

A new approach to river bank retreat and advance in 2D numerical models of fluvial morphodynamics

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ABSTRACT: River bank retreat and advance are modes of morphological evolution in addition to bed level changes and changes in bed sediment composition. They produce planform changes such as width adjustment and meander bend migration. However, their reproduction in a 2D numerical model still remains a challenge. Defining bank-lines along the nearest grid lines of a rectangular computational grid leads to staircase lines that impede any reasonable determination of the hydraulic loads on the banks. An adaptive curvilinear boundary-fitted grid may seem to solve this problem, but arbitrary bank retreat and advance appear to deform such a grid prohibitively within a few bank-line update steps. We therefore present a new approach in which shifting bank-lines are followed as separate moving objects on a fixed grid, using local immersed-boundary techniques to solve the flow and sediment transport in the vicinity of the bank-lines. This means that the grid itself remains stationary but the flow domain is adapted each (morphological) time step. The use of separate moving objects also gives the opportunity to track bank-lines that are not on the border of the computational domain, but somewhere inside this domain, e.g. the main river channel between floodplains or the channels in an estuary.

The immersed-boundary approach with moving bank lines has been implemented in the existing framework of Delft3D, which allows us to reuse all advanced features that are already present in this code, such as advanced bed level updating and sediment transport formulations. The distance and the direction of bank retreat are computed using a simple bank erosion formula, but can be easily extended to incorporate other bank erosion mechanisms.

We analyze the performance of this approach for a simple meandering river. The results show that the new approach is capable of reproducing complex river planform changes without grid deformation problems and without the need to employ a very fine grid.

1 INTRODUCTION

River bank retreat and advance are modes of morphological evolution in addition to bed level changes and changes in bed sediment composition. Mosselman (1995) distinguishes three classes of morphological models capable of simulating river planform changes through bank retreat and advance: meander models based on linearized physics-based equations (Ikeda et al, 1981; Crosato, 1987, 1989, 2008), braided-river planform changes based on empirical evolution rules (Klaassen et al, 1993; Jagers, 2001, 2003, 2004), and 2D or 3D nonlinear physics-based morphological models with erodible banks (Mosselman, 1992, 1998; Darby et al, 2002). The 2D models of Mosselman (1992, 1998) and Darby et al (2002) employ an adaptive curvilinear boundary-fitted grid to avoid the occurrence of staircase-shaped

bank-lines as the river planform evolves. However, arbitrary bank retreat and advance appear to deform the boundary-fitted grid prohibitively too within a few bank-line update steps. We therefore present a new approach in which shifting bank-lines are followed as separate moving objects on a fixed grid, using local immersed-boundary techniques to solve the flow and sediment transport in the vicinity of the bank-lines. We incorporated this method into the existing depth-averaged modelling framework of Delft3D (Lesser et al., 2004). This framework is quasi 3D in the sense that the intensity of 3D helical flow is computed by means of a parameterization based on streamline curvature.

The outline of our work is as follows. We first investigate possible methods to deal with the moving boundaries and motivate our choice for a so-called immersed-boundary method with discrete forcing. We then show some results with this approach for flow through a curved channel with stationary boundaries. The actual mechanisms that are used to model the bank erosion process and the update of the bank lines are given in section 3. After that, we present some preliminary results for a simple meandering river and formulate some conclusions and recommendations for future work.

2 BOUNDARY TREATMENT

During the bank erosion process the bank lines retreat. Limiting the position of the bank lines to grid lines results in staircase-shaped bank lines and the flow field will not be well represented in the very area where an accurate flow field is needed for proper evaluation of bank erosion. This shortcoming can be improved by using an adaptive boundary-fitted grid or an immersed-boundary approach. A schematic difference between the use of an adaptive boundary-fitted grid and an immersed-boundary technique is shown in Figure 1. Both methods are further explained in the next two sections.

