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Absolute and Relative Orbit Determination for the CHAMP/GRACE Constellation

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Abstract

Precise orbit determination was investigated for a satellite constellation comprised of two different missions, the CHAllenging Minisatellite Payload (CHAMP) satellite and the Gravity Recovery And Climate Experiment (GRACE) twin satellites. The orbital planes of these two missions aligned closely during March to May 2005, allowing precise baseline determinations between the associated three satellites based on their onboard BlackJack Global Positioning System (GPS) receivers. The GRACE-A/B satellites fly in tandem formation with a baseline of around 220 km, whereas the baselines between CHAMP and the GRACE tandem vary from about 110 to 7500 km during 24-hr orbital arcs centered around the points of closest approaches. A number of factors had to be dealt with for orbit determinations, including the cross-talk between the CHAMP GPS main navigation and occultation antennas, the different levels of non-gravitational accelerations, and the rapidly changing geometry that complicates the fixing of integer ambiguities for the GPS carrier-phase observations.

Quality assessments of the orbit solutions were based on comparisons with Satellite Laser Ranging (SLR) observations, best orbit solutions had a precision of typically 1.7-2.3 cm. Consistency checks between reduced-dynamic and kinematic orbit solutions were done. For the GRACE baselines, the reduced-dynamic/kinematic baseline consistency was typically better than 1 cm, with an ambiguity fixing success rate of around 94%. The agreement with the K/Ka-Band Radar Ranging (KBR) measurements was about 0.6 mm. For the CHAMP/GRACE pairs, the reduced-dynamic/kinematic baseline consistency varied from 0.5 to 2.5 cm, where better consistency was obtained for shorter arcs.

Keywords: Precise Orbit Determination, Precise Baseline Determination, High-dynamic Baseline, Satellite Constellation, Integer Ambiguity, Antenna Pattern

1. Introduction

Precise Orbit Determination (POD) is a prerequisite for many Low Earth Orbit (LEO) satellites for meeting their mission objectives. Often, absolute orbit precision levels of a few cm are required (Vetter, 2007). Nowadays, the Global Positioning System (GPS) becomes the prime system for cm-level POD of LEO satellites (Wu et al., 1991). In addition, formation flying missions

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like the tandem GRACE and the bistatic TerraSAR-X/TanDEM-X have shown the capability of GPS-based mm-level Precise Baseline Determination (PBD) (Kroes et al., 2005; Montenbruck et al., 2011), where the best precision is obtained after fixing the integer ambiguities of Double-Differenced (DD) carrier-phase observations (Teunissen, 1999). For GRACE, a PBD precision level of better than 1 mm has been achieved. In addition, few mm precision levels have been claimed for the TerraSAR/TanDEM-X satellites (Allende-Alba and Montenbruck, 2016). The Swarm constellation has been used as a test bed for also PBD, where two of the three identical Swarm satellites fly in a pendulum formation (Friis-Christensen et al., 2008). It has been shown that great care has to be taken with GPS observation pre-processing, characterizing carrier-phase center and code residual variations, estimating and constraining (relative) empirical accelerations that absorb force model errors and orbital arc length, and fixing DD integer carrier-phase cycle ambiguities (Allende-Alba et al., 2017; Mao et al., 2018). In general, the present PBD research focus mostly on satellite formations with relatively stable baselines between hundreds of meters up to 220 km (GRACE).

At present many space projects call for the implementation of more complex constellations to fulfill corresponding mission requirements and PBD for those satellites might support their mission objectives (Sabol et al., 2001). The European Copernicus Program has been building a constellation comprised of diverse satellites functioning in different orbits until the end of the next decade (Butler, 2014). One of its composed sub-missions, Sentinel-1, will consist of two or even four satellites flying in formation to provide continuous global radar imaging observations (Torres et al., 2012). By the end of 2018, an Iridium-next constellation will be completed with 66 functioning satellites. A simulation shows its capacity of making use of their GPS-based orbits for recovering large-scale gravity variations and especially for tracking large scale movement of water mass (Gunter et al., 2012). Moreover, the next generation gravity field recovery satellite mission will probably consist of two separate formations operating in different orbital planes (Elsaka et al., 2014). These space missions flying in large constellations definitely require more robust absolute and relative orbit reconstructions, for which the POD and PBD methods need to be further investigated.

Van Barneveld (2012) initiated an innovative research for a complex constellation comprised of the CHAMP (Reigber et al., 2002) and the GRACE twin satellites (Tapley et al., 2004a). After the launch of CHAMP on 15 July 2000 and GRACE on 17 March 2002, a favorable geometry existed in the first half of 2005 when the two orbital planes were closely aligned. In this period, the CHAMP and GRACE satellites regularly closely approached each other allowing to serve as an ideal test bed for assessing the performance of POD and PBD methods. The GRACE formation baseline length was always varying around 220 km, whereas each CHAMP/GRACE baseline length varied rapidly from 110 to more than 7500 km in a 24-hr period around the point of closest approach. Nonetheless, Van Barneveld (2012) showed that it is very challenging to have a completely correct carrier-phase integer cycle ambiguity fixing. In fact, the precision of PBD solutions often significantly deteriorated because of wrongly fixed ambiguities, particularly for the ground-based PBD which experienced more challenging disturbances such as troposphere (Blewitt, 1989). The work presented in this study builds on the work initiated by Van Barneveld (2012). It is assessed if PBD solutions can be enhanced by considering the following modifications based on the implementation outlined by Van Barneveld (2012):

- A different approach is adopted to select orbital arcs. The CHAMP satellite and the GRACE twin satellites baseline lengths reach a local minimum in every 2.7 days in the favorable time period. The epoch of the closest approach - or encounter - is determined and then a 24-hr orbital arc is defined starting 12 hr before and ending 12 hr after. This differs with Van Barneveld (2012) in which orbital arcs started and ended at midnight and thus resulted in significantly different constellation geometries. In fact, also the impact of the length of the orbital arc on the reliability of DD carrier-phase ambiguity fixing and associated precision of baseline solutions is assessed. Typically, 24-hr arcs are used in GPS-based POD and PBD, but due to the rapidly changing geometry for the CHAMP/GRACE baselines, it is interesting to assess the impact of the arc length which has been varied from 2 to 24 hr.

- The so-called Code Residual Variation (CRV) patterns are applied to correct GPS code observations (Mao et al., 2017). The need of using antenna Phase Center Variation (PCV) patterns in both POD (Haines et al., 2004; Jäggi et al., 2009; Van den IJssel et al., 2015) and PBD (Allende-Alba and Montenbruck, 2016; Mao et al., 2017) has been well demonstrated for many satellite missions. In addition to these PCV patterns, CRV patterns were found to further improve POD and PBD when GPS signal cross-talk and large multi-path existed on the GPS main navigation antenna of the trailing GRACE satellite (Mao et al., 2017). During the selected period, the CHAMP satellite had kept its space-borne GPS occultation antenna activated and therefore signal interference occurred on the main antenna. No GPS antenna code patterns were applied in Van Barneveld (2012).

- The GRACE DD carrier-phase ambiguities are fixed before introducing the DD carrier-phase observations between the CHAMP and GRACE satellites. The so-called empirical accelerations that absorb GRACE force model errors can be more heavily constrained in a relative sense than for the CHAMP/GRACE combinations leading to more reliable DD ambiguity fixing. These then serve as prior information to compute the CHAMP/GRACE baselines. For the CHAMP/GRACE baselines, both solutions with and without ambiguity fixing (the latter referred to as *float* solution), are generated. In the *float* solution, the GRACE solution will then not be influenced by the additional CHAMP-relevant ambiguity resolution. In Van Barneveld (2012) no float solution was computed. As stated above, Van Barneveld (2012) showed that especially wrongly-fixed integer ambiguities affect the precision of both GRACE and CHAMP/GRACE baselines. In this research, the comparison between two solutions will show the detailed impact of CHAMP/GRACE baseline ambiguity fixing on the GRACE baseline.

This research selects 30 24-hr orbital arcs from March to May 2005, or the period indexed by Day-Of-Year (DOY): 061-152. Each orbital arc provides one relatively stable GRACE baseline and two highly variable baselines, *i.e.* CHAMP/GRACE-A and CHAMP/GRACE-B. The quality of POD and PBD solutions will be assessed by comparison with independent Satellite Laser Ranging (SLR) observations (Degnan, 1993) and, for GRACE, also K-Band Radar ranging (KBR) system (Tapley et al., 2004b). In addition, quality assessments will be done by comparison with external orbit solutions and by comparing kinematic with reduced-dynamic POD and PBD solutions (Allende-Alba and Montenbruck, 2016; Mao et al., 2018).

This study is organized as follows. Section 2 introduces the selected satellite constellation and observational data, including a quality assessment of the latter. Section 3 briefly presents the kinematic and reduced-dynamic PBD methods and associated parameterizations. Section 4 includes the assessment of GPS observations processing and antenna CRV patterns. PBD is done for each dual-satellite pair and the triple-satellite constellation. Moreover, the results of quality assessments are discussed. In Section 5 conclusions and future recommendations are made resulting from this research.

2. Satellite Constellation

As outlined in the previous Section, a constellation is formed by selecting CHAMP and the two GRACE satellites. For this constellation, 30 orbital 24-hr arcs are selected during which CHAMP closely approaches the GRACE tandem. This section includes a quality assessment of the associated GPS data.

2.1. Orbital Arc Selection

The CHAMP and GRACE satellites were initially deployed into different polar orbits, which aligned closely in April 2005. In this period, the difference between the CHAMP and GRACE altitudes is around 110 km on the average and the alignment of the orbital planes follows from the adjacent values for the inclination and right ascension of ascending node (see Table 1 for the relevant Keplerian orbital elements). The lower CHAMP satellite experiences stronger non-gravitational perturbations than GRACE mostly due to the denser neutral thermosphere along its in-flight direction (Doornbos, 2012). The best alignment of the orbital planes occurs on 7 April 2005.

Table 1: Three selected Keplerian orbital elements for the CHAMP and GRACE satellites during March-May 2005. a represents the semi-major axis, I the orbit inclination and *RAAN* the right ascension of the ascending node (Credit: satellite two line elements data is obtained from www.space-track.org).

Satellite	a [km]	I [deg]	*RAAN* [deg]
GRACE-A	6849.3-6848.3	89.0	211.4-199.5
GRACE-B	6849.3-6848.3	89.0	211.4-199.5
CHAMP	6739.9-6734.4	87.2	221.1-185.4

The altitude difference leads to an orbital period difference of around 2.2 min. Therefore, CHAMP will have a closest approach with the GRACE twin satellites approximately every 2.7 days. Eventually 35 24-hr orbital arcs can be selected based on these orbit encounters (Table 2). Please note that the orbital arc indexed by DOY 096 is excluded because of maneuvers by GRACE-B, DOYs 061, 064, 074 and 077 are excluded due to large gaps in the GPS data of at least one of the three satellites. A representative orbital arc is illustrated in Figure 1. It can be clearly seen that the GRACE baseline is maintained at a value of around 220 km. Two CHAMP/GRACE baseline lengths vary dramatically from 110 to 7500 km in a 12-hr period close to the encounter.

