beta = 2.5$. Simulation results are shown in Fig. 1. It is seen from Fig. 1 that the proposed approach is sufficient to make the systems output follow the desired reference trajectory and fairly good tracking performance has been achieved. In addition, the boundedness of variable x_2 and control input u can also be observed from Fig. 1.

Note that the non-affine function $h(\boldsymbol{x}, v(u(t)))$ contains dead zone nonlinearity and is therefore obviously non-differentiable, which implies that the existing methods cannot work, while our approach is adequate to control this system.

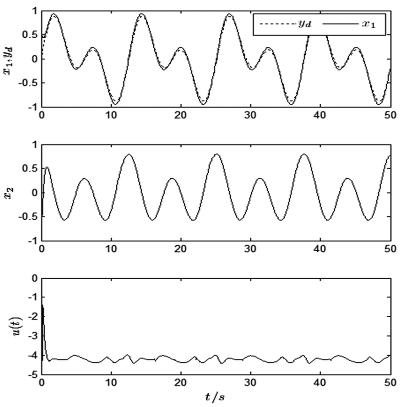

Fig. 1. The response curves of Example 1

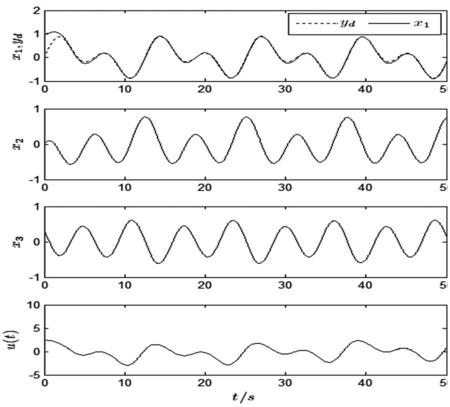

Fig. 2. The responses of Genesio system

Moreover, when designing the controller, all the existing methods have not considered the influence of actuator input nonlinearity, while our scheme has taken it into account.

Example 2: Consider a Genesio system with parametric perturbations and actuator input backlash nonlinearity as follows

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -cx_1 - bx_2 - ax_3 + x_1^2 + h(\boldsymbol{x}, v(u(t))) + d(\boldsymbol{x}, t) \\ y = x_1 \end{cases}$$

where $h(\boldsymbol{x}, v(u(t))) = (2 + \cos x_1)(1 + 0.2\sin^2 v(u(t)))\times\ v(u(t))$ is non-affine function; $d(\mathbf{x}, t) = \sin(0.1t)$ denotes external interference and $a = 1.6$; $b = 3$; $c = 6$; initial states are set as $x_1(0) = 1$, $x_2(0) = 0$, $x_3(0) = 0$. Set the desired output trajectory as $y_d = 0.5(\sin t + \sin(0.5t))$.

The model of actuator backlash nonlinearity is expressed as

$$v(u(t)) = \begin{cases} 1.2(u - 0.5), & \dot{u} > 0 \ and \ \varphi(u) = 1.2(u - 0.5) \\ 1.2(u + 0.6), & \dot{u} < 0 \ and \ \varphi(u) = 1.2(u + 0.6) \\ v(t\_) & otherwises \end{cases}$$

In the simulation, the design parameters are selected as: $\boldsymbol{\Gamma} = diag[0.5]$, $\lambda_1 = 2$, $\lambda_2 = 1$, $\sigma_1 = \sigma_2 = 0.3$, $\gamma = 1.2$, $\tau = 0.5$, $\beta = 5$ and the remaining parameters and the controller structure keep unchanged. The results are depicted in Fig. 2.

It can be easily seen from Fig. 2 that the proposed scheme can not only achieve good tracking performance even in the presence of actuator input dead zone nonlinearity while taking actuator input backlash nonlinearity, but also in the presence of non-differentiable non-affine functions.

5 Conclusions

By modeling the non-affine function and actuator dead zone and backlash model appropriately, a novel adaptive neural tracking control scheme is presented for a more general class of uncertain pure feedback systems. The proposed method is not only applicable to the actuator dead zone nonlinearity, but also is suitable for the backlash nonlinearity. In addition, the assumption that the actuator function must be known has been canceled and the restrictive differentiability condition of non-affine function has been relaxed. Finally, the performance of the proposed approach has been verified through two simulation examples.

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