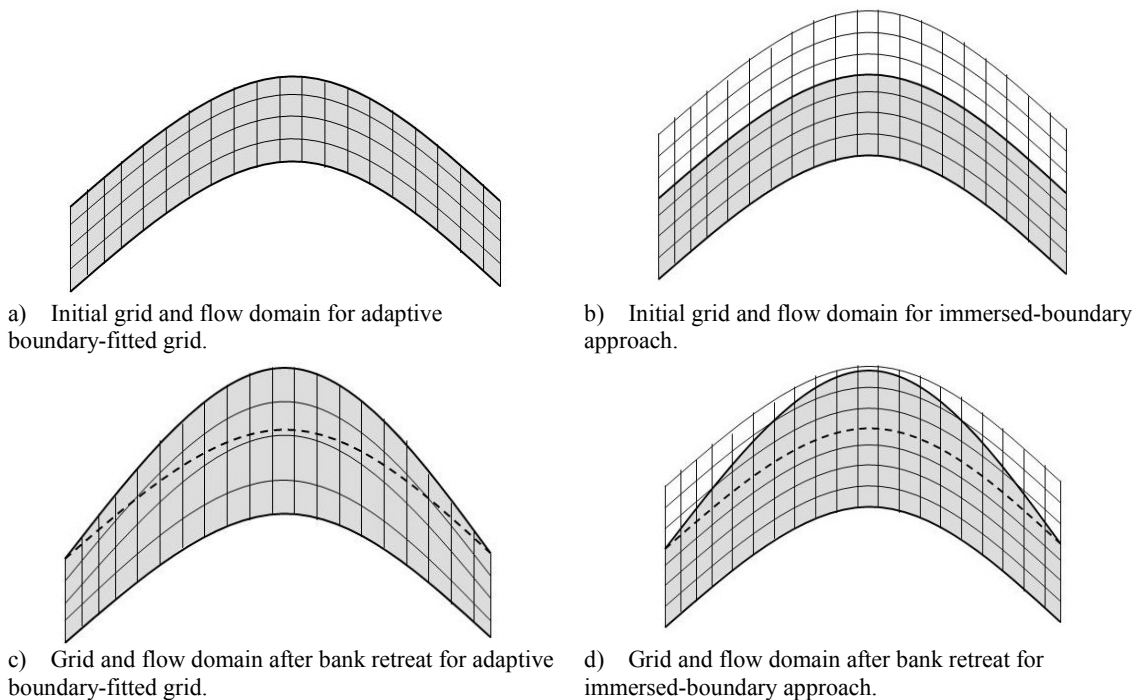


Figure 1 Schematic difference between adaptive boundary-fitted grid and immersed-boundary technique

2.1 Adaptive boundary-fitted grid approach

In the adaptive boundary-fitted grid approach the grid deforms during the computation such that it follows the moving boundaries. In this way the (curvilinear) grid is maintained and the boundary conditions are easy to implement. However, when the deformations become too large, the changes in geometry produce grid adaptation problems, such as the necessity to add or delete grid points and the loss of grid smoothness and orthogonality. The most extreme deformations occur when cut-offs are formed in a meandering river. Also, a so-called ALE-formulation has to be implemented to account for the

deformation of the cells in the numerical implementation. Furthermore, in practical applications, the banks are often not located along the borders of the computational domain, but somewhere inside this domain, e.g. the main river channel between floodplains or the channels in an estuary. Therefore the adaptive boundary-fitted grid approach is abandoned and a method that keeps track of migrating banks inside the computational domain is used, as explained in the next section.

2.2 Immersed-boundary approach

In the immersed-boundary approach, the grid is kept stationary and the boundary moves through the grid as an independent object. In this way arbitrary domains are possible and the grid remains orthogonal. However, you have to keep track of the boundary and this boundary is not aligned with the grid lines. This means that the flow is not aligned with the grid and in the cells that are cut by the boundary the numerical implementation has to account for this. Also, the implementation of boundary conditions is not straightforward anymore.