2.2. Data Quality Assessment

Figure 2 shows the number of GPS satellites that are simultaneously tracked by each satellite pair. For a dual-satellite PBD, a minimum of 5 GPS satellites is required that are simultaneously in view (Kroes, 2006). The GRACE pair shares on average 7 satellites that are in view simultaneously. For the high-low CHAMP/GRACE satellite pairs this number drops rapidly as the baseline lengths increase. Surprisingly, for the CHAMP/GRACE-A satellite pair this number is larger than that for the GRACE tandem when the distance is smaller than 500 km.

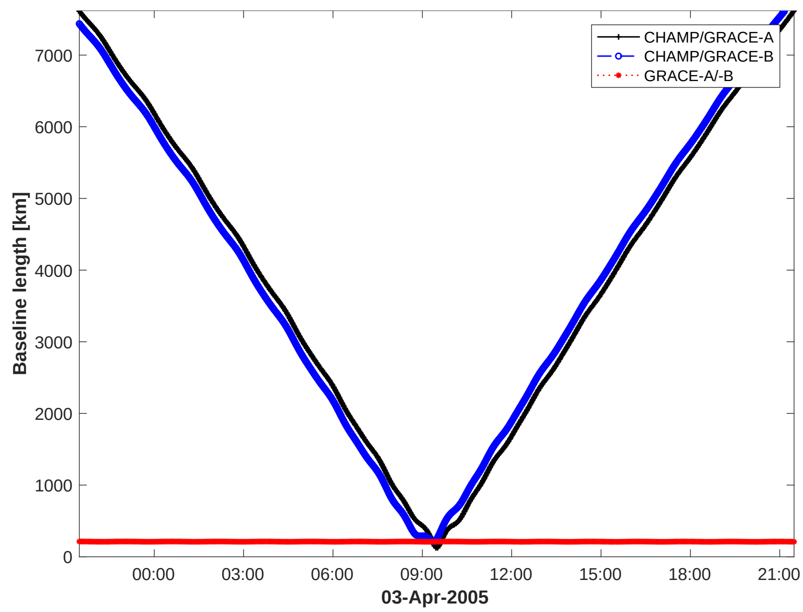

Figure 1: Baseline length variations for each dual-satellite pair during a representative 24-hr orbital arc (DOY 093, 2005). The GRACE baseline has been stabilized at around 220 km, the two CHAMP/GRACE baselines are rapidly changing from 110 to 7500 km in 24 hours.

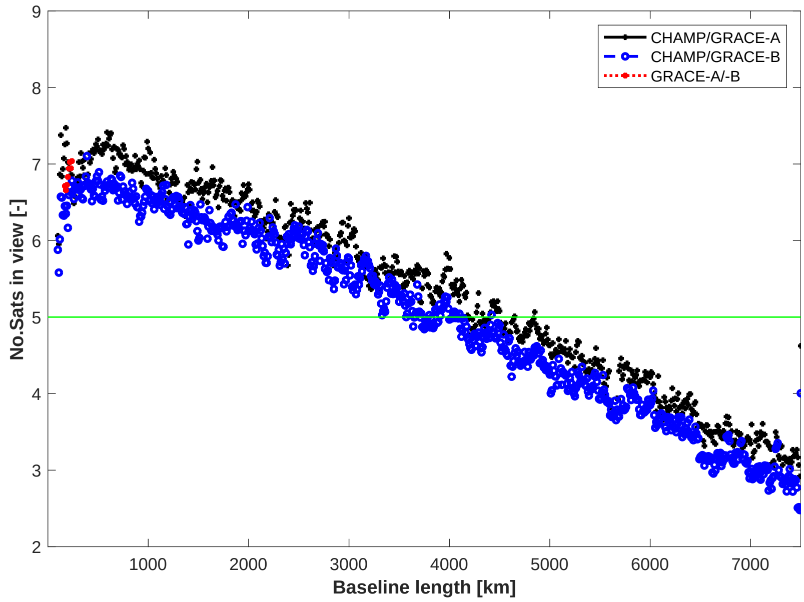

Figure 2: The number of GPS satellites simultaneously tracked by two space-borne GPS receivers as a function of distance (every 10 km) for each dual-satellite pair (analysis for 30 24-hr orbital arcs).

Table 2: Thirty-five identified 24-hr orbital arcs for the CHAMP/GRACE constellation. Please note that DOY specifies the day of the middle epoch of each orbital arc. This DOY number will be used as orbital arc identifier in this research (the 5 orbital arcs marked by * are excluded).

Date (YYYY-MM-DD)	DOY	Middle of the arc	Minimum distance (km)
2005-03-02	061	13:37:30	235.97*
2005-03-05	064	05:09:30	208.80*
2005-03-07	066	21:27:00	122.47
2005-03-10	069	13:44:00	191.49
2005-03-13	072	05:16:00	183.20
2005-03-15	074	21:32:20	195.28*
2005-03-18	077	13:03:50	132.32*
2005-03-21	080	04:35:30	119.71
2005-03-23	082	20:06:40	102.98
2005-03-26	085	11:37:40	121.40
2005-03-29	088	03:07:50	171.49
2005-03-31	090	18:00:10	220.47
2005-04-03	093	09:29:00	121.92
2005-04-06	096	00:26:20	205.76*
2005-04-08	098	15:50:10	126.39
2005-04-11	101	06:42:50	123.35
2005-04-13	103	21:31:50	167.06
2005-04-16	106	12:17:40	197.76
2005-04-19	109	03:01:40	201.95
2005-04-21	111	17:45:00	176.84
2005-04-24	114	08:28:00	135.27
2005-04-26	116	23:10:50	129.65
2005-04-29	119	13:53:40	218.93
2005-05-02	122	03:52:00	225.44
2005-05-04	124	18:34:20	111.55
2005-05-07	127	08:31:30	219.99
2005-05-09	129	23:13:20	172.97
2005-05-12	132	13:09:50	130.90
2005-05-15	135	03:06:20	153.27
2005-05-17	137	17:02:20	208.57
2005-05-20	140	06:58:10	151.70
2005-05-22	142	20:53:50	139.24
2005-05-25	145	10:49:30	121.03
2005-05-28	148	00:44:50	227.62
2005-05-30	150	13:54:30	205.15

This is because the CHAMP GPS receiver tracks GPS satellites down to lower elevations than GRACE-B resulting in more DD carrier-phase combinations. It is noted that the number for the CHAMP/GRACE-A pair is nearly always larger than CHAMP/GRACE-B, which again is due to the lower elevation cut-off angle for GRACE-A than for GRACE-B. A smaller number of possible DD carrier-phase combinations results in a larger Geometric Dilution Of Position (GDOP) negatively affecting the quality of PBD solutions. The combination of Figure 1 and 2 reveals that PBD for longer CHAMP/GRACE baselines will be more challenging than shorter baselines, since with increasing baseline lengths less and less GPS satellites are simultaneously in view. It can be also observed that for time instances less than 5 hr away from the point of closest approach (thus arc length of around 10 hr), on the average more than 5 GPS satellites are in view simultaneously.

As stated in Section 1, the application of PCV and CRV maps are essential to fully exploit the precision of GPS observations for POD and PBD. The GPS antenna PCV patterns are created for each individual satellite using the *Residual Approach* method (Jäggi et al., 2009). The use of PCV maps has been widely recognized as standard processing step for satellite POD. In addition, GPS code observations can also be affected by various random and systematic error sources, *e.g.* thermal noise and multipath. The CHAMP GPS main navigation receiver suffers from significant superposition of the direct signal with interfering signals taking a different signal path, referred to as *cross-talk* (Montenbruck and Kroes, 2003; Ho et al., 2012). A GPS data quality assessment has been done based on the methods outlined in (Kroes, 2006). Figure 3 displays estimated code noise levels for the CHAMP and GRACE GPS receivers as a function of elevation. The CHAMP and GRACE-A GPS receivers experience larger patterns at lower elevations, where the collected GPS signal will be influenced more by the ionosphere. Besides, for particularly the L_2 frequency, it can be observed that an irregular pattern exists for CHAMP. This agrees with Montenbruck and Kroes (2003) who reported pronounced code patterns for the aft-looking hemisphere of the navigation antenna. The *cross-talk* was found to impact the P_2 code observations more than the P_1 code observations. This cross-talk signal interference is due to the activated occultation antenna and characterizing the associated impact is expected to improve POD and PBD solutions. A similar phenomenon occurred during the later phases of the GRACE mission lifetime, but not during the selected period for the research described in this study. For the CHAMP/GRACE POD and PBD solutions of this research, GPS antenna CRV patterns were derived and implemented (Section 4).

3. Precise Baseline Determination

This research makes use of the GPS High Precision Orbit Determination Software Tools (GHOST) add-on tool called Multiple Orbit Determination using Kalman filtering (MODK) (Wermuth et al., 2010; Van Barneveld, 2012). The MODK tool relies on an Iterative Extended Kalman Filter (IEKF) and allows the so-called subset ambiguity fixing. MODK is also capable of providing a kinematic baseline solution in addition to the reduced-dynamic baseline solution produced by the IEKF (Mao et al., 2018). Another GHOST software module - KInematic Point Positioning (KIPP) - is used for computing single-satellite kinematic orbit solutions for comparison purposes.

The primary MODK module output consists of reduced-dynamic POD and PBD solutions based on an IEKF, where both gravitational and non-gravitational force models are employed. Force model errors are compensated by the estimation of so-called empirical accelerations. Since the CHAMP and GRACE satellites fly at different altitudes, the level of - especially non-gravitational - force model errors is different. The empirical accelerations are defined in the local-horizontal, local-vertical reference frame, *i.e.* they act in the radial, along-track and cross-track direction, and are defined as first-order Gauss-Markov processes (Bierman, 2006). These are characterized by a correlation time (τ), standard deviation of a-priori values (σ_a) and process noise (σ_p). These parameters are to be set according to predicted force model error level for each satellite. In addition, it is possible to not only set these parameters for absolute, but also relative empirical accelerations. This has been proved to be very beneficial for estimating the

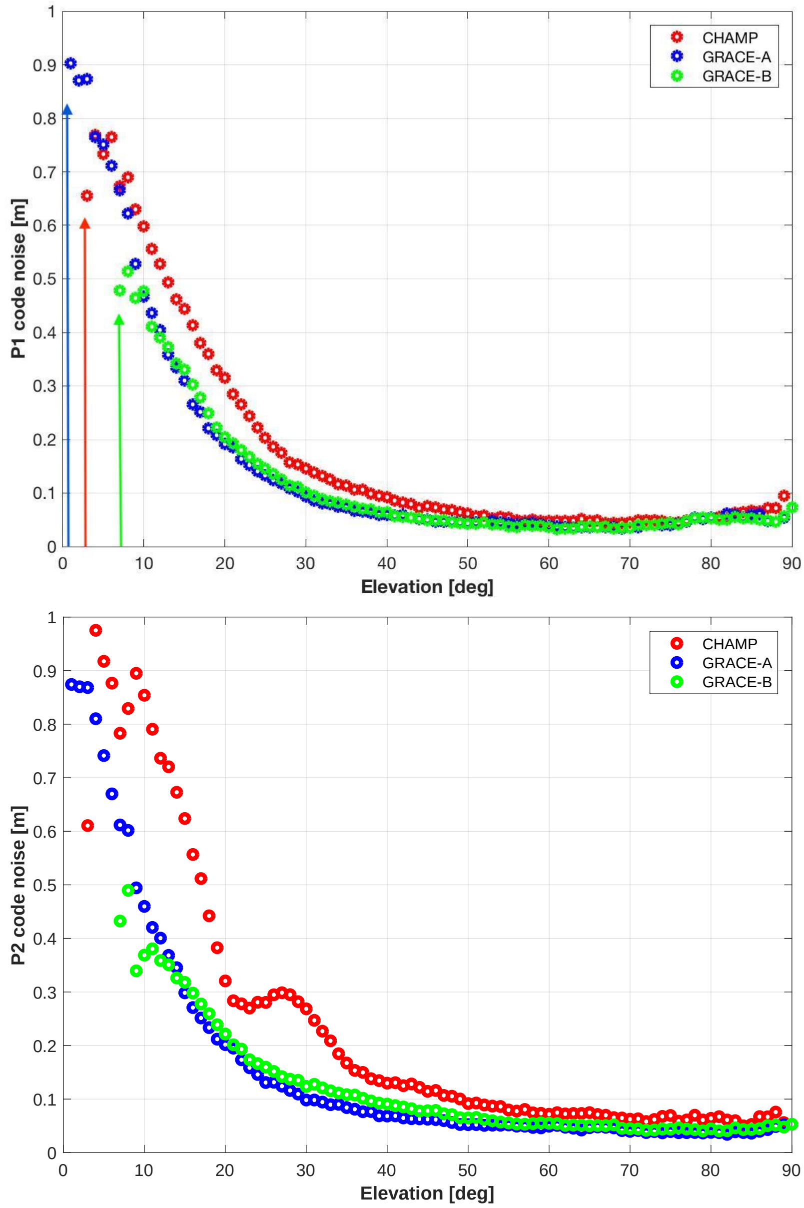

Figure 3: Root-Mean-Square (RMS) of code residuals (mostly multi-path effect, interference, systematic errors, etc.) as a function of elevation for the GPS L_1 (top) and L_2 (bottom) frequencies for the associated GPS receiver RINEX data (selected day: DOY 091, 2005).

empirical accelerations for the GRACE satellites, since they fly almost identical orbits and it is fair to assume that their orbit determination suffer from similar force model errors. It has been demonstrated that constraining the relative empirical accelerations results in improved GRACE and Swarm-A/C baseline solutions (Mao et al., 2017, 2018). For this research, it was found by trial and error that for τ a value of 600 s works well. Values for the other parameters, dynamic models and data sets are included in Table 3.

For the CHAMP orbit, relatively large non-gravitational force model errors are to be absorbed by the empirical accelerations, especially in the along-track direction, for which atmospheric drag dominates. As mentioned above, the empirical accelerations for the two GRACE satellites are constrained such that they more or less take identical values: the differences between these accelerations have values of 0.1, 1.0, and 0.3 nm/s^2 for the process noise σ_p in three directions. The associated values for the CHAMP/GRACE satellite pair combinations is two orders of magnitude larger to make sure the GRACE POD and PBD solutions are not adversely affected. The CHAMP and GRACE GPS receiver carrier-phase observations typically have a precision at the mm-level, compared to dm for the code observations (Montenbruck and Kroes, 2003). When using GPS code and carrier-phase observations in the PBD, the latter should be exploited with higher weight. For the CHAMP and GRACE single-satellite POD the precision level is set at 0.5 m for the code and 4 mm for the carrier-phase observations, comparable to the actual noise levels (see *e.g.* Figure 3), leading to a ratio of 125. When constructing the DD combination between two GPS satellites and two GPS receivers, DD carrier-phase integer ambiguities can be resolved for each GPS frequency, and most common errors such as GPS clock and ephemeris errors are eliminated to the maximal extent. For the single-difference combination the code/carrier-phase weight ratio for GRACE tandem is set as 350 (0.7/0.002 m) to facilitate its pre-constrained ambiguity resolution, whilst for CHAMP/GRACE a much smaller ratio is set (0.7/4000 m, Table 3) to minimize the influence of more challenging CHAMP/GRACE carrier-phase ambiguity fixing on GRACE baseline determination.

The so called Least-squares Ambiguity De-correlation Adjustment (LAMBDA) is selected for ambiguity resolution (Teunissen, 1999). It has been used to determine baseline solutions for several satellite missions (Kroes et al., 2005; Allende-Alba and Montenbruck, 2016). To maximize the ambiguity fixing success rate, a subset fixing process is implemented. It allows to fix only those integer ambiguities that pass several tests (Verhagen, 2005). This is a modification of the conventional use of LAMBDA which only accepts epochs when the entire set of ambiguities pass the tests (Van Barneveld, 2012). Unfixed ambiguities are retained at their float values, but might be fixed in next iterations (provided the IEKF did not yet converge). An extra test was included that results in keeping the float values if one (or more) of the GPS carrier-phase observation residuals that contribute to the DD is (are) above 5 cm.

As mentioned above, MODK has the capability to provide a second baseline solution, referred to as kinematic. To generate this solution, the position of one of the satellites is kept fixed (and thus its absolute position is in fact a reduced-dynamic solution), but the position of the other satellites is then derived kinematically from the GPS observations including the integer ambiguity fixes that were made in the generation of the reduced-dynamic PBD. Kinematic baseline solutions are only generated if at least 5 GPS satellites are in common view of the reference and other satellite for each possible combination pair. Also here an additional test is employed: epochs for which the RMS of GPS observation residuals after kinematic PBD is above 5 cm will be excluded. The kinematic baseline acts as an internal consistency check for the reduced-dynamic PBD. For detailed information regarding the differences between the kinematic and reduced-dynamic approaches please see Mao et al. (2018).

Table 3: Overview of dynamic models, data sets and IEKF parameter settings employed in the MODK PBD package for the CHAMP/GRACE constellation.

Spacecraft model	500 kg canon-ball with cross-section of 1.0 m^2 for GRACE and 0.5 m^2 for CHAMP
Gravitational forces	GOCO03S 120×120 (selectable, maximum 250×250) static gravity field, plus linear trends for spherical harmonic degree 2 terms according to IERS2003 (Mayer-Gürr et al., 2012; McCarthy and Petit, 2004) Luni-solar third body perturbations CSR Ocean tides model based on TOPEX/GRACE data (FES2004 as reference) (Lyard et al., 2006)
Non-gravitational forces	Atmospheric drag: Jacchia 71 density model (Jacchia, 1972) Solar radiation pressure: Conical Earth shadow, Sun flux data
Earth parameters	Leap second data table of TAI-UTC CODE Earth rotation parameters, version 2.0 (Dach et al., 2009)
GPS products	CODE 5s GPS final products and clocks (Dach et al., 2018) IGS transmitter antenna phase center offsets and variations (Schmid et al., 2016) CODE global ionosphere maps (also P1-P2 differential code bias for GPS satellites) (Schaer, 1999)
Antenna patterns	Frequency-dependent PCV and CRV patterns (Mao et al., 2017)
Orbit arc length	2 to 24 hr (selectable with an interval: 2 hr)
Reference orbits	GRACE: Jet Propulsion Laboratory (JPL) L1B orbit products (Bertiger et al., 2002), CHAMP: reduced-dynamic POD orbit computed by MODK
Attitude data	GRACE: JPL L1B (Case et al., 2002), CHAMP: GeoForschungsZentrum (GFZ) ISDC (Reigber et al., 2002)
GPS observations	GRACE: JPL L1B, CHAMP: GFZ ISDC
IEKF iterations	Maximum: 15
GPS data weighting	Code/Carrier-phase: 0.5/0.004 m
Amb. Resolution	LAMBDA subset fixing (Teunissen, 1999; Van Barneveld, 2012)
Amb. Validation (Verhagen, 2005)	Probability test : 99.9% Integer test: 5% Discrimination test: 5 Widelane test: 5%, 0.2 cycles Ionosphere free test: 5%, 0.01 m
GPS data editing Zero-difference	Minimum SNR (signal to noise ratio): 5 Minimum cut-off elevation : 0 ° Ionospheric delay change threshold : 2.0 m Ionosphere-free (IF) code editing outliers : 1.0 m IF carrier-phase editing outliers : 0.015 m
C_D Zero-difference	1 per 24 hr, GRACE: σ_a =2.3, σ_p =1.0, CHAMP: σ_a =4.5, σ_p =1.0
C_R Zero-difference	1 per 24 hr, GRACE: σ_a =1.3, σ_p =1.0, CHAMP: σ_a =1.6, σ_p =1.0
Empirical acc. (τ =600 s unit: nm/s^2)	Radial : σ_a =10, GRACE: σ_p =2, CHAMP: σ_p =4 Along-track: σ_a =40, GRACE: σ_p =8, CHAMP: σ_p =16 Cross-track: σ_a =20, GRACE: σ_p =4, CHAMP: σ_p =8
Ionospheric delay Clock offset	τ =10 s, σ_a =100 m, σ_p =1 m τ =100 s, σ_a =500 m, σ_p =500 m

Single-difference settings	GRACE-A/B	CHAMP/GRACE
Ionospheric delay change threshold	0.5 m	2.5 m
IF code editing outlier	1.0 m	5.0 m
IF carrier-phase editing outlier	0.02 m	0.20 m
C_D	σ_p =0.1	σ_p =1.0
C_R	σ_p =0.1	σ_p =1.0
Empirical acc. (τ =600 s unit: nm/s^2)	Radial: σ_a = 5 , σ_p =0.1 Along-track: σ_a = 30 , σ_p =1 Cross-track: σ_a = 10 , σ_p =0.3	σ_a = 50 , σ_p =10 σ_a = 300 , σ_p =100 σ_a = 100 , σ_p =30
GPS data weighting: code/carrier-phase	0.7/0.002 m	0.7/4000 m
Ionospheric delay (τ =10 s)	σ_p =10 m	σ_p =10 m
Clock offset (τ =100 s)	σ_p =5000 m	σ_p =5000 m

4. Results and Discussions

Before discussing single-, dual-, and triple-satellite POD and PBD solutions and associated consistency checks (Sections 4.2 and 4.3), attention is paid to the impact of processing GPS code observations and correcting them by introducing CRV maps (Section 4.1). Validation of POD and PBD solutions by comparison with SLR observations is addressed in Section 4.4. This Section is concluded by an analysis of the impact of the arc length (Section 4.5).