There are several ways to deal with boundary conditions at boundaries that cut through grid cells, the so called Immersed Boundary Methods (IBMs). In his general overview of these methods, Bandringa (2010) distinguishes three classes:

- 1) *Continuous forcing*: a forcing function is included into the momentum equation and the equations are solved in the entire domain (including the 'dry' part).
- 2) *Discrete forcing*: the discretization in the cells near the boundaries is adjusted to account for their presence.
- 3) *Cut cell approach*: the cells near the boundary are truncated to create cells which conform to the shape of the surface.

The main disadvantage of using continuous forcing is that the flow equations have to be solved in the 'dry' part of the domain. This is not consistent with the current implementation of Delft3D. Also the smoothing of the continuous forcing function leads to an inability to provide a sharp representation of the boundary. Furthermore it is difficult to ensure conservation of mass.

Methods using a cut cell approach have mainly been developed for finite-volume methods, as they adapt the shapes of the cells near the boundary. Since Delft3D is based on a finite-difference method, these methods are less suitable and therefore a method based on discrete forcing will be used.

2.3 Numerical adjustments

Using an immersed-boundary method with discrete forcing implies that the numerical discretization has to be adjusted in the cells that are cut by the bank line. To define the locations of the bank lines, the 'dry' areas of the domain, that are surrounded by these bank lines, are described by polygons consisting of straight line segments. The grid parameters (grid cell area, Δx and Δy) of the cells that are cut by these segments are adjusted to ensure that the continuity equation is solved correctly. This is done as follows:

For each polygon:

- 1) Define the cells inside the polygon as 'dry'
- 2) Find the cells that are cut by the polygon (light blue cells in Figure 2)
- 3) For each cut-cell:
 - a. Find the intersection points of the polygon with the edges of the cell.
 - b. Determine the 'wet' side of the cell.
 - c. Compute the factor with which the grid parameters (grid cell area, Δx and Δy) of the cut cells have to be adjusted.

The continuity equation and momentum equation are then solved using the adjusted grid parameters. An example of a polygon and the grid parameters that have to be adjusted is shown in Figure 2 for a simple Cartesian grid. To avoid numerical problems, grid cells with areas that are smaller than a certain threshold are also set to 'dry'.

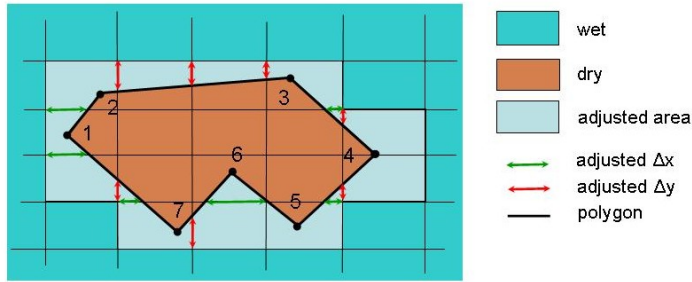


Figure 2 Example of a polygon and the corresponding grid parameters that have to be adjusted

During the bank erosion process the polygons are updated each time the locations of the bank lines change and this implies that the grid parameters have to be recomputed as well. The exact method used to update the location of the bank lines is described in section 3.

2.4 Results on a curved channel

We analyze the results of using a cut-cell boundary for a simple test case consisting of flow through a curved channel. The width of the channel is 100 m and after 50 m the channel has an offset angle of 5.7 degrees to the left. At 50 m before the end, the channel bends back again. The initial depth of the channel is 4 m everywhere. At the lower left inflow boundary a total discharge of $Q = 400 \text{ m}^3/\text{s}$ is imposed and at the outflow boundary a water level of 0 m. A constant Chézy value of $35 \text{ m}^{1/2}/\text{s}$ and a free slip boundary condition at the solid walls are imposed.

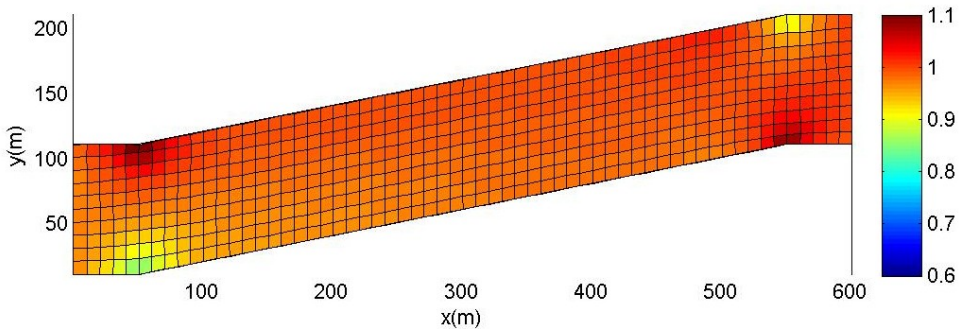


Figure 3 Flow through a curved channel on a curvilinear grid

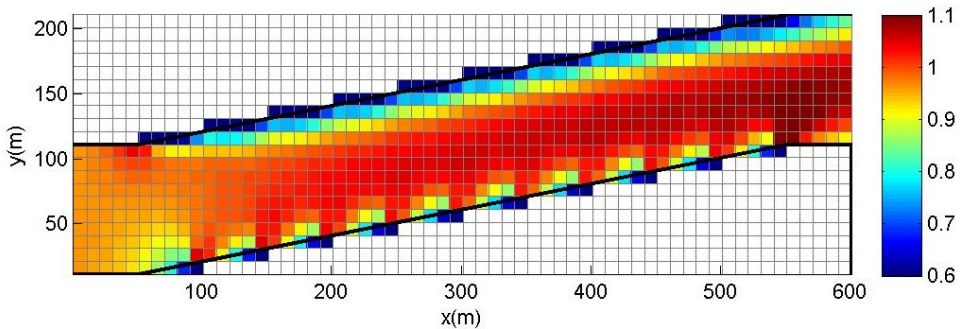


Figure 4 Flow through a curved channel on a Cartesian grid with staircase boundary

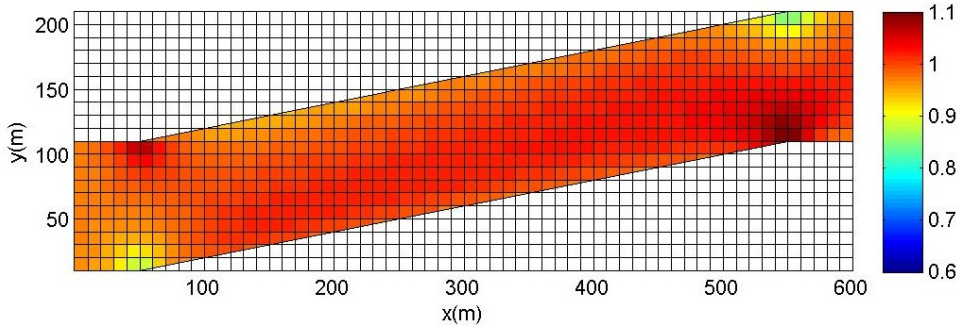


Figure 5 Flow through a curved channel on a Cartesian grid with adaptation of grid parameters in the cut cells

Figure 3 shows the magnitude of the flow velocity on a curvilinear grid of 10x60 cells. The same simulation is performed on a 20x60 Cartesian grid with a staircase boundary and the results are depicted in Figure 4. It can be seen that the staircase boundary results in unphysical fluctuations in the flow velocity. Adapting the grid parameters in the cells that are cut by the boundary gives the results as shown in Figure 5. The fluctuations caused by the staircase boundary almost disappeared. Remaining deviations with respect to the results on the curvilinear grid are caused by the fact that only the grid parameters are adjusted. To account for the presence of the boundary more accurately, also the discretization of the momentum equation has to be adjusted in the cells that are cut by the boundary.