4.1. GPS Observations Processing

The tracking channels of the BlackJack main navigation GPS receiver are allocated in sets of three to track C/A, P_1 (on the first frequency) and P_2 (on the second frequency) code observations accompanied by the corresponding C/A, L_1, and L_2 carrier-phase observations. This research only makes use of the dual-frequency P-code and the associated carrier-phase measurements. It has to be noted that for the CHAMP GPS receiver a small percentage of rather adverse carrier-phase outliers are identified (Montenbruck and Kroes, 2003). Its unfavorable impact on PBD has to be minimized. Figure 4 presents the time series of CHAMP single-satellite POD carrier-phase residuals on the first frequency for DOY 101. Irregular residual outliers exist frequently even after the default data processing scheme. To get rid of these outliers, the setting for the data editing item *ionosphere-free combination carrier-phase editing outliers* (as stated in Table 3) is realized to be extremely important. This item is described by the following equations,

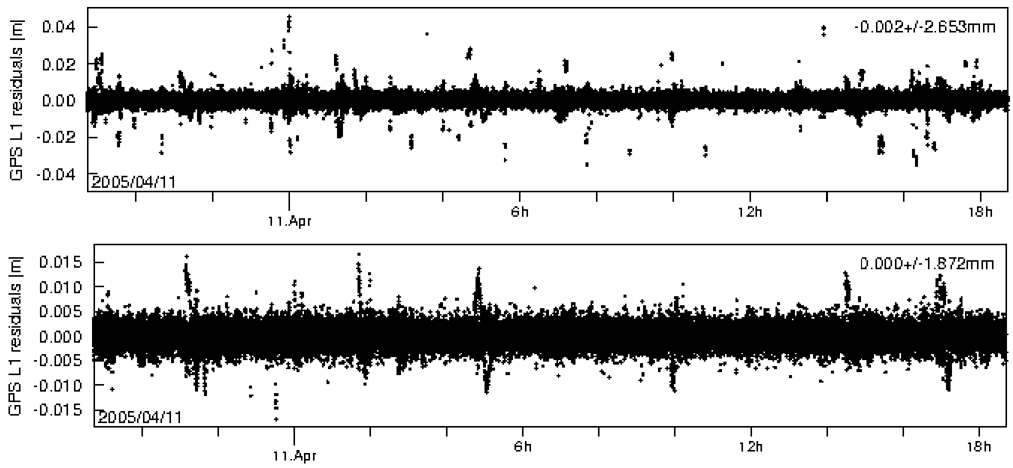

Figure 4: The CHAMP GPS receiver carrier-phase residuals on the first frequency for the old (top) and new (bottom) data processing schemes (Orbit arc DOY 101 2005, please note the difference in scale between top and bottom plots).

$$\frac{1}{n-1}\sum_{i=1}^{n}\left\{[R_{IF}^{obs}(t_i) - R_{IF}^{mod}(t_i,\mathbf{r})] - [R_{IF}^{obs}(t_{i-1}) - R_{IF}^{mod}(t_{i-1},\mathbf{r})]\right\}^2$$

$$\leq \sigma_{res}^2$$

$$R_{IF} = \frac{f_1^2 R_{f_1} - f_2^2 R_{f_2}}{f_1^2 - f_2^2}$$

where, R is the carrier-phase converted range computed either by the IF combination or on each frequency (f_1=1575.42 MHz and f_2=1227.60 MHz). It can be modeled (*mod*) based on the reference coordinates of the LEOs (**r**) and the corresponding GPS satellites at certain epochs (t_i). The modeled range will be compared with the real observed value (*obs*) to obtain the modeling residuals. The mean of the residual change between two adjacent epochs for n GPS satellite measurements is required to pass a certain threshold σ_{res}, otherwise the associated GPS measurements will be excluded for the following POD steps. The original threshold is set as 0.05 m, which accepts 92.6%, 98.0% and 90.9% data for the GRACE-A, GRACE-B and CHAMP POD respectively for the selected 30 orbital arcs. A reduction from 0.05 to 0.015 m results into 1.4%, 0.7% and 0.5% less data passing the data processing scheme for the associated satellites (Table 4, indexed by *Old* and *New*). Figure 4 displays that the new processing scheme screens out most of the existing outliers. This change is found to be of great importance to improve the overall POD and PBD performance. Table 4 displays the residual level statistics comparison between the *Old* (0.05 m) and *New* (0.015 m) sets of data. The carrier-phase residual statistics on the first frequency is significantly reduced by 0.9, 0.6 and 1.5 mm for GRACE-A, GRACE-B and CHAMP satellites, respectively. Reduction for the other frequency is however less pronounced. Smaller carrier-phase residuals are considerably beneficial for the PBD performance (Mao et al., 2018). Therefore the *New* scheme has been adopted for the following research.

Table 4: The RMS of GPS observation residuals and the percentage of used GPS observations in the MODK single-satellite POD mode. Three sets of data are displayed: −*Old* represents the data set using the old data processing scheme and −*New* means the new one, *CRV* means the results by including the antenna CRV patterns. For all the solutions PCV patterns are included (statistics for 30 orbital arcs).

Satellite	Solution	P1	P2	L1	L2	Perc.
		Code		Carrier-phase		
		[m]	[m]	[mm]	[mm]	[%]
GRACE-A	Old	0.21	0.22	2.85	1.73	92.60
	New	0.20	0.21	1.94	1.17	91.26
	+CRV	0.20	0.20	1.94	1.18	91.26
GRACE-B	Old	0.17	0.17	2.41	1.46	97.96
	New	0.17	0.17	1.77	1.08	97.29
	+CRV	0.17	0.15	1.77	1.08	97.29
CHAMP	Old	0.23	0.29	3.46	2.12	90.93
	New	0.23	0.29	2.00	1.24	90.44
	+CRV	0.23	0.19	2.03	1.24	91.90

The CHAMP and GRACE satellites are equipped with different generation JPL BlackJack GPS receivers leading to different performance. Figure 5 shows the CHAMP GPS antenna CRV patterns created for this research, note that the creation of them is only based on single-satellite POD with MODK. The patterns clearly depict the signal interference on the rear side of the CHAMP GPS navigation antenna, which is close to the GPS occultation antenna (see Figure 3 in (Montenbruck and Kroes, 2003)). The CRV patterns reach a level of nearly 1 meter on especially the GPS L_2 frequency. This level is much above the L_2 wavelength and will thus impact the ambiguity fixing for DD carrier-phase combinations that include CHAMP. No similar interference patterns are observed for the two GRACE satellites during this period, see *e.g.* (Mao et al., 2017).

Table 4 displays statistics for the GPS observation residuals obtained by first excluding (in-

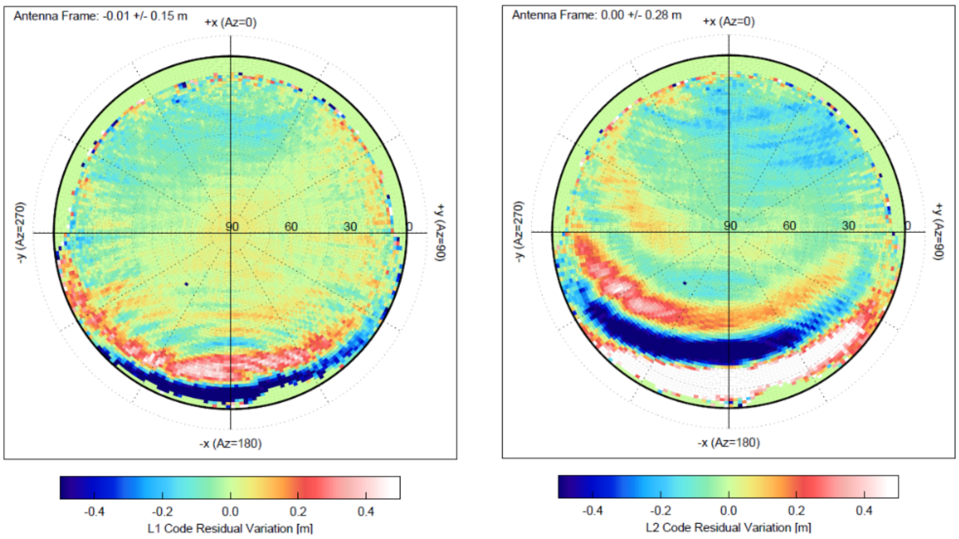

Figure 5: The CHAMP GPS receiver code residual variation patterns on the first (left) and the second (right) GPS frequency in the antenna-fixed reference system, for which the North axis coincides with the satellite body-fixed +X axis (0° azimuth), the Up and bore-sight axis coincides with the -Z axis, and the East axis completes the right-handed system. The April 2005 (DOY 091-120) data were used for generating the CRV maps.

dicated as *New*) and then including CRV maps (indicated as *+CRV*). The implementation of CRV patterns significantly reduces the RMS of the CHAMP code residuals on the second frequency from 29 to 19 cm. This also agrees with Mao et al. (2017) presenting similar analysis for GRACE-B satellite during when its radio occultation was activated. A similar but smaller effect of 1-2 cm reduction can be seen for the two GRACE satellites. The RMS of the carrier-phase residuals slightly increases on the first GPS frequency for CHAMP when including the CRV maps, which increases the percentage of GPS observations passing data editing process from 90.4% to 91.9%. Changes to both GRACE satellites are hardly visible since only less than 0.01% of GPS observations is influenced.

Another assessment is done to evaluate the impact of CRV patterns on the DD ambiguity fixing for a typical 24-hr arc dual-satellite PBD, as displayed in Figure 6. The top part of this Figure shows that the GRACE CRV patterns slightly increase the ambiguity fixing success rate by 0.5%, indicated by the sparse blue stripes at edges of a few tracking passes. Thus especially for DD carrier-phase combinations that include observations at the begin and end of tracking passes at low elevations the ambiguity fixing success rate is increased. This corresponds to the conclusions in (Mao et al., 2017) for GRACE-only PBD. The impact for CHAMP is bigger: its CRV patterns change the values of many fixed integers (the red stripes), also lead to extra fixed integer ambiguities (blue stripes), but also causes some integer ambiguities not to be fixed (yellow stripes). However, in total around 5% more ambiguities are fixed. Changes predominantly occur when the GRACE and CHAMP satellites are further away from the point of closest approach (distance typically larger than 1000 km). Although several stringent ambiguity tests were done (as Table 3 describes), including a 99.9% probability test (Verhagen, 2005), it was found that several DD ambiguities are likely wrongly fixed. Attention to this will be paid in the next Sections.

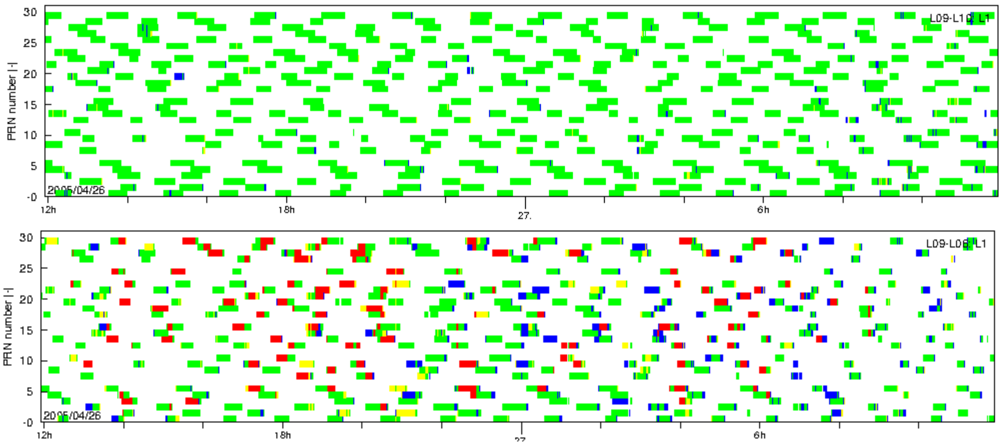

Figure 6: The fixed ambiguities comparison between the solution with and without GPS receiver antenna CRV patterns for the GRACE-A/B (top) and the CHAMP/GRACE-A (bottom) baselines, only the result for the first frequency is shown. Green: the same integers, Red: different integers, Yellow: fixed integers that exist in the solution using only antenna PCV patterns, Blue: newly fixed integers that only exist in the solution additionally including antenna CRV patterns (DOY 116, 2005).

4.2. Dual-satellite PBD

MODK is capable of running orbit determinations in different modes, *i.e.* single- (see Section 4.1), dual- and triple-satellite. In this section first dual-satellite POD and PBD is addressed, followed by triple-satellite POD and PBD in Section 4.3. As mentioned above, PBD for the close GRACE formation benefits from constraining the differences between empirical accelerations since they fly almost identical orbits, and enhances the ambiguity fixing success rate. Moreover, its onboard KBR instrument provides the data for an independent validation of the GPS-derived baseline at μm precision level. The dual-satellite PBD approach is used as well for the two CHAMP/GRACE satellite pair combinations. However, for these combinations the differences between the estimated empirical accelerations are very loosely constrained. Unfortunately, for these baselines no independent validation can be done as for GRACE with KBR observations. For these baselines, quality assessment will be based on comparisons with other external PBD and POD solutions (Allende-Alba and Montenbruck, 2016; Jäggi et al., 2016), and in this case the *KIPP* orbits will be used. In addition, a consistency check will be included by comparison with the earlier mentioned MODK kinematic PBD solutions.

Table 5 shows the baseline consistency between the kinematic and reduced-dynamic solutions for each possible satellite pair. For the GRACE tandem, baseline consistency is obtained at a level of 7.6/3.1/2.2 mm in the radial/along-track/cross-track directions for the ambiguity fixed solution. The consistency is thus displayed in the local-horizontal, local-vertical reference frame, where for baselines the radial, along-track and cross-track direction of a reference satellite is taken (GRACE-A for GRACE-A/B and CHAMP/GRACE-A satellite pair, and GRACE-B for CHAMP/GRACE-B pair). The worst consistency is obtained for the radial direction, which results from the geometry, *i.e.* a large value for the Radial Dilution Of Precision (RDOP). As expected, for the line-of-sight (pointing from the reference satellite to the other satellite) direction, the value is almost identical to the value for the along-track direction. Fixing the ambiguities, clearly significantly improves the consistency with the KBR down to a level of 0.64 mm com-

Table 5: Consistency between MODK dual-satellite reduced-dynamic and kinematic baseline solutions (RMS) for the radial, along-track, cross-track and line-of-sight direction. Two reduced-dynamic baseline solutions were obtained, one with (*Integer*) and one without (*Float*) ambiguity fixing. In addition, the consistency between MODK float baseline solutions and baselines derived from single-satellite kinematic POD using the GHOST *KIPP* module was assessed. The ambiguity success rate (Amb.), percentage of epochs covered by kinematic PBD solutions (Ava.), and consistency with KBR observations (for GRACE) are indicated as well. The values hold for the selected 30 24-hr arcs.

Solution	Radial	Along-track	Cross-track	Line-of-sight	Amb.	Ava.	KBR
Unit	[mm]	[mm]	[mm]	[mm]	[%]	[%]	[mm]
GRACE-A/B							
Float	6.88	3.89	1.87	3.91	NA	92.40	3.00
Integer	7.60	3.08	2.17	3.11	94.56	92.49	0.64
KIPP	30.80	23.15	20.55	23.34	NA	99.33	NA
CHAMP/GRACE-A							
Float	24.31	18.56	5.56	17.79	NA	61.42	NA
Integer	23.49	14.12	6.32	11.76	80.02	62.00	NA
KIPP	41.44	37.48	24.81	34.14	NA	98.65	NA
CHAMP/GRACE-B							
Float	24.24	19.08	5.64	17.83	NA	57.36	NA
Integer	22.62	13.60	6.25	10.68	85.61	57.36	NA
KIPP	43.72	38.14	33.87	35.50	NA	98.59	NA

pared to 3.00 mm for the *Float* solution, which agrees well with precision levels achieved by different PBD software (Kang et al., 2006; Jäggi et al., 2007; Allende-Alba and Montenbruck, 2016).

Note that ambiguity fixing will generally increase the carrier-phase residuals for two satellites, since the fixing allows less freedom for the ambiguities to accommodate the carrier-phase observations and in additions wrongly-fixed integer ambiguities probably occasionally occur (Van Barneveld, 2012). In this research, the RMS-of-fit of carrier-phase residuals for two satellites slightly increased from 1.95/1.82 (L_1, GRACE-A/B) and 1.18/1.10 mm (L_2) to 1.97/1.84 and 1.19/1.12 mm respectively. Incorrectly fixed ambiguities are expected to deteriorate the kinematic baseline precision. As discussed in Section 3, the kinematic baseline is the subtraction between the reduced-dynamic orbit of a reference satellite and the kinematic orbit of the other satellite, all computations are done in the MODK module. This differs with the *KIPP* solution, which is the subtraction of two independent kinematic orbits computed by the KIPP module of the GHOST software package. The reduced-dynamic PBD is influenced to a much smaller extent due to its inherent smoothing capacity. It can be seen from Table 5 that for the GRACE tandem the ambiguity fixing slightly deteriorates the baseline consistency in the radial and cross-track directions by 0.7 and 0.3 mm, respectively. Nevertheless the baseline consistency is improved by 0.8 mm in the along-track direction, where the GRACE baseline geometry and thus the fixed ambiguities align.

For the CHAMP/GRACE-A PBD solution, the kinematic and reduced-dynamic baseline consistency is changed from a level of 24.3/18.6/5.6 to 23.5/14.1/6.3 mm after ambiguity fixing. A clear improvement of 4.5 mm is seen in the along-track direction, only small deterioration exists for the cross-track direction. Improvement in the line-of-sight direction is more profound, showing a change from 17.8 to 11.8 mm. The result for the CHAMP/GRACE-B pair is similar. In fact, the dominating geometric component for the CHAMP/GRACE satellite pairs for 24-hr arcs aligns in the along-track direction since two orbital planes were aligned closely. A smaller component will be in the radial direction due to the orbital altitude difference of around 110 km (Table 1). The ambiguity fixing again improves the consistency in the radial and along-track directions. For the CHAMP/GRACE pairs Van Barneveld (2012) did an additional analysis of full-set ambiguity fixing in LAMBDA, showing that only $1-2\%$ epochs experience successful full-set ambiguity fixing. In this research the subset ambiguity fixing acquires again a much higher proportion of 80.0% and 85.6% for CHAMP/GRACE-A and CHAMP/GRACE-B, respectively. The latter satellite pair benefits further from the better GRACE-B GPS measurements (Table 4). It will be shown in Section 4.5 that there is also a strong dependency of the consistency on the arc length. For longer arc lengths the CHAMP/GRACE baselines grow rapidly leading to a less favorable geometry with a reduced number of GPS satellites in common view (see also Figure 2). The next step should be enhancing the CHAMP/GRACE PBD with more strict triple-satellite dynamic constraints and make use of the more reliable GRACE ambiguity fixing as much as possible.

The baselines derived from the single-satellite kinematic POD *KIPP* have been compared with the *Float* MODK reduced-dynamic baseline solutions as well. The *KIPP*-based baselines are available for $98.6-99.3\%$ of the epochs, much higher than for the MODK PBD solutions. In fact, during the selected period both versions of the BlackJack GPS receivers are tracking a large number (around 8) of GPS satellites, which results in this high availability for single-satellite POD. This differs significantly with MODK PBD which requires to have at least 5 GPS satellites in common view by two GPS receivers. However, the associated *KIPP* baseline consistency with the MODK PBD solutions is much worse, clearly showing the benefit of adopting a dual-satellite *vs.* single-satellite POD approach. PBD has the benefit of using DD carrier-phase observations and, particular for GRACE, the use of constraints for limiting the differences between estimated empirical accelerations (see Figure 7, in the next Section, where for GRACE-A and -B indeed almost identical values are obtained in an MODK triple-satellite solution).

4.3. Triple-satellite PBD

In a next step, MODK is used to generate PBD solutions for the full CHAMP/GRACE constellation in one go, *i.e.* by triple-satellite POD and PBD. First, PBD solutions are obtained without ambiguity fixing (again referred to as *Float*). Second, solutions are produced for which DD ambiguities are fixed for all GRACE/CHAMP satellite pairs (*Integer*). The triple-satellite PBD offers the possibility to define a so-called preferred baseline and then assess if this leads to improved solutions for the other baselines. Since the GRACE baseline can be determined with high precision by GPS, it can be investigated if the CHAMP/GRACE baselines and in conjunction the CHAMP absolute orbit solution can benefit from this. In a first step, the MODK IEKF estimates the GRACE baselines until the DD ambiguity fixing has converged (typically requiring 5-6 iterations). After this, the two CHAMP/GRACE baselines are included as well and a new DD integer ambiguity fixing is done (including the GRACE-A/B DD ones). The crucial question is if this new ambiguity fixing can be done correctly and indeed improved baseline and absolute CHAMP (and GRACE) positions can be obtained. The first attempts reported in Van Barneveld (2012) show that the risk of incorrectly fixing the ambiguities and thereby decreasing the quality of the orbit and PBD solutions is high. Therefore, a third MODK triple-satellite solution is done by freezing the GRACE-related DD ambiguities obtained in the first step and not fixing the DD ambiguities for CHAMP/GRACE combinations, referred to as *GAB-pref*.

Table 6: Consistency between MODK triple-satellite reduced-dynamic and kinematic baseline solutions (RMS) for the radial, along-track, cross-track and line-of-sight direction. Three reduced-dynamic baseline solutions were obtained adopting different handling of the DD carrier-phase ambiguities (see main text for details), referred to as *Integer*, *Float* and *GAB-pref*. In addition, the consistency between MODK float baseline solutions and baselines derived from single-satellite kinematic POD using the GHOST *KIPP* module was assessed. The ambiguity success rate (Amb.), percentage of epochs covered by kinematic PBD solutions (Ava.), and consistency with KBR observations (for GRACE) are indicated as well. The values hold for the selected 30 24-hr arcs.