3 BANK EROSION

3.1 Bank retreat

The lateral erosion of the banks is computed by the following simple Partheniades-type formula (Partheniades, 1965) for the erosion of cohesive soils:

$$\begin{cases} \frac{\partial n_B}{\partial t} = E(\alpha\tau - \tau_c) & \text{for } \alpha\tau > \tau_c \\ \frac{\partial n_B}{\partial t} = 0 & \text{for } \alpha\tau < \tau_c \end{cases}$$

where $\partial n_B / \partial t$ [m/s] is the rate of bank retreat, E [m³/N.s] the erodibility coefficient, τ [N/m²] the shear stress along the bank, τ_c [N/m²] the critical flow shear stress below which no bank erosion occurs and α [-] a factor which converts bed shear stresses to shear stresses on the bank. The values of E , α and τ_c have to be defined by the user and the shear stress along the bank line is computed according to

$$\tau = \frac{u_b^2 \rho_w g}{C^2},$$

with u_b [m/s] the flow velocity along the bank line, ρ_w [kg/m³] the density of water, g [m/s²] the gravitational acceleration and C [m^{1/2}/s] the Chézy coefficient.

3.2 Eroded volume of sediment

Due to the bank erosion process a certain amount of sediment becomes available from the retreating banks. This volume of sediment has to be added to the overall sediment balance and the local bed level has to be updated accordingly. The amount of sediment that has to be added to a certain grid cell is determined by the change in cell volume, V_{eros} , due to the movement of the bank line given by:

$$V_{eros} = (A^n - A^{n-1})(h_{bank} + d^n),$$

with A^n [m²] the current area of a grid cell and A^{n-1} [m²] the area at the previous time step, h_{bank} [m] the height of the bank and d [m] the depth in the grid cell (positive down). To update the sediment balance, the eroded volume is added to the relevant grid cell, using the sediment properties available for this cell.

To be able to compute the eroded volume accurately, the movement of the bank line is limited to one grid cell per time step.

The amount of local underwater bed level increase is equal to the eroded volume divided by the local grid cell area. Since bank erosion products are often reduced by a washload factor to account for the fraction of fine bank material that disappears as washload without playing any role in the morphology of the river bed, strict mass conservation is not required.

3.3 Bank line movement

During the bank erosion process, the positions of the bank lines need to be updated as well. We model a bank line as a series of straight line segments. The new location of a bank line is determined by moving each line segment according to the computed local lateral retreat. The updated position of a bank line point is determined by computing the intersection point of neighbouring updated bank line segments, see Figure 6. However, in some situations this can result in very large displacements of bank line points, especially when neighbouring bank lines are almost in a straight line. In these cases, first, two new locations of a point are found based on the displacement of the neighbouring bank line segments (red dots in Figure 7). The final location of a point is then determined by taking the average of these two locations (green dot in Figure 7).

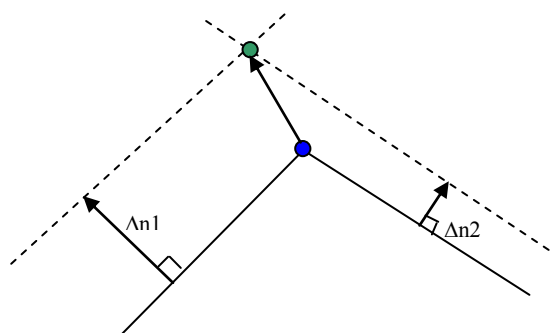


Figure 6 Moving a vertex of a bank line based on the intersection point of two bank line segments.
Blue: original location, Green: new location

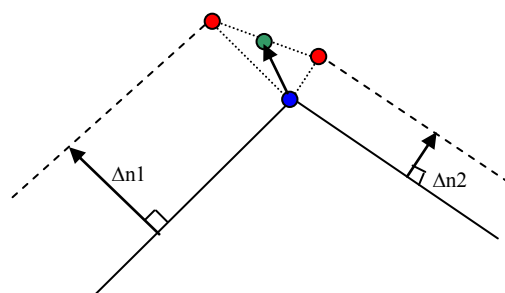


Figure 7 Moving a vertex of a bank line based on the average displacement of two bank line segments.
Blue: original location, Red: New location based on movement of separate segments, Green: new location (average of the two red points)

To avoid problems with the adjustment of grid parameters, bank line segments that become too small are joined with neighbouring bank lines. On the other hand, if bank line segments become too large, they are split in two to increase the flexibility of the bank line.

4 TEST COMPUTATIONS

We analyze the performance of the new approach for a simple meandering river. We start off with a 1490 m long and 45 m wide straight channel consisting of 149 x 9 grid cells. The slope of the channel is $6.7 \cdot 10^{-4}$ and a constant Chézy value of $35 \text{ m}^{1/2}/\text{s}$ is used. At the inflow boundary a discharge of $76.68 \text{ m}^3/\text{s}$ is imposed. The inflow boundary is located at the left 2/3 of the channel. This ensures that an asymmetric flow pattern will develop to initiate meandering behaviour. At the outflow boundary a constant water level of 2 m is imposed. The initial flow conditions are obtained by performing a computation without bank erosion and morphology updates. Figure 8 depicts the flow pattern thus obtained.

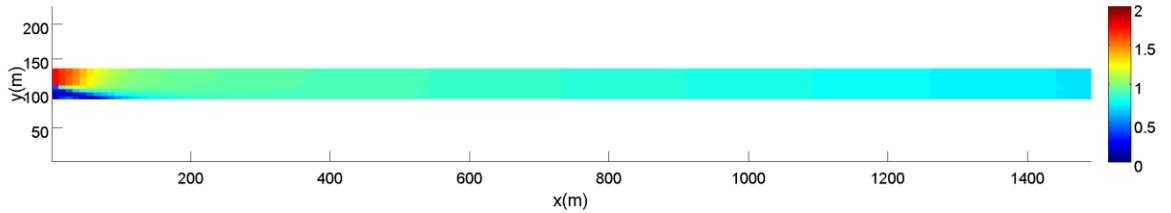


Figure 8 Initial magnitude of the flow velocity

The morphological simulation is then started with the following values for the bank erosion parameters: $E = 5.0 \cdot 10^{-5} \text{ m}^3/\text{N}\cdot\text{s}$, $\tau_{cr} = 0.2 \text{ N/m}^2$ and $\alpha = 0.1$. For the computation of the eroded volume the height of the bank, h_{bank} , varies linearly from 3.0 m at the inflow to 2.5 m at the outflow.

The sediment transport rate is described using a standard transport model based on the formula of Meyer-Peter & Müller (1948):

$$S = \beta D_{50} \sqrt{\Delta g D_{50}} (\mu \theta - \theta_{cr})^{3/2}$$

With S the total bed-load transport rate per unit width, $\beta = 5$ a calibration coefficient (equal to 8 in Meyer-Peter-Müller's original formula), $D_{50} = 2 \text{ mm}$ the mean sediment diameter, $\mu = 0.7$ the ripple factor, $\Delta = 1.65$ the relative density and $\theta_{cr} = 0.025$ the critical mobility parameter. The Shields mobility parameter θ is given by

$$\theta = \left(\frac{u}{C} \right)^2 \frac{1}{\Delta D_{50}}$$

in which u is the magnitude of the flow velocity and C the Chézy coefficient.

To speed up the computation a morphological acceleration factor of 144 is used at a hydrodynamic time step of 0.6 s. Furthermore, the location of the bank line is updated every 100 time steps to avoid the generation of too small grid cells. To avoid problems with imposing boundary conditions, the bank lines are fixed at the inflow boundary.

The evolving differences between the actual and initial bed level are depicted in Figure 9a-g, at different moments in morphological time. Due to bank erosion and sedimentation of eroded material at the beginning of the channel, the initially asymmetric inflow becomes almost symmetrical, albeit not exactly at the centre of the initial channel. This leads to the development of an island in the first part of the channel (although it does not become completely dry). Behind this island, a meandering pattern evolves, due to bank retreat and the development of point bars. This pattern finally becomes a bit distorted, because the island is not located in the centre of the channel.