Solution Unit	Radial [mm]	Along-track [mm]	Cross-track [mm]	Line-of-sight [mm]	Amb. [%]	Ava. [%]	KBR [mm]
			GRACE-A/B				
Float	6.95	4.00	1.90	4.02	NA	90.84	3.22
GAB-pref	7.70	3.18	2.21	3.21	93.89	90.91	0.63
Integer	8.29	3.31	2.29	3.34	93.70	90.91	0.66
KIPP	30.82	23.23	20.55	23.36	NA	99.33	NA
			CHAMP/GRACE-A				
Float	24.08	18.63	5.51	17.40	NA	61.19	NA
GAB-pref	24.14	18.50	5.49	17.31	NA	61.17	NA
Integer	23.61	13.96	6.57	11.45	82.92	61.05	NA
KIPP	41.47	37.92	24.91	34.53	NA	98.65	NA
			CHAMP/GRACE-B				
Float	23.91	18.22	5.56	16.98	NA	56.61	NA
GAB-pref	24.07	18.02	5.63	16.76	NA	56.61	NA
Integer	25.02	15.16	6.69	11.96	86.67	56.47	NA
KIPP	43.89	38.89	28.87	36.13	NA	98.60	NA

When comparing the results for the MODK dual- and triple-satellite PBD solutions, it can be seen that the consistency between the reduced-dynamic and kinematic baseline solutions in general hardly changes when the third satellite is introduced (Table 5 and Table 6). For the GRACE *Float* solution, the baseline consistency slightly deteriorates from 6.9/3.9/1.9 to 7.0/4.0/1.9 mm when CHAMP is introduced. Also a deterioration of 0.1 mm for the line-of-sight is obtained. These are caused by the fact that the availability of GRACE kinematic baselines is reduced by around 1.6%. Actually the additional data editing after including the third satellite eliminates around 1.5% GRACE GPS observations extra before the PBD computation. The kinematic baseline availability reductions for CHAMP/GRACE-A and CHAMP/GRACE-B are only 0.2% and 0.7%, indicating that more kinematic solutions could pass the 5 cm GPS residuals criterion (Section 3) even after the exclusion of more GPS data. The inclusion of the lower flying CHAMP satellite and the more relaxed relative dynamic constraints between CHAMP and GRACE thus do not improve the baseline consistency for all *Float* baselines. The impact of stringent and relaxed constraints on the differences between estimated GRACE and CHAMP empirical accelerations can be clearly seen in Figure 7. Moreover, it is clear that including the CHAMP satellite relevant ambiguity fixing negatively affects the GRACE DD ambiguity fixing. The baseline precision displays a slightly worse level for the RMS of differences with the KBR observations from 0.63 to 0.66 mm (in comparison with 0.64 mm in Table 5 for the dual-satellite PBD).

Compared to the *Float* and *GAB-pref* solutions, the third MODK triple-satellite approach *Integer* results in a significantly improved consistency in the line-of-sight between the reduced-dynamic and kinematic baseline solutions for the two CHAMP/GRACE satellite pairs. This corresponds to the conclusions drawn from Table 5. Please note that when comparing with the MODK GRACE dual-satellite ambiguity fixed solution, the kinematic and reduced-dynamic baseline consistency for the *Integer* solution is worse (0.7/0.2/0.1 mm for the radial, along-track and cross-track directions) and around 0.9% fewer integer ambiguities are fixed. Small changes again result from additional data editing to take into account the lower CHAMP satellite (Table 3). It has to be noted that in comparison with Van Barneveld (2012) the adverse influence of CHAMP/GRACE ambiguity fixing for the GRACE PBD is hardly visible in this research. The newly implemented modifications as introduced in Section 1 obtain more stable POD and PBD performance for each of the satellites or satellite pairs.

4.4. Satellite Laser Ranging Validation

SLR observations for three satellites allow an independent validation of MODK GPS-based orbits in the line-of-sight direction between each SLR ground station and a LEO satellite (Pearlman et al., 2002). To get rid of occasionally poor quality tracking observations, an editing threshold of 30 cm - which is more than an order of magnitude above the RMS of fit levels - is used. Observations below 10° elevation angle are also excluded to eliminate observations that are relatively more affected by atmospheric delay. The SLR retro-reflector modeling pattern from GFZ is included as corrections (Neubert et al., 1998). Table 7 shows that 93.1%, 92.5% and 93.8% of the SLR observations are used for the orbit validations for GRACE-A, GRACE-B and CHAMP, respectively. In total 22 SLR stations are included in the validations. The reference orbits of GRACE are obtained from JPL, for which use was made of so-called single receiver ambiguity fixing (Bertiger et al., 2010).

Table 7: Mean and standard deviation of SLR observations for different orbit solutions for the 30 24-hr arcs (unit: [mm]). Please note that all solutions identified by Single, Dual and Triple were obtained by using MODK.

Satellite	GRACE-A	GRACE-B	CHAMP
JPL	−3.1 ± 14.3	−1.9 ± 16.5	NA
Single	−5.1 ± 15.2	−3.3 ± 18.8	−7.4 ± 23.5
Dual:GRACE	−5.1 ± 14.9	−3.6 ± 17.9	NA
Dual:CHAMP/GRA	−4.7 ± 16.7	NA	−8.2 ± 23.7
Dual:CHAMP/GRB	NA	−0.7 ± 22.3	−8.7 ± 23.2
Triple:Float	−5.4 ± 15.0	−3.4 ± 18.5	−7.4 ± 24.0
Triple:GAB-pref	−5.2 ± 15.1	−3.3 ± 18.4	−7.8 ± 23.8
Triple:Integer	−5.7 ± 17.2	−3.2 ± 18.8	−7.3 ± 23.2
Nr.SLR Obs.	3031(93.1%)	2937(92.5%)	2921(93.8%)

For the single-satellite MODK and external orbit solutions, the mean of the SLR observation residuals is in general at the level of a few mm , where the RMS varies between 14 and 24 mm (Table 7). For GRACE, the best consistency is obtained for the JPL orbits, which - as mentioned above - make use of single receiver ambiguity fixing. In the mean time, single receiver ambiguity fixing and more sophisticated non-gravitational force modeling have been tested and implemented in several GHOST modules (Hackel et al., 2017; Montenbruck et al., 2017). However they are not yet applied to MODK.

It is clear to see that the SLR consistency improves for the MODK dual-satellite GRACE orbit solutions (0.3 to 0.9 mm reduction in the standard deviation). The correlations between

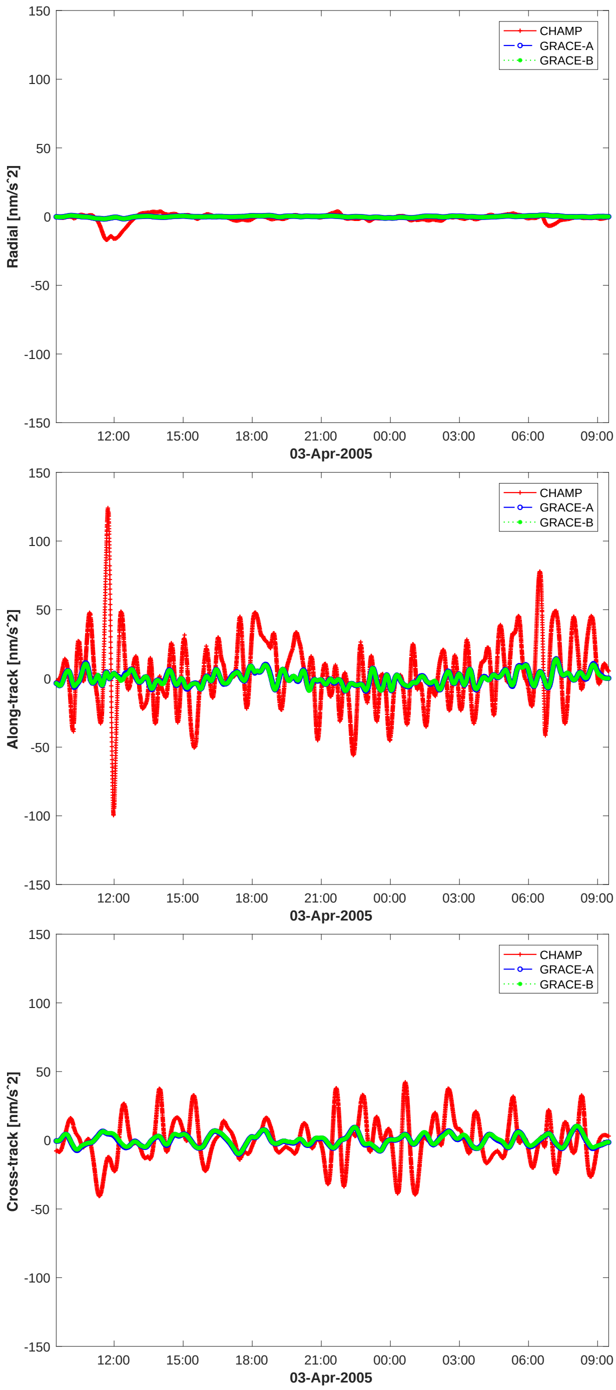

Figure 7: Time series of estimated empirical accelerations in the radial (top), along-track (middle) and cross-track (bottom) directions for each satellite based on triple-satellite PBD (no ambiguity fixing) for a typical day (DOY 093, 2005). Same scales are set for the vertical axes. Please note that the curves for GRACE-A and -B almost completely overlap.

the daily RMS of empirical accelerations for two GRACE satellites are analyzed for the single-satellite POD solutions and the dual-satellite PBD solutions. The correlations significantly increase from 0.945/0.999/0.980 to 1.000/0.999/0.991 in the radial, along-track and cross-track directions, respectively. The precision of the GRACE orbits thus benefits from their joint estimation, the ambiguity fixing and the use of constraints for the differences between the empirical accelerations.

For the two MODK triple-satellite solutions (*Float* and *GAB-Pref*), SLR consistency levels become slightly better (between 0.1 and 0.4 mm) than the single-satellite orbits for the GRACE satellites. However, slight GRACE satellites orbit precision reductions for especially GRACE-A are seen for the MODK triple-satellite solution where all ambiguities are fixed (*Integer*). This agrees with the conclusions in Van Barneveld (2012) that fixing all integer ambiguities will downgrade the GRACE-A orbit precision. It has to be noted that in this research no reduction of precision is seen for the GRACE-B orbit and the CHAMP orbit precision is even improved by 0.3 mm, whilst Van Barneveld (2012) reported downgraded orbit precision for all three satellites in comparison with the single-satellite solutions (3.4, 6.7 and 4.2 mm for GRACE-A, GRACE-B and CHAMP respectively). The dual-satellite CHAMP/GRB integer ambiguity fixing also slightly improves the CHAMP orbit. However, the *Dual:CHAMP/GRA* solution slightly deteriorates the CHAMP orbit. For both MODK CHAMP/GRACE dual-satellite orbit solutions, the SLR consistency deteriorates for the GRACE satellites. They indicate the existence of wrongly-fixed integer ambiguities for particularly the CHAMP/GRACE satellite pairs, which is a very interesting point for further research.