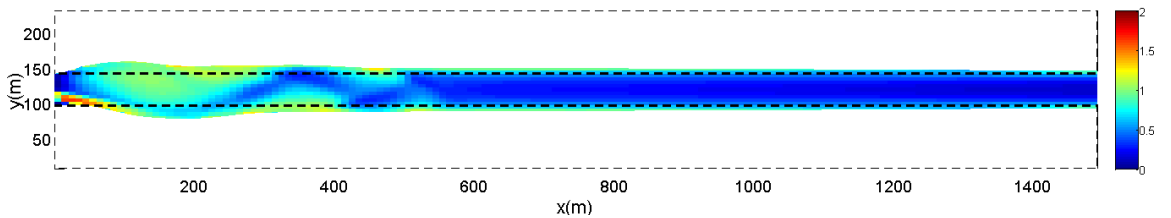


Figure 9a Cumulative erosion and sedimentation after 10 days

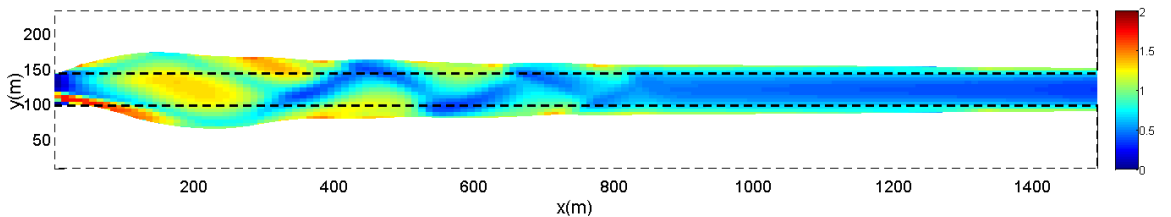


Figure 9b Cumulative erosion and sedimentation after 20 days

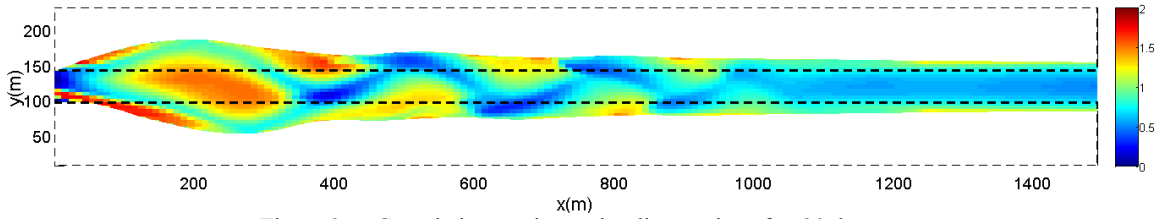


Figure 9c Cumulative erosion and sedimentation after 30 days

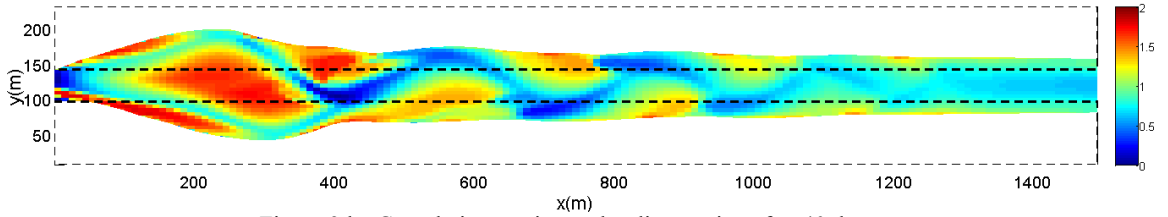


Figure 9d Cumulative erosion and sedimentation after 40 days

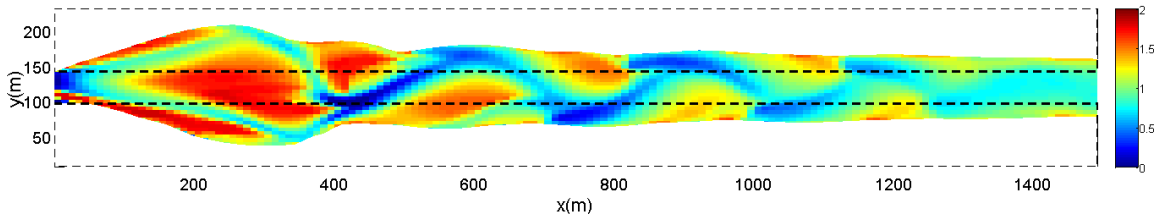


Figure 9e Cumulative erosion and sedimentation after 50 days

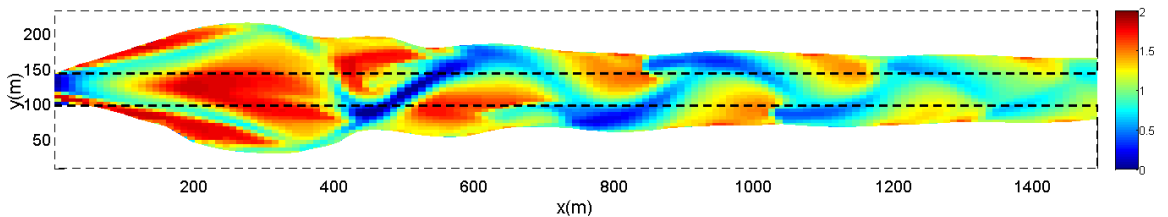


Figure 9f Cumulative erosion and sedimentation after 60 days

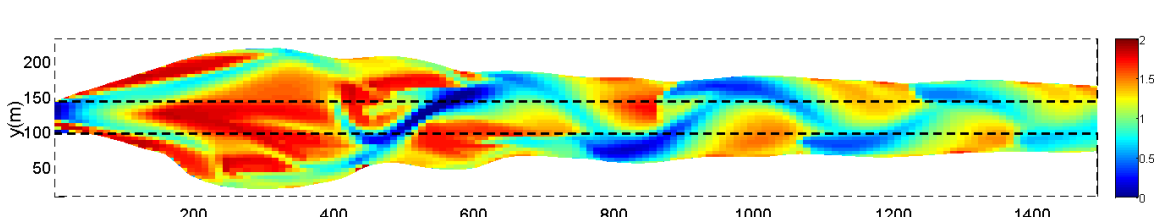


Figure 9g Cumulative erosion and sedimentation after 70 days

5 CONCLUSIONS AND RECOMMENDATIONS

In order to be able to accurately simulate the interaction between bank erosion and changes in bed topography, a model has been developed that can deal with these processes simultaneously. The model has been based on the existing quasi 3D framework of Delft3D and an immersed-boundary technique has been implemented to account for the moving bank lines due to bank erosion. The distance and the direction of bank retreat are computed using a simple bank erosion formula, but this formula can easily be extended to incorporate other bank erosion mechanisms. The eroded volume from the banks is added to the overall sediment balance and locally the bed levels are updated accordingly. The performance of this approach has been shown for a simple meandering river. The results show that the new approach is capable of reproducing complex river planform changes without grid deformation problems and without the need to employ a very fine grid.

The following recommendations are given for further research:

- To account for the presence of the boundary more accurately, also the discretization of the momentum equation has to be adjusted in the cells that are cut by the boundary.
- The efficiency of the bank line movement can be improved by moving the bank line points along grid lines. This implies that the adaptation of grid parameters can be determined directly instead of having to be recomputed each time the bank lines are updated.
- The use of separate moving objects gives the opportunity to track bank-lines that are not on the border of the computational domain, but somewhere inside this domain, e.g. along the main river channel between floodplains or along the channels in an estuary.
- The performance of the new approach for river bank retreat has only been shown for the morphological evolution of an initially straight channel. The next step is to compare model results with experimental data and to simulate the development of a real-life natural river.

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