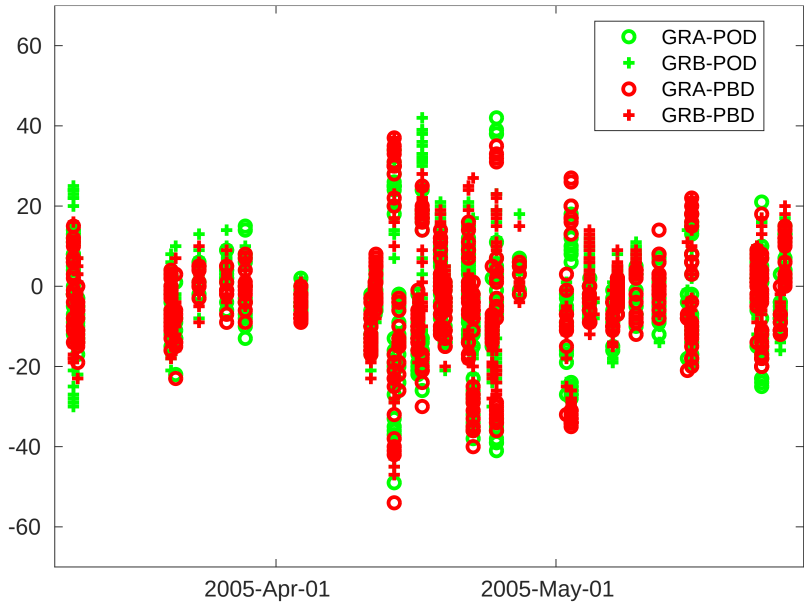

Figure 8: SLR observation residuals (unit:mm) for the GRACE-A/B MODK single-satellite POD and dual-satellite PBD orbit solutions for all passes of the Yarragadee station in Australia. The RMS of all green marks (reference) is 13.2 mm, the RMS of all red marks is 13.0 mm.

For some SLR passes, an interesting extra validation can be done for GRACE. For these passes, the associated SLR tracking station is switching between the GRACE-A and -B satellites and it is interesting to see if time series of remaining SLR observation residuals are continuous or display jumps. To this aim, all tracking passes by the high-quality Yarragadee station in Australia are selected. In the selected period, this station collected 547 and 632 observations for GRACE-A and -B satellites. The SLR validation residuals for two satellite orbits are displayed in Figure 8. It can be seen that in general two satellite orbits align slightly better in the GRACE-A/B dual-satellite PBD solution. Figure 10 shows three representative tracking passes on DOY 093, 101 and 109. When tracking the GRACE-A/B formation, normally the tracking of Yarragadee station switches 1-6 times during one satellite overpass that lasts for a few minutes. For these three days, smaller jumps occur at the switching times for the dual-satellite solutions. To a lesser extent this is the case for the other selected days as well. The GRACE-A/B orbit alignment assessment for the triple-satellite PBD is similar. Thus, this is an indication that the PBD seems to improve the consistency between the GRACE-A and -B orbit solutions. These results agree well with Allende-Alba et al. (2017) and Arnold et al. (2018) who report similar conclusions for the Swarm-A/C and TerraSAR-X/TanDEM-X formations, respectively.

Further assessment is done to check the alignment of GRACE-A/B satellites based on the CHAMP/GRACE-A and CHAMP/GRACE-B PBD solutions. Surprisingly better alignment can be also obtained for a few days, as Figure 9 displays. However when comparing with Figure 8 more unstable days exist: Table 7 shows worse RMS of fits for the GRACE-A and GRACE-B orbits for the two associated CHAMP/GRACE PBD solutions. This means the CHAMP satellite occasionally constrains the GRACE orbits through dual-satellite PBD however it does not necessarily always improve the other satellite orbit due to its more in-flight perturbations and problematic data quality. This however implies that the use of a high-quality satellite as the third

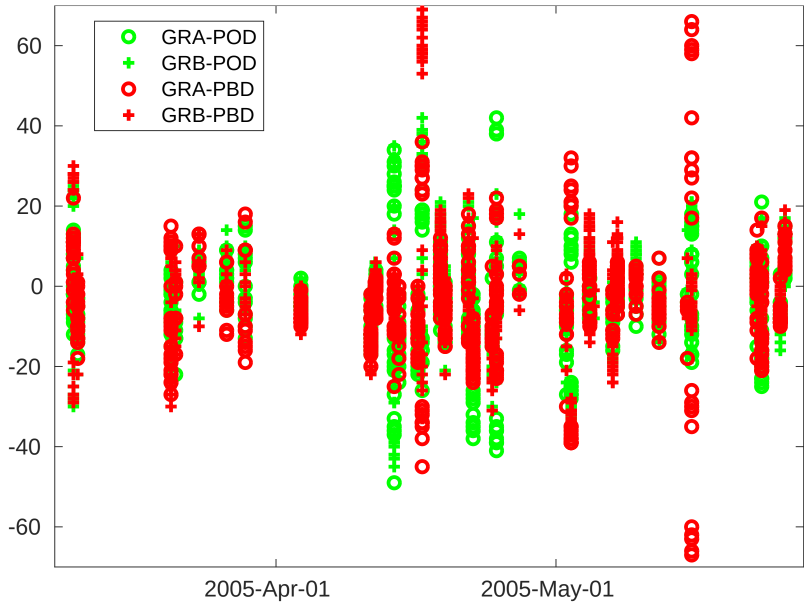

Figure 9: SLR observation residuals (unit:mm) for the GRACE-A/B MODK single-satellite POD and GRACE-A/B orbits obtained from CHAMP/GRACE-A and CHAMP/GRACE-B PBD orbit solutions for all passes of the Yarragadee station in Australia. The RMS of all green marks (reference) is 13.2 mm, the RMS of all red marks is 15.3 mm.

satellite might be more beneficial for all satellites.

4.5. Orbital Arc Length Analysis

Therefore, it is interesting to assess the impact of selecting shorter periods around the point of closest approach between CHAMP and the GRACE satellites. MODK triple-satellite PBD solutions have been produced for arc lengths from 2 to 24 hr, *i.e.* between 1 and 12 hr from the point of closest approach (all within the selected 30 24-hr periods specified in Table 2). For all arc lengths, the triple-satellite *GAB-pref* and *Integer* solutions were produced. Figure 12 clearly reveals the impact of increasing the arc length for all 30 orbital arcs: the line-of-sight baseline consistency deteriorates for longer CHAMP/GRACE arcs however not for the stable GRACE baseline. The integer ambiguity fixing for both CHAMP/GRACE baselines significantly improves the agreement between the kinematic and reduced-dynamic solutions, however it is found that the GRACE integer ambiguity fixing might be also perturbed occasionally by CHAMP (in this research, only the 10 hours GRACE-A/B PBD on DOY 111 is influenced).

Figure 13 displays the statistics of the kinematic and reduced-dynamic baseline consistency as a function of maximum orbital arc length for two MODK triple-satellite solutions, *GAB-pref* and *Integer*. The orbit arc has a significant impact on the PBD performance, for instance the consistency in the radial direction worsens by a factor of 2 when the arc is increased from 2 to 24 hours. It can be seen again that the baseline consistency for the *Integer* solution is in general worse than for the *GAB-pref* solution for the cross-track directions. However, clearly better consistency is obtained for the radial and along-track directions. The only difference between two solutions is the fixing of CHAMP/GRACE DD integer ambiguities. A similar conclusion is drawn based for the GRACE baseline results displayed in Table 5. Thus, results indicate that the

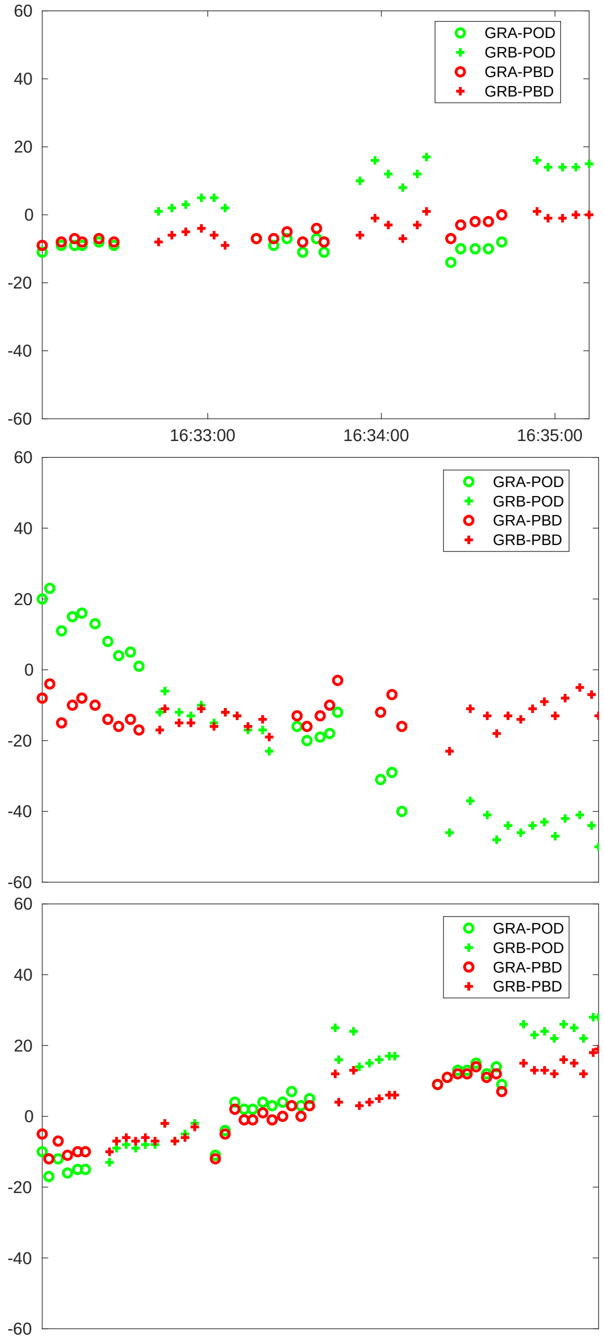

Figure 10: SLR observation residuals (unit:mm) for the GRACE-A/B MODK single-satellite POD and dual-satellite PBD orbit solutions for three representative passes (top: DOY 093; middle: DOY 101; bottom: DOY 103) of the Yarragadee station in Australia. For each pass the DOY in 2005 is indicated.

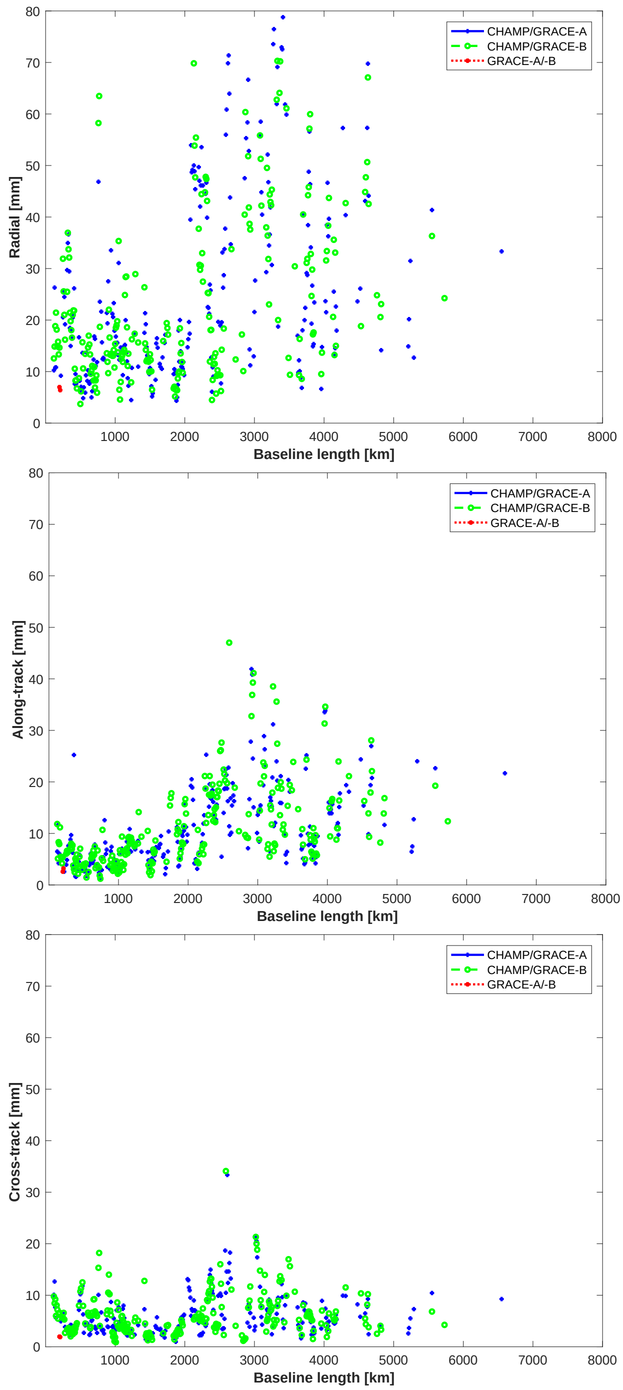

Figure 11: RMS of differences between MODK triple-satellite kinematic and reduced-dynamic solutions as a function of distance (in steps of 10 km) in the radial (left), along-track (middle) and cross-track (right) directions for the three baselines (DOY 093, 2005).

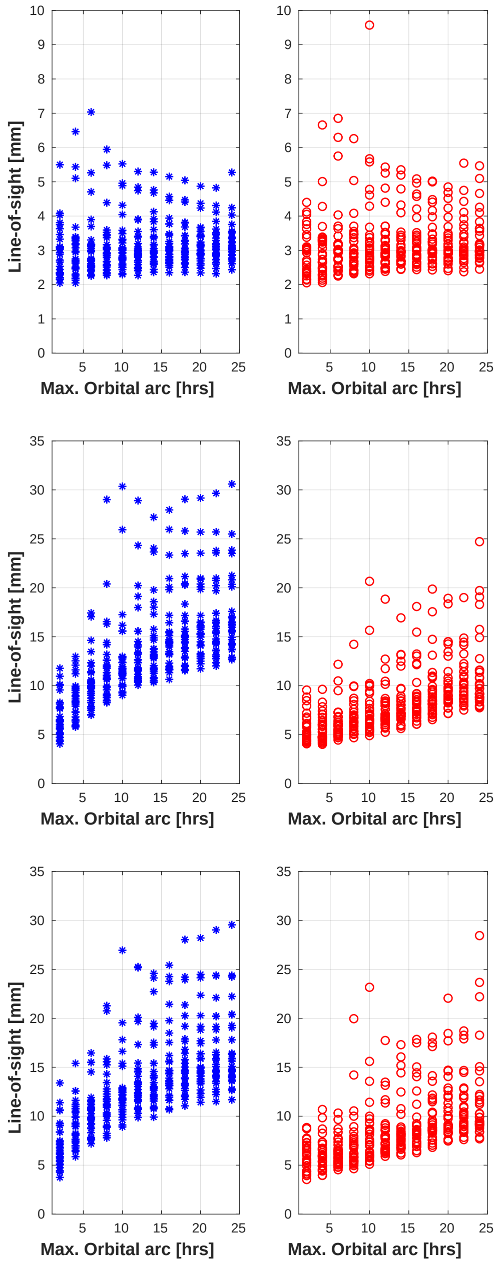

Figure 12: RMS of MODK triple-satellite kinematic and reduced-dynamic baseline solution differences in the line-of-sight direction for the *GAB-pref* and *Integer* approaches, as a function of the maximum orbital arc length for all 30 arcs of the GRACE-A/B (top), CHAMP/GRACE-A (middle) and CHAMP/GRACE-B (bottom) satellite pairs. Note that different axis scales are used for GRACE-A/B and the other two baselines. For each direction, the *GAB-pref* solution is displayed on the left as blue stars, and the *Integer* solution on the right as red circles.

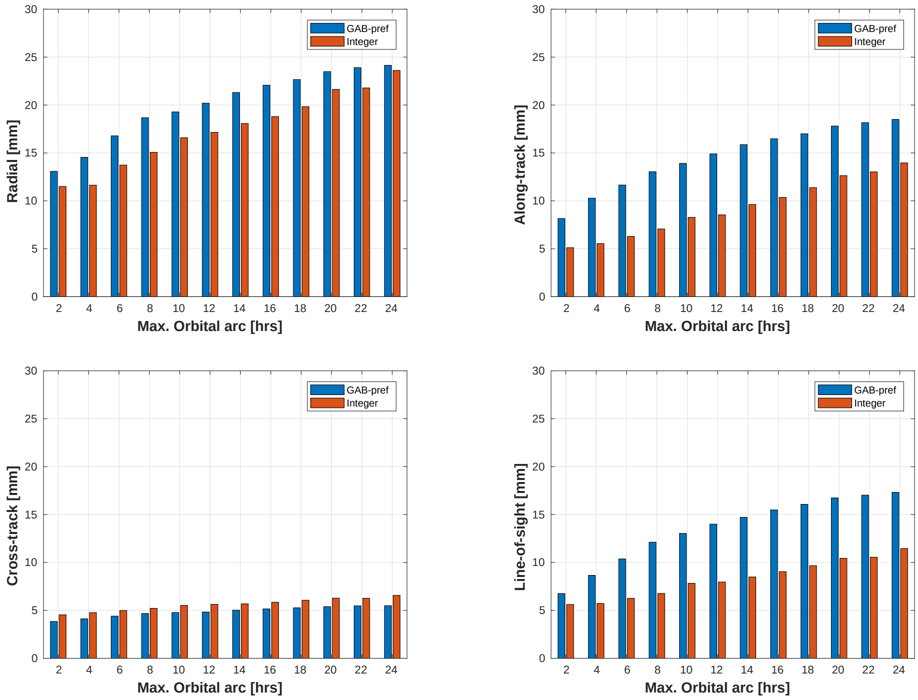

Figure 13: RMS of MODK triple-satellite kinematic and reduced-dynamic baseline differences for the *Integer* and the *GAB-pref* approaches as a function of the maximum orbital arc length in the radial (top-left), along-track (top-right), cross-track (bottom-left) and line-of-sight (bottom-right) directions for the CHAMP/GRACE-A baseline. Similar results are obtained for the CHAMP/GRACE-B baseline.

ambiguity fixing will enhance the baseline solution in the radial, along-track and line-of-sight direction, but slightly deteriorate the cross-track direction.

5. Summary and Outlook

This study investigates a challenging satellite constellation formed by the CHAMP satellite and the GRACE twin satellites. Two different types of satellite baselines are available: one stable tandem baseline (GRACE-A/B) and two high-dynamic high-low baselines (CHAMP/GRACE-A and CHAMP/GRACE-B). Each selected 24-hr CHAMP/GRACE orbital arc has a baseline length ranging from 110 to 7500 km. The GRACE satellites on the one hand and the CHAMP satellite on the other hand show quite different levels of non-gravitational force model uncertainties. This makes the triple-satellite Precise Baseline Determination (PBD) more challenging. An iterative extended Kalman filter and a subset ambiguities fixing strategy are conducted for the reduced-dynamic PBD, whereas a kinematic PBD solution can be generated by using the fixed integer ambiguities and satellite coordinates obtained from the reduced-dynamic solutions. The MODK module allows different strategies for ambiguity fixing, namely no fixing at all *float*, full integer ambiguity fixing *Integer*, and only fixing the GRACE DD ambiguities *GAB-pref*.

Compared with Van Barneveld (2012) who initially proposed this research case, a few innovations were made that led to improvements. First, orbital arcs (thirty in total) were selected that center around the point of closed approach between the GRACE and CHAMP satellites (occurring every 2.7 days). This approach leads to a more homogeneous observation geometry for the investigated arcs and allows to better study the possibility of PBD as a function of baseline length. The GPS data processing scheme has also been carefully tailored for especially editing out spurious carrier-phase observations. In addition, Code Residual Variation (CRV) maps are generated to correct GPS code observations. These maps mitigate the effect of so-called cross-talk between the CHAMP GPS POD and radio occultation antennas. The single-, dual- and triple-satellite PBDs are done based on either float or fixed integer ambiguities. It can be also done such that the GRACE baseline is first computed based on fixed integer ambiguities, and then subsequently fix the CHAMP/GRACE DD carrier-phase ambiguities. By this approach, the influence of the more challenging CHAMP/GRACE ambiguity fixing could be assessed. Furthermore, the sensitivity of PBD precision to arc length (and thus baseline length) was assessed by generating solutions for different orbital arcs ranging from 2 to 24 hrs.

The above mentioned changes have resulted in improved PBD solutions for the high-dynamic GRACE/CHAMP constellation in comparison with Van Barneveld (2012). As for the more stable GRACE baseline, the consistency between its kinematic and reduced-dynamic solutions are at a level of 8.3/3.3/2.3 mm (radial/along-track/cross-track). The ambiguity fixing success rate reaches 93.7% and the K-Band Radar ranging (KBR) system proves a baseline precision at a level of 0.6 mm. The baseline consistency for the CHAMP/GRACE-A baseline are at a level of 23.6/14.0/6.6 mm due to rapidly changing and less favorable geometry, which is also shown by the rapidly deteriorating CHAMP/GRACE consistency with arc length or baseline length. The result for the CHAMP/GRACE-B baseline is similar. The consistency of PBD orbit solutions with Satellite Laser Ranging (SLR) observations is at the level of a few cm, 17.2, 18.8 and 23.2 mm RMS of fit for GRACE-A, GRACE-B, and CHAMP, respectively. Compared to the single-satellite Precise Orbit Determination (POD) solutions, the CHAMP orbit precision slightly improved by 0.3 mm. No improvement was obtained for the GRACE-B orbit solution, and the SLR RMS-of-fit deteriorated by 2.0 mm for GRACE-A. Finally, SLR passes for which the laser station is alternating between the two GRACE satellites were used to show improved consistency between the orbits of the GRACE satellites. The Yarragadee SLR station was selected to this aim. Results show that indeed a better agreement for the orbit solutions is obtained when using dual- or triple-satellite PBD solutions. It was shown that also the inclusion of the CHAMP satellite in the constellation occasionally helps to improve the GRACE-A/B orbit alignment, nevertheless it is not as effective as the dual-satellite GRACE PBD. The results also show that more robust and precise PBD results are obtained for the high-dynamic baselines as compared to the initial work done by Van Barneveld (2012). Despite the significantly improved high-dynamic baseline determinations, results indicate there is still room for improvement. Possibly, results can be further enhanced by revisiting and enhancing the ambiguity fixing method, which now fully relies on the LAMBDA method. Moreover, it will be interesting to assess if precise high-dynamic baseline solutions can indeed support improved observation of gravity by space-borne GPS receivers.